Linear Algebra - 2

Slope, intercept (weight bias)

$y = mx + c$ $y = wx + b$

vector

classifier / decision boundary

misclassification

Halfspace

Plane

Context : Orange Vs Tangerine
Fish 1 Vs Fish 2
IPL → win Vs defeat

scatter plot from "features"

len
width
color

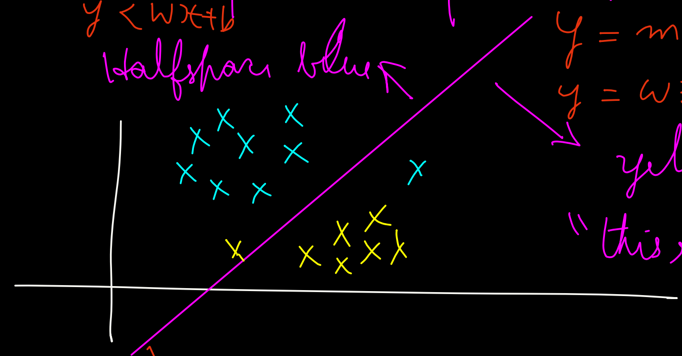
(score, overs, wicket, ground)

dataset

f1	f2	f3	...	Target
x	x	x	x	

"row" row 1
"vector"
"data point"

$y < wx+b$
Halfspace blue

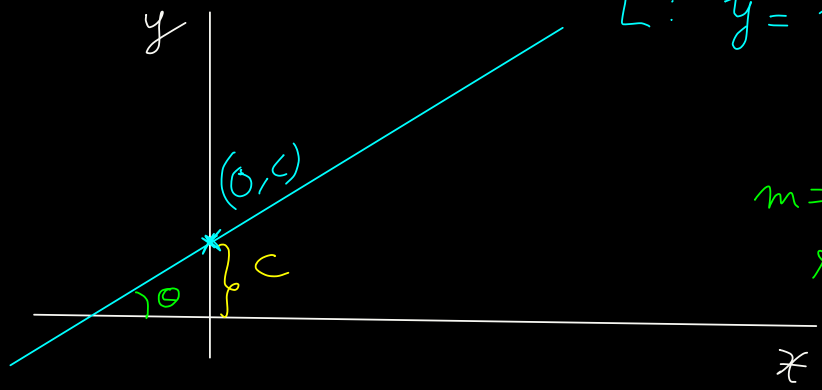


$y = mx + c$
$y = wx + b$
yellow
"this side"
Halfspace
$y > wx + b$

"decision boundary"

Minimize "misclassification" ("error")
among all possible values of w & b

Get "best" w and b



L : $y = \underline{m}x + \underline{c}$

$m = \tan\theta$ slope

intercept

when $x = 0$, $y = c$ $\quad (0, c) \in L$

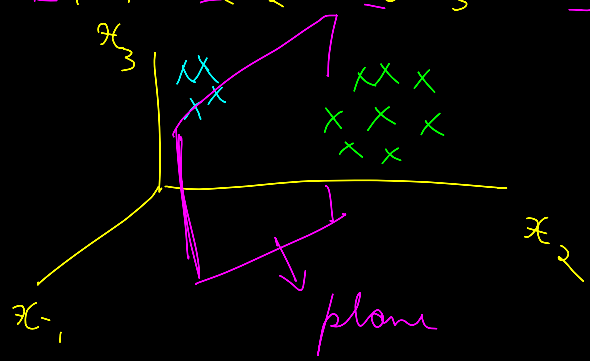
$y = mx + c \iff 2y = 2mx + 2c$

$$\boxed{w_1 x_1 + w_2 x_2 + w_0 = 0}$$

2 features

(w_0 or b mean the same)

3 features : $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$



n- dimension $\quad \omega_1 x_1 + \omega_2 x_2 + \cdots + \omega_n x_n + \omega_0$

"Hyperplane"

Matrix Multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 2 \qquad 2 \times 2$

$1 \times 5 + 2 \times 7 = 19$

$1 \times 6 + 2 \times 8 = 22$

$3 \times 5 + 7 \times 4 = 43$

row $\rightarrow$ column

$$\underset{1 \times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2 \times 1}{\begin{bmatrix} 5 \\ 7 \end{bmatrix}} = \underset{1 \times 1}{19}$$

"dot product" "scalar"

dot product of $\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= x_1 y_1 + x_2 y_2$

dot product $\begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\omega_1 x_1 + W_2 x_2 = -W_0 \longrightarrow \omega_1 x_1 + \omega_2 x_2 + \omega_0 = 0$

dot product $\begin{bmatrix} m & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$mx - y = -c$

$\hookrightarrow y = mx + c$

Dot product Notation

$x^T y$ or $x \cdot y$ or $\langle x, y \rangle$

"vector" $\longrightarrow$ default assume column vector

$(1, 2) \longrightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

"row" of dataset $\longrightarrow$ "column"

$\begin{bmatrix} \text{income} \\ \text{def} \\ \text{properties} \\ \vdots \end{bmatrix}$

dot product of $x = (1, 2)$ and $y = (3, 4)$

$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad y = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$x^T y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 3 + 8 = 11$

Our parameter : $(w_1, w_2, w_3 \cdots w_n, w_0)$ is itself some sort of a "vector"

parameter & feature are engaged as

$\underbrace{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n} + w_0 = 0$

$w^T x + w_0 = 0$

$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} \quad w_0$: number / scalar n X 1

Loan example

x_1 : applicant income "features"
x_2 : Co-app income
x_3 : credit history Loan status "target"
x_4 : prop area ..

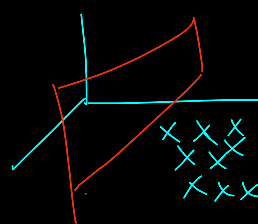
$$\underline{10}\, x_1 + \underline{4}\, x_2 + \underline{700}\, x_3 \longrightarrow$$ Can this predict target

w_1 w_2 w_3

$$w = \begin{bmatrix} 10 \\ 4 \\ 700 \\ \vdots \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix}$$

"new feature" $w^T x$

finding this is the biggest challenge



"best decision boundary"

↳ depends on the parameters w & w_0

↳ finding the "best" w & w_0 is the main part of "learning" in machine learning

Relation between parameter (w & w_0) & decision boundary

Recap Line:

L1: $y = mx + c$

$m = -2$, $c = 0$

$y = -2x$

$2x + y = 0$

$2x_1 + x_2 = 0$

$w_1 = 2$, $w_2 = 1$, $w_0 = 0$

L2: $y = mx + c$

$m = -2$, $c = 1$

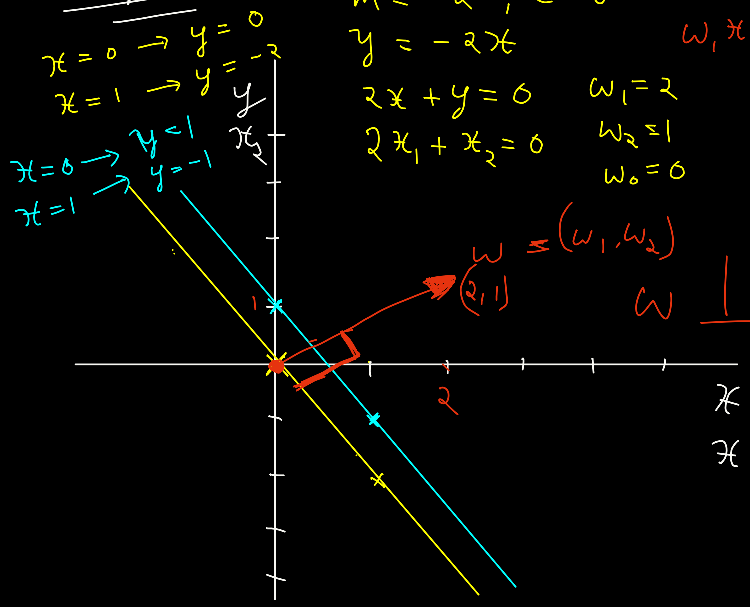
$y = -2x + 1$

$2x + y - 1 = 0$

$2x_1 + x_2 - 1 = 0$

$w_1 = 2$, $w_2 = 1$, $w_0 = -1$

$w_1 x_1 + w_2 x_2 + w_0 = 0$ → slides the line



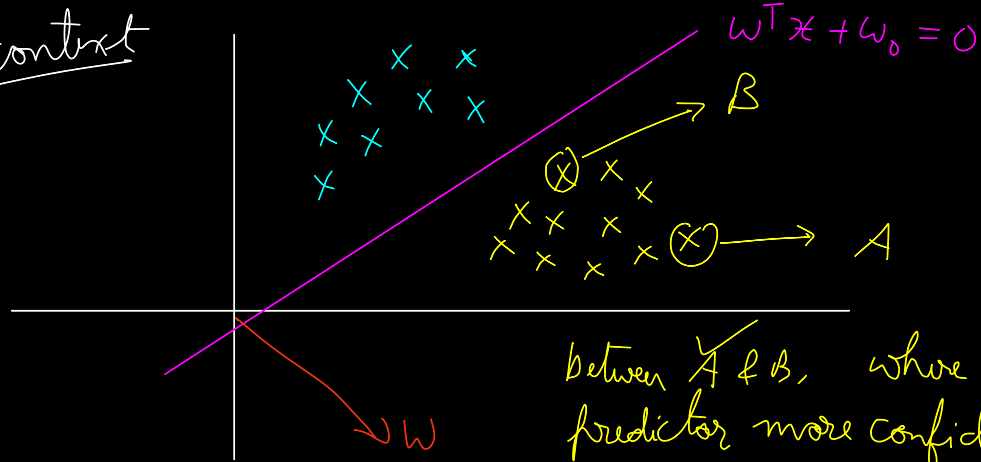
$w = (w_1, w_2)$

$w \perp L1$

$w \perp L2$

$$\begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0 = 0$$

Recap context



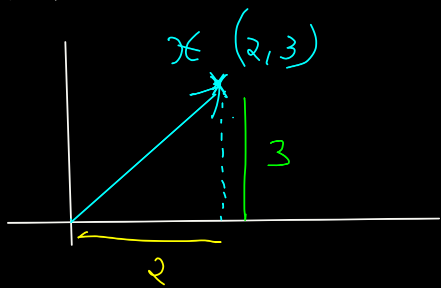
between A & B, where is the predictor more confident?

Which side does the point lie (next class Halfspace)

new $x \Rightarrow w^T x + w_0 > 0$?

$\Rightarrow w^T x + w_0 < 0$

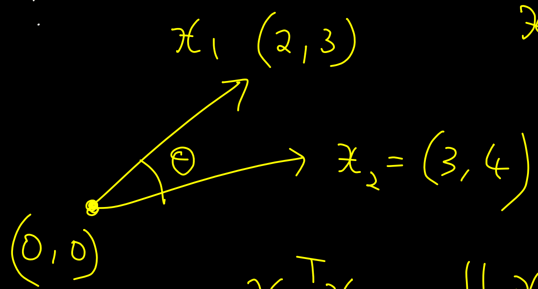
Length "norm"



"norm" $\|\cdot\|$

$$\|x\| = \sqrt{2^2 + 3^2}$$

Angle:



$$x_1^T x_2 = 2\cdot 3 + 3\cdot 4 = 18$$

$$\|x_1\| = \sqrt{2^2+3^2}$$

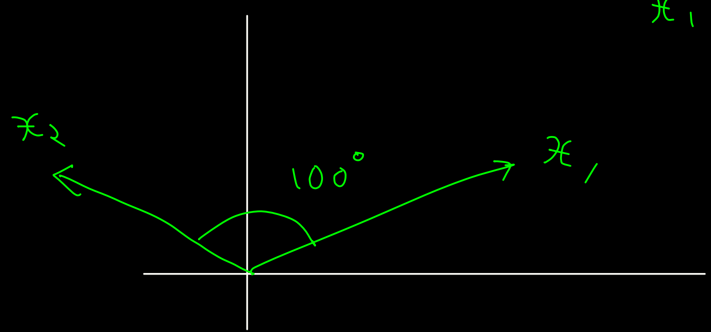
$$\|x_2\| = \sqrt{3^2+4^2}$$

$$x_1^T x_2 = \|x_1\| \, \|x_2\| \cos\theta$$

$$\cos\theta = \frac{18}{\sqrt{2^2+3^2}\,\sqrt{3^2+4^2}}$$

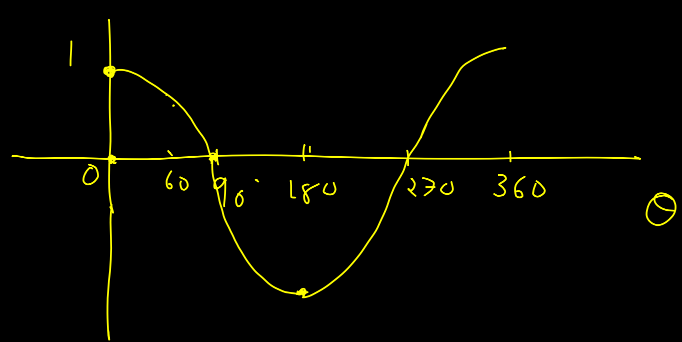
$$\boxed{\cos\theta = \frac{x_1^T x_2}{\|x_1\| \, \|x_2\|}}$$

$$x_1^T x_2 \begin{cases} \to +ve? \\ \to -ve? \end{cases}$$

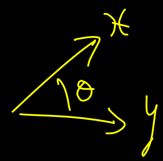


$\cos\theta$:

θ	$\cos\theta$
0	1
60°	1/2
90°	0

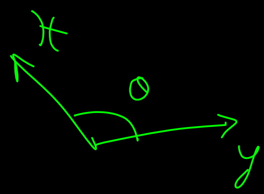


$$x^T y = \|x\| \, \|y\| \cos\theta$$

1) 

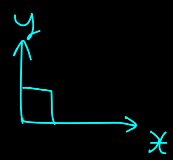
$\theta < 90^\circ$ "acute"

then $x^T y > 0$

2) 

$\theta > 90^\circ$ "obtuse"

then $x^T y < 0$

3) 

$\theta = 90^\circ$

then $x^T y = 0$ $\qquad \cos 90 = 0$