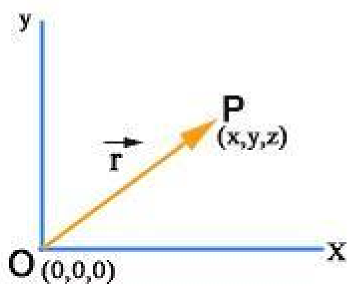
The vector form of a hyperplane is: $w^T x + w_0 = 0$

- **Vectors** can be interpreted as coordinates as well as a line segment from the origin to the coordinate.

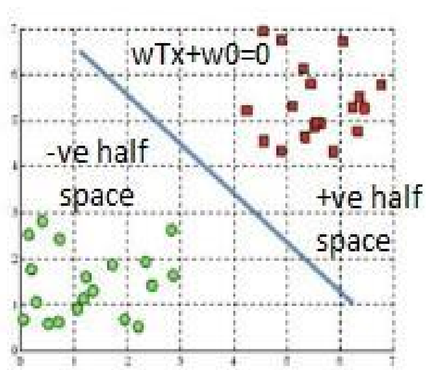
$$\text{Where, } w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix} \qquad \text{and, } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

For example, the below-given vector $\vec{r}$ can be considered as coordinates of point P(x,y,z) as well as a line segment from the origin to point P(x,y,z).



- **Half Spaces:** In geometry, a half-space is either of the two parts into which a plane divides the three-dimensional Euclidean space.

Example: Let's assume that a hyperplane $w^T x + w_0 = 0$ is classifying the data points of two different classes in a space.



Let's say we got a point x_0 in the space.

Now, **If:**

$w^T x_0 + w_0 > 0$ => The point is in the +ve halfspace

$w^T x_0 + w_0 < 0$ => The point is in the -ve halfspace.

- The **transpose** operation changes a column vector into a row vector and vice versa.

For example,

if $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_n \end{bmatrix}$ then, $\vec{a}^T = \begin{bmatrix} a_1 & a_2 & a_3 & . & . & . & a_n \end{bmatrix}$

- **The dot product** of two vectors $\vec{a}$ and $\vec{b}$ is given as :

$$\vec{a} . \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + ... + a_n b_n$$

Where, $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_n \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 & b_2 & b_3 & . & . & . & b_n \end{bmatrix}$

Also, $\vec{a} . \vec{b} = \vec{b} . \vec{a}$

Geometrically, it is the product of the magnitudes of the two vectors and the cosine of the angle between them.

i.e. $\vec{a} . \vec{b} = |\vec{a}|.|\vec{b}|.cos(\theta)$

where θ is the angle between the two vectors.

If the dot product of two vectors is **zero**, then the vectors are **perpendicular** to each other.

- A **unit vector** is a vector that has a magnitude of 1.

To convert a vector $\vec{u}$ into a unit vector, we divide the vector by its magnitude.

i.e. unit vector = $\hat{u} = \dfrac{\vec{u}}{||\vec{u}||}$

➔ We can multiply any scalar value with the unit vector to get the desired magnitude (equal to that scalar value) in the same direction.

➔ All vectors with the same unit vector are **parallel**

- **Distance between two points** having coordinates (x_1, y_1) and (x_2, y_2) in an x-y plane is given as:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **Norm** or **Magnitude** of a vector is calculated by taking the square root of dot product with itself.

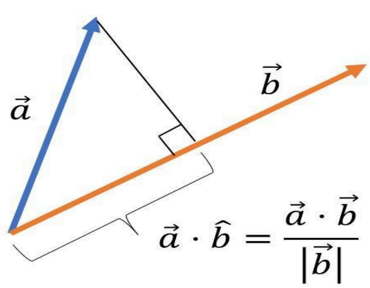
i.e. $||\overrightarrow{a}|| = \sqrt{\overrightarrow{a}.\overrightarrow{a}}$

It represents the **length** of a vector or **distance** of $\overrightarrow{a}$ coordinate from the origin.

- **Angle between two vectors** is given as :

$$\theta = cos^{-}\left(\frac{\overrightarrow{a}.\overrightarrow{b}}{||\overrightarrow{a}||.||\overrightarrow{b}||}\right)$$

- **Projection of vector** $\overrightarrow{a}$ on $\overrightarrow{b}$ = $\frac{\overrightarrow{a}.\overrightarrow{b}}{||b||}$



- At the **point of intersection** of two lines, both lines will have the same coordinates.

Example:

Let's say we have two lines, y = x+2 and y = 2x+1. We need to find the point of intersection of these two lines.

We assume that the lines intersect at a single point (a,b). Therefore, this point will satisfy both the line's equations.

i.e. b = a+2 — i) b = 2a+1 — ii)

Solving the above two equations, we get a = 1 and b = 3.

Therefore, the given two lines intersect at the point (1,3).