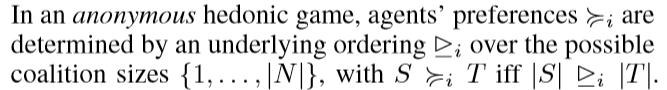
# **2016:**

### **1. Complexity of Hedonic Games with Dichotomous Preferences:**

Dichotomous Hedonic Games are hedonic games in which each agent either approves or disapproves of a given coalition. In this work, we study the computational complexity of questions related to ﬁnding optimal and stable partitions in dichotomous hedonic games under various ways of restricting and representing the collection of approved coalitions.

Definitions:

* Lists: In a context in which it is sensible to presume that agents will only approve at most polynomially many coalitions, we can represent their preferences by merely listing all approved coalitions. The complexity of stability problems for lists in the non-dichotomous case is studied by Ballester (2004). We consider here an even more restricted variant: in the k-list representation, every agent submits a list of at most k approved coalitions.
* Anonymous Preferences:

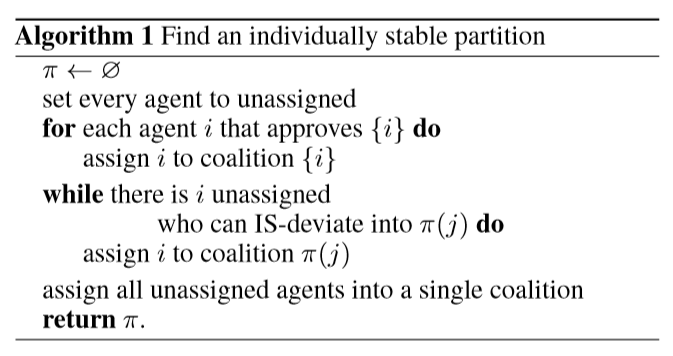


* Roommates: Consider the restriction of dichotomous hedonic games where agents only approve coalitions of size at most 2. This case could also be referred to as a (stable) roommate problem with dichotomous preferences.
* Majority Games: Consider agents that approve those coalitions in which the agent is friends with majority of members.

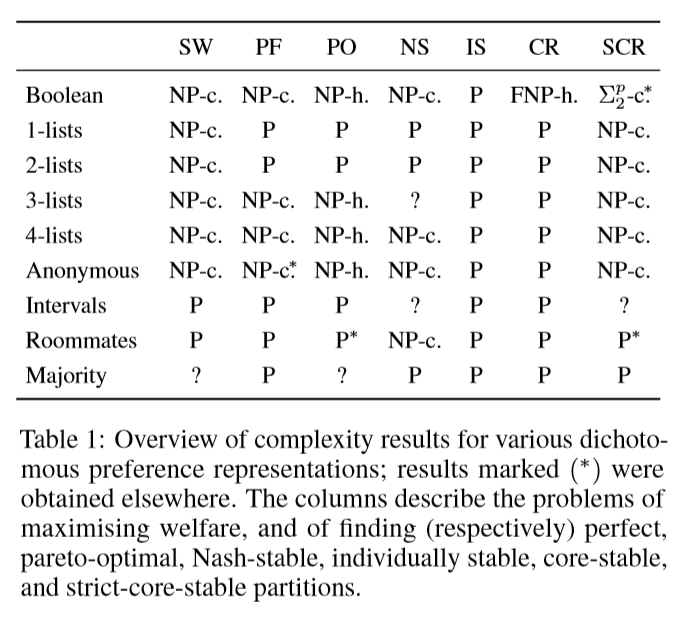
Results:

* Every dichotomous hedonic game admits a partition that is both core-stable and individually stable.
* For every dichotomous hedonic game, we can ﬁnd an individually stable partition in O(n3) calls to an oracle that decides whether a given coalition S ⊆ N is approved by a given agent i ∈ N.

Algorithm 1 finds an individually stable partition in O(n3). Note that the while loop executes at most n times, since each agent is assigned only once.



* It is FNP-hard to ﬁnd a core-stable partition in a boolean hedonic game.
* It is coNP-complete to decide whether a given partition π is core-stable in a boolean hedonic game.
* Maximizing social welfare is NP-complete even for 1-lists.
* Deciding existence of a strict-core-stable partition is NP-complete even for 1-lists.
* Finding a perfect partition or a Nash-stable partition is easy for 2-lists.
* Deciding existence of a perfect partition or a strict-core-stable partition is NP-complete for 3-lists.
* The problems of deciding existence of a perfect, a strict-core-stable, or a Nash-stable partition are NP-complete for anonymous preferences, even if at most 4 sizes are approved.
* If every agent only approves intervals, we can ﬁnd a welfare-maximizing partition in polynomial time.
* Deciding the existence of a Nash stable partition for dichotomous roommates is NP-complete, even in the bipartite (marriage) case, and even if each agent approves at most 4 coalitions.
* In a majority game, a partition that is both Nash-stable and core-stable is guaranteed to exist and can be found in polynomial time.
* Let δ(G) be minimum degree of G where G is graph representing majority game with n agents. Then if δ(G) >= n/2 then G is Hamiltonian.
* Let Δ(G) be maximum degree of G where G is graph representing majority game with n agents. If Δ(G) <= k−1 then G has an equitable k-colouring.
* If G is a graph with δ(G) >= n/2 then the vertices of G can be partitioned into edges and triangles.
* In majority games, perfect and strict-core stable partitions coincide. If a perfect partition exists, then a perfect partition consisting of edges and triangles exists. Hence there is a polynomial time algorithm which will produce a perfect and strict-core-stable partition if it exists.



Open Problems:

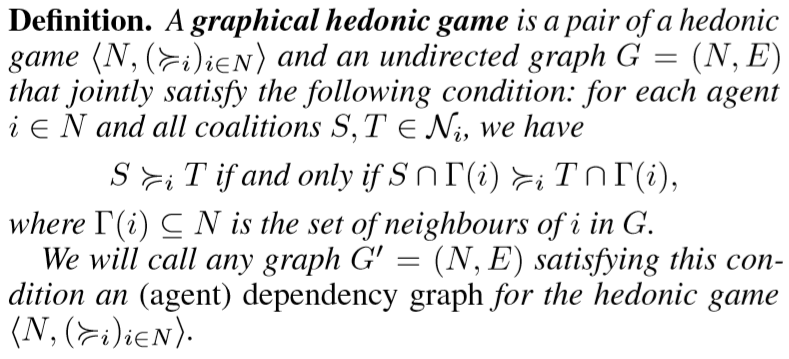
* Settle the ‘frontier of tractability’ for Nash-stability in k-lists.
* Decide the existence of Nash and strict-core-stable partitions in the interval case.
* Empirical evaluation of the power of boolean hedonic games when solved by modern SAT solvers.

# **2. Graphical Hedonic Games of Bounded Tree width:**

Hedonic games are a well-studied model of coalition formation, in which selﬁsh agents are partitioned into disjoint sets and agents care about the make-up of the coalition they end up in. The computational problems of ﬁnding stable, optimal, or fair outcomes tend to be computationally intractable in even severely restricted instances of hedonic games. We introduce the notion of a graphical hedonic game and show that, in contrast, on classes of graphical hedonic games whose underlying graphs are of bounded treewidth and degree, such problems become easy.

Acronyms and Definitions:

* Graphical Hedonic Games:



* Let (<N, (≥ i) i∈N>, G) be a graphical hedonic game. Its treewidth is the treewidth of G, and its degree is the maximum degree of G.
* In a graphical hedonic game with dependency graph G, a coalition S ⊆ N is connected if G[S] is connected, that is if S induces a connected subgraph in G. A partition π of N is connected if each S ∈ π is connected.
* The formulas of hedonic game logic (HG-logic) are deﬁned recursively as:

1. atomic formulas: i = j, i ∈ S, S = π(i), S ≥ i T
2. boolean combinations of formulas: ¬φ, (φ ∨ ψ), (φ ∧ ψ)
3. quantiﬁcation over agents: ∀i φ, ∃i φ
4. quantiﬁcation over coalitions: ∀S φ, ∃S φ
5. quantiﬁcation over partitions: ∀π φ, ∃π φ

* φ-HEDONIC GAMES Instance: a graphical hedonic game (<N, (≥ i) i∈N>, G) and a formula φ of HG-logic.
* MSO[σ]: Given a signature σ, the language MSO[σ] of monadic second order logic is given by the grammar

φ ::= x=y | Rix1 ......xar(Ri)| Xx| (φ ∨ φ) | (φ ∧ φ) | ¬φ |∃x φ|∀x φ|∃x φ|∀x φ, where x,y,x1,x2,... are ﬁrst-order variables, and X denotes set variables.

* Hedonic Coalition Nets(or HC-nets): Each agent speciﬁes a set of weighted propositional formulas, called rules, with propositional atoms given by the agents.

Results:

* For each hedonic game, there exists a unique edge-minimal agent dependency graph.
* The problem φ-HEDONIC GAMES is ﬁxed parameter tractable with respect to the length |φ| of the formula φ, and the treewidth k and degree d of the graph G. That is, the problem can be solved in time O(f(|φ|, k, d)·n) where f is a computable function, and n is the number of agents. Here we assume that the relation “S ≥ i T” can be decided in time only depending on d, but not on n.
* For every class of graphical hedonic games of bounded treewidth and degree, there exist linear-time algorithms that can decide whether a given such game admits a partition that is (i) core-stable, (ii) strict-core-stable, (iii) Pareto-optimal, (iv) perfect (v) Nash-stable, (vi) individually stable, (vii) envy-free, or that satisﬁes any combination of these properties.
* Given a formula φ of MSO[σ] and a σ-structure A, we can in time g(|φ|, tw(A))·|A|+O(||A||) decide whether A |= φ, where g is a computable function.
* There is an O(2kd\*dn) algorithm that, given a graphical hedonic game and a tree decomposition, decides whether there exists a connected partition π of the agent set that satisﬁes (a combination of) (i) individual rationality, (ii) Nash stability, (iii) individual stability, (iv) envy-freeness. Subject to any combination (or none) of the preceding conditions, we can also maximize utilitarian, egalitarian, or Nash social welfare under π.
* There is an O(2kd\*dn) algorithm that given a hedonic game, an associated dependency graph, a tree decomposition, and a partition π of N, decides whether π is (i) Pareto optimal, (ii) core-stable, (iii) strict-core-stable.
* CORE-EXISTENCE is NP-hard even for graphical hedonic games of tree width 2 that are given by an HC-net.
* NASH-STABLE-EXISTENCE is NP-hard even for graphical hedonic games of treewidth 1 that are given by an HC-net.

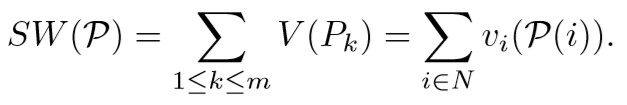
Open Problems:

* Are there alternative conditions on graph topology that yield tractability?
* Can we say anything about the structure of stable outcomes in dependence on the structure of the graphical hedonic game?
* Find faster algorithms than those provided through HG-logic for Σp 2-type questions like the existence of a core-stable partition or of ﬁnding a Pareto-optimal partition.

# **3. Price of Pareto Optimality in Hedonic Games:**

Price of Anarchy measures the welfare loss caused by selfish behavior: it is deﬁned as the ratio of the social welfare in a socially optimal outcome and in a worst Nash equilibrium. A similar measure can be derived for other classes of stable outcomes. In this paper, we argue that Pareto optimality can be seen as a notion of stability, and introduce the concept of Price of Pareto Optimality: this is an analogue of the Price of Anarchy, where the maximum is computed over the class of Pareto optimal outcomes, i.e., outcomes that do not permit a deviation by the grand coalition that makes all players weakly better off and some players strictly better off.

Acronyms and Definitions:

* Social Welfare (SW): Social Welfare of partition P is defined as follows  where vi is the utility function of player i
* Given a hedonic game (N, (vi)i∈N), the Price of Pareto Optimality (PPO) is deﬁned as

PPO = maxp∈PSW(P\*)/SW(P) if SW(P) > 0 for all p∈P and

PPO = +∞ otherwise.

* ΔG: maximum degree of a node in Graph G.

Results:

* For any M > 0 there is a symmetric fractional hedonic game F(G) where G =(N, E, w) is a tree and all weights are positive such that PPO(F(G)) >M.
* Let G =(N, E) be asymmetric unweighted graph with |N| ≥ 2, and let P be a Pareto optimal partition for F(G). Then

1. every coalition in P is connected,
2. if E != ∅, then P contains at least one non-singleton coalition.

* Let G = (N, E) be a symmetric unweighted graph with |N| ≥ 2. Then PPO(F(G)) ≤ 2ΔG(ΔG + 1).
* Let G = (N, E) be an unweighted tree with |N| ≥ 2 and let P\* be an optimal partition for the fractional hedonic game F(G). Then every Pk\* ∈P\* is a dk-star for some dk ≥ 1.
* Let G = (N, E) be an unweighted tree with |N| ≥ 2 and let C be a minimum vertex cover of G. The social welfare of any optimal partition for the fractional hedonic game F(G) is at most (2ΔG/(ΔG+1))|C|.
* Let G = (N, E) be an unweighted tree with |N|≥2, and let P be a Pareto optimal partition for the fractional hedonic game F(G). Then every coalition in P is either a star or a superstar.
* Let G =(N,E) be an unweighted tree with |N|≥2. Let P be a Pareto optimal partition for the fractional hedonic game F(G). If P contains a singleton P = {i} then every j such that (i, j) ∈ E satisﬁes the following conditions:

1. j is not in a singleton,
2. j is not the center of a superstar,
3. j is not the leaf of a superstar.

* Let G = (N, E) be an unweighted tree with |N| ≥ 2. Then PPO(F(G)) ≤ ΔG +2.
* There exists a fractional hedonic game on an unweighted tree G = (N, E) for which the Price of Pareto Optimality is strictly greater than ΔG −1/3.
* Let G = (N, E) be a symmetric unweighted graph with |N| ≥ 2. Let C be a minimum vertex cover of G. The social welfare of any optimal partition for the modiﬁed fractional hedonic game MF(G) is at most 2|C|.
* Let G = (N, E) be a symmetric unweighted graph with |N| ≥ 2. Let P be a Pareto optimal partition for the modiﬁed fractional hedonic game MF(G). Then every coalition in P is either a star or a clique.
* Let G = (N, E) be a symmetric unweighted graph with |N| ≥ 2. Let P be a Pareto optimal partition for the modiﬁed fractional hedonic game MF(G). For every edge (i, j) ∈ E with P(i) != P(j), it holds that if i in P forms a singleton, is a leaf of a multi-degree star or a node in a triangle, then j in P is either the center of a multi-degree star or a node in a 1-star.
* Let G = (N, E) be a symmetric unweighted graph with |N|≥2, and let P be a Pareto optimal partition for MF(G). Then

1. every coalition in P is connected,
2. if E != ∅, then P contains at least one non-singleton coalition.

* Let G = (N, E) be a symmetric unweighted graph with |N| ≥ 2. Then PPO(MF(G)) ≤ 2.
* Let G = (N, E) be a symmetric unweighted bipartite graph with |N| ≥ 2. Then PPO(MF(G)) ≤ 1.

Open Problems:

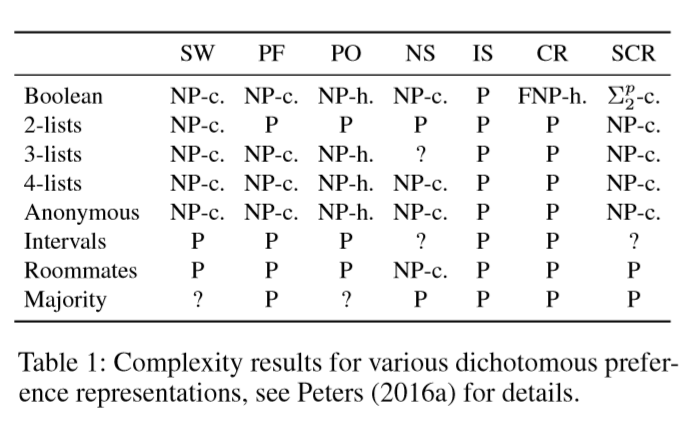
* It is not clear if the upper bound of PPO(F(G), where G is symmetric unweighted graph with |N|>2, is tight; in fact, we do not have examples of fractional hedonic games on symmetric unweighted graphs whose PPO exceeds ΔG.
* It would be interesting to compute or bound PPO measure for other classes of (cooperative and non-cooperative) games.

# **4. Towards Structural Tractability in Hedonic Games:**

Hedonic games are a well-studied model of coalition formation, in which selﬁsh agents are partitioned into disjoint sets, and agents care about the make-up of the coalition they end up in. The computational problem of ﬁnding a stable outcome tends to be computationally intractable, even after severely restricting the types of preferences that agents are allowed to report. We investigate a structural way of achieving tractability, by requiring that agents’ preferences interact in a well-behaved manner.

Results:

* The problems of deciding whether a given additively separable hedonic game admits a core or a strict-core-stable partition are Σp2-complete, even if valuations are symmetric and sparse: no agent has non-zero valuations for more than 10 agents.
* For any logical sentence φ of HG-logic that may quantify over(connected) partitions, coalitions, and agents, the problem of deciding whether a hedonic game given by an HC-net satisﬁes φ is ﬁxed-parameter tractable with parameters the treewidth and degree of the dependency graph of the game.



Open Problems:

* Suppose agents are ordered along a line and have additively separable preferences that are single-peaked on that line: agents prefer those agents closer to them on the line. Does every such game admit a core-stable outcome?
* If so, can we ﬁnd it in polynomial time?
* What happens if all individually rational coalitions are intervals with respect to this line?
* Can we dispense with the bound on G’s maximum degree when only considering additively separable games?

# **5. Altruistic Hedonic Games:**

All models of representing hedonic games studied so far are based upon selfish players only. Among the known ways of representing hedonic games compactly, we focus on friend-oriented hedonic games and propose a novel model for them that considers not only a player’s own preferences but also her friends’ preferences under three degrees of altruism.

Definitions:

* Weak Friend-Orientedness: If coalition A is acceptable for i, then A∪{f} is also acceptable for i, where f ∈ Fi\A.
* Favoring Friends: If x ∈ Fi and y ∈ Ei then {x, i}­ >i {y, i}.
* In difference between Friends: If x, y ∈ Fi then {x, i} ∼i {y, i}.
* In difference between Enemies: If x, y ∈ Ei then{ x, i} ∼i {y, i}.
* Sovereignty of Players: For a ﬁxed player i and each C ∈ Ni, there exists a network of friends such that C ends up as i’s most preferred coalition.
* Monotonicity: Let j != i be a player with j ∈ Ei and A, B ∈ Ni, and >=i be the preference relation resulting from >=i when j turns from being i’s enemy to being i’s friend (all else being equal).
* Symmetry: Let j and k be two distinct players with j != i !=k. We say that >=i is symmetric if it holds that if swapping the positions of j and k in G is an automorphism then

(∀C ∈Ni \ (Ni ∪ Nk))[C ∪ {j} ∼i C ∪ {k}].

* Local Friend Dependence: The preference order >=i can depend on the sets of friends F1,...,Fn. Let A, B ∈ Ni. We say that comparison (A,B) is
  + friend-dependent in >=i if (1) A >=i i B is true (false) and (2)can be made false(true) by changing the set of friends of some players (except for i);
  + locally friend-dependent in >=i if (1) A >=i B is true(false), (2) can be made false(true) by changing the set of friends of some players that are in A or B and are i’s friends, and (3) changing the set of friends of all other players in N\({i}∪(Fi∩(A∪B)) does not affect the status of the comparison.
* Friend-Oriented Unanimity: Let A, B ∈ Ni with A ∩ Fi = B ∩ Fi. We say that >=i is friend-orientedly unanimous if A­ >jF B for each j ∈ (Fi ∪ {i})∩A implies that A­ >i B.
* Let(N, >=) be a hedonic game and Γ be a coalition structure. A coalition C ⊆ N blocks Γ if for each i ∈ C it holds that C­ >=i Γ(i). If there is at least one I ∈ C with C­ >i Γ(i) while C >=j Γ(j) holds for the other players j != i in C, we call C weakly blocking. A coalition structure Γ is said to be

1. Individually rational if for all i∈N, Γ(i) is acceptable;
2. Nash-stable if for all i ∈ N and for each C ∈ Γ∪{∅} with Γ(i) != C, it holds that Γ(i) >=i C∪{i};
3. Individually stable if for all i ∈ N and for each C ∈ Γ∪{∅}, it either holds that Γ(i) >=i C∪{i} or there is a player j ∈ C with C ­>j C∪{i};
4. Contractually individually stable if for all i ∈ N and for each C ∈ Γ∪{∅}, it either holds that Γ(i) >=i C∪{i}, or there is a player j ∈C with C­ >j C∪{i}, or there is a player k ∈ Γ(i)with i != k and Γ(i)­ >k Γ(i)\{i};
5. Strictly popular if it beats every other coalition structure Γ’ != Γ in pairwise comparison, that is, if |{ i∈N | Γ(i)­ >i Γ’(i)}| > |{ i∈N | Γ’(i)­ >i Γ(i)}|;
6. (Strictly) core-stable if there is no (weakly) blocking coalition;
7. Perfect if for all i ∈ N and for all C ∈ Ni, it holds that Γ(i) >=i C.

Results:

* Under all three degrees of altruism, weak friend-orientedness, favoring friends, indifference between friends, indifference between enemies, sovereignty of players, symmetry, and friend-oriented unanimity are satisﬁed.
* Equal-treatment preferences and altruistic treatment preferences are not type-II-monotonic.
* For all three degrees of altruism, it can be tested in polynomial time whether a given coalition structure in a given game is Nash-stable, individually stable, or contractually individually stable.
* For all three degrees of altruism, there always exist Nash-stable, individually stable, and contractually individually stable coalition structures.
* Under selﬁsh-ﬁrst preferences, the problem of whether a given coalition structure in a given game is strictly popular is coNP-complete and the problem of whether there exists a strictly popular coalition structure in a given game is coNP-hard.
* In games with selﬁsh-ﬁrst preferences, there always exists a (strictly) core-stable coalition structure.

Open Problems:

* Completely characterize when certain properties hold or stable coalition structures exist.
* Extend the model and normalize by the size of the coalition to consider only relative contributions of friend-of-a friend relationships.

# **6. Hedonic Games with Graph-restricted Communication:**

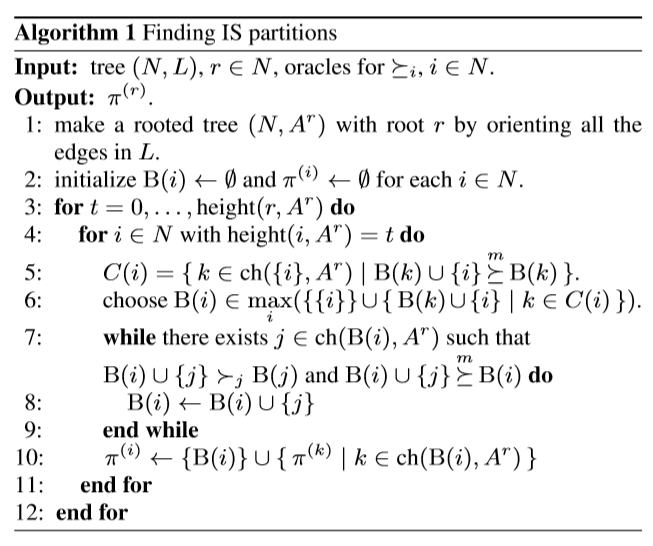
We study hedonic coalition formation games in which cooperation among the players is restricted by a graph structure: a subset of players can form a coalition if and only if they are connected in the given graph. We investigate the complexity of ﬁnding stable outcomes in such games, for several notions of stability.

Definitions:

* A hedonic game with graph structure, or a hedonic graph game, is a triple (N,(>i)i∈N, L) where (N,(>i)i∈N) is a hedonic game and L ⊆ { {i, j}| i != j, i, j ∈ N } is the set of communication links between players. A coalition X ⊆ N is said to be feasible if it is connected in (N, L).
* Given a hedonic graph game(N, (>i)i∈N, L), we say that j is a neighbor of i if {i, j} ∈ L. A feasible deviation of a player i to X 6∈Ni is called
  + in-neighbor feasible if it is NS feasible and accepted by all of i’s neighbors in X.
  + IR-in-neighbor feasible if it is in-neighbor feasible and for all j ∈ X it holds that X ∪{i} >j {j}.

Results:

* Suppose that we are given oracle access to the preference relations >i of all players in a hedonic graph game G = (N, (>i)i∈N, L), where (N, L) is a forest. Then we can ﬁnd an individually stable feasible outcome of G in time polynomial in |N|.
* Algorithm 1 finds IS partitions



* For each i ∈ N, j ∈ succ(i, Ar) and all X ∈ π(i)∪{∅} there is no IS feasible deviation of j from π(i)(j) to X.
* Suppose that the graph(N, L) contains a cycle C = {i1,i2,...,ik} with k ≥ 3, {ih, ih+1} ∈ L for h = 1,2,...,k, where ik+1 := i1. Then, we can choose preference relations (>i)i∈N so that the set of IS feasible partitions of the game (N, (>i)i∈N, L) is empty.
* For the class of hedonic graph games, the following statements are equivalent.

1. (N, L) is a forest.
2. For every hedonic graph game (N, (>i)i∈N, L) there exists an individually stable feasible partition of N.

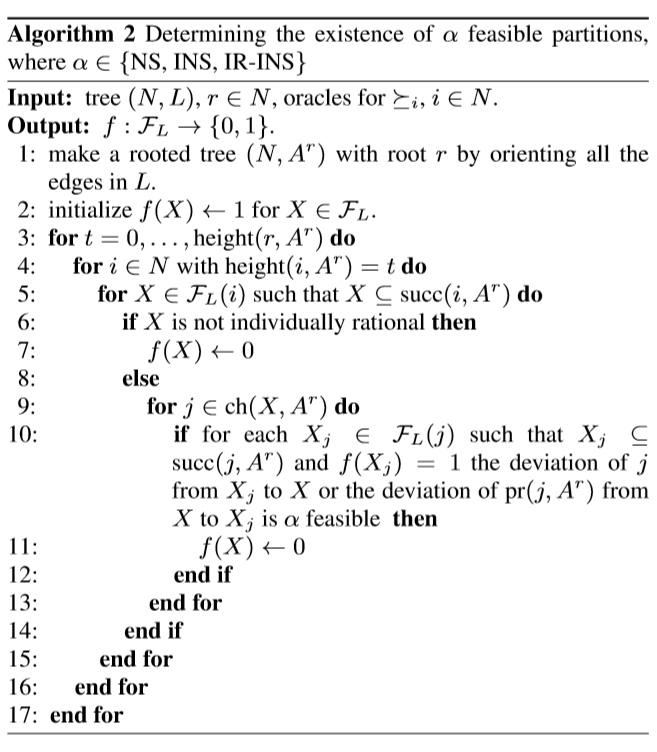
* For the class of hedonic games with graph structure, the following statements are equivalent.

1. (N, L) is a forest.
2. For every hedonic graph game (N, (>i)i∈N, L) there exists a core stable feasible partition of N.

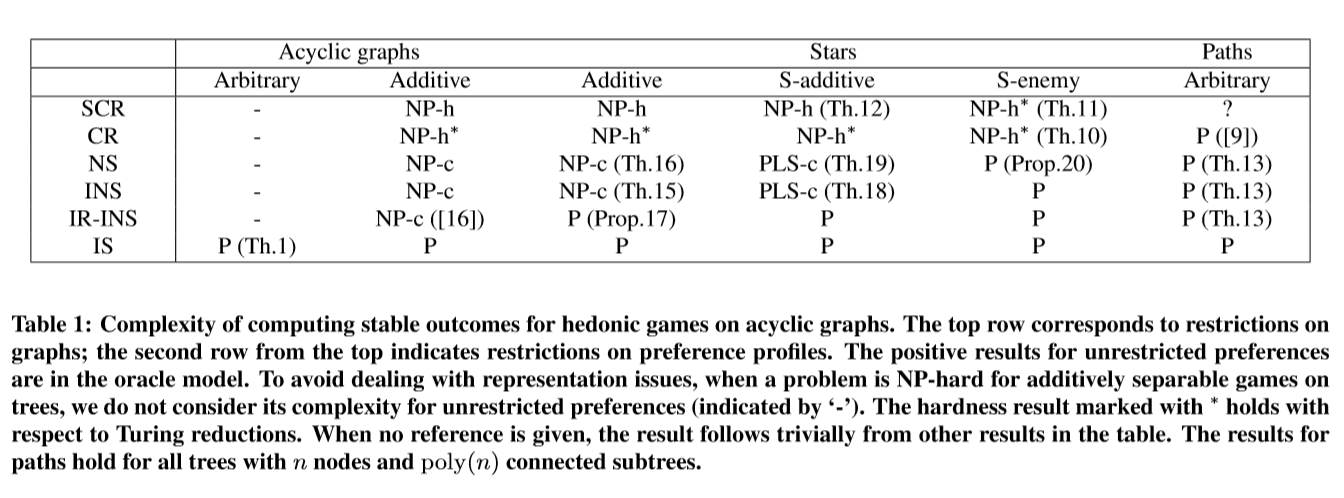
* For the class of hedonic games with graph structure, the following statements are equivalent.

1. (N,L) is a forest.
2. For every hedonic graph game (N, (>i)i∈N, L), there exists a feasible partition of N that belongs to the core and is individually stable.

* Suppose that we are given oracle access to the preference relations >i of all players in a hedonic graph game G = (N, (>i)i∈N, L),where (N, L) is a forest. Then we can ﬁnd a core stable feasible outcome of G in time polynomial in the number of connected subsets of (N, L).
* Suppose that we are given oracle access to the preference relations >i of all players in a hedonic graph game G = (N, (>i)i∈N, L), where (N, L) is a forest. Then we can ﬁnd a feasible outcome of G that belongs to the core and is individually stable in time polynomial in the number of connected subsets of (N, L).
* If one can ﬁnd a core stable feasible partition in a symmetric enemy-oriented graph game whose underlying graph is a star in time polynomial in the number of players then P=NP.
* If there exists a polynomial-time algorithm that, given a symmetric enemy-oriented graph game whose underlying graph is a star, decides whether this game has a strictly core stable feasible partition then P=NP.
* Given a symmetric additively separable hedonic graph game whose underlying graph is a star, it is NP-hard to determine whether it has a strictly core stable feasible partition.
* Suppose that we are given oracle access to the preference relations >i of all players in a hedonic graph game G = (N, (>i)i∈N, L), where (N, L) is a forest. Then we can decide whether G admits a Nash stable, in-neighbor stable or IR in-neighbor stable feasible outcome (and ﬁnd one if it exists) in time polynomial in the number of connected subsets of (N, L).



* Given an additively separable hedonic graph game whose underlying graph is a star, it is NP-complete to determine whether it has an in-neighbor stable feasible partition.
* Given an additively separable hedonic graph game whose underlying graph is a star, it is NP-complete to determine whether it has a Nash stable feasible partition.
* Every hedonic graph game (N, (>i)i∈N, L) where (N, L) is a star has an IR-in-neighbor stable partition, and givenoracleaccesstotheplayers’preferencerelations,suchapartition can be found using O(|N|3) oracle calls.
* Given a symmetric additively separable hedonic graph game whose under lying graph is a star, it is PLS-complete to ﬁnd an in-neighbor stable feasible partition.
* Given a symmetric additively separable hedonic graph game whose underlying graph is a star, it is PLS-complete to ﬁnd a Nash stable feasible partition.
* A Nash stable feasible outcome of a symmetric enemy-oriented game on a star can be computed in polynomial time.



Open Problems:

* It remains unknown whether a strictly core stable partition for a hedonic game on a tree can be computed in time polynomial in the number of connected coalitions.
* Checking if our algorithms can be extended to graphs that are “almost” acyclic.
* if there are constraints on the communication structure other than acyclicity that lead to existence/tractability results for common hedonic games stability concepts.

# **7. Local Fairness in Hedonic Games via Individual Threshold Coalitions:**

We introduce and systematically study local fairness notions in hedonic games by suitably adapting fairness notions from fair division. In particular, we introduce three notions that assign to each player a threshold coalition that only depends on the player’s individual preferences. A coalition structure (i.e., a partition of the players into coalitions) is considered locally fair if all players’ coalitions in this structure are each at least as good as their threshold coalitions. We relate our notions to previously studied concepts and show that our fairness notions form a proper hierarchy. We also study the computational aspects of ﬁnding threshold coalitions and of deciding whether fair coalition structures exist in additively separable hedonic games. At last, we investigate the price of fairness.

Definitions:

* The grand-coalition threshold of i ∈ N is deﬁned as

GCi = max{{i}, N}, where we maximize with respect to >=i. A coalition structure satisﬁes grand-coalition fairness (GC) if π(i) >=i GCi for every i ∈ N.

* The max-min threshold of i ∈ N is deﬁned as
* MaxMini = maxπ∈Π(N\{i}) max{{i}, minC∈πC∪{i}}, where maximization and minimization are with respect to >=i. A coalition structure π satisﬁes max-min fairness (MAX-MIN) if π(i) >=i MaxMini for every i ∈ N.
* Let G = (N, v) be an additively separable hedonic game and let π\* denote a coalition structure maximizing utilitarian social welfare. Deﬁne the maximum price of min-max fairness by

Max-PoMMF(G) = max π∈Π(N), π is min-max fairSW(π∗)/SW(π)

Results:

* An individually rational or core-stable coalition structure does not necessarily satisfy min-max fairness.
* Every Nash-stable coalition structure satisﬁes min-max fairness.
* Every grand-coalition fair coalition structure satisﬁes min-max fairness, yet a min-max fair coalition structure does not necessarily satisfy grand-coalition fairness.
* A strictly strong Nash-stable coalition structure does not necessarily satisfy grand-coalition fairness.
* An individually rational, Nash-stable, core stable, or strictly core-stable coalition structure does not necessarily satisfy grand-coalition fairness.
* Every max-min fair coalition structure satisﬁes grand-coalition fairness, yet a grand-coalition fair coalition structure does not necessarily satisfy max-min fairness.
* A max-min fair coalition structure does not necessarily satisfy contractually individual stability or core stability.
* A grand-coalition fair or min-max fair coalition structure does not necessarily satisfy contractually individual stability or core stability.
* A max-min fair, grand-coalition fair, or min-max fair coalition structure does not necessarily satisfy Nash stability, Pareto optimality, strictly strong Nash stability, strict core stability, utilitarian social welfare, or perfectness.
* A max-min fair coalition structure does not necessarily satisfy envy-freeness by replacement or egalitarian social welfare.
* A grand-coalition fair or min-max fair coalition structure does not necessarily satisfy envy-freeness by replacement or egalitarian social welfare.
* An individually rational, Nash-stable, core stable, strictly strong Nash-stable, or strictly core-stable coalition structure does not necessarily satisfy max-min fairness.
* MIN-MAX-THRESHOLD is coNP-complete.
* MIN-MAX-EXIST is NP-hard and in Σp2.
* In additively separable hedonic games, for every i ∈ N we have MaxMini = GCi.
* MAX-MIN-THRESHOLD and GRAND-COALITION-THRESHOLD are in P.
* The problems GRAND-COALITION-EXIST and MAX-MIN-EXIST are NP-complete.
* Let G=(N, v) be a symmetric ASHG of n players with vi(j) ≥ 0 for every i, j ∈ N. Then Max-PoMMF(G) ≤ n−1. In addition, this bound is tight.
* Let G = (N, v) be a symmetric ASHG. Then every coalition structure π that maximizes utilitarian social welfare satisﬁes min-max fairness.

Open Problems:

* Finding suitable restrictions to players’ valuation functions such that the maximum price of min-max fairness is bounded by a nontrivial constant.
* Identify sufﬁcient conditions that imply the existence of a fair coalition structure, determining the complexity of searching for a min-max fair coalition structure in symmetric additively separable hedonic games, and showing Σp2-hardness of MIN-MAX-EXIST.

# **8. Browsing Regularities in Hedonic Content Systems:**

Various hedonic content systems (e.g. mobile apps for video, music, news, jokes, pictures, social networks etc.,) increasingly dominates people’s daily spare life. This paper studies common regularities of browsing behaviors in these systems, based on a large data set of user logs.

Acronyms and Definitions:

