



MATHEMATICS *for* UPSC CSE MAINS

TOPIC: 2017 Solution (Analytical Geometry)

Analytical Geometry

UPSC - 2017.

Que:- 1(d) find the equation of the tangent plane at the point $(1, 1, 1)$ to the conicoid $3x^2 - y^2 = 2z$.

Sol. Point passing from $(1, 1, 1)$ is being given by

$$\frac{x-1}{l} = \frac{y-1}{m} = \frac{z-1}{n} = \gamma$$

Gen. Point $(\lambda l + 1, m\lambda + 1, n\lambda + 1)$
Since this point lies on this conicoid :-

$$\Rightarrow 3(\lambda l + 1)^2 - (m\lambda + 1)^2 = \alpha(n\lambda + 1)$$

$$\Rightarrow 3(\lambda^2 l^2 + 1 + 2\lambda l) - (m^2 \lambda^2 + 1 + 2m\lambda) = 2n\lambda + 2$$

$$\Rightarrow \lambda^2(3l^2 - m^2) + \lambda(6l - 2m - 2n) + 3 - 1 = 2$$

$$\Rightarrow \lambda^2(3l^2 - m^2) + \lambda(6l - 2m - 2n) = 0$$

\therefore at Tangent we will have one value of λ & it is obtained by putting the coeff. of λ is equal to zero. i.e. sum of two roots is zero.

$$6l - 2m - 2n = 0 \Rightarrow 3l - m - n = 0$$

now put l, m, n as $(x-1, y-1, z-1)$

$$\Rightarrow 3(x-1) - (y-1) - (z-1) = 0$$

$$\Rightarrow 3x - 3 - y + 1 - z + 1 = 0$$

$$\Rightarrow \boxed{3x - y = z + 1} \quad \text{Ans}$$

Que. 1(e) Find the shortest distance b/w the skew line :

$$\frac{x-3}{3} = \frac{y-7}{1} = \frac{z-3}{1} \quad \& \quad \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

$$\text{Sol.} \quad \vec{a}_1 = 3\hat{i} + 8\hat{j} + 3\hat{k}; \quad \vec{b}_1 = 3\hat{i} + (\hat{j}) + \hat{k}$$

$$\vec{a}_2 = -3\hat{i} - 7\hat{j} + 6\hat{k}; \quad \vec{b}_2 = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \underline{(-6\hat{i} - 15\hat{j} + 3\hat{k})}; \quad \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

Now

$$S.D. = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \hat{i}(-6) - \hat{j}(15) + \hat{k}(3)$$

$$\vec{b}_1 \times \vec{b}_2 = \underline{(-6\hat{i} - 15\hat{j} + 3\hat{k})}$$

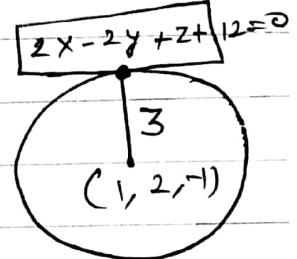
$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{36 + 225 + 9}$$

$$= \frac{36 + 225 + 9}{\sqrt{36 + 225 + 9}} = \sqrt{270} = \sqrt{9 \times 30}$$

$$= 3\sqrt{30} \text{ units.}$$

2(c) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

$$\text{Sol.} \quad x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$$



$$\text{Centre} = (1, 2, -1)$$

$$\text{radius} = \sqrt{1+4+1+3} = \sqrt{9} = 3$$

Now, if it (plane) touches the sphere then the distance of plane from the centre should be equal to radius of the sphere.

so L dist. from Centre to plane is :

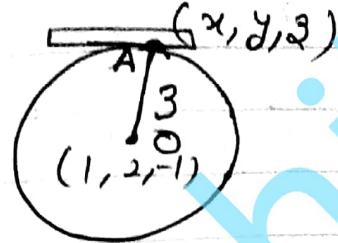
$$P = \left| \frac{2(1) - 2(2) + (-1) + 12}{\sqrt{4+4+1}} \right| = \left| \frac{2 - 4 - 1 + 12}{3} \right|$$

$$= \frac{9}{3} = 3$$

Hence it touches the sphere.

Now the point of contact; let it be (x_1, y_1, z)

so dir. ratio of this line is
(radius)
 $(2, -2, 1)$



And the eq.ⁿ of line is given

$$\frac{x-1}{2} = \frac{y+2}{-2} = \frac{z+1}{1} = \gamma$$

Gen. point $(x = 2\gamma + 1, y = -2\gamma + 2, z = \gamma + 1)$

Since this pt. is on the plane, hence it will satisfy the equation

$$2(2\gamma + 1) - 2(-2\gamma + 2) + (\gamma + 1) + 12 = 0$$

$$\Rightarrow 4\gamma + 2 + 4\gamma - 4 + \gamma + 1 + 12 = 0$$

$$\Rightarrow 9\gamma + 9 = 0 \Rightarrow \boxed{\gamma = -1}$$

$\Rightarrow (-1, 4, -2)$ point of contact. Ans.

Que: 3(d) Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$.
10 marks.

Sol.ⁿ Given conicoid $ax^2 + by^2 + cz^2 = 1 \dots \textcircled{1}$

Let one of the 3 mutually \perp tangent plane is

$$lx + my + nz = p \dots \textcircled{2}$$

By the condition of tangency \Rightarrow

$$\frac{l_1^2}{a} + \frac{m_1^2}{b} + \frac{n_1^2}{c} = P_1^2$$

$$\Rightarrow P_1 = \pm \sqrt{\frac{l_1^2}{a} + \frac{m_1^2}{b} + \frac{n_1^2}{c}}$$

Now eq. (2) becomes :-

$$l_1 x + m_1 y + n_1 z = \sqrt{\frac{l_1^2}{a} + \frac{m_1^2}{b} + \frac{n_1^2}{c}} \quad \text{--- (3)}$$

Similarly

$$l_2 x + m_2 y + n_2 z = \sqrt{\frac{l_2^2}{a} + \frac{m_2^2}{b} + \frac{n_2^2}{c}} \quad \text{--- (4)}$$

$$l_3 x + m_3 y + n_3 z = \sqrt{\frac{l_3^2}{a} + \frac{m_3^2}{b} + \frac{n_3^2}{c}} \quad \text{--- (5)}$$

Now to find the locus we need to eliminate
the l, m, n ; i.e. both side :-

$$\begin{aligned} & \Rightarrow x^2 \left(\cancel{l_1^2} + \cancel{l_2^2} + \cancel{l_3^2} \right) + y^2 \left(\cancel{m_1^2} + \cancel{m_2^2} + \cancel{m_3^2} \right) + z^2 \left(\cancel{n_1^2} + \cancel{n_2^2} + \cancel{n_3^2} \right) \\ & + 2xy(l_1 m_1 + l_2 m_2 + l_3 m_3) + 2yz(m_1 n_1 + m_2 n_2 + m_3 n_3) + \\ & 2zx(n_1 l_1 + n_2 l_2 + n_3 l_3) = \frac{1}{a} \left(\cancel{l_1^2} + \cancel{l_2^2} + \cancel{l_3^2} \right) + \frac{1}{b} \left(\cancel{m_1^2} + \cancel{m_2^2} + \cancel{m_3^2} \right) \\ & + \frac{1}{c} \left(\cancel{n_1^2} + \cancel{n_2^2} + \cancel{n_3^2} \right) \\ & \Rightarrow \boxed{x^2 + y^2 + z^2 = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}} \quad \text{Ans} \end{aligned}$$

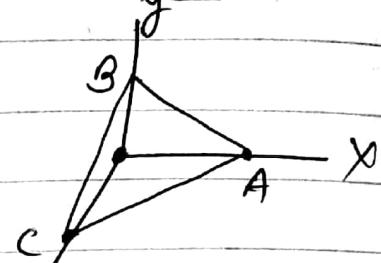
Ques(b) A plane passes through a fixed point
15 Marks (a, b, c) and cuts the axes at the points
 A, B, C respectively. Find the locus of the
 centre of the sphere which passes through
 the origin O & A, B, C.

Sol. Let the eq.ⁿ of the sphere OABC, through
 O(0,0,0) be

$$[x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0]$$

Now at pt. A $\Rightarrow y = z = 0$

$$\Rightarrow x^2 + 2ux = 0 \Rightarrow x = -2u$$



Similarly at pt. B & pt. C
 we get $y = -2v$ & $z = -2w$

Now the eq.ⁿ of Plane ABC is

$$\Rightarrow \frac{x}{-2u} + \frac{y}{-2v} + \frac{z}{-2w} = 1$$

$$\Rightarrow \frac{x}{-u} + \frac{y}{-v} + \frac{z}{-w} = 2$$

Now this plane passes through (a, b, c)

$$\Rightarrow \frac{a}{-u} + \frac{b}{-v} + \frac{c}{-w} = 2$$

Now here $(-u, -v, -w)$ are the centre
 of the sphere, so the locus is given by

$$\Rightarrow \boxed{\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2}$$

Ans

(2) Analytical Geomet

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SOLVED EXAMPLES

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Example 1. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

Solution. The centre of the sphere is $(1, 2, -1)$ and its radius $= \sqrt{1+4+1+3} = 3$

\therefore Perpendicular distance of the centre $(1, 2, -1)$ from the plane

$$= \frac{|2(1) - 2(2) + (-1) + 12|}{\sqrt{4+4+1}} = \frac{|2-4-1+12|}{3} = 3, \text{ which is the radius}$$

Since the distance of the centre from the plane = radius of the sphere, hence the plane touches the sphere.

To find the point of contact:

The point of contact is the foot of perpendicular from the centre to the plane.

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(2) Analytical Geomet

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Q 4(d) 2016
15 Marks

Find the Locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$.

Conicoid $\Rightarrow \alpha x^2 + \beta y^2 + \gamma z^2 = 1 \quad \text{--- (1)}$

Let one of the mutually \perp tangent plane to eq.(1) is

$$l_1 x + m_1 y + n_1 z = P \quad \text{--- (2)}$$

Acc. to the condition of tangency :

$$\frac{l_1^2}{\alpha} + \frac{m_1^2}{\beta} + \frac{n_1^2}{\gamma} = P^2$$

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