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MATHEMATICS FOR UPSC CSE MAINS

⑦ (Part-1) Dated:- 23/08/2016

Runge-Kutta Method :-

✓ Euler method → 1st Order
modified Euler method → 2nd "
Runge - method → 3rd "

Taylor Series :- $h^{(n)}$ ← order

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \ldots\ldots$$

✓ Runge-Kutta Method → 4th Order

Working Rule :-

for finding increment K of y ; Corresponding to an increment h of x by Runge-Kutta method from

$$\left[\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0\right]$$

Calculate :

$$K_1 = hf(x_0, y_0)$$

$$K_2 = hf(x_0 + \tfrac{1}{2}h,\ y_0 + \tfrac{1}{2}K_1)$$

$$K_3 = hf(x_0 + \tfrac{h}{2},\ y_0 + \tfrac{1}{2}K_2)$$

$$K_4 = hf(x_0 + h,\ y_0 + K_3)$$

and finally Compute

$$\boxed{K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)}$$

$\Rightarrow \boxed{y_1 = y_0 + K}$

⑧

Que:- Apply "R-K method (4th order)"; To find Approxi. value of y when $h = 0.2$.
Given $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$

Sol.ⁿ $x_0 = 0,\ y_0 = 1,\ h = 0.2;\quad f(x_0, y_0) = x_0 + y_0 = 1$

$$K_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2$$

$$K_2 = h(f(x_0 + \tfrac{1}{2}h,\ y_0 + \tfrac{1}{2}K_1)) = 0.2 \times f(0.1, 1.1)$$
$$\Rightarrow 0.2 \times 1.2 = 0.24$$

$$K_3 = hf(x_0 + \tfrac{h}{2},\ y_0 + \tfrac{1}{2}K_2) = 0.2 \times f(0.1, 1.12)$$
$$= 0.2 \times (0.1 + 1.12) = 0.244$$

$$K_4 = hf(x_0 + h,\ y_0 + K_3) = 0.2\, f(0.2, 1.244)$$
$$= 0.2 \times (0.2 + 1.244)$$
$$= 0.2888$$

$$\Rightarrow K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = \frac{1}{2}(0.2 + 0.24 + 0.244 + 0.2888)$$
$$= 0.2428$$

So, Required Approximate value of $y = 1 + 0.2428$
$= \boxed{1.2428}$ Ans

Que:- (2015 Mains); 15 marks :-
Solve the initial value problem $\frac{dy}{dx} = x(y - x)$, $y(2) = 3$ in the interval $[2, 2.4]$ using Rugne-Kutta fourth order method with step size $h = 0.2$.

Sol.ⁿ $x_0 = 2,\ y_0 = 3,\ h = 0.2\quad f(x_0, y_0) = x(y - x)$

i) for $y(2)$: $\Rightarrow K_1 = hf(x_0, y_0) = 0.2 \times f(2, 3)$
$= 0.2 \times 2(3 - 2) = 0.4$

$\Rightarrow K_2 = hf(x_0 + \frac{h}{2},\ y_0 + \frac{K_1}{2}) = 0.2 \times f(2.1, 3.2)$
$= 0.2 \times 2.1(1.1) = 0.462$

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(9)

$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.2 \times f(2.1, 3.231)$

$= 0.2 \times 2.1 \times 1.31 = 0.475$

$K_4 = h f(x_0 + h, y_0 + K_3) = 0.2 \times f(2.2, 3.475)$

$= 0.2 \times 2.2 \times 1.275 = 0.561$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= \frac{1}{6}(0.4 + 2(0.462) + 2(0.475) + 0.561)$

$= 0.4725$

so $y(2) = y_0 + K = 3 + 0.4725 = \boxed{3.4725}$

ii) $y(2.2)$

$x_0 = 2.2;\ y_0 = 3,\ h = 0.2$

$K_1 = 0.2 \times 2.2 \times 0.8 = 0.352$

$K_2 = 0.2 \times 2.3 \times 0.876 = 0.402$

$K_3 = 0.2 \times 2.3 \times 0.901 = 0.414$

$K_4 = 0.2 \times 2.4 \times 1.014 = 0.4870$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= 0.4118$

$\Rightarrow y(2.2) = 3 + 0.4118 = 3.4118$

iii) $y(2.4)$; $x_0 = 2.4,\ y_0 = 3,\ h = 0.2$

$K_1 = 0.2 \times 2.4 \times 0.6 = 0.288$

$K_2 = 2.5 \times 0.2 \times 0.644 = 0.322$

$K_3 = 2.5 \times 0.2 \times 0.661 = 0.330$

$K_4 = 2.6 \times 0.2 \times 0.730 = 0.3799$

$y(2.4) = y_0 + K$; where $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= 0.3287$

$y(2.4) = 3 + 0.32487 = \boxed{3.32487}$ Ans.

Jai श्री राम

Dated: 23 Aug. 2016

Numerical Method

(Part - II)

① "COMPUTER PROGRAMMING"

Binary system:-

Base = 2. eg. $(1011)_r$

⇓

only two digit i.e. $\boxed{0}$ or $\boxed{1}$

if $r = 2 \Rightarrow$ Binary

$r = 8 \Rightarrow$ octal

$r = 10 \Rightarrow$ decimal

$r = 16 \Rightarrow$ Hexadecimal

Counting

(0, 1, 10, 11, 100, 101, 111...)

• Addition:-

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$

eg.

$$\begin{array}{r} 10110 \\ +\ 10011 \\ \hline \boxed{1}\ \boxed{01001} \end{array}$$

Carry $\Rightarrow$

Subtraction:-

$$\begin{array}{rl} 8\,4\,2\,1 & \\ 1011 & (11)_{10} \\ -\ 1101 & (13)_{10} \\ \hline \boxed{1110} & -2 \\ & \boxed{1010} \end{array}$$

• Multiplication:-

$$\begin{array}{r} 1101 \\ \times\ 1011 \\ \hline 1101 \\ 1101\times \\ 0000\times\times \\ 1101\times\times\times \\ \hline ①\,\boxed{0001111} \end{array}$$

Octal s/s:- Base = 8

0, 1, 2, 3, 4, 5, 6, 7 = 8 digit

Arithmetic operation :-

(1) Octal s/s :

$1+1 = 2$,
$1+6 = 7$
$1+7 = 10$
$2+7 = 11$

eg. 1.

$$\begin{array}{r} {}^{1\ 1}\quad \\ (7654)_8 \\ +\,(2742)_8 \\ \hline 12616 \end{array}$$

Eg. 2

$$\begin{array}{r} 7653 \\ -\,2412 \\ \hline 5241 \end{array}$$

(2) Decimal :- Base = 10 i.e. (0, 1, 2, ... 9)

(3) Hexa-Decimal :- base = 16 [0, 1, ... 9, A, B, C, D, E, F]
A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

$$\begin{array}{r} (A9C4F)_{16} \\ +\,(5E2B4)_{16} \\ \hline \boxed{1}\ \boxed{07F03} \end{array}$$

Carry → 1

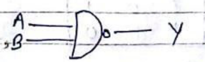
Logic Gate

(1) AND Gate : $Y = A \cdot B$

T.T : –

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(2) NAND Gate : $Y = \overline{A \cdot B}$



T.T : –

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

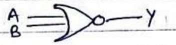
(3)

3.) OR Gate :- $Y = A+B$

T.T : –

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

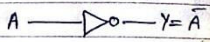
4.) NOR Gate : $Y = \overline{A+B}$



T.T :

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

5) NOT GATE : $Y = \bar{A}$ i.e

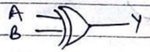


T.T : –

A	Y
0	1
1	0

6.) EXOR

$Y = A\bar{B} + \bar{A}B$
$= A \oplus B$
$= \overline{A \odot B}$

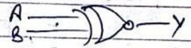


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

odd No. one = 1
Even No. One = 0

7.) EX-NOR

$Y = AB + \bar{A}\bar{B}$
$= \overline{A \oplus B}$
$= A \odot B$

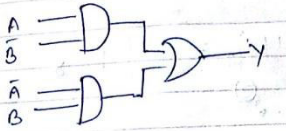


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

One → odd No. = 0
One → even No. = 1

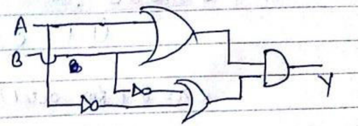
★ **SOP**
(Sum of Product)

$Y = A\bar{B} + \bar{A}B$



POS
(Product of Sum)

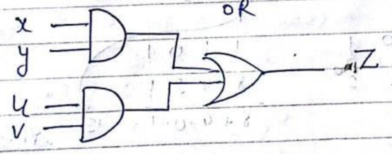
$Y = (A+B)\cdot(\bar{A}+\bar{B})$



Q (2014) Use only "AND" and "OR" logic gates
10 Marks to construct for boolean exp.

$$Z = xy + uv$$

Sol.n



Logic Gate : $\underbrace{\text{NOT AND OR}}_{\text{BASIC}}$ $\underbrace{\text{NAND NOR}}_{\text{UNIVERSAL}}$ $\underbrace{\text{ExOR ExNOR}}_{\text{Arithmatic device}}$

Arithmatic device const.r like Comparator, adder, Mux, Parity Generator

★ De Morgan Theorem:-

i) $\overline{A+B} = \bar{A}\cdot\bar{B}$

ii) $\overline{A\cdot B} = \bar{A}+\bar{B}$

★ Conversion of no s/s in one another.

① Binary To Octal, decimal, Hexadecimal

✓A) B to O :-

$$(11101101111)_2 = (?)_8$$

$r = 8$ (for octal) $= 8 = 2^{③}$

$\underbrace{001}_{1}\ \underbrace{110}_{6}\ \underbrace{110}_{6}\ \underbrace{111}_{7} \Rightarrow (1667)_8$

✓B) B to D :-

$$(1101)_2 = (?)_{10}$$

$\begin{matrix}1 & 1 & 0 & 1\\ 2^3 & 2^2 & 2^1 & 2^0\end{matrix}$; $\begin{matrix}1 & 1 & 0 & 1\\ 8 & 4 & 2 & 1\end{matrix}$

$8+4+0+1 = 13$ Ans.

✓C) B to H :-

$$(\underbrace{11}_{3}\underbrace{0101}_{5}\underbrace{1011}_{B}\underbrace{0101}_{5}\underbrace{1100}_{C})_2 = (?)_{16}$$

Hexa $= 16 = 2^{④}$

$$(35B5C)_{16}$$

② Decimal to Binary, Octal and Hexadecimal

✓ a) D to B :

$$(25)_{10} = (?)_2$$

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$$\begin{array}{r|l l}
2 & 25 & \\ \hline
2 & 12 & 1 \\
2 & 6 & 0 \\
2 & 3 & 0 \\
 & 1 & 1
\end{array}$$

$(11001)_2$

B.) D to O

$(108)_{10} = (?)_8$

$$\begin{array}{r|l l}
8 & 108 & \\ \hline
8 & 13 & 4 \\
 & 1 & 5
\end{array}$$

$(154)_8$

C.) D to H

$(2754)_{10} = (?)_{16}$

~~2754~~

$$\begin{array}{r|l l}
16 & 2754 & \\ \hline
16 & 172 & 2 \\
 & 10 & 12 = C \\
 & A &
\end{array}$$

$(AC2)_{16}$

3. Hexadecimal to Binary, Octal & Decimal

A) H to B :- $(AB03)_{16} = (1010\ 1011\ 0000\ 0011)_2$

B) H to O :- $(2E6F9)_{16} = (000\ 101\ 110\ 011\ 011\ 111\ 001)_2$

5 6 3 3 7 1

$= (563371)_8$

C.) H to D :- $(AC2)_{16} = (?)_{10}$

$$\begin{array}{ccc}
A & C & 2 \\
16^2 & 16^1 & 16^0
\end{array}
\qquad
\begin{array}{ccc}
10 & 12 & 2 \\
256 & 16 & 1
\end{array}$$

$2560 + 192 + 2 = (2754)_{10}$

4) Octal to Binary, decimal, Hexadecimal

a) O to B :- $(72)_8 = (111\ 010)_2$

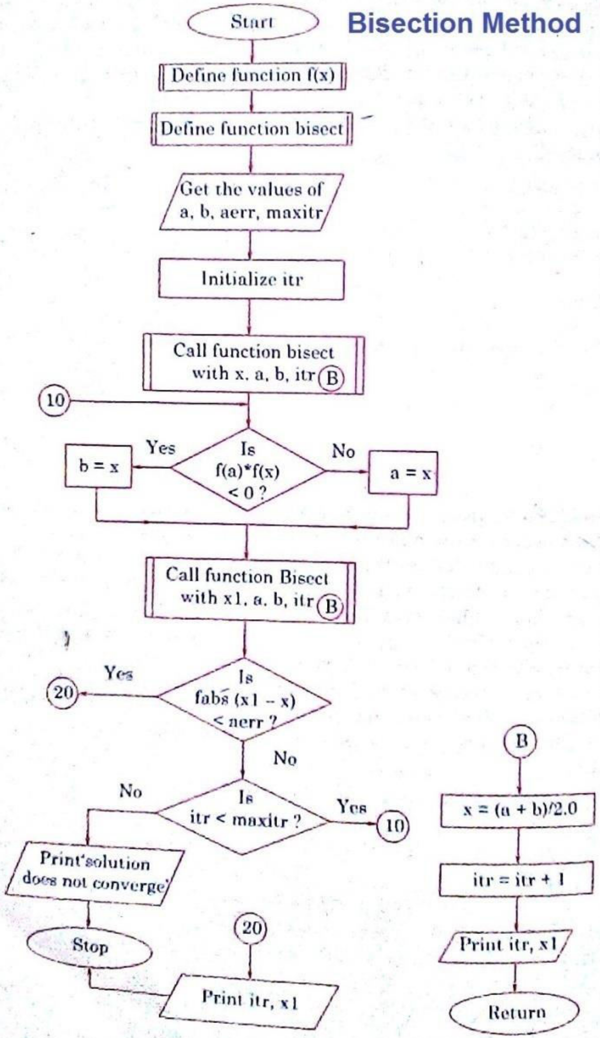
b) O to D :- $(72)_8 = (\ )_{10} \Rightarrow \begin{array}{cc} 7 & 2 \\ 8^1 & 8^0 \end{array} \Rightarrow \begin{array}{cc} 7 & 2 \\ 8 & 1 \end{array}$

$56 + 2 = (58)_{10}$

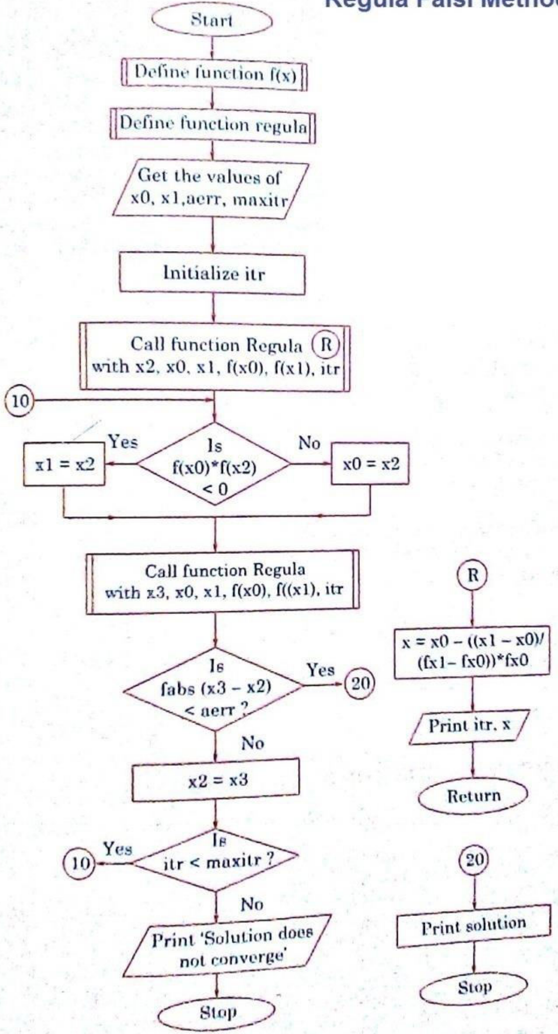
c.) O to H :- $(72)_8 = (11|1010)$ ⇒ $(3A)_{16}$

3 A

Bisection Method



Regula Falsi Method



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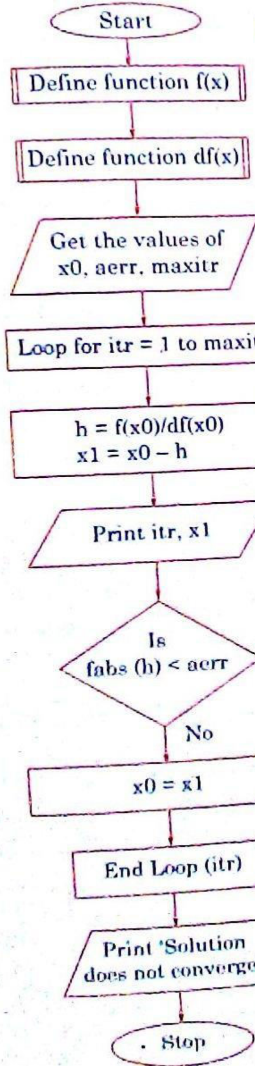
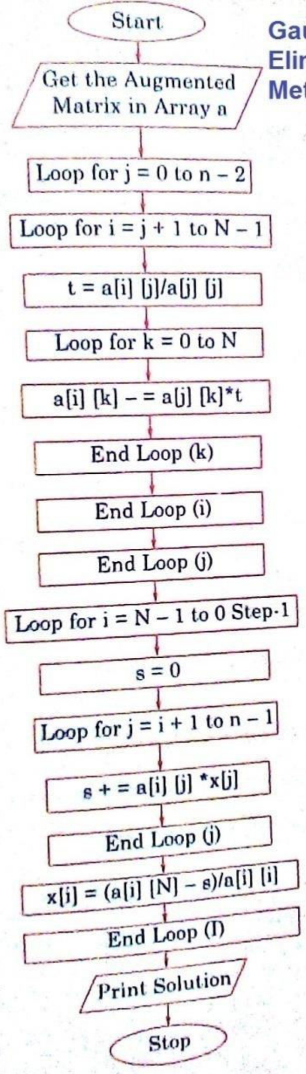
FLOW CHARTS

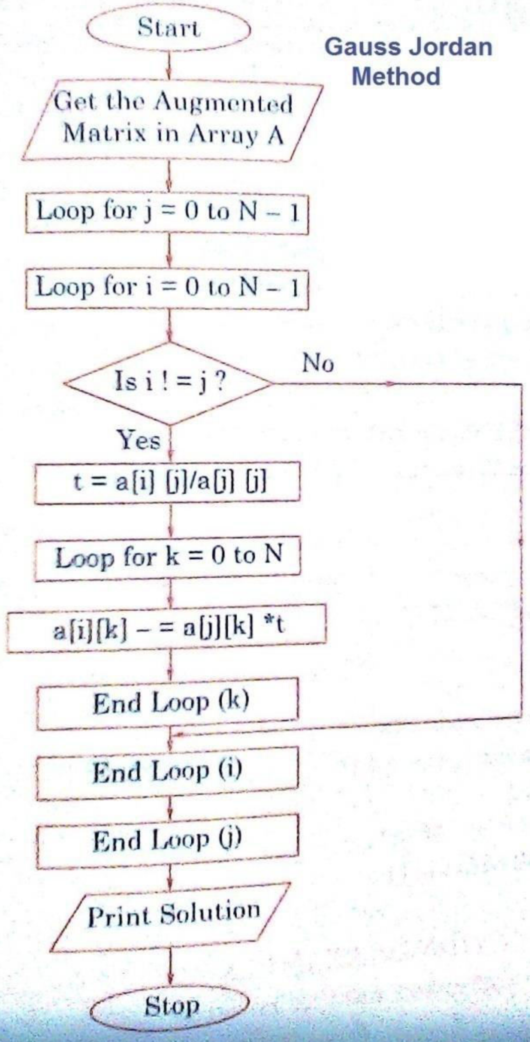
Newton-Raphson Method



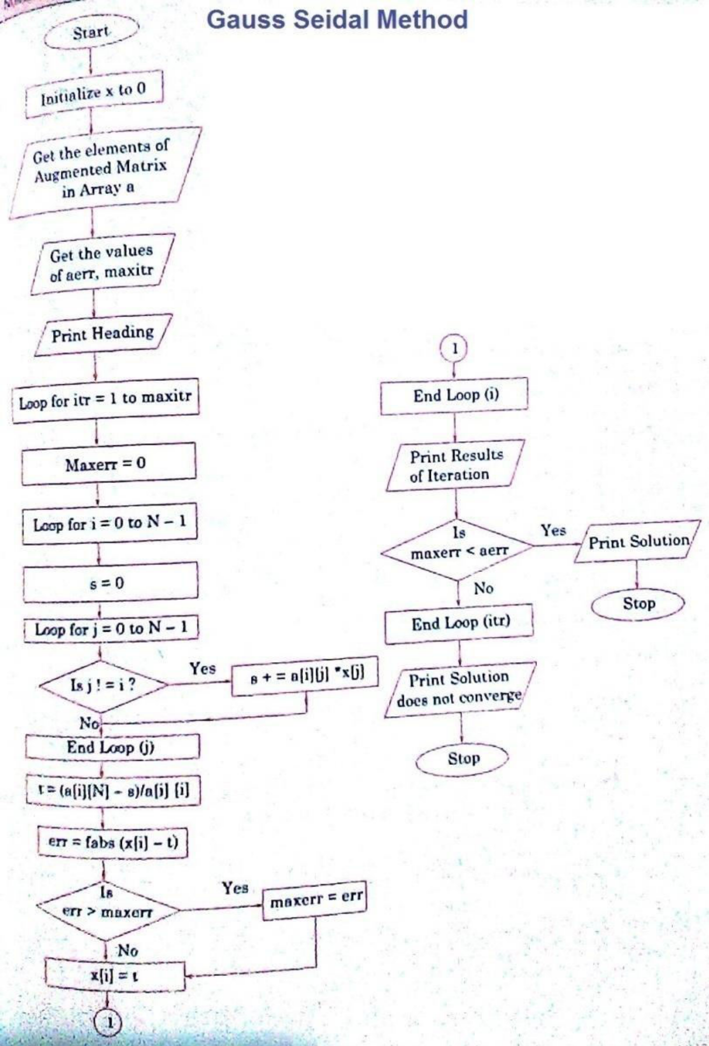
Gauss Elimination Method



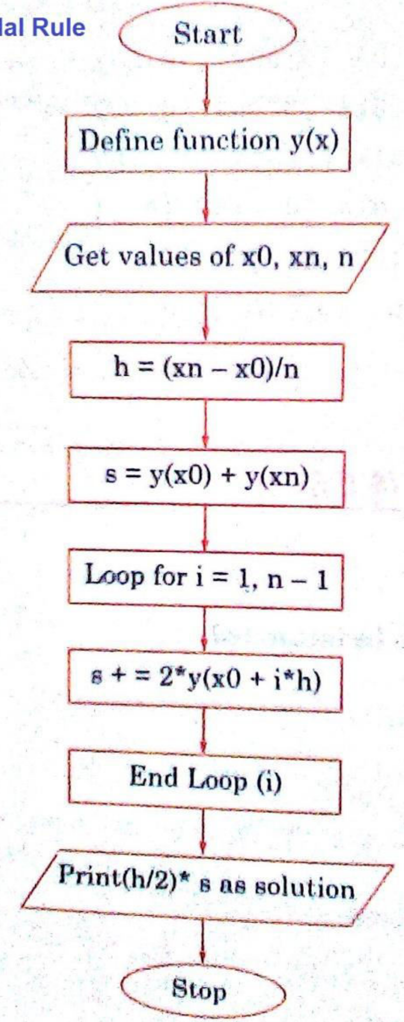
Gauss Jordan Method



Gauss Seidal Method



Trapezoidal Rule



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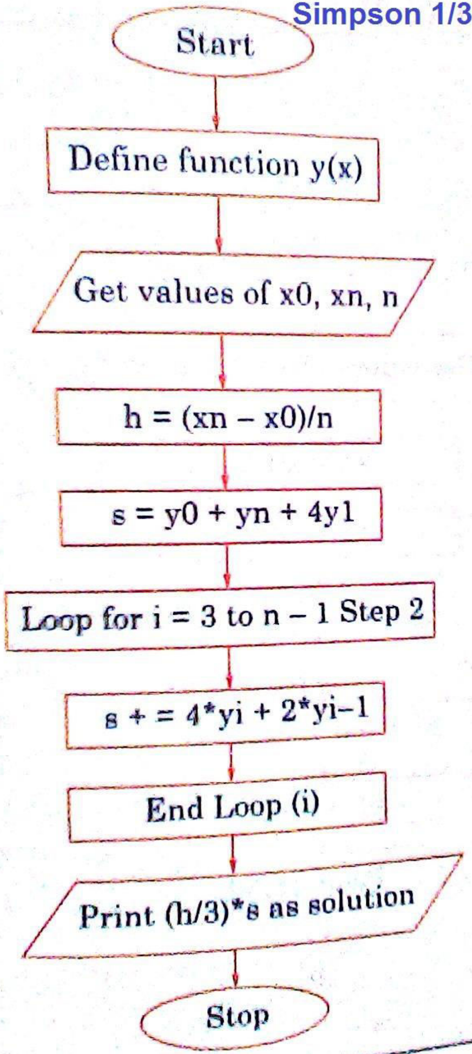
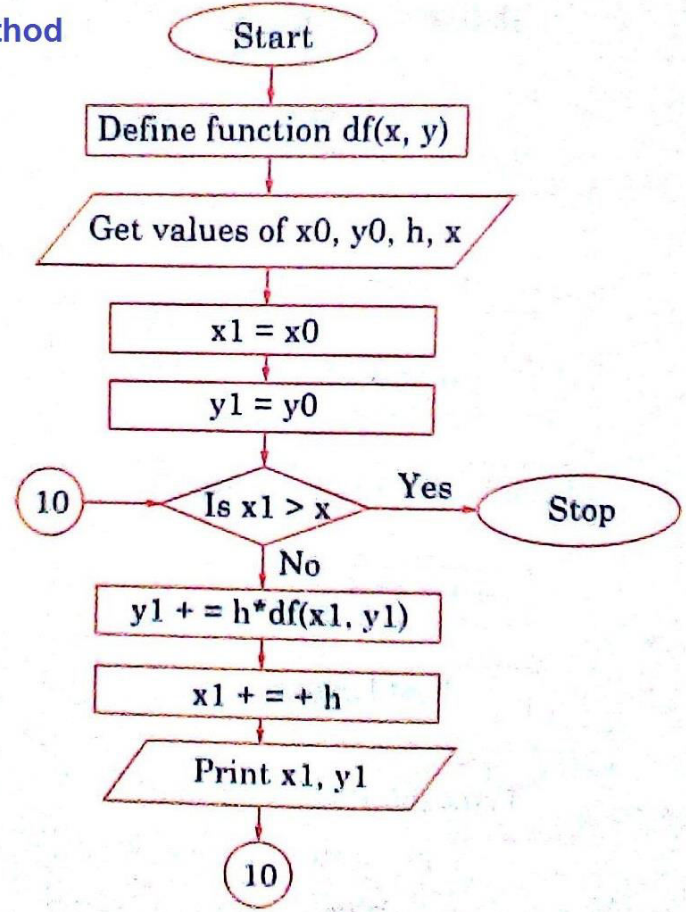
Simpson 1/3rd Rule



Euler's Method



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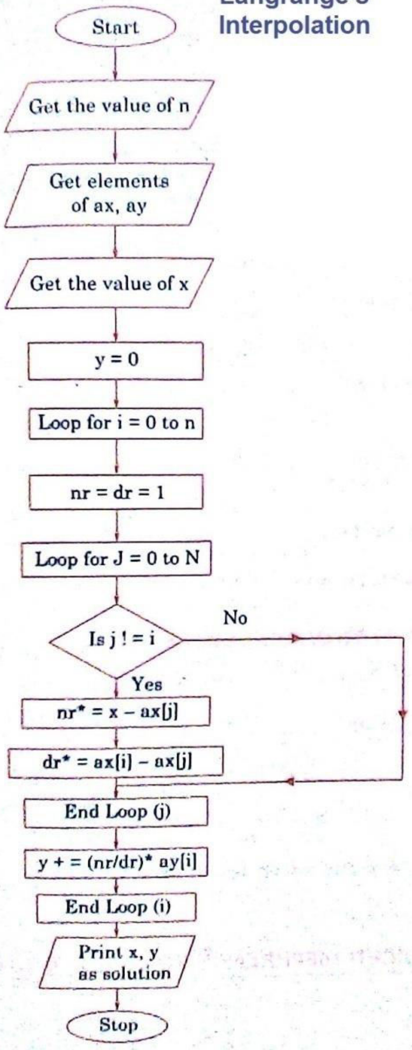
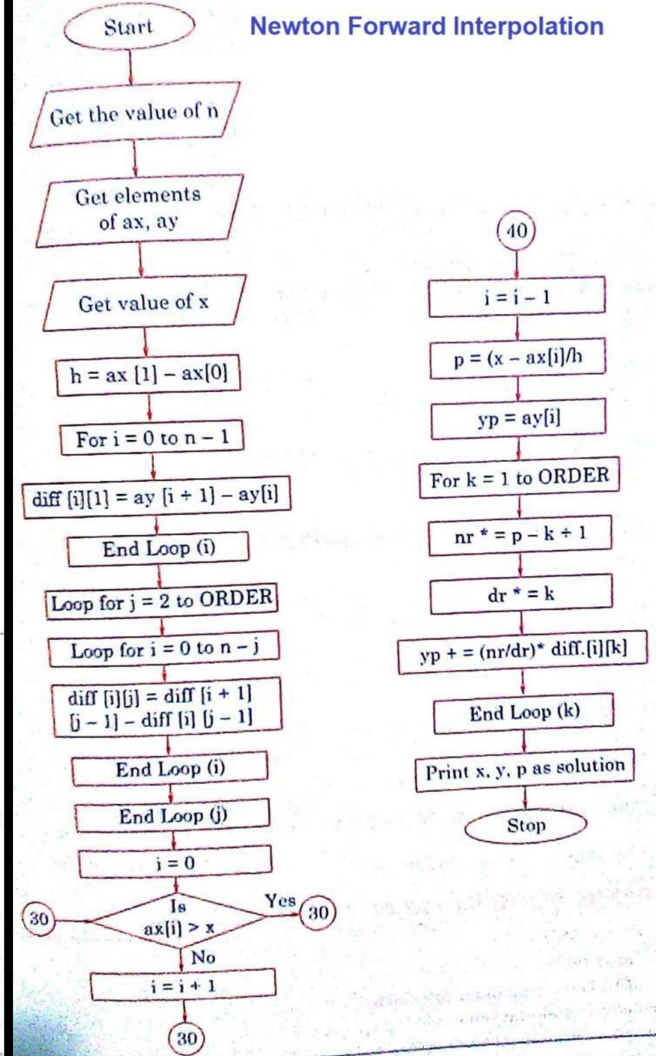
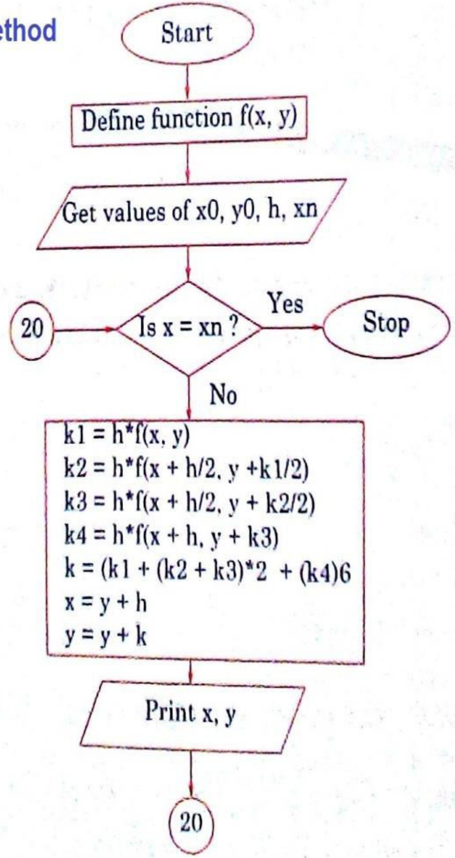
Langrange's Interpolation



Newton Forward Interpolation



Runge-Kutta Method



THANKS

NOTE: You can download the PPTS of Lectures at the link given in Description