MATHEMATICS FOR UPSC CSE MAINS

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SYLLABUS

Numerical methods:

1. Solution of algebraic and transcendental equations of one variable by bisection, Regula-Falsi and Newton-Raphson methods.
2. Solution of system of linear equations by Gaussian elimination and Gauss-Jordan (direct), Gauss-Seidel (iterative) methods.
3. Newton's (forward and backward) interpolation, Lagrange's interpolation.
4. Numerical integration: Trapezoidal rule, Simpson's rules, Gaussian quadrature formula.
5. Numerical solution of ordinary differential equations: Euler and Runga Kutta-methods.

Non - Linear Equations

Solution of Algebraic & Transcedental Equations –

Polynomials :-

Algebraic f^n of the form –

$$f_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots\; a_{n-1}x + a_n$$ are

Called Polynomials.

Transcedental Eq^n :-

Non- algebraic eq^n are called transcedental equation.

e.g – $\log x^3 - 0.7$

$\psi(x) = e^{-0.5x} - 5x$

$\psi(x) = \sin^2 x - x^2 - 2$

Intermediate Value theorem :- if $f(a) \times f(b) < 0$ then there exists atleast one root of the equation $f(x) = 0$ in the interval (a, b)

Bisection Method :- This method is based on a th^m which states that if a f^n $f(x)$ is continuous b/w a & b & $f(a)$ & $f(b)$ are of opposite signs, then there exists atleast one root b/w a & b.

This method is useful for determining the root of algebraic & transcedental equations.

Bisection Method :-

Que :- $f(x) = x^3 - 4x - 9 = 0$

find roots of the eqn. using Bisection Method.

Sol.^{n}

$f(2) = 8 - 8 - 9 = -ve$
$f(3) = 27 - 12 - 9 = +ve$
So Root lies b/w 2 to 3

$x_1 = \frac{2+3}{2} = 2.5$

$f(x_1) = (2.5)^3 - 4(2.5) - 9 = -3.375 = -ve$

so, the root lies b/w x_1 and 3. i.e. 2.5 to 3

$x_2 = \frac{1}{2}(2.5+3) = 2.75$ ~~+ve~~

$f(x_2) = (2.75)^3 - 4(2.75) - 9 = 0.7969$ i.e. +ve

2.75

Hence the root lies b/w x_1 & x_2. i.e. 2.5 & 2.75

$\Rightarrow x_3 = \frac{1}{2}(2.5 + 2.75) = 2.625$

$f(x_3) = (2.625)^3 - 4(2.625) - 9 = -1.4121$ i.e. -ve

Hence Root lies b/w x_3 & x_2 i.e.

$x_4 = \frac{1}{2}(x_2 + x_3) = 2.6875$

Repeating this process, the successive approximation will be

$x_5 = 2.71875$, $x_6 = 2.70313$, $x_7 = 2.71094$
$x_8 = 2.70703$, $x_9 = 2.70508$, $x_{10} = 2.70605$
$x_{11} = 2.70654$, $x_{12} = 2.70642$

Hence Root is $\boxed{2.7064}$ Ans.

Que. $f(x) = x^3 - 2x + (-5) = 0$

Regula - Falsi Method ; Three Decimal Point

Sol.^{n}

$f(2) = 8 - 4 - 5 = -ve$
$f(3) = 27 - 6 - 5 = +ve$
$\}$ Root lies b/w 2 to 3.

$\longrightarrow$ $x_0 = 2$, $x_1 = 3$

$f(2) = -1$; $f(3) = 16$

$x_2 = x_0 - \frac{(x_1 - x_0) f(x_0)}{f(x_1) - f(x_0)} = 2 - \left(\frac{3-2}{+1+16}\right)(-1) = 2 + \frac{1}{17}$

$= 2.0588$

$f(x_2) = f(2.0588) = -0.3908$

$\longrightarrow$ the root lies b/w 2.0588 & 3.

$x_0 = 2.0588$; $x_1 = 3$

$f(x_0) = -0.3908$, $f(x_1) = 16$

$x_3 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} = 2.0588 - \left(\frac{3 - 2.0588}{16 + 0.3908}\right) \times (-0.3908)$

$= 2.0813$

Repeating this process, the successive approximations are

$x_4 = 2.0862$, $x_5 = 2.0915$, $x_6 = 2.0934$

$$\boxed{x_7 = 2.0941}$$

The Root is 2.094 Correct to decimal places.

Q (2013); 10 Marks. Newton Forward Interpolation

Marks	30-40	40-50	50-60	60-70	70-80
No. of Student	31	42	51	35	31

Find the number of student whose marks lies b/w 40 to 45.

Sol.

x	y_x	Δy_x	$\Delta^2 y_x$	$\Delta^3 y_x$	$\Delta^4 y_x$
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

$h = 10$; $P = \frac{x - x_0}{h} = \frac{45-40}{10} = \frac{5}{10} = 0.5$

[Since y_{45} i.e. number of student marks less than 45, So Take $x_0 = 40$]

formula:

$$y_{45} = y_{40} + P\,\Delta y_{40} + \frac{P(P-1)}{2!}\Delta^2 y_{40} + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_{40} + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_{40}$$

$$y_{45} = 31 + 0.5(42) + \frac{0.5(-0.5)}{2}\times 9 + \frac{0.5(-0.5)(-1.5)}{6}\times(-25) + \frac{0.5(-0.5)(-1.5)(-2.5)}{24}\times 37$$

$$= 31 + 21 - 1.125 - 1.5625 - 1.4463$$

$$\boxed{y_{45} = 47.87}$$ Ans.

Newton forward Interpolation formula –

$$y_x = y_0 + P(\Delta y_0) + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \ldots$$

$$\boxed{P = \frac{x - x_0}{h}}$$

Backward formula –

$$y_x = y_n + P\nabla y_n + \frac{P(P+1)}{2}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \ldots$$

$$\boxed{P = \frac{x - x_n}{h}}$$

Que (2015) Find the Lagrange Interpolating polynomial that fits the following that:

let

		x_0	x_1	x_2	x_3
x	:	-1	2	3	4
$f(x)$ (y)	:	-1	11	31	69
		y_0	y_1	y_2	y_3

Find = $f(1.5)$. $\Rightarrow$ $(y = 1.5)$ & $x = ?$

Sol.

Formula :-

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} \times x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \times x_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} \times x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} \times x_3$$

$$x = \frac{(1.5-11)(1.5-31)(1.5-69)}{(-1-11)(-1-31)(-1-69)} \times (-1) + \frac{(1.5+1)(1.5-31)(1.5-69)}{(11+1)(11-31)(11-69)} \times 2$$

$$+ \frac{(1.5+1)(1.5-11)(1.5-69)}{(31+1)(31-11)(31-69)} \times 3 + \frac{(1.5+1)(1.5-11)(1.5-31)}{(69+1)(69-11)(69-31)} \times 4$$

$= -0.703 + 0.715 - 0.198 + 0.018 = \boxed{-0.168}$ Ans

Newton - Rapson Method :-

Used for the solution of algebraic & transcedental equation. This method is useful to improve the result obtained by one of the previous methods.

Let x_0 be an appx. root of $f(x)=0$ & Let $x_1 = x_0 + h$ be the correct root so that $f(x_1)=0$, Expanding $f(x_0+h)$ by Taylor's series-

$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \dots\dots = 0$$

Nelecting the 2nd & Higher - Order derivatives -

$$f(x_0) + hf'(x_0) = 0$$

Hence $\boxed{h = -\frac{f(x_0)}{f'(x_0)}}$

$$x_1 = x_0 + h = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Successive approximations are given by $x_2, x_3 \dots x_{n+1}$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$ — Newton Rapson or Iteration Formula.

This method requires more computing Time.

Que (2014); 10 Marks; Newton - Rapson Method

$f(x) = \cos x - xe^x$; find root up to 4 decimal point

Sol.^{n}

$$f'(x) = -\sin x - xe^x - e^{+x}$$

$f(0) = 1 - 0 = +ve$; $f(1) = 0.54 - e = -ve$

let $\boxed{x_0 = 0.5}$

Newton Rapson's formula :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad ; (n = 0, 1, 2, 3)$$

Put $n=0$; $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)}$

$$= 0.5 - \frac{0.8776 - 0.8244}{-0.4794 - .8244 - 1.6487}$$

$$= 0.5 + \frac{0.0532}{2.9525} = 0.5 + 0.0180$$

$$= \boxed{0.5180}$$

Put $n=1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)} = 0.5180 - \frac{f(0.5180)}{f'(0.5180)}$$

$$= 0.5180 - \frac{0.8688 - 0.8695}{-0.4951 - 0.8695 - 1.6787}$$

$$= 0.5180 - \frac{-0.0007}{-3.0433} = 0.5180 - 0.0002$$

$$= \boxed{0.5178} \text{ Ans.}$$

Simultaneous Linear Equations -

Elimination method :-

In this method, unknowns are eliminated successively & the system is reduced to an upper triangular system from which the unknowns are found by back substitution.

Consider the equations -

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Step I : To eliminate x from 2nd & 3rd equations.

Assuming $a_1 \neq 0$, we eliminate x from 2nd equation by subtracting (a_2/a_1) times the Ist equation from 2nd equation. IIly x is eliminated from 3rd equation by eliminating (a_3/a_1) times the Ist equation from 3rd equation.

New system -
$$a_1x + b_1y + c_1z = d_1$$
$$b_2'y + c_2'z = d_2'$$
$$b_3'y + c_3'z = d_3'$$

Ist equation is called pivotel equation & a_1 is called Ist Pivot.

Step II : To eliminate y from 3rd equation.

Assuming $b_2' \neq 0$, y is eliminated from 3rd eqn by subtracting (b_3'/b_2') times the 2nd eqn from 3rd equation.

New system -
$$a_1x + b_1y + c_1z = d_1$$
$$b_2'y + c_2'z = d_2'$$
$$c_3''z = d_3''$$

2nd equation is called Pivotal equation & b_2' is the new pivot.

Step III : To eliminate the unknowns.

The values of x, y, z are from the new system by back substitution.

The method will fail if any one of the pivots a_1, b_2' or c_3'' becomes zero.

Q. Apply Gauss - elimination method, to solve the equations
$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = 4$$

Sol.

	Check sum	
$x + 4y - z = -5$	-1	— i)
$x + y - 6z = -12$	-16	— ii)
$3x - y - z = 4$	5	— iii)

Step I : To eliminate x, ii) − i), iii) − 3 i)

Check sum

$-3y - 5z = -7$ — -15 — iv)

$-13y + 2z = 19$ — 8 — v)

Step II : To eliminate y, v) − $13/3$ (iv)

Check sum

$\frac{71}{3} z = \frac{148}{3}$ — 73

$$\boxed{z = \frac{148}{71}}$$

Step III : By back substitution −

$z = 148/71$

$y = \frac{7}{3} - \frac{5}{3}\left(\frac{148}{71}\right) = -\frac{81}{71}$

$x = -5 - 4\left(-\frac{81}{71}\right) + \frac{148}{71} = \frac{117}{71}$ Ans.

Gauss - Jordan method :-

This is a modification of gauss elimination method. In this method, elimination of unknowns is performed not in the equations below but but in the equations above also, ultimately reducing the system to a diagonal matrix form i.e each eqn involving only one unknown.

Q. Apply gauss - Jordan method to solve the equa[tions]

$x + y + z = 9$; $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$

Sol. Step I : To eliminate x from ii) & iii)

$\Rightarrow$ ii) − 2 i) & iii) − 3 i)

$x + y + z = 9$ — iv)

$-5y + 2z = -5$ — v)

$y + 2z = 13$ — vi)

Step II : To eliminate y from iv) & vi).

$\Rightarrow$ iv) + $\frac{1}{5}$(v) & vi) + $\frac{1}{5}$ (v)

$x + \frac{7}{5} z = 8$ — vii)

$-5y + 2z = -5$ — viii)

$12/5\, z = 12$ — ix)

Step III : To eliminate z from vii) & viii)

$\Rightarrow$ vii) − $\frac{7}{12}$ (ix) & viii) − $5/6$ (ix)

$x = 1$

$-5y = -15$

$12/5\, z = 12$

Hence solution is − $x = 1$, $y = 3$, $z = 5$ Ans.

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Gauss - Seidal method :-

This is a modification of Jacobi's method.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Written as -

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y)$$

Substituting $y = y_0$ & $z = z_0$ -

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$

Putting, $x = x_1$ & $z = z_0$ -

$$y_1 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_0)$$

Putting, $x = x_1$, $y = y_1$ -

$$z_1 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

⇒ Jacobi & Gauss-Siedal methods for any choice of the initial approximation if in each eq^n of system, the absolute value of the largest coefficient is almost equal to or in atleast one equation greater than the sum of the absolute values of all the remaining coefficients.

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Numerical Method

Dated: 18/08/2016

Que:- (2015) : 15 Marks

"Gauss - Seidel Method"

$$10w - 2x - y - z = 3$$
$$-2w + 10x - y - z = 15$$
$$-w - x + 10y - 2z = 27$$
$$-w - x - 2y + 10z = -9$$

Find the solution of the system

Sol.

$$w = \frac{1}{10}(2x + y + z + 3)$$
$$x = \frac{1}{10}(15 + 2w + y + z)$$
$$y = \frac{1}{10}(27 + w + x + 2z)$$
$$z = \frac{1}{10}(-9 + w + x + 2y)$$

1st Iteration :

Putting : $w = w_0$, $x = x_0$, $y = y_0$, $z = z_0$ & All are initially Zero

$$w_1 = \frac{1}{10}(3) = 0.3$$

Putting $w = w_1$, $y = y_0$, $z = z_0$

$$x_1 = \frac{1}{10}(15 + 2(0.3) + 0 + 0) = \boxed{1.56}$$

Putting $w_0 = 0.3$, $x = 1.56$, & $z = z_0$

$$y_1 = \frac{1}{10}(27 + 0.3 + 1.56 + 0) = \boxed{2.886}$$

Putting $w = 0.3$, $x = 1.56$, $y = 2.886$,

$$z_1 = \frac{1}{10}(-9 + 0.3 + 1.56 + 2(2.886)) = \boxed{-0.1368}$$

2nd Iteration : Putting $x = x_1$, $y = y_1$, $z = z_1$

$$w_2 = \frac{1}{10}(2(1.56) + 2.886 - 0.1368 + 3) = 0.88692$$

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Putting: $\omega = \omega_2,\ y = y_1,\ z = z_1$

$$x_2 = \frac{1}{10}\left(15 + 2(0.88692) + 2.866 - 0.1368\right) = 1.9503$$

Putting: $\omega = \omega_2,\ x = x_2,\ z = z_1$

$$y_2 = \frac{1}{10}\left(27 + 0.88692 + 1.9503 + 2(-0.1368)\right)$$

$$\Rightarrow y_2 = 2.956362$$

Putting $\omega = \omega_2,\ x = x_2,\ y = y_2$

$$z_2 = \frac{1}{10}\left(-9 + 0.88692 + 1.9503 + 2(2.9564)\right)$$

$$\Rightarrow z_2 = -0.024998$$

3rd Iteration:-

Putting $x = x_2,\ y = y_2,\ z = z_2$

$$\omega_3 = \frac{1}{10}\left(3 + 2(1.9503) + 2.956362 - 0.024998\right)$$

$$\Rightarrow \boxed{\omega_3 = 0.9831}$$

Putting $\omega = \omega_3,\ y = y_2,\ z = z_2$

$$x_3 = \frac{1}{10}\left(15 + 2(0.9831) + 2.95636 - 0.024998\right)$$

$$\Rightarrow \boxed{x_3 = 1.9898}$$

Putting $\omega = \omega_3,\ x = x_3,\ z = z_2$

$$y_3 = \frac{1}{10}\left(27 + 0.931 + 1.9898 - 2(0.024998)\right)$$

$$= 2.9871$$

Putting $\omega = \omega_3,\ x = x_3,\ y = y_3$

$$z_3 = \frac{1}{10}\left(-9 + 0.9831 + 1.9898 + 2(2.9871)\right)$$

$$= -0.00529$$

4th iteration

Putting: $x = x_3,\ y = y_3,\ z = z_3$

$$\omega_4 = \frac{1}{10}\left(3 + 2(1.9898) + 2.9871 - 0.00529\right)$$

$$\Rightarrow \boxed{\omega_4 = 0.9961}$$

Putting: $\omega = \omega_4,\ y = y_3,\ z = z_3$

$$x_4 = \frac{1}{10}\left(15 + 2(0.9961) + 2.9871 + (-0.00529)\right)$$

$$\Rightarrow \boxed{x_4 = 1.9974}$$

Putting: $\omega = \omega_4,\ x = x_4,\ z = z_3$

$$y_4 = \frac{1}{10}\left(27 + 0.9961 + 1.9974 + 2(-0.00529)\right)$$

$$\boxed{y_4 = 2.99829}$$

Putting: $\omega = \omega_4,\ x = x_4,\ y = y_4$

$$z_4 = \frac{1}{10}\left(-9 + 0.9961 + 1.9974 + 2(2.9983)\right)$$

$$z_4 = -0.00099$$

Hence $\boxed{\omega_4 = 1}\ \boxed{x_4 = 2}\ \boxed{y_4 = 3}\ \boxed{z_4 = 0}$

$$\Rightarrow \boxed{x_1 = 1,\ x_2 = 2,\ x_3 = 3,\ x_4 = 0}$$ Ans.

Que:- (2014) 10 Marks Trapezoidal Rule.

$$\int_0^1 \frac{1}{1+x^2}\,dx \quad \text{using 5-Sub interval} \quad 0-1 \Rightarrow \frac{1-0}{5} = \boxed{0.2}$$

Sol.

x	0	0.2	0.4	0.6	0.8	1
y	1	0.962	0.862	0.735	0.610	0.5
	y_0	y_1	y_2	y_3	y_4	y_5

Trapezoida Rule:-

$$y = \frac{h}{2}\left(y_0 + 2(y_1+y_2+y_3+y_4) + y_5\right)$$

$$\Rightarrow \; = \frac{0.2}{2}\left[(1+0.5) + 2(0.962+0.862+0.735+0.610)\right]$$

$$= 0.1\,[1.5 + 6.338] = 0.7838 \text{ Ans.}$$

Normal. $\tan^{-1}x \Big|_0^1 = \tan^{-1}1 - \tan^{-1}0 = \frac{\pi}{4} - 0$

$$= \frac{3.14}{4} = 0.7857$$

$$\boxed{\text{Error} = 0.0019}$$

Simpson $\frac{1}{3}$rd Rule :

$$y = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+y_7+y_{n-1}) + 2(y_2+y_4+y_6+\cdots y_{n-2})\right]$$

Simpson $3/8$th Rule :-

$$y = \frac{3h}{8}\left((y_0+y_n) + 3(y_1+y_2+y_4+y_5+\cdots\cdots+y_{n-1}) + 2(y_3+y_6+y_9+\cdots+y_{n-3})\right)$$

Ans.

The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28·8	40	46·4	51·2	32·0	17·6	8	3·2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule. 15

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Que:- Gaussian Quadrature.

Formula: $\left[I = \int_{-1}^{1} f(u)\,du = \frac{8}{9} f(0) + \frac{5}{9}\left[f(-\sqrt{3/5}) + f(\sqrt{3/5}) \right] \right]$

$\int_{0.2}^{1.5} e^{-x^2}\,dx$ using 3-point Gaussian Quadrature.

Sol.

(1.) Change the limits i.e -1 to 1.

(2.) $x = \frac{1}{2}(b-a)u + \frac{1}{2}(b+a) = \frac{1}{2}(1.5-0.2)u + \frac{(1.5+0.2)}{2}$

$\Rightarrow \boxed{x = 0.65u + 0.85}$

(3) $I = \int_{0.2}^{1.5} e^{-x^2}\,dx = 0.65 \int \underbrace{e^{-(0.65u+0.85)^2}}_{f(u)} \cdot du$

$f(u) = e^{-(0.65u+0.85)^2}$

$f(0) = e^{-(0+.85)^2} = \boxed{0.4855}$

$f(-\sqrt{3/5}) = \boxed{0.8869}$

$f(+\sqrt{3/5}) = \boxed{0.1601}$

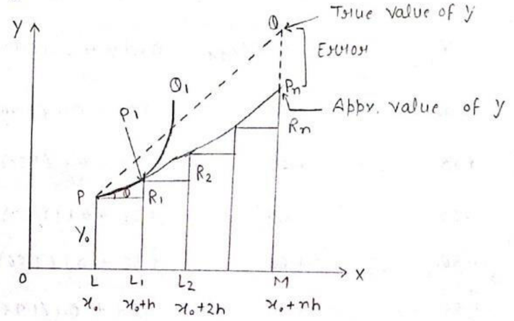
$I = \left[\frac{8}{9}(0.4855) + \frac{5}{9}(0.8869 + 0.1601)\right] \times 0.65$

$= 0.65865$ Ans.

Euler's method :-

Consider the eq^n $\frac{dy}{dx} = f(x, y)$ —— ①

given that $y(x_0) = y_0$. We have to find the Ordinate of any point Q on the curve.



Let us divide LM into n sub-intervals each of width h. if the Ordinate through L_1 meets the tangent at P in $P_1(x_0+h, y_1)$ then -

$$y_1 = y_0 + h\left(\frac{dy}{dx}\right)_P$$

$$y_1 = y_0 + h\, f(x_0, y_0)$$

Let P_1Q_1 be the curve of solution of (1) through P_1 & let its tangent at P_1 meet the Ordinate through L_2 in $P_2(x_0+2h, y_2)$ then -

$$y_2 = y_1 + hf(x_0+h, y_1)$$

$$\vdots$$

llly $\quad y_n = y_{n-1} + hf(x_0 + \overline{n-1}\,h, y_{n-1})$

MAINS 2013 Que.

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Use Euler's method with step size h = 0·15 to compute the approximate value of y(0·6), correct up to five decimal places from the initial value problem

$$y' = x(y + x) - 2$$

$$y(0) = 2$$

x	y	$y' = x(x+y)-2$	$y_{old} + h(y') = y_{new}$
0.00	2	-2	$2 + 0.5(-2) = 1$
0.15	1	-1.8275	$1 - 0.5(0.8275) = 0.08625$
0.30	0.08625	-1.884125	$0.08625 - 0.5(1.884125) = -0.8558125$
0.45	(-0.8558125)	-2.1826156	$-0.8558125 - 0.5(2.1826156) = -1.9471203$
0.60	(-1.9471203)	-2.8082722	$-1.9471203 - 0.5(2.8082722) = -3.3512564$

$$\boxed{y(0.6) = -3.35126}$$ Ans.

THANKS