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MATHEMATICS
for
UPSC CSE MAINS

TOPIC: 2017 Solution (Analytical Geometry)-Part 2

Reduction of 2nd Degree Eq^n to stand. form.

Procedure :-

$$F(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$$

(i.) Identify the coeffi. like $\{a, b, c, f, g, h, u, v, w \ \& \ d\}$

(ii) Make Discriminating Cube :-

$$\begin{vmatrix} a-\lambda & h & g \\ h & b-\lambda & f \\ g & g & c-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \lambda^2(a+b+c) + \lambda(bc+ca+ab-f^2-g^2-h^2) - D = 0$$

where $D = \{abc + 2fgh - af^2 - bg^2 - ch^2\}$

$\Rightarrow$ Three values of λ; $\lambda_1, \lambda_2, \lambda_3$

Now Cases :-

(1.) All threee values are NON-Zero & diff.^{r}

(i) Now Calculate $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0$

$\Rightarrow$ 3 eq^n in 3Variable $\Rightarrow$ (x, y, z) Centre

(ii) $\Rightarrow$ $[d' = u\alpha + v\beta + w\gamma + d]$

iii) Req. Eq^n = $[\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 + d' = 0]$

(2.) If one value of λ is zero, & other two are diff.^{r}

$$\left\{\begin{array}{l} al + hm + gn = 0 \\ hl + bm + fn = 0 \\ gl + fm + cn = 0 \end{array}\right\} \Leftarrow \begin{vmatrix} a-\lambda & h & g \\ h & b-\lambda & f \\ g & f & c-\lambda \end{vmatrix} = 0$$

from any two eq.^{n} : calculate

$$\frac{l}{(-)} = \frac{m}{(-)} = \frac{n}{(-)}$$

ii) $K = ul + vm + wn = \boxed{}$ → Zero ✓ → Repeat step 1.

→ Zero X

$$\{\lambda_1 x^2 + \lambda_2 y^2 + 2KZ = 0\}$$

Repeat step 1.

Que:- 4 (a) Reduce the following eq^n to the stand. form and hence determine the Nature of the conicoid :

15 Marks.

$$x^2+y^2+z^2-yz-zx-xy-3x-6y-9z+21=0$$

Sol.^{n}

Given Conicoid: $x^2+y^2+z^2-yz-zx-xy-3x-6y-9z+21=0$

$a=b=c=+1$, $f=g=h=\frac{-1}{2}$, $u=\frac{-3}{2}$, $v=-2$, $w=\frac{-9}{2}$

$d=21$.

Now

Discriminating cube is given by

$$= \begin{vmatrix} a-\lambda & h & g \\ h & b-\lambda & f \\ g & f & c-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1-\lambda & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & 1-\lambda \end{vmatrix}$$

$$\Rightarrow \lambda^3-\lambda^2(3)+\lambda\left(3-\frac{3}{4}\right)-\left(\cancel{1}+\cancel{\left(\frac{-1}{4}\right)}-\cancel{\frac{3}{4}}\right)=0$$

$$\Rightarrow \lambda^3-3\lambda^2+\frac{9}{4}\lambda=0 \Rightarrow 4\lambda^3-12\lambda^2+9\lambda=0$$

$$\lambda(4\lambda^2-12\lambda^2+9)=0$$

$$\lambda=0,\quad 4\lambda^2-12\lambda^2+9=0$$

$$4\lambda^2-6\lambda-6\lambda+9=0$$

$$2\lambda(2\lambda-3)-3(2\lambda-3)=0$$

$$(2\lambda-3)^2=0$$

$$\Rightarrow \lambda=\frac{3}{2},\frac{3}{2}$$

$$\boxed{\lambda=0,\ \frac{3}{2},\ \frac{3}{2}}$$

Now one value of λ is zero, so the dir. Ratio in ~~the~~ Correspond to it.

$$\begin{vmatrix} a-\lambda & h & g \\ h & b-\lambda & f \\ g & f & c-\lambda \end{vmatrix}=0$$

$$al+hm+gn=0$$
$$hl+bm+fn=0$$
$$gl+fm+cn=0$$

$\Rightarrow 2l-m-n=0$ —— (1)

$\Rightarrow -l+2m-n=0$ —— (2)

$\Rightarrow$

$$\frac{l}{3}=\frac{m}{3}=\frac{n}{3}$$

$$\langle l,m,n\rangle=\langle 1,1,1\rangle$$

Now $K=ul+mv+nw=\frac{-3}{2}-2-\frac{9}{2}$

$=-8$

so Reduced eq^n:

$$\lambda_1 x^2+\lambda_2 y^2+2Kz=0$$

$$\Rightarrow \frac{3}{2}(x^2+y^2)+(-16)z=0$$

$$\Rightarrow \boxed{3(x^2+y^2)-32z=0} \text{ Ans}$$

The general equation of second degree

$$F(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$$

can be reduced to any of the following forms :

1.	$Ax^2 + By^2 + Cz^2 = 1$	:	Ellipsoid
2.	$Ax^2 + By^2 - Cz^2 = 1$	:	Hyperboloid of one sheet
3.	$Ax^2 - By^2 - Cz^2 = 1$	:	Hyperboloid of two sheets
4.	$Ax^2 + By^2 + Cz^2 = 0$	:	Cone
5.	$Ax^2 + By^2 = 2Cz$	:	Elliptic paraboloid
6.	$Ax^2 - By^2 = 2Cz$	:	Hyperbolic paraboloid
7.	$A(x^2 + y^2) + Cz^2 = 1$	:	Ellipsoid of revolution
8.	$A(x^2 - y^2) + Cz^2 = 1$	:	Hyperboloid of revolution
9.	$A(x^2 + y^2) = 2Cz$	:	Paraboloid of revolution
10.	$Ax^2 + By^2 + d = 0$	:	Elliptic cylinder
11.	$Ax^2 - By^2 + d = 0$	:	Hyperbolic cylinder
12.	$Ax^2 - By^2 = 0$	:	Pair of intersecting planes
13.	$Ax^2 + Bx + C = 0$	:	Pair of parallel lines
14.	$y^2 = Ax$	:	Parabolic cylinder

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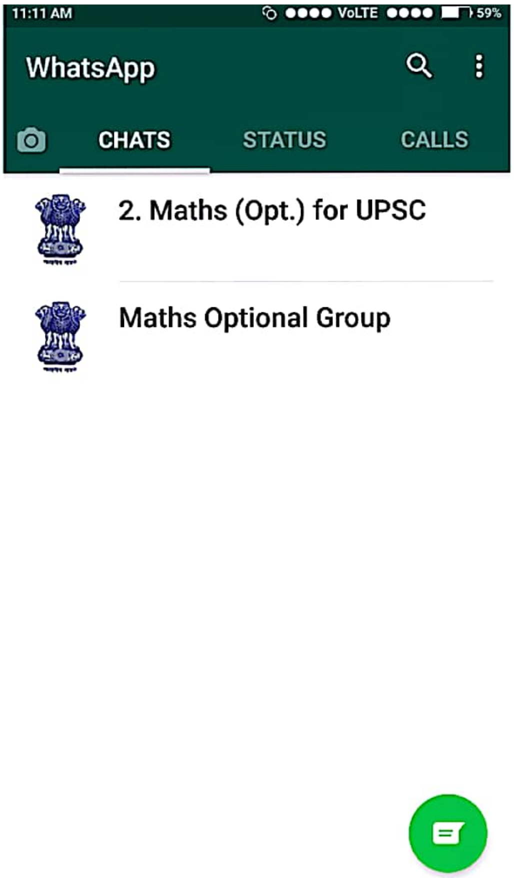
Contact



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- https://chat.whatsapp.com/IJHg2IZtihRGwNiaBc9L2B