DATE:

195/250

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

[INSTITUTE OF MATHEMATICAL SCIENCES]

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-06: IAS(M)/22-JULY.-2018

Maximum Marks: 250

Time: Three Hours

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 34PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name: K. VARUN REDDY

Roll No. 631486

Test Centre: OLD RAJENDRA NAGAR

Medium: ENGLISH

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Varunreddy

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

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			Total Marks	195/250	

DO NOT WRITE ON
THIS SPACE

SECTION – A

1. (a) Union of two subgroups is a subgroup iff one of them is contained in the other. **[10]**

First let us prove ~~ne~~sufficient condition

Given w_1, w_2 be two subgroups

· One of them is contained in another

$\Rightarrow$ let $w_1 \subseteq w_2$ – ①

$\Rightarrow w_1 \cup w_2 = w_2$ (from ①)

$\because w_2$ is a subgroup $\Rightarrow$ union of two subgroups is a subgroup

Necessary conditions

Union of w_1, w_2 is a subgroup – ②

Assume one is not contained in another

$\Rightarrow \exists\, a \in w_1$ such that $a \notin w_2$
$\exists\, b \in w_2$ such that $b \notin w_1$ } ③

but $a, b \in w_1 \cup w_2$ and $w_1 \cup w_2$ is a subgroup

08

$\Rightarrow a+b \in w_1 \cup w_2$ (closure property)

$\Rightarrow a+b \in w_1$ or $a+b \in w_2$ (union property)

$\Rightarrow b \in w_1$ or $a \in w_2$ ($\because a \in w_1$ & $b \in w_2$)

both are false from (3) $\because$ $b \notin W_1$; $a \notin W_2$

$\therefore$ contradiction

Hence the assumption that one is not contained in another is wrong

$\Rightarrow$ One is contained in another

Hence the result

1. (b) Let R be the ring of 3 × 3 matrices over reals. Show that $S = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \middle| x \text{ real} \right\}$ is a subring of R and has unity different from unity of R. **[10]**

Given R - ring of all 3x3 matrices over reals

Unity of R - Identity - $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

now take subset $S = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \middle| x \in R \right\} = \left\{ x \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \middle| x \in R \right\}$

To prove it is subring

Let $X = x \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$; $Y = y \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \in S$

(i) $X-Y \in S$ $\because$ $X-Y = (x-y)\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ and $(x-y) \in R$

(ii) $XY \in S$ $\because$ $XY = (xy)\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} = (xy)\begin{bmatrix}3&3&3\\3&3&3\\3&3&3\end{bmatrix}$

$= (3xy)\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ and $3xy \in R$ — (1)

$\therefore$ S is a subring of R

(b) Unity of S is - from (1) $XY = X$ if $y = 1/3$

i.e. $\boxed{3xy = x}$

($\because$ matrix $\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ cancel out)

08

$\therefore$ Unity of S is $\begin{bmatrix}1/3&1/3&1/3\\1/3&1/3&1/3\\1/3&1/3&1/3\end{bmatrix}$ different from Unity of R

Hence the result

1. (c) Prove that every infinite bounded subset of real numbers has a limit point. **[10]**

Weierstrass - Boltzano Theorem :- Every Infinite bounded subset of real numbers has a limit point

Proof :- Let subset be S

define subset T of S = $\{x \in S \mid x >$ finite number of elements of S$\}$

we shall prove supremum (T) is a limit point

08

By order completeness theory, subset T shall have a supremum since it is bounded

Let sup (T) be M

Take a small neighbourhood of M $(M-\epsilon, M+\epsilon)$

we have, $M-\epsilon \in T$; $M+\epsilon \notin T$
(by definition of supremum)

$\Rightarrow$ $M-\epsilon$ > finite number of elements of S | $\Rightarrow$ $M+\epsilon$ > Infinite number of elements of S

$\Rightarrow$ between $M-\epsilon$, $M+\epsilon$ there are Infinite number of elements of S

$\therefore$ By definition of limit point ; M is a limit point of S

$\therefore$ Infinite bounded subset of R has a limit point

Hence proved

1. (d) Use Canchy's theorem/Cauchy integral formule evaluate

(i) $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$ where $C: |z-i| = 2$ (ii) $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$ where C is the circle $|z| = 1$

[10]

(i) Given $\int_C \frac{z-1}{(z+1)^2(z-2)}$ where $C: |z-i|=2$

take $f(z) = \left(\frac{z-1}{z-2}\right)$ → analytic inside $C: |z-i|=2$
($\because z=2$ is outside C)

$\Rightarrow$ $\int_C \frac{f(z)}{(z+1)^2} = f'(-1)\,2\pi i$ (Cauchy's theorem $\frac{n!}{2\pi i}\oint \frac{f(z)}{(z-z_0)^{n+1}} = f^n(z_0)$)

$f'(z) = \frac{(z-2)-(z-1)}{(z-2)^2} = \frac{-1}{(z-2)^2}$

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$\therefore$ $\int_C \frac{f(z)}{(z+1)^2} = 2\pi i \frac{(-1)}{(z-2)^2}$ at $z=-1$ $= \frac{2\pi i\,(-1)}{9} = \boxed{\frac{-2\pi i}{9}}$

(ii) $\int_c \frac{\sin^6 z \, dz}{(z-\pi/6)^3}$ where $c: |z| = 1$ $\quad z = \pi/6$ lies inside the circle of unit radius

$\therefore$ it is a pole

take $f(z) = \sin^6 z \Rightarrow f'(z) = 6\sin^5 z \cos z$

$\Rightarrow f''(z) = 30\sin^4 z \cos^2 z - 6\sin^6 z$

$\Rightarrow f''(\pi/6) = 30 \times \frac{1}{16} \times \frac{3}{4} - 6 \times \frac{1}{64} = \frac{39}{32}$

now $\int_c \frac{f(z)}{(z-\pi/6)^3} = \frac{2\pi i}{2!} \cdot f''(\pi/6)$ $\quad \left(\because \frac{n!}{2\pi i} \oint_c \frac{f(z)}{(z-z_0)^{n+1}} = f^n(z_0)\right)$

$= \pi i \, f''(\pi/6)$ $\quad$ cauchy's Integral formula

$= \boxed{\frac{39\pi i}{32}}$ $\quad \frac{21\pi i}{16}$

Hence the result

1. (e) Write the dual of the following problem.

Min. $z = x_1 + x_2 + x_3$, subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_2 - x_3 \geq 4$; $x_1, x_3 \geq 0$ and x_2 is unrestricted. **[10]**

The given LPP equation can be rewritten as

$\min z = x_1 + x_2 + x_3$

s.c

$x_1 - 3x_2 + 4x_3 \geq 5$

$-x_1 + 3x_2 - 4x_3 \geq -5$

$-x_1 + 2x_2 \geq -3$

$2x_2 - x_3 \geq 4$

$x_1, x_3 \geq 0$; x_2 - unrestricted

write x_2 as $x_2' - x_2''$ where $x_2', x_2'' \geq 0$

08

$\Rightarrow$

$\min z = x_1 + x_2' - x_2'' + x_3$

s.c

$$\left.\begin{array}{l} x_1 - 3x_2' + 3x_2'' + 4x_3 \geq 5 \\ -x_1 + 3x_2' - 3x_2'' - 4x_3 \geq -5 \\ -x_1 + 2x_2' - 2x_2'' \geq -3 \\ 2x_2' - 2x_2'' - x_3 \geq 4 \end{array}\right\} - (1)$$

$x_1, x_2', x_2'', x_3 \geq 0$

• Dual of (1) is

$\max z = 5y_1 - 5y_2 - 3y_3 + 4y_4$

s-t

$\boxed{y_1 - y_2} - y_3 \le 1$

$-3y_1 + 3y_2 + 2y_3 + 2y_4 \le 1$

$3y_1 - 3y_2 - 2y_3 - 2y_4 \le -1$

$4y_1 - 4y_2 - y_4 \le 1$

s $y_1, y_2, y_3, y_4 \geq 0$

further can be written as

$\max z = 5(y_1') - 3y_3 + 4y_4$

s-t

$y_1' - y_3 \le 1$

$-3y_1' + 2y_3 + 2y_4 \boxed{=1}$

$4y_1' - y_4 \le 1$

$y_1' = y_1 - y_2$ (unrestricted)

$y_3, y_4 \geq 0$

Hence the Dual LPP

2. (a) Let H be a subgroup of a group G. Then $W = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G. [15]

Given H- subgroup of group G

$\omega = \bigcap_{g \in G} gHg^{-1}$ — (1)

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Let $e \in G$, be Identity element

Now, $eHe^{-1} = H \Rightarrow \omega \subseteq H$ ($\because \omega = eHe^{-1} \cap \dots$)

$\boxed{\text{To prove: } \omega \text{ is normal subgroup of } G}$

$\Rightarrow$ elements Let $y \in \omega$ —

ω is normal subgroup if $g\omega g^{-1} \subseteq \omega \quad \forall g \in G$

$\Rightarrow \because y \in W \Rightarrow y \in H \cap g_1 H g_1^{-1} \cap g_2 H g_2^{-1} \cdots$

$(g_1, g_2, \cdots \in R)$

$\Rightarrow$ now take for any $g \in R$

~~gkg^{-1}~~ $gyg^{-1} = K$ (to show $K \in W$)

now, since $y \in H \cap g_1 H g_1^{-1} \cap g_2 H g_2^{-1} \cdots$

$gyg^{-1} \in gHg^{-1} \cap gg_1 H g_1^{-1} g^{-1} \cap gg_2 H g_2^{-1} g^{-1} \cdots$

$\Rightarrow gyg^{-1} \in gHg^{-1} \cap (gg_1) H (gg_1)^{-1} \cap (gg_2) H (gg_2)^{-1} \cdots$

$\because$ all elements in the group are distinct we know that $\forall g \in G$; $gG = G$ and are binary operations $gg_1, gg_2 \cdots$ are distinct

$\therefore$ Based on property $gG = G$, we get

$\Rightarrow gyg^{-1} \in H \cap g_1 H g_1^{-1} \cap g_2 H g_2^{-1} \cdots$

$\in W$

$\therefore gyg^{-1} \in W \; \forall g \in R$

$\boxed{\therefore W \text{ is a normal subgroup}}$

Hence the result

2. (b) If $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and f is continuous at a point of $\mathbb{R}$, prove that f is uniformly continuous on $\mathbb{R}$. [15]

Given $f(x+y) = f(x) + f(y)$ $\forall x,y \in R$ and f is continuous at a point of R

Let's take $x_0 \in R$, where $f(x)$ is continuous

$\Rightarrow \lim_{x \to x_0} f(x) = f(x_0)$

$\Rightarrow |f(x_0+h) - f(x_0)| < \epsilon$ whenever $|h| < \delta$ for any $\epsilon > 0$ (however small)

$\Rightarrow |f(h)| < \epsilon$ whenever $|h| < \delta$ — (1) ($\because f(x_0+h) = f(x_0) + f(h)$)

To prove u.c.

Now take any random x_1, y_1 - we shall prove

$|f(x_1) - f(y_1)| < \epsilon$ whenever $|x_1 - y_1| < \delta$

$\because x_1 = y_1 + (x_1 - y_1)$ (δ, ϵ are from above)

$\Rightarrow |f(x_1) - f(y_1)| =$ $\Rightarrow \delta$ is independent of x_1, y_1

$= |f(y_1 + (x_1 - y_1)) - f(y_1)|$

$= |f(x_1 - y_1)| < \epsilon$ if $|x_1 - y_1| < \delta$ (from (1))

here $h = (x_1 - y_1)$

Therefore δ is only dependent on ϵ

$\boxed{|f(\delta)| < \epsilon}$ → only criteria

$\therefore f(x)$ is uniformly continuous on R

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2. (c) The integral function f(z) satisfies everywhere the inequality $|f(z)| \le A|z|^k$ where A and k are positive constants. Prove that f(z) is a polynomial of degree not exceeding k. **[06]**

Integral function $f(z)$ satisfies everywhere

$|f(z)| \le A|z|^k$ A, K are positive coefficients

$\boxed{\text{~~By Rouche theorem we have~~ } A|z|^k}$ $\Rightarrow |f(z)| \le |Az^k|$

By Rouche theoem, we have

Az^k, $Az^k + f(z)$ have same number of roots in the complex plane

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Assume

If $f(z)$ is polynomial of degree exceeding K

$\Rightarrow f(z)$ has roots more than K

$\Rightarrow Az^k + f(z)$ has roots more than K

But Az^k has only K roots

$\therefore$ contradiction

$\therefore$ assumption is wrong

$f(z)$ cannot be a ~~degre~~ polynomial of degree Exceeding K

Hence the result

2. (d) Prove that

$$\int_0^{2\pi} \frac{\cos^2 3\theta}{1-2p\cos 2\theta+p^2}\,d\theta = \pi\frac{1-p+p^2}{1-p}, 0<p<1.$$

[14]

To find $I = \int_0^{2\pi} \frac{\cos^2 3\theta}{1-2p\cos 2\theta+p^2} = \int_0^{2\pi} \frac{1+\cos 6\theta}{2(1-2p\cos 2\theta+p^2)}$

now take the region $c: \underline{\underline{|z|=1}}$

we have $\underline{\underline{z=e^{i\theta}}}$ $\quad dz = i.e^{i\theta}.d\theta \Rightarrow \frac{dz}{iz} = d\theta$

$z^2 = e^{i(2\theta)}$

$\Rightarrow \cos 2\theta = \frac{z^2+\frac{1}{z^2}}{2}$

$\therefore I =$ real part of $\oint_c \frac{1+e^{i(6\theta)}}{2(1-2p\cos 2\theta+p^2)}\,d\theta$

$=$ real part of $\oint_c \frac{(1+z^6)}{2(1-p(z^2+\frac{1}{z^2})+p^2)}\cdot dz/iz$

$=$ real part of $\oint_c \frac{z(1+z^6)}{2i(-pz^4+(p^2+1)z^2-p)}\,dz$

Take denominator

it is $\underline{\underline{\text{singular}}}$ when $z^2 = \frac{-(p^2+1)\pm\sqrt{(p^2+1)^2-4p^2}}{-2p}$

$\Rightarrow z^2 = \frac{-(p^2+1)\pm(p^2-1)}{-2p}$

$\Rightarrow z^2 = \frac{1}{p}$ (or) p

$\therefore 0<p<1$

$\Rightarrow z = \underline{\underline{\pm\sqrt{p}}}$ lies inside the circle $\underline{\underline{|z|=1}}$

$I =$ real part of $\oint_c \frac{z(1+z^6)}{-2ip(\underline{\underline{z^2-p}})(\underline{\underline{z^2-1/p}})}\,dz$

calculating residue at $z=\sqrt{p}$

$$\lim_{z\to\sqrt{p}} \frac{z(1+z^6)}{-2ip(z^2-p)(z^2-1/p)}(z-\sqrt{p})$$

$$\Rightarrow \lim_{z\to\sqrt{p}} \frac{z(1+z^6)}{-2ip(z+\sqrt{p})(z^2-1/p)}$$

$$= \frac{\sqrt{p}(1+p^3)}{-2ip(2\sqrt{p})(p-1/p)} = \frac{(1+p^3)}{-4i(p^2-1)}$$

residue at $z=-\sqrt{p}$

$$\lim_{z\to-\sqrt{p}} \frac{z(1+z^6)}{-2ip(z^2-p)(z^2-1/p)}(z+\sqrt{p})$$

$$\Rightarrow \lim_{z\to-\sqrt{p}} \frac{z(1+z^6)}{-2ip(z-\sqrt{p})(z^2-1/p)}$$

$$= \frac{(-\sqrt{p})(1+p^3)}{-2ip(-2\sqrt{p})(p-1/p)}$$

$$= \frac{(1+p^3)}{-4i(p^2-1)}$$

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$$\therefore I = \text{real part of } 2\pi i\left[\frac{1+p^3}{-4i(p^2-1)} + \frac{1+p^3}{-4i(p^2-1)}\right] = \frac{\pi[1+p^3]}{-(p^2-1)}$$

$$= \frac{\pi(p+1)(1+p^2-p)}{(1-p)(p+1)} = \frac{\pi(1+p^2-p)}{1-p}$$

$$\boxed{\therefore \int_0^{2\pi} \frac{\cos^2 3\theta}{1-2p\cos 2\theta+p^2}\,d\theta = \frac{\pi(1-p+p^2)}{1-p}}$$

Hence the result

3. (a) (i) If in a ring R, with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$ then show that R is commutative.

(ii) Show that the ring R of real valued continuous functions on [0, 1] has zero divisors. **[9 + 9 = 18]**

3(a)(i) Given R - ring with unity

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$(xy)^2 = x^2y^2 \quad \forall\, x, y \in R$ $\qquad \because 1 \in R \Rightarrow x+1 \in R;\ y+1 \in R$

put $(x+1)$ in place of x, we get

$$\Rightarrow ((x+1)y)^2 = (x+1)^2y^2 \quad \Rightarrow (xy+y)^2 = (x^2+1+2x)y^2$$

$$\Rightarrow (xy)^2 + y^2 + xy^2 + yxy = x^2y^2 + y^2 + 2xy^2$$

$$\Rightarrow \boxed{yxy = xy^2} \quad -(1)$$

put $(y+1)$ in place of y, we get

$\Rightarrow (y+1)\,x(y+1) = x(y+1)^2 = x(y^2+1+2y)$

$\Rightarrow (yx+x)(y+1) = (\cancel{yxy}+xy+yx+\cancel{x}) = \cancel{xy^2}+\cancel{x}+2xy$

(from ①, we have $yxy = xy^2$)

$\Rightarrow \boxed{yx = xy}$

$\therefore$ the Ring R is commutative

(ii) Ring R of real valued continuous functions on $[0,1]$

To show it has zero divisors

Take $f(x) = \begin{cases} 1 & \text{if } 0 \le x < \frac{1}{2} \\ 0 & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$ $\Rightarrow f(x)$ is not a zero function

$g(x) = \begin{cases} 0 & \text{if } 0 \le x < \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$ $\Rightarrow g(x)$ is not a zero function

It is not a conti func

we define product $(fg)(x) = f(x)\,g(x)$

now Based on above definition of f, g

we have

$(fg)(x) = f(x)\,g(x) = 0 \quad \forall\, x \in [0,1]$

But $f(x) \neq 0;\ g(x) \neq 0$ $\Rightarrow$ The ring R has zero divisors

Hence the result

$f : [0,1] \to \mathbb{R}$ s.t

$f(x) = \begin{cases} \frac{1}{2} - x, & 0 \le x \le \frac{1}{2} \\ 0, & \frac{1}{2} \le x \le 1 \end{cases}$

$g : [0,1] \to \mathbb{R}$ s.t

$g(x) = \begin{cases} 0, & 0 \le x \le \frac{1}{2} \\ x - \frac{1}{2}, & \frac{1}{2} \le x \le 1 \end{cases}$

both are non-zero continuous but the product is zero

$\therefore R$ has zero divisors

3. (b) For the series $\sum_{1}^{\infty} f_n(x)$ where

$$f_n(x) = n^2 x e^{-n^2x^2} - (n-1)^2 x e^{-(n-1)^2 x^2},\ x \in [0, 1]$$

show that $\sum_{1}^{\infty} \int_0^1 f_n(x)dx \neq \int_0^1 \left(\sum_{1}^{\infty} f_n(x) \right) dx.$

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Is the series $\sum_{1}^{\infty} f_n(x)$ uniformly convergent on [0, 1]? **[15]**

Given series $S_n = \sum_{1}^{\infty} f_n(x)$ where

$f_n(x) = n^2 x e^{-n^2x^2} - (n-1)^2 x \cdot e^{-(n-1)^2 x^2}$; $x \in [0,1]$

calculating $\sum^{\infty} \int_0^1 f_n(x)\,dx$

$= \sum_{n=1}^{\infty} \left[\int_0^1 n^2 x e^{-n^2x^2} dx - \int_0^1 (n-1)^2 x\, e^{-(n-1)^2 x^2} dx \right]$

$= \sum_{n=1}^{\infty} \left(-\frac{1}{2}\right) \left| e^{-n^2x^2} \right|_0^1 - \left(-\frac{1}{2}\right) \left| e^{-(n-1)^2 x^2} \right|_0^1 = \frac{1}{2} \left[\sum_{n=1}^{\infty} \left(e^{-(n-1)^2} - e^{-n^2} \right) \right]$

P.T.O.

$\Rightarrow \frac{1}{2}\left[1 - \cancel{e^{-1}} + \cancel{e^{-1}} - \cancel{e^{-4}} + \cancel{e^{-4}} - e^{-9} \dots \quad e^{-n^2}\right]$

$\uparrow (0) \; n \to \infty$

$= \boxed{\boxed{1/2} = \sum_{n=1}^{\infty} \int_0^1 f_n(x)}$

calculating $\int_0^1 \sum_{n=1}^{\infty} f_n(x)\, dx$

Take $\boxed{\sum_{n=1}^{\infty} f_n(x) \leq \frac{n^3 x}{e^{n^2x^2}} - \frac{(n-1)^3 x}{e^{(n-1)^2x^2}}}$ ✓ multiplying with n

$\sum_{n=1}^{\infty} f_n(x) = x e^{-x^2} - (0) + 2^2 \cdot \cancel{x} e^{-2^2x^2} - \cancel{x} e^{-x^2} + 3^2 x e^{-3^2x^2} - \cancel{2^2} x e^{-2^2x^2} \dots\dots$

$\uparrow (n=1)$

$= \lim_{n \to \infty} n^2 x e^{-n^2x^2} = \lim_{n \to \infty} \frac{n^2 x}{e^{n^2x^2}}$ ∞/∞ form

L-Hospital rule

$= \lim_{n \to \infty} \frac{2n \cdot x}{e^{n^2x^2} \cdot 2n x^2}$

$\therefore \boxed{\int_0^1 \sum_{n=1}^{\infty} f_n(x) = 0} \quad = 0$

$\therefore \sum_{n=1}^{\infty} \int_0^1 f_n(x)\, dx \neq \int_0^1 \sum_{n=1}^{\infty} f_n(x)\, dx$

(b) series is not uniformly convergent on $[0,1]$ as it is not term by term Integrable.

Also $S_n = n^2 x e^{-n^2x^2}$

3. (c) Using the simplex method solve the LPP problem: Minimize $z = x_1 + x_2$, subject to $2x_1 + x_2 \ge 4$, $x_1 + 7x_2 \ge 7$, and $x_1, x_2 \ge 0$. **[17]**

Given LPP can be rewritten as

Maximize $-z = -x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

s.t

$2x_1 + x_2 - S_1 + 0S_2 + A_1 + 0A_2 = 4$

$x_1 + 7x_2 + 0S_1 - S_2 + 0A_2 + A_2 = 7$

(S_1 - surplus variable, A_1 - artificial variable)

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$x_1, x_2, S_1, S_2, A_1, A_2 \ge 0$

Initial basic solution $(x_1, x_2, S_1, S_2, A_1, A_2) = (0, 0, 0, 0, 4, 7)$

Forming simplex table

C_B	basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
c_j		-1	-1	0	0	$-M$	$-M$		
$-M$	A_1	2	1	-1	0	1	0	4	4
$-M$	A_2	1	7	0	-1	0	1	7	1 →
	$z_j = \sum a_{ij} c_j$	$-3M$	$-8M$	M	M	$-M$	$-M$		
	$C_j = c_j - z_j$	$3M-1$	$8M-1$ ↑	$-M$	$-M$	0	0		

From the table Incoming variable - x_2; outgoing variable A_2

C_B	basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
$-M$	A_1	$13/7$	0	-1	$\frac{1}{7}$	1	–	3	$21/13$ →
-1	x_2	$1/7$	1	0	$-1/7$	0	–	1	7
	z_j	$\frac{-13}{7}M - \frac{1}{7}$	-1	$+M$	$\frac{-M}{7} + \frac{1}{7}$	$-M$			
	$C_j = c_j - z_j$	$\frac{13M}{7} - \frac{6}{7}$ ↑	0	$-M$	$\frac{M}{7} - 1/7$	0			

(Strike off A_2 column)

$-\frac{1}{7} - \frac{1}{7} \cdot \frac{1}{13}$

$-\frac{1}{7}\left[1 + \frac{1}{13}\right]$

$-\frac{1}{7} \cdot \frac{14}{13}$

$= -\frac{2}{13}$

From the table Incoming variable x_1; outgoing - A_1

-1	x_1	1	0	$-7/13$	$1/13$	~~7/13~~	–	$21/13$	
-1	x_2	0	1	$1/13$	$\frac{-2}{13}$	–	–	$\frac{10}{13}$	
	z_j	-1	-1	$6/13$	$1/13$	–	–		
	C_j	0	0	$-6/13$	$-1/13$	–	–		

(Strike off A_1 column)

since all c_j (net evaluation) are ≤ 0

we can say optimality condition has arrived

values of $\left.\begin{array}{l} x_1 = 21/13 \\ x_2 = 10/13 \end{array}\right\}$ are positive ∴ feasible solution

∴ value of $z = x_1 + x_2$

$= \frac{31}{13}$ is the minimum value satisfying the constraints

∴ $\boxed{z = \frac{31}{13}\ ;\ (x_1, x_2)^T = (21/13 \quad 10/13)^T}$

Hence the result

4. (a) If R and S are two rings, then
$ch(R \times S) = 0$ if $ch\,R = 0$ or $ch\,S = 0$
$= k$ where $k = l.c.m.\,(ch\,R, ch\,S)$ **[15]**

4. (b) A function f is defined on [0, 1] by $f(0) = 0$ and

$f(x) = 0$, if x be irrational

$= \frac{1}{q}$, if $x = \frac{p}{q}$ where p, q are positive integers prime to each other.

Show that f is integrable on [0, 1] and $\int_0^1 f = 0$. **[13]**

4. (c) If w = u + iv represents the complex potential for an electric field and $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function u. **[12]**

4. (d) A methods Engineer wants to assign four new methods to three work centres. The assignment of the new methods will increase production and they are given below. If only one method can be assigned to a work centre, determine the optimum assignment :

Increase in production (unit)

		Work centres		
		A	B	C
Method	1	10	7	8
	2	8	9	7
	3	7	12	6
	4	10	10	8

[10]

SECTION – B

5. (a) Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also the particular integral which passes through the line $x = 1, y = 0$. **[10]**

Given P.D.E $(2xy-1)p + (z-2x^2)q = 2(x-yz)$

(comparing with standard equation) $(P)p + (Q)q = (R)$

writing Lagrange Auxiliary Equation

(multipliers $x, y, 0$)

$$\frac{dx}{2xy-1} = \frac{dy}{z-2x^2} = \underbrace{\frac{dz}{2(x-yz)} = \frac{x\,dx + y\,dy}{2x^2y - x + zy - 2x^2y}}_{\text{Take these two}}$$

$\Rightarrow \frac{dz}{2} = \frac{x\,dx + y\,dy}{(-1)}$ $\Rightarrow$ on Integration we get

$\boxed{x^2 + y^2 + z = c_1}$ —(1)

Also

$$\frac{dx}{2xy-1} = \frac{2y\,dy + dz}{2zy - 4x^2y + 2x - 2yz}$$ (multipliers $0, 2y, 1$)

$\Rightarrow dx = \frac{2y\,dy + dz}{(-2)(x)}$ on Integration we get

$\boxed{x^2 - y^2 - z = c_2}$ —(2)

$\therefore$ $\boxed{\text{General Integral}}$ is $\phi(x^2+y^2+z,\ x^2-y^2-z) = 0$ —(3)

$\boxed{\text{Particular Integral}}$ – passing through line $x = 1; y = 0$

$\Rightarrow$ from (1) we get $1 + z = c_1$; (2) gives $1 - z = c_2$

$\Rightarrow$ $c_1 + c_2 = 2$ —(4) (substitute c_1, c_2 from (1), (2))

we get

$2x^2 = 2$ $\Rightarrow$ $\boxed{x^2 = 1}$ pair of lines $x = 1$, $x = -1$

Particular Integral

Hence the result

5. (b) Find complete integral of $(x^2 - y^2)\,pq - xy(p^2 - q^2) = 1$. **[10]**

Given PDE $(x^2-y^2)pq - xy(p^2-q^2) = 1$ — ① let it be $f(x,y,z,p,q) = 0$

charpits auxiliary equation

$$\frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z} = \frac{dz}{-pf_p - qf_q} = \frac{dx}{-f_p} = \frac{dy}{-f_q}$$

$\Rightarrow$

$$\frac{dp}{2xpq - yp^2} = \frac{dq}{-2ypq + xq^2} = \frac{dz}{-(x^2-y^2)pq + 2xyp^2 - (x^2-y^2)pq + 2xyq^2}$$

$$= \frac{dx}{-(x^2-y^2)q + 2xyp} = \frac{dy}{-(x^2-y^2)p - 2xyq}$$

-02-

(p^2+q^2) $-(x^2-y^2)$

$y^2p^2 + x^2q^2$

yd

from ①,②,④,⑤ equalities we get

$ydp + xdq + qdx + pdy$

5. (c) Given that f(0) = 1, f(1) = 3, f(3) = 55, find the unique polynomial of degree 2 or less, which fits the given data. find the bound on the error. **[10]**

Given data

	x	$f(x)$
x_0	0	1
x_1	1	3
x_2	3	55

→ we can fit a polynomial of degree ≤ 2

By Lagrange Interpolation formula we have

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

substituting we get

$$f(x) = \frac{(x-1)(x-3)}{3}(1) + \frac{(x)(x-3)}{(-2)}(3) + \frac{(x)(x-1)}{6}(55)$$

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$$= \frac{1}{6}\left[2(x^2-4x+3) - 9(x^2-3x) + 55(x^2-x) \right]$$

$$= \frac{1}{6}\left[48x^2 - 36x + 6 \right] = \boxed{8x^2 - 6x + 1}$$

$\boxed{f(x) = 8x^2 - 6x + 1}$ – unique polynomial which fits the data

Bound of Error

Given by formula $E_n(x) = \frac{f^{n+1}(x)}{(n+1)!} \times (x-x_0)(x-x_1)\cdots(x-x_n)$

In this case $f^3(x) = 0$ ($\because$ f(x) is 2nd degree polynomial)

$\therefore$ Bound of error = 0

Hence the result

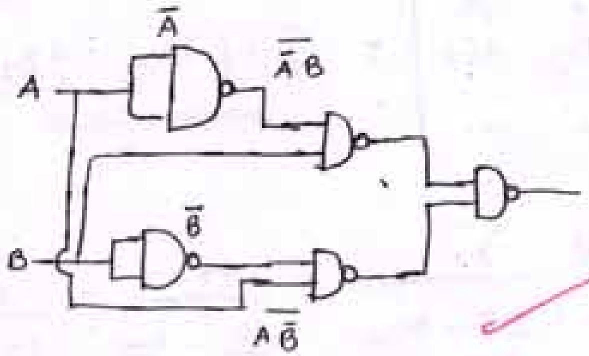
5. (d) (i) Implement $Y = \bar{A}B + A\bar{B}$ using NAND gates only
(ii) Find the hexadecimal equivalent of the decimal number $(587632)_{10}$. **[10]**

(i) $Y = \bar{A}B + A\bar{B} = \overline{\overline{\bar{A}B} \cdot \overline{A\bar{B}}}$ $\left(\because \overline{A+B} = \bar{A}\cdot\bar{B}\right)$

De-Morgan theorem

$= ((\underbrace{A \text{ nand } A}_{(\bar{A})}) \text{ nand } B) \text{ nand } (A \text{ nand } (\underbrace{B \text{ nand } B}_{\bar{B}}))$

logic circuit



$\overline{\overline{\bar{A}B} \cdot \overline{A\bar{B}}} = \bar{A}B + A\bar{B} = Y$

08

(ii) Hexadecimal equivalent of $(587632)_{10}$

Division algorithm

16	587632	
16	36727	– 0
16	2295	– 7
16	143	– 7
16	8	– 15 (F
	8	

$\therefore$ Hexadecimal representation is

$(8F770)_{16}$

$(\because F = 15)$

Hence the result

5. (c) Prove that the necessary and sufficient condition that vortex lines may be at right angles to the streamlines are $\mu, v, w = \mu\left(\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}\right)$, where μ and ϕ are functions of x, y, z, t.

[10]

Let the velocity vector $\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$

vortex vector or vorticity is given by $\text{curl}\,\vec{q}$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix} = \sum i\left(\partial w/\partial y - \partial v/\partial z\right)$$

vortex lines are given by $\dfrac{dx}{\frac{\partial w}{\partial y} - \partial v/\partial z} = \dfrac{dy}{\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}} = \dfrac{dz}{\frac{\partial v}{\partial x} - \partial u/\partial y}$ — ①

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stream lines are given by $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$ — ②

Both ①, ② are perpendicular if

$$\sum u\left(\partial w/\partial y - \partial v/\partial z\right) = 0 \quad -③$$

for equation ③ to be true

$u\,dx + v\,dy + w\,dz$ must be Exact differential

$\Rightarrow u\,dx + v\,dy + w\,dz = \mu\,d\phi = \mu\left(\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz\right)$

$\Rightarrow \boxed{(u, v, w) = \mu\left(\partial\phi/\partial x, \partial\phi/\partial y, \partial\phi/\partial z\right)}$

Hence the result

6. (a) Solve $(D^2 - DD' - 2D'^2)\, z = (2x^2 + xy - y^2) \sin xy - \cos xy$. **[10]**

6. (b) Find a partial differentail equation by eliminating a, b, c from $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. **[07]**

6. (c) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method

$$x_{k+1} = -(ax_k + b)/x_k$$

is convergent near $x = \alpha$ if $|\alpha| > |\beta|$ and that

$$x_{k+1} = -b/(x_k + a)$$

is convergent near $x = \alpha$ if $|\alpha| < |\beta|$.

Show also that the iteration method

$$x_{k+1} = -(x_k^2 + b)/a$$

is convergent near $x = \alpha$ is $2|\alpha| < |\alpha + \beta|$. **[15]**

6. (d) Two equal rods AB and BC, each of length l smoothly joined at B are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$, where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$. **[18]**

7. (a) Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution. **[15]**

7. (b) Find the inverse of **[10]**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

by Gauss-Jordan method.

7. (c) The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite simpson's $\frac{1}{3}$ rule. **[10]**

7. (d) A sphere of radius a and mass M rolls down a rough plane inclined at an angle α to the horizontal.

If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[15]**

8. (a) The ends A and B of a rod 20 cm long have the temperature at 30° and 80° until steady state prevails. The temperatures of the ends are changed to 40° and 60° respectively. Find the temperature distribution in the rod at time t. **[18]**

20cm

Initial temperature of rod $30°, 80°$ at steady state

$u(x,t)$ be Temperature function

$\Rightarrow u(x,0) = 30 + \frac{50}{20} \cdot x = \boxed{30 + \frac{5}{2}x}$ — ①

$u(x,t) = X(x)T(t)$ by separation of variables

NOW $u(0,t) = 40°$, $u(20,t) = 60°$ } ends temperature change — ②

Heat equation (1D) is

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{K} \cdot \frac{\partial u}{\partial t}$ — ③ putting $u(x,t) = X(x)\,T(t)$ we get

$\boxed{\frac{X''}{X} = \frac{1}{K} \cdot \frac{T'}{T}}$ — ③

• take $\boxed{v(x,t) = u(x,t) - 40 - x}$ — ④

$\Rightarrow u(x,t) = v(x,t) + x + 40$

weget $v(0,t) = 0$; $v(20,t) = 0$ — ⑤

$v(x,0) = 30 + \frac{5}{2}x - 40 - x = \boxed{\frac{3}{2}x - 10}$ — ⑥

} → Initial and boundary conditions

Now let $v(x,t) = X(x)\,T(t)$ — ⑦

putting in ③, we get $\boxed{\frac{X''}{X} = \frac{1}{K} \cdot \frac{T'}{T} = \lambda}$ — ⑧

-1b

from boundary conditions ⑤ we have

$X(0) = 0$; $X(20) = 0$ $(\because T(t) \neq 0)$ — ⑨

for ⑧, we have 3 cases λ = 0, m^2 (+ve), $-m^2$ (−ve)

case(i) $\lambda = 0 \Rightarrow X'' = 0$

$\Rightarrow X = ax + b \Rightarrow$ on substituting ⑨ we get $a = b = 0$

$\Rightarrow u(x,t) = 0$ (not possible)

case(ii) $\lambda = m^2 \Rightarrow X'' = m^2 X$; $T' = m^2 K T$

$\Rightarrow X = c_1 e^{-mx} + c_2 e^{mx}$; $T = c_1 e^{m^2 K t}$

T is Exponential power of e ; and positive $\Rightarrow$ Temperature Increases Exponentially with time (false for physical nature of problem)

Case iii) $\lambda = -m^2$

$\Rightarrow \quad X'' + m^2 X = 0 \quad ; \quad T' = -m^2 K T$

on Integration

$X = E_1 \cos(mx) + F_1 \sin(mx) \qquad T = G_1 e^{-m^2 Kt}$

$\because X(0) = 0 \Rightarrow E_1 = 0$

$X(20) = 0 \Rightarrow \sin(20m) = 0 \Rightarrow m = \frac{n\pi}{20} \quad (n = 1, 2, 3 \dots)$

$\Rightarrow \quad v_n(x,t) = F_1 \sin\left(\frac{n\pi}{20} x\right) e^{-(n\pi/20)^2 K t}$

General solution

$$v(x,t) = \sum_{n=1}^{\infty} F_1 \sin\left(\frac{n\pi}{20} x\right) e^{-(n\pi/20)^2 K t}$$

Initial value $\quad v(x,0) = \frac{3x}{2} - 10 = \sum_{n=1}^{\infty} F_1 \sin\left(\frac{n\pi x}{20}\right)$

By fourier transform

$$F_1 = \frac{2}{20} \int_0^{20} \left(\frac{3x}{2} - 10\right) \sin\left(\frac{n\pi}{20} x\right) = \frac{1}{10}\left(\left(\frac{3x}{2} - 10\right)\left(\frac{-20}{n\pi}\right) \cos\left(\frac{n\pi x}{20}\right)\Big|_0^{20} - \left(\frac{3}{2}\right)\left(\frac{-20}{n\pi}\right)\left(\frac{20}{n\pi}\right) \sin\left(\frac{n\pi x}{20}\right)\Big|_0^{20}\right)$$

$\uparrow$ (zero)

$$= \frac{1}{10}\left((10)\left(\frac{-20}{n\pi}\right)(-1)^n + (10)\left(\frac{-20}{n\pi}\right)(1)\right)$$

$$= -\left(\frac{20}{n\pi}\right)\left(1 + (-1)^n\right) \quad \text{value} \rightarrow \frac{-40}{n\pi} \text{ if } n \text{ is Even}$$

 0 if n is odd.

$$\Rightarrow \quad v(x,t) = \sum_{m=1}^{\infty} \frac{-40}{(2m)\pi} \sin\left(\frac{2m\pi}{20} x\right) e^{-\left(\frac{m\pi \cdot 2}{20}\right)^2 \cdot Kt}$$

$$\Rightarrow \boxed{u(x,t) = \frac{x+40}{} + \sum_{m=1}^{\infty} \frac{-20}{m\pi} \sin\left(\frac{m\pi}{10} x\right) e^{-\left(\frac{m\pi}{10}\right)^2 K t}}$$

Hence the result

8. (b) Solve the initial value problem

$u' = -2tu^2,\ u(0) = 1$

whith h = 0.2 on the interval [0, 0.4]. Use the fourth order classical Runge-Kutta method. compare with the exact solution. **[15]**

given $u'(t) = -2tu^2 = \boxed{f(u,t)}$; $u(0) = 1$ ($f(t,u)$)

$\frac{dy}{dt} = -2tu^2$

$\Rightarrow$ on Integration

$\frac{1}{u} = t^2 + 1$

$\Rightarrow \boxed{u = \frac{1}{t^2+1}}$

To calculate $u(0.4)$ at $h = 0.2$ (steps)

Now first $u(0.2) = u(0) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = h\,f(0, u(0)) = 0$

$k_2 = h\,f(0+0.1,\ u(0)+0) = (0.2)\,f(0.1, 1)$

$= (0.2)(-2\cdot(0.1)\cdot 1)$

$= -0.04$

$k_3 = h\,f(0.1,\ u(0) + (-0.02)) = (0.2)\,f(0.1, 0.98)$

$= -0.038416$

$k_4 = h\,f(0.2,\ u(0) - 0.01928) = 0.2 \times f(0.2, 0.9807)$

$= -0.076941$

$\therefore u(0.2) = 1 + \frac{1}{6}(0 - 0.08 - 0.07682 - 0.076941)$

$= 0.96104$

$\boxed{\text{Now } u(0.4)} = u(0.2) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = h\,f(0.2, 0.96104) = -0.07388$

$k_2 = h\,f(0.3, 0.92409) = -0.1024$

$k_3 = h\,f(0.3, 0.9098) = -0.09932$

$k_4 = h\,f(0.4, 0.91137) = -0.1329$

$u(0.4) = 0.96104 + \frac{1}{6}(-0.07388 - 2\times 0.1024 - 2\times 0.09932 - 0.1328)$

$\boxed{u(0.4) = 0.85935}$ $\rightarrow$ Hence approximation

Exact solution

$\because u = \frac{1}{t^2+1} \rightarrow u(0.4) = \frac{1}{1+(0.4)^2} = 0.86206$

difference $= 0.00271$ (Error)

Hence the result

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8. (c) Prove that liquid motion is possible when velocity at (x, y, z) is given by

$u = \frac{3x^2 - r^2}{r^5}, v = \frac{3xy}{r^5}, w = \frac{3xz}{r^5}$, where $r^2 = x^2 + y^2 + z^2$ and the stream lines are the intersection of the surfaces, $(x^2 + y^2 + z^2)^3 = c(y^2 + z^2)^2$, by the planes passing through Ox. Is this irrotational?

[17]

Given velocity $u = \frac{3x^2 - r^2}{r^5}$; $v = \frac{3xy}{r^5}$; $w = \frac{3xz}{r^5}$

liquid motion is possible if $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ (Incompressible fluid)

$\frac{\partial u}{\partial x} = \frac{6x}{r^5} - \frac{15x^3}{r^7} + \frac{3 \cdot x}{r^5}$ | $\frac{\partial v}{\partial y} = \frac{3x}{r^5} - \frac{15xy^2}{r^7}$

$\frac{\partial w}{\partial z} = \frac{3x}{r^5} - \frac{15xz^2}{r^7}$

$\sum \frac{\partial u}{\partial x} = \frac{15x}{r^5} - \frac{15x(x^2+y^2+z^2)}{r^7} = 0$

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∴ Possible fluid motion

$\boxed{\text{Stream lines}}$ are given by

$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \Rightarrow \frac{dx}{3x^2 - r^2} = \frac{dy}{3xy} = \frac{dz}{3xz}$

from last two equations we get

$\frac{dy}{y} = \frac{dz}{z} \Rightarrow$ on Integration $y = c_1 z$ —① $\boxed{\text{(planes passing through ox)}}$

And taking multipliers x, y, z ; y, z we get

$$\frac{x dx + y dy + z dz}{2(x^2+y^2+z^2)x} = \frac{y dy + z dz}{3x(y^2+z^2)}$$

on Integration we get

$$(x^2+y^2+z^2)^3 = c_2(y^2+z^2)^2 \quad \text{—②}$$

Together ①, ② gives Stream lines ; note ① passes through ox

$\boxed{\text{Irrotational}}$ if ~~curl~~ curl $\vec{q} = 0$

$$\Rightarrow \text{curl}\,\vec{q} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{3x^2-r^2}{r^5} & \frac{3xy}{r^5} & \frac{3xz}{r^5} \end{vmatrix}$$

$$= i\left(\frac{-15xyz}{r^7} + \frac{15xyz}{r^5}\right) + j\left(\frac{-15x^2z}{r^7} + \frac{3z}{r^5} - \frac{3z}{r^5} - \frac{15x^2z}{r^7}\right)$$

$$+ k\left(\frac{3y}{r^5} - \frac{15x^2y}{r^7} - \frac{15x^2y}{r^7} + \frac{3y}{r^5}\right)$$

$$= \underline{\underline{0}}$$

$\therefore$ The fluid motion is irrotational

Hence the result

END OF THE EXAMINATION

ROUGH SPACE