



TARUN KAUSHIK



MATHEMATICS FOR UPSC CSE

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SYLLABUS

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PAPER-I

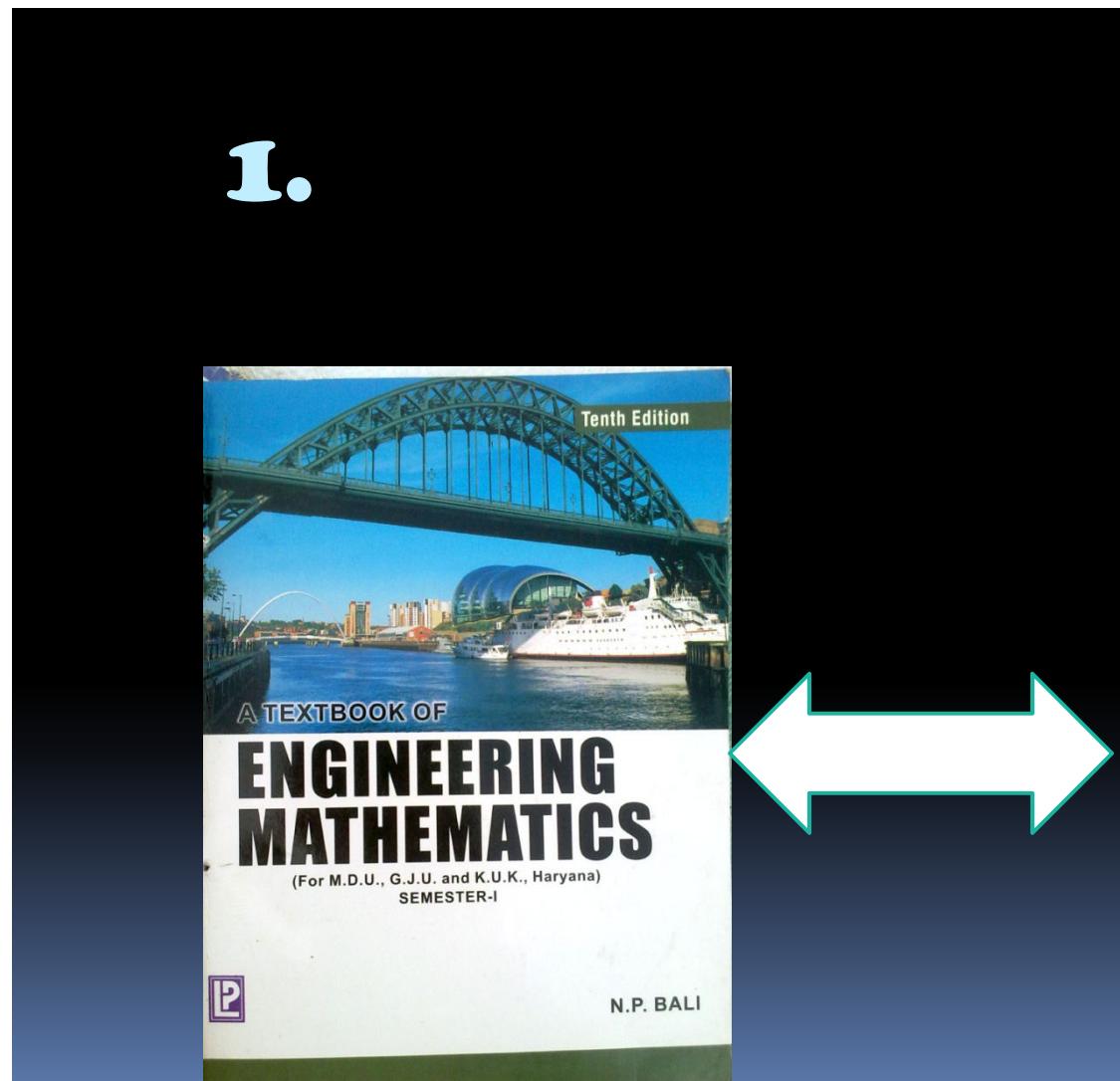
250
Marks

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PAPER-II

1. **LINEAR ALGEBRA**
2. Calculus
3. Ordinary Differential Equation
4. Analytical Geometry
5. Vector Analysis
6. Dynamics & Statics

1. Abstract Algebra
2. Real Analysis
3. **Complex Analysis**
4. **Linear Programming**
5. Partial Differential Equations
6. Numerical Analysis
7. Mechanics & Fluid Dynamics



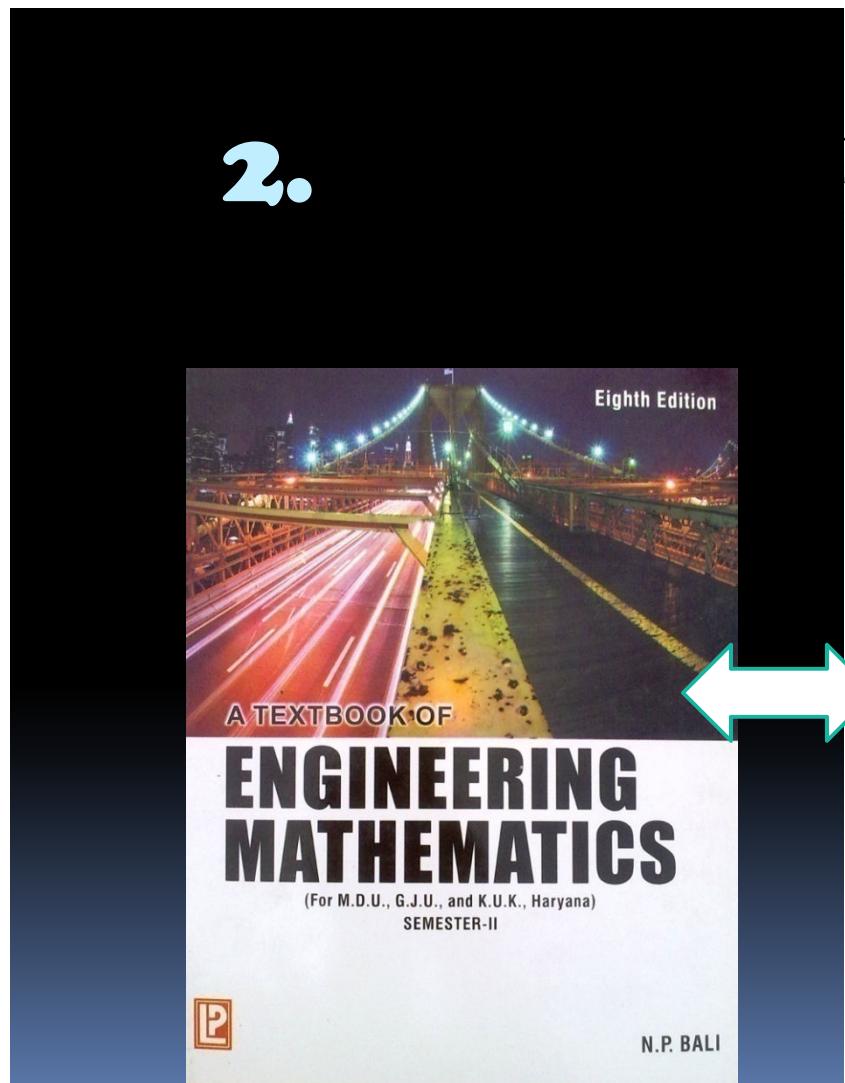
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*Chapters 2, 3 and 4 are not included in the syllabus of KU, Kurukshetra.

**Chapter 13 is not included in the syllabus of MDU, Rohtak.



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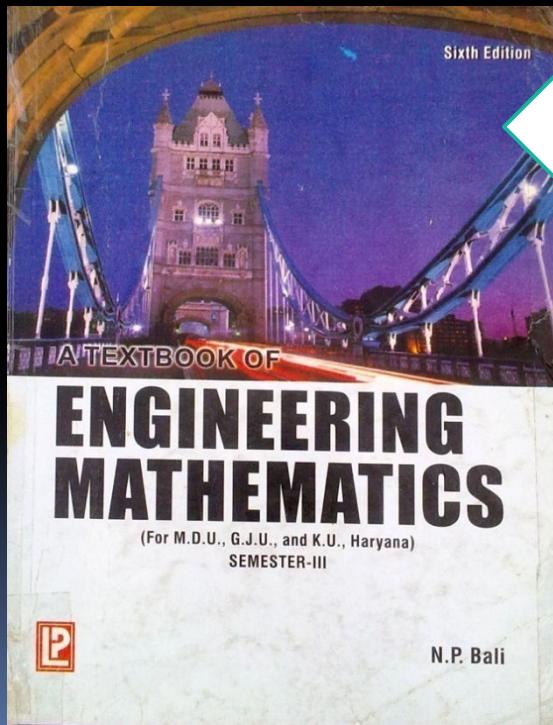


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3.



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Linear Algebra Syllabus

- Vector spaces over \mathbb{R} and \mathbb{C} , linear dependence and independence, subspaces, bases, dimension; Linear transformations, rank and nullity,.
- Algebra of Matrices; Row and column reduction, Echelon form, congruence's and similarity; Rank of a matrix; Inverse of a matrix;
- Solution of system of linear equations; Eigen values and eigenvectors, characteristic polynomial, Cayley- Hamilton theorem,
- Symmetric, skew symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices and their Eigen values.



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Matrix

Linear-Algebra

Dated: 10/03/2016

Linearly Independent :-

Ques: (2015-Mains) The vector : (10 Marks)

$v_1 = (1, 1, 2, 4)$; $v_2 = (2, -1, -5, 2)$, $v_3 = (1, -1, 4, 0)$ and $v_4 = (2, 1, 1, 6)$ are linearly independent.

Sol.⁴

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & -5 & 2 \\ 1 & -1 & 4 & 0 \\ 2 & 1 & 1 & 6 \end{bmatrix} \xrightarrow{\substack{R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \leftrightarrow R_3 - R_1 \\ R_4 \leftrightarrow R_4 - 2R_1}} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -7 & -2 \\ 0 & -2 & -6 & -4 \\ 0 & -1 & -3 & -2 \end{bmatrix} \xrightarrow{\substack{R_2 \leftrightarrow R_2 - 3 \\ R_3 \leftrightarrow R_3 - 2 \\ R_4 \leftrightarrow R_4 - 2R_1}} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{bmatrix} \xrightarrow{\substack{R_2 \leftrightarrow R_2 - R_3 \\ R_4 \leftrightarrow R_4 + R_3}} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ Hence this shows that the vector v_1 and v_2 are linearly independent vector & v_3 & v_4 are linearly dependent.

(Mains 2015)

Ques:- Reduce the following matrix to the Row Echelon form and hence find its Rank.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

Sol. $R_2 \leftrightarrow R_2 - 2R_1$, $R_3 \leftrightarrow R_3 - R_1$, $R_4 \leftrightarrow R_4 - 8R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 3 & 2 & 3 \\ 0 & -15 & -10 & -15 \end{bmatrix} \text{ Now } R_4 \leftrightarrow \frac{R_4}{-5}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_3 - R_2 \\ R_4 \leftrightarrow R_4 - R_2$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \boxed{\text{Rank} = 2}$$

↑ Reduced Echelon matrix.

Ques: (Mains 2015) (12 Marks)

$$\text{If } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \text{ find } A^{30}$$

Sol?

Using Cayley Hamilton theorem.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)(\lambda^2 - 1) = 0 \\ \lambda^3 - 2\lambda^2 - \lambda^3 - 1 + 1 = 0 \\ A - A - A + I = 0$$

$$A^3 = A^3 + A - I \quad \text{--- (1)}$$

$$A^4 = A^3 + A^2 - A = A^2 + A - I + A^2 - A = 2A^2 - I$$

$$A^8 = 4A^4 + I - 4A^2 = 4(2A^2 - I) + I - 4A^2 = 4A^2 - 3I$$

$$A^{16} = 16A^4 + 9I - 24A^2 = 32A^2 - 16I + 9I - 24A^2 = 8A^2 - 17I$$

$$A^{32} = 64A^4 + 49I - 112I = 128A^2 - 64I + 49I - 112I$$

$$\begin{array}{lcl} A^4 = 2A^2 - I & ; & A^{64} = 38A^2 - 31A \\ A^8 = 4A^2 - 3I & ; & A^{16} = 64A^2 - 63A \\ A^{16} = 8A^2 - 7I & ; & A^{256} = 128A^2 - 129I \\ A^{32} = 16A^2 - 15I & ; & A^{512} = 256A^2 - 255I \end{array} \Rightarrow \boxed{A^{30} = 16I - 15A^2}$$



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$$A^3 = A + A - I \Rightarrow A = I + A' - A^2$$

$$\Rightarrow A^2 = I + A' - A$$

$$\rightarrow A^{30} = 16I - 15A^2$$

$$= 16I - 15(I + A' - A) = 16I - 15I + 15(A - A')$$

$$A^{30} = I + 15(A - A')$$

Shortcut to calculate inverse of a matrix:-

$$\begin{array}{|c|ccccc|} \hline & 1 & 0 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ \hline \end{array}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A| = (-1)$$

$$A' = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A^{30} = 15 \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \right) + I$$

$$= 15 \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 15 & 0 \\ 15 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 15 & 0 \\ 15 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Ans}$$

Ques (Main 2015) (12 Marks)

$\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}$ \leftarrow vector

i) Basis

ii) dimension.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_3 - R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\xrightarrow{R_3 \leftrightarrow R_3 - 2R_2}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 \leftrightarrow R_4 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Ans}}$$

Basis = $\{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 0, 1)\}$

Dim = 3. Ans

Ques (Main 2014) (12 Marks)

System of Linear Equation:-

$$\begin{aligned} x + y + z &= 6 & \text{i)} \quad \text{No solution} \\ x + 2y + 3z &= 10 & \text{ii)} \quad \text{unique solution} \\ x + 2y + 4z &= 11 & \text{iii)} \quad \text{Infinite no. of soln.} \end{aligned}$$

$$\stackrel{\text{Soln}}{=} A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}; B = \begin{bmatrix} 6 \\ 10 \\ 11 \end{bmatrix}; \boxed{P(A); P(A:B)}$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & 4 & : & 11 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2 - R_1} \xrightarrow{R_3 \leftrightarrow R_3 - R_1}$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & 3 & : & 5 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_3 - R_2}$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 1 & : & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_1 - R_2}$$



Case: I (No solution)
 $P(A) \neq P(A:B)$

for this:

$$1=3, \mu \neq 10$$

ii) unique solution: \oplus

$$\lambda \neq 3, \mu \neq 10$$

(iii) Many solution :-

$$t=3 \text{ and } u=10$$

Que: (2014) (8-Marks) "Eigen Values & Eigen Vectors"

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Sat.

$$|A - \lambda I| = 0 \quad ; \quad \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -1 \end{vmatrix}$$

$$\textcircled{1} \quad P(A) = P(A:B) = \frac{\text{No. of favorable outcomes}}{\text{Total number of outcomes}}$$

$$\textcircled{2} \quad \rho(A) \neq \rho(A:B) \\ \Rightarrow \text{No sol.}$$

$$\textcircled{3} \quad A(A) = P(A:B) < \text{No. of variable}$$

\Rightarrow Many solution

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad |A - \lambda I| = 0$$

$$\begin{pmatrix} -2 & 1 & 2 \\ 2 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix} \xrightarrow{\text{the } 1^{\text{st}} \text{ column out}} \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ -2 & 1 & 2 \end{pmatrix}$$

$$(-2 - \lambda) \left[(1 - \lambda)(\lambda + 1) + 12 \right] - 2 \left[-2\lambda - 6 \right] - 3 \left[-4 + 1 - \lambda \right]$$

$$+ (2 + \lambda) (1 - \lambda^2 + 12)$$

$$= -\lambda^3 - \lambda^2 + 19\lambda + 45 = 0$$



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$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0 \quad (\lambda = -3)$$

$$(\lambda + 3)(\lambda^2 - 2\lambda - 15) = 0 \quad \lambda = -3, -3, 5$$

$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 + 5x_3 = 0 ; \quad x_2 + 2x_3 = 0$$

$$x_2 = -2x_3$$

$$x_1 + 2(-2x_3) + 5x_3 = 0$$

$$x_1 - 4x_3 + 5x_3 = 0$$

$$x_1 = x_3$$

$$\frac{x_1}{1} - \frac{x_2}{2} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

JAN-July

Laxmikant

* Orthogonal Matrix :

$$\text{if } A \cdot A^T = I \Rightarrow$$

* Hermitian Matrix:

$$\Downarrow$$

$$a_{ij} = \bar{a}_{ji}$$

Complex No. Conjugate

Skew-Hermitian Matrix:-

$$a_{ij} = -\bar{a}_{ji}$$

Complex Number
Conjugate

Que:- (2015) Find Eigen values & eigen vectors of Matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} = A$$

$$\text{Solution} \quad [A - \lambda I] = 0 \Rightarrow \begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix} = [A - \lambda I]$$

$$\begin{aligned}
 & |A - \lambda I| = 0 \\
 \Rightarrow & (-\lambda) [((5-\lambda)(1-\lambda) - 1)] - 1 [(-\lambda) - 3] + 3 (-15 + 3\lambda) \\
 & ((-\lambda)(5 - 5\lambda - \lambda + \lambda^2 - 1) - (-2 - \lambda)) + 3(-14 + 3\lambda) \\
 & (-\lambda)(4 - 6\lambda + \lambda^2) + 2 + \lambda - 42 + 9\lambda \\
 = & 4 - 6\cancel{\lambda} + \cancel{\lambda^2} - 4\cancel{\lambda} + 6\cancel{\lambda^2} - \cancel{\lambda^3} + 2 + \cancel{\lambda} - 42 + 9\cancel{\lambda} \\
 = & -\lambda^3 + 7\lambda^2 - 36 = 0 \quad \Rightarrow \quad \lambda^3 - 7\lambda^2 + 36 = 0
 \end{aligned}$$

by trial method $\lambda = -2$ is a factor.

$$\begin{array}{r}
 (\lambda+2) \overline{) \lambda^3 - 7\lambda^2 + 36} \\
 \underline{-\lambda^3 - 2\lambda^2} \\
 \hline
 -9\lambda^2 + 36 \\
 \underline{-9\lambda^2 + 18\lambda} \\
 \hline
 18\lambda + 36 \\
 \underline{18\lambda + 36} \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{l}
 \lambda^2 - 9\lambda + 18 \\
 \quad \quad \quad \boxed{\lambda^2 - 9\lambda + 18} \\
 (\lambda-6)(\lambda-3) = 0
 \end{array}$$

$$\Rightarrow \text{Eigen value} = \lambda = -2, +3, +6$$

Eigen vectors:-

$$\therefore \text{for } \lambda = -2 \Rightarrow (\bar{A} - \lambda I) = \begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$R_3 \leftrightarrow R_3 - R_4 \Rightarrow \begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\curvearrowleft R_2 \longleftrightarrow R_2 - 3R_1$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 0 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow 3x_4 + x_2 + 3x_3 = 0 \quad \& \quad x_2 = 0$$

$$3x_1 + 3x_3 = 0 \Rightarrow 3x_4 = -3x_3$$

$$x_4 = -x_3$$

vector $(1, 0, -1)$

iii) for $\lambda = 6$

ii) for $\lambda = 3$

$$[A - \lambda I] = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\sim_{R_1} \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 24 \\ -2 & 1 & 3 & x_1 \\ 3 & 1 & -2 & x_3 \end{array} \right] = 0 \quad \left[\begin{array}{ccc|c} -5 & 1 & 3 & x_2 \\ 3 & 1 & -5 & x_3 \end{array} \right] = 0$$

$\hookrightarrow R_2 \leftrightarrow R_2 + 5R_1$ & $R_3 \leftrightarrow$

$$\begin{array}{l} R_2 \leftrightarrow R_2 + 2R_1, R_3 \leftrightarrow R_3 - 3R_1 \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & x_1 \\ 0 & 5 & 5 & x_2 \\ 0 & -5 & -5 & x_3 \end{array} \right] = 0 \end{array} \quad \left(\begin{array}{ccc|c} 1 & -1 & 1 & x_4 \\ 0 & -4 & 8 & x_2 \\ 0 & 4 & -8 & x_3 \end{array} \right) = 0$$

$$r \quad R_3 \longleftrightarrow R_3 + R_2$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \quad \left| \quad \begin{pmatrix} 0 & -4 & 8 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = 0 \right.$$

$$x_1 + x_2 + x_3 = 0 \quad \boxed{x_4 = x_2} \quad \boxed{x_2 = 2x_3}$$

$$\begin{array}{c} x_2 + x_3 = 0 \\ \left[\begin{array}{cc} 1 & -1 \\ 0 & 1 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \end{array} \quad \text{1, 2, 1 Ans}$$

$$x_2 = -x_3 \quad (1, -1, 1) \quad \text{vector}$$

1 - 3 | New |