National Institute of Technology Mizoram

Mid- Semester Examination, Odd Semester (2022-23)

Discrete Mathematics (CSL – 1302)

Semester - 5th    Full Marks - 30    Duration - 1:30 hours

---

**Answer all 3(Three) Questions. All Questions carry same marks**

**(3 * 10 = 30 Marks)**

1(A) Answer yes or no with one line of justification

(i) Is $1 \in \{1\}$?

(ii) Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?

(iii) Is $\{1\} \subseteq \{1, 2\}$?

(iv) Is $1 \in \{\{1\}, 2\}$?

(v) Is $\{1, 2\} \subseteq \{1, \{2\}, \{3\}\}$?

(B) Find the sets A and B if $A-B = \{1, 5, 7, 8\}$, $B-A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

(5 + 5)

OR

Let A, B, C, and D be sets.

Define:

$UX = (A \cup B) \times (C \cup D)$

$XU = (A \times C) \cup (B \times D)$

Here is a proof that claims XU = UX.

$(x, y) \in XU \iff (x, y) \in (A \times C) \cup (B \times D)$ (1)

$\iff (x, y) \in ((A \times C) \text{ or } (B \times D))$ (2)

$\iff (x \in A \text{ and } y \in C) \text{ or } (x \in B \text{ and } y \in D)$ (3)

$\iff (x \in A \text{ or } x \in B) \text{ and } (y \in C \text{ or } y \in D)$ (4)

$\iff (x \in A \cup B) \text{ and } (y \in C \cup D)$ (5)

$\iff (x, y) \in UX$ (6)

(A) Indicate the line/lines in the proof that are erroneous and the actual error.

(B) Establish a relationship between UX and XU.

(5 +5)

2(A) Let R be a binary relation on sets X and Y. We can define the inverse of R, written $R^{-1}$, as $y\, R^{-1}\, x$ holds iff $x\, R\, y$ holds, where $x \in X$ and $y \in Y$. Fill the third column below with the weakest appropriate property such that each item is true (there may be situations "can't say" also):

| No. | R−1 is | iff R is |
|---|---|---|
| 1 | Total | |
| 2 | Injection | |
| 3 | a Surjection | |

| | |
|---|---|
| 4 | a Bijection |
| 5 | a Function |

(B)Let f : R → R be a total, injective function which is not a bijection. Give a concrete example of such a function.

(5 + 5)

OR

(A) Suppose that the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Express each of the following sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.

(i) {1, 3, 6, 10};

(ii) {2, 3, 4, 7, 8, 9}

(B) Show that the composition of relations is associative.

(5 + 5)

3(A)A survey of 500 adults found that 190 played golf, 200 skied, 95 played tennis, 100 played golf but did not ski or play tennis, 120 skied but did not play golf or tennis, 30 played golf and skied but did not play tennis, and 40 did all three.

(i) How many played golf and tennis but did not ski?

(ii) How many played tennis but did not play golf or ski?

(iii) How many participated in at least one of the three sports?

(B)Let U be the universal set consisting of the set of all students taking classes at the University and B = {x|x is currently taking a business course}

E = {x|x is currently taking an English course}

M = {x|x is currently taking a math course}

Write an expression using set operations and show the region on a Venn diagram for each of the following:

(i) The set of students at the University taking a course in at least one of the above three fields.

(ii) The set of all students at the University taking both an English course and a math course but not a business course.

(5 + 5)

OR

(A)Let U = {1,2,3,4,5,6,7,8,9,10}, A = {1,2,3,4,5}, B = {4,5,6,7}, C = {5,6,7,8,9,10}. Verify that the identities are true for these sets.

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $(A \cup B)^c = A^c \cap B^c$

(B)(i) Prove the identity without Venn diagram $(A \cup B) \cap (A \cup B^c) = A$

(ii) Find all partitions of set S = {1, 2, 3}

(5 + 5)

**National Institute of Technology Mizoram**

**Mid – Semester Examination, Odd Semester - 2021**

**DISCRETE MATHEMATICS (CSL 1302)**

**B.Tech 3rd Semester CSE** | **Full Marks: 15 marks** | **Duration 1:00 hour**

---

**Answer All Questions**

1. Write the set A = {1, 2, 3, 4, 5, ...} in set-builder form. **(1 Marks)**

2. Find $A \cup (B \cup C)$, if A = {1, 3, 5}, B = {2, 4, 6} and C = {1, 5, 7}. **(2 Marks)**

3. Prove the following by: Definition and Venn diagram
a. $(A \cap B) \cap C = A \cap (B \cap C)$ **(2 Marks)**
b. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(2 Marks)**

4. Let A = {a, b, c, d }, B = { p, q, r, s } denote sets. R : A -> B, R is a function from A to B. Then which of the following relations are not functions ? **(2 Marks)**
(i) { (a, p) (b, q) (c, r) }
(ii) { (a, p) (b, q) (c, s) (d, r) }
(iii) { (a, p) (b, s) (b, r) (c, q) }
a. (i) and (ii)
b. (ii) and (iii)
c. (i) and (iii)
d. None of these

5. Show that the two statements $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.? **(3 Marks)**

6. Suppose that the statement $p \rightarrow \neg q$ is false. Find all combinations of truth values of r and s for which $(\neg q \rightarrow r) \wedge (\neg p \vee s)$ is true? **(3 Marks)**

NATIONAL INSTITUTE OF TECHNOLOGY MIZORAM

End–Semester Examination, Odd Semester (2022-2023)

Discrete Mathematics (CSL – 1302)

3rd Semester Maximum Marks: 50 Time: 3 hours

Answer all 5 (Five) Questions. All Questions Carry the Same Marks

($5 \times 10 = 50$ Marks)

1. a) How to confirm that the identity element in a group (G, *) is unique?

b) Is there a finite set that is a poset and totally ordered set but not a well-ordered one? Justify

c) Show that if $n^2$ is even, then n is even.

d) Is $(p \to q) \to [(p \to q) \to q]$ a tautology?

e) A simple non-directed graph G contains 21 edges, 3 vertices of degree 4, and the other vertices of degree 2. Find the number of vertices in graph G. = 18

f) Is there a full binary tree that has 10 internal vertices and 13 terminal vertices?

g) Let G = (V, E) be a connected simple planar graph, then find the maximum degree of a vertex in G. = 2

h) What do you mean by the statement – "There are different sizes of infinite sets"?

i) What is the reflexive closure of the relation $R=\{(a,b) \mid a < b\}$ on the set of integers ?

j) Differentiate range and codomain of a function

$1 \times 10 = 10$

2.a)(i)Show that set of all non-zero real numbers is a group with respect to multiplication

(ii)Show that $G = \{1, w, w^2\}$ is an abelian group under multiplication. Where $1, w, w^2$ are cube roots of unity $2 + 3$

b)Consider the groups $(G_1, *)$ and $(G_2, \oplus)$ with identity elements $e_1$ and $e_2$ respectively.

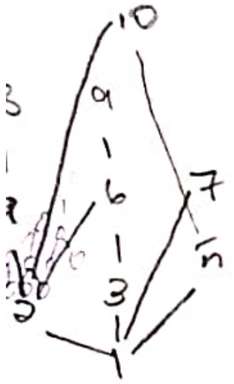
If $f: G_1 \to G_2$ is a group homomorphism, then prove that

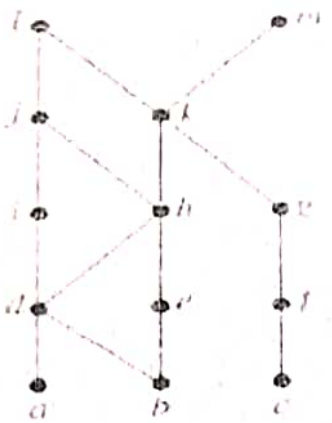
i) $f(a^{-1}) = [f(a)]^{-1}$

ii) If f is an isomorphism from $G_1$ onto $G_2$, then $f^{-1}$ is an isomorphism from $G_2$ onto $G_1$ $2+3$

**OR**

c) For the Hasse diagram given below; find greatest, least, LB, glb, UB for the subsets {d, k, f}





5

d) (i) Prove that if each of the posets (A1, $\preceq$1) and (A2, $\preceq$2) is a chain and $\preceq$L is the lexicographic order on A1 × A2, then (A1 × A2, $\preceq$L) is also a chain.

(ii) Draw the Hasse diagram for the divisibility relation on the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

Ans:- R = {(1,2) (1,3) (1,4) (1,5) (1,6) (1,7) (1,8), (1,9), (1,10) 2

(2,4), (2,6) (2,8) (2,10)

(3,6), (3,9)

(4,8)

(5,10)}

3/3

3. a) (i) Prove that the sum of two rational numbers is rational.

(ii) Prove that—"A simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into (n-2) triangles." 2 + 3

b) (i) Show that—"There are infinitely many prime numbers."

(ii) Prove that – "There exist irrational numbers x and y such that $x^y$ is rational" 3 + 2

**OR**

c) Consider the statement, "if the goods are unsatisfactory, then your money will be refunded." This was an advertising slogan of the Haldiani Company. Is the given statement logically equivalent to "goods satisfactory or money refunded"? What about to "if your money is not refunded, then the goods are satisfactory"? And what about to "if the goods are satisfactory, then your money will not be refunded"? 5

d) Translate the following statements into logical expressions using predicates, quantifiers, and logical connectives. 5

Predicates: C(x): x is a CSE 1302 student; L(x): x loves music

The universe of discourse for the variable x is all students.

(a) Every student loves music.

(b) No student loves music.

(c) Some students love music.

(d) Every CSE 1302 student loves music.

(e) Some CSE 1302 students love music.

4. a)(i) Show that a degree sequence with all distinct elements cannot represent a simple non-directed graph.

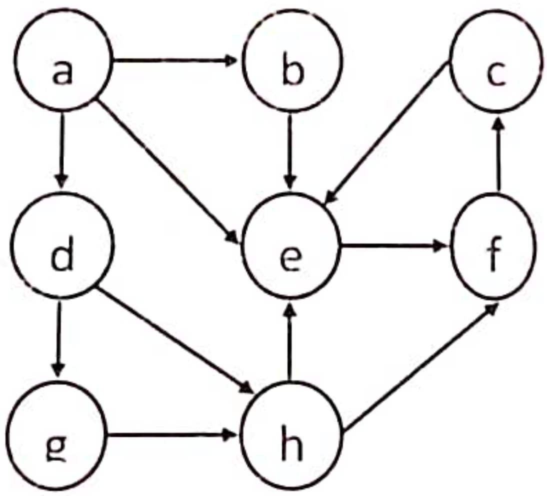



Figure 1

(ii) If G is any connected graph, C is any circuit in G, and one of the edges of C is removed from G, the graph that remains is still connected. 2 + 3

b) Compare DFS and BFS traversal sequences for the graph given in Figure 1. 5

**OR**

c) Find a shortest path from a given source (vertex S) to each of the vertices using Dijkstra's method, for the graph shown in Figure 2. 5

d) A town has a set of houses and a set of roads, shown in Figure 3. A road connects two houses only with a specific cost. Find the minimum repair cost required to repair roads so that everyone stays connected. 5

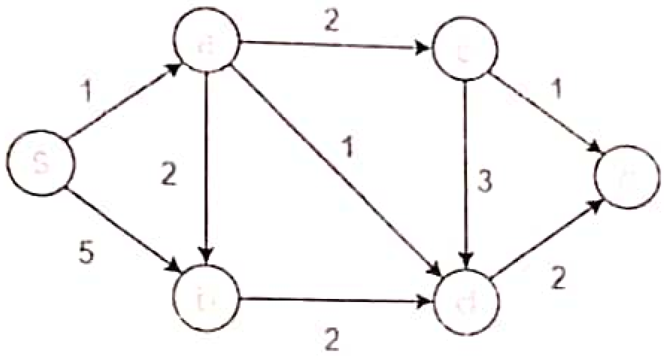



Figure 2

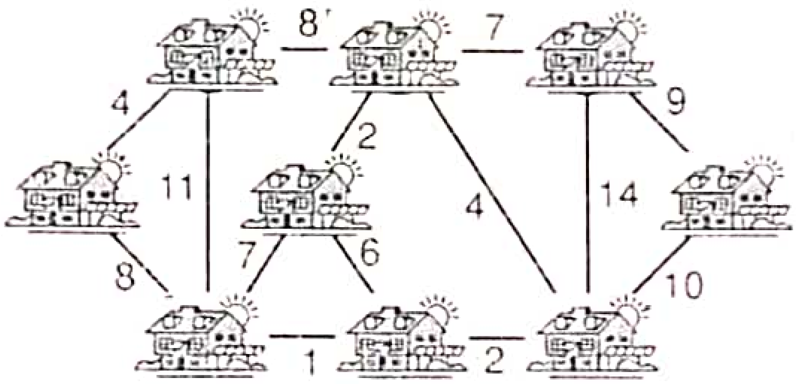



Figure 3

5. a) (i) Show that subpaths of shortest paths are shortest paths.
(ii) Find a topological ordering of nodes for the graph in Figure 4. **3 + 2**

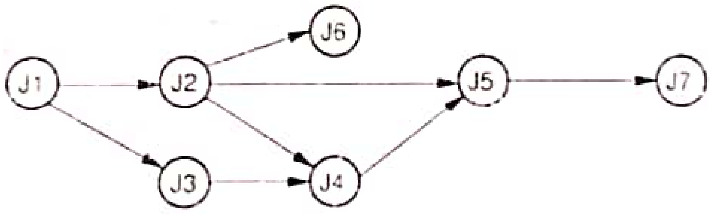



Figure 4

b) (i) Prove that $[A \cup (B \cap C)] \cap ([A' \cup (B \cap C)] \cap (B \cap C)') = \varnothing$
(ii) Show that – "The set of real numbers is uncountable." **2 + 3**

**OR**

c) (i) How many different reflexive relations can be defined on a set A containing n elements?
(ii) Suppose that R is the relation on the set of strings that consist of English letters such that aRb if and only if l(a) = l(b), where l(x) is the length of the string x. Is R an equivalence relation? **2 + 3**

d) (i) Functions f and g are both defined on the set of real numbers and c is a constant.
$f(x) = cx - 3$ and $g(x) = cx + 5$
If $(f \circ g)(x) = (g \circ f)(x)$ for all values of x, what is the value of c?
(ii) What is the solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$? **2 + 3**