National Institute of Technology Mizoram
Mid – Semester Examination, Odd Semester (2022)
LINEAR ALGEBRA AND APPLICATION (MAL 1303)

III[rd] Sem CSE | **Full Marks: 30 marks** | **Duration: 1:30 hours**

Answer all 3 (Three) Questions. All Questions Carry Same Marks
(3 * 10 = 30 Marks)

1(a) If operation defined by $x * y = \frac{x+y}{1+xy}$ where x, y are real number.
Find $\{[(2 * 3) * 4] * \ldots\} * 2021$

OR

Find at least three different ways to express vector $B = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$ as linear combination of matrices $A_1 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

(b) Check whether the set of 2×2 Matrices of determinant zero is subspace of the space of all 2×2 Matrices.

OR

Let $\alpha_1, \alpha_2, \alpha_3$ be linearly independent vector. For what values of k are the vector $\alpha_1 - \alpha_3, \alpha_2 - \alpha_1, k\alpha_3 - \alpha_2$ are linearly independent.

2. Find all real number x for which there is linear map $T: R^3 \to R^3$ such that $T(1,1,1) = (1, x, 1)$, $T(1,0,-1) = (1,0,1)$, $T(-1,-1,0) = (1,2,3)$ and $T(1,-1,-1) = (1, x, -2)$.

OR

Let $T: R^3 \to R^3$ is linear transformation such that $T(x, y, z) = (-3x + 2y + 3z, x - \alpha y + z, -4x + 4y + \alpha z)$. Find α so the nullity of T is equal to 1.

3. Let R^+ be the set of all positive real numbers. List all the vector space axioms which fail to hold with respect to following operations
$x + y = xy$ and $c.x = x^c$

OR

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$x + y = xy$ and $c.x = c^x$

NATIONAL INSTITUTE OF TECHNOLOGY MIZORAM

End-Semester Re–Examination, Odd Semester (2022-23)

Linear Algebra & Application (MAL 1302)

3^{rd} Semester (CSE) Maximum Marks: 50 Time: 3:00 hrs

(Answer all questions. All questions are compulsory.)

$\mathbb{R}$ and $\mathbb{C}$ denote Real and Complex Number Systems, respectively.

1. (a) Let V be a set of ordered pairs(a, b) of real numbers. Show that V is not a vector space over $\mathbb{R}$ with addition and scalar multiplication defined by $(a, b) + (c, d) = (a + c, b + d)$ and $k(a, b) = (a, b)$. [10M]

2. (a) Suppose the mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $f(x, y) = (x + y, x)$. Show that f is linear. [3M]

   (b) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear mapping for which $f(1, 2) = (2, 3)$ and $f(0, 1) = (1, 4)$. Find a formula for $f(x, y)$ if $\{(1, 2), (0, 1)\}$ is a basis of $\mathbb{R}^2$. [3M]

   (c) Let $G : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by $G(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimension of the kernel of map G. [4M]

3. (a) Determine whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x - 4y, 3x - 6y)$ is non–singular? [4M]

   (b) If $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $B' = \{(1, 0, 1), (2, 1, 2), (1, 2, 2)\}$ are two bases, find the change–of–basis matrix P from B to the basis B'. [4M]

   (c) If T is a linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (2x + y, 3x + 2y)$, show that T is invertible. [2M]

4. (a) If $u = (2, 3, 5)$, $v = (1, -4, 3)$ are vectors in $\mathbb{R}^3$, then find the angle between the vectors u and v using inner products. [4M]

   (b) Let V be the vector space of polynomials $f(t)$ with inner product $< f, g >= \int_{-1}^{1} f(t)g(t)dt$. Apply the Gram–Schmidt orthogonalization process to $\{1, t, t^2, t^3\}$ to find an orthogonal basis $\{f_0, f_1, f_2, f_3\}$ with integer coefficients for $\mathbb{P}_3(t)$. [6M]

5. (a) If characteristic and minimal polynomials of an operator T are , respectively, $F(x) = (x-2)^4(x-5)^3$ and $m(x) = (x - 2)^2(x - 5)^3$, then write down the possible Jordon canonical forms. [4M]

   (b) If $A = \begin{pmatrix} 1 & 1 \\ i & 3 + 2i \end{pmatrix}$ is a matrix of linear operator T, then show that T is normal. [3M]

   (c) Find the adjoint of $T : \mathbb{C}^3 \to \mathbb{C}^3$ defined by $T(x, y, z) = (2x + (1 - i)y, (3 + 2i)x - 4iz, 2ix + (4 - 3i)y - 3z)$. Find adjoint operator T^* of T. [3M]

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NATIONAL INSTITUTE OF TECHNOLOGY MIZORAM

End Semester Examination, Odd Semester (2022-23)

Linear Algebra & Application (MAL 1302)

3^{rd} Semester (CSE) Maximum Marks: 50 Time: 3:00 hrs

(Answer all questions. All questions are compulsory.)

$\mathbb{R}$ and $\mathbb{C}$ denote Real and Complex Number Systems, respectively.

1. (a) Consider the subspaces U, V of $\mathbb{R}^4$ defined by $U = \{(x, y, z, w) \in \mathbb{R}^4 : y + z + w = 0\}$ and $V = \{(x, y, z, w) \in \mathbb{R}^4 : x = -y, z = 2w\}$. Find the basis for each of U, V and $U \cap V$. [10M]

2. (a) Consider $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f(x, y, z) = (|x|, 0)$. Show that f is not linear. [3M]

   (b) If $f : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear mapping defined by $f(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t)$, find

   (i) basis and the dimension of the kernel of map f. [4M]

   (ii) basis and the dimension of the range of map f. [3M]

3. (a) If a linear operator T on $\mathbb{R}^3$ is defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$, then

   (i) show that T is invertible, [4M]

   (ii) find $T^{-1}(2, 4, 6)$, [3M]

   (iii) find a formula for T^{-2}. [3M]

4. (a) If $u = (x_1, x_2), v = (y_1, y_2)$, then verify whether $< u, v >= x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ is an inner product in $\mathbb{R}^2$? [5M]

   (b) Let V be a vector space of polynomials over $\mathbb{R}$ with inner product defined by $< f, g >= \int_0^1 f(t)g(t)dt$. Given $f(t) = t + 2, g(t) = 3t - 2$, find $< f, g >$ and $\|f\|$. [2M]

   (c) If W consists of vectors $u = (1, 2, 3, -1, 2), v = (2, 4, 7, 2, -1)$ in $\mathbb{R}^5$, find a basis of orthogonal compliment $W^\perp$ of W. [3M]

5. (a) Find an orthonormal basis for the subspace U of $\mathbb{R}^4$ spanned by vectors $v_1 = (1, 1, 1, 1), v_2 = (1, 2, 4, 5), v_3 = (1, -3, -4, -2)$. [4M]

   (b) Find all possible Jordon canonical forms for a transformation with characteristic polynomial $(x - 7)^4$ and find minimal polynomial in each case. [4M]

   (c) Let T be a linear operator on $\mathbb{C}^3$ defined by $T(x, y, z) = (2x + iy, y - 5iz, x + (1 - i)y + 3z)$. Find adjoint operator T^* of T. [2M]

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