National Institute of Technology Mizoram
Online Mid sem exam even sem 2021
CSE
Subject : convex optimization: theory and algorithms MAL 1405

1. Prove that Intersection of two convex sets is also a convex set.
2. Show that $C = \{(x_1, x_2) : 2x_1 + 3x_2 = 7\} \subset R^2$ is a convex set.
3. Find the values of $x$, at which the cubic function $f(x) = x^3 + ax^2 + bx + c$ is convex function.
4. Find the intervals of convexity and concavity of the function $f(x) = 2x^3 - 18x^2$

| END SEM MAL 1405 |
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Instruction: only upload answer copy in google classroom only

1. (a) Consider $f(x) = 10(x_2 - x_1)^2 + (1 - x_1)^2$, $x = 9x_1, x_2) \in R^2$ compute the gradient vector and Hessian Matrix of $f$ at any $x \in R^2$. Find All stationary Points of $f$. Show that $x^* = (1, 1)$ is the unique global minimizer of $f$ and that the Hessian of $f$ at $x^*$ is positive definite.

   (b) Show that the Hessian matrix of $f$ is singular if and only if $x$ satisfies the condition $x_2 - x_1^2 = 0.05$. Hence show that Hessian matrix of $f$ is positive definite for all $x$ such that $f(x) < 0.025$.

2. Let $f(x_1, x_2, x_3) = 4x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_1 + 8$ Find the gradient vector and Hessian matrix for this function. Determine where the function is convex. Find its stationary point(s) and classify each as relative min or relative max or saddle point. Does this function have an absolute minimum value on $R^3$? (if so, where, and what is the value?)

**National Institute of Technology Mizoram**
**End – Semester Examination, Even- 2022-23**
**Convex Optimizaion: Theory and Algorithm (MAL 1405)**
**B. Tech 4thSem CSE    Full Marks: 50 marks    Duration: 2:30 hours**

1. (a) Prove or disprove that the set $S = \{(x,y,z) \in R^3 / (xz-y^2) \geq 0\}$ is convex
   (b) Check the convexity of the function
   $f(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3 - 6x_1 - 4x_2 - 2x_3$

2. (a) Why is the steepest descent method not efficient in practice although the directions used are the best direction?
   (b) How is the value of Lambda updated if the direction search $S_i$ is not equal to zero in Rosen's Projection method.
   (c) What is the difference between Newton and Quasi Newton methods?
   (d) What is the basic difference in the search equation of DFP and BFGS method?
   (e) Write the formula for Projection matrix $P_i$ used in Rosen's Projection Gradient method. Under what condition on $g_i(x_i)$ it is calculated.

3. (a) How are search directions generated in the Fletcher Reeves method. Derive it.
   (b) Minimize $f(x_1, x_2) = x_1^2 + 3x_2^2 - x_1x_2$ using Fletcher Reeves method using the starting point $X_1 = [1 \ \ 2]^T$

4. Minimize $f(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2 + x_3^2 + x_4^2$
   Subject to
   $$x_1 + x_2 + x_3 + x_4 = 1$$
   $$x_4 \leq A$$
   where A is a parameter. Find the possible values of A for which it attains the minima.

5. (a) Derive the iterative equation for Newton's method considering a quadratic function
   (b) Minimize $f(x_1, x_2) = x_1^2 + 3x_2^2 - x_1x_2$ using Newton's method using the starting point using the starting point $X_1 = [1 \ \ 2]^T$