

DAA

ASSIGNMENT - 5

Back Tracking

(i) N-Queen's problem

The problem of trying to n queens on a $N \times N$ chessboard, such that no queen can attack another queen

→ Generally queens can attack

- (i) Horizontally
- (ii) Vertically
- (iii) Diagonally.

This means that no 2 queens can share the same row, column or diagonal

Back Tracking Approach:

• It involves placing queens one by one in different columns and rows.

• If placing a queen in a specific position leads to a conflict, we backtrack by removing the queen & trying the next possible position.
Steps to solve:

- (i) Initialize the board: Start with an empty $N \times N$ board.
- (ii) Place the queen: Begin by placing the 1st queen in the first column of the 1st row.
- (iii) Check for conflicts: Ensure no 2 queens threaten each other. If a conflict occurs backtrack to the previous step.
- (iv) Move to the next row: Place the next queen in the next row. Continue this process for all rows.
- (v) Backtrack as needed: If you cannot place a queen in a row without a conflict, remove the last placed queen and try the next side position in the previous row.

vi) Repeat: continue this process until all queens are placed on the board or all possibilities are exhausted.
(standard chess board 8x8)

16c4

state space Tree

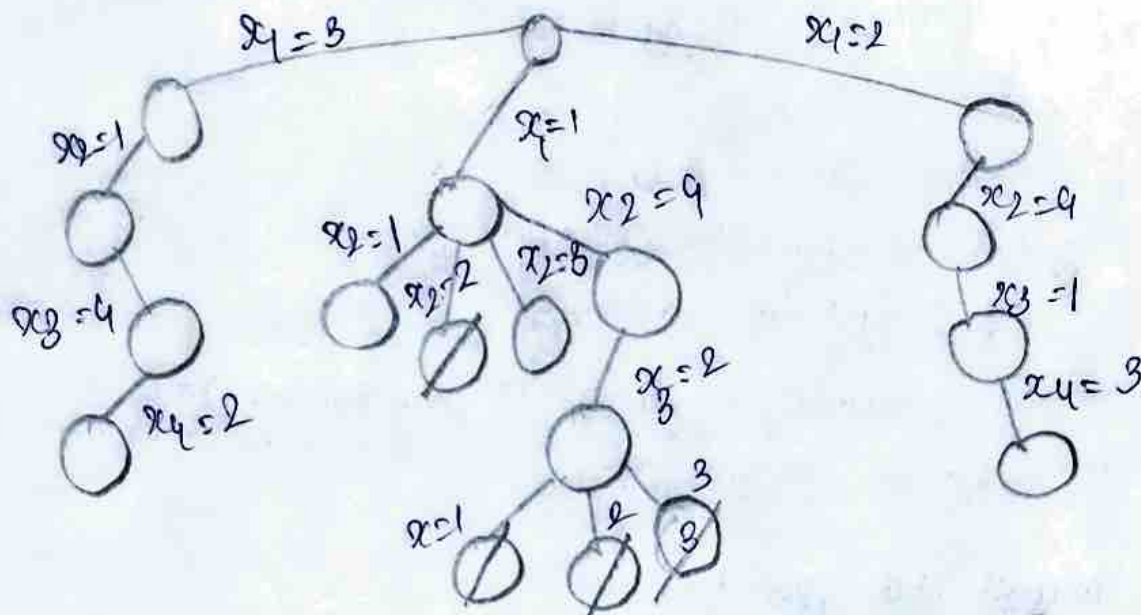
$$1 + \sum_{i=0}^3 \left[\frac{1}{n} (4-i) \right]$$

ALGORITHM:

- Initialize a $N \times N$ board with all positions set to 0.
- Start from the 1st row ($i=0$)
- Try to place the queen in each column ($j=0$ to $N-1$) of the current row.
- Check if the queen can be placed at the position (i, j) without any conflict; check rows - ensure no queen exist in the same column
check diagonals - ensure no queen is ~~also~~ placed in the same diagonal
- If the queen can be placed set board $[i][j] = 1$
recursively call the algorithm for $i+1$.
- Recursive call returns true, then all queens are placed so, return true.
- if the r.c returns false backtrack set board $[i][j] = 0$
Try the next column $j+1$
- If all columns are tried & no solution is found return false

	1	2	3	4
1			Q ₁	
2	Q ₂			
3				Q ₃
4				

State Space Tree



2	4	1	3
1	2	3	4

3	1	4	2
1	2	3	4

1	2	3	4
	Q_1		
			Q_2
Q_3			
			Q_4

1	2	3	4
		Q_1	
Q_2			
			Q_3
	Q_4		

→ Graph colouring

- ' G ' be a graph and ' m ' be an integer.
- The m -colorability decision problem is finding whether the nodes of ' G ' can be coloured using ' m ' colours in such a way that no two adjacent nodes have the same colour.
- The m -colorability optimization problem is finding the smallest integer ' m ' for which the graph can be coloured.
- The integer m is referred to as the chromatic number.

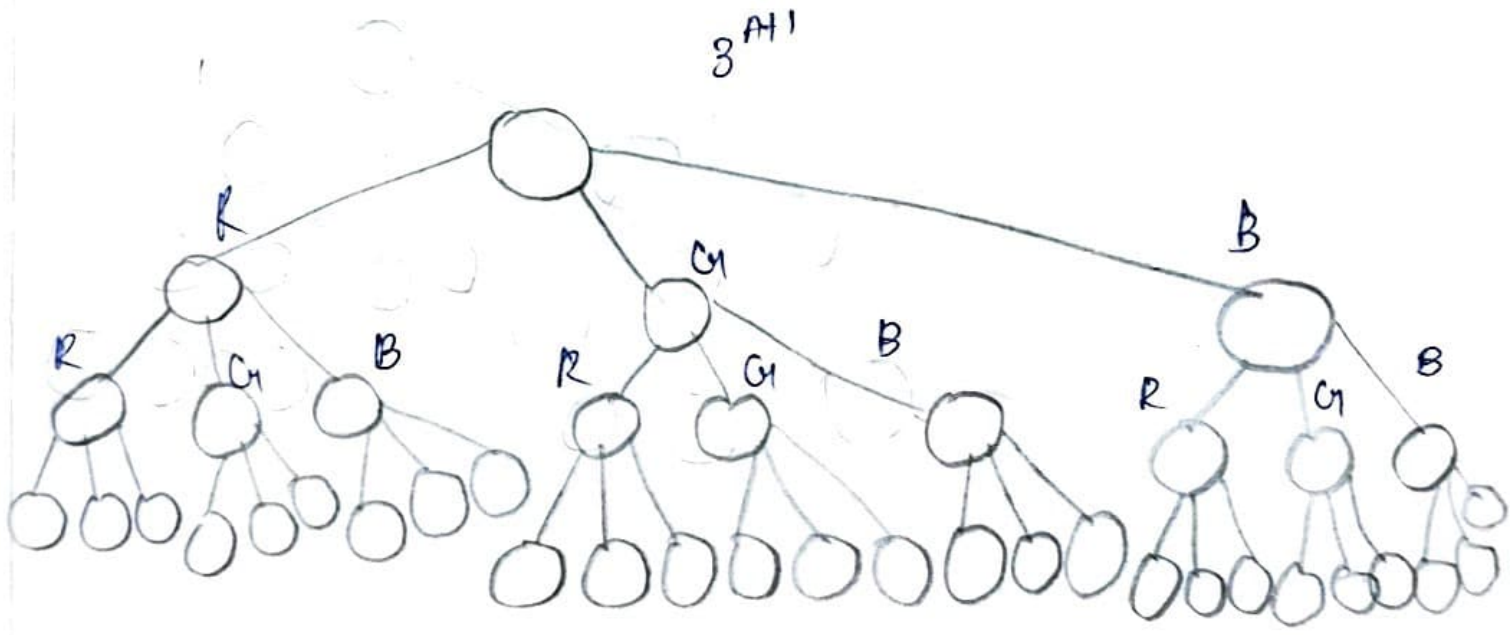
⇒ steps are followed to perform valid colouring for the Graph $G=(V,E)$

- Order the nodes arbitrarily.
- Assign the 1st node with a color.
- Give the partial assignments of colors $C_1, C_2, C_3, \dots, C_{i-1}$ to the $i-1$ nodes, try to find the colour for the i th node.
- If there are no possible color for the i th node then backtrack & change the option for the previous node

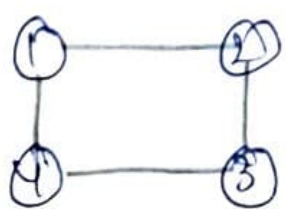
Normal state space tree



$m = \{R, G, B\}$



Now Using Back tracking



$m = \{R, G, B\}$

State space Tree

