

ASSIGNMENT - I

1. To prove: $100n + 5 \in O(n^2)$

n.k.t, if $f(n) \in O(g(n))$,

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

here, let $f(n) = 100n + 5$; $g(n) = n^2$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} \Rightarrow \lim_{n \rightarrow \infty} \frac{100n + 5}{n^2}$$

Using L. Hospital rule,

$$\lim_{n \rightarrow \infty} \frac{100}{2n} \Rightarrow \frac{100}{\infty} = 0.$$

$f(n)$ has smaller Order of growth than $g(n)$. Hence $f(n) \in O(g(n))$ hence proved.

2. Matrix multiplication ($A[0, \dots, n-1, 0, \dots, n-1]$, $B[0, n-1, 0, \dots, n-1]$)

for $i \leftarrow 0$ to $n-1$ do:

for $j \leftarrow 0$ to $n-1$ do

$C[i, j] \leftarrow 0.0$

for $k \leftarrow 0$ to $n-1$ do

$C[i, j] \leftarrow C[i, j] + A[i, k] * B[k, j]$

return C.

input size: n^2

basic operation: $C[i, j] \leftarrow C[i, j] + A[i, k] * B[k, j]$

iterations: 0 to $(n-1)^3$ (or) 1 to n^3

$$\Rightarrow \sum_{i=1}^n 1 \Rightarrow (n^3 - 1) + 1 = n^3$$

$$T(n) \in O(n^3)$$

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3 Binary(n) {
    count ← 1
    while n > 1 do
        count ← count + 1
        n ← [n/2]
    return count
}

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input size: 4 (integer)

basic operation: $n > 1$

iterations: $\frac{n}{2}$ times to $n/2$

$$\Rightarrow \sum_{i=1}^{n/2} 1 = \left(\frac{n}{2} - 1\right) + 1 \Rightarrow \frac{n}{2} = \frac{1}{2}n$$

$T(n) \in O(n)$

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4 Secret(A[0..n-1]) {

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for $n \geq 2$ do

minval ← A[0];

maxval ← A[0];

for $i \leftarrow 1$ to $n-1$ do

if $A[i] < \text{minval}$

minval ← A[i]

if $A[i] > \text{maxval}$

maxval ← A[i]

return maxval - minval

}

input size: n

basic operation: $A[i] < \text{minval}$

iterations: 1 to $n-1$

$$\Rightarrow \sum_{i=1}^{n-1} 1 \Rightarrow (n-1-1) + 1 = n-1$$

$T(n) \in O(n)$

- a) This algorithm compute the difference between maximum and minimum value of an array
- b) the basic operation is
 $A[i] < \text{minval}$ (or) $A[i] > \text{maxval}$
 because these repeat the most and prioritise above assignment statements
- c) basic operation is executed n times
- d) Efficiency class of this algorithm is $O(n)$
- e) There are no such Alternative methods that has better efficiency class than this algorithm because to find min and max element of an array we have 2 methods
- i) sort the array
 and take the first
 and last elements
 (time (least): $O(n \log n)$)

ii) use brute force
 approach
 (time: $O(n)$)
- since $n \log n > n$, efficient method is brute force approach, which is followed in the given algorithm.