

PROBABILITY

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13.1.11 ¹ A fair die is rolled. Consider events $E=1,3,5$ $F=2,3$ and $G=2,3,4,5$. Find

- (a) $\Pr(E | F)$ and $\Pr(F | E)$
- (b) $\Pr(E | G)$ and $\Pr(G | E)$
- (c) $\Pr(E \cup F | G)$ and $\Pr(E \cap F | G)$

Solution:

Event	Probability
P(E)	$\frac{1}{2}$
P(F)	$\frac{1}{3}$
P(G)	$\frac{2}{3}$
P(EF)	$\frac{1}{6}$
P(EG)	$\frac{1}{3}$
P(FG)	$\frac{1}{3}$
P(EFG)	$\frac{1}{6}$

Table 2:

- (a) $\Pr(E | F)$ and $\Pr(F | E)$

$$\Pr(E | F) = \frac{\Pr(EF)}{\Pr(F)} \quad (13.1.3.1)$$

$$\implies \Pr(E | F) = \frac{\frac{1}{6}}{\frac{1}{3}} \quad (13.1.3.2)$$

$$\implies \Pr(E | F) = \frac{1}{2} \quad (13.1.3.3)$$

$$\Pr(F | E) = \frac{\Pr(FE)}{\Pr(E)} \quad (13.1.3.4)$$

$$\implies \Pr(F | E) = \frac{\frac{1}{6}}{\frac{1}{2}} \quad (13.1.3.5)$$

$$\implies \Pr(F | E) = \frac{1}{3} \quad (13.1.3.6)$$

¹Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

(b) $\Pr(E \mid G)$ and $\Pr(G \mid E)$

$$\Pr(E \mid G) = \frac{\Pr(EG)}{\Pr(G)} \quad (13.1.3.7)$$

$$\implies \Pr(E \mid G) = \frac{\frac{1}{2}}{\frac{3}{2}} \quad (13.1.3.8)$$

$$\implies \Pr(E \mid G) = \frac{1}{3} \quad (13.1.3.9)$$

$$\Pr(G \mid E) = \frac{\Pr(GE)}{\Pr(E)} \quad (13.1.3.10)$$

$$\implies \Pr(G \mid E) = \frac{\frac{1}{2}}{\frac{1}{2}} \quad (13.1.3.11)$$

$$\implies \Pr(G \mid E) = 1 \quad (13.1.3.12)$$

(c) $\Pr(E \cup F \mid G)$ and $\Pr(E \cap F \mid G)$

$$\Pr(E + F \mid G) = \frac{\Pr((E + F)G)}{\Pr(G)} \quad (13.1.3.13)$$

$$\implies \Pr(E + F \mid G) = \frac{\Pr(EG + FG)}{\Pr(G)} \quad (13.1.3.14)$$

$$\implies \Pr(E + F \mid G) = \frac{\Pr(EG) + \Pr(FG) - \Pr(EFG)}{\Pr(G)} \quad (13.1.3.15)$$

$$\implies \Pr(E + F \mid G) = \frac{3}{4} \quad (13.1.3.16)$$

$$\Pr(EF \mid G) = \frac{\Pr(EFG)}{\Pr(G)} \quad (13.1.3.17)$$

$$\implies \Pr(EF \mid G) = \frac{1}{4} \quad (13.1.3.18)$$