PROBABILITY

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13.1.11 [1] A fair die is rolled.Consider events E=1,3,5 F=2,3 and G=2,3,4,5.Find

    (a) $\Pr(E \mid F) \, and \, \Pr(F \mid E)$

    (b) $\Pr(E \mid G) \, and \, \Pr(G \mid E)$

    (c) $\Pr(E \cup F \mid G) \, and \, \Pr(E \cap F \mid G)$

Solution:

Event	Probability
P(E)	$\frac{1}{2}$
P(F)	$\frac{1}{3}$
P(G)	$\frac{2}{3}$
P(EF)	$\frac{1}{6}$
P(EG)	$\frac{1}{3}$
P(FG)	$\frac{1}{3}$
P(EFG)	$\frac{1}{6}$

Table 2:

(a) $\Pr(E \mid F)$ and $\Pr(F \mid E)$

$$\Pr(E \mid F) = \frac{\Pr(EF)}{\Pr(F)} \tag{13.1.3.1}$$

$$= \frac{\frac{1}{6}}{\frac{1}{3}} \tag{13.1.3.2}$$

$$= \frac{1}{2} \tag{13.1.3.3}$$

$$\Pr(F \mid E) = \frac{\Pr(FE)}{\Pr(E)} \tag{13.1.3.4}$$

$$= \frac{\frac{1}{6}}{\frac{1}{2}} \tag{13.1.3.5}$$

$$= \frac{1}{3} \tag{13.1.3.6}$$

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[1]Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

(b) $\Pr\left(E \mid G\right)$ and $\Pr\left(G \mid E\right)$

$$\Pr\left(E \mid G\right) = \frac{\Pr\left(EG\right)}{\Pr\left(G\right)} \tag{13.1.3.7}$$

$$= \frac{\frac{1}{3}}{\frac{2}{3}} \tag{13.1.3.8}$$

$$= \frac{1}{2} \tag{13.1.3.9}$$

$$\Pr\left(G \mid E\right) = \frac{\Pr\left(GE\right)}{\Pr\left(G\right)} \tag{13.1.3.10}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}} \tag{13.1.3.11}$$

$$= \frac{2}{3} \tag{13.1.3.12}$$

(c) $\Pr\left(E \cup F \mid G\right)$ and $\Pr\left(E \cap F \mid G\right)$

$$\Pr\left(E + F \mid G\right) = \frac{\Pr\left((E+F)G\right)}{\Pr\left(G\right)} \tag{13.1.3.13}$$

$$= \frac{\Pr\left(EG + FG\right)}{\Pr\left(G\right)} \tag{13.1.3.14}$$

$$= \frac{\Pr\left(EG\right) + \Pr\left(FG\right) - \Pr\left(EFG\right)}{\Pr\left(G\right)} \tag{13.1.3.15}$$

$$= \frac{3}{4} \tag{13.1.3.16}$$

$$\Pr\left(EF \mid G\right) = \frac{\Pr\left(EFG\right)}{\Pr\left(G\right)} \tag{13.1.3.17}$$

$$= \frac{1}{4} \tag{13.1.3.18}$$