PROBABILITY

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13.1.11 [1] A fair die is rolled.Consider events E=1,3,5 F=2,3 and G=2,3,4,5. Find

(a) $\Pr(E \mid F)$ *and* $\Pr(F \mid E)$

(b) $\Pr(E \mid G)$ *and* $\Pr(G \mid E)$

(c) $\Pr(E \cup F \mid G)$ *and* $\Pr(E \cap F \mid G)$

Solution:

Event	**Probability**
$\Pr(E)$	$\frac{1}{2}$
$\Pr(F)$	$\frac{1}{3}$
$\Pr(G)$	$\frac{2}{3}$
$\Pr(EF)$	$\frac{1}{6}$
$\Pr(EG)$	$\frac{1}{3}$
$\Pr(FG)$	$\frac{1}{3}$
$\Pr(EFG)$	$\frac{1}{6}$

Table 13.1.3.2: Probability of Events.

(a) $\Pr(E \mid F)$ and $\Pr(F \mid E)$

$$\implies \Pr(E \mid F) = \frac{\Pr(EF)}{\Pr(F)} \tag{13.1.3.1}$$

$$= \frac{\frac{1}{6}}{\frac{1}{3}} \tag{13.1.3.2}$$

$$= \frac{1}{2} \tag{13.1.3.3}$$

$$\implies \Pr(F \mid E) = \frac{\Pr(FE)}{\Pr(E)} \tag{13.1.3.4}$$

$$= \frac{\frac{1}{6}}{\frac{1}{2}} \tag{13.1.3.5}$$

$$= \frac{1}{3} \tag{13.1.3.6}$$

[1] Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

(b) $\Pr(E \mid G)$ and $\Pr(G \mid E)$

$$\implies \Pr(E \mid G) = \frac{\Pr(EG)}{\Pr(G)} \tag{13.1.3.7}$$

$$= \frac{\frac{1}{3}}{\frac{2}{3}} \tag{13.1.3.8}$$

$$= \frac{1}{2} \tag{13.1.3.9}$$

$$\implies \Pr(G \mid E) = \frac{\Pr(GE)}{\Pr(G)} \tag{13.1.3.10}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}} \tag{13.1.3.11}$$

$$= \frac{2}{3} \tag{13.1.3.12}$$

(c) $\Pr(E \cup F \mid G)$ and $\Pr(E \cap F \mid G)$

$$\implies \Pr(E + F \mid G) = \frac{\Pr((E + F)G)}{\Pr(G)} \tag{13.1.3.13}$$

$$= \frac{\Pr(EG + FG)}{\Pr(G)} \tag{13.1.3.14}$$

$$= \frac{\Pr(EG) + \Pr(FG) - \Pr(EFG)}{\Pr(G)} \tag{13.1.3.15}$$

$$= \frac{3}{4} \tag{13.1.3.16}$$

$$\implies \Pr(EF \mid G) = \frac{\Pr(EFG)}{\Pr(G)} \tag{13.1.3.17}$$

$$= \frac{1}{4} \tag{13.1.3.18}$$