PROBABILITY

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13.1.11 1 A fair die is rolled. Consider events E=1,3,5 F=2,3 and G=2,3,4,5. Find

- (a) $Pr(E \mid F)$ and $Pr(F \mid E)$
- **(b)** $Pr(E \mid G)$ and $Pr(G \mid E)$
- (c) $Pr(E \cup F \mid G)$ and $Pr(E \cap F \mid G)$

Solution:

Event	Probability
P(E)	$\frac{1}{2}$
P(F)	$\frac{1}{3}$
P(G)	$\frac{2}{3}$
P(EF)	$\frac{1}{6}$
P(EG)	$\frac{1}{3}$
P(FG)	$\frac{1}{3}$
P(EFG)	$\frac{1}{6}$

Table 2:

(a) $Pr(E \mid F)$ and $Pr(F \mid E)$

$$\Pr\left(E \mid F\right) = \frac{\Pr\left(EF\right)}{\Pr\left(F\right)} \tag{13.1.3.1}$$

$$=\frac{\frac{1}{6}}{\frac{1}{2}}\tag{13.1.3.2}$$

$$=\frac{1}{2} \tag{13.1.3.3}$$

$$\Pr(F \mid E) = \frac{\Pr(FE)}{\Pr(E)}$$
(13.1.3.4)

$$=\frac{\frac{1}{6}}{\frac{1}{2}}\tag{13.1.3.5}$$

$$=\frac{1}{3} \tag{13.1.3.6}$$

¹Read question numbers as (CHAPTER NUMBER).(EXERCISE NUMBER).(QUESTION NUMBER)

(b) $Pr(E \mid G)$ and $Pr(G \mid E)$

$$\Pr\left(E \mid G\right) = \frac{\Pr\left(EG\right)}{\Pr\left(G\right)} \tag{13.1.3.7}$$

$$=\frac{\frac{1}{3}}{\frac{2}{3}}\tag{13.1.3.8}$$

$$=\frac{1}{2} \tag{13.1.3.9}$$

$$\Pr\left(G \mid E\right) = \frac{\Pr\left(GE\right)}{\Pr\left(G\right)} \tag{13.1.3.10}$$

$$\begin{aligned}
&= \frac{1}{3} \\
&= \frac{1}{2} \\
&= \frac{1}{2} \\
&= \frac{1}{2} \\
&= \frac{1}{2} \\
&= \frac{\Pr(GE)}{\Pr(G)} \\
&= \frac{1}{3} \\
&= \frac{1}{2} \\
&= \frac{1}{3} \\
&= \frac{1}{3} \\
&= \frac{2}{3} \end{aligned} (13.1.3.10)$$

$$=\frac{2}{3} \tag{13.1.3.12}$$

(c) $Pr(E \cup F \mid G)$ and $Pr(E \cap F \mid G)$

$$\Pr\left(E + F \mid G\right) = \frac{\Pr\left((E + F)G\right)}{\Pr\left(G\right)} \tag{13.1.3.13}$$

$$= \frac{\Pr(EG + FG)}{\Pr(G)}$$

$$= \frac{\Pr(EG) + \Pr(FG) - \Pr(EFG)}{\Pr(G)}$$

$$= \frac{3}{4}$$
(13.1.3.14)
(13.1.3.15)

$$=\frac{\Pr\left(EG\right)+\Pr\left(FG\right)-\Pr\left(EFG\right)}{\Pr\left(G\right)}\tag{13.1.3.15}$$

$$=\frac{3}{4} \tag{13.1.3.16}$$

$$\Pr\left(EF \mid G\right) = \frac{\Pr\left(EFG\right)}{\Pr\left(G\right)} \tag{13.1.3.17}$$

$$=\frac{1}{4} \tag{13.1.3.18}$$