Optimization Assignment

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Problem Statement - Show that the shortest distance from a given point to a given straight line is the perpendicular distance.

Solution

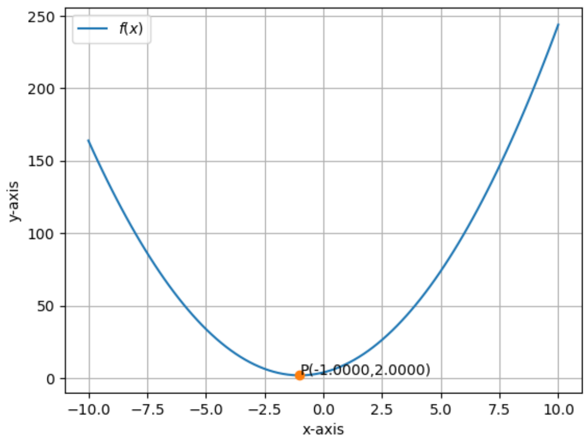

Figure 1: Graph

Assumptions : Let us assume (2,2) be the given point and x+y=2 be the given line,
line and point in vector form

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{n}^T\mathbf{x} = c \tag{2}$$

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \tag{3}$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4}$$

$$d^2 = \|\mathbf{P} - \mathbf{x}\|^2 \tag{5}$$

$$d^2 = \|\mathbf{P} - (\mathbf{A} + \lambda\mathbf{m})\|^2 \tag{6}$$

$$d^2 = \lambda^2\|\mathbf{m}\|^2 - \lambda(2(\mathbf{P} - \mathbf{A})^T\mathbf{m}) + \|\mathbf{P} - \mathbf{A}\|^2 \tag{7}$$

by substituting P,A,m in equ(7)

$$d^2 = \lambda^2\left\|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\|^2 - 2\lambda\left[\left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\right)^T\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right] + \left\|\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\right\|^2 \tag{8}$$

by sollving equ(8) we get,

$$d^2 = 2(\lambda^2 + 2\lambda + 2) \tag{9}$$

Obective Function :

$$d^2 = 2(\lambda^2 + 2\lambda + 2) \tag{10}$$

Gradient descent method

$$f(x) = 2(x^2 + 2x + 2) \tag{11}$$

$$f'(x) = 2(2x + 2) \tag{12}$$

we have to attain the maximum value of m. This can be seen in Figure.Using gradient descent method we can find its minima value.

$$x_{n+1} = x_n - \alpha\nabla f(x_n) \tag{13}$$

$$\implies x_{n+1} = x_n + \alpha 2(2x + 2) \tag{14}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 1.414 \tag{15}$$

$$\text{Minima Point} = -1 \tag{16}$$

$$\implies \lambda = -1 \tag{17}$$

$$d_p = \frac{|n^T\mathbf{P} - c|}{\|\mathbf{n}\| + ++} \tag{18}$$

$$d_p = 1.414 \tag{19}$$

Hence,the shortest distance from a given point to a given straight line is perpendicular distance.