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Optimization Assignment

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IITH - Future Wireless Communication (FWC)

Problem Statement - Show that the shortest distance from a given point to a given straight line is the perpendicular distance.

we have to attain the maximum value of m . This can be seen in Figure. Using gradient descent method we can find its minima value.

Solution

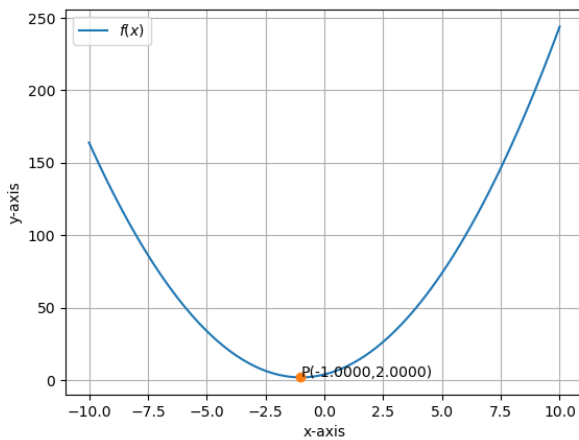


Figure 1: Graph

Assumptions : Let us assume $(2,2)$ be the given point and $x+y=2$ be the given line, line and point in vector form

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{n}^T \mathbf{x} = c \quad (2)$$

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (3)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (4)$$

$$d^2 = \|\mathbf{P} - \mathbf{x}\|^2 \quad (5)$$

$$d^2 = \|\mathbf{P} - (\mathbf{A} + \lambda \mathbf{m})\|^2 \quad (6)$$

Obective Function :

$$d^2 = 2(\lambda^2 + 2\lambda + 2) \quad (7)$$

Gradient descent method

$$f(x) = 2(\lambda^2 + 2\lambda + 2) \quad (8)$$

$$f'(x) = 2(2\lambda + 2) \quad (9)$$

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (10)$$

$$\Rightarrow x_{n+1} = x_n + \alpha 2(2\lambda + 2) \quad (11)$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 1.414 \quad (12)$$

$$\text{Minima Point} = -1 \quad (13)$$

$$\Rightarrow \lambda = -1 \quad (14)$$

$$d = \frac{|n^T P - c|}{\|n\|} \quad (15)$$

$$d = 1.414 \quad (16)$$

Hence, the shortest distance from a given point to a given straight line is perpendicular distance.