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# Optimization Assignment

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**Problem Statement** - Show that the shortest distance from a given point to a given straight line is the perpendicular distance.

## Solution

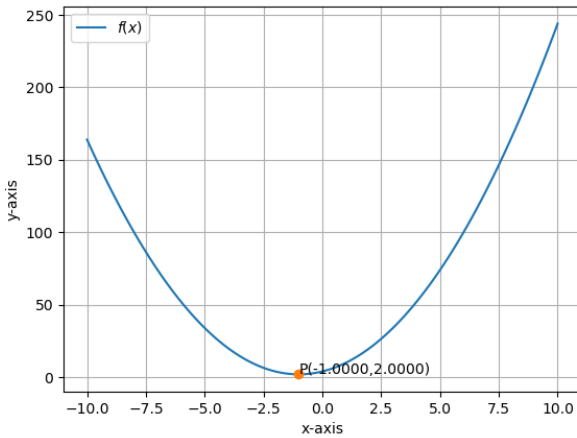


Figure 1: Graph

**Assumptions :** Let us assume (2,2) be the given point and  $x+y=2$  be the given line, line and point in vector form

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{n}^T \mathbf{x} = c \quad (2)$$

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (3)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (4)$$

$$d^2 = \|\mathbf{P} - \mathbf{x}\|^2 \quad (5)$$

$$d^2 = \|\mathbf{P} - (\mathbf{A} + \lambda \mathbf{m})\|^2 \quad (6)$$

$$d^2 = \lambda^2 \|\mathbf{m}\|^2 - \lambda(2(\mathbf{P} - \mathbf{A})^T \mathbf{m}) + \|\mathbf{P} - \mathbf{A}\|^2 \quad (7)$$

by substituting P,A,m in (7)

$$d^2 = \lambda^2 \left\| \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\|^2 - 2\lambda \left[ \left( \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right)^T \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right] + \left\| \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\|^2 \quad (8)$$

by solving (8) we get,

$$d^2 = 2(\lambda^2 + 2\lambda + 2) \quad (9)$$

**Obective Function :**

$$d^2 = 2(\lambda^2 + 2\lambda + 2) \quad (10)$$

**Gradient descent method**

$$f(x) = 2(x^2 + 2x + 2) \quad (11)$$

$$f'(x) = 2(2x + 2) \quad (12)$$

we have to attain the maximum value of m. This can be seen in Figure. Using gradient descent method we can find its minima value.

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (13)$$

$$\Rightarrow x_{n+1} = x_n + \alpha 2(2x + 2) \quad (14)$$

Taking  $x_0 = 0.5, \alpha = 0.001$  and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 1.414 \quad (15)$$

$$\text{Minima Point} = -1 \quad (16)$$

$$\Rightarrow \lambda = -1 \quad (17)$$

$$d_p = \frac{|\mathbf{n}^T \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (18)$$

$$d_p = 1.414 \quad (19)$$

Hence, the shortest distance from a given point to a given straight line is perpendicular distance.