भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Optimization Assignment

Hari Venkateswarlu Annam
hariannam99@gmail.com
IITH - Future Wireless Communication (FWC)

Problem Statement - The line $y = mx + 1$ is a tangent to the curvr $y^2 = 4x$ if the value of m is.

Solution

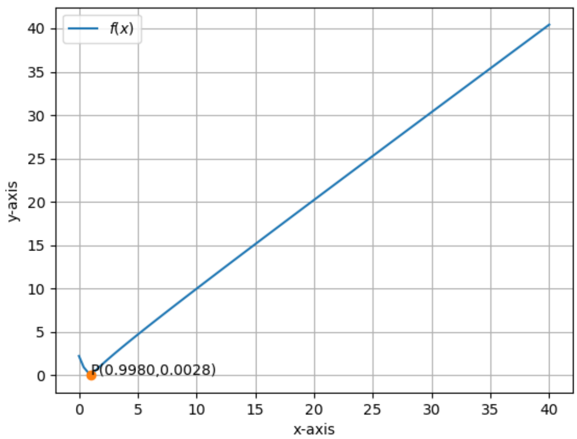

Figure 1: Graph of f(x)

$$\mathbf{x}^\top v\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

$$\mathbf{n}^\top\mathbf{x} = 1 \tag{2}$$

$$\mathbf{x} = \mathbf{e_2} + \mu\mathbf{m} \tag{3}$$

$$d = \|\mathbf{nx} - \mathbf{e_2} - \mu\mathbf{m}\| \tag{4}$$

Gradient descent

$$f(x) = \sqrt{\left(\frac{m^2x - 2 + m}{m^2}\right)^2 + \left(\frac{2m\sqrt{x} - m - 2 + m}{m}\right)^2} \tag{5}$$

$$f'(x) = \frac{(mx - \frac{2}{m} + 1 + 2m - \frac{2}{\sqrt{x}})}{\sqrt{(mx - \frac{2}{m} + 1)^2 + (2m\sqrt{x} - 2)^2}} \tag{6}$$

we have to attain the maximum value of f(x). This can be seen in Figure f(x).Using gradient descent method we can find its minima value.

$$x_{n+1} = x_n - \alpha\nabla f(x_n) \tag{7}$$

$$\implies x_{n+1} = x_n + \alpha\frac{(mx - \frac{2}{m} + 1 + 2m - \frac{2}{\sqrt{x}})}{\sqrt{(mx - \frac{2}{m} + 1)^2 + (2m\sqrt{x} - 2)^2}} \tag{8}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\text{Minima} = 0.002824134040434986 \tag{9}$$

$$\implies \boxed{\text{Minima} = 0.002824134040434986} \tag{10}$$

$$\boxed{\text{Minima Point} = 0.9980035344636117} \tag{11}$$

$$\therefore \textit{Hence Proved}$$