# ECE269-FA21 Homework 3

Hari Sankar

TOTAL POINTS

**27.5 / 45**

QUESTION 1

## 1 Problem 2 **8 / 15**

✓ **- 0 pts part a correct**

**- 1 pts** part a minor mistakes

**- 2 pts** part a some effort

**- 3 pts** part a error/missing/no sufficient effort

**- 0 pts** part b correct

**- 1 pts** part b minor mistakes

**- 2 pts** part b some effort

✓ **- 3 pts part b error/missing/no sufficient effort**

**- 0 pts** part c correct

✓ **- 1 pts part c minor mistakes**

**- 2 pts** part c some effort

**- 3 pts** part c error/missing/no sufficient effort

**- 0 pts** part d correct

✓ **- 1 pts part d minor mistakes**

**- 2 pts** part d some effort

**- 3 pts** part d error/missing/no sufficient effort

**- 0 pts** part e correct

**- 1 pts** part e minor mistakes

✓ **- 2 pts part e some effort**

**- 3 pts** part e error/missing/no sufficient effort

1 please check. It is U.TU

2 show if A is full rank A.TA is invertible using simple rank equation.

3 though your meaning is correct, your notation is incorrect mathematically. Please correct. You cannot say inner product of a matrix with a vector in notation.

4 incorrect.

5 If P =I then in the intial step of <x-Px, .> = <x-x, .> = <0, .> = 0.

QUESTION 2

## 2 Programming Assignment Report **19.5 / 30**

**- 0 pts** Correct

**- 0 pts** Noiseless case

**- 1 pts** Noiseless: plot lack of necessary details, for example, colorbar, axis label...

**- 4.5 pts** Noiseless: x6 plots; change not consistent (0.75 pt each plot)

**- 0.5 pts** Noiseless: x1; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)

**- 1.5 pts** Noiseless: x6 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)

**- 1 pts** Noiseless: comments lack of important details

**- 3 pts** Noiseless: x4 plots; did not perform enough number of realizations (0.75 pt each plot)

**- 1.5 pts** Noiseless: comments lack of important details regarding the change in M,s

**- 2 pts** Noiseless: x4 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)

**- 0.5 pts** Noiseless: x1 plots; change not consistent (0.5 pt each plot)

**- 1.5 pts** Noiseless: x3 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)

**- 2.25 pts** Noiseless: x3 plots; change not consistent (0.75 pt each plot)

**- 4 pts** Noiseless: x4; too few M tested; and results not consistent (1 pt)

**- 3 pts** Noiselss: x3; loss of required plots (1 pt each)

**- 1.5 pts** Noiseless: x3 plots; plot scale does not look right (0.5 pt each plot)

**- 1.5 pts** Noiseless: x3 plots; change not consistent

(0.5 pt each plot)

**- 1 pts** Noiseless: x2 plots; change not consistent (0.5 pt each plot)

**- 3 pts** Noiseless: x6 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)

**- 6 pts** Noiseless: x6 plots; wrong/missing plot (1 pt each)

**- 1 pts** Noiseless: x2 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)

**- 0.25 pts** Noiseless: plot missing colorbar

**- 0.25 pts** Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)

**- 2 pts** Noiseless: x4 plots; plots do not look consistent (0.5 pt each plot)

**- 1 pts** Noiseless: x2 plots; plot scale does not look right (0.5 pt each plot)

**- 0.5 pts** Noiseless: comment missing details

**- 1.5 pts** Noiseless: x2 plots; cannot see the trend/too few M,s performed (0.75 pt each plot)

✓ **- 0.5 pts Noiseless: x1 plot; change not consistent (0.5 pt each plot)**

✓ **- 0.5 pts Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)**

✓ **- 2 pts Noiselss: x2; loss of required plots (1 pt each)**

**- 0.5 pts** Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)

**- 2 pts** Noiseless: no comment

**- 0.75 pts** Noiseless: x1; plots not look correct (0.75 pt each plot)

**- 0.5 pts** Noiselss: x2; too few M, s (0.25 pt each)

**- 4 pts** Noiseless: x4; missing plot (1 pt)

**- 2.5 pts** Noiseless: x5 plots; change not consistent (0.5 pt each plot)

**- 1 pts** Noiseless: x1; missing plot (1 pt)

**- 0.75 pts** Noiselss: x3; minor inconsistency in the phase transition plots (0.25 pt each)

**- 1 pts** Noiseless: x4 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)

**- 0.75 pts** Noiselss: x3; minor inconsistency in the phase transition plots (0.25 pt each)

**- 1.25 pts** Noiselss: x5; minor inconsistency in the phase transition plots (0.25 pt each)

**- 0.5 pts** Noiseless: x1 plots; plot scale not correct (0.5 pt each plot)

**- 1 pts** Noiseless: comments missing details

**- 1 pts** Noiseless: x4 plots; x/y-axis do not have enough values to observe a complete view of trend/minor inconsistency (0.25 pt each plot)

**- 1 pts** Noiseless: plots have error in colorbar/axis label...

**- 3 pts** Noiseless: x4 plots; plots not look correct (0.75 pt each plot)

**- 1.5 pts** Noiseless: x6 plots; minor inconsistency in the transition plots (0.25 pt each plot)

**- 1 pts** Noisy: plot lack of necessary details, for example, colorbar, axis label...

**- 3 pts** Noisy: x6 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)

**- 4 pts** Noisy: x8 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)

**- 2.25 pts** Noisy: x9 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)

**- 3 pts** Noisy: x4; did not perform enough number of realizations (0.75 pt each plot)

**- 12 pts** Noise case: b) plots wrong

**- 4 pts** Noisy: x4; missing plots (1 pt each)

**- 6 pts** Noisy: x12 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)

**- 9 pts** Noisy: x9; wrong plots (1 pt each)

**- 3 pts** Noisy: x3; lack of required plots

**- 9 pts** Noisy: x12; plots wrong (0.75 pt each)

**- 1.5 pts** Noisy: x3 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)

**- 2 pts** Noisy: x4 plots; cannot see the expected trend clear enough; or did not perform enough number of  realizations (0.5 pt each plot)

- **3 pts** Noisy: x3; missing 3 plots in part(a) or (b)
- **2 pts** Noisy: no comment
- **1.5 pts** Noisy: x3 plots; change not consistent (0.5 pt each plot)
- **3 pts** Noisy: x6 plots; part(a), performance change not consistent (0.5 pts each)
- **1.5 pts** Noisy: x2 plots; cannot observe the trend (0.75 pt each plot)
- **4.5 pts** Noisy: x6; plots wrong (0.75 pt each)
- **1 pts** Noisy: x2 plots; change not consistent (0.5 pt each plot)
- **4.5 pts** Noisy: x6 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.75 pts each)
- **2 pts** Noise case: no comments
- **6 pts** Noisy: x6; missing 6 plots in part(a) or (b)
- **1.5 pts** Noisy: x6 plots; change not consistent (0.25 pt each plot)
- **1.5 pts** Noisy: x6 plots; part(a)/part(b), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.25 pts each)
- **4.5 pts** Noisy: x6 plots; change on the phase transition plots not consistent (0.75 pts each)
- **9 pts** Noisy: x12; wrong setting of plots(0.75 pt each)
- **1.5 pts** Noise case: comments not complete

✓ **- 2 pts Noisy: x4 plots; change not consistent (0.5 pt each plot)**

- **5 pts** Noisy: x10 plots; cannot see the expected trend clear enough; or did not perform enough number of  realizations (0.5 pt each plot)
- **0.5 pts** Noisy: x2; minor inconsistency in the phase transition plots (0.25 pt each)
- **7 pts** Noisy: x7; missing/wrong plots (1 pt each)
- **4.5 pts** Noisy: x6 plots; cannot observe the trend (0.75 pts each)
- **3 pts** Noisy: x4 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.75 pts each)

✓ **- 4 pts Noisy: x4; missing plots (1 pt each)**

- **5 pts** Noisy: x10 plots; cannot see the expected trend clear enough; or did not perform enough number of  realizations (0.5 pt each plot)
- **1.5 pts** Noisy: x6 plots; minor inconsistency (0.25 pt each plot)
- **1 pts** Noisy: x4 plots; x/y-axis do not have enough values to observe a complete view of trend/minor inconsistency (0.25 pt each plot)
- **3 pts** Noisy: x6 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.5 pts each)
- **1 pts** Noisy: x4 plots; part(a)/part(b), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.25 pts each)
- **7 pts** Noisy: x7; wrong/missing plots(1 pt each)
- **0.75 pts** Noisy: x3; minor inconsistency in the phase transition plots (0.25 pt each)
- **0.5 pts** Noisy: plot lack of necessary details, for example, colorbar, axis label...
- **9 pts** Noisy: x9; wrong plots(1 pt each)
- **9 pts** Noisy: x12 plots; cannot observe the trend (0.75 pt each plot)
- **4 pts** Noisy: x4; missing 4 plots in part(a) or (b)
- **1.5 pts** Noisy: x6 plots; cannot see the expected trend clear enough; or did not perform enough number of  realizations (0.25 pt each plot)
- **6 pts** Noisy: x8; plots wrong (0.75 pt each)
- **1 pts** Noise case: comments not complete
- **1.25 pts** Noisy: x5 plots; change not consistent (0.25 pt each plot)
- **0.5 pts** Noise case: comments not complete
- **0.5 pts** Decode: (a) wrong conclusion
- **1.5 pts** Decode: x3; wrong LS images (with 0.5pt deducted each)
- **3 pts** Decode: x3; output image wrong (1pt each)
- **3 pts** Decode: x3; missing output image (1pt each)
- **1 pts** Decode: Comment lacks important details
- **6 pts** Decode: x6; decode image wrong (1pt each)
- **4 pts** Decode: x4; output image wrong (1pt each)
- **2 pts** Decode: x2; missing output image (1pt each)
- **2 pts** Decode: x2; output image wrong (1pt each)
- **8 pts** Decode: missing the whole sub-problem

✓ **- 1.5 pts Decode: x3; output does not look correct**

**(0.5 pt each)**

- **1.5 pts** Decode: comments missing important observations
- **1 pts** Decode: comments missing details
- **0.5 pts** Decode: wrong display of images
- **1 pts** Decode: x2; output image quality not correct (0.5 pt each)
- **2 pts** Decode: no comments
- **0.5 pts** Decode: Comment not complete; did not mention LS
- **1.5 pts** Decode: comments missing important details
- **5 pts** Decode: x5; missing plots (1 pt each)
- **30 pts** Missing programming assignment
- **6 pts** Decode: x6; missing plots (1 pt each)
- **6 pts** Noiseless: x6; no plots (1 pt each)
- **2 pts** Noiseless: no comments
- **12 pts** Noisy: x12; no plots (1 pt each)
- **2 pts** Noisy: no comments
- **1.25 pts** Noiselss: x5; minor inconsistency in the phase transition plots (0.25 pt each)

6 LS outputs do not look correct.

7 This pixel does not look consistent with the trend.

8 For normalized error, the scale of N=50 is not consistent with N=20

9 Missing N=100 two plots

10 The probability of recovery rate should be better with a sigma=1e-9

11 Missing one plot for a different noise level.

12 Missing 3 plots.

13 What's your metric to plot the phase transition? The probability of recovery rate should be better with a sigma=1e-9.

2 (a) Projection Matrix: $P = P^T \in \mathbb{R}^{n \times n}$ : — Symmetric; $P = P^2$.

(a) If $P$ is Projection Matrix, then $I-P$ is also one.

Since $I^T = I$ and $P^T = P$ (due to $P$ being Projection Matrix),

$(I-P)^T = I-P$.

$\Rightarrow I-P$ is symmetric.

Consider $(I-P)^2 = I^2 + P^2 - 2 \cdot I \cdot P$

$= I + P - 2P$

$= I - P$.

$\Rightarrow I-P$ is Projection Matrix.

(b) Since columns of $U$ are Orthonormal, $U \in \mathbb{R}^{n \times k}$

$U = [u_1 | u_2 | u_3 \cdots | u_k]$ ; $\langle u_i, u_j \rangle = 0 \ \forall \ i \neq j$ and magnitude of each $u_i = 1$.

$$U \cdot U^T = [u_1 | u_2 | u_3 | \cdots | u_k] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{bmatrix}$$

$= [u_1^2 | u_2^2 | u_3^2 \cdots | u_k^2]$ | $\langle u_i, u_j \rangle = 0$ as they are Orthogonal

But since magnitude of $u_i = 1$, $u_i^2 = 1$.

$\Rightarrow U \cdot U^T = I$.

Similarly, $U^T \cdot U = I$.

$\therefore U \cdot U^T = U^T \cdot U = (U \cdot U^T)^T$ //

To show that $(U \cdot U^T)^2 = U \cdot U^T$. — (2)

Since product of U, and its Transpose is $I$,
$I^2 = I$;
$\Rightarrow$ (2) is satisfied.
This happens because, for an orthonormal matrix, A, $A^{-1} = A^T$.

Thus, $U \cdot U^T$ is a Projection matrix.

(c) $A \in \mathbb{R}^{n \times k}$.
$k \leq n$.
A is full rank.

To show: $A(A^TA)^{-1}A^T$ is a projection matrix.

condition 1: $P^T = P$.

$$[A(A^TA)^{-1}A^T]^T = (A^T)^T((A^TA)^{-1})^T A^T$$
$$= A\,((A^TA)^T)^{-1}\,A^T$$
$$= A\,(A^TA)^{-1}\,A^T$$
$\Rightarrow$ $A(A^TA)^{-1} \cdot A^T$ is symmetric.

Condition 2: $P^2 = P$.

$$[A \cdot (A^T \cdot A)^{-1} A^T]^2 = A \cdot (A^T \cdot A)^{-1} A^T \cdot A \cdot (A^T \cdot A)^{-1} A^T$$
$$= A \underbrace{\{(A^T \cdot A)^{-1} \cdot (A^T \cdot A)\}}_{= I} (A^T \cdot A)^{-1} A^T$$
$$= A \cdot (A^T \cdot A)^{-1} \cdot A^T.$$

$\therefore$ Both conditions are solved. Thus, $A(A^TA)^{-1}A^T$ is Projection Matrix. It is also $((A^TA)^{-1}A^T)$ The pseudo Inverse of A.

2 (d) $y$: orthogonal projection of $x$ onto $S$.

To prove:

If $P$ is projection matrix, $y = Px$ is projection to $R(P)$.

Proof:

$dim(y) \leq n$.

$dim(x) = n$.

let $P \in \mathbb{R}^{k \times n}$ $k \leq n$.

Given $P$ is a projection matrix;

$Y = Px$.

$R(P)$ : subspace $R(P) \subseteq R^{k}$

and $R(P) = \{Px \mid x \in \mathbb{R}^{n}\} \subseteq R^{k}$.

$\Rightarrow Px \in R^{k}$

now we have to show $Px$ is a projection of $x$.

Condition for orthogonal projection.

$$\langle x - Px, p \rangle = 0 \ \forall \ p \in R(P)$$

But, $p$ is nothing but all a linear combination of columns of $P$.

$\Rightarrow$ If, we show,

$\langle x - Px, P \rangle = 0$ $\Rightarrow$ Then our proof is done.

$\Rightarrow P^{T} \cdot (x - Px) = 0$.

$P^{T}(x - Px) = P^{T}x - P^{2}x$. $(\because P^{T} = P, \ P^{T} \cdot P = P^{2})$

$= Px - Px$ $(\because P^{2} = P)$

$= 0$ //

Hence proved.

2) (i) $u$: Unit vector.

To find $P$: Such that $y = Px$ is projection of $x$ onto span $(u)$.

$Span(U) = \alpha_i u$.

If the orthogonal projection of vector $x$ is $\hat{x}$ over $V$, then

$$\langle \hat{x} - x, v \rangle = 0 \quad \forall\, v \in V;\ x \in V.$$

Similarly, If $y = Px$ is orthogonal to $x$,

$\Rightarrow \langle x - Px, \alpha_i u \rangle = 0$

$\Rightarrow \langle \alpha_i u, x - Px \rangle = 0.$

$\Rightarrow \langle \alpha_i u, x \rangle - \langle \alpha_i u, Px \rangle = 0$

$\Rightarrow \alpha_i \langle u, x \rangle - \alpha_i \langle u, Px \rangle = 0.$

$\Rightarrow \langle u, x \rangle - \langle u, Px \rangle = 0.$

$\Rightarrow \langle u, x \rangle = \langle u, Px \rangle$

$\Rightarrow u^T \cdot x = u^T \cdot Px.$

$\Rightarrow u^T (x - Px) = 0$

$\Rightarrow u^T x (1 - P) = 0.$

Since $u$ is Unit Vector, $u^T \neq 0$; $x \neq 0$

$\Rightarrow I - P = 0 \Rightarrow P = I$ will give the required projection of $x$ onto span $(u)$.

## 1 Problem 2 8 / 15

- ✓ **- 0 pts** part a correct
- **- 1 pts** part a minor mistakes
- **- 2 pts** part a some effort
- **- 3 pts** part a error/missing/no sufficient effort
- **- 0 pts** part b correct
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- ✓ **- 3 pts** part b error/missing/no sufficient effort
- **- 0 pts** part c correct
- ✓ **- 1 pts** part c minor mistakes
- **- 2 pts** part c some effort
- **- 3 pts** part c error/missing/no sufficient effort
- **- 0 pts** part d correct
- ✓ **- 1 pts** part d minor mistakes
- **- 2 pts** part d some effort
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- **- 0 pts** part e correct
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1 please check. It is U.TU

2 show if A is full rank A.TA is invertible using simple rank equation.

3 though your meaning is correct, your notation is incorrect mathematically. Please correct. You cannot say inner product of a matrix with a vector in notation.

4 incorrect.

5 If P =I then in the intial step of <x-Px, .> = <x-x, .> = <0, .> = 0.

**Programming assignment- HW3 : OMP**

**Instructions to run code :**

1. For case without noise, run omp_experimental_setup.m
2. For case with noise,
   a. With known sparsity, run omp_expt_Setup_withnoise.m. Here, you need to toggle "sparsity_unknown" variable found in line 45 of the code to 0.
   b. With unknown sparsity, run omp_expt_Setup_withnoise.m. Here, you need to toggle "sparsity_unknown" variable found in line 45 of the code to 1.
3. Please change the "itr" value found in both "omp_experimental_setup.m" and "omp_expt_Setup_withnoise.m" to pick the respective N.
4. Error support recovery vector is in variable : "ESR_cols".
5. Normalized error can be found in variable : "norm_mat" ( only in problem 3).
6. Heatmap can be generated by using the variables in 4 and 5.
7. Code for question 5 is separately placed as "question5.m". Please add the data file with the same name given as under files section into the same folder. I have not uploaded it owing to size constraints.

5. **Decode a Compressed Message:** In this part of the assignment, you will uncover a hidden message from their compressed sketches (generated using random measurement matrices) using the OMP algorithm that seeks the sparsest solution. An unknown and sparse image **X**, containing a message, has been compressed using three different random matrices (of different sizes) **A**1,**A**2,**A**3 to produce three corrupted images as follows

$\mathbf{Y}_i = \mathbf{A}_i\mathbf{X}$

The corrupted images and the measurement matrices are provided to you under Files>Homework.

(a) Can you guess the message by simply displaying the compressed images?

**Ans :** No. We cannot guess the message from the compressed image

(b) Using OMP, recover **X** from **Y**1, **Y**2, **Y**3. Figure out the appropriate stopping criteria.

Reshape the recovered **X** into a 2D image of size 90 $\times$ 160 and decode the message.

Show your results for each case. Compare these results with the Least Squares Solution.

Ans : The least square solution shows a grainy image for the 3 combinations of A and Y and the message is not visible at all.

*Figure 1 A1-Y1*

*Figure 2A2-Y2*

*Figure 3 A3-Y3*

However, with omp, the first combination did not produce any meaningful output. But the second and third cases of A2-y2/A3-y3 produced valid output and the message could be decoded.

**The message was M > Cs log(N/S).**

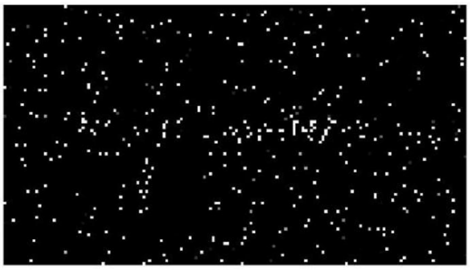


*Figure 4 A1-Y1 OMP*


*Figure 5 A2-Y2 OMP*


*Figure 6 A3-Y3 OMP*

c) Which corrupted image gave you the best result for OMP? Can you explain why?

**Ans:** Both A2-Y2 and A3-Y3 combo gave good results but A3-Y3 had better resolution.

(d) (For Fun) Can you make an (educated) guess about the meaning of this message?

**Ans:** Condition which M and S has to satisfy for OMP to work properly.

3. **Noiseless case: (n = 0)**

Implement OMP (you may stop the OMP iterations once $\|\mathbf{y} - \mathbf{Ax}(k)\|2$ is close to 0 and evaluate its performance. Calculate the probability of Exact Support Recovery (i.e. the fraction of runs when $\hat{S} = S$) by averaging over 2000 random realizations of **A**, as a function of M and smax (for different fixed values of N). For each N, the probability of exact support recovery is a two dimensional plot (function of M and smax) and you can display it as an image. The resulting plot is called the "noiseless phase transition" plot, and it shows how many measurements (M) are needed for OMP to successfully recover the sparse signal, as a function of smax. Do you observe a sharp transition region where the probability quickly transitions from a large value (close to 1) to a very small value (close to 0)? Generate different phase transition plots for the following values of N: 20, 50 and 100. Regenerate phase transition plots for average Normalized Error (instead of probability of successful recovery). Comment on both kinds of plots.

**Answer : Plots for ESR.**

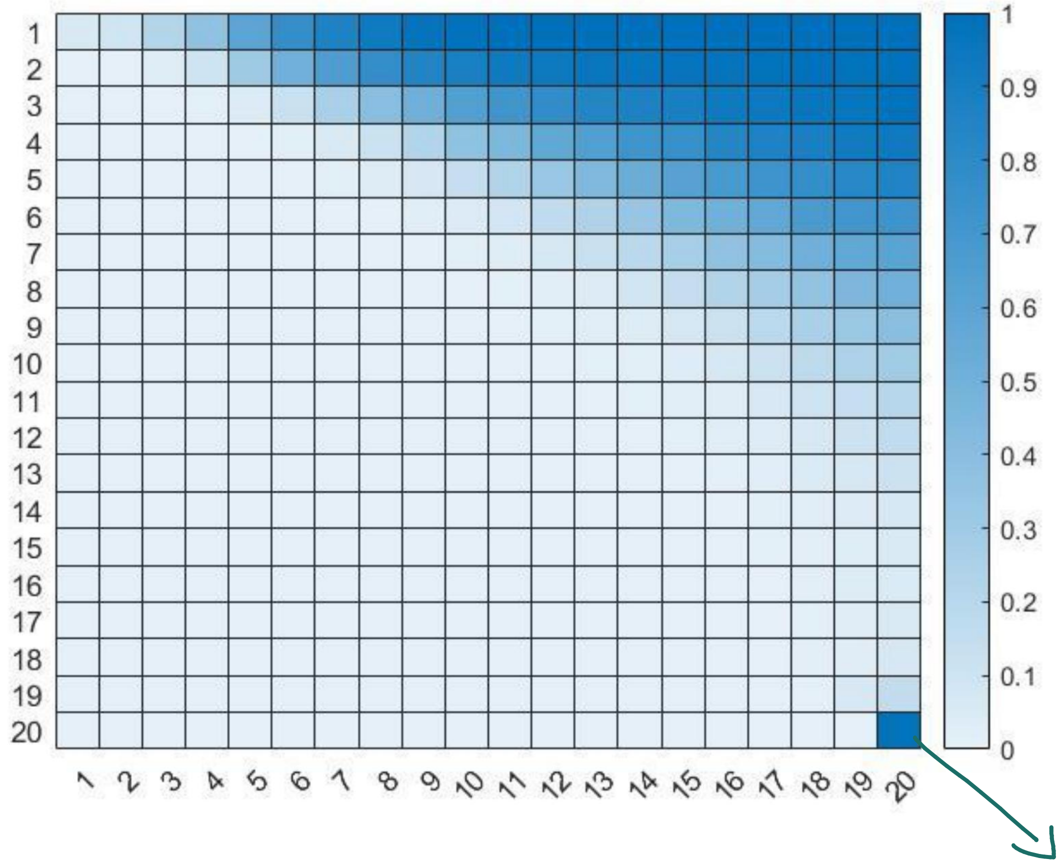


*Figure 7 ESR for N=20. **X-axis : Rows. Y-axis : Sparsity.***

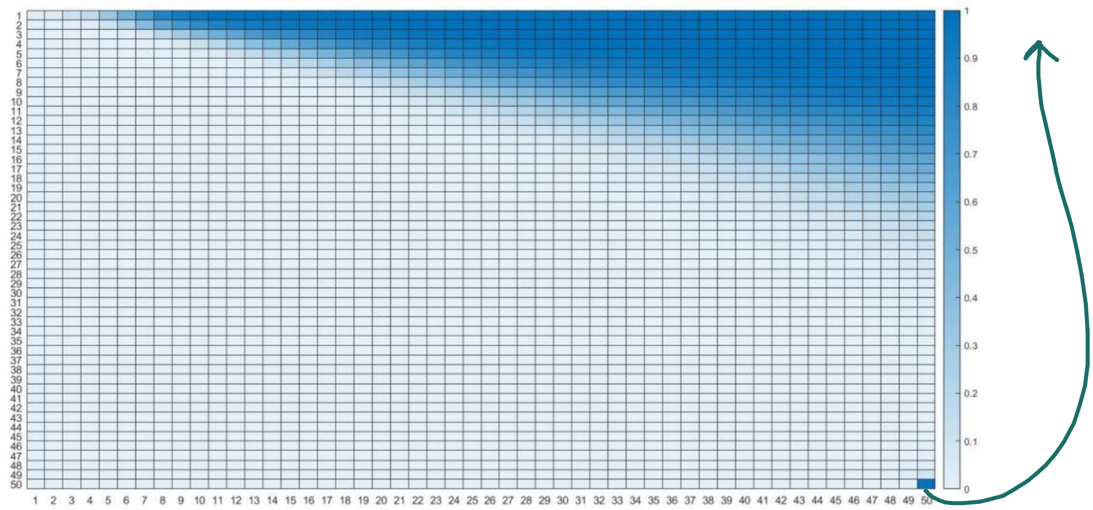
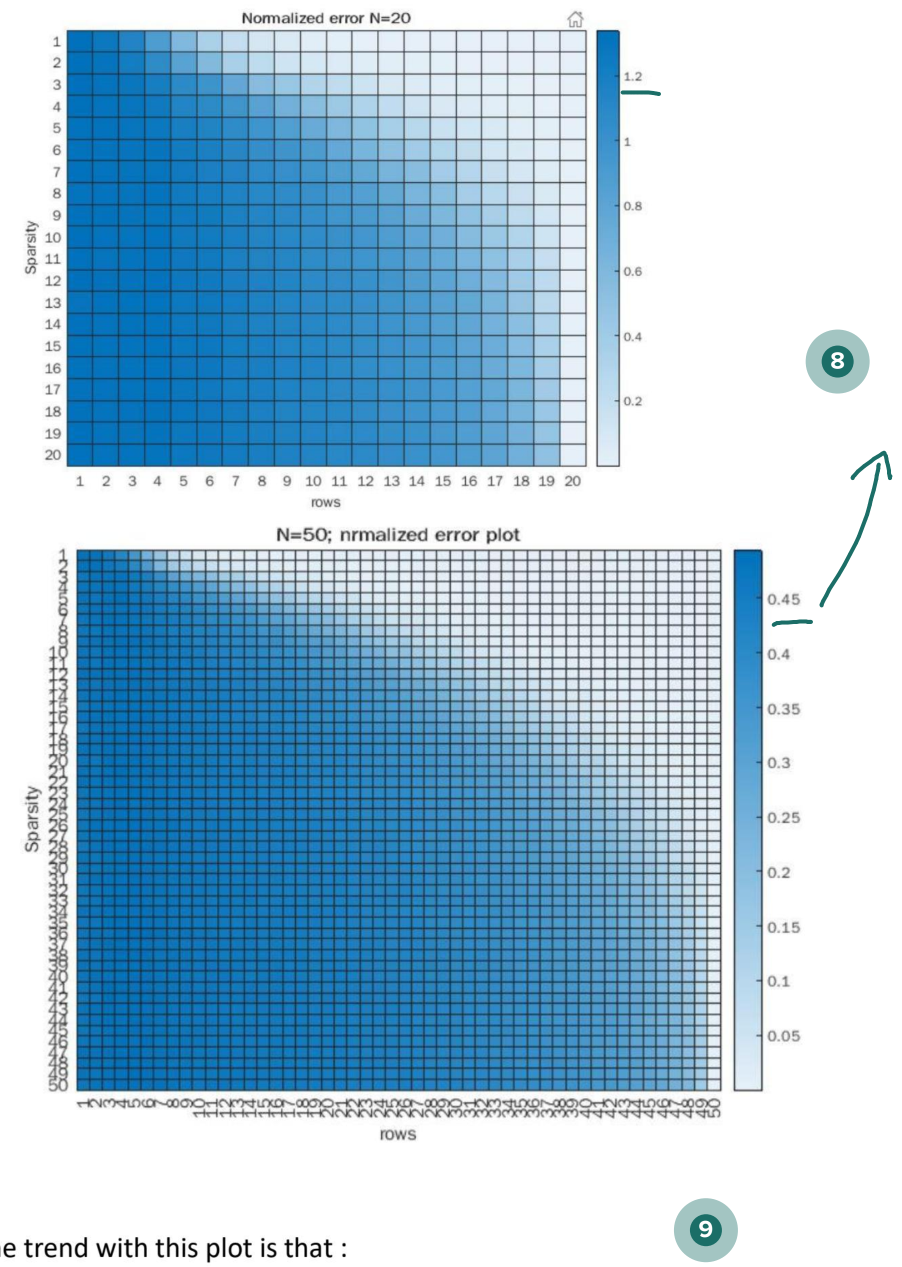


*Figure 8 ESR for N=50, X-axis : Rows, Y axis : Sparsity.*

From the heatmap we can see that when sparsity increases, for lower number of rows, OMP is not able to recover X properly. However, where M is greater than 10, the OMP starts to perform well for low sparse vectors. With a high number of rows, the OMP stops performing well for over sparsity of 20. But a unique case was observed with M and S_max, where OMP performed really well. So, I think OMP can be put to good usage when we want to reduce the number of columns by less than 50% of the columns and with sparsity upto 50% of S_max.

The Normalized error plots is as follows :



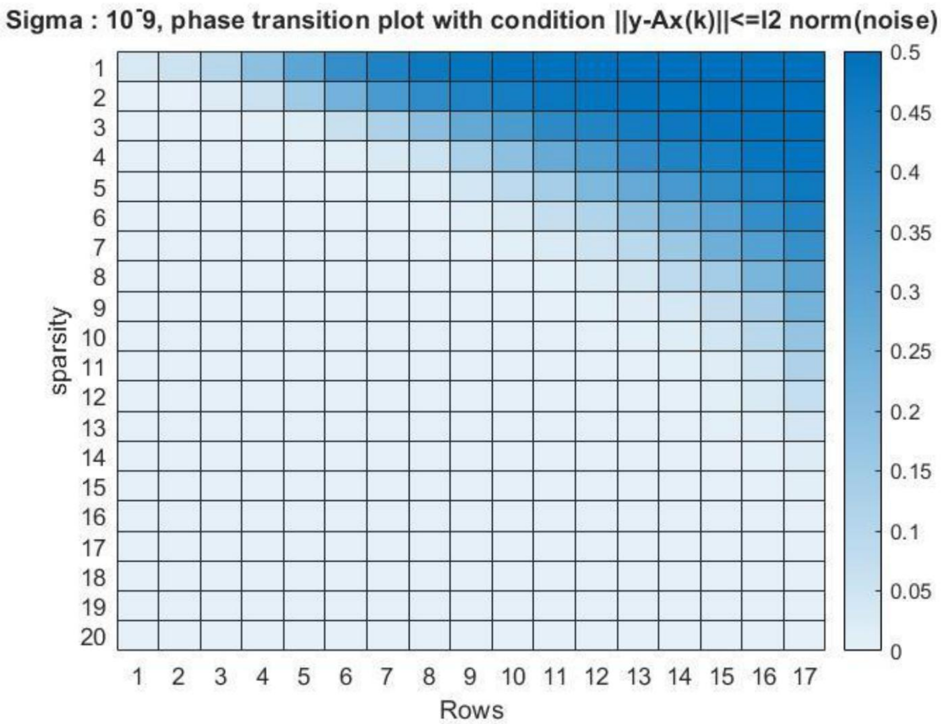

The trend with this plot is that :

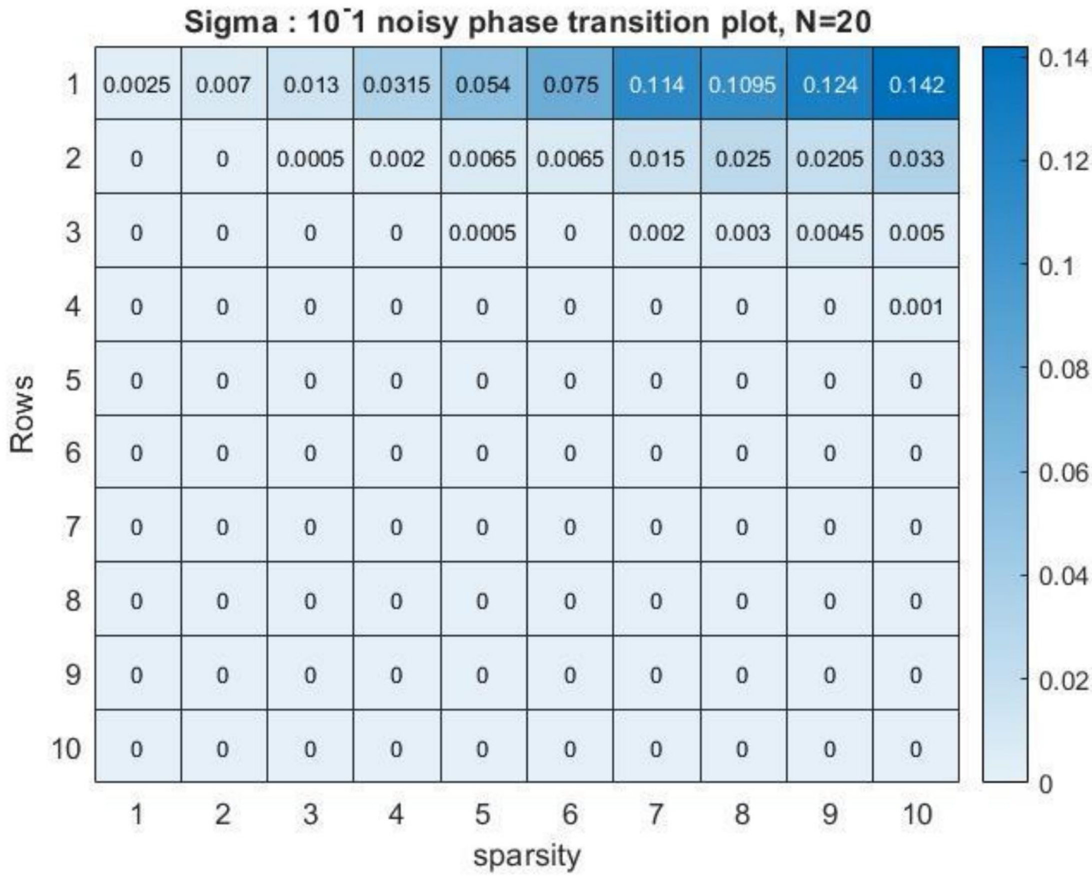
1. Normalized error reduces with increase in M.
2. Normalized error reduces with Increase In S.
3. Since the maximum normalized error achieved is 0.45, we can say that for each (S,M) condition, the OMP estimates upto 50% of the time the correct indices and value.
4. Approximately, if we draw a diagonal on the heat map, for higher value of M, we are seeing lesser error.

5. Again, this implies, OMP can be used to reduce the size of matrix by 50% at max without giving levy to errors with a good amount of sparsity.
6. The value of N doesn’t seem to affect OMP to a greater extent.

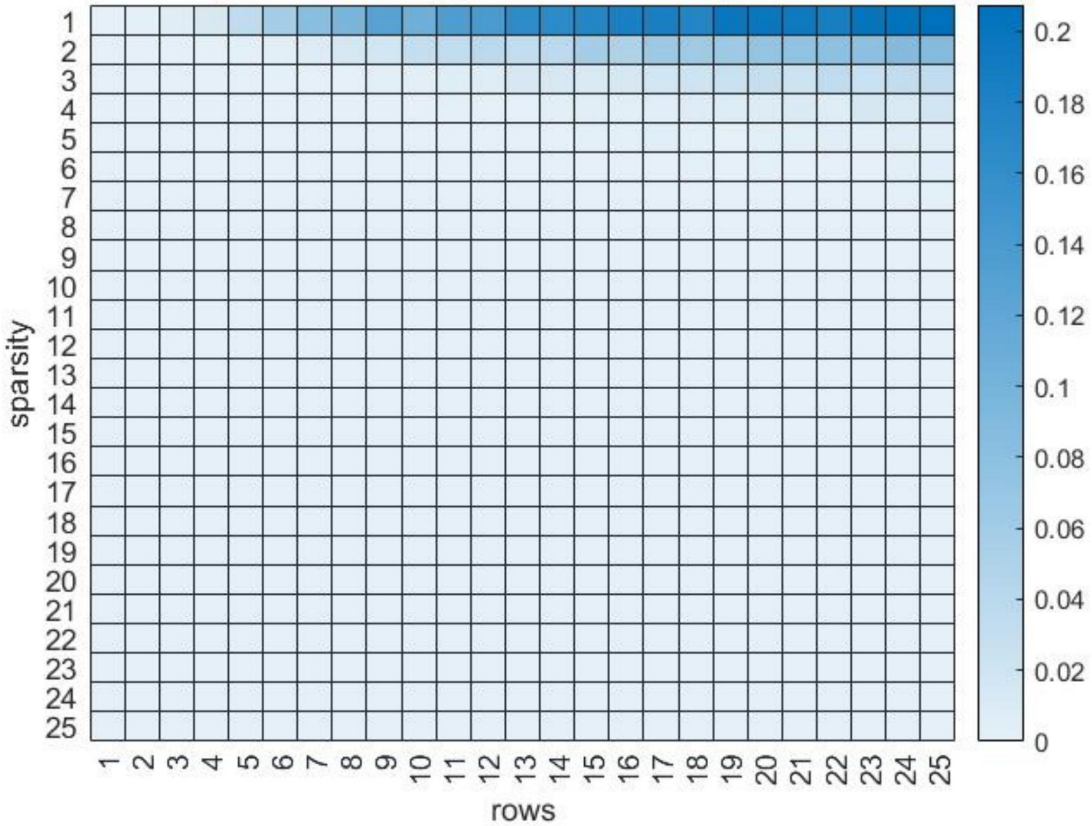
4. **Noisy case: (n** $/=$ **0)**

(a) Assume that sparsity s is known. Implement OMP (terminate the algorithm after first s columns of **A** are selected). Generate “noisy phase transition" plots (for fixed N and $\sigma$) where success is defined as the event that the Normalized Error is less than 10–3. Repeat the experiment for two values of $\sigma$ (one small and one large) and choose N as 20, 50 and 100. Comment on the results.

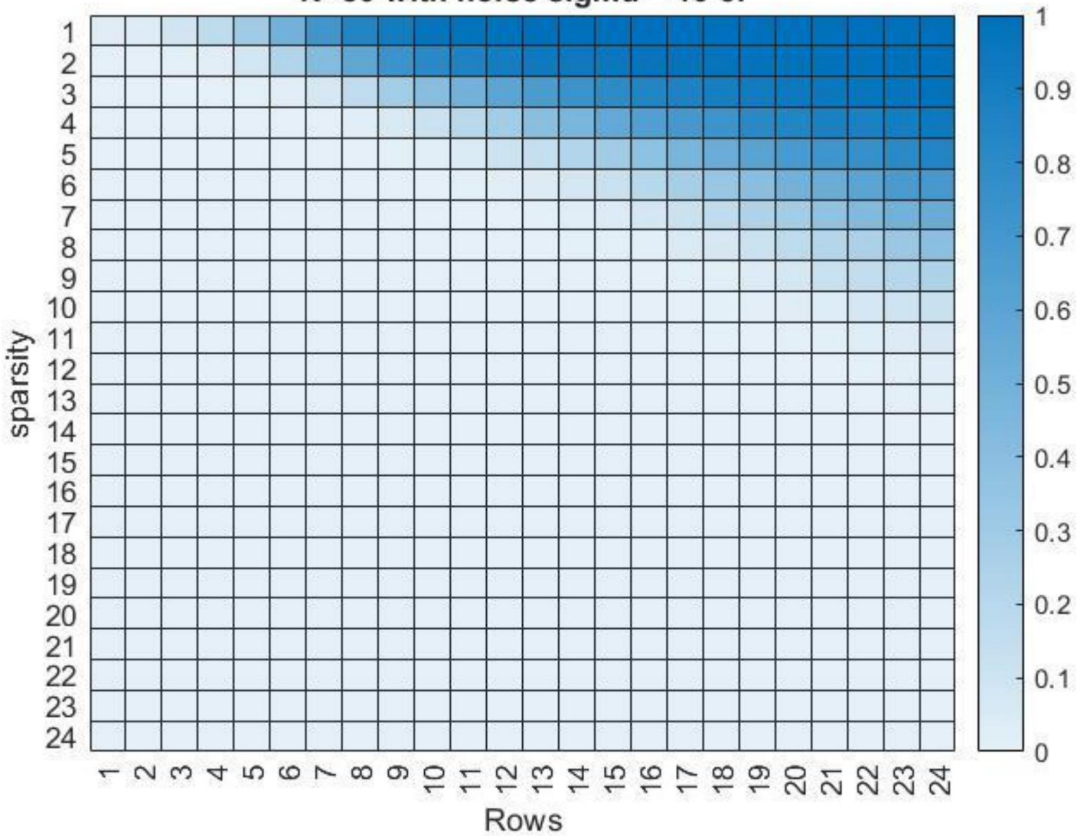

Sigma : 10ˉ9, phase transition plot with condition ||y-Ax(k)||<=l2 norm(noise)
sparsity
Rows
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5


Sigma : 10ˉ1 noisy phase transition plot, N=20
Rows
sparsity
0.0025 0.007 0.013 0.0315 0.054 0.075 0.114 0.1095 0.124 0.142
0 0 0.0005 0.002 0.0065 0.0065 0.015 0.025 0.0205 0.033
0 0 0 0 0.0005 0 0.002 0.003 0.0045 0.005
0 0 0 0 0 0 0 0 0 0.001
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8 9 10
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14

**Sigma : 10ˉ1 noisy phase transition plot, N=50**




**N=50 with noise sigma = 10ˉ9.**



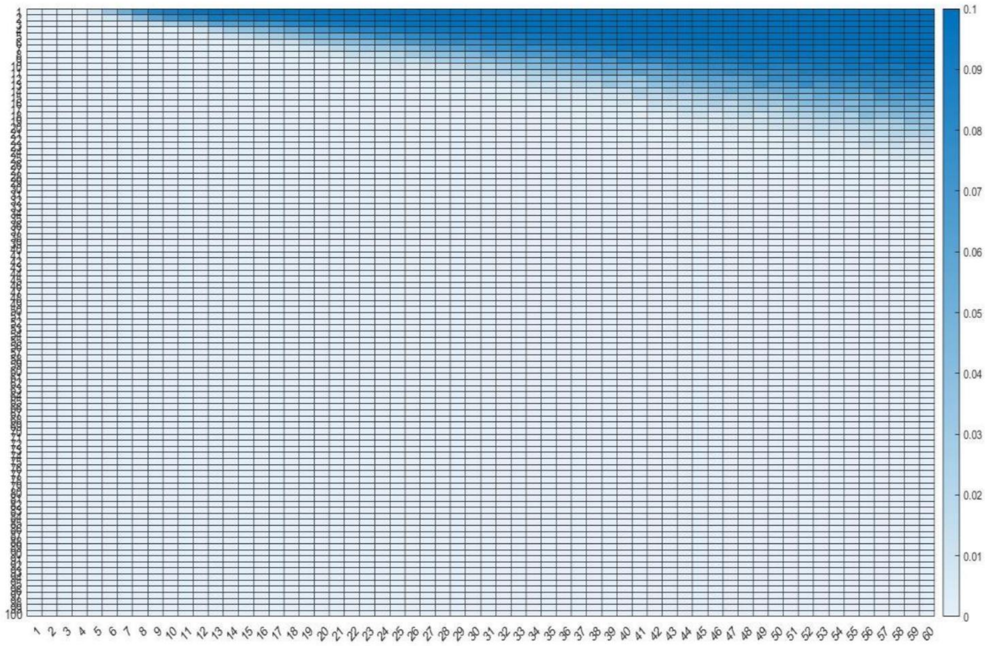


*Figure 9 Known sparsity with noise 10^-9. X-axis: Rows/ Y-axis : sparsity. N=100*



In this case, I had generated two cases of noise with sigma = 10^-9 and 10^-1 and had tested OMP's performance.

Case a: (Low noise ) The OMP was still able to perform well with the ESR matching the noiseless case in most part of the grid. So, in low noise scenario, OMP still manages to extract X with a reasonable accuracy. The trends doesn't seem to vary across N.

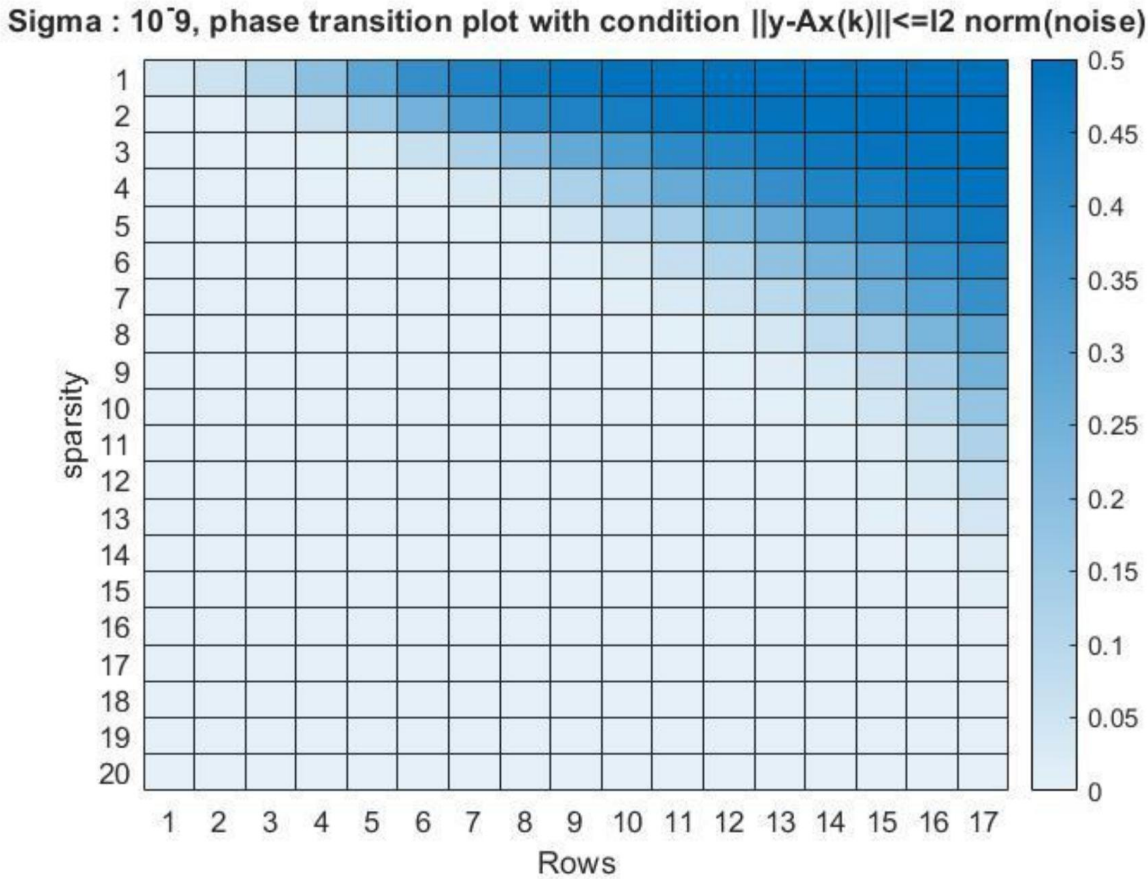
Case b: ( High Noise) In this case, the performance reduced drastically with only low sparse signals being able to be estimated with high confidence for M greater than 10. In other cases, the ESR probability is almost equal to 0.1, rendering OMP inefficient in this case.

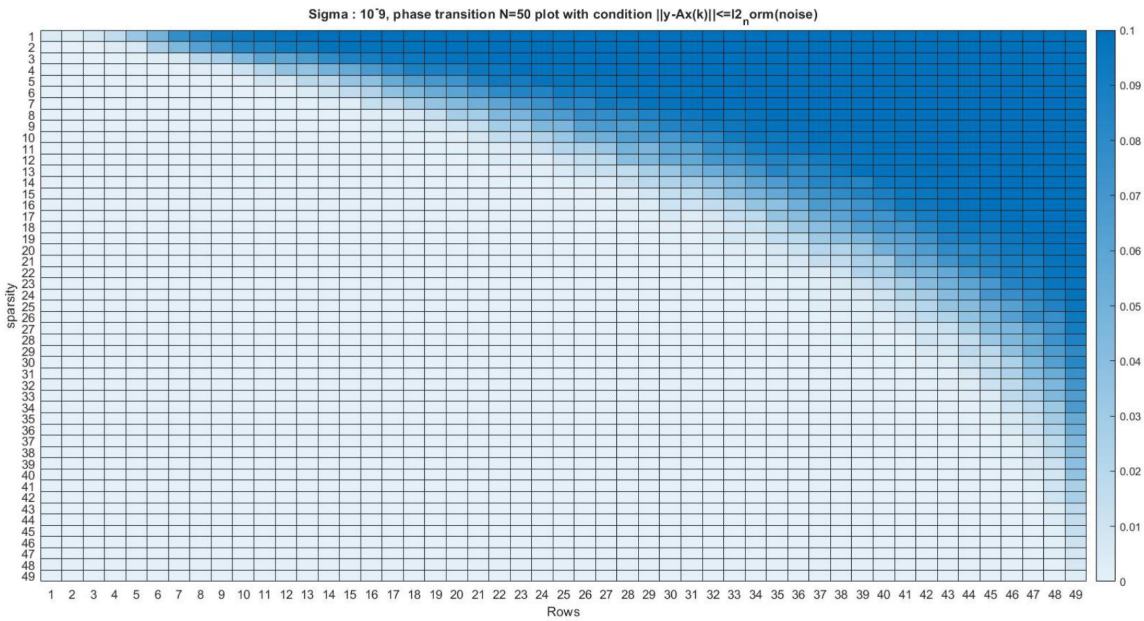
(b) Assume the sparsity s is NOT known, but $\|\mathbf{n}\|2$ is known. Implement OMP where

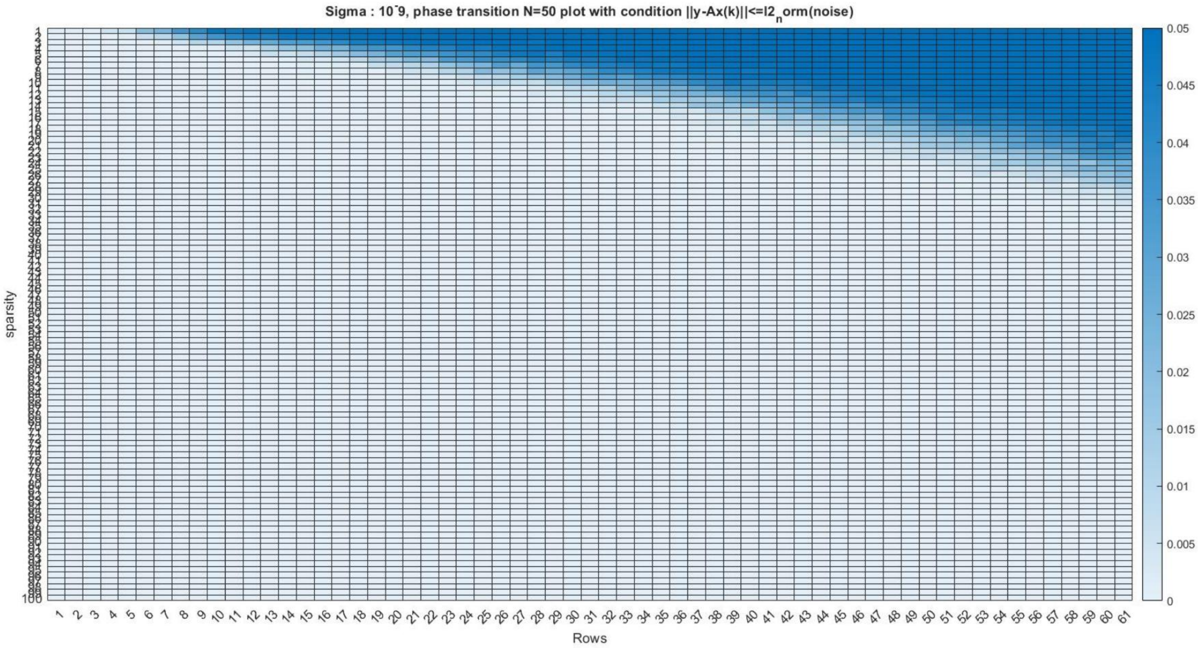
you may stop the OMP iterations once $\|\mathbf{y} - \mathbf{Ax}(k)\| \leq \|\mathbf{n}\|2$). Generate phase transition

plots using the same criterion for success as the previous part. Comment on the

results.


Sigma : 10ˉ9, phase transition plot with condition ||y-Ax(k)||<=l2 norm(noise)
sparsity
Rows


Sigma : 10ˉ9, phase transition N=50 plot with condition ||y-Ax(k)||<=l2_norm(noise)
sparsity
Rows




Answer:  Trends from the simulation :

a. The performance seems to have been improved In this case with OMP able to identify X upto a higher number of sparsity when compared to the normal exit condition used In the previous simulation of Residue being less than 0.001.
b. The performance across M doesn't seem to have changed when compared to previous simulation also.

## 2 Programming Assignment Report 19.5 / 30

- **- 0 pts** Correct
- **- 0 pts** Noiseless case
- **- 1 pts** Noiseless: plot lack of necessary details, for example, colorbar, axis label...
- **- 4.5 pts** Noiseless: x6 plots; change not consistent (0.75 pt each plot)
- **- 0.5 pts** Noiseless: x1; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)
- **- 1.5 pts** Noiseless: x6 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)
- **- 1 pts** Noiseless: comments lack of important details
- **- 3 pts** Noiseless: x4 plots; did not perform enough number of realizations (0.75 pt each plot)
- **- 1.5 pts** Noiseless: comments lack of important details regarding the change in M,s
- **- 2 pts** Noiseless: x4 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)
- **- 0.5 pts** Noiseless: x1 plots; change not consistent (0.5 pt each plot)
- **- 1.5 pts** Noiseless: x3 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)
- **- 2.25 pts** Noiseless: x3 plots; change not consistent (0.75 pt each plot)
- **- 4 pts** Noiseless: x4; too few M tested; and results not consistent (1 pt)
- **- 3 pts** Noiselss: x3; loss of required plots (1 pt each)
- **- 1.5 pts** Noiseless: x3 plots; plot scale does not look right (0.5 pt each plot)
- **- 1.5 pts** Noiseless: x3 plots; change not consistent (0.5 pt each plot)
- **- 1 pts** Noiseless: x2 plots; change not consistent (0.5 pt each plot)
- **- 3 pts** Noiseless: x6 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)
- **- 6 pts** Noiseless: x6 plots; wrong/missing plot (1 pt each)
- **- 1 pts** Noiseless: x2 plots; cannot see the expected trend; or did not perform enough number of realizations (0.5 pt each plot)
- **- 0.25 pts** Noiseless: plot missing colorbar
- **- 0.25 pts** Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)
- **- 2 pts** Noiseless: x4 plots; plots do not look consistent (0.5 pt each plot)
- **- 1 pts** Noiseless: x2 plots; plot scale does not look right (0.5 pt each plot)
- **- 0.5 pts** Noiseless: comment missing details
- **- 1.5 pts** Noiseless: x2 plots; cannot see the trend/too few M,s performed (0.75 pt each plot)

✓ **- 0.5 pts Noiseless: x1 plot; change not consistent (0.5 pt each plot)**

✓ **- 0.5 pts Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)**

✓ **- 2 pts Noiselss: x2; loss of required plots (1 pt each)**

- **- 0.5 pts** Noiselss: x2; minor inconsistency in the phase transition plots (0.25 pt each)
- **- 2 pts** Noiseless: no comment
- **- 0.75 pts** Noiseless: x1; plots not look correct (0.75 pt each plot)
- **- 0.5 pts** Noiselss: x2; too few M, s (0.25 pt each)
- **- 4 pts** Noiseless: x4; missing plot (1 pt)
- **- 2.5 pts** Noiseless: x5 plots; change not consistent (0.5 pt each plot)
- **- 1 pts** Noiseless: x1; missing plot (1 pt)

- **0.75 pts** Noiselss: x3; minor inconsistency in the phase transition plots (0.25 pt each)
- **1 pts** Noiseless: x4 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)
- **0.75 pts** Noiselss: x3; minor inconsistency in the phase transition plots (0.25 pt each)
- **1.25 pts** Noiselss: x5; minor inconsistency in the phase transition plots (0.25 pt each)
- **0.5 pts** Noiseless: x1 plots; plot scale not correct (0.5 pt each plot)
- **1 pts** Noiseless: comments missing details
- **1 pts** Noiseless: x4 plots; x/y-axis do not have enough values to observe a complete view of trend/minor inconsistency (0.25 pt each plot)
- **1 pts** Noiseless: plots have error in colorbar/axis label...
- **3 pts** Noiseless: x4 plots; plots not look correct (0.75 pt each plot)
- **1.5 pts** Noiseless: x6 plots; minor inconsistency in the transition plots (0.25 pt each plot)
- **1 pts** Noisy: plot lack of necessary details, for example, colorbar, axis label...
- **3 pts** Noisy: x6 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **4 pts** Noisy: x8 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **2.25 pts** Noisy: x9 plots; x/y-axis do not have enough values to observe a complete view of trend (0.25 pt each plot)
- **3 pts** Noisy: x4;  did not perform enough number of  realizations (0.75 pt each plot)
- **12 pts** Noise case: b) plots wrong
- **4 pts** Noisy: x4; missing plots (1 pt each)
- **6 pts** Noisy: x12 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **9 pts** Noisy: x9; wrong plots (1 pt each)
- **3 pts** Noisy: x3; lack of required plots
- **9 pts** Noisy: x12; plots wrong (0.75 pt each)
- **1.5 pts** Noisy: x3 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **2 pts** Noisy: x4 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **3 pts** Noisy: x3; missing 3 plots in part(a) or (b)
- **2 pts** Noisy: no comment
- **1.5 pts** Noisy: x3 plots; change not consistent (0.5 pt each plot)
- **3 pts** Noisy: x6 plots; part(a), performance change not consistent (0.5 pts each)
- **1.5 pts** Noisy: x2 plots; cannot observe the trend (0.75 pt each plot)
- **4.5 pts** Noisy: x6; plots wrong (0.75 pt each)
- **1 pts** Noisy: x2 plots; change not consistent (0.5 pt each plot)
- **4.5 pts** Noisy: x6 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.75 pts each)
- **2 pts** Noise case: no comments
- **6 pts** Noisy: x6; missing 6 plots in part(a) or (b)
- **1.5 pts** Noisy: x6 plots; change not consistent (0.25 pt each plot)
- **1.5 pts** Noisy: x6 plots; part(a)/part(b), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.25 pts each)

- **4.5 pts** Noisy: x6 plots; change on the phase transition plots not consistent (0.75 pts each)
- **9 pts** Noisy: x12; wrong setting of plots(0.75 pt each)
- **1.5 pts** Noise case: comments not complete

✓ - **2 pts** **Noisy: x4 plots; change not consistent (0.5 pt each plot)**

- **5 pts** Noisy: x10 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **0.5 pts** Noisy: x2; minor inconsistency in the phase transition plots (0.25 pt each)
- **7 pts** Noisy: x7; missing/wrong plots (1 pt each)
- **4.5 pts** Noisy: x6 plots; cannot observe the trend (0.75 pts each)
- **3 pts** Noisy: x4 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.75 pts each)

✓ - **4 pts** **Noisy: x4; missing plots (1 pt each)**

- **5 pts** Noisy: x10 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.5 pt each plot)
- **1.5 pts** Noisy: x6 plots; minor inconsistency (0.25 pt each plot)
- **1 pts** Noisy: x4 plots; x/y-axis do not have enough values to observe a complete view of trend/minor inconsistency (0.25 pt each plot)
- **3 pts** Noisy: x6 plots; part(a), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.5 pts each)
- **1 pts** Noisy: x4 plots; part(a)/part(b), cannot observe significant difference between the two noise levels/choice of sigma not appropriate (0.25 pts each)
- **7 pts** Noisy: x7; wrong/missing plots(1 pt each)
- **0.75 pts** Noisy: x3; minor inconsistency in the phase transition plots (0.25 pt each)
- **0.5 pts** Noisy: plot lack of necessary details, for example, colorbar, axis label...
- **9 pts** Noisy: x9; wrong plots(1 pt each)
- **9 pts** Noisy: x12 plots; cannot observe the trend (0.75 pt each plot)
- **4 pts** Noisy: x4; missing 4 plots in part(a) or (b)
- **1.5 pts** Noisy: x6 plots; cannot see the expected trend clear enough; or did not perform enough number of realizations (0.25 pt each plot)
- **6 pts** Noisy: x8; plots wrong (0.75 pt each)
- **1 pts** Noise case: comments not complete
- **1.25 pts** Noisy: x5 plots; change not consistent (0.25 pt each plot)
- **0.5 pts** Noise case: comments not complete
- **0.5 pts** Decode: (a) wrong conclusion
- **1.5 pts** Decode: x3; wrong LS images (with 0.5pt deducted each)
- **3 pts** Decode: x3; output image wrong (1pt each)
- **3 pts** Decode: x3; missing output image (1pt each)
- **1 pts** Decode: Comment lacks important details
- **6 pts** Decode: x6; decode image wrong (1pt each)
- **4 pts** Decode: x4; output image wrong (1pt each)
- **2 pts** Decode: x2; missing output image (1pt each)
- **2 pts** Decode: x2; output image wrong (1pt each)
- **8 pts** Decode: missing the whole sub-problem

✓ - **1.5 pts** **Decode: x3; output does not look correct (0.5 pt each)**

- **1.5 pts** Decode: comments missing important observations

- **1 pts** Decode: comments missing details
- **0.5 pts** Decode: wrong display of images
- **1 pts** Decode: x2; output image quality not correct (0.5 pt each)
- **2 pts** Decode: no comments
- **0.5 pts** Decode: Comment not complete; did not mention LS
- **1.5 pts** Decode: comments missing important details
- **5 pts** Decode: x5; missing plots (1 pt each)
- **30 pts** Missing programming assignment
- **6 pts** Decode: x6; missing plots (1 pt each)
- **6 pts** Noiseless: x6; no plots (1 pt each)
- **2 pts** Noiseless: no comments
- **12 pts** Noisy: x12; no plots (1 pt each)
- **2 pts** Noisy: no comments
- **1.25 pts** Noiselss: x5; minor inconsistency in the phase transition plots (0.25 pt each)

6 LS outputs do not look correct.

7 This pixel does not look consistent with the trend.

8 For normalized error, the scale of N=50 is not consistent with N=20

9 Missing N=100 two plots

10 The probability of recovery rate should be better with a sigma=1e-9

11 Missing one plot for a different noise level.

12 Missing 3 plots.

13 What's your metric to plot the phase transition? The probability of recovery rate should be better with a sigma=1e-9.