ANNEXURE - I

ANNA UNIVERSITY
CHENNAI - 25

College Code	9	6	2	5								
College Name	Rajas Engineering College											
Register Number	9	6	2	5	1	6	1	0	4	0	0	1
Name of the Candidate	H. Ajithkumar											
Degree	B.E											
Branch	CSE				Semester	5th						
Question Paper Code	X	2	0	7	8	8						
Subject Code	M	A	6	5	6	6						
Subject Name	Discrete Mathematics											
Date	08	07	2021	Session	FN	AN ✓						
No. of Pages used	20	In words	Twenty									

All particulars given above by me are verified and found to be correct

Signature of the Student with date	H. Ajith. 08/07/2021

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Instructions to the Candidate: Put Tick mark (✓) for the questions attended in the tick mark column against each question

PART – A			PART – B & C								Grand Total (in words)
Question No.	✓	Marks	Question No.		(i) ✓	(i) Marks	(ii) ✓	(ii) Marks	(iii) ✓	(iii) Marks	
1	✓		11	a	✓		✓				
2	✓			b							
3	✓		12	a							
4	✓			b	✓		✓				
5	✓		13	a							
6	✓			b	✓		✓				
7	✓		14	a	✓		✓				
8	✓			b							
9	✓		15	a							
10	✓			b	✓		✓				Grand Total
			16	a							
				b							
Total											

Declaration by the Examiner: Verified that all the questions attended by the student are valued and the total is found to be correct

Date	Name of the Examiner	Signature of the Examiner

Reg.No: 962516104001 Sub code: MA6566

Name: H.Ajithkumar Sub name: Discrete Mathematics

Part-A

1) $T \leftrightarrow T \wedge F$

Soln

$T \leftrightarrow F$

$= (T \to F) \wedge (F \to T)$

$= (\neg T \vee F) \wedge (\neg F \vee T)$

$= (F \vee F) \wedge (T \vee T)$

$= F \wedge T$

$= F.$

2)

Soln

R: It rains today

U: I buy an umbrella

Then, the given statement in symbolic term is $R \to U$

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2) Field

A Commutative ring with identity $(R,+,\cdot)$ is called a field if every non-zero element has a multiplicative inverse.

Thus $(R,+,\cdot)$ is a field if

1) $(R,+)$ is abelian group and
2) $(R-\{0\},\cdot)$ is also abelian group

3) The word 'ENGINEERING' contain 11 terms, namely 3N, 3E, 3N, 2G & 1R

$\therefore$ Required number of arrangements $= \dfrac{11!}{(3!)(3!)(2!)(2!)(1!)} = 277200$

4) If n Pigeonholes are occupied by n+1 or more Pigeons then at least one Pigeonhole is occupied by greater than one Pigeon. Generalized Pigeonhole Principle is: if n Pigeonholes are occupied by kn+1 or more Pigeons, where k is a positive integer, then at least one pigeonhole is so occupied by k+1 or more Pigeons.

5) complete graph

A graph G is said to be complete if every vertex in G is connected to every other vertex in G. Thus a complete graph G must be connected in G.

7)

let $(G, *)$ be a group

Let $a \in G$ and e be the identify of G. Let a_1^{-1} and a_2^{-1} the two different inverse of the same element

$$a_1^{-1} * a = a * a_1^{-1} = e$$

$$a_2^{-1} * a = a * a_2^{-1} = e$$

Now $(a_1^{-1} * a) * a_2^{-1} = e * a_2^{-1} = a_2^{-1}$ — (1)

$a_1^{-1} * (a * a_2^{-1}) = a_1^{-1} * e = a_1^{-1}$ — (2)

From (1) & (2), we get

$$a_1^{-1} = a_2^{-1}$$

There will be no two different inverse for the same element.

9)

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$
$(1,4), (2,5), (3,6), (4,1), (5,2), (6,3)$

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10) Soln

Since $a \wedge b$ is the GLB of $\{a, b\}$, we have

$$a \wedge b \leq a \quad \text{--- (1)}$$

Obviously $a \leq a$ — (2)

From (1) & (2), we have

$$a \vee (a \wedge b) \leq a \quad \text{--- (3)}$$

By the difination of LUB, we have

$$a \leq a \vee (a \wedge b) \quad \text{--- (4)}$$

By combining (3) & (4), we have

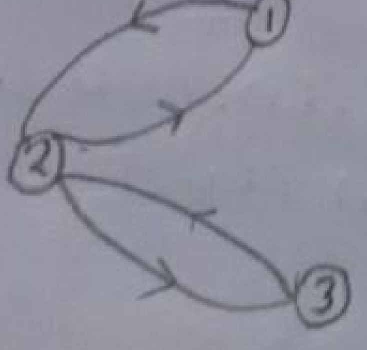
$$a \vee (a \wedge b) = a$$

Similary we can prove

$$a \wedge (a \vee b) = a$$

∴ absorption law is valid in a Boolean Algebra.

b) Soln



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Part-B

11) a) i) $(P \wedge Q) \vee (\sim P \wedge R)$

soln.

P	Q	R	$\neg P$	$P \wedge Q$	$\neg P \wedge R$	$(P \wedge Q) \vee (\neg P \wedge R)$	Minterms	Maxterms
T	T	T	F	T	F	T	$P \wedge Q \wedge R$	-
T	T	F	F	T	F	T	$P \wedge Q \wedge \neg R$	-
T	F	T	F	F	F	F	-	$\neg P \vee Q \vee \neg R$
T	F	F	F	F	F	F	-	$\neg P \vee Q \vee R$
F	T	T	T	F	T	T	$\neg P \wedge Q \wedge R$	-
F	T	F	T	F	F	F	-	$P \vee \neg Q \vee R$
F	F	T	T	F	T	T	$\neg P \wedge \neg Q \wedge R$	-
F	F	F	T	F	F	F	-	$P \vee Q \vee R$

The PDNF is

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

$(P \wedge Q) \vee (\neg P \wedge R)$

$\Rightarrow ((P \wedge Q) \wedge T) \vee ((\neg P \wedge R) \wedge T)$ $(\because P \wedge T = P)$

$\Rightarrow ((P \wedge Q) \wedge (R \wedge \neg R)) \vee ((\neg P \wedge R) \wedge (Q \vee \neg Q))$ $(\because P \vee \neg P = T)$

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (P \wedge R \vee \neg Q))$

$\Rightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

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is the required PDNF

The required PCNF is

$(\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$

11) a) ii) soln

{1}	1) $P \rightarrow M$	Rule P
{2}	2) $\neg M$	Rule P
{1,2}	3) $\neg P$	Rule T $(\neg Q, P \rightarrow Q \Rightarrow \neg P)$
{4}	4) $P \vee Q$	Rule P
{4}	5) $\neg P \rightarrow Q$	Rule T $(P \rightarrow Q \Leftrightarrow \neg P \vee Q)$
{1,2,4}	6) Q	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
{7}	7) $Q \rightarrow R$	Rule P
{1,2,4,7}	8) R	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
{1,2,4,7}	9) $R \wedge (P \vee Q)$	Rule T $(P, Q \Rightarrow P \wedge Q)$

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12)
b) i) Soln

Let A be one of the six People
The Remaining 5 People can be accommodate into
2 groups
i.e. 1. Friends of A and
2. Enemies of A
Now, by generalized pigeon hole principle atleast
one of the group must contain

$$\left\lceil \frac{5-1}{2} \right\rceil + 1 = 3 \text{ People}$$

$\Rightarrow$ Atleast three must be mutual friends
or atleast three must be mutual strangers.
Hence Proved.

1a)
b)ii) Soln

1) $6C_3 \times 7C_4 = 700$

2) Four the committe of atleast one, women, the following are the Possibilities

i) $1W + 3M = 4P$
ii) $2W + 2M = 4P$
iii) $3W + 1M = 4P$
iv) $4W + 0M = 4P$

Therefore the selection can be done in

$= (7C_1)(6C_3) + (7C_2)(6C_2) + (7C_3)(6C_1) + (7C_4)(6C_0)$

$= 140 + 315 + 210 + 35$

$= 700$

3) 4 Person that has atmost one man

i) $1M + 3W = 4P$
ii) $0M + 4W = 4P$

Therefore, the selection can be done.

$= (6C_1)(7C_3) + (6C_0)(7C_4)$

$= 245$

4) 4 Person that has children of the both sexes

i) $1M + 3W = 4P$

ii) $2M + 2W = 4P$

iii) $3M + 1W = 4P$

Therefore the selections can be done

$$= (6C_1)(7C_3) + (6C_2)(7C_2) + (6C_3) + (7C_1)$$

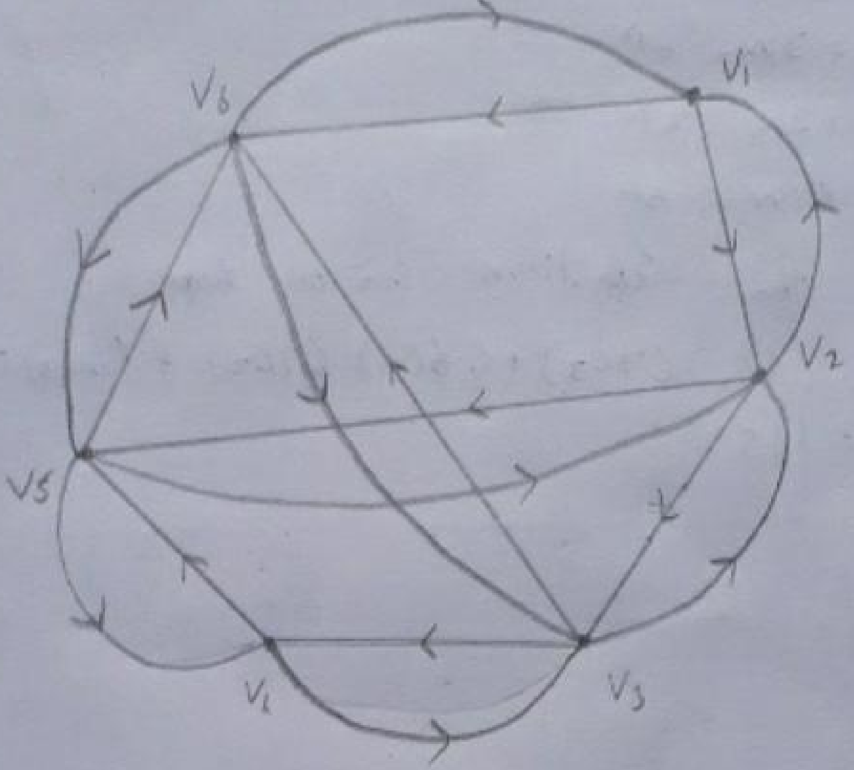
$$= 665$$

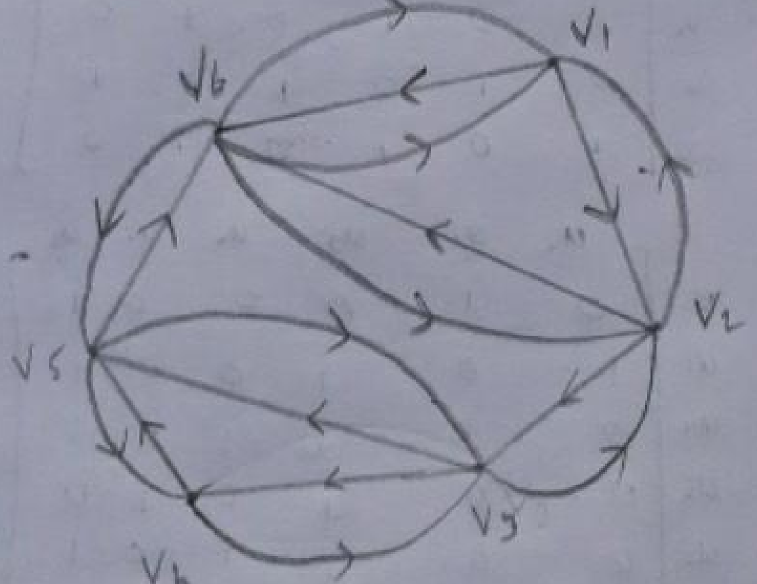
13) b) i) Soln

$$G_1 = \begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 0 & 0 & 0 & 1 \\ v_2 & 1 & 0 & 1 & 0 & 1 & 0 \\ v_3 & 0 & 1 & 0 & 1 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ v_5 & 0 & 1 & 0 & 1 & 0 & 1 \\ v_6 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$$

$$G_2 = \begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline u_1 & 0 & 1 & 0 & 0 & 0 & 1 \\ u_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ u_3 & 0 & 1 & 0 & 1 & 1 & 0 \\ u_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ u_5 & 0 & 0 & 1 & 1 & 0 & 1 \\ u_6 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$A(G_1) \neq A(G_2)$





13)b)
ii) Soln

Let $G = \{V, E\}$ be any graph with 'n' number of vertices and 'e' number of edges

Let $V_1, V_2, \dots V_k$ be the vertices of odd degree and $V_1', V_2', \dots, V_m'$ be the vertices of even degree.

To Prove, k is even

we know that $\sum_{i=1}^{n} d(V_i) = 2|E| = 2e$

$$\Rightarrow \sum_{i=1}^{k} d(V_i) + \sum_{j=1}^{m} d(V_j') = 2e$$

Each of $d(V_j)$ is even $\Rightarrow \sum_{j=1}^{m} d(V_j')$ and $2e$ are even numbers (being the sum of even numbers)

$\therefore \sum_{i=1}^{k} d(V_i) +$ an even number = an even number

$\sum_{i=1}^{k} d(V_i)$ = an even number

Since, each term $d(v_i)$ is odd

Therefore, the number of terms in the LHS sum must be even.

$\Rightarrow$ K is even

Hence the theorem.

14)
a) i) $(a * b)^{-1} = b^{-1} * a^{-1}$

soln

let a, b be only two elements of a group $(G, *)$ By closure axiom $a * b \in G$ By the Existence of inverse element axiom, $a^{-1}, b^{-1}, (a * b)^{-1}$ Exist in G.

To Prove : $(a * b)^{-1} = b^{-1} * a^{-1}$

Proof: By Closure axioms $b^{-1}, a^{-1} \in G \Rightarrow b^{-1} * a^{-1} \in G$

Consider $(b^{-1} * a^{-1}) * (a * b) = b^{-1} * (a^{-1} * a) * b$

$= b^{-1} * e * b$

$= b^{-1} * e * b$

$= b^{-1} * b \quad [\therefore e * b = b]$

$= e$ — ①

Also

$(a * b) * (b^{-1} * a^{-1}) = a * (b * b^{-1}) * a^{-1}$

$= a * e * a^{-1}$

$= a * a^{-1} \quad [e * a^{-1} = a^{-1}]$

$= e$ — ②

By ① & ② we get

$(a * b) * (b^{-1} * a^{-1}) = (b^{-1} * a^{-1}) * (a * b) = e$

$[\text{ie } a * a^{-1} = a^{-1} * a = e]$

$\therefore (a * b)^{-1} = b^{-1} * a^{-1}$

14) a) ii) Proof

let $(M, *)$ be a given Monoid and S be the set of all idempotent elements of M

Claim

$(S, *)$ is submonoid

Since the element of S are taken form of M, we have $S \subseteq M$

Subclaim-1 : $e \in S$

Since the identity element 'e' satisfies $e * e = e$. becomes idempotents

$\therefore e \in S$

Sub claim-2: $a, b \in S \Rightarrow a * b \in S$

Assume $a, b \in S$

$\therefore a * a = a$ and $b * b = b$ — ①

Now $(a * b) * (a * b) = a * (b * (a * b))$

$= a * (a * b * b)$

$= a * (a * b)$

$= (a * a) * b)$

$= a * b$

$(a * b) * (a * b) = a * b$

Theer fore $(a * b)$ is idempotent

$a * b \in G$

we can easily prove $*$ is associative in S

$\therefore S$ is submonoid

The set of all idempotents forms a submonoid.

15)
b) i) Boolean Algebra

Soln

Demorgan's law:

3. $(a\wedge b)' = a' \vee b'$ (or) $\overline{(a\wedge b)} = \bar{a} \vee \bar{b}$

4. $(a \vee b)' = a' \wedge b'$ (or) $\overline{(a \vee b)} = \bar{a} \wedge \bar{b}$

proof

Claim-1 : $(a\wedge b)' = a' \vee b'$

To Prove the above, it is enought to Prove that

1. $(a\wedge b) \wedge (a' \vee b') = 0$

2. $(a\wedge b) \vee (a' \vee b') = 1$

1. $(a\wedge b) \wedge (a' \vee b')$

$\Rightarrow ((a\wedge b)\wedge a') \vee ((a\wedge b)\wedge b')$ (Distributive Rule)

$\Rightarrow (a\wedge a' \wedge b) \vee (a\wedge b\wedge b')$ (Associative Rule)

$\Rightarrow (0\wedge b) \vee (a\wedge 0)$ $(a\wedge a' = 0)$

$\Rightarrow 0 \vee 0$

$\Rightarrow 0$

$\therefore (a \wedge b) \wedge (a' \vee b') = 0$ —— ①

2. $(a \wedge b) \vee (a' \wedge b')$

$\Rightarrow (a \vee (a' \vee b')) \wedge (b \vee (a' \vee b'))$ (Distributive Rule)

$\Rightarrow (a \vee a' \vee b) \wedge (a' \vee b \vee b')$ (Associative rule)

$\Rightarrow (1 \vee b) \wedge (a' \vee 1)$ $(a \vee a' = 1)$

$\Rightarrow (1 \wedge 1)$ $(a \vee 1 = 1)$

$\Rightarrow 1$

$\therefore (a \wedge b) \vee (a' \vee b')$ —— ②

From ① & ②

$(a \wedge b)' = a' \vee b$

claim-2: $(a \vee b)' = a' \wedge b'$

It is enough to prove that

1. $(a \vee b) \wedge (a' \wedge b') = 0$

2. $(a \vee b) \vee (a' \wedge b') = 1$

1. $(a \vee b) \wedge (a' \wedge b')$

$\Rightarrow (a \wedge (a' \wedge b')) \vee (b \wedge (a' \wedge b'))$ (Distributive Rule)

$\Rightarrow (a \wedge a' \wedge b') \vee (b \wedge b' \wedge a')$ (Associative Rule)

$\Rightarrow (0 \wedge b') \vee (0 \wedge a')$ $(a \wedge a' = 0)$

$\Rightarrow 0 \vee 0 = 0$

$\therefore (a \vee b) \wedge (a' \wedge b') = 0$ —— ③

2. $(a \vee b) \vee (a' \wedge b')$

$\Rightarrow ((a \vee b) \vee a') \wedge ((a \vee b) \vee b')$ (Distributive Rule)

$\Rightarrow (a \vee a' \vee b) \wedge (a \vee b \vee b')$ (Associative Rule)

$\Rightarrow (1 \vee b) \wedge (a \vee 1)$ $(a \vee a' = 0)$

$\Rightarrow 1 \wedge 1 = 1$ (idempotent law)

$\therefore (a \vee b) \vee (a' \wedge b') = 1$ —— ④

From ③ and ④

$(a \vee b)' = a' \wedge b'$

15)
b) ii) Soln

We know that every chain is a distributive lattices ~~and~~ consider a chain $(L, \wedge, \vee)$

Hence, assume let $(L, \wedge, \vee)$ be the given distributive lattice

D_1 : $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ holds good —— ①

$\forall a, b, c \in L$

Now, if $a \leq c$, then $a \vee c = c$

$$(1) \Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$= (a \vee b) \wedge c$$

if $a \leq c$ then $a \vee (b \wedge c) = (a \vee b) \wedge c$

Every distributive Lattice is Modular

But, converse is not true

i.e. Every modular lattices need not be distributive

For example, M_5 (Diamond) Lattices is modular but it is not distributive.