Computer Science 452 - Homework Assignment #3

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Problem 1.2.1: Convert the following finite automata to regular expressions.

(a) The given automaton computes the regular expression ba^*.

(b) The given automaton computes the regular expression $((a \cup b)a^*b(bb)^*a)^*$.

Problem 1.29b, Claim: The language $A = \{www \mid w \in \{a, b\}^*\}$ is not regular.

Proof. Suppose for contradiction that A is regular. Let p be the Pumping length of A, and $w = a^p b^p$. Then, $x = www \in A$.

The string x is sufficiently large, so the decomposition $x = qrs$ must exist as per the Pumping lemma. Since $|qr| \leq p$ and $r \neq \varepsilon$, r must consist only of the symbol "a". Then, the string qr^2s has more "a"s than "b"s, so it cannot be in A.

This result contradicts the Pumping lemma; thus, it is true that A is not regular. □

Problem 1.32, Claim: The language B as defined is regular.

NOTE: Arithmetic is done in $\mathbb{Z}_2$ when describing δ.

Proof. It suffices to show that B^R is regular. We construct a DFA to recognize B^R.

We assume w.l.o.g. that $\varepsilon \in B^R$. The DFA $M_B = (Q, \sum_3, \delta, q_0, F)$, where:

$$Q = \{q_0, q_1, q_e\}; \tag{1}$$

$$F = \{q_0\}; \tag{2}$$

$$\delta\left(q_i, \begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{cases} q_{a+b+i} & q_i \neq q_e \text{ and } a + b + i = c \\ q_e & \text{otherwise} \end{cases} \tag{3}$$

recognizes B. Thus, the claim holds. □

Problem 1.35, Claim: The language E as given is not regular.

Proof. Suppose for contradiction that E is regular. Let p be the Pumping length of E, and choose $s = \begin{bmatrix}0\\1\end{bmatrix}^p \begin{bmatrix}1\\0\end{bmatrix}^p$.

By definition, $s \in E$. Furthermore, since $|s| > p$, it holds that $s = xyz$ as per the Pumping lemma. That is, xy must have length at most p. Therefore, it holds that y only contains symbols $\begin{bmatrix}0\\1\end{bmatrix}$.

So, the number of symbols of the form $\begin{bmatrix}0\\1\end{bmatrix}$ and those of the form $\begin{bmatrix}1\\0\end{bmatrix}$ must be unbalanced in xy^2z, so $xy^2z \notin E$. This contradicts the Pumping lemma. Thus, the claim holds. $\square$

Problem 1.40: Prove the following claims.

(a) $NOPREFIX$ **closes the set of regular languages.**

Proof. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite-state machine, i.e. a DFA, that computes a regular language A. Define $M' = (Q, \Sigma, \delta', q_0, F)$ be a finite-state machine with $\delta'(q, a) = \delta(r, a)$ if $r \notin F$ and $\delta'(q, a) = \emptyset$ otherwise (remove all edges from the accepting states). Then, M' computes the language $NOPREFIX(A)$.

Hence, $NOPREFIX(A)$ is regular, as required. $\square$

(b) $NOEXTEND$ **closes the set of regular languages.**

Proof. Suppose A is a regular language and that the DFA $M = (Q, \Sigma, \delta, q_0, F)$ recognizes A. Let $M' = (Q, \Sigma, \delta, q_0, F \setminus F')$, where F' is the set of accepting states that have a path of length at least 1 to another accepting state.

Here, M' recognizes $NOEXTEND(A)$, so $NOEXTEND(A)$ is regular, as required. $\square$

Problem 1.4.6: Prove that the following languages are not regular.

(a) $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$

Proof. Suppose for contradiction that L is regular. Let p be the Pumping length of L, and $s = 0^p 1^p 0^p \in L$. Then, $s = xyz$ as per the Pumping lemma.

Now, since $|xy| \leq p$, it holds that y consists entirely of 0s. Then, $s' = xy^2z$ is of the form $0^q 1^p 0^p$, where $q > p$. Thus, it holds that $s' \notin L$.

This contradicts the Pumping lemma; thus, the claim holds. $\square$

(b) $L = \{w \mid w$**is not a palindrome**$\}$

Proof. It suffices to show that $L^\complement$ is not regular. Suppose for contradiction that $L^\complement$ is regular, and let p be the Pumping length of $L^\complement$.

Take $s = aa^R \in L^\complement$ where $|a| \geq p$ and a^R is the reversal of a. We know that $s = xyz$ as per the Pumping lemma. Since $|xy| \leq p$, we know that xy is some prefix of a. So, $s' = xy^2z \notin L^\complement$.

This is a contradiction of the Pumping lemma. Thus, the claim holds. $\square$

(c) $L = \{wtw \mid w, t \in \{0,1\}^+\}$

Proof. Suppose for contradiction that L is regular, and let p be the Pumping length of L. Fix $w = 0^p1^p$, and let $s = wtw \in L$ with $t \in L$ arbitrary.

We know that $s = xyz$ with $|xy| \leq p$, as per the Pumping lemma, so xy consists entirely of 0s. Thus, $s' = xy^2z$ has an unbalanced number of 0s; consequently, $s' \notin L$.

This contradicts the Pumping lemma; thus, the claim holds. $\square$

Problem 1.49: Prove the following.

(a) **The language B as given is regular.**

Proof. Let k in the context of B be arbitrary. We know there exists a regular language L_1 of strings of the form 1^k, and that there exists a regular language $L2$ of strings y s.t. y contains at least k instances of the symbol "1". Observe that B is the concatenation of L_1 and L_2, which implies that B is regular since regular languages are closed under concatenation. $\square$

(b) **The language C as given is not regular.**

Proof. Suppose for contradiction that C is regular. Let p be the Pumping length of C.

Let $s = 1^py \in C$, and suppose that y has $k \leq p$ instances of the symbol 1. We know that $s = xyz$ with $|xy| \leq p$, as per the Pumping lemma, so y consists of all 1s.

Now, consider $s' = xy^nz$ for $n > p$. It must be the case that $s' \notin C$. This contradicts the Pumping lemma; thus, the claim holds. $\square$