Computer Science 452 - Homework Assignment #4

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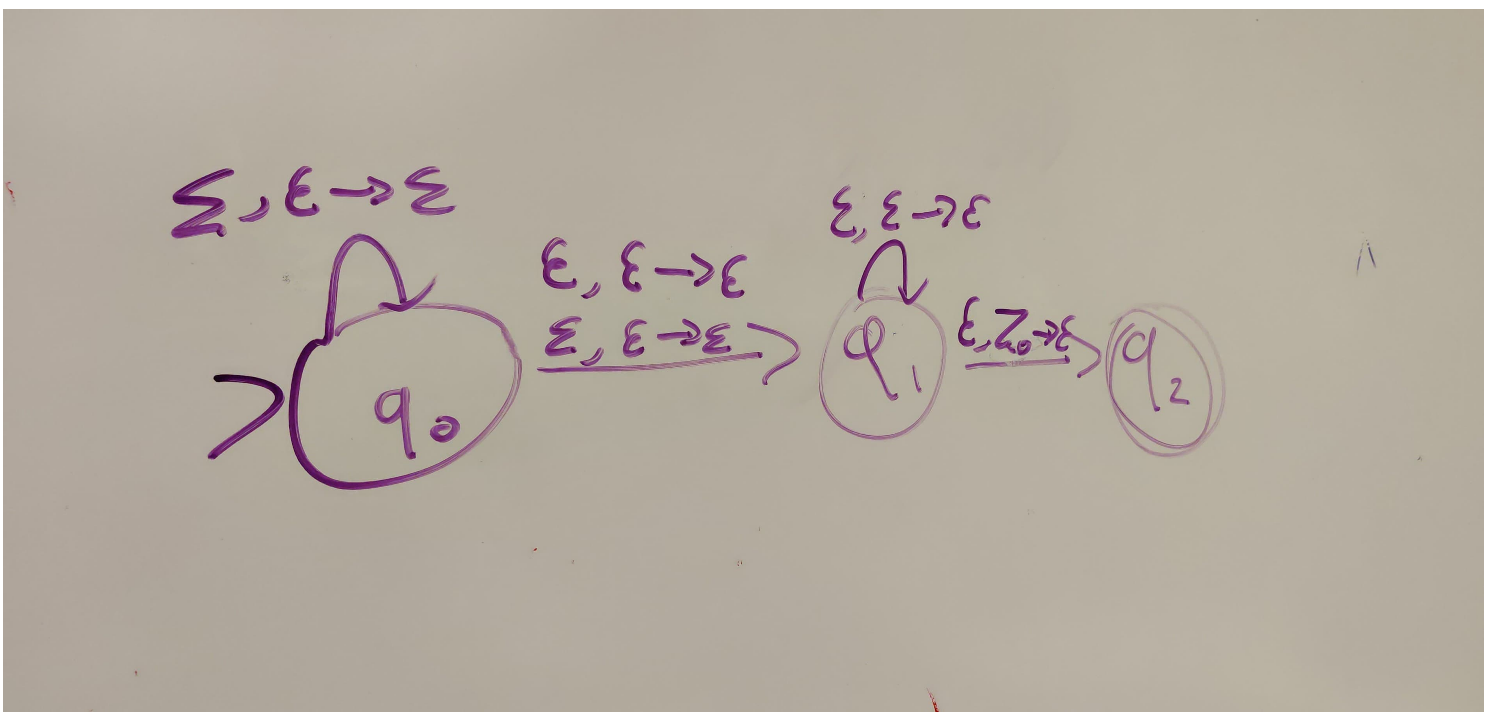
Problem 2.4e: Generate a CFG for the language of palindromes.

The CFG required is as follows, for $\alpha \in \Sigma$:

$$
\begin{aligned}
A &\to \epsilon \\
A &\to \alpha \\
S &\to \alpha S \alpha
\end{aligned}
$$

Problem 2.5: Give informal descriptions and state diagrams of PDAs for Problem 2.4e.

The PDA M that recognizes the language of bit-wise palindromes is as follows:



Problem 2.9: Provide a CFG that computes $A = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$.

The CFG to compute the superset A_1 of the language where $i = j$ is as follows:

$$\begin{aligned} S_1 &\to aS_2bS_3 \mid \epsilon \\ S_2 &\to aS_2b \mid \epsilon \\ S_3 &\to cS_3 \mid \epsilon \end{aligned}$$

The CFG to compute the superset A_2 of the language where $j = k$ is as follows:

$$\begin{aligned} S_4 &\to S_5bS_6c \mid \epsilon \\ S_5 &\to cS_5 \mid \epsilon \\ S_6 &\to bS_6c \mid \epsilon \end{aligned}$$

A is the language given by the grammar:

$$S_0 \to S_1 \mid S_4$$

Problem 2.10: Informally describe a PDA of the language described in Problem 2.9.

The PDAs to describe the languages A_1 and A_2 are trivial. We observe that $A = A_1 \cup A_2$, and extending PDAs with union over their languages is also trivial.

Problem 2.14: Convert the given CFG to Chomsky-Normal Form.

The equivalent description of the CFG is as follows:

$$\begin{aligned} S &\to BC \mid AB \mid BA \mid ZZ \mid \epsilon \\ A &\to BC \mid AB \mid BA \mid ZZ \\ B &\to ZZ \\ C &\to AB \\ Z &\to 0 \end{aligned}$$

Problem 2.35, Claim: Let G be a CFG that contains at least b variables. Then, if G generates some string with a derivation of at least 2^b steps, then $L(G)$ is infinite.

Proof. W.l.o.g., assume G is in Chomsky-Normal Form. Then, every derivation can generate at most two terminal symbols, so the parse tree using G must have height at most $2^b - 1$.

Trivially, it must be the case that the parse tree has height at least $b + 1$. Thus, there is a path from root to leaf of length $b + 1$. By the Pigeonhole Principle, at least one variable A occurs twice.

This is sufficient to show that the CFL generated by G is infinite. □