

# Math 351 - Homework Assignment #1

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September 16, 2019

## Problem 0.7: Supply a relation that satisfies each constraint.

- (a) **Reflexive and symmetric, but not transitive.**

The relation between two people  $x, y$  given iff  $x$  is blood-related to  $y$  is reflexive and symmetric, but not transitive.

- (b) **Reflexive and transitive, but not symmetric.**

$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \geq b\}$  is reflexive and transitive, but not symmetric.

- (c) **Symmetric and transitive, but not reflexive.**

$R = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$  in  $X = \{0, 1, 2\}$ . This is symmetric and transitive, but not reflexive since  $(2, 2) \notin R$ .

## Problem 0.9: Formally describe the given graph.

The given graph is the complete bipartite graph  $K_{3,3}$  with formal description  $(\{1, 2, 3, 4, 5, 6\}, \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5), (1, 6), (2, 6), (3, 6)\})$ .

## Problem 0.10: Find the error in the given proof.

The error in the given proof is that the writer defines  $a = b = 1$ . Thus, when he transforms the equality  $(a + b)(a - b) = b(a + b)$  to  $a + b = b$ , he divides by  $a - b = 0$ . Thus, the proof is incorrect.

## Problem 0.12: Find the error in the given proof.

Let  $P(h)$  be the predicate that all horses in a set of  $h$  horses are the same color. As per the base case,  $P(1)$  is true. However,  $P(2)$  is not implied by the base case  $P(1)$ .

Take two horses  $x, y$  of different color. By the inductive hypothesis, all the horses in  $\{x\}$  and those in  $\{y\}$  are the same color. However, the horses in  $\{x, y\}$  are not the same color by definition. Thus, the claim of the author that  $P(k)$  implies  $P(k + 1)$  is untrue.

**Problem 0.13, Claim: It is not true that every graph of two or more vertices contain two vertices of equal vertices when self-loops are allowed.**

*Proof.* The graph  $G$  with two vertices  $x, y$  where  $x$  has a self-loop and  $y$  has no edge is a counterexample.  $\square$

**Problem 0.14, Claim: Ramsey's Theorem.**

*Proof.* Let  $G$  be a graph of size  $n$ . Now, assume a 2-coloring on the edges of the complete graph  $K_n$  of size  $n$  s.t. red edges are in  $G$  and blue edges are not in  $G$ .

If there are  $k \geq \frac{1}{2} \lg n$  red edges, then we are done. So, assume that this is not the case, i.e.  $k \leq \frac{1}{2} \lg n$ . Therefore, there are  $n^2 - n - k$  blue edges in our coloring of  $K_n$ .

Finally, observe that  $k + \frac{1}{2} \lg n \leq \lg n \leq n(n-1) = n^2 - n$ . Thus,  $n^2 - n - k \geq \frac{1}{2} \lg n$ ; so, it follows that there are more than  $\frac{1}{2} \lg n$  blue edges.

We know that the blue edges of  $K_n$  are not connected by an edge in  $G$ . Thus, the set of vertices connected with blue edges in  $K_n$  forms an independent set in  $G$ , as required.  $\square$

**Extra Problem, Claim: The relation  $S$  defined by  $(1, 1) \in S$  and  $(a + 1, b + 2a + 1) \in S$  for all  $(a, b) \in S$  is equivalent to the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  induced by  $n \mapsto n^2$ .**

*Proof.* Let  $P(n)$  be the predicate that  $(n, n^2) \in S$  for a particular  $n$ . We prove the claim with mathematical induction.

Base Case: It is given that  $(1, 1) = (1, 1^2) \in S$ . Thus,  $P(1)$  holds.

Inductive Step: Suppose that  $P(k)$  holds for all  $k$  s.t.  $1 \leq k \leq m$ , where  $m \geq 1$ . It follows from the inductive hypothesis that  $m + 1 \mapsto m^2 + 2m + 1 = (m + 1)^2$ . Thus,  $P(m + 1)$  holds.

Hence, by the Principle of Mathematical Induction, the claim holds.  $\square$