Computer Science 452 - Homework Assignment #3

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Problem 1.2.1: Convert the following finite automata to regular expressions.

(a) The given automaton computes the regular expression ba^*.

(b) The given automaton computes the regular expression $((a \cup b)a^*b(bb)^*a)^*$.

Problem 1.29b, Claim: The language $A = \{www \mid w \in \{a, b\}^*\}$ is not regular.

Proof. Suppose for contradiction that A is regular. Let p be the Pumping length of A, and $w = a^p b^p$. Then, $x = www \in A$.

The string x is sufficiently large, so the decomposition $x = qrs$ must exist as per the Pumping lemma. Since $|qr| \leq p$ and $r \neq \varepsilon$, r must consist only of the symbol "a". Then, the string qr^2s has more "a"s than "b"s, so it cannot be in A.

This result contradicts the Pumping lemma; thus, it is true that A is not regular. □

Problem 1.32, Claim: The language B as defined is regular.

NOTE: Arithmetic is done in $\mathbb{Z}_2$ when describing δ.

Proof. It suffices to show that B^R is regular. We construct a DFA to recognize B^R.

We assume w.l.o.g. that $\varepsilon \in B^R$. The DFA $M_B = (Q, \sum_3, \delta, q_0, F)$, where:

$$Q = \{q_0, q_1, q_e\}; \tag{1}$$

$$F = \{q_0\}; \tag{2}$$

$$\delta\left(q_i, \begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{cases} q_{a+b+i} & q_i \neq q_e \text{ and } a + b + i = c \\ q_e & \text{otherwise} \end{cases} \tag{3}$$

recognizes B. Thus, the claim holds. □