Problem 9.7: Supply regular expressions that generate the following languages.

- We supply $(0^\star 10^\star)^{500}$.
- We supply $\cup_{i=1}^{500}(0^\star 10^\star)^i$.
- We supply $\cup_{i=1}^{500}(0^\star 10^\star)^i(0^\star 10^\star)^\star$.

Problem 9.9, Claim: If NP = $\mathbf{P}^{\mathbf{SAT}}$, then NP = co-NP.

Proof. Suppose that $NP = P^{SAT}$. Now, let L be a language in $coNP$; then, its complement must be in NP. There must exist an oracle machine M that applies M^{SAT} to solve L; the machine that accepts input iff M rejects it is sufficient to show that $L^{\complement} \in P^{SAT} = NP$. Thus, L must be an element of $coNP$.

It can be similarly shown that $NP \supseteq coNP$. Thus, the claim holds. □

Problem 9.12: Describe the problems with the given erroneous proof.

The proof claims that it follows from the Cook-Levin Theorem that $NP \subseteq TIME(n^k)$, where SAT is decided with time-complexity $\mathcal{O}(n^k)$.

However, the Cook-Levin Theorem only guarantees a *polynomial-time* reduction from $L \in NP$ to SAT; thus, it does not follow from this that $NP \subseteq TIME(n^k)$, as the author claims.

Problem 9.13, Claim: If $A \in$ TIME(n^6), then pad$(A, n^2) \in$ TIME(n^3).

Proof. Let M be a machine that decides A with time-complexity $\mathcal{O}(n^6)$. We define the machine M' that decides the specified padding of L as follows:

On input x, check if x is of the format $pad(w,|w|^2)$. If so, simulate M on w; otherwise, reject.

This decides $pad(A, n^2)$ with time-complexity $\mathcal{O}(n^6)$ as required. □

Problem 9.14, Claim: If EXPTIME $\neq$ NEXPTIME, then P $\neq$ NP.

Proof. Suppose for contraposition that P = NP. Consider $L \in$ NEXPTIME, and let c be a positive integer s.t. $L \in \text{NTIME}(2^{n^c})$. Clearly, $\text{pad}(A, 2^n) \in$ NP, so by assumption, it is also in P. Therefore, $L \in \text{TIME}(2^{\text{O}(n^c)}) \subseteq$ EXPTIME.

Consequently, EXPTIME = NEXPTIME, as required. $\square$

Supplementary Problem 1, Claim: If A is complete for $EXPTIME$ under $\leq_m^p$ restrictions, then there exists $\epsilon > 0$ s.t. $A \notin$ DTIME(2^{n^ϵ}).

This cannot be shown, because $EXPTIME$ can be written as $\cup_{k \in \mathbb{N}} DTIME(2^{n^k})$. If A is complete for $EXPTIME$ under polynomial-time m-reductions, then it must be in $DTIME(2^{n^\epsilon})$ for all ϵ. Thus, the converse is true.

Supplementary Problem 2, Claim: IF B is complete for $PSPACE$ under logarithmic-time m-reductions then there exists $\epsilon > 0$ s.t. $A \notin$ DSPACE(2^{n^ϵ}).

Similarly with Problelm 1, this is not true; the opposite can be proven.

Supplementary Problem 3, Claim: The language A, as defined, is complete in NP under logarithmic-time m-reductions.

Proof. First, we provide a reduction f from A to the Hamiltonian path problem as follows:

On input (x, y), accept iff M^{PATH} accepts x and rejects y.

Similarly, we provide a reudction g from the Hamiltonian path problem to A:

On input x, accept x iff M^A accepts (x, y), where y is the complement of x.

Both reductions are in logarithmic-time, as required. Thus, A is Turing-equivalent to $PATH$. Since $PATH$ is NP-complete, then so too must be A, as required. $\square$