Math 351 - Homework Assignment #1

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September 16, 2019

Problem 0.7: Supply a relation that satisfies each constraint.

(a) Reflexive and symmetric, but not transitive.

The relation between two people x, y given iff x is blood-related to y is reflexive and symmetric, but not transitive.

(b) Reflexive and transitive, but not symmetric.

 $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \geq b\}$ is reflexive and transitive, but not symmetric.

(c) Symmetric and transitive, but not reflexive.

 $R = \{(0,0),(0,1),(1,0),(1,1)\}$ in $X = \{0,1,2\}$. This is symmetric and transitive, but not reflexive since $(2,2) \notin R$.

Problem 0.9: Formally describe the given graph.

The given graph is the complete bipartite graph $K_{3,3}$.

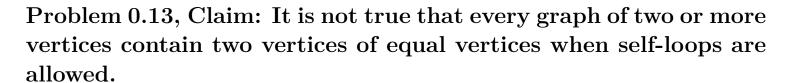
Problem 0.10: Find the error in the given proof.

The error in the given proof is that the writer defines a = b = 1. Thus, when he transforms the equality (a + b)(a - b) = b(a + b) to a + b = b, he divides by a - b = 0. Thus, the proof is incorrect.

Problem 0.12: Find the error in the given proof.

Let P(h) be the predicate that all horses in a set of h horses are the same color. As per the base case, P(1) is true. However, P(2) is not implied by the base case P(1).

Take two horses x, y of different color. By the inductive hypothesis, all the horses in $\{x\}$ and those in $\{y\}$ are the same color. However, the horses in $\{x, y\}$ are not the same color by definition. Thus, the claim of the author that P(k) implies P(k+1) is untrue.



Proof. The graph G with two vertices x, y where x has a self-loop and y has no edge is a counterexample. \Box

Problem 0.14, Claim: Ramsey's Theorem.

Proof. Let R(s,t) be the least integer k s.t. every graph on k or more vertices contains an s-clique or t-size independent set. It suffices to show that $R(t,t) \leq 2^{2t}$. In fact, R(s,t) is well-defined as the Ramsey Numbers, which have the well-known upper-bound $k' = \binom{r+s-2}{r-1}$. In particular, $R(t,t) \leq k' \leq k$, so the claim holds. \square

Extra Problem, Claim: The relation S defined by $(1,1) \in S$ and $(a+1,b+2a+1) \in S$ for all $(a,b) \in S$ is equivalent to the function $f: \mathbb{N} \to \mathbb{N}$ induced by $n \mapsto n^2$.

Proof. Let P(n) be the predicate that $(n, n^2)inS$ for a particular n. We prove the claim with mathematical induction.

Base Case: It is given that $(1,1) = (1,1^2) \in S$. Thus, P(1) holds.

<u>Inductive Step:</u> Suppose that P(k) holds for all k s.t. $1 \le k \le m$, where $m \ge 1$. It follows from the inductive hypothesis that $m+1 \mapsto m^2+2m+1=(m+1)^2$. Thus, P(m+1) holds.

Hence, by the Principle of Mathematical Induction, the claim holds.