

Random Variables:

A random variable is a measurable function defined on a probability space that maps from the sample space to the real numbers.

In simple words random variable is a mapping from an event to \mathbb{R}

Examples:

(1) Events of tossing 3 coins ($|S| = 2^3 = 8$)

Random Variable $\leftarrow X = \text{number of heads}$

$$P(X=0) = \frac{1}{2^3}$$

$$P(X=1) = {}^3C_1 \left(\frac{1}{2} \cdot \frac{1}{2^2} \right)$$

$$P(X=2) = {}^3C_2 \left(\frac{1}{2^2} \cdot \frac{1}{2} \right)$$

$$P(X=3) = \frac{1}{2^3}$$

$X \in \{0, 1, 2, 3\} \Rightarrow \text{Discrete R.V.}$

(2) $X = \text{amount of rainfall on a given day.}$

$$P(X \geq 2 \text{ cm}) = 0.95$$

$$P(X \geq 1.5 \text{ cm}) = 0.96$$

$$P(X \leq 1 \text{ cm}) = 0.99$$

$X \in [0, \infty) \Rightarrow \text{Continuous R.V.}$

Discrete R.V	Continuous R.V
# of possible values a R.V can take is countable	Set of possible outcomes are not countable
$X \in \{1, 2, 3, 4, 5, 6\}$	$X \in [0, 1]$