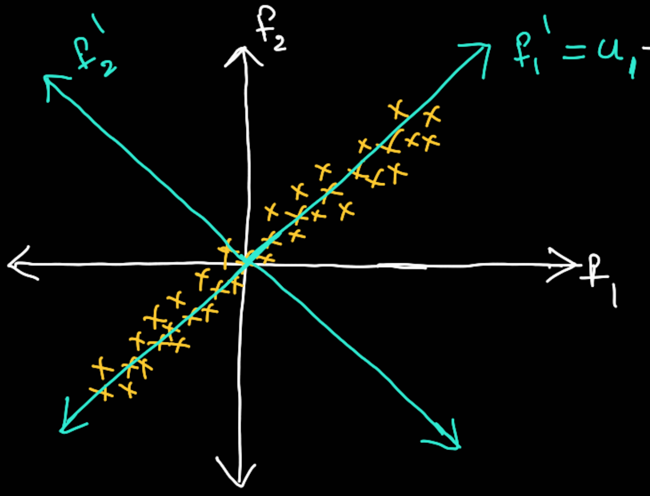
Mathematical Objective function of PCA:



$\longrightarrow$ Since we are only intrested in direction of u_1 but not on its magnitude we will consider it as unit vector

$$\|u_1\| = 1$$

And let projections of x_i on u_i be x_i'

$$x_i' = \frac{u_1 \cdot x_i}{\|u_1\|^2} = u_1^T \cdot x_i$$

$$\boxed{x_i' = u_1^T \cdot x_i}$$

$$\bar{x}_i' = u^T \cdot \bar{x}_i$$

$\bar{x}_i'$: mean $\{x_i'\}_{i=1}^{n}$

$\bar{x}_i$: mean $\{x_i\}_{i=1}^{n}$

Our task is to find u_1 s.t $\text{Var}\{\text{proj}_{u_1} x_i\}_{i=1}^{n}$ is maximal

$$\text{Var}\{\overbrace{\text{Proj}_{u_1} x_i}^{x_i'}\}_{i=1}^{n} = \text{Var}\{u_1^T x_i\}_{i=1}^{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(u_1^T x_i - u_1^T \bar{x}\right)^2$$

if our data X is column standardized $(\bar{x} = 0)$

$$\text{Var}\{x_i'\}_{i=1}^{n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n}\left(u_1^T x_i\right)^2}$$

$\longrightarrow$ This value should be maximum so the optimization problem is as follows.

Find $\max\limits_{u_1} \frac{1}{n}\sum_{i=1}^{n}\left(u_1^T x_i\right)^2$ such that $u_1^T \cdot u_1 = 1 = \|u\|^2$ //