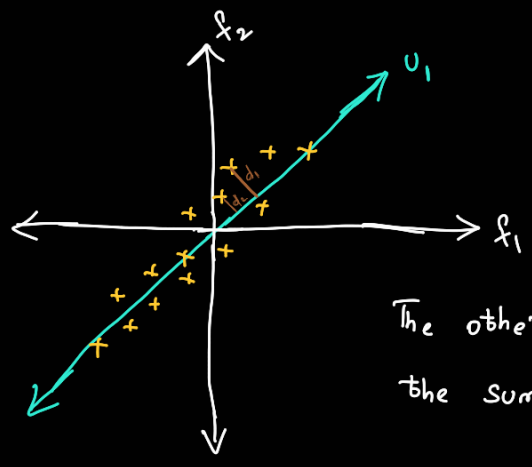
Alternative formulation of PCA: distance-minimization

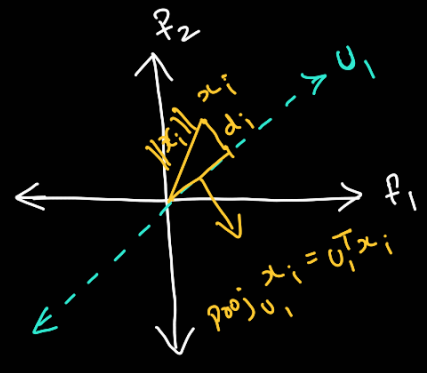
Another alternative to write the optimization problem for PCA is as follows



d_i : Distance from x_i to u_1

The other way of finding Optimal u_1 is by minimizing the sum of all d_i

$$\min_{u_1} \sum_{i=1}^{n} d_i^2$$



$$d_i^2 = \|x_i\|^2 - (u_1^T x_i)^2$$
$$d_i^2 = x_i^T x_i - (u_1^T x_i)^2$$

So, the problem can be written as

$$\min_{u_1} \sum_{i=1}^{n} \left(x_i^T x_i - (u_1^T x_i)^2 \right) \quad \text{s.t} \quad u_1^T u = 1 = \|u\|$$