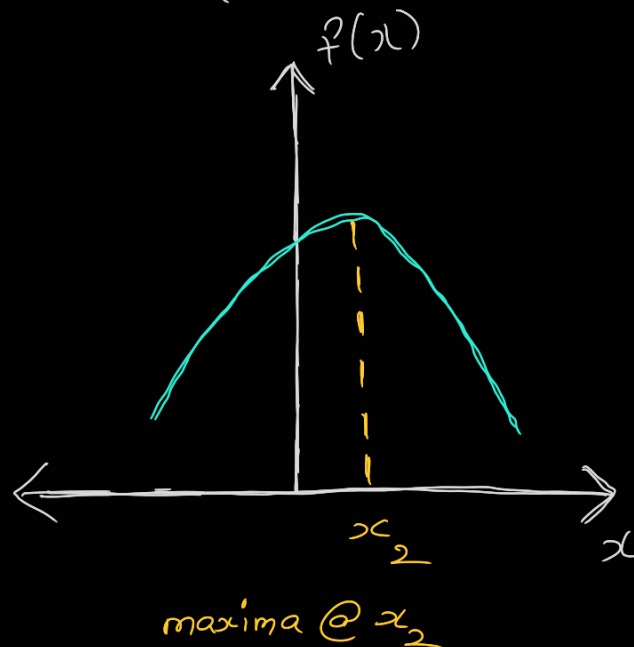
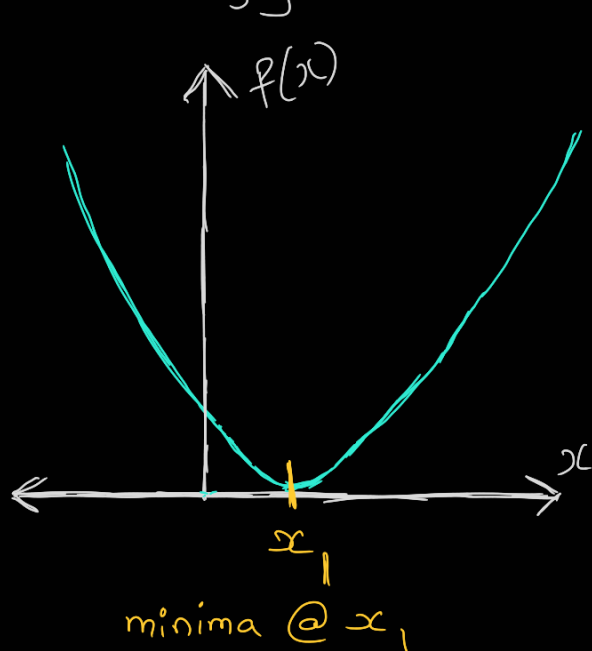


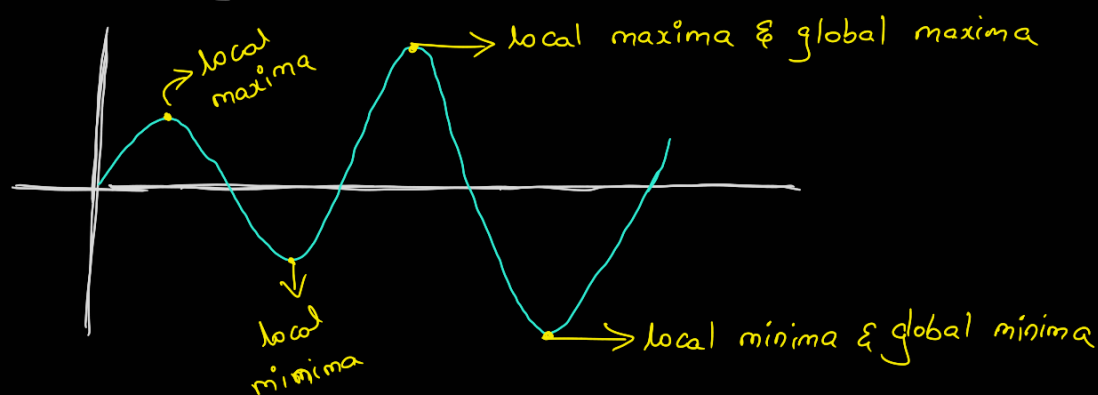
Maxima and Minima

All the calculus we have covered till now is to find minima and maxima, going back to our problem of predicting incomes we are trying to minimize the error i.e. $(y - \hat{y})^2$



Point to be noted is functions can have both minima and maxima or one of minima and maxima or neither of them

lets look a slightly more complex looking example



Global minima = $\min(\text{all local minima})$

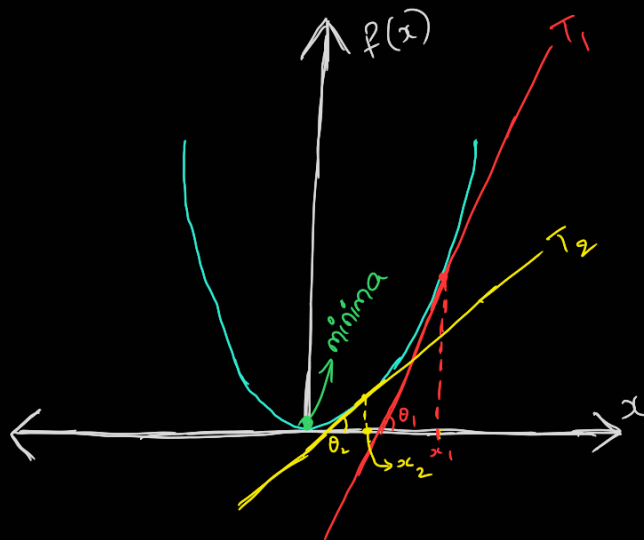
Global maxima = $\max(\text{all local maxima})$

Regardless of minima and maxima, all these points are called optima.

local minima & maxima \Rightarrow local optima

Global minima & maxima \Rightarrow Global optima

How to find them?



In our example

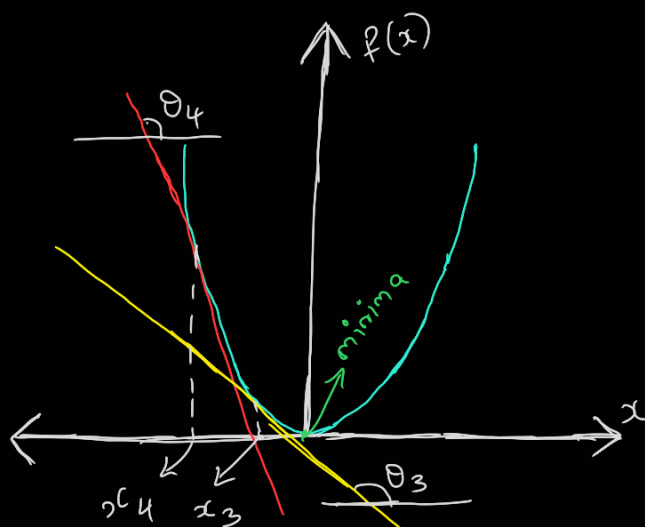
$$0^\circ < \theta_2 < \theta_1 < 90^\circ$$

$$0 < \tan(\theta_2) < \tan(\theta_1) < \infty$$

at origin

$$\text{slope @ } x_2 < \text{slope @ } x_1$$

which means slope is \downarrow while travelling towards origin from +ve side



$$\text{Here } 180^\circ > \theta_3 > \theta_4 > 90^\circ$$

$$\Rightarrow \tan(\theta_3) < 0 \quad \tan(\theta_4) < 0 \quad \rightarrow \text{Both are -ve}$$

$$\tan(\theta_3) > \tan(\theta_4)$$

Key observations Here are

→ At minima slope = 0

→ Right side of minima slope is +ve lly for maxima

→ Left side of minima slope is -ve

→ Around minima slope is increasing

$$\Rightarrow \text{At minima } \frac{d}{dx} \left(\frac{df(x)}{dx} \right) > 0$$

→ At maxima slope = 0

→ Right side of maxima slope is -ve

→ Left side of maxima slope is +ve

→ Around maxima slope is decreasing

$$\Rightarrow \text{At maxima } \frac{d}{dx} \left(\frac{df(x)}{dx} \right) < 0$$