21.5 Axioms of Probability, Properties and Examples-

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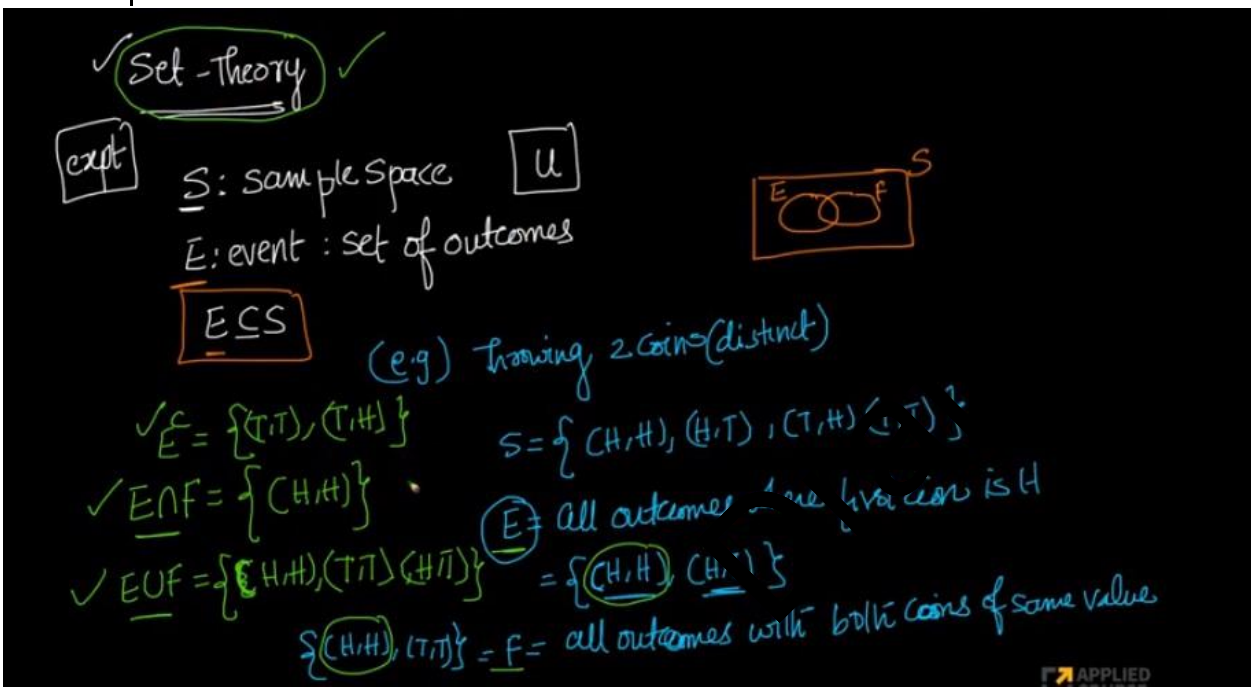

Let’s study probability in set theoretic perspective

1. If we consider an experiment, we define a sample space(S) which is a set of all outcomes. An event E which is a set of outcomes (subset of S).
2. Let’s say we are flipping two coins and we have our sample space as s= {HH, HT,TH,HH}
3. Consider an event E of getting outcomes where first coin is H (E={HH, HT})
4. Also consider another event F of getting outcomes where both coins are of same value
5. Now we can think of these events E and F as sets and can perform all operations which we can perform on sets. i.e.) set union, intersection, complement etc as shown above.

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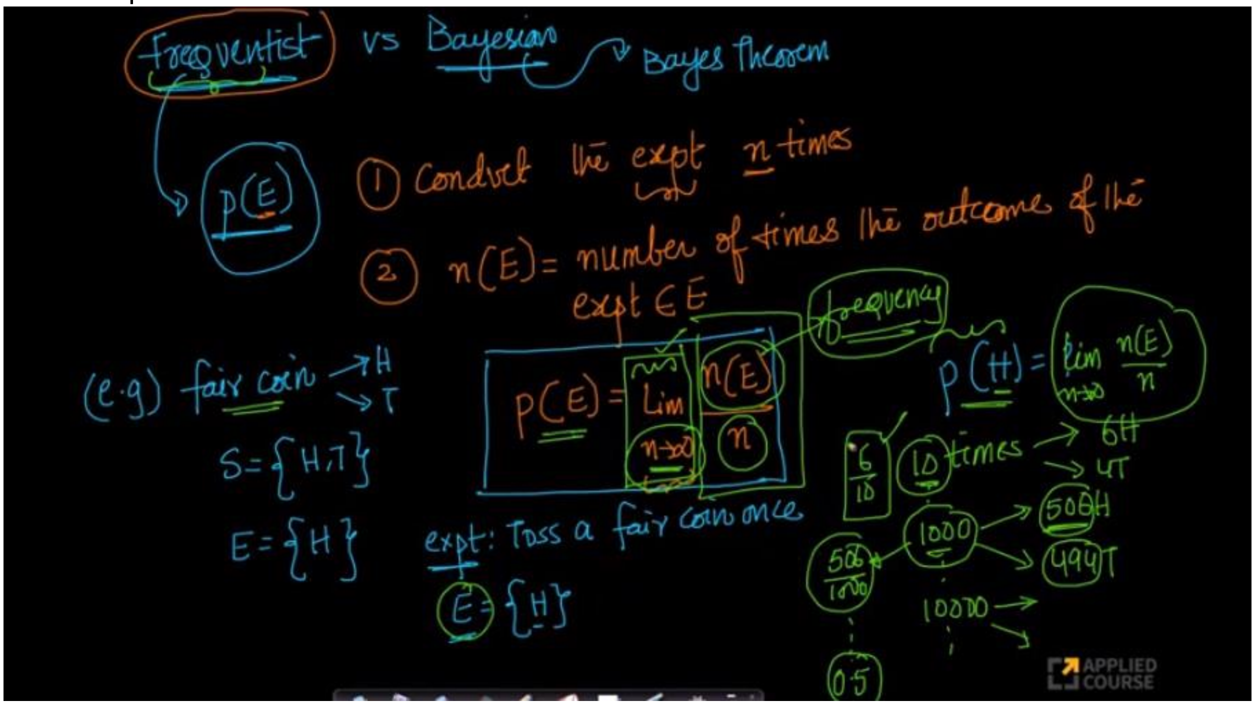


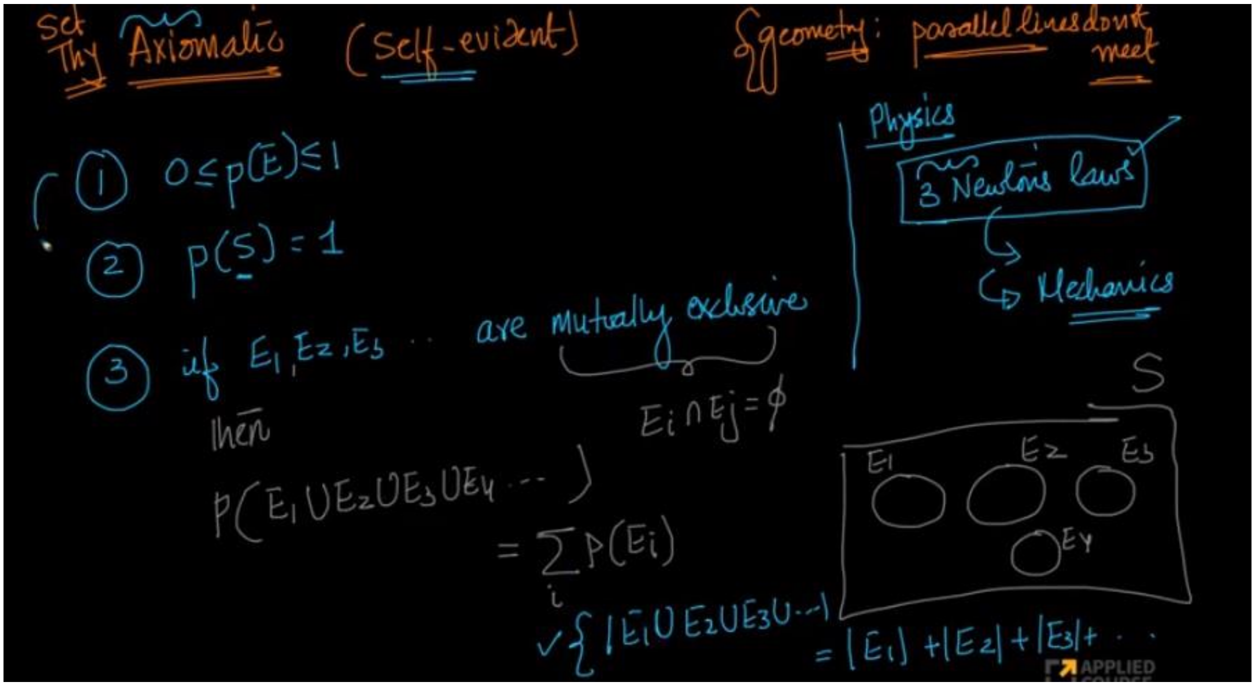
Let's understand the Frequentist and Bayesian approach of probability

From a frequentist approach probability of an event p(E) can be defined as

1. let's say we are conducting an experiment n times(assume the experiment of tossing two fair coins)
2. n(E)be the number of times the outcome of the experiment belongs to event E

Then p(E) = $\lim_{n \to \infty} n(E)/n$

n(E) is the frequency of occurrence of the event and n is total number of times we are conducting the experiment as n tends to infinity (we conduct the experiment infinite times)



Axiomatic (self-evident) approach of probability says

1. The probability of any event lies between 0 and 1i.e) 0≤p(E)≤ 1
2. The probability of sample space is p (S)=1
3. If there are events E1, E2, E3……. etc which are mutually exclusive(E1∩E2 =Φ) then
   $p\,(E1UE2UE3UE1....) = \sum i{=}1\ \mathrm{p(Ei)}$

Using the above axioms and properties in set theory we can actually prove all theorems in probability

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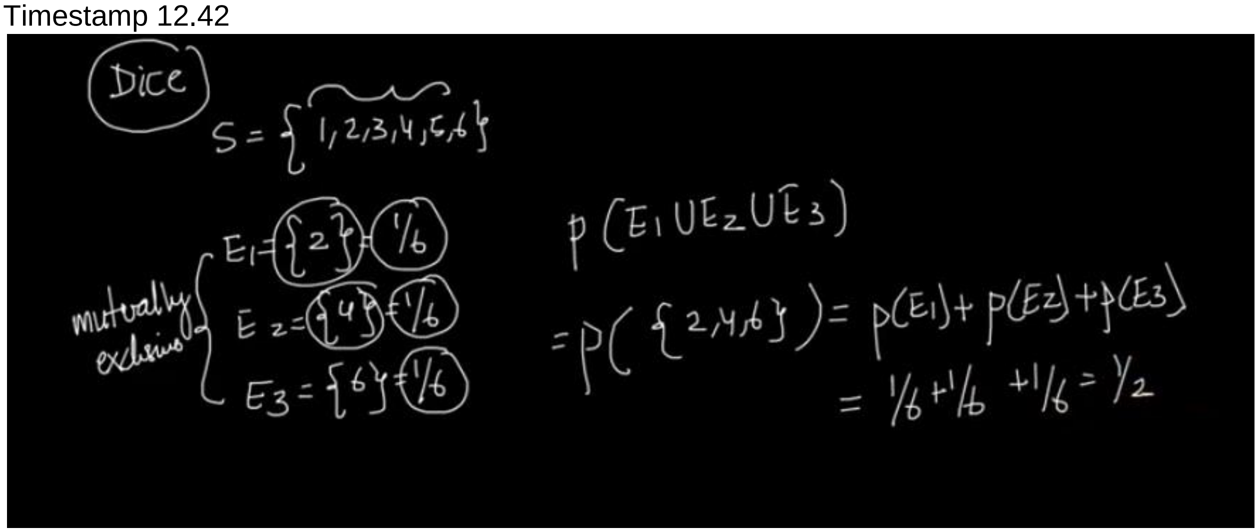

the sum of the events.
Above is an example where we have three mutually exclusive events and their union is equal to

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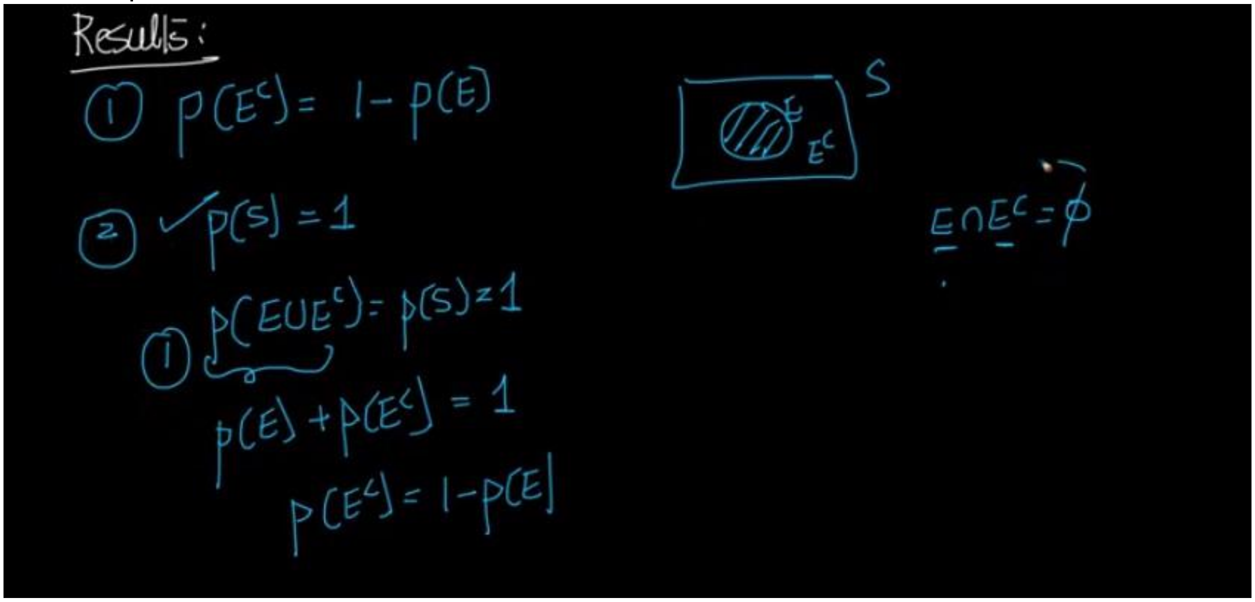

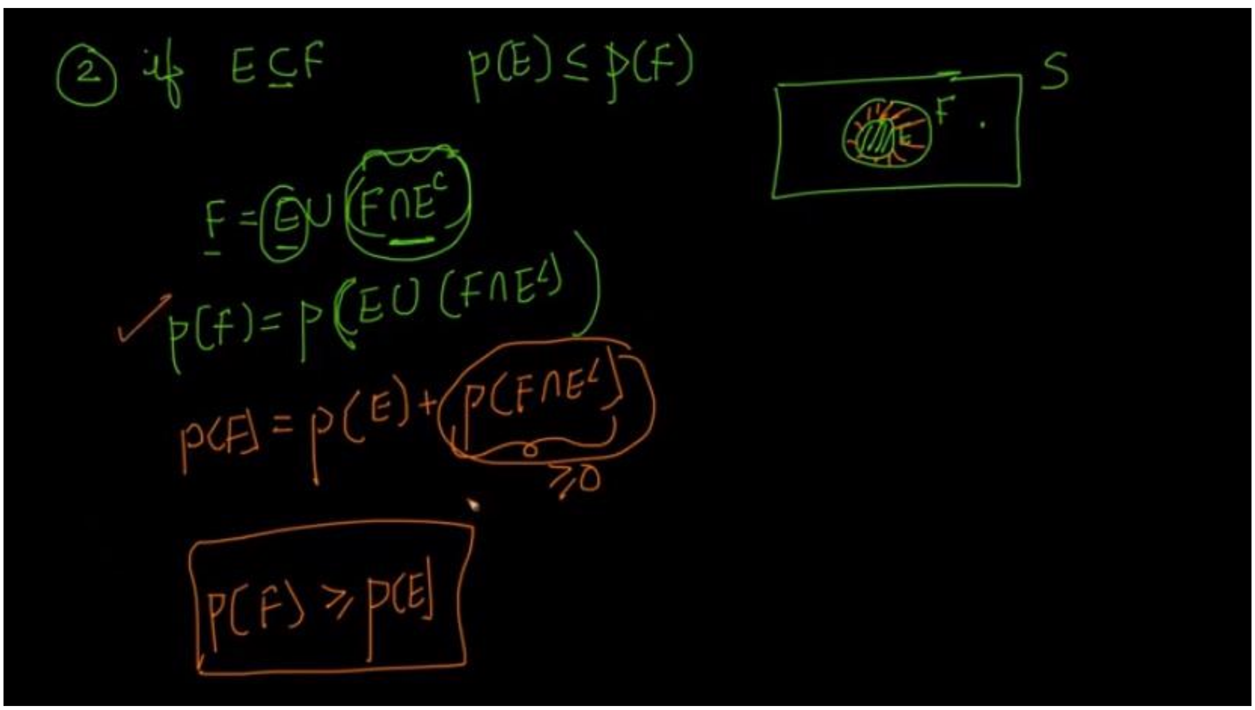

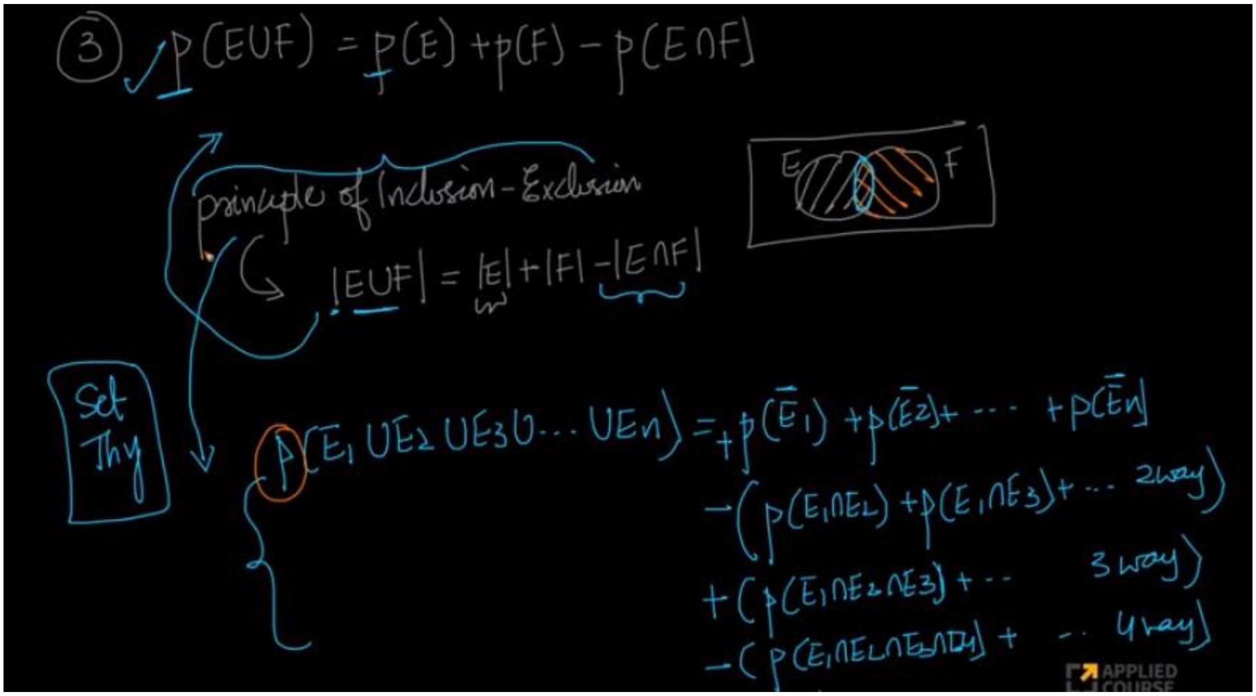

- Above are few examples of proof that we can solve all theorems in probability using these axioms and properties in set theory

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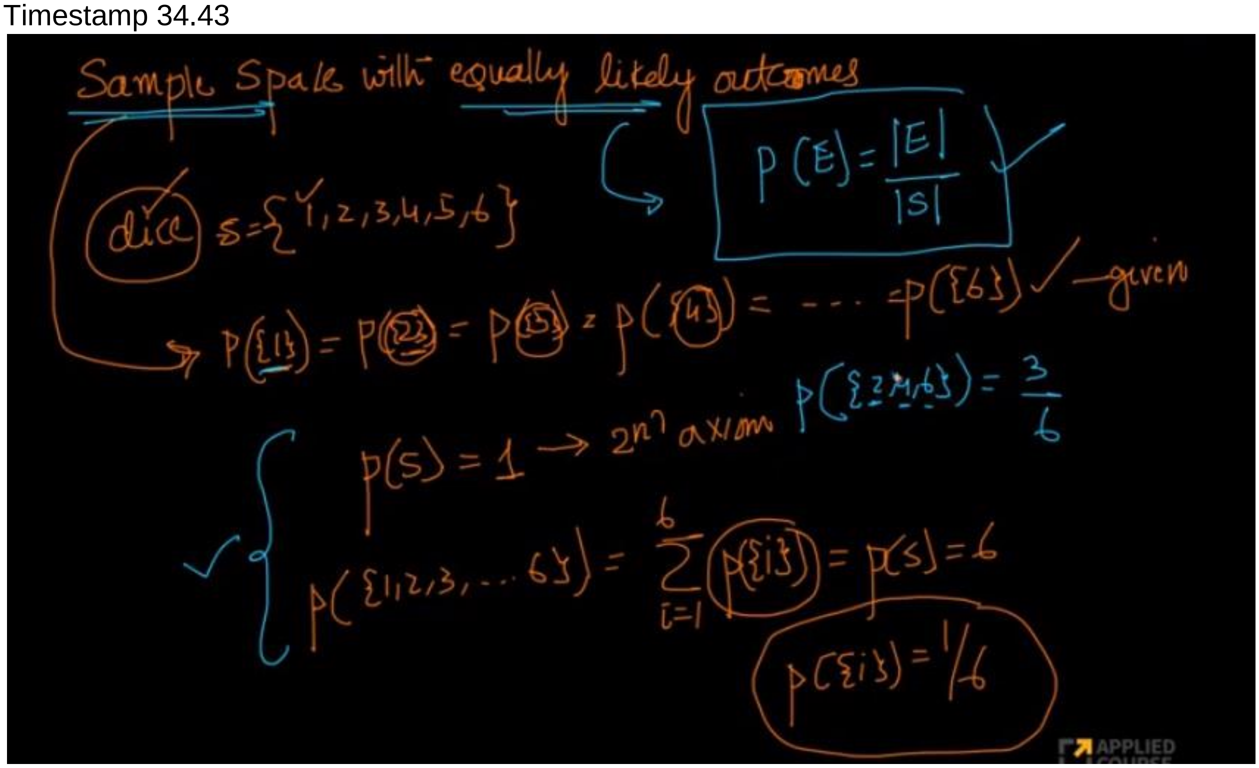

- In an experiment of rolling a dice, we have a sample space with equally likely outcomes as shown above.

S= {1,2,3,4,5,6}

p({1}) =p({2})=p({3})=p({4})=p({5})=p({6})=⅙ and We know that probability of sample space is p(S)=1

- Hence, p(E)=|E|/|S| this is true for sample space with equally likely events

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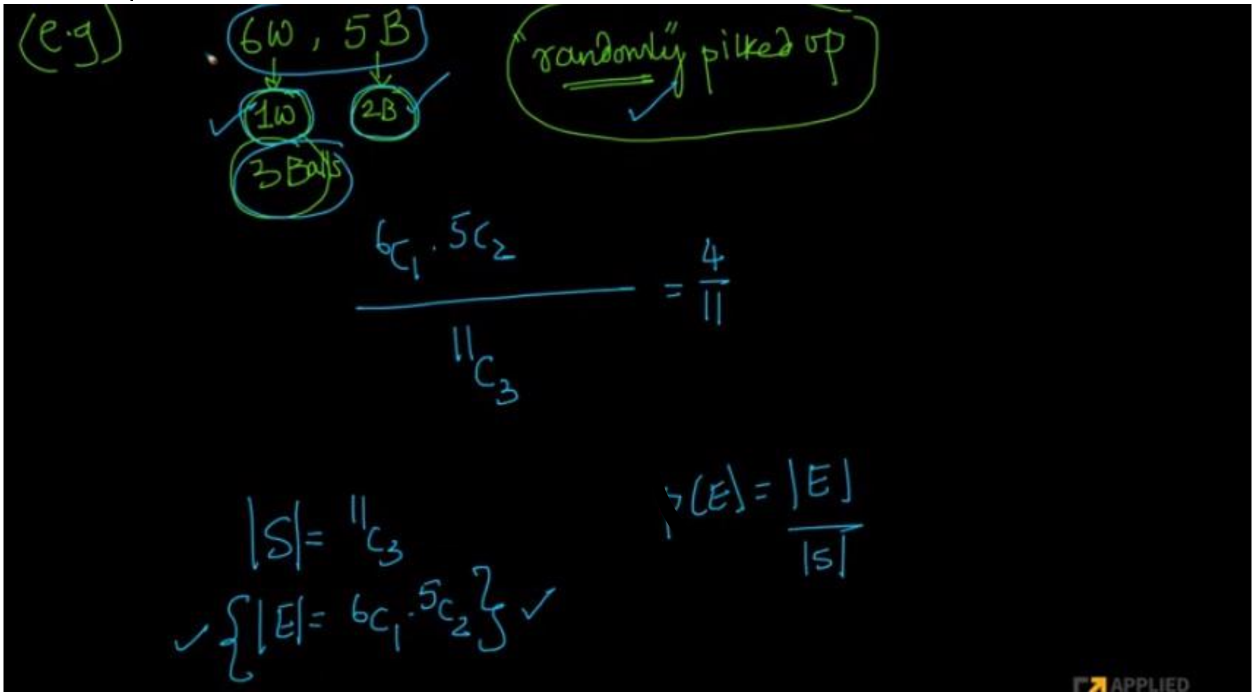
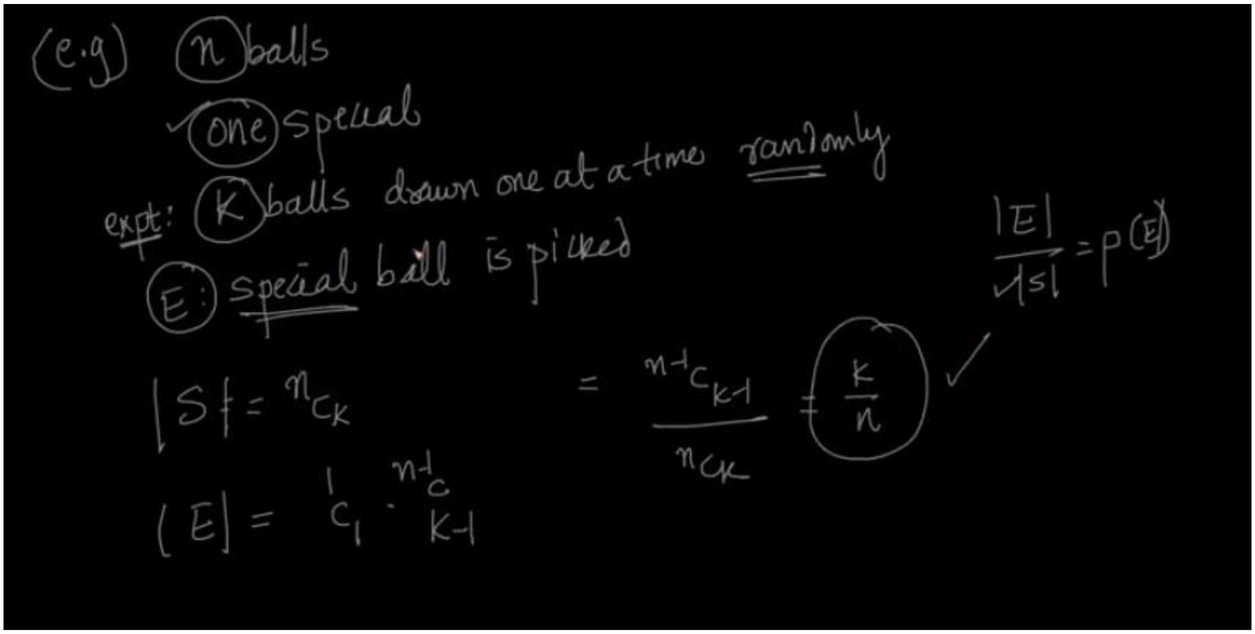
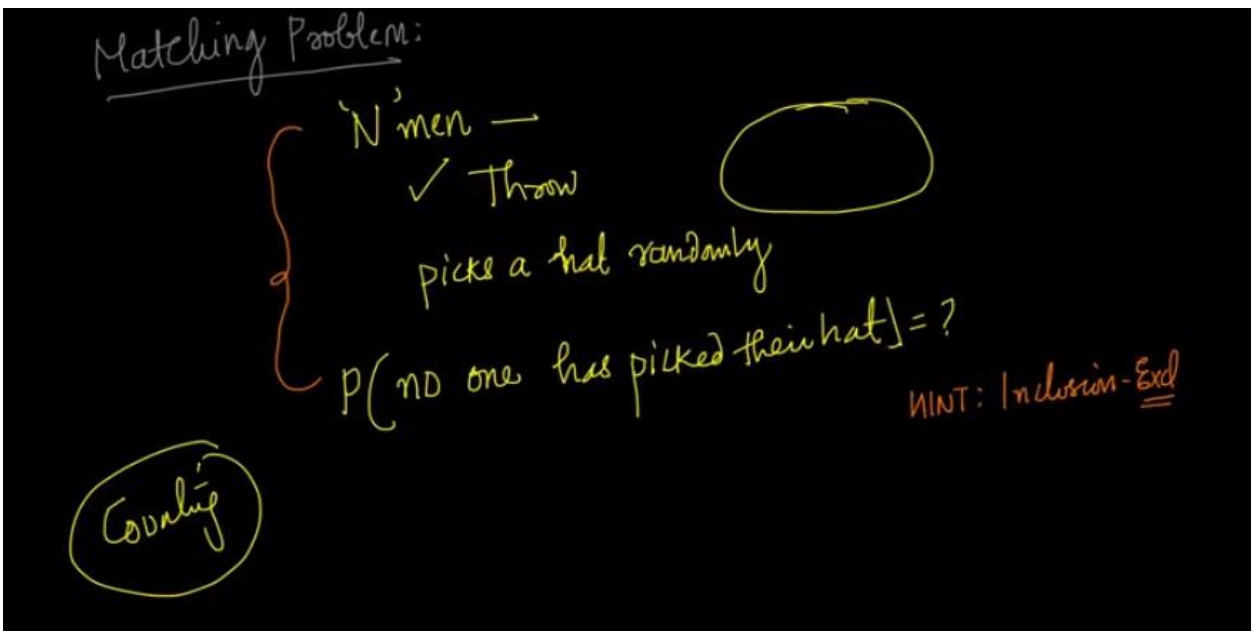


- In the above example we have to find probability that we have to choose 3 balls out of 11 balls such that we need to choose exactly 1 white and 2 black. Out of 11 balls we have 6 are white and 5 are black.

- As shown above using combinatorics we can find the probability is 4/11



1. Above example we have n balls out of which one ball is special. let's say in an experiment of drawing k balls out of n balls randomly.
2. Let say event E where special ball is picked
3. p(E) can be calculated using combinatorics as shown above



- In a party where N men throw their hats into the bag and then each of them picks a hat randomly. We have to find probability that No one picks their own hat. It can be solved using principle of inclusion and exclusion as shown below

E_i = i^{th} person has picked the correct hat

$P(E_1 \cup E_2 \cup \cdots E_n)$ = prob. of atleast one person picking their hat

$1 - P(E_1 \cup E_2 \cdots \cup E_n)$ = prob of no person picking their hat

$\frac{1}{n} + \frac{1}{n} + \cdots \frac{1}{n} = \frac{n}{n} = 1/1!$

$P(E_1) + P(E_2) + \cdots P(E_n)$

$\left(\frac{1}{n} \cdot \frac{1}{n-1}\right) {}^{n}C_2 = \cancel{\frac{n \times n-1}{2!}} \times \frac{1}{\cancel{n}} \cdot \frac{1}{\cancel{n-1}}$

$-\left(P(E_1 \cap E_2) + P(E_1 \cap E_3) + \cdots\right) \to \frac{1}{2!}$

$+\left(P(E_1 \cap E_2 \cap E_3) + \cdots\right) \to \frac{1}{\cancel{n}} \cdot \frac{1}{\cancel{n-1}} \cdot \frac{1}{\cancel{n-2}} \quad \frac{\cancel{n}\ \cancel{n-1}\ \cancel{n-2}}{3!} = \frac{1}{3!}$

P 3

$P\left(\bigcup_{i=1}^{n} E_i\right) = \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots$

$1 - P\left(\bigcup_{i=1}^{n} E_i\right) = \underbrace{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \cdots}_{} + \frac{1}{n!}$

∞-series

if $n = \infty$ $\quad e^{-1} = 0.36788$