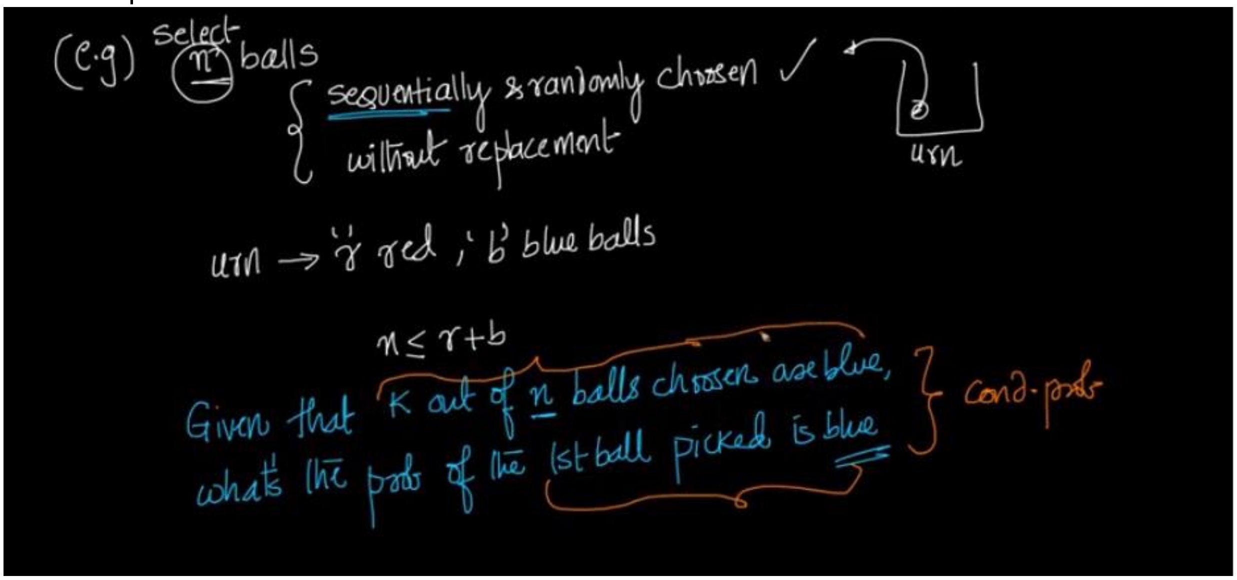
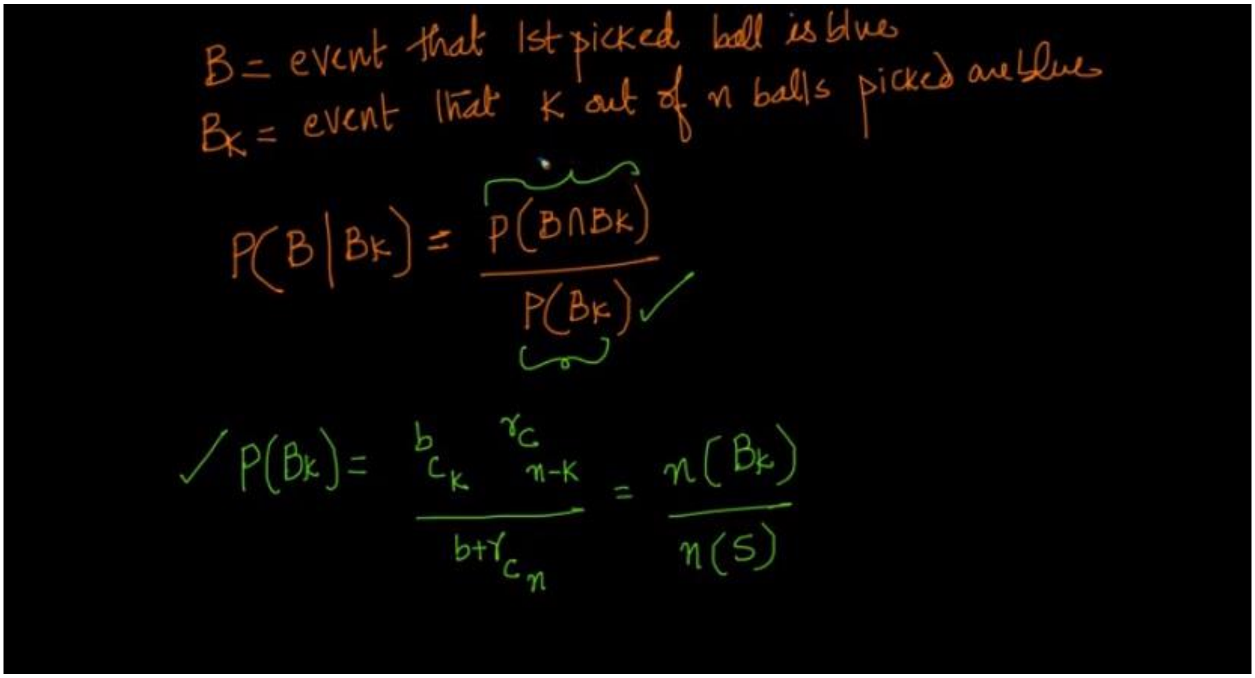
21.7 Multiplication theorem

Timestamp



- For understanding conditional probability let's consider the above example
- There are r red balls and b blue balls in an urn, we have to select n balls sequentially and randomly chosen without replacement.

- Given that out of n balls chosen K balls are blue. We need to find the probability of the 1 st ball being picked is blue.



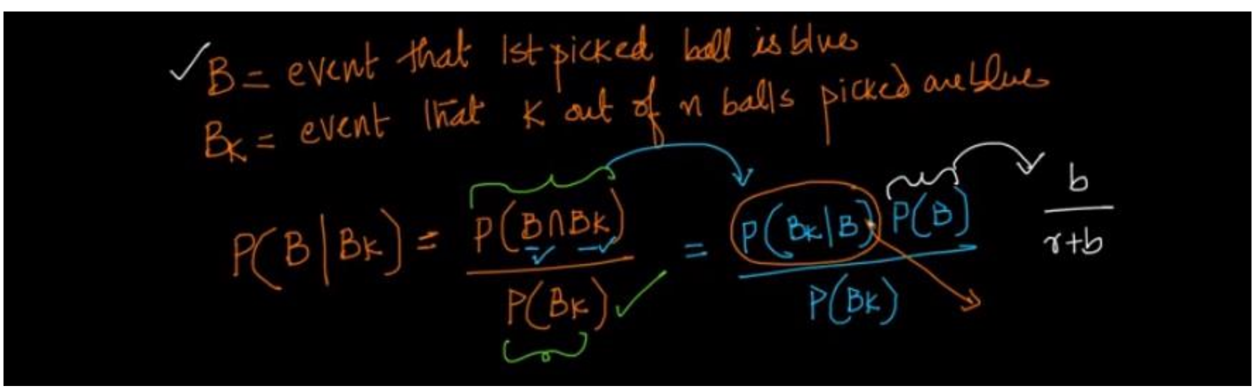
We define events as shown above and the we need to find.

P(B|BK) = P (B ∩ BK)/P(BK)

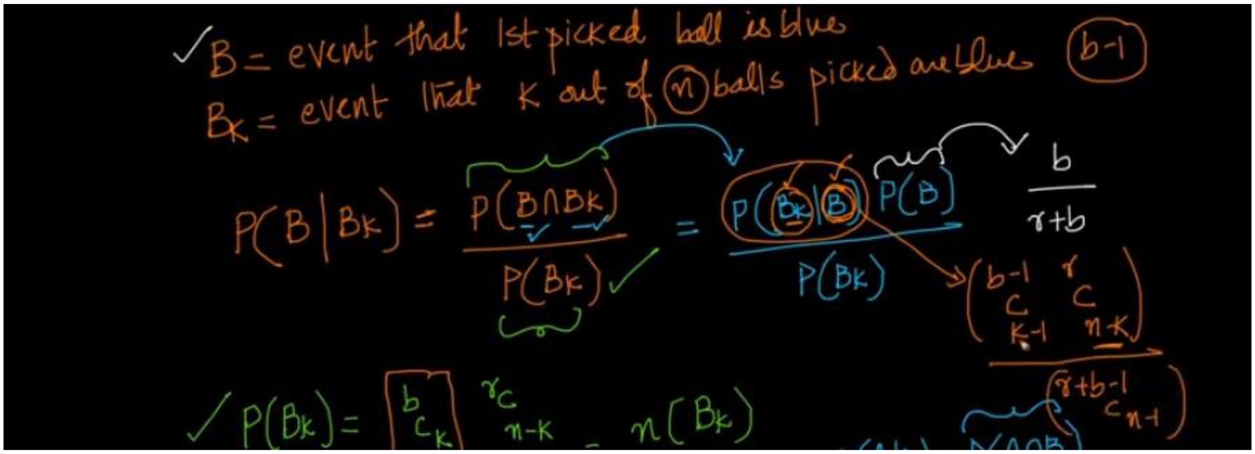
We calculate the probability of event BK (k out of n, balls picked are blue) as shown above.

- We are choosing n balls out of b+r balls, this is our sample space.
- We select k balls out of b blue balls and n-k balls out of r red balls

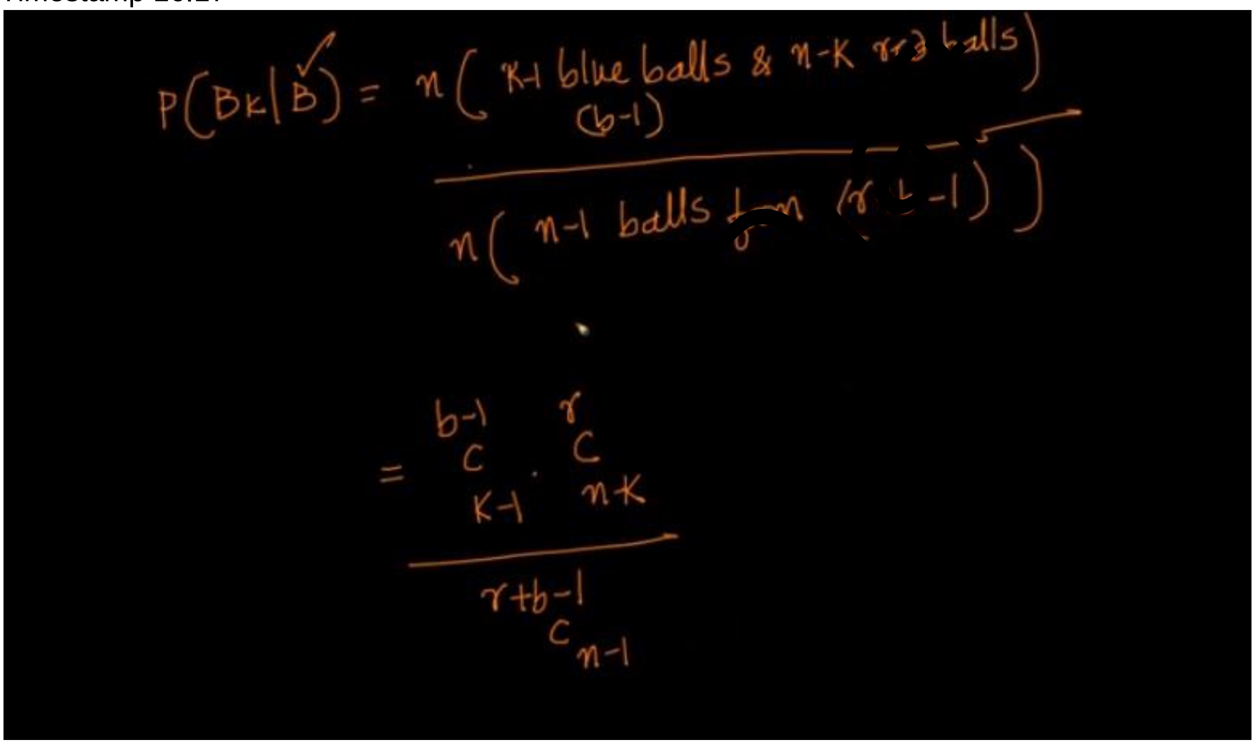
Timestamp 10.19



Using conditional probability, we can rewrite ***P (B∩ BK)*** . Now we find P(B), the probability that first picked ball is blue i.e.) b/r+b as shown above.



Timestamp 16.17



- Given that we have already picked first ball blue, now we have to find the probability that we pick k blue balls out of n balls ***i.e.) P (Bk / B)***
- Since we have already picked first ball blue we choose k-1 balls out of b-1 blue balls and n-k balls out of r red balls as shown above

Now we have our numerator and denominator, we just substitute the values and we get
P(B|BK) = P (B ∩ BK)/P(BK) =k/n

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \quad \text{if } P(E_2) \neq 0$$

Above is the definition of conditional probability, We can think of multiplication rule as a generalization of conditional probability

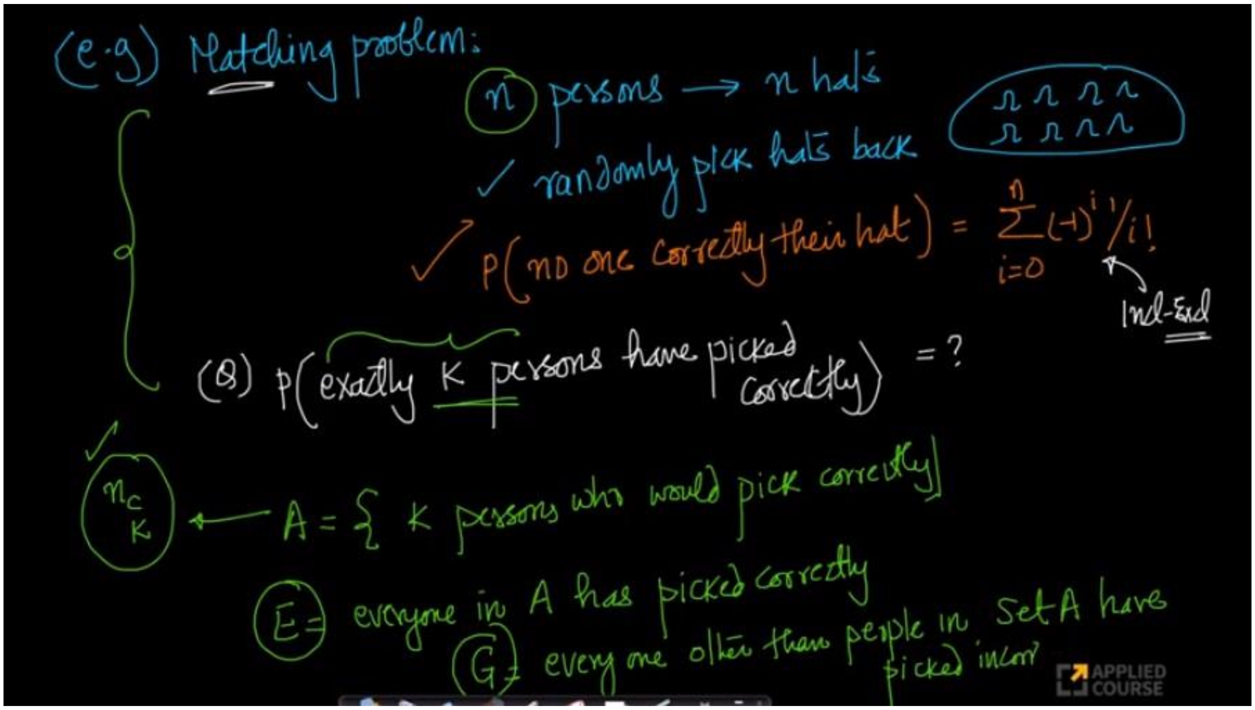
Timestamp 22.44

$$\Rightarrow P(E_1 \cap E_2) = P(E_1|E_2)\,P(E_2)$$

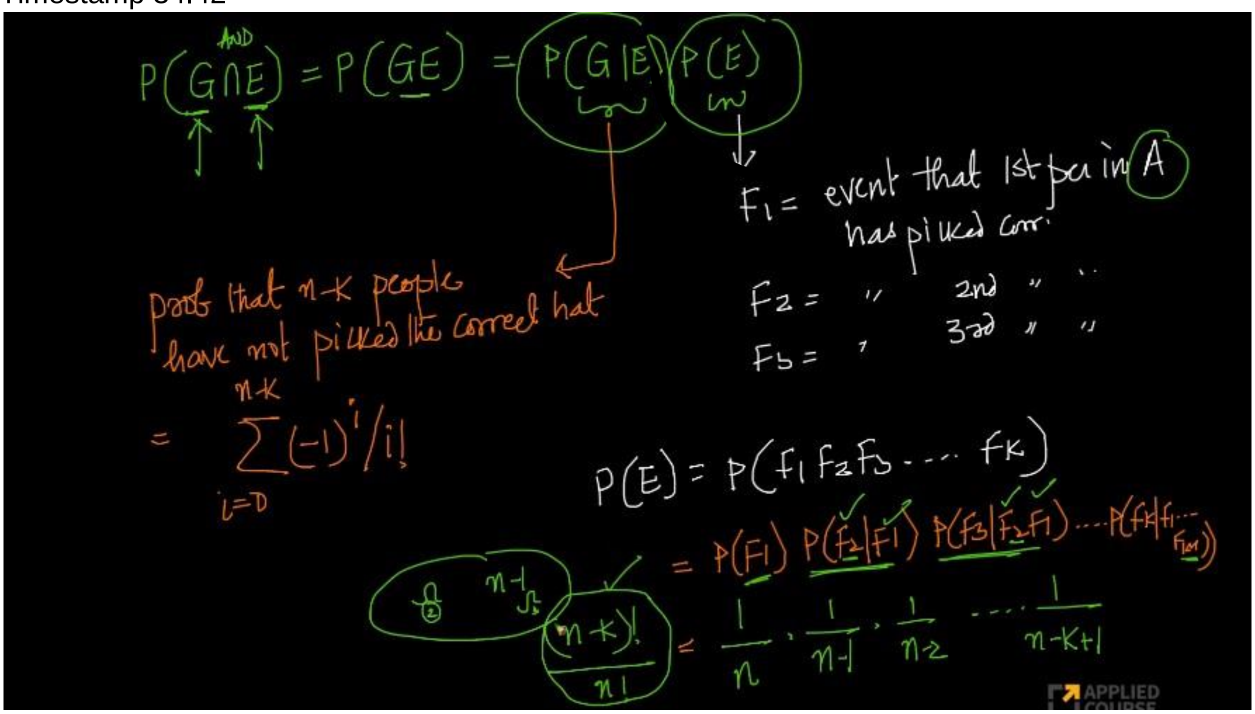
$$P(E_1E_2) = P(E_1|E_2)\,P(E_2)$$

$$\text{Gen: } P(E_1E_2E_3\cdots E_n) = P(E_1)\cdot P(E_2|E_1)\cdot P(E_3|E_2E_1)\cdots P(E_n|E_{n-1}E_{n-2}\cdots E_1)$$

By repeatedly applying conditional probability we obtain the multiplication rule as shown above.



Timestamp 34.42



- Above is a problem where n persons randomly pick their hats will be solved as shown above