Optima using Partial derivatives

We have seen how to find multi-variant derivative. Now lets try to find optima of a multi-variable function

let $f(x,y) = z = x^2 + y^2 - 2x + 2y + 6$

$$\nabla z = \begin{bmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x - 2 \\ 2y + 2 \end{bmatrix}$$

At optima $\nabla z = 0$

$$\Rightarrow \begin{bmatrix} 2x - 2 \\ 2y + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x = 1;\ y = -1$$

At $x = 1$ and $y = -1$ we have a optima

Techniques for determining if an optima is a minima or a maxima in case of multivariable function will be discussed later

By looking at the plot of $f(x,y)$ we can say there does not exist a maxima for $f(x,y)$. So, the optima which we computed will definetly be minima.

minima of $f(x,y) = x^2 + y^2 - 2x + 2y + 6$ lies at $x = 1$ and $y = -1$

which is $z = f(x,y) = 1 + 1 - 2 - 2 + 6$

$$\boxed{z = 4}$$

Note: There exists cases where $\frac{df}{dy} = 0$ (or) $\nabla f = 0$ at some places other than minima & maxima, such cases will be covered in next lecture.