

## 21.9 Law of total Probability

Timestamp 3.00

Law of total probability

(e.g) Manufacturing

✓ Two factories  $X$  &  $Y$  (bulbs)

✓ bulbs made in  $X$ , work for 5000hrs in 99% cases

✓ " " "  $Y$ , " " " in 95% cases

$p =$  prob that a bulb works for 5000hrs

60%	40%
X	Y
98%	

1. Above is an example of use case where we use Law of total probability
2. We have two manufacturing factories X and Y. As shown above if we have a package where we have 60% of bulbs from factory X and rest 40% from factory Y. Now we need to find the probability that the bulb works for 5000hours.

A single bulb can't be manufactured in both factory A and B. So, this is a mutually exclusive event

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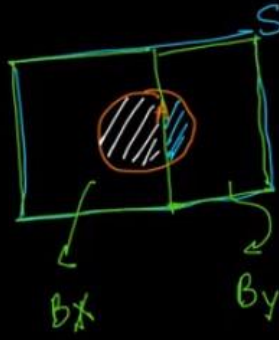
✓  $A$ : event that a bulb works for 5000 hrs  
 $B_x$ : event that a bulb made at  $x$   
 $B_y$ : " " " "  $y$

$$P(A) = P(A \cap B_x) + P(A \cap B_y)$$

$$= P(A|B_x) P(B_x) + P(A|B_y) P(B_y)$$

$$(0.99 \times 0.6) + (0.95 \times 0.4)$$

$$\left\{ \begin{array}{l} S = B_x \cup B_y \\ B_x \cap B_y = \phi \end{array} \right\}$$



- We can solve the problem as shown above using total probability.

if  $B_1, B_2, B_3, \dots$

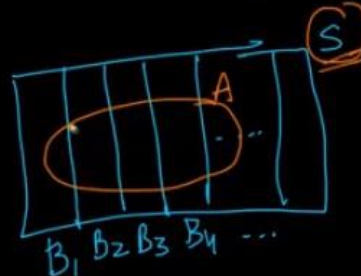
$$\left\{ \begin{array}{l} B_i \cap B_j = \phi \rightarrow \text{mutually exclusive} \\ B_1 \cup B_2 \cup B_3 \cup \dots = S \end{array} \right.$$

then,

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + \dots$$

$$= P(A|B_1) P(B_1) + P(A|B_2) P(B_2) + \dots$$

$$P(A) = \sum_i P(A|B_i) P(B_i)$$



- If you have several events  $B_1, B_2, B_3, B_4, \dots$  etc. such that all are mutually exclusive events and union of all the events is sample space  $S$  (mutually exhaustive events). Then probability of any event  $A$  can be written as shown above using law of total probability.