

21.6 Conditional Probability & Examples

Timestamp

Handwritten notes on a blackboard illustrating conditional probability using the example of throwing two distinct dice.

- Conditional probability
- (e.g.) Throw 2 distinct dice
- Sample space $S = \{(1,1), (1,2), \dots, (6,6)\}$
- Event F : first die is 3 = $\{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$
- Event E : sum of two dice = 7 = $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$
- Intersection of events E and F : $\{(3,4)\}$
- Probability of event E has occurred given that event F has already occurred:

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/36}{6/36} = \frac{1}{6}$$

where $P(F) \neq 0$.

Let's try to understand conditional probability using an example

- Consider an experiment of throwing 2 dice, we have Sample space $S = \{(1,1), (1,2), \dots, (6,6)\}$
- Let's say
 - Event F where we get 3 on the first die - $\{(3,1), (3,2), (3,4), (3,3), (3,5), (3,6)\}$
 - Event E where sum of two dice is 7 - $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

Now we can find probability of event E given that event F has already occurred using conditional probability

$$P(E|F) = P(E \cap F) / P(F) \text{ as shown above}$$

Conditioned on the fact that F has already occurred the probability of event E occurring will be given by $P(E \cap F) / P(F)$ and probability holds only when $P(F) \neq 0$.

Timestamp 13.30

(8) Student taking one-hr exam

✓ $P(\text{student finishes the exam in under } x \text{ hrs}) = \frac{x}{2} \quad \text{--- (1)}$

Given a student is working at 0.75 hr what is the prob that the student uses the full 1 hr

✓ $F = \text{student uses the full one hr}$
 $F^c = \text{student finishes exam in under 1 hr}$

$P(F^c) = 1/2$ $F \cup F^c = S$
 $P(F) = 1 - 1/2 = 1/2 \quad \text{--- (2)}$

$L_x = \text{student finishes in } x \text{ hrs}$
 $L_x^c = \text{student is still working at } x \text{ hrs}$

$L_{0.75}^c$ ✓

$P(F | L_{0.75}^c) = \frac{P(F \cap L_{0.75}^c)}{P(L_{0.75}^c)} = \frac{P(F)}{1 - P(L_{0.75})}$

$= \frac{1/2}{1 - 0.75/2}$
 $= 0.8$ ✓

$P(L_x) = \frac{x}{2} \quad 0 \leq x \leq 1$

{ eqns are poetry } ✓

Consider the above example where a student is taking one hour exam.

- Given probability that student finishes the exam in under x hrs $= x/2$
- Given a student is working at 0.75 hrs We need to find the probability that the student uses full hour.

We can solve the problem as shown above using conditional probability.