Bayes Theorem:

Bayes theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the event

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

Proof:

w.k.T from Conditional probability

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{P(B \cap A)}{P(B)} \quad \left[\because P(A \cap B) = P(B \cap A)\right]$$

$$\Rightarrow P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \quad \left[\text{from Conditional prob.}\right]$$

$$P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B|A)P(A) + P(B|A^c)\,P(A^c)} \Bigg\} \text{ Alternative representation.}$$

Where is Bayes theorem useful:

lets see a example of medical diagnosis of Breast Cancer

Here are

→ Approx 1% of women in 40-50 have breast Cancer

→ Mammogram (x-ray) → Chap but not perfect

↳ Technique for identifying Cancer.

→ lets Consider that the stats of mammogram are as follows

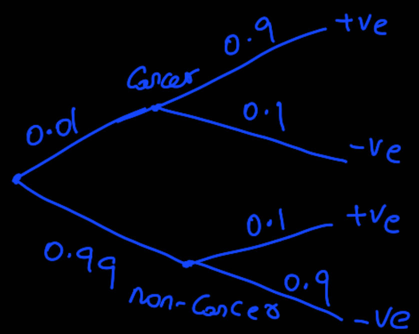
1. If a woman has breast cancer, the test will result in +ve value 90% of times.
2. If a woman doesnot have breast Cancer, the test results in +ve value 10% of times.

So, now if a woman of age 42 with tested +ve in mammogram comes to a doctor, then what is the probability that woman has cancer

$$P(\text{Cancer} \mid +\text{ve}) = \frac{P(+\text{ve} \mid \text{Cancer})\, P(\text{Cancer})}{P(+\text{ve})}$$

$$P(\text{cancer} \mid +\text{ve}) = \frac{0.9 \times 0.01}{0.108}$$

$$= \frac{9}{108} = \underline{\underline{8.35\,\%}}$$



$$P(+\text{ve}) = P(+\text{ve} \cap \text{Cancer}) + P(+\text{ve} \cap \text{not-Cancer})$$

$$\Rightarrow P(+\text{ve} \mid \text{Cancer})\, P(\text{Cancer}) + P(+\text{ve} \mid \text{non-Cancer})\, P(\text{Cancer})$$

$$\Rightarrow (0.9)(0.1) + (0.1)(0.99) = \underline{\underline{0.108}}$$