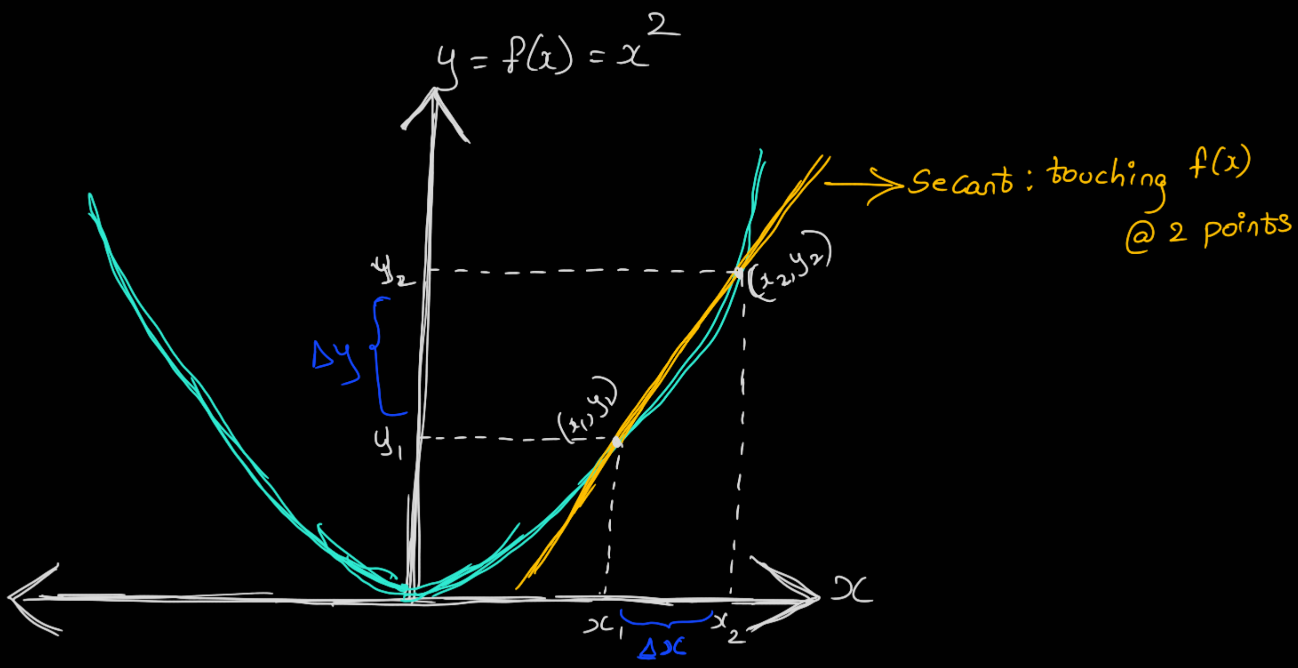
Rate of change:



Now if we want to find slope of this secant line

$$a = x_2 - x_1$$

$$b = y_2 - y_1$$

$$\tan(\theta) = b/a = \Delta y / \Delta x$$

↳ slope of secant

What if $\Delta x \to 0$: x_2 will move closer towards x_1 and

Secant will become tangent @ x_1

$$\left.\frac{dy}{dx}\right|_{x=x_1} = \lim_{(x_2 - x_1) \to 0} \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

→ How much is y changing for a small change in x

⇓

Rate of change

Upto now we have been Computing $\left(\frac{dy}{dx}\right)$ at Some point. So, insted of Computing at Some point Can we Compute at all Points

The answer is yes.

Let $f(x) = x^2$

$$\frac{d(x^2)}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{(x+\Delta x) - x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + \Delta x^2 + 2 \cdot x \cdot \Delta x - \cancel{x^2}}{\cancel{x} + \Delta x - \cancel{x}}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x^2 + 2x \cdot \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \Delta x + 2x \Rightarrow 0 + 2x = 2x$$

$$\boxed{\frac{d(x^2)}{dx} = 2x}$$

now, if we need derivative at $x=4$

$$\left.\frac{d(x^2)}{dx}\right|_{x=4} = 2 \cdot 4 = \underline{\underline{8}}$$