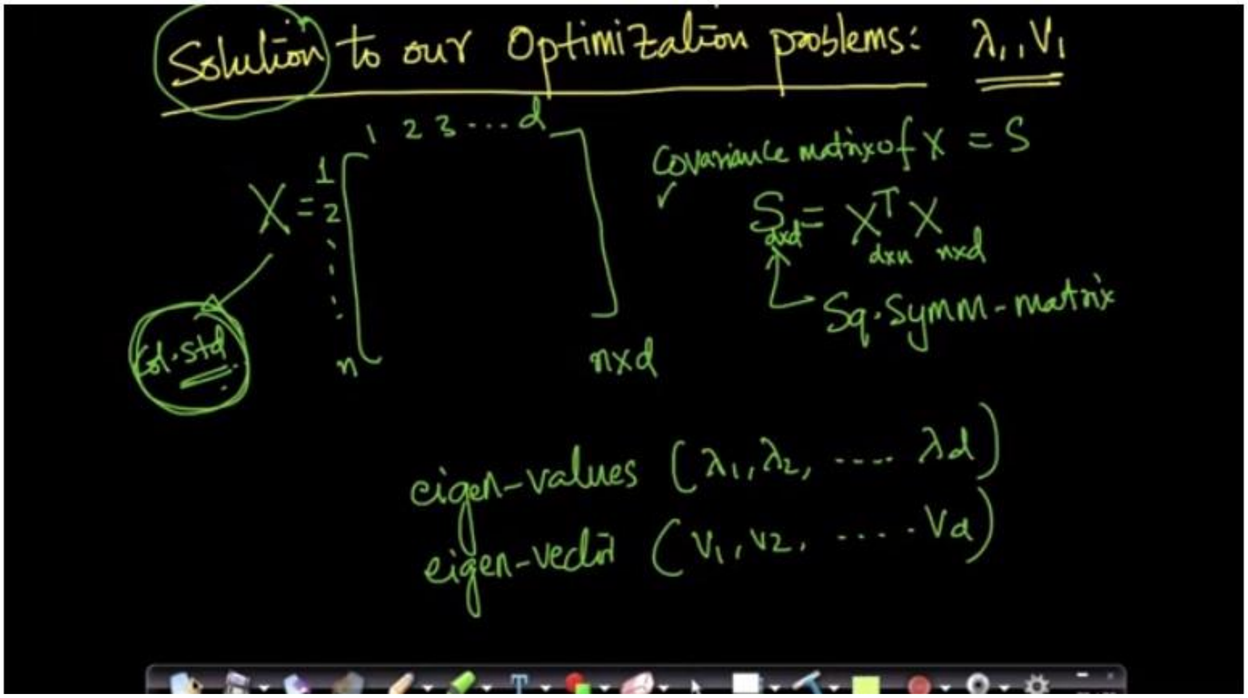
Eigen Values and Eigen Vectors



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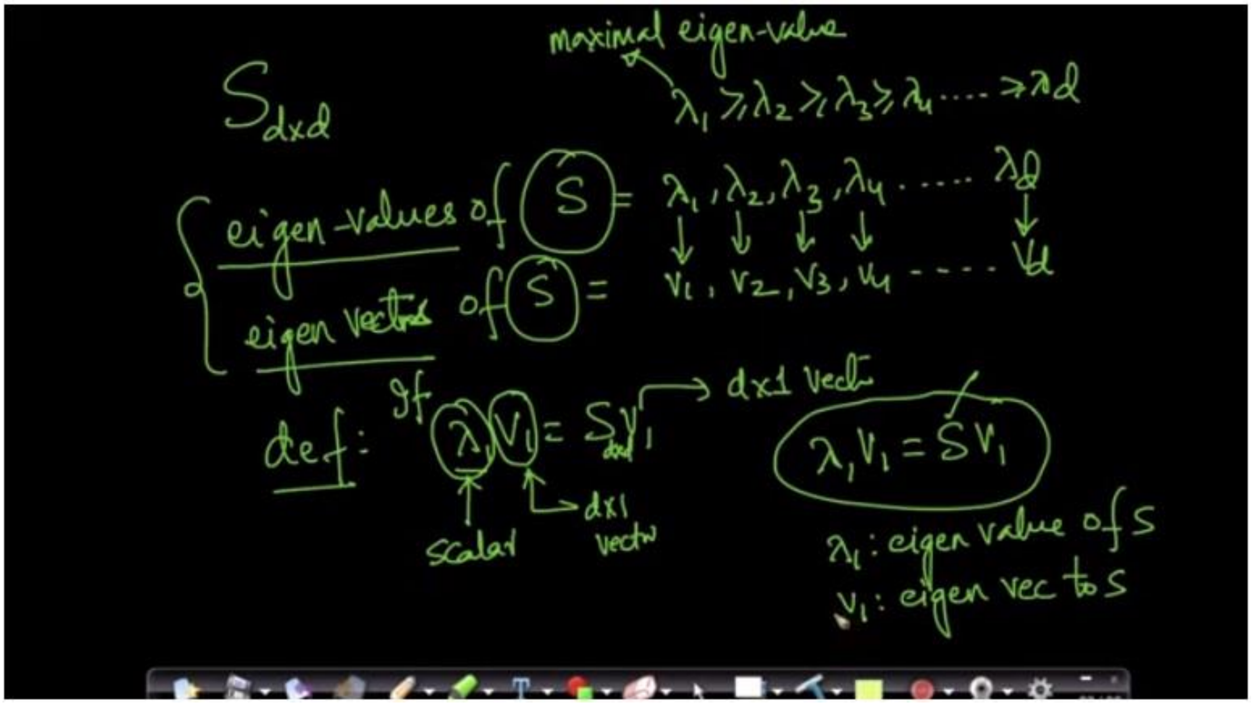
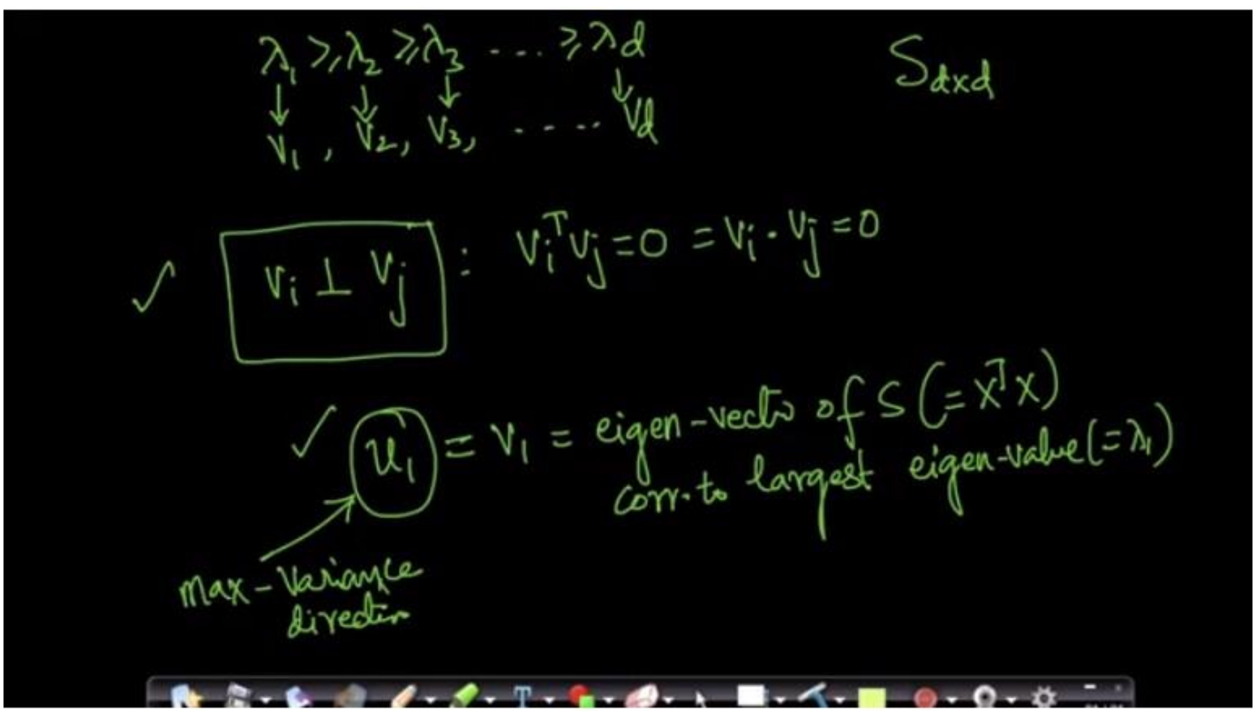
Solution to our optimization problem will be to find the eigenvalues and eigenvectors of the covariance matrix S of our data matrix X. (Correction in image S= (X^TX)/n) [S=(XTX)/(n-1) for unbiased estimate. What is meant by biased estimate and unbiased estimate will be covered in the upcoming live session]



Finding eigenvalues and eigen vectors of any matrix S is pretty straightforward and we might learnt earlier. We have to find eigen vectors and eigenvalues such that they satisfy the condition highlighted.



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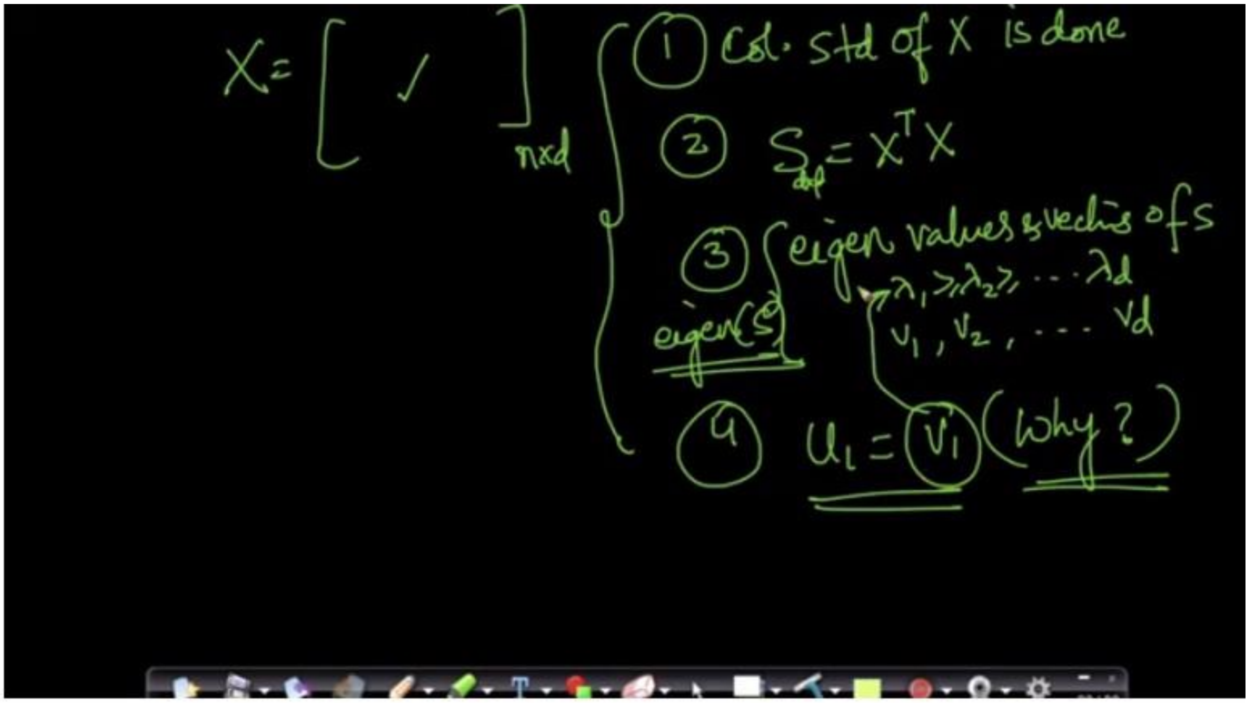
Eigenvectors have a property that every eigen vector is perpendicular to every other eigenvector. If we can find the eigenvectors and eigenvalues of the covariance matrix S, then we can easily get the direction u1 which maximizes the variance as it is equal to the eigenvector of S that corresponds to the largest eigenvalue.



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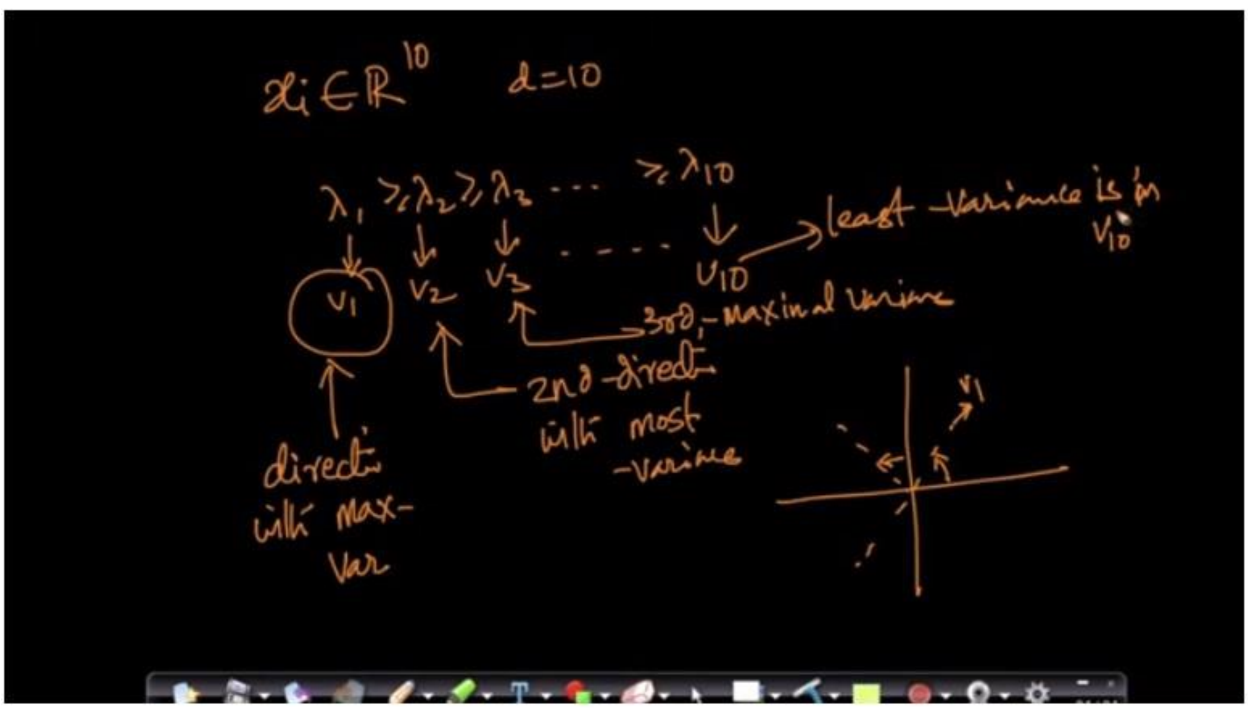
So given a matrix X, in order to find u1, the max-variance direction, we need to ensure that X is column standardized, post this we can calculate the covariance matrix S. Following that you can just find the eigenvalues and eigenvectors of S andu1 will just be the eigenvector with max eigenvalue.



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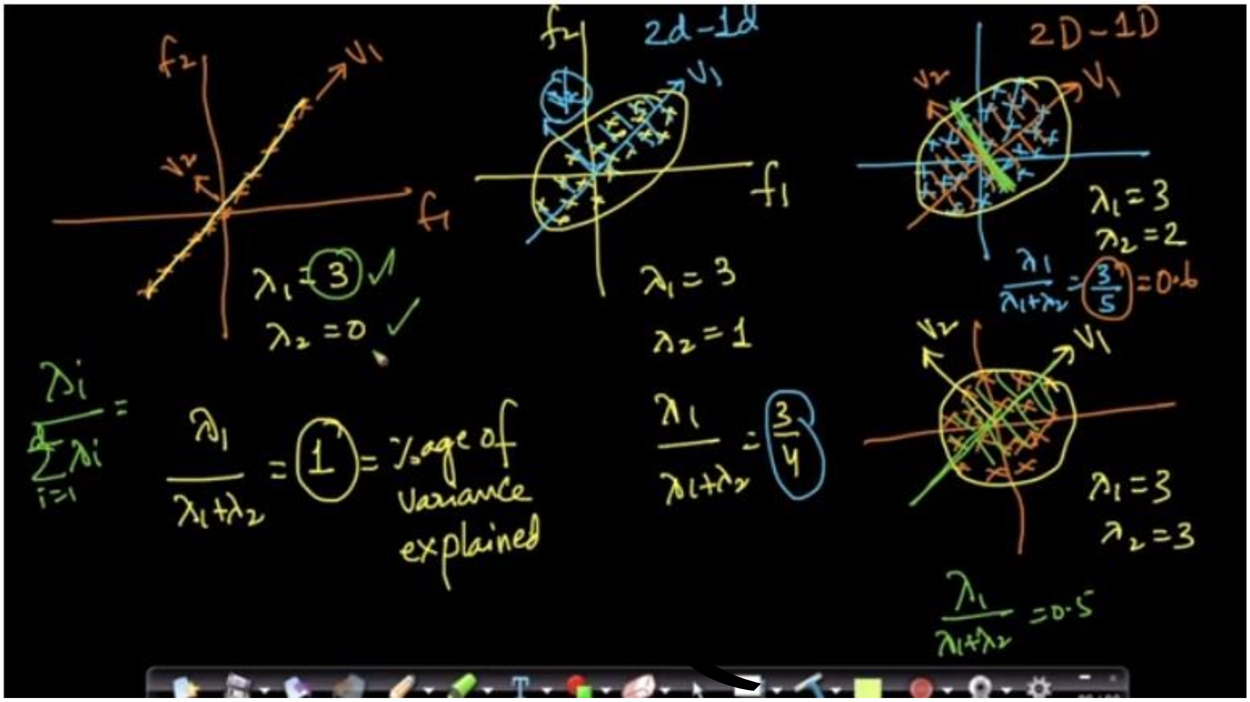
So, if we sort of our eigenvectors of the covariance matrix S in the decreasing order of their corresponding eigenvalues, then v1 represents the direction with max-variance, v2 represents the direction with second max-variance and so on.



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While eigenvectors v1, v2, etc tell us the directions of most-variance, second most variance etc. ƛ1, ƛ2, etc tell us the variance explained in that direction. As shown in the above figure ƛ1/(ƛ1+ƛ2) tells us the percentage of variance explained by the direction of v1.