

## 21.4 Probability from a set theoretic view

Timestamp 9,45

Sample space (S): Set of all the possible outcomes of an expt

Expt: flipping 2 coins (distinct)

Outcomes:  $\{(H,H), (H,T), (T,H), (T,T)\} = S$

Expt: conduct a 7-way horse race

Outcomes: ordering of 7 horses  
 $2, 1, 3, 6, 5, 4, 7$

horse-race  $(7)$   $1, 2, 3, \dots, 7$

$(7!)$   
 $S = \{7! \text{ possible orderings}\}$

Expt: 2 dice (distinct)  $1, 2, 3, \dots, 6$

Outcomes:  $\{(1,2), (1,3), (1,4), \dots\} = S = \{36 \text{ possible outcomes}\}$

$6 + 6 = (36)$  ✓

Expt: light bulb, measuring the #hrs the light bulb works.

Outcomes:  $(S) = \{x : 0 \leq x < \infty\}$   
 $x \in \mathbb{R}$

$\left. \begin{matrix} 0.1 \\ 0.2 \\ 0.11 \end{matrix} \right\}$

1. Let's study probability from set theoretic view
2. Let's say an experiment of flipping 2 coins the possible outcomes are as shown above. Set of all possible outcomes of an experiments called the sample space(S).

3. Imagine another experiment where we conduct a seven horse race, the set of all possible outcomes are the ordering of horses after the race is sample space i.e.)  $7!$  possible orderings are the sample space
4. In experiment if throwing two distinct dice the set of all possible outcomes are  $36$  ( $6 \times 6$ ) which is sample space.
5. Consider another experiment where we have a lightbulb and we will be measuring the number of hours the light bulb works. In this case our possible outcomes lie in range  $0 \leq X < \infty$  which is sample space.

The possible outcomes need not be discrete finite sets, it can also be an infinite set of outcomes. Sample space is the space of all the possible outcomes that we care about as experiment is concerned.

Timestamp 16.57

Event {E} Any subset of S is an event

(e.g)  $E = \{x : 0 \leq x \leq 5\} \Rightarrow$  light bulb works for 0 to 5 hrs

$E \subseteq S$

$\begin{Bmatrix} l_1 \\ l_2 \\ l_3 \\ \vdots \\ l_m \end{Bmatrix} \rightarrow \leq 5 \text{ hrs}$

Expt: 2 dice (distinct)  $1, 2, 3, \dots, 6$

Outcomes:  $\{(1,2), (1,3), (1,4), \dots\} = \underline{S} = \{36 \text{ possible outcomes}\}$

$6 \times 6 = 36$

$S \supseteq \underline{E} = \{(3,3), (1,5), (5,1), (4,2), (2,4)\}$

1. An event is any subset of S (Sample space) including null set.
2. Consider the experiment of the light bulb discussed above and we can have an event E such that the light bulb works for 5 hours as shown.  $E: 0 \leq X \leq 5$
3. In the experiment of throwing two dice let's define an event E such that the sum of the dice is 6  $E: \{(3,3), (2,4), (4,2), (1,5), (5,1)\}$
4. Above are all possible ways in which the event (getting sum 6 when two dice are rolled) can occur.

Similarly, we can define events on experiments where  $E \subseteq S$ .

