Solved Examples on finding optima:

① $f(x) = \log(1+e^{x})$ → This function is useful later in ML

Sol: → lets find derivative of $f(x)$

$$\Rightarrow f'(x) = e^{x}\cdot\left(\frac{1}{1+e^{x}}\right) = \frac{e^{x}}{1+e^{x}}$$

At optima $f'(x) = 0$

$$\Rightarrow \frac{e^{x}}{1+e^{x}} = 0 \Rightarrow \underbrace{e^{x} = 0 \left[\text{There is no such } x \text{ for which } e^{x} = 0\right]}$$

So, $f(x)$ do not have any optima

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② $f(x) = (10-(2x+3))^{2}$

Sol:

$$f'(x) = 2\cdot\left(2\left(10-(2x+3)\right)\right)$$

$$= -4\left(10-(2x+3)\right)$$

$$= -40 + 8x + 12$$

$$= 8x - 28$$

At optima $f'(x) = 0$

$$\Rightarrow 8x - 28 = 0$$

$$x = 28/8 = 3.5 \quad (\text{optima occurs @ } 3.5)$$

→ But is it maxima or minima

→ slope @ 3 = $(8\times3) - 28 = -4$

→ slope @ 4 = $(8\times4) - 28 = +4$

On right side of optima slope is +ve & on left side of optima slope is −ve } So, The optima (3.5) is the point where minima occurs

$$\text{minima} = f(3.5) = (10-(2\times3.5+3))^{2}$$

$$= (10-10)^{2} = \underline{0}$$