

21.8 Independent events

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✓ Independence of events:

(e.g) Expt: Toss a coin & throw a dice

$\begin{cases} E: \text{the coin is H} \\ F: \text{the dice is 3} \end{cases}$ ✓

$p(E \cap F) = \overbrace{p(E|F)}^{= p(E)} p(F) \rightarrow \text{def. cond. probs}$

$= p(E) \cdot \frac{1}{6}$

$= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

✓ Independent events

$\begin{cases} p(\check{E}|\check{F}) = p(\check{E}) \text{ --- (1)} \\ p(\check{E} \cap \check{F}) = p(\check{E}) p(\check{F}) \checkmark \end{cases}$

Diagrams: A circle with 'H' and a cube with '3' on top, both with checkmarks.

1. Consider an experiment of tossing a coin and throwing a dice. As shown above define two events E and F
E = Probability of getting heads
F = Probability of getting 3
2. The probability of both events happening $E \cap F$ can be obtained using conditional probability.
 $p(E \cap F) = p(E|F) \cdot p(F)$
3. The fact that F has already happened has no impact on E happening. Event E happening doesn't depend on event F (E is independent of F), so $p(E|F) = p(E)$.
Then
 $p(E \cap F) = p(E) \cdot p(F)$
4. Two events are said to be independent events when their intersection is equal to the product of the product of events.

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NOTE: Independence vs Mutually Exclusive

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|---|--|
| $P(E \cap F) = P(E)P(F)$ $P(E F) = P(E)$ | $E \cap F = \phi$ $P(E \cap F) = P(\phi) = \underline{0}$ |
|---|--|

- For mutually exclusive events the intersection of events is null, independent events when their intersection is equal to the product of the product of events.

(e.g.) 52 Cards

expt: { pick a 1st card randomly
replace the 1st card
pick the 2nd card randomly }

$P(\text{jack and } 8) = P(\text{jack} \cap 8)$
 $= P(\text{jack}) \cdot P(8)$
 $= \frac{4}{52} \cdot \frac{4}{52}$

- In the above example probability of picking a second card doesn't depend on the event of picking the first card since we are replacing the first card. They both are independent events.

(e.g) (2) Coins (distinct)

✓ E: 1st coin is H
 ✓ F: 2nd coin is T

$P(EF) = P(E) P(F)$
 $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Gen: $P(E_1 E_2 \dots E_n) = P(E_1) P(E_2) P(E_3) \dots = \prod_{i=1}^n P(E_i)$

indep. of each other

- We can extend the concept of independent events and generalize it for n events as shown above. It's nothing but the product of all the events which are independent.



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NOTE: if E & F are indep then E & F^c are also indep

$P(E) = P(E \cap F) + P(E \cap F^c)$
 $= P(E) P(F) + P(E \cap F^c)$

$P(E) \{1 - P(F)\} = P(E \cap F^c)$

$P(E) P(F^c) = P(E \cap F^c)$

- As shown above if two events E and F are independent then E and F^c are also independent.

(e.g.) { An infinite seq. of trials expt (indep) is performed
 a success prob = p

(a) at least one success in n indep trials

$$= 1 - P(\text{no success in } n \text{ trials})$$

$$= 1 - P(\checkmark F_1 \checkmark F_2 F_3 F_4 \dots F_n)$$

$$= 1 - P(F_1)P(F_2) \dots P(F_n) = \underline{\underline{1 - (1-p)^n}}$$

{ trial: throwing a die
 success: outcome is 1
 (p=1/6)
 failure: outcome is not 1
 (1-p)}

(b) exactly K successes in n trials

$$= \binom{n}{K} p^K (1-p)^{n-K}$$

(c) all n trials are succ.

$$= p \cdot p \cdot \dots \text{ n times } = \underline{\underline{p^n}}$$

- Above is an example of using independent events to solve problems.