

Exploring Infectious Disease Dynamics: A Mathematical Approach with Python

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Background and History

- widely used in the study of how diseases spread through populations
- describes the dynamics of an infectious disease outbreak over time
- crucial for implementing effective control measures
- a powerful tool that simplifies the complex processes of disease transmission into a set of mathematical equations

Basics

- 1 Susceptible (S): Individuals who are unaffected now but are vulnerable to getting infected
 - 2 Infectious (I): Individuals who are currently infected and are able to spread the disease
 - 3 Recovered or Removed (R): Individuals who have recovered from the disease or who have passed away from the disease.
- The model then traces the movement of individuals between these compartments over time.

Significance

- 1 Predictive Insights
- 2 Public Health Planning
- 3 Resource Allocation
- 4 Research and Policy Evaluation

Furthermore, the SIR model's analytical capabilities extend to addressing critical aspects of an epidemic, including:

- 1 Peak Infection Rate
- 2 Total Attack Rate
- 3 Herd Immunity
- 4 Effects of Transmission, Recovery, and Death Rates

Assumptions

- 1 Homogeneous Mixing
- 2 Closed Population
- 3 Permanent Immunity
- 4 Constant Parameters
- 5 Exponential Growth at the Start
- 6 No Disease-Induced Death

Limitations

- 1 Simplification of human behavior
- 2 Homogeneous mixing
- 3 Closed population
- 4 Deterministic nature
- 5 Limited data requirements
- 6 Inability to capture complex interventions

Disease Parameters

- 1 β (**Transmission Rate**): average number of people an infected individual will transmit the disease to per unit time.
- 2 γ (**Recovery Rate**): rate at which infected individuals recover and move to the recovered compartment. inverse of the average infectious period

Size Parameters

- 1 **S:** total number of individuals in the population susceptible to catching the disease.
- 2 **I:** total number of individuals in the population who are currently infected and can spread the disease.
- 3 **R:** total number of individuals in the population who are recovered or have passed away due to the disease and are considered no longer susceptible to the disease

- 1 **Initial Conditions:** The model starts with specified initial numbers of susceptible (S_0), infected (I_0), and recovered (R_0) individuals.
- 2 **N (Total Population Size):** total number of individuals in the population being modeled. It's used to calculate proportions of susceptible, infected, and recovered individuals.

Transitions

- 1 Susceptible (S) to Infected (I) β
- 2 Infected (I) to Recovered (R) γ

There are no direct transitions between Susceptible (S) and Recovered (R) in the basic SIR model

Model Equations

system of three ordinary differential equations that describe the rates of change for each compartment over time.

$$dS/dt = -\beta * S * I/N$$

$$dI/dt = \beta * S * I/N - \gamma * I$$

$$dR/dt = \gamma * I$$

Susceptible Individuals

$$dS/dt = -\beta * S * I/N$$

- rate of change in the susceptible population is proportional to the product of S, I and β
- represents the number of contacts between susceptible and infected individuals
- β determines the probability of infection per contact
- negative because the number of susceptible individuals decreases as they become infected.

Infected Individuals

$$dI/dt = \beta * S * I/N - \gamma * I$$

they can either recover from the disease or remain infected
rate of change is determined by two factors:

- 1 **Gaining new infections:** represented by the same term as in the susceptible population change, $\beta * S * I/N$.
- 2 **Losing individuals to recovery:** proportional to the infected population (I) and the recovery rate (γ).

Recovered Individuals

$$dR/dt = \gamma * I$$

- Recovered individuals gain immunity and cannot be infected again
- rate of change in the recovered population is simply the number of individuals recovering from the disease
- directly proportional to I and γ

Equilibrium Points

state where the values of S , I , and R remain constant over time, indicating that the epidemic has reached a steady state

- 1 **Disease-free equilibrium ($S = N$, $I = 0$, $R = 0$):** This occurs when there are no infectious individuals in the population
- 2 **Endemic equilibrium (S^* , I^* , R^*):** This occurs when the disease persists in the population at a constant level

Basic Reproduction Number (R_0)

average number of secondary infections caused by a single infected individual in a fully susceptible population.

$$R_0 = \beta/\gamma$$

If $R_0 > 1$, the disease can spread and cause an epidemic.

If $R_0 < 1$, the disease will eventually die out.

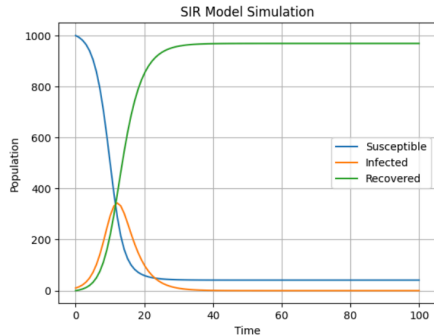
Building the Model

AIM:

- 1 use Python to run simulations for a hypothetical disease spread model
- 2 study the visualization under normal parameter values approximated from literature
- 3 study the effects of disease parameters on disease spread
- 4 effect of the number of initial infected individuals

High Transmission, Low Recovery

$$\beta = 0.75, \gamma = 0.25, n = 1000, t = 100, I_0 = 10$$



Result:

Basic Reproduction Number (R_0): 3.00

Peak infection time: 12th day

Peak infection size: 335 individual(s)

Number of people recovered: 957 individual(s)

Percentage of total population infected at peak: 33 %

Final epidemic size: 957 individual(s)

Number of people infected at the end of the simulation: 0 individual(s)

Attack Rate: 95.70%

Herd Immunity Threshold: 66.67%

Duration of epidemic: 39 day(s)

Highest Infection Rate: 51 individual(s) infected per day

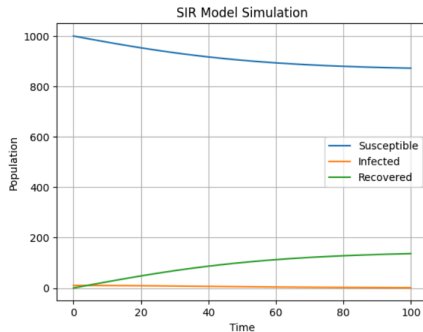
The disease does not get to 50% of the population at any given time

Inference:

- 1 concerning scenario of a highly transmissible and slow-recovery disease with a R_0 of 3.00
- 2 prolonged duration of 39 days
- 3 disease spreading to a significant portion of the population
- 4 peak infection on the 12th day affected 33% of individuals, with a strikingly high attack rate of 95.70%
- 5 slow recovery is evident in the sustained peak infection size of 343 individuals

Low Transmission, Low Recovery

$$\beta = 0.25, \gamma = 0.25, n = 1000, t = 100, I_0 = 10$$



Result:

Basic Reproduction Number (R_0): 1.00

Peak infection time: 0th day

Peak infection size: 10 individual(s)

Number of people recovered: 127 individual(s)

Percentage of total population infected at peak: 1 %

Final epidemic size: 127 individual(s)

Number of people infected at the end of the simulation: 1 individual(s)

Attack Rate: 12.70%

Herd Immunity Threshold: 0.00%

Disease spread does not end in the given simulation period

Highest infection rate is less than 0, so not significant

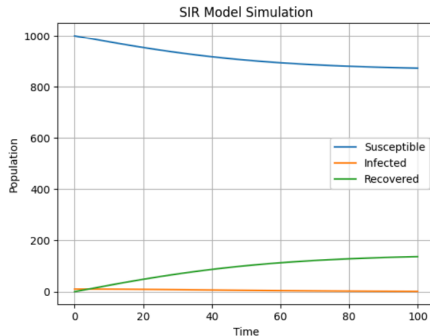
The disease does not get to 50% of the population at any given time

Inference:

- 1 reduction of the R_0 from 3 to 1, indicating a substantial decrease in the disease's transmission rate
- 2 despite this lower transmission, the disease persists in the population for an extended period
- 3 lower R_0 , while indicative of a weaker disease, paradoxically leads to a more prolonged presence within the population

Low Transmission, High Recovery

$$\beta = 0.25, \gamma = 0.75, n = 1000, t = 100, I_0 = 10$$



Result:

Basic Reproduction Number (R_0): 0.33

Peak infection time: 0th day

Peak infection size: 10 individual(s)

Number of people recovered: 14 individual(s)

Percentage of total population infected at peak: 1 %

Final epidemic size: 14 individual(s)

Number of people infected at the end of the simulation: 0 individual(s)

Attack Rate: 1.40%

Herd Immunity Threshold: -200.00%

Duration of epidemic: 4 day(s)

Highest infection rate is less than 0, so not significant

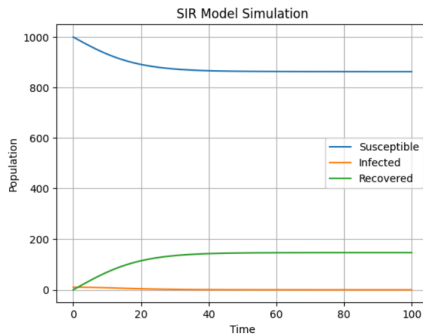
The disease does not get to 50% of the population at any given time

Inference:

- 1 remarkably low attack rate of 1.4%
- 2 resolution within just 4 days
- 3 minimal impact is attributed to the combination of a low R_0 of 0.33
- 4 signifying a modest transmission rate, and a higher recovery rate
- 5 concise duration and limited spread underscore the effectiveness of these factors in rapidly containing and resolving the disease within the population.

High Transmission, High Recovery

$$\beta = 0.75, \gamma = 0.75, n = 1000, t = 100, I_0 = 10$$



Result:

Basic Reproduction Number (R_0): 1.00

Peak infection time: 0th day

Peak infection size: 10 individual(s)

Number of people recovered: 137 individual(s)

Percentage of total population infected at peak: 1 %

Final epidemic size: 137 individual(s)

Number of people infected at the end of the simulation: 0 individual(s)

Attack Rate: 13.70%

Herd Immunity Threshold: 0.00%

Duration of epidemic: 33 day(s)

Highest infection rate is less than 0, so not significant

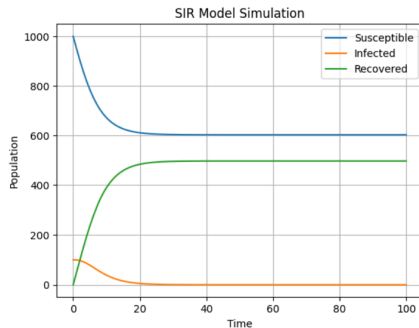
The disease does not get to 50% of the population at any given time

Inference:

- 1 R0 of 1.00 reflects a moderate transmission rate
- 2 peak infection on the 0th day, affects 1% of the population, with a final epidemic size of 137 individuals
- 3 the attack rate is relatively higher(13.70%)
- 4 the duration of the epidemic is shorter(33 days)
- 5 indicates a balance between transmission and recovery rates
- 6 intermediate R0 value results in a more extended but still manageable epidemic
- 7 controlled nature of the outbreak

Higher I0

$$\beta = 0.5, \gamma = 0.5, n = 1000, I_0 = 100, T = 100$$



Result:

Basic Reproduction Number (R_0): 1.00

Peak infection time: 0th day

Peak infection size: 100 individual(s)

Number of people recovered: 401 individual(s)

Percentage of total population infected at peak: 10 %

Final epidemic size: 401 individual(s)

Number of people infected at the end of the simulation: 0 individual(s)

Attack Rate: 40.10%

Herd Immunity Threshold: 0.00%

Duration of epidemic: 26 day(s)

Highest infection rate is less than 0, so not significant

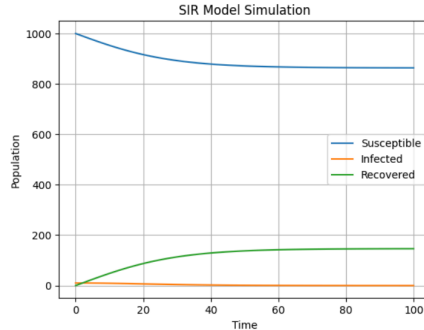
The disease does not get to 50% of the population at any given time

Inference:

- 1 immediate and substantial peak infection size (10% of the population)
- 2 higher Attack Rate (40.10%)
- 3 relatively short-lived (26 days)

Lower I_0

$$\beta = 0.5, \gamma = 0.5, n = 1000, I_0 = 10, T = 100$$



Result:

Basic Reproduction Number (R_0): 1.00

Peak infection time: 0th day

Peak infection size: 10 individual(s)

Number of people recovered: 136 individual(s)

Percentage of total population infected at peak: 1 %

Final epidemic size: 136 individual(s)

Number of people infected at the end of the simulation: 0 individual(s)

Attack Rate: 13.60%

Herd Immunity Threshold: 0.00%

Duration of epidemic: 50 day(s)

Highest infection rate is less than 0, so not significant

The disease does not get to 50% of the population at any given time

Inference:

- 1 A smaller initial infected population results in a more controlled outbreak
- 2 lower peak infection size of 1%
- 3 lower attack rate (13.60%)
- 4 slower but manageable spread
- 5 final epidemic size exceeds the initial infected population, emphasizing a prolonged but contained outbreak

Effects of the initial condition

- 1 higher initial infected population catalyzes exponential growth, leading to a larger peak infection size
- 2 higher initial cases result in an early peak
- 3 the effectiveness of control measures is influenced by the initial infected population's magnitude
- 4 smaller initial population enhances manageability
- 5 the initial infected population plays a pivotal role in determining the herd immunity threshold, affecting the percentage of the population needed for sufficient immunity

Effects of disease characteristics:

- 1 higher R_0 and transmission rate can lead to more widespread infections and a greater peak infection size
- 2 conversely, a lower R_0 and higher recovery rate contribute to a milder and shorter-lived epidemic
- 3 a moderate R_0 of 1.00, while resulting in a higher attack rate, can still yield a manageable epidemic with a shorter duration.
- 4 underscore the critical role of transmission and recovery rates in shaping the dynamics of disease outbreaks

Implications for disease control:

- 1 healthcare infrastructure and early intervention become critical in mitigating the impact of diseases with higher transmission rates
- 2 diseases with lower transmission rates may benefit from targeted vaccination and community awareness campaigns
- 3 implications for preventative measures
- 4 importance of early intervention

Shortcomings:

- 1 simplification of reality
- 2 does not take into account all of the factors that can affect the spread of a disease

For example, the SIR model does not take into account the fact that some individuals may be more susceptible to infection than others, or that the transmission rate may vary depending on the environment.

Enhancements:

- SEIR (Susceptible-Exposed-Infectious-Recovered) model introduce an exposed compartment, accounting for the latent period before individuals become infectious
- further advancements include incorporating demographic factors, spatial considerations, and heterogeneous populations, recognizing that different subgroups may experience varied transmission and recovery rates.

Conclusion:

- an ever-evolving model in the field of epidemiology
- simple yet powerful framework has provided invaluable insights into disease dynamics
- continues to shape public health
- foundation for complex models, which solve many issues that the original model did not address.