Effect Of A Dampener On The Vibrations Of A Tennis Racket

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1 Problem statement

The goals of the project are the following:

- To model a tennis racquet.
- To model a dampener.
- To look at the oscillations at the handle of the racket when a dampener is placed at different points.

2 Our model of a tennis racquet head

The head of the racket is modelled as a 2-D array of springs as shown in the figure below. The size of the racket (length:breadth proportion) has been chosen to match that of an actual tennis racket. But the distance between the 'nodes' (points where springs are connected) is slightly lesser than the diameter of a real tennis ball. It is assumed that the ball hits the racket between the nodes and the impact of the ball is distributed equally on the four nearest springs and nodes around the ball. The racket is symmetric about the vertical axis. Hence, even the spring constants of symmetric springs (about the vertical axis) have been taken to be equal. The racket is not symmetrical about the horizontal axis due to the presence of the handle. Therefore, each horizontal line of springs has a different spring constant. The springs on the edges are actually along the beam. Hence, their spring constants are all equal and are much higher (taken to be two orders of magnitude higher in this model) than all other springs. The natural lengths of all springs except the ones at the corner are the same.

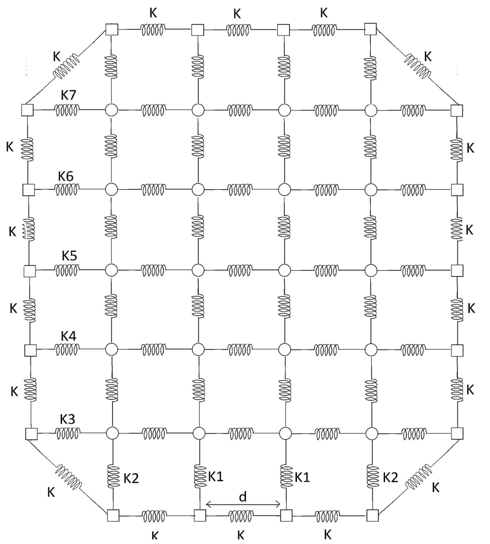

Diagram of the model with necessary spring constants marked

3 Displacement equations of the nodes

The nearest neighbour approximation has been applied to derive the equations of the nodes, i.e., each node's dynamics are affected only by those of the neighbouring nodes.
Let us have the 2-D array in the x-y plane. If :
x = displacement of node in consideration along z-axis
x_{Rt} (x_{Lt}, x_{up}, x_{dn}) = displacement of node on the right along z-axis (left, above, below)
k_{Hor} = spring constant of nodes along that horizontal line
k_{vert} = spring constant of nodes along that vertical line
k = spring constant of springs along the edges
m = mass of each node
d = natural length of each spring

then the motion of the entire system of nodes is characterized by the following equations:

- For all the nodes having 4 neighbours:

$$m\ddot{x} = \frac{-k_{Hor}}{2d^2}((x-x_{Rt})^3 + (x-x_{Left})^3) + \frac{-k_{Vert}}{2d^2}((x-x_{Up})^3 + (x-x_{Down})^3) \tag{1}$$

- For the nodes on the edges of the grid, which have one neighbouring node to the side, and two vertically up and down:

$$m\ddot{x} = \frac{-k}{2d^2}((x-x_{Down})^3 + (x-x_{Up})^3) + \frac{-k_{Hor}}{2d^2}(x-x_{Hor})^3 \tag{2}$$

- For the nodes which are in the corners and have one to the side, one vertically up or down and one aligned at 45o to the vertical axis :

$$m\ddot{x} = \frac{-k}{4d^2}(x-x_{Diag})^3 + \frac{-k}{2d^2}(x-x_{Vert})^3 + \frac{-k_{Hor}}{2d^2}(x-x_{Hor})^3 \tag{3}$$

3.1 Derivation

If k_n is a neighbouring node, and x_n is its displacement along z-axis, and the spring constant of the spring attached to the two nodes is k, then :

m $\ddot{x} = -k(\sqrt{d^2+(x-x_n)^2} - d)(\frac{x-x_n}{\sqrt{d^2+(x-x_n)^2}})$
$= -k(x-x_n)(1 - \frac{d}{\sqrt{d^2+(x-x_n)^2}})$
$= -k(x-x_n)(1 - (1 + \frac{(x-x_n)^2}{d^2})^{-1/2}))$
$= -k(x-x_n)(1 - (1 - \frac{(x-x_n)^2}{2(d^2)}))$
$= -k(x-x_n)\frac{(x-x_n)^2}{2(d^2)}$
$= -k\frac{(x-x_n)^3}{2(d^2)}$

By the nearest neighbour approximation and superposition principle, we get equations (1) and (2).
For diagonally neighbouring node, d is replaced by $\sqrt{2}$d in one of the terms of (2) that contain spring constant k, to get equation (3).
These are now solved using Euler's method.

4 Results and Discussion

4.1 Plot of displacements v/s time

The values of the different spring constants are :
k=[3.241,3.241,4.723,4.703,4.684,4.703,4.723,500]ergs-cm^{-2}
For the given impact function, the displacements at some points are:

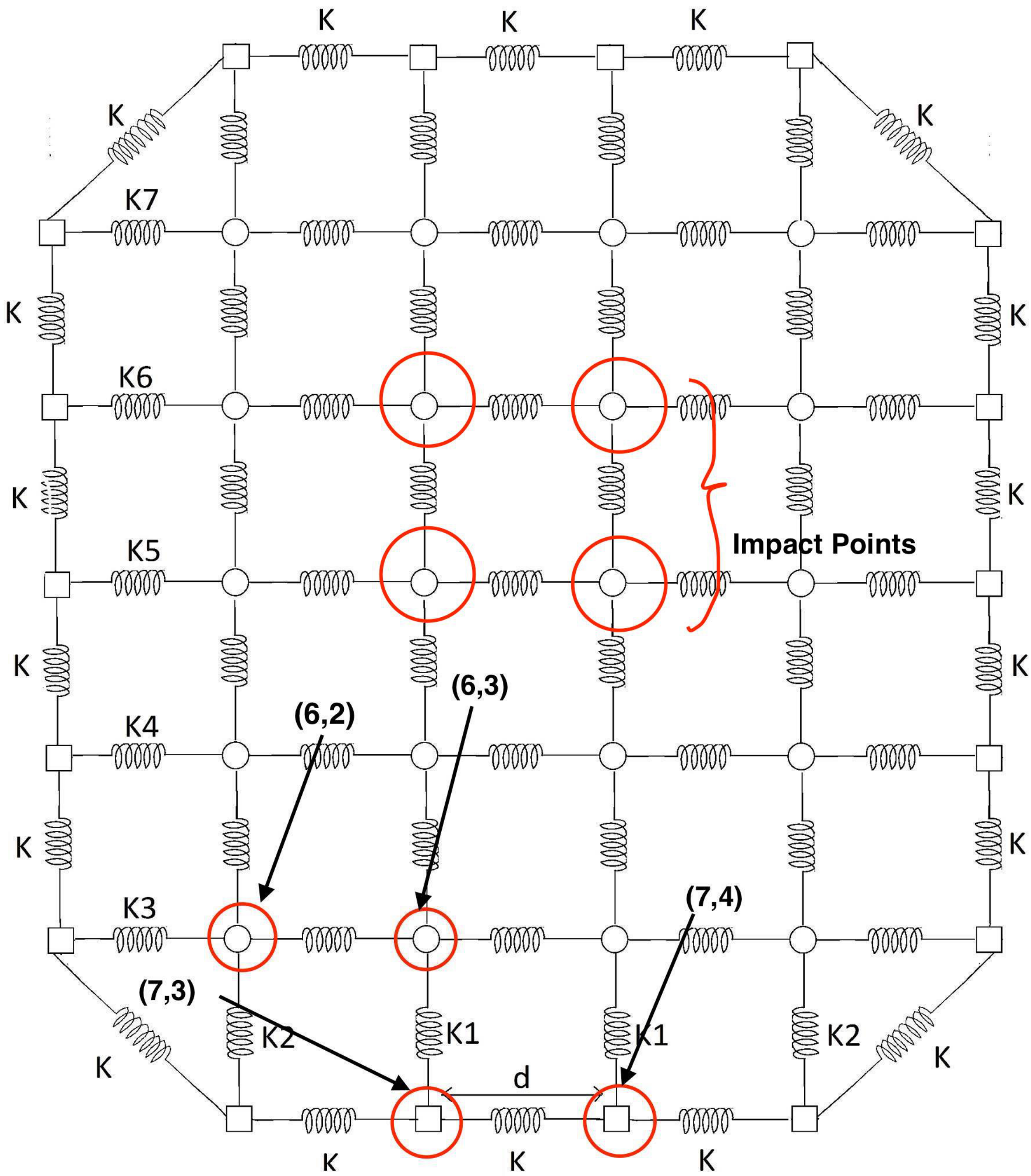

Figure : All relevant points under consideration

Y-Axis: Normalised Amplitude
X-Axis: Suitable time parameter
For all the graphs

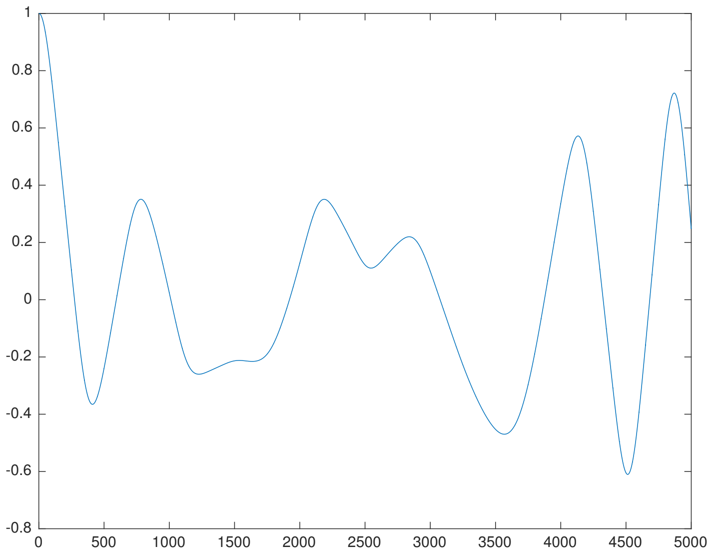

Figure: Undamped displacement of one impact point (3,3)

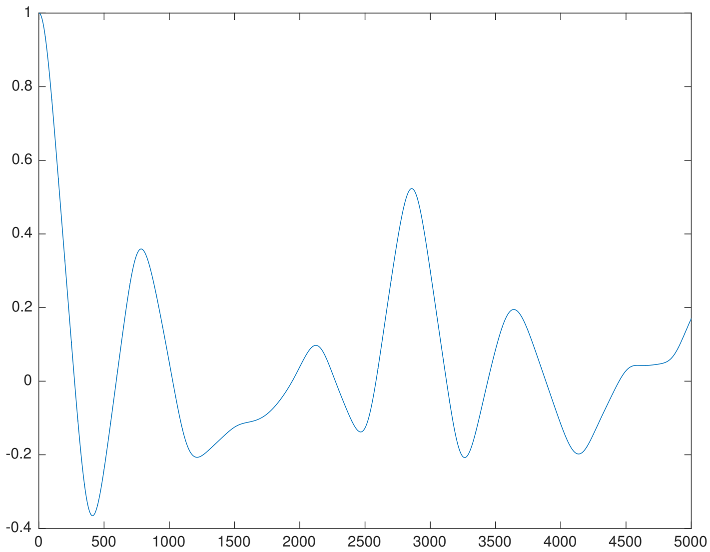

Figure: Undamped displacement of one impact point (4,3)

One point is vertically below the other. The lack of horizontal symmetry is clearly seen here.

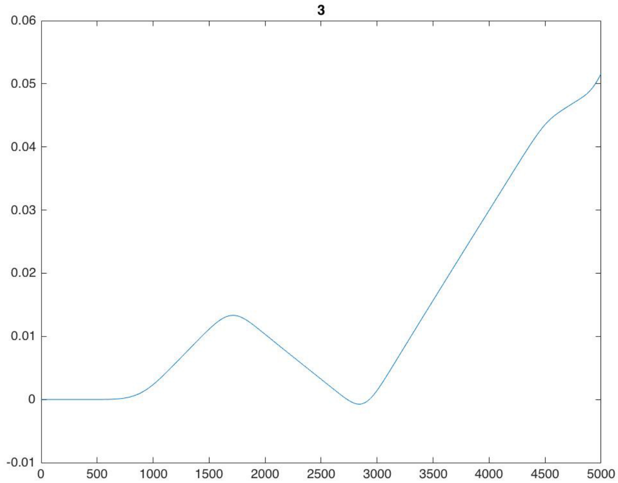

Figure: Undamped displacement of one handle point (7,3)

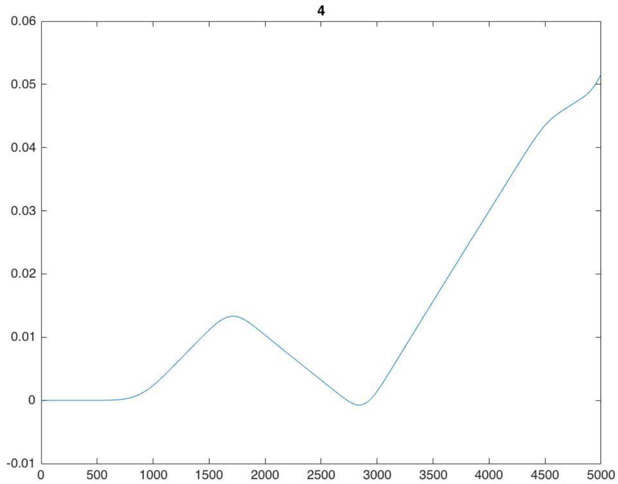

Figure: Undamped displacement of another handle point (7,4)
These graphs are identical. This is due to the vertical symmetry.

5 Effects of introducing a dampener

5.1 Modification of eqns when dampener is placed at different locations

Depending on where the dampener is placed, the corresponding node will have an extra $-\gamma\vec{u}$ on the RHS where $\vec{u}$ is the velocity of the particular junction. The value of b (where $\gamma = b/m$) is taken to be $2g - s^{-1}$. m , where (mass of node, m = 0.75g)

5.2 Plots and discussion

Dampener placed at (6,2)

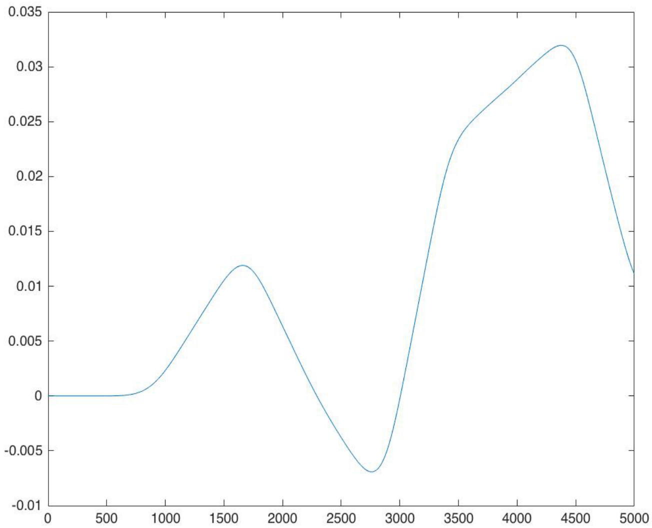

Figure 1: Damped oscillation of handle point (7,3)

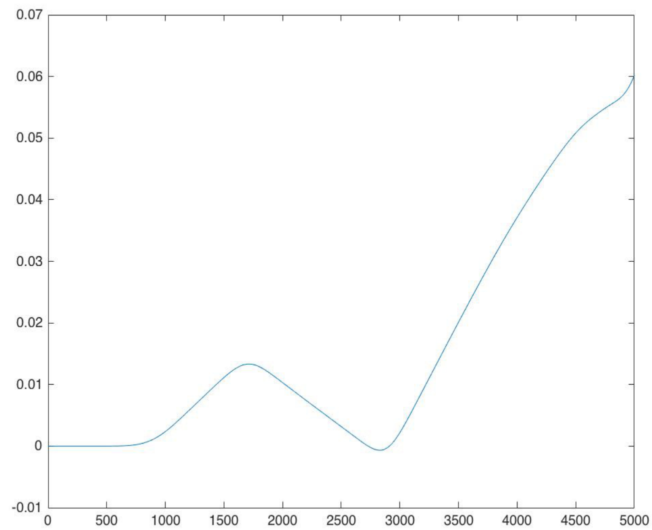

Figure 2: Damped oscillation of handle point (7,4)

Dampener placed at (6,3)

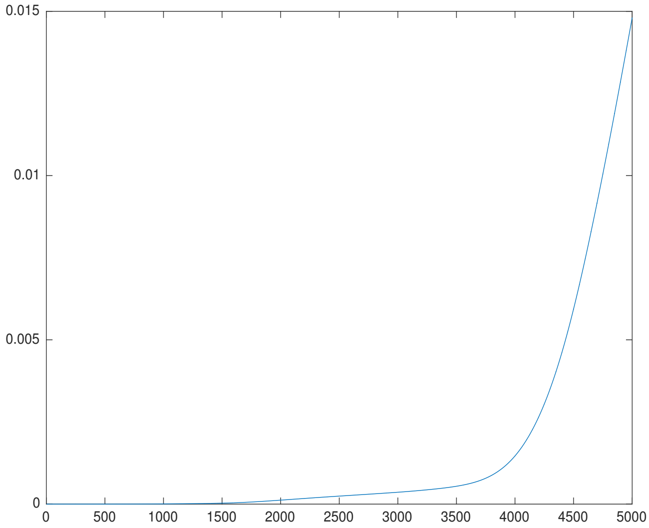

Figure 3: Damped oscillation of handle point (7,3)

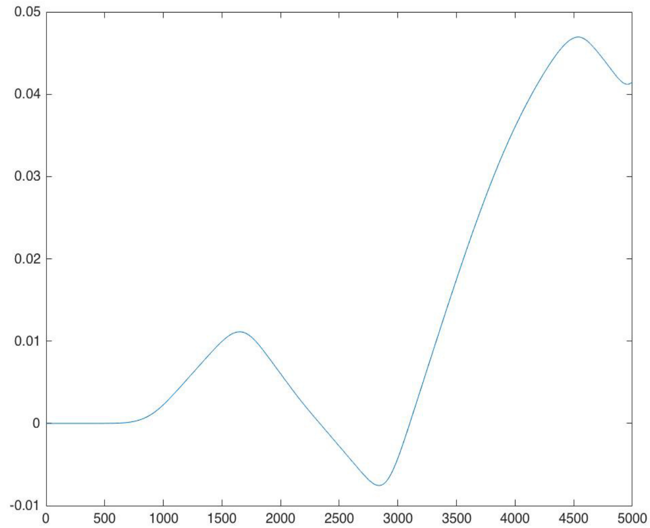

Figure 4: Damped oscillation of handle point (7,4)

A simulation of the time evolution of this system with a given set of initial conditions was performed and presented.

6 Conclusion

We observed that at places other than (6,3) i.e more distant from the handle points there is a slight *increase* in amplitude for either junction.This is clearly seen in the figure for undamped oscillation at (7,4) of v/s damped oscillation with dampener at (6,2). This is strange and we couldn't figure out whether this is normal behaviour or not. When we place the dampener at some distance from the handle junctions, then, the amplitude at the junctions were found to increase. But locally, near the dampener, the amplitude is found to decrease as expected. And at (6,3), there is a significant decrease in amplitude as compared to the undamped system. Hence, we can conclude that (6,3) is the best spot to place the dampener.

7 Limitations

- Circular Shape: The tennis racquet has a circular head. We found it difficult to model the rim if it's circular.
- Handle joint: The racquet has a handle attached to the head and it's properties influence the damping and transmission of vibrations.
- String pattern: In an actual racquet, the strings are crossed only in the middle. Moreover tennis rules require that a dampener be placed only at the place where the strings *don't* cross.
- Inherent damping of string: As mentioned before, the inherent damping co-efficient of the strings has not been considered.

References

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[3] S R Goodwill and S J Haake. Spring damper model of an impact between a tennis ball and racket. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 215(11):1331–1341, 2001.