Effect of a Dampener

An exploration of vibrations of a tennis racquet

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- Model a dampener.
- Look at the oscillations at the handle when the dampener is placed at different points.

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- The length:breadth proportion has been chosen to match an actual tennis racquet.
- We assume that the ball will hit the racquet between the nodes.
- The distance between the nodes equals the diameter of the ball (practically, a little less than the diameter).
- The impact due to the ball is distributed equally amongst the 4 nearest nodes.

Diagram

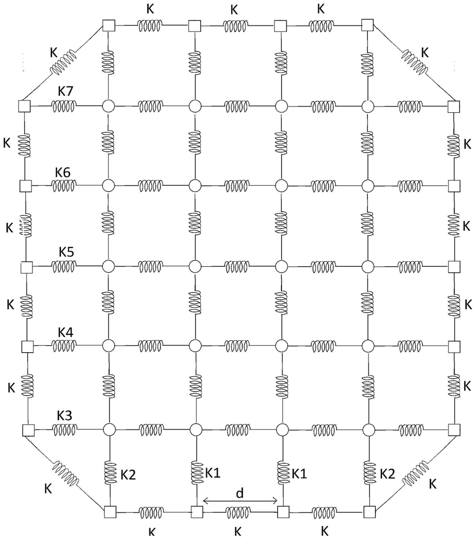

Diagram of the model with necessary spring constants marked

Modelling a Tennis Racquet (contd.)

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- The racquet is symmetric about the vertical axis. Hence, correspondingly, even the spring constants are same about the axis.
- About the horizontal it is not symmetric due to the presence of the handle.
- The springs at the rim have very high spring constant in comparison to the rest.

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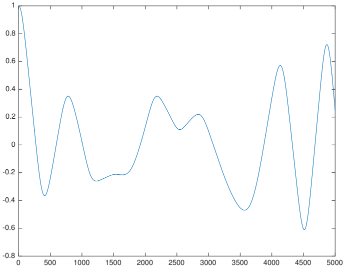
- k is the spring constant of the springs on the rim.
- Nearest neighbour approximation has been applied to derive these equations (each circle's dynamics is affected only by those of the neighbouring circles).
- No damping has been included as yet.

These are now solved using Euler's method.

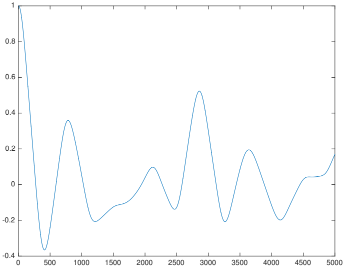
Plot of displacements v/s time

The values of the different spring constants are
k=[3.241,3.241,4.723,4.703,4.684,4.703,4.723,500]ergs-cm^{-2}
For the given impact function, the displacements at some points are:

Displacement of one impact point(3,3)



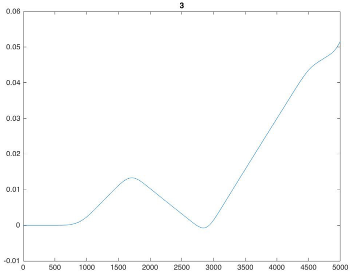
Displacement of another impact point(4,3)



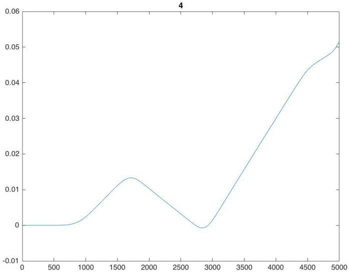
Note that one point is vertically below the other. The lack of horizontal symmetry is clearly seen here.

Plot of displacements v/s time (contd)

Displacement of one handle point (7,3)



Displacement of another handle point (7,4)



Note that the graphs are identical. This is due to the vertical symmetry.

Modification of eqns when dampener is placed at different locations

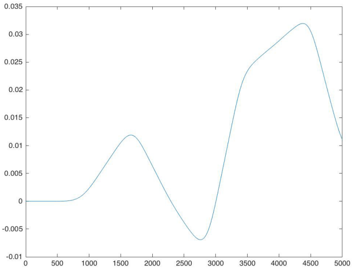
Depending on where the dampener is placed the corresponding junction will have an extra $-\gamma\vec{u}$ on the RHS where $\vec{u}$ is the velocity of the particular junction. The value of b ($\gamma = b/m$) is taken to be $2g - s^{-1}$. m (mass of gut) = 0.75g

Result

The effect of the dampener (at the handle) when placed at different points on the frame is shown here:

Dampener located at (6,2)

Displacement of one handle point (7,3)



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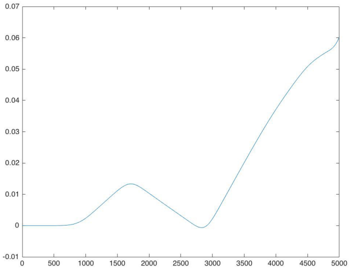

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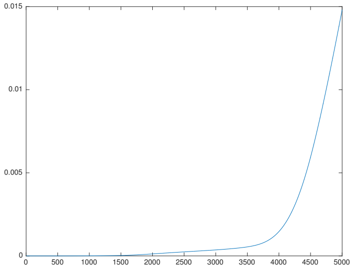

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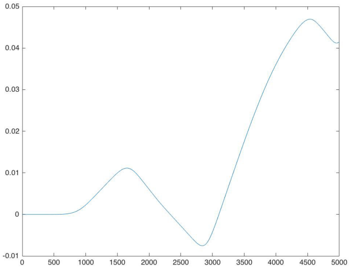

We observed that at places other than (6,3) there is a slight increase in amplitude for either junction. And at (6,3), there is a significant and consistent decrease in amplitude. Hence, we can conclude that (6,3) is the best spot to place the dampener.

- Circular Shape.
- Handle joint.
- String pattern:No crossing at the ends.
- Inherent damping of string.

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