Harikrishnan Gopal Janakiraman
01372580

91.502
Foundations of Computer Science
Fall 2013 / Take-home Midterm

1) a) M1: $q_1 \to q_2 \to q_3 \to q_1 \to q_1$

M2: $q_1 \to q_1 \to q_1 \to q_2 \to q_4$

b) M1: Doesn't accept string aabb

M2: Accept the string aabb, Final state (q_4).

c) Both M1 & M2 doesn't accept the string ϵ.

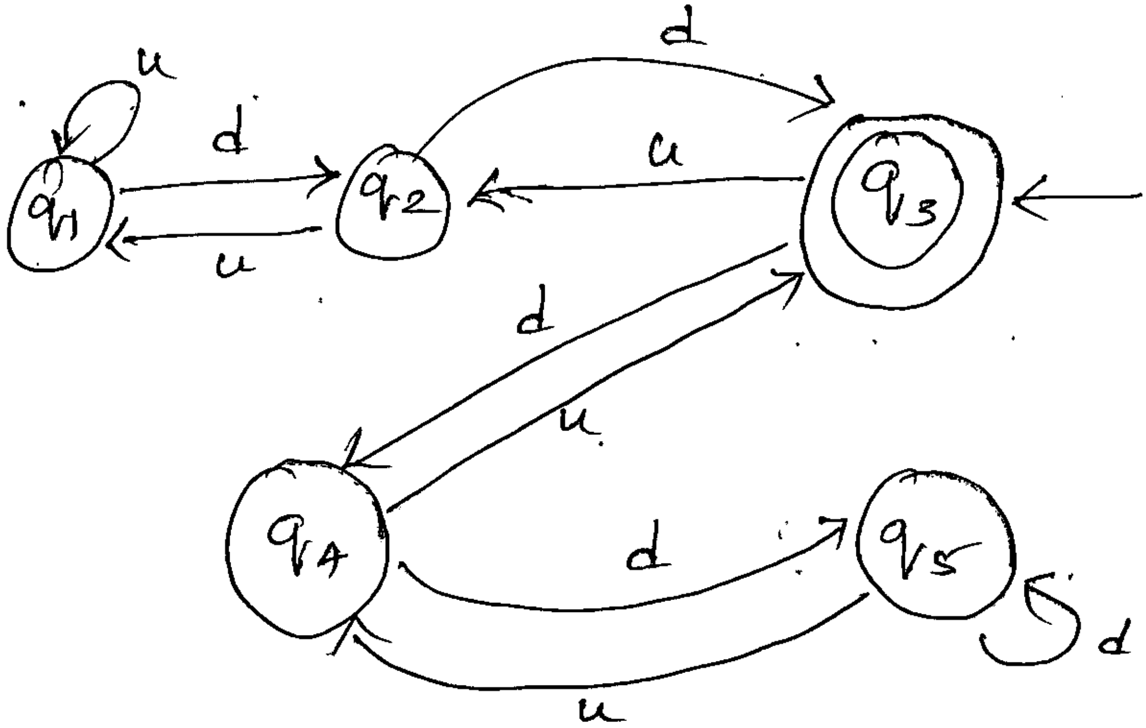
d) M1:

δ	a	b
$\to q_1$	q_2	q_1
$\to q_2$	q_3	q_3
q_3	q_2	q_1

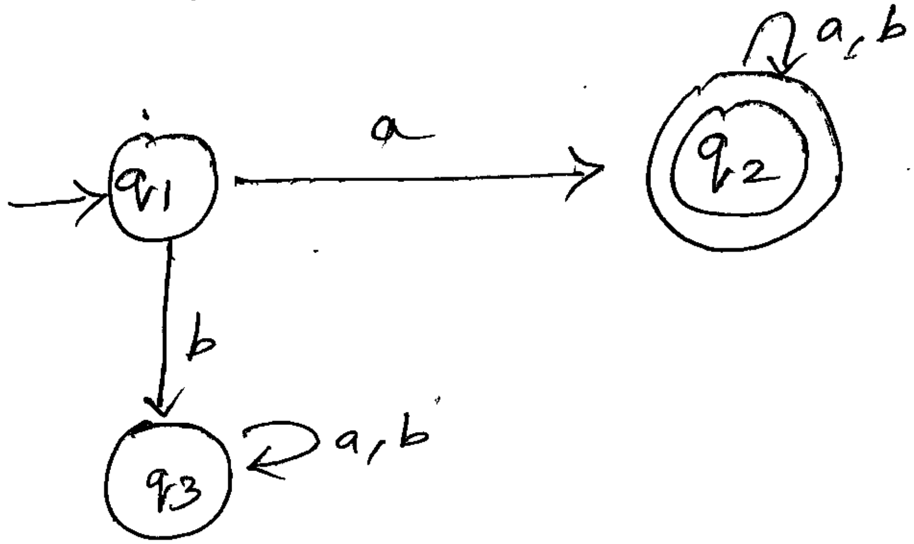
M2:

δ	a	b
q_1	$\to q_1$	q_2
q_2	q_3	q_4
q_3	q_2	q_1
q_4	q_3	q_4

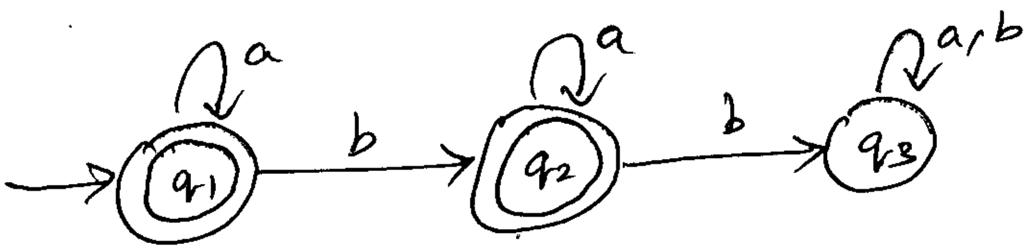
2)



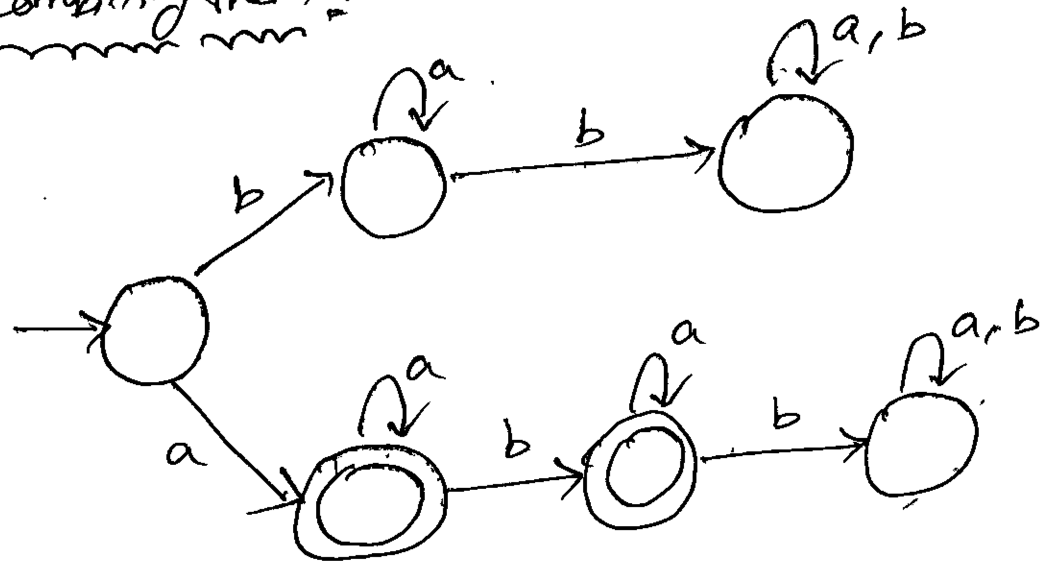
3 (a) $L_1 = \{ w | w$ starts with an $a \}$



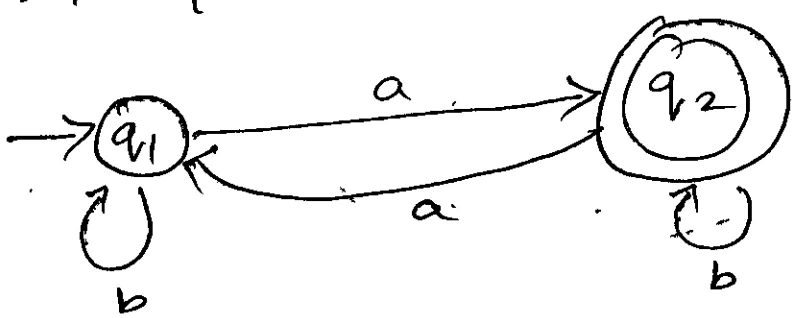
$L_2 = \{ w | w$ has atmost one $b \}$



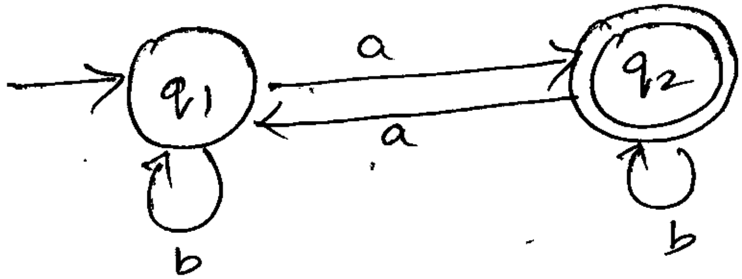
Combining them :-



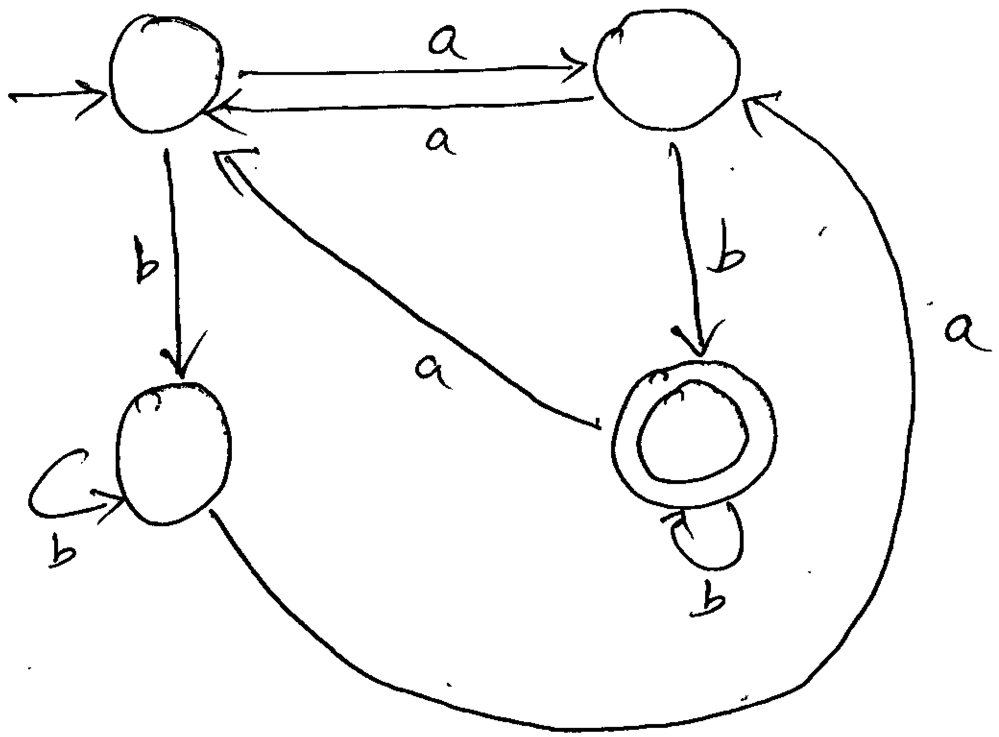
(b) $L_1 = \{ w \mid w$ has an odd number of a's $\}$



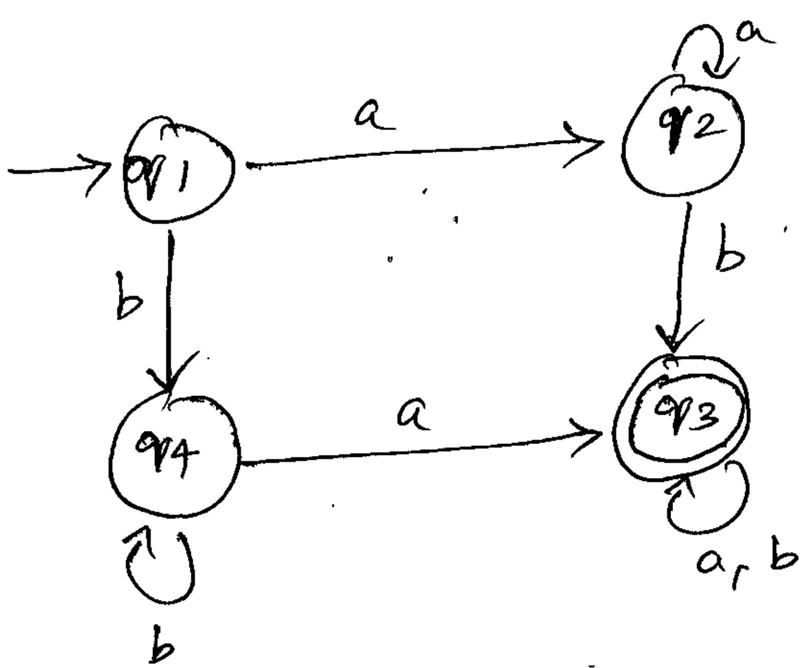
$L_2 = \{ w \mid w$ ends with b $\}$



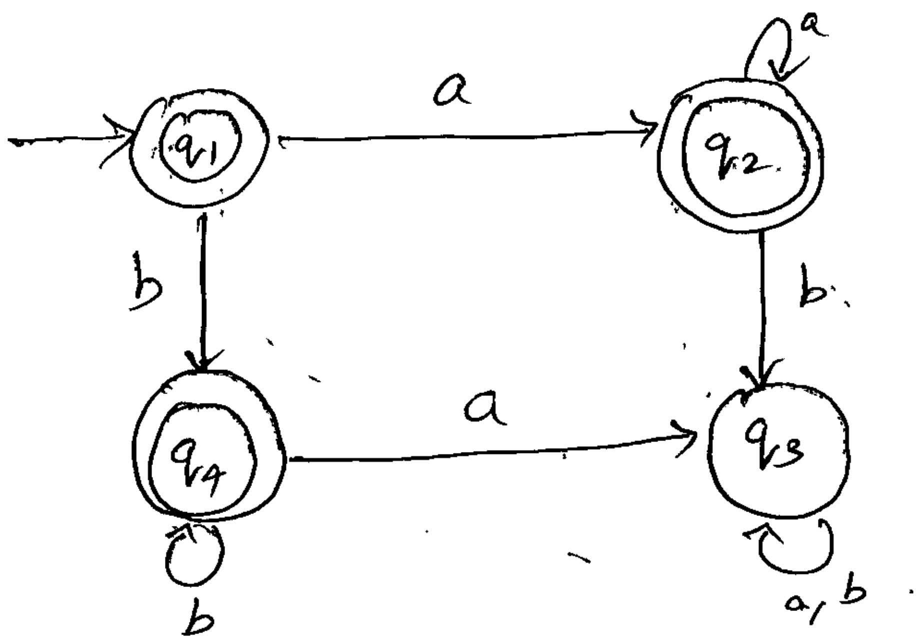
Combining them :-



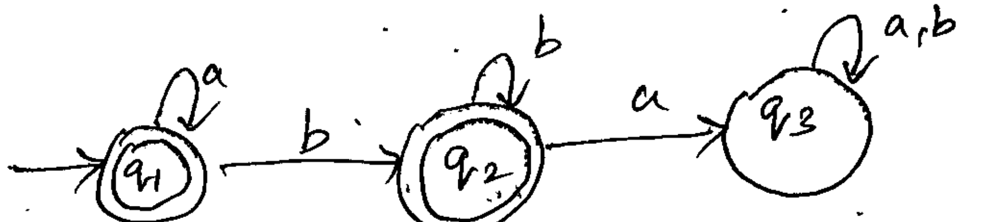
4) (a) L_1 = {w | w contains substrings ab or ba}



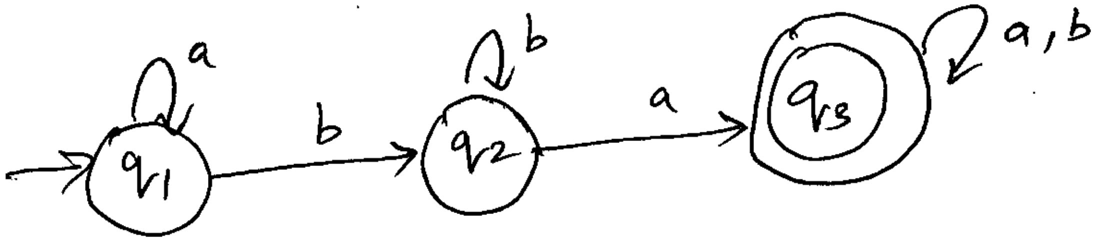
$L_2 = \{ w | w$ Contains neither the substrings ab nor ba $\}$



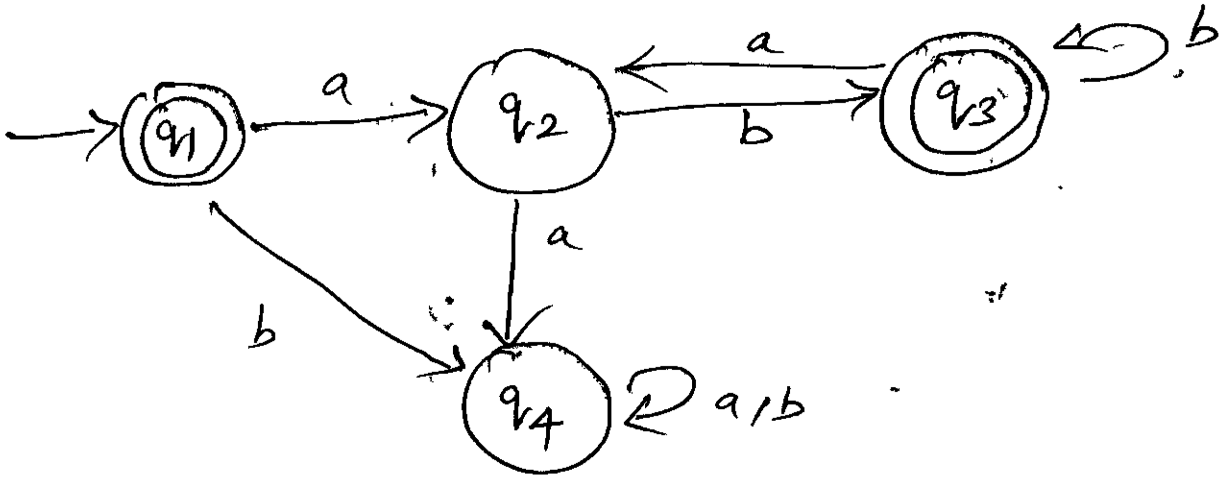
(b) $L_1 = \{ w | w$ is any string in $a^* b^* \}$



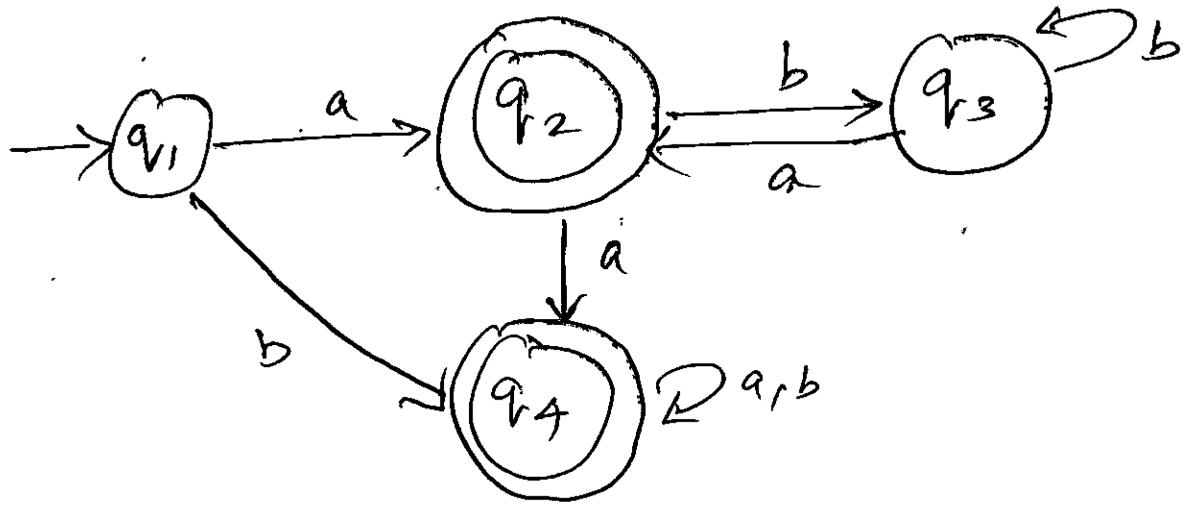
$L_2 = \{ w | w$ is any string not in $a^* b^* \}$



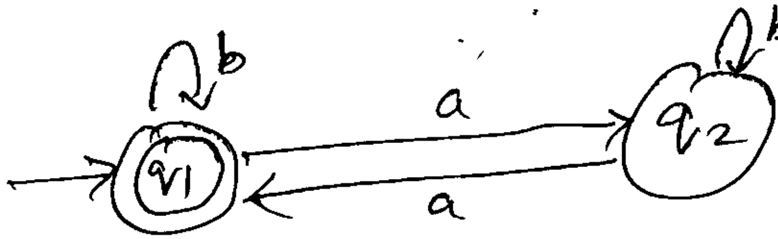
c) $L_1 = \{ w | w$ is any string in $(ab^+)^* \}$



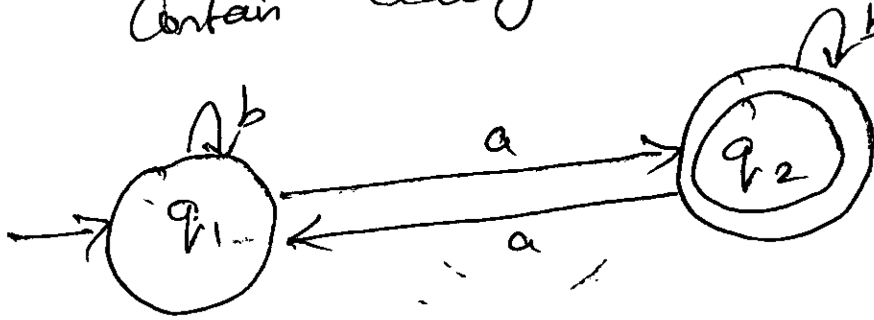
$L_2 = \{ w | w$ is any string not in $(ab^+)^* \}$



d) $L_1 = \{ w \mid w$ is any string that contain exactly two a's $\}$



$L_2 = \{ w \mid w$ is any string that doesn't contain exactly two a's $\}$



5) Prove that $L(S)^R$ is regular, whenever S is a regular expression.

Theorem :-

$L = S$.

Let $S = (Q_S, \Sigma_S, \delta_S, q_S, F_S)$ be a NFA and let $L = L(S)$.

The ϵ-NFA S^R defined below accepts the language L^R.

1. $S^R = (Q_S \cup \{q_A\}, \Sigma_S, \delta_{S^R}, q_A, \{q_s\})$ & $q_A \notin Q_S$.

2. $p \in \delta_S(q, s) \Leftrightarrow q \in \delta_{S^R}(p, s)$

where $s \in \Sigma_S$ & $q, p \in Q_S$.

3. ϵ-closure $(q_A) = F_S$.

Proof:-

First, we prove the following statement.
$\exists$ a path from q to p in S labeled with w if and only if $\exists$ a path from p to q in S^R labeled with w^R for $q, p \in Q_S$.
The proof is by induction on the length of w.

Base case:-
$|w| = 1$ (from definition of δ_{S^R}.

Induction Case:-

Assuming the Statement holds for words of length $< n$ & let $|\omega| = n$ & $\omega = xs$

Let $p \in \delta_s^*(q_1, \omega) = \delta_s^*(q_1, xs)$

knowing that $\delta_s^*(q_1, x_s) = \cup_{p'} \delta_s(s', a)$

$\forall p' \in \delta_s^*(q_1, x)$

By Induction hypothesis:-

$p' \in \delta_{s^R}(P, s)$ & $q \in \delta_{s^R}^*(p', x^R)$

$p \in \delta_{s^R}^*(P, sx^R) \Leftrightarrow p \in \delta_s^*(q, xs)$

$\leadsto q \in \delta_{s^R}^*(P, sx^R)$

Letting $q = q_s$ & $P = a$ for some $a \in F_s$

& substituting them ω^R for $x^R s$ gives that

$P \in \delta_{s^R}^*(a, \omega^R)$ $\forall a \in F_s$ there is a path labeled with ϵ from q_a to every state in F_s & a path from every state in F_S to the State q_{rs} labeled with w^R.

Then there is a Path labeled with $\epsilon w^R = w^R$

from q_a to q_{rs}..

There by it Proves, $L(S)^R$ is regular whenever

$L(S)$ is regular..

6) Prove that the language $L = \{ wxw^R \mid w, x \in \{0,1\}^*$ and $|w| \geq 1$ and $|x| \geq 1\}$

is regular (w^R is the string w written

backwards).

PROOF:-

Assume L, is defined by expression

wxw. The proof is a structural induction

on the size of >1. we show that

$L(wxw) = L(wxw^R)$, that is

Reversal of w with Concatenation of w & w

results the regular language when done

without reversal.

BASIS:-

If L_1 is ϵ, ϕ, or, a for some symbol a, then L_1 is same as L. That is,

$\phi . \phi . \phi^R = \phi$,

$\epsilon . \epsilon . \epsilon^R = \epsilon$ and

$a . a . a^R = a . a . a$

INDUCTION:-

Lets take $wx = E1 \, \& \, E2 = w$.

There by, concatenating

$L_1 = E1E2 = wxw$.

note that $L_1 = E_1E_2 = E_1E_2^R = L$

(i.e) reversal of order of E_2 results the some regular language. For instance

$L(E_1) = \{11 \; 01\}$

$L(E_2) = \{11\}$

$\therefore$ Reversal of $L(E_2) = \{11\}$

If we concatenate both the reversal,

we get

$L_1 \doteq E_1 E_2 = \{11\ 01\}, \{11\}$
$= \{11\ 01\ 11\}$

which is the same language of $L = \omega x \omega$
$= \{11\ 0111\}$

Thereby, we have shown that $L_1 = L$ and both are under same regular language.

7) Proof:-

Since M is a PDA and there is no computation on every string x causes the stack size $C(t)$ to be constant.

Then the input string must be x^* eg:- $\{0\}^*$ i.e the set of all string of 0's, which causes no transitions on M, (i.e) state in M has a loop of x. since we know that the input string x^* is a regular language, thereby $L(M)$ must be regular

8) a) $S \rightarrow 0A0 \mid 1A \mid 0 \mid 1$

$A \rightarrow 0A \mid 1A \mid \epsilon$

b) $S \rightarrow 0A \mid 1A$

$A \rightarrow 0S \mid 1S \mid \epsilon$

c) $S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$

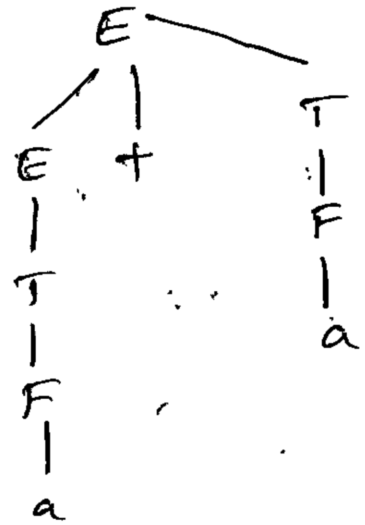
d) $S \rightarrow S$

9) a) $E \rightarrow T \rightarrow F \rightarrow a$

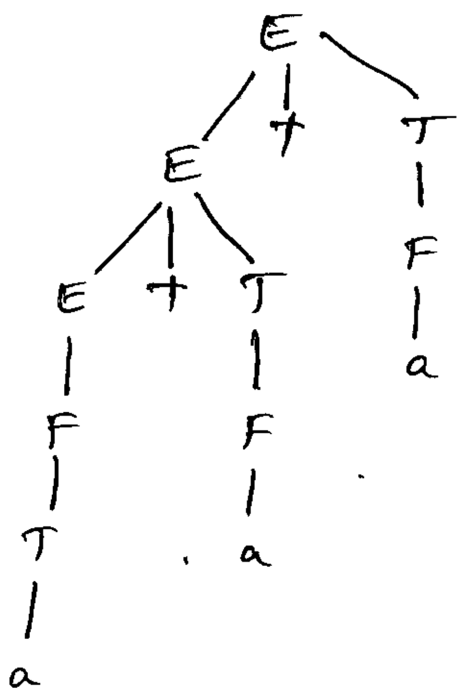
$$
\begin{array}{c}
E \\
| \\
T \\
| \\
F \\
| \\
a
\end{array}
$$

b) $E \rightarrow E+T \rightarrow E+F \rightarrow E+a \rightarrow T+a \rightarrow$

$F+a \rightarrow a+a$



(c) $E \rightarrow E+T \rightarrow E+F \rightarrow E+a \rightarrow E+T+a \rightarrow$
$E+F+a \rightarrow E+a+a \rightarrow T+a+a \rightarrow$
$F+a+a \rightarrow a+a+a$



(d) $E \to T \to F \to (E) \to (T) \to (F) \to$
$((E)) \to ((T)) \to ((F)) \to ((a))$