PS-2

1. Partition matrices

$$y = \beta_0 + \beta_1 x_1 + \dots \beta_k x_k + \varepsilon$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \dots & x_{1k} \\ 1 & x_{21} & & x_{2k} \\ 1 & x_{31} & & \vdots \\ \vdots & & & \vdots \\ 1 & x_{T1} & & x_{Tk} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}$$

$$\boxed{y = \beta X + \varepsilon}$$

$$y = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \varepsilon$$

$$y = X_1 \beta_1 + X_2 \beta_2 + \varepsilon$$

Multiply by X_1' & X_2' on both sides

$$X_1'y = X_1'X_1\beta_1 + X_1'X_2\beta_2 + X_1'\varepsilon_1 \quad \text{— (II)}$$

$$X_2'y = X_2'X_1\beta_1 + X_2'X_2\beta_2 + X_2'\varepsilon \quad \text{— (III)}$$

$$\begin{bmatrix} X_1'\varepsilon = 0 \\ X_2'\varepsilon = 0 \end{bmatrix} \text{ As they are } \perp$$

$$\Rightarrow X_1'X_1\beta_1 = X_1'Y - X_1'X_2\beta_2$$

$$\underline{\hat{\beta}_1 = X_1'(Y - X_2\beta_2)(X_1'X_1)} \quad \text{— } \cancel{(3)} \text{ (IV)}$$

$\cancel{3 \text{ in}}$ (IV) in (III)

$$X_2'X_2 - X_2'X_1(X_1'X_1)^{-1}X_1'X_2\beta_2$$

$$= X_2'X_1(X_1'X_1)^{-1}X_1'Y$$

$$X_2'\{(I - P_1)X_2\}\beta_2 = X_2'(I - P_1)y$$

$$\underline{\hat{\beta}_2 = \{X_2'(I - P_1)X_2\}^{-1}X_2'(I - P_1)y} \quad \text{— (V)}$$

(IV) & (V) are the answers