__/__/__

Problem set 1
BUAN 6340

1. $P = X(X'X)^{-1}X'$ $\quad M = I_n - P$

Given $y = X\beta + e$ $\quad \hat{\beta} = (X'X)^{-1}X'y$

i) P is idempotent $(PP = P)$

$$\text{LHS} = PP = (X(X'X)^{-1}X')(X(X'X)^{-1}X')$$

$$= X\underbrace{(X'X)^{-1}(X'X)}_{I}(X'X)^{-1}X'$$

$$= X I_n (X'X)^{-1}X'$$

$$= X(X'X)^{-1}X' = P = \text{RHS}$$

$\therefore$ P is idempotent

ii) M is idempotent i.e $MM = M$

$$\text{LHS} = MM = (I_n - P)(I_n - P)$$

$$= I_n I_n - I_n P - P I_n + PP$$

$$= I_n - P - P + P = I_n - P = M$$

$\therefore$ M is idempotent

c) $Py = \hat{y}$

$LHS = Py$

$= [X(X'X)^{-1}X']\, y$

$= X\,[(X'X)^{-1}X'y]$

$= X\hat{\beta} = \hat{y} = RHS$

$Py = \hat{y}$

d) $My = \hat{e}$

$LHS = My$

$= (I_n - P)(X\beta + e)$

$= I_n X\beta - I_n e - PX\beta - Pe$

$= X\beta - e - PX\beta - Pe$

$\therefore$ ~~e is identity~~

$y = X\beta + e$ $\quad Py = X(X'X)^{-}X'y$

$y - Py = e$

__/__/____

d) $My = \hat{e}$

$$LHS = My = (I_n - P)y$$
$$= I_n y - Py$$
$$= y - Py \quad (\because Py = \hat{y} \text{ from ©})$$
$$= y - \hat{y} = \hat{e} = RHS$$

e) $Py + My = y$

$$LHS = Py + My = \hat{y} + \hat{e} \quad (\text{from © \& ⓓ})$$
$$= \hat{y} + y - \hat{y}$$
$$= y = RHS$$

f) $\hat{y} \perp \hat{e} \Rightarrow \hat{y}.\hat{e} = 0.$

$$LHS = \hat{y}.\hat{e} = Py.My$$
$$= Py(I_n - P)y$$
$$= Py^2 - P.Py^2 = (P.P = P)$$
$$= Py^2 - Py^2 = 0 = RHS$$

$\therefore \hat{y} \perp \hat{e}$