Solutions

Math 1300-005 - Spring 2017

More on the Chain Rule - 2/23/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the next midterm. The Chain Rule is *easily* the most important tool we have in this class so make sure you feel comfortable using it! These are all very hard bordering on ridiculous, but they are good practice nonetheless.

out: $f(\;\;)$
in: $3f(4f(x))$

1. If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.

$$F'(x) = f'(3f(4f(x))) \cdot \frac{d}{dx}\left(3f(4f(x))\right)$$

out: $f(\;\;)$
in: $4f(x)$

$$= f'(3f(4f(x))) \cdot 3f'(4f(x)) \cdot \frac{d}{dx}(4f(x))$$

$$= f'(3f(4f(x))) \cdot 3 f'(4f(x)) \cdot 4 f'(x).$$

so
$$F'(0) = 12 f'(3f(4f(0))) \cdot f'(4f(0)) \cdot f'(0)$$
$$= 12 f'(3f(0)) \cdot f'(0) \cdot f'(0)$$
$$= 12 f'(0) \cdot f'(0) \cdot f'(0)$$
$$= 12 \cdot 8 = \boxed{96}$$

out: $f(\;\;)$
in: $xf(xf(x))$

2. If $H(x) = f(xf(xf(x)))$, where $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 5$, and $f'(3) = 6$, find $H'(1)$. [This is not as simple as problem 1 and involves the product rule.]

Remember: $x \cdot f(x)$ requires the product rule!

$$H'(x) = f'(xf(xf(x))) \cdot \frac{d}{dx}\overbrace{\left(x \cdot f(xf(x))\right)}^{PR}$$

out: $f(\;\;)$
in: $xf(x)$

$$= f'(xf(xf(x))) \cdot \left[f(xf(x)) + x \cdot f'(xf(x)) \cdot \frac{d}{dx}\overbrace{(xf(x))}^{PR}\right]$$

$$= f'(xf(xf(x))) \cdot \left(f(xf(x)) + x \cdot f'(xf(x)) \cdot [f(x) + xf'(x)]\right)$$

So
$$H'(1) = f'(f(f(1))) \cdot \left(f(f(1)) + f'(f(1)) \cdot [f(1) + f'(1)]\right)$$
$$= f'(f(2)) \cdot \left(f(2) + f'(2) \cdot [2 + 4]\right)$$
$$= f'(3) \cdot \left(3 + 5 \cdot [2 + 4]\right)$$
$$= 6 \cdot (3 + 30)$$
$$= 6 \cdot 33 = \boxed{198}$$

3. Find dy/dx. All of these problems involve multiple iterations of the chain rule.

(a) $y = \sin(\cos(\tan(\sec(x))))$ out: $\sin(\quad)$ in: $\cos(\tan(\sec(x)))$

$\frac{dy}{dx} = \cos(\cos(\tan(\sec(x)))) \cdot \frac{d}{dx}\cos(\tan(\sec(x)))$ out: $\cos(\quad)$ in: $\tan(\sec(x))$

$= \cos(\cos(\tan(\sec(x)))) \cdot \left[-\sin(\tan(\sec(x))) \cdot \frac{d}{dx}\tan(\sec(x))\right]$ → out: $\tan(\quad)$ in: $\sec(x)$

$= -\cos(\cos(\tan(\sec(x)))) \cdot \sin(\tan(\sec(x))) \cdot \sec^2(\sec(x)) \cdot \frac{d}{dx}\sec(x)$

$= \boxed{-\cos(\cos(\tan(\sec(x)))) \cdot \sin(\tan(\sec(x))) \cdot \sec^2(\sec(x)) \cdot \sec(x)\tan(x)}$

(b) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ out: $\sqrt{\quad} = (\quad)^{1/2}$ in: $x + (x + x^{1/2})^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}\left(x + (x+x^{1/2})^{1/2}\right)^{-1/2} \cdot \frac{d}{dx}\left(x + (x+x^{1/2})^{1/2}\right)$ → out: $(\quad)^{1/2}$ in: $(x+x^{1/2})$

$= \frac{1}{2}\left(x + (x+x^{1/2})^{1/2}\right)^{-1/2} \cdot \left[1 + \frac{1}{2}(x+x^{1/2})^{-1/2} \cdot \frac{d}{dx}(x+x^{1/2})\right]$

$= \boxed{\frac{1}{2}\left(x + (x+x^{1/2})^{1/2}\right)^{-1/2} \cdot \left[1 + \frac{1}{2}(x+x^{1/2})^{-1/2} \cdot \left(1+\frac{1}{2}x^{-1/2}\right)\right]}$

(c) $y = \cos\left(\sqrt{\sin(\cot(x))}\right)$ → out: $\cos(\quad)$ in: $(\sin(\cot(x)))^{1/2}$

$\frac{dy}{dx} = -\sin\left(\sqrt{\sin(\cot(x))}\right) \cdot \frac{d}{dx}(\sin(\cot(x)))^{1/2}$ → out: $(\quad)^{1/2}$ in: $\sin(\cot(x))$

$= -\sin\left(\sqrt{\sin(\cot(x))}\right) \cdot \frac{1}{2}(\sin(\cot(x)))^{-1/2} \cdot \frac{d}{dx}\sin(\cot(x))$ → out: $\sin(\quad)$ in: $\cot(x)$

$= -\sin\left(\sqrt{\sin(\cot(x))}\right) \cdot \frac{1}{2}(\sin(\cot(x)))^{-1/2} \cdot \cos(\cot(x)) \cdot \frac{d}{dx}\cot(x)$

$= \boxed{-\sin\left(\sqrt{\sin(\cot(x))}\right) \cdot \frac{1}{2}(\sin(\cot(x)))^{-1/2} \cdot \cos(\cot(x)) \cdot (-\csc^2(x))}$

(d) $y = e^{(5^{x^2})}$ → out: $e^{(\quad)}$ in: $5^{(x^2)}$

$\frac{dy}{dx} = e^{(5^{x^2})} \cdot \frac{d}{dx}5^{x^2}$ → out: $5^{(\quad)}$ in: x^2

$= e^{(5^{x^2})} \cdot 5^{x^2} \cdot \ln(5) \cdot \frac{d}{dx}(x^2) = \boxed{e^{(5^{x^2})} \cdot 5^{x^2}\ln(5) \cdot 2x}$