(Solutions)

Math 1300-005 - Spring 2017

Implicit Differentiation Intro - 2/27/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 2.

1. Our goal in this problem is to use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + xy = 3 - y^2$$

at the point (1,1).

(a) First, apply d/dx to both sides of $x^2 + xy = 3 - y^2$ and use the chain rule and the guidelines from lecture. Notice the second term on the left-hand side is a product.

$$\frac{1}{3}(x^3 + xy) = \frac{1}{3}(3-y^3)$$

 $3x + y + xy' = -2yy'$

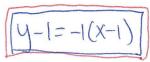
(b) Move all terms that have a y' to the left-hand side of the equation from (a) and move terms that do NOT have a y' to the right-hand side of the equation from (a). Solve for y'.

(c) Plug in x = 1 and y = 1 to your expression for y' to get the slope of the tangent line to the curve at (1, 1).

$$y'$$
 at $(1,1)$: $y' = -\frac{2(1)-(1)}{1+2(1)} = \frac{-3}{3} = \boxed{-1}$

(d) Write an equation of the tangent line based on your work in (a), (b), and (c).

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2. Following the same procedure outline in problem 1, find an equation of the tangent line to the curve

$$y\sin(2x) = x\cos(2y)$$

at the point $(\pi/2, \pi/4)$. Notice that each side of the equation above involves a product.

(a)
$$\frac{1}{2}(y\sin(2x)) = \frac{1}{2}(x\cos(2y))$$
 [Product rule to each side, then chain rule as needed]
 $y\sin(2x) + y\cos(2x) \cdot \lambda = \cos(2y) - x\sin(2y) \cdot 2y'$

(b) Rearranging:
$$y'\sin(2x) + 2xy'\sin(3y) = \cos(3y) - 3y\cos(2x)$$

 $y'(\sin(2x) + 2x\sin(3y)) = \cos(3y) - 3y\cos(2x)$
 $y' = \cos(3y) - 3y\cos(2x)$
 $y' = \cos(3y) - 3y\cos(2x)$

(c) Plly in (
$$\Pi_{2} \pi/4$$
):

$$y' = \frac{(3(2\pi) - 2\pi)}{\sin(2(\pi))} + 2(\pi) \sin(2(\pi))} = \frac{0 - \pi(1)}{0 + \pi(1)} = \frac{\pi}{2}$$

3. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

4. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

$$e^y \cos(x) = 1 + \sin(xy)$$

$$e^{y} \cdot y'(\cos(x) + e^{y}(-\sin(x)) = \cos(xy) \cdot \frac{1}{2}(xy)$$
 $e^{y} \cdot y'(\cos(x) - e^{y} \sin(x) = (\cos(xy)(y + xy))$
 $e^{y} \cdot y'(\cos(x) - e^{y} \sin(x) = y \cos(xy) + xy'(\cos(xy))$
 $e^{y} \cdot y'(\cos(x) - xy'(\cos(xy) = y \cos(xy) + e^{y} \sin(x))$

$$y'(e^{y}\cos(x) - x\cos(xy)) = y(\cos(xy) + e^{y}\sin(x))$$

$$y' = \frac{y\cos(xy) + e^{y}\sin(x)}{e^{y}\cos(xy) - x\cos(xy)}$$