Math 1300, Midterm 3

April 10, 2017

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Brendt Gerics	8:00-8:50
☐	Section 002	Leo Herr	9:00-9:50
☐	Section 003	Tyler Schrock	9:00-9:50
☐	Section 004	Lee Roberson	10:00-10:50
☐	Section 005	Braden Balentine	10:00-10:50
☐	Section 006	Xingzhou Yang	10:00 - 10:50
☐	Section 007	Lee Roberson	11:00 - 11:50
☐	Section 008	Shen Lu	11:00 - 11:50
☐	Section 009	Suzanne Craig	12:00 - 12:50
☐	Section 010	Carlos Pinilla-Suarez	12:00 - 12:50
☐	Section 011	Nathan Davidoff	1:00 - 1:50
☐	Section 012	Sion Ledbetter	1:00 - 1:50
☐	Section 013	Ruofan Li	2:00 - 2:50
☐	Section 014	Daniel Martin	2:00 - 2:50
☐	Section 015	Isabel Corona	3:00 - 3:50
☐	Section 016	Ira Becker	3:00 - 3:50
☐	Section 017	Ira Becker	4:00 -4:50
☐	Section 430R	Patrick Newberry	11:00 -11:50

Question	Points	Score
1	9	
2	8	
3	8	
4	12	
5	8	
6	12	
7	22	
8	9	
9	12	
Total:	100	

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 80 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. **Multiple Choice.** Evaluate the following derivatives. Circle **the** correct answer.

(a) (3 points) $\dfrac{d}{dx}[\log_2(x)]$

I) $\dfrac{2}{x}$ II) $\dfrac{\ln(2)}{x}$ III) $\dfrac{1}{\ln(2)x}$

IV) $\dfrac{1}{2x}$ V) $\ln(2)2^x$

(b) (3 points) $\dfrac{d}{dx}\left[\ln(2x^3-4x)\right]$

I) $6x^2-4$ II) $\dfrac{1}{2x^3-4x}$ III) $\dfrac{2x^3-4x}{6x^2-4}$

IV) $\dfrac{1}{6x^2-4x}$ V) $\dfrac{6x^2-4}{2x^3-4x}$

(c) (3 points) $\dfrac{d}{dx}\left[\arctan(2x)\cdot\sqrt{x}\right]$

I) $\dfrac{2\sqrt{x}}{\sqrt{1-4x^2}}+\dfrac{\arctan(2x)}{2\sqrt{x}}$ II) $-\dfrac{2\sqrt{x}}{\sqrt{1-4x^2}}+\dfrac{\arctan(2x)}{2\sqrt{x}}$

III) $\dfrac{2\sqrt{x}}{1+2x^2}+\dfrac{\arctan(2x)}{2\sqrt{x}}$ IV) $\dfrac{2\sqrt{x}}{1+4x^2}+\dfrac{\arctan(2x)}{2\sqrt{x}}$

V) $\dfrac{1}{\sqrt{x}(1+4x^2)}$

2. (8 points) Use logarithmic differentiation and the properties of logarithms to find the derivative of

$$y = \left(3x^2 e^x\right)^x$$

3. (a) (6 points) Use the linearization of the function $f(x) = \sin\left(\frac{1}{2}x\right)$ at the x value $a = 0$ to find an estimate for $\sin\left(\frac{1}{2}(.01)\right)$.

(b) (2 points) Is this estimation an over/under estimate? Justify your answer.

4. The following statements are all false. For **each** statement, justify why it is false by providing an explanation that includes a picture.

   (a) (4 points) If a function $f(x)$ is decreasing on the interval $[-1, 1]$, then $f'(x)$ must also be decreasing on $[-1, 1]$.

   (b) (4 points) If a function $g(x)$ is continuous on an open interval (a, b), then $g(x)$ has an absolute maximum and minimum on the interval (a, b).

   (c) (4 points) If a function $h(x)$ is defined on the closed interval $[a, b]$, then $h(x)$ has an absolute maximum and minimum value on $[a, b]$.

5. (8 points) Find the absolute minimum value and the absolute maximum value of the function

$$h(x) = e^{x^2-4x+3}$$

on the interval $[1, 5]$.

6. Suppose two police officers are at two consecutive exits 10 miles apart on a straight highway. Assume the speed limit on the highway is 60 mph, and a speeding ticket costs \$100 plus \$10 for each mph above 60.

   (a) (6 points) If a car passes the first police officer at 3:00 and passes the second police officer 12 minutes later, can the second officer issue a ticket to the car using the mean value theorem as justification?

   (b) (6 points) Answer *one* of the following:

      i) If your answer to part (a) was yes, for how much can the cop write the ticket ?

      ii) If your answer to part (a) was no, can we conclude that the car never broke the speed limit?

7. Consider the following function $f(x)$ and its derivatives:

- $f(x) = \dfrac{-2x(x-1)}{(x+1)^2}$
- $f'(x) = \dfrac{2(1-3x)}{(x+1)^3}$
- $f''(x) = \dfrac{12(x-1)}{(x+1)^4}$

(a) (2 points) What are the x and y intercepts of $f(x)$?

(b) (2 points) What are equations of the vertical asymptotes (if any exist) of $f(x)$?

(c) (2 points) What are the equations of the horizontal asymptotes of $f(x)$?

(d) (2 points) On which interval(s) is $f(x)$ increasing?

(e) (2 points) On which interval(s) is $f(x)$ decreasing?

- $f(x) = \dfrac{-2x(x-1)}{(x+1)^2}$
- $f'(x) = \dfrac{2(1-3x)}{(x+1)^3}$
- $f''(x) = \dfrac{12(x-1)}{(x+1)^4}$

(f) (2 points) On which interval(s) is $f(x)$ concave up?

(g) (2 points) On which interval(s) is $f(x)$ concave down?

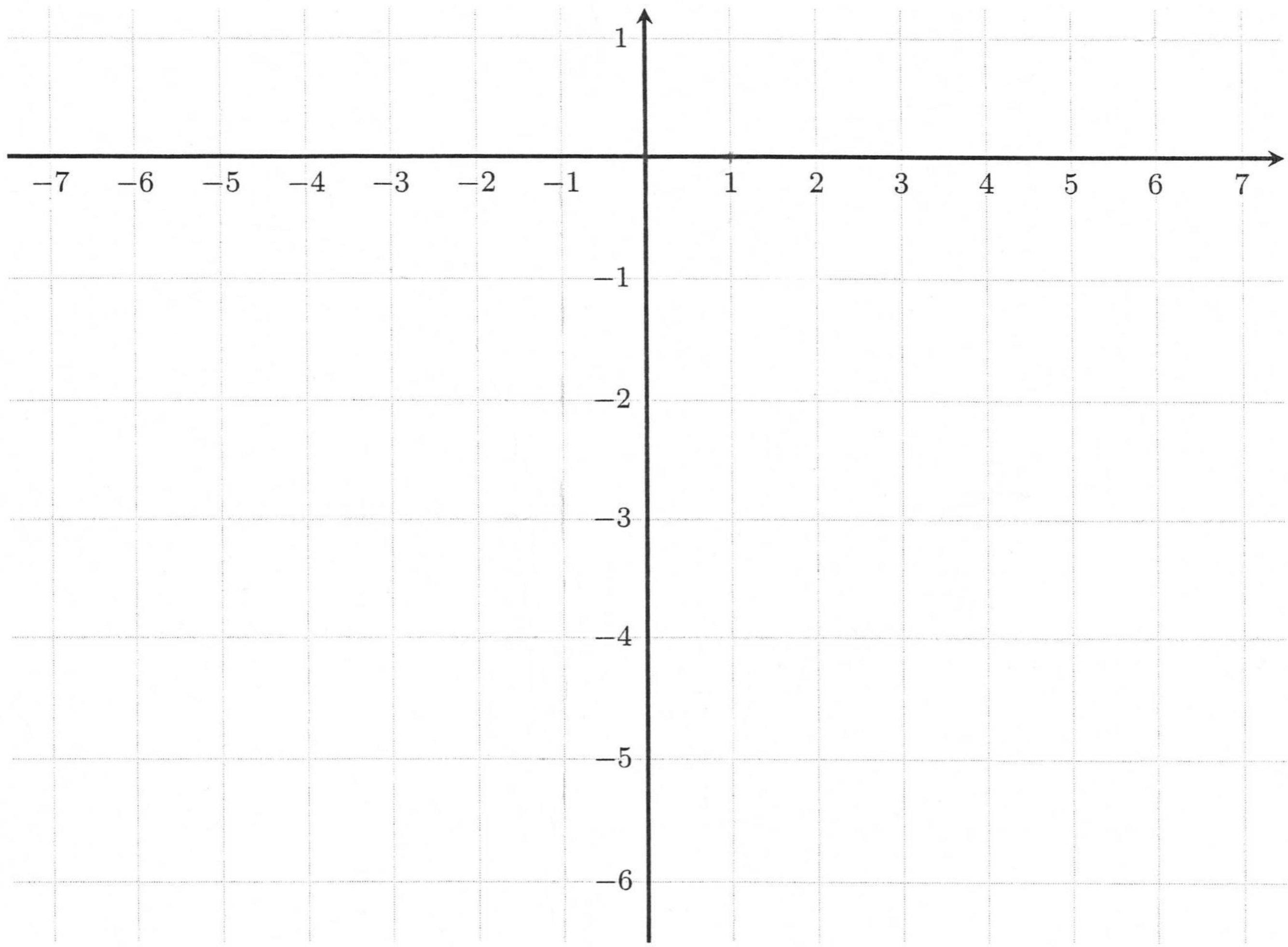
(h) (2 points) At which point(s) (x, y) does $f(x)$ have a local maximum?

(i) (2 points) At which point(s) (x, y) does $f(x)$ have a local minimum?

(j) (2 points) At which point(s) (x, y) does $f(x)$ have an inflection point?

(k) (2 points) Using the information from parts (a) through (j) on the previous pages, draw a sketch of the graph of $f(x)$ below. Be sure to label any local min and/or max and inflection points.

$$\text{Graph of } f(x) = \frac{-2x(x-1)}{(x+1)^2}$$



$f(x)$

$f'(x)$

$f''(x)$

8. **Multiple Choice.** Evaluate the following limits. Circle **the** correct answer.

(a) (3 points) $\lim_{x\to\infty}\left[x-\sqrt{x}\right]$

I) $-\infty$ II) -1 III) 0 IV) 2 V) ∞

(b) (3 points) $\lim_{x\to 1}\dfrac{\sin(x-1)}{\ln(x)}$

I) $-\infty$ II) -1 III) 0 IV) 1 V) ∞

(c) (3 points) $\lim_{x\to 0^+}\left(\sqrt{x}\right)^x$

I) $-\infty$ II) 0 III) $\frac{1}{2}$ IV) 1 V) ∞

9. (a) (8 points) A cylindrical soup can has volume 128π cm^3. What radius will minimize the surface area? Recall that the surface area of a cylinder is given by $S = 2\pi r^2 + 2\pi rh$ and the volume by $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

(b) (4 points) Justify that your answer in part (a) is, in fact, the radius that minimizes surface area.