

## Math 1300-005 - Spring 2017

Implicit Differentiation Intro - 2/27/17



*Guidelines:* Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 2.

1. Our goal in this problem is to use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + xy = 3 - y^2$$

at the point  $(1, 1)$ .

- (a) First, apply  $d/dx$  to both sides of  $x^2 + xy = 3 - y^2$  and use the chain rule and the guidelines from lecture. Notice the second term on the left-hand side is a product.

$$\begin{aligned}\frac{d}{dx}(x^2 + xy) &= \frac{d}{dx}(3 - y^2) \\ 2x + \cancel{y} + xy' &= -2yy'\end{aligned}$$

- (b) Move all terms that have a  $y'$  to the left-hand side of the equation from (a) and move terms that do NOT have a  $y'$  to the right-hand side of the equation from (a). Solve for  $y'$ .

$$\begin{aligned}xy' + 2yy' &= -2x - y \\ y'(x + 2y) &= -2x - y \rightarrow \boxed{y' = \frac{-2x - y}{x + 2y}}\end{aligned}$$

- (c) Plug in  $x = 1$  and  $y = 1$  to your expression for  $y'$  to get the slope of the tangent line to the curve at  $(1, 1)$ .

$$y' \text{ at } (1, 1): y' = \frac{-2(1) - (1)}{1 + 2(1)} = \frac{-3}{3} = \boxed{-1}$$

- (d) Write an equation of the tangent line based on your work in (a), (b), and (c).

$$\boxed{y - 1 = -1(x - 1)}$$

2. Following the same procedure outline in problem 1, find an equation of the tangent line to the curve

$$y \sin(2x) = x \cos(2y)$$

at the point  $(\pi/2, \pi/4)$ . Notice that each side of the equation above involves a product.

(a)  $\frac{d}{dx}(y \sin(2x)) = \frac{d}{dx}(x \cos(2y))$  [Product rule to each side, then chain rule as needed]

$$y' \sin(2x) + y \cos(2x) \cdot 2 = \cos(2y) - x \sin(2y) \cdot 2y'$$

(b) Rearranging:  $y' \sin(2x) + 2xy' \sin(2y) = \cos(2y) - 2y \cos(2x)$

$$y'(\sin(2x) + 2x \sin(2y)) = \cos(2y) - 2y \cos(2x)$$

$$y' = \frac{\cos(2y) - 2y \cos(2x)}{\sin(2x) + 2x \sin(2y)}$$

(c) Plug in  $(\pi/2, \pi/4)$ :

$$y' = \frac{\cos(\frac{\pi}{2}) - 2(\frac{\pi}{4})\cos(\frac{\pi}{2})}{\sin(\frac{\pi}{2}) + 2(\frac{\pi}{2})\sin(\frac{\pi}{4})} = \frac{0 - \frac{\pi}{2}(-1)}{0 + \pi(1)} = \frac{\pi/2}{\pi} = \frac{1}{2}$$

(d)  $y - \pi/4 = \frac{1}{2}(x - \pi/2)$

3. Find  $dy/dx$  by implicit differentiation according to steps (a) and (b) in problem 1.

$$\sec^2(x-y) - y' \sec^2(x-y) = \frac{y'(1+x^2) - 2xy}{(1+x^2)^2}$$

$$\frac{d}{dx}(\tan(x-y)) \stackrel{\text{Chain}}{=} \frac{d}{dx}\left(\frac{y}{1+x^2}\right) \text{ or}$$

$$\sec^2(x-y) \cdot \frac{d}{dx}(x-y) = \frac{y'(1+x^2) - y \cdot 2x}{(1+x^2)^2}$$

$$\sec^2(x-y) \cdot (1-y') = \frac{y'(1+x^2) - 2xy}{(1+x^2)^2}$$

$$y' = \frac{-(1+x^2)^2 \sec^2(x-y) - 2xy}{-(1+x^2)^2 \sec^2(x-y) - (1+x^2)}$$

$$y' = \frac{(1+x^2)^2 \sec^2(x-y) + 2xy}{(1+x^2)^2 \sec^2(x-y) + (1+x^2)}$$

4. Find  $dy/dx$  by implicit differentiation according to steps (a) and (b) in problem 1.

$$e^y \cos(x) = 1 + \sin(xy)$$

$$\frac{d}{dx}(e^y \cos(x)) = \frac{d}{dx}(1 + \sin(xy))$$

$$e^y \cdot y' \cos(x) + e^y (-\sin(x)) = \cos(xy) \cdot \frac{d}{dx}(xy)$$

$$e^y \cdot y' \cos(x) - e^y \sin(x) = \cos(xy)(y + xy')$$

$$e^y \cdot y' \cos(x) - e^y \sin(x) = y \cos(xy) + xy' \cos(xy)$$

$$e^y \cdot y' \cos(x) - xy' \cos(xy) = y \cos(xy) + e^y \sin(x)$$

$$y'(e^y \cos(x) - x \cos(xy)) = y \cos(xy) + e^y \sin(x)$$

$$y' = \frac{y \cos(xy) + e^y \sin(x)}{e^y \cos(x) - x \cos(xy)}$$