MATH 1300: HW #6

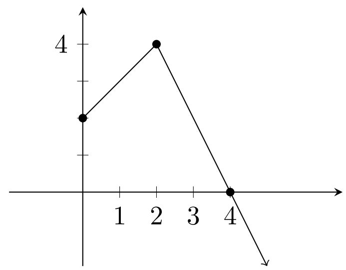
Due on February 23, 2017 at 10:00am

Professor Braden Balentine Section 005

John Keller

Additional Problems for Homework 6

A graph of $f(x)$ is shown below.
It is piecewise linear.



The table below gives values of $g(x)$ and $g'(x)$.

x	0	1	2	3	4
$g(x)$	2	5	9	11	8
$g'(x)$	3	4	3	-3	-4

1. Given $h(x) = f(x)g(x)$, find $h'(1)$.

$$\begin{aligned} h'(1) &= f'(1)g(1) + f(1)g'(1) \\ &= 1 \cdot 5 + 3 \cdot 4 \\ &= 5 + 12 \\ &= 17 \end{aligned}$$

2. Given $k(x) = \frac{f(x)}{g(x)}$, find $k'(3)$.

$$\begin{aligned} k'(3) &= \frac{f'(3)g(3) - f(3)g'(3)}{[g(3)]^2} \\ &= \frac{-2 \cdot 11 - 2 \cdot -3}{11^2} \\ &= \frac{-22 + 6}{11^2} \\ &= \frac{-16}{121} \end{aligned}$$

3. Given $\ell(x) = \frac{g(x)}{\sqrt{x}}$, find $\ell'(4)$.

$$\begin{aligned} \ell'(4) &= \frac{g'(4)\sqrt{4} - g(4) \cdot 0}{4} \\ &= \frac{-4 \cdot 2 - 8 \cdot 0}{4} \\ &= \frac{-8 - 0}{4} \\ &= -2 \end{aligned}$$

Section 3.2

48. If f is a differentiable function, find an expression for the derivative:

    (d) $y = \frac{1+xf(x)}{\sqrt{x}}$.

$$\begin{aligned} y &= \frac{\big(0 + f(x) + xf'(x)\big)\sqrt{x} - \big(\frac{1}{2}x^{-\frac{1}{2}}(1 + xf(x))\big)}{(\sqrt{x})^2} \\ &= \frac{(f(x) + xf'(x))\sqrt{x} - \big(\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{\frac{1}{2}}f(x)\big)}{x} \\ &= \frac{x^{\frac{1}{2}}f(x) + x^{\frac{1}{2}}f'(x) - \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}}f(x)}{x} \\ &= \frac{\frac{1}{2}x^{\frac{1}{2}}f(x) + x^{\frac{1}{2}}f'(x) - \frac{1}{2}x^{-\frac{1}{2}}}{x} \\ &= \frac{2xf'(x) + xf(x) - 1}{2x^{\frac{2}{3}}} \end{aligned}$$

50. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in years) that is sold is a unction of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

    (a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$?

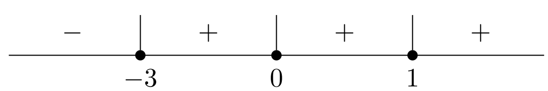
    - When the selling price is \$20, then the quantity is 10,000 yards.
    - At the price of \$20, the slope is -350 per dollar, meaning that for every dollar cheaper the selling price, the quantity goes down by 350 (at the exact price of \$20).

    (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

$$\begin{aligned} R'(p) &= p \cdot f'(p) + p' \cdot f(p) \\ &= p \cdot f'(p) + f(p) \\ R'(20) &= 20 \cdot f(20) + f'(20) \\ &= 20(-350) + 10,000 \\ &= 9,300 \text{ yards} \end{aligned}$$

    Because R' is the slope at only one specific point, not much can be interpreted from the value, but the positive slope can be somewhat associated with a higher revenue.

51. On what interval is the function $f(x) = x^3e^x$ increasing? $\mathbf{(-3, \infty)}$



52. On what interval is the function $f(x) = x^2e^x$ concave downward? $\mathbf{(-1.5, 0)}$

58. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that $F'' = f''g + 2f'g' + fg''$.

$$\begin{aligned} F' &= f' \cdot g + f \cdot g' \\ F'' &= \big(f'' \cdot g' \cdot f \cdot g'\big) + \big(f' \cdot g \cdot f' \cdot g''\big) \\ &= f'' \cdot g + 2f' \cdot g' + f \cdot g'' \end{aligned}$$

    (b) Find similar formulas for F''' and $F^{(4)}$.

$$\begin{aligned} F''' &= f^3g + 3f''g' + 3f'g'' + fg^3 \\ F^{(4)} &= f^4g(x) + 4f^3g' + 6f''g'' + 4g^3f' + fg^4 \end{aligned}$$

(c) Guess a formula for $F^{(n)}$.

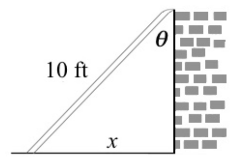
$$F^n = f^n \cdot g + nf^{n-1} \cdot g' + n^{\frac{1}{2}} f^{n-2} g^{n-2} + ng^{n-1} f' + fg^n$$

Section 3.3

16. Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

$$\begin{aligned} \sec x &= \frac{1}{\cos x} \\ \frac{d}{dx}(\sec x) &= \frac{0 \cdot \cos x - (-\sin x)}{\cos^2 x} \\ &= \frac{\sin x}{\cos x \cdot \cos x} \\ &= \boxed{\sec x \tan x} \end{aligned}$$

37. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{3}$?



$$\begin{aligned} 10^2 &= x^2 + \cos\theta \\ x &= (100 - \cos\theta)^{\frac{1}{2}} \\ \frac{dx}{d\theta} &= \frac{1}{2}(100 - \cos\theta)\sin\theta \\ \frac{dx}{d\theta} &= \frac{1}{2}\Big(100 - \frac{1}{2}\Big)\frac{\sqrt{3}}{2} \\ \frac{dx}{d\theta} &= \frac{\sqrt{3}}{4}(99.5) \\ \frac{dx}{d\theta} &= \boxed{\frac{\sqrt{3} \cdot 99.5}{4}} \end{aligned}$$

Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

39. $\frac{d^{99}}{dx^{99}}(\sin x)$

$$\begin{aligned} \frac{d}{dx} &= \boldsymbol{\cos x} \\ \frac{d^2}{dx^2} &= -\sin x \\ \frac{d^3}{dx^3} &= -\cos x \\ \frac{d^4}{dx^4} &= \sin x \\ \frac{d^5}{dx^5} &= \boldsymbol{\cos x} \\ \frac{d^9}{dx^9} &= \frac{d}{dx} \\ \frac{d^{97}}{dx^{97}} &= \frac{d}{dx} \\ \frac{d^{99}}{dx^{99}} &= \frac{d^3}{dx^3} = \boxed{-\cos x} \end{aligned}$$

40. $\frac{d^{35}}{dx^{35}}(x \sin x)$

$$\begin{aligned} \frac{d}{dx} &= \mathbf{1} \sin x + x \cos x \\ \frac{d^2}{dx^2} &= \mathbf{2} \cos x - \sin x \\ \frac{d^3}{dx^3} &= \mathbf{-3} \sin x - x \cos x \\ \frac{d^4}{dx^4} &= x \sin x \mathbf{-4} \cos x \\ \frac{d^5}{dx^5} &= \mathbf{5} \sin x + x \cos x \\ \frac{d^6}{dx^6} &= \mathbf{6} \cos x - x \sin x \\ \frac{d^{35}}{dx^{35}} &= \boxed{-x \cos x \mathbf{-35} \sin x} \end{aligned}$$