INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/Test 3**, (3) **lecture number/instructor name** and (4) **SUMMER 2015** on the front of your bluebook. Also make a **grading table** with room for 4 problems and a total score. **Start each problem on a new page.** **Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK! JUSTIFY ALL YOUR ANSWERS!**

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1. The following are not related:

   (a) (7 pts) Differentiate $y = \int_{2x}^{2/x} \frac{3+t}{1+5t^2} dt$ [Do not simplify.]

   (b) (6 pts) Evaluate the integral $\int (1 + \cot^2 x) dx$

   (c) (6 pts) Evaluate the integral $\int \frac{7x^3+5x^2-3}{\sqrt[4]{x}} dx$

   (d) (6 pts) Evaluate the integral $\int_1^3 |x^2 - 1| dx$

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2. The following problems are not related:

   (a) (12 pts) The function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 5$ has two local extrema. Estimate the value of x where one of the local extreme values of $f(x)$ occur using one iteration of Newton's method (in other words, find x_2). Use $x_1 = 0$ as an initial approximation.

   (b) (12 pts)

      i. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm respectively, if two sides of the rectangle lie along the legs.

      ii. How do you know your answer is a maximum? Justify your answer based on the theories of this class.

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3. (a) (6 pts) Using the definition for area using right hand endpoints,

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x]$$

   find an expression for the area under the curve $y = -3x^2 + 6x$ from 0 to 2 as a limit.
   Note: The following formulas may be useful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

   (b) (6 pts) Evaluate the limit.

   (c) (6 pts) Now express the area as an integral and find the average value, f_{avg}.

   (d) (6 pts) Find all c between $x = 0$ and $x = 2$ so that $f(c) = f_{avg}$.

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4. Let the function f be defined by $f(x) = \int_1^x \frac{1}{t} dt$ for $x > 0$.

   (a) (5 pts) What is $f(1)$? What is $f'(x)$? What is $f'(1)$?

   (b) (5 pts) f is differentiable. Why?

   (c) (6 pts) Show that $f'(5x) = f'(x)$.

   (d) (5 pts) Using the definition of f, show that $f(x+h) - f(x) = \int_x^{x+h} \frac{1}{t} dt$

   (e) (6 pts) Now suppose $h(x) = \int_0^{\cos(x-2)} 3t^2 dt$ and $f(s) = \int_\pi^{4s} h(x) dx$.
   Find $f''(1/2)$.
