MATH 1300: HW #5

Due on February 16, 2017 at 10:00am

Professor Braden Balentine Section 005

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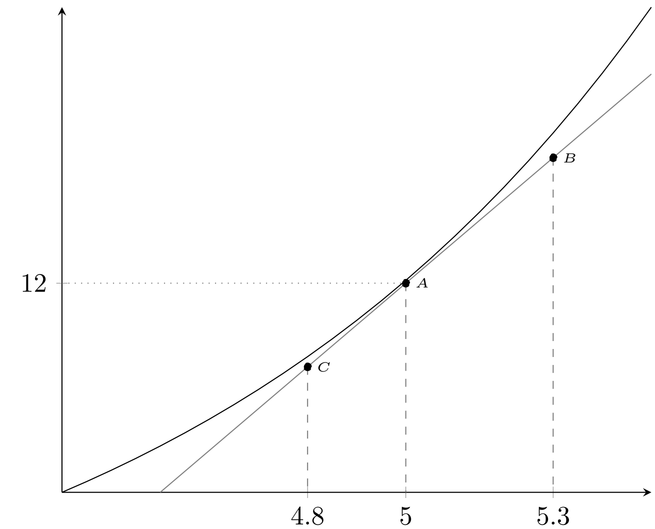
Graphical Problems

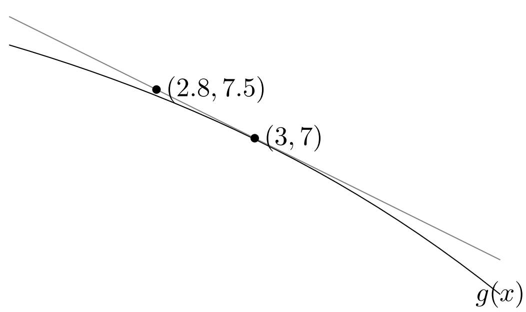
1. Find the coordinates of A, B, and C.



$$f(5) = 12 \qquad f'(5) = 2$$

- Equation for tangent line: $y = 2x + 2$
- A = (5,12)
- B = (4.8,11.6); $2(4.8) + 2 = 9.6 + 2 = 11.6$
- C = (5.3,12.6); $2(5.3) + 2 = 10.6 + 2 = 12.6$

2. If possible, find each of the following values. Write "not enough information" where appropriate.



(a) $g(3) = 7$
(b) $g(2.8) =$ Not enough information
(c) $g(7) =$ Not enough information
(d) $g^{-1}(7) = g\left(\frac{1}{7}\right) =$ Not enough information
(e) $g'(7) =$ Not enough information
(f) $g'(3) = -2.5$
(g) $g'(2.8) =$ Not enough information

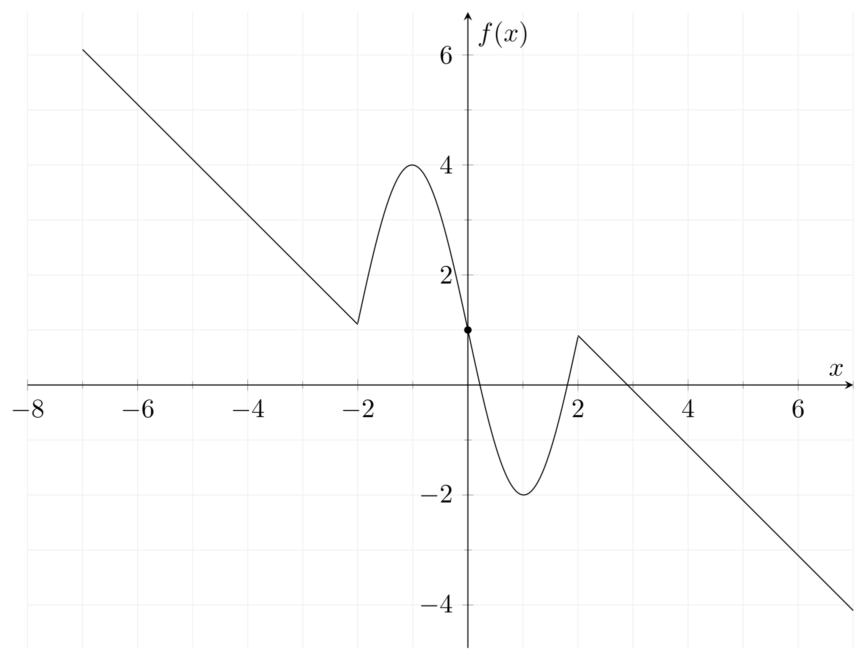
Section 2.8

9. The president announces that the national deficit is increasing, but at a decreasing rate. Interpret this statement in terms of a function and its derivatives.
   The president's statement can be described by mathematical terms by saying the derivative of the national deficit is slowly decreasing, due to the slope of the deficit lessening as time goes on.

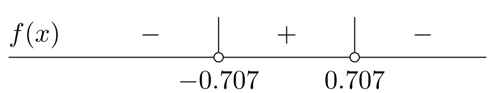
22. Sketch the graph of a function that satisfies all of the given conditions:

$$f'(1) = f'(-1) = 0$$
$$f'(x) < 0 \text{ if } |x| < 1$$
$$f'(x) > 0 \text{ if } 1 < |x| < 2$$
$$f'(x) = -1 \text{ if } |x| > 2$$
$$f''(x) < 0 \text{ if } -2 < x < 0$$

inflection point (0,1)



25. Suppose $f'(x) = xe^{-x^2}$.

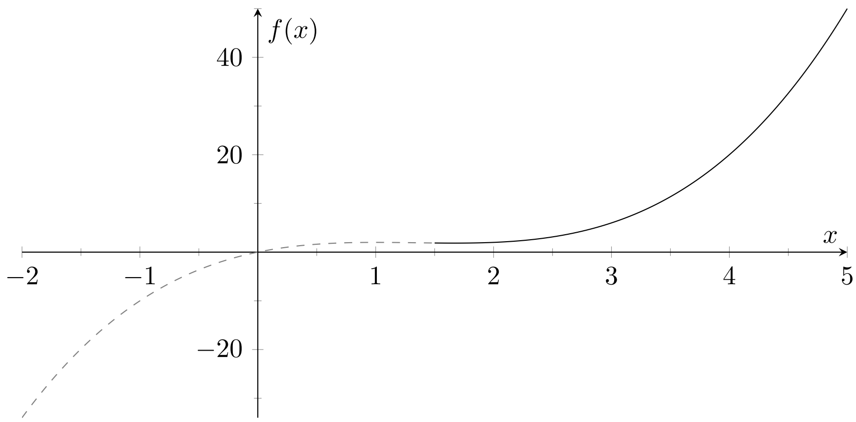


   (a) On what interval is f increasing? On what interval is f decreasing?
   Increasing: $(-\infty, -0.707) \cup (0.707, \infty)$ Decreasing: $(-0.707, 0.707)$

   (b) Does f have a maximum value? Minimum value?
   The minimum is -0.707, and the maximum is 0.707.

Section 3.1

48. On what interval is the function $f(x) = x^3 - 4x^2 + 5x$ concave upward? $(1.5, \infty)$



62. The equation $y'' + y' - 2y = x^2$ is called a **differential equation** because it involves an unknown function y and its derivatives y' and y''. Find constants A, B, and C which that the function $y = Ax^2 + Bx + C$ satisfies this equation.

$$\begin{aligned} y'' + y' - 2y &= x^2 \\ -2y &= x^2 - y' - y'' \\ y &= -\frac{2}{3}x^2 + \frac{1}{2}y' + \frac{1}{2}y'' \end{aligned}$$

$$\boxed{A = -\frac{1}{2} \quad B = \left(\frac{1}{2}y' + \frac{1}{2}y''\right) \quad C = 0}$$

66. Suppose the curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a tangent line when $x = 0$ with equation $y = 2x + 1$ and a tangent line when $x = 1$ with equation $y = 2 - 3x$. Find the values of a, b, c, and d.

$$\begin{aligned} &2 = 4(0)^3 + 3a(0)^2 + 2b(0) + c \\ &c = 2 \end{aligned}$$

$$\begin{aligned} &1 = 0^4 + a(0)^3 + b(0)^2 + c(0) + d \\ &d = 1 \end{aligned}$$

$$\begin{aligned} -3 &= 4(1)^3 + 3a(1)^2 + 2b(1) + c \\ 3a + 2b + c &= -3 \\ 3a + 2b + 2 &= -3 \\ 3a + 2b &= -5 \\ 3a &= -5 - 2b \\ a &= -\frac{5}{3} - \frac{2}{3}b \end{aligned}$$

$$\begin{aligned} -1 &= (-1)^4 + a(-1)^3 + b(-1)^2 + c(-1) + d \\ -a + b - c + d + 1 &= -1 \\ -a + b - 2 + 1 &= -2 \\ -a + b - 2 + 1 &= -2 \\ -a + b &= -1 \end{aligned}$$

Now combining the two:

$$a = -\frac{3}{5} \qquad b = -\frac{8}{5}$$