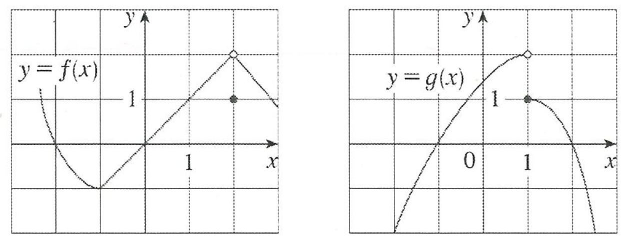
solutions

Math 1300-005 - Spring 2017

Using the Limit Laws - 1/25/17

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



(a) $\lim_{x\to 2}[f(x) + 3g(x)]$

$= \lim_{x\to 2} f(x) + 3\lim_{x\to 2} g(x)$ [Law 1+3]

$= 2 + 3\cdot(0)$

$= \boxed{2}$

(b) $\lim_{x\to 1}[2f(x) + g(x)]$ [Law 1+3]

$2\cdot\lim_{x\to 1^+} f(x) + \lim_{x\to 1^+} g(x) = 2\cdot 1 + 1 = 3$

$2\cdot\lim_{x\to 1^-} f(x) + \lim_{x\to 1^-} g(x) = 2\cdot 1 + 2 = 4$

RHL $\neq$ LHL so $\boxed{\text{DNE}}$

(c) $\lim_{x\to 0}[f(x)g(x)]$

$= \lim_{x\to 0} f(x)\cdot \lim_{x\to 0} g(x)$ [law 4]

$= 0\cdot\frac{1}{3} = \boxed{0}$

(d) $\lim_{x\to -1}\frac{f(x)}{g(x)}$

$\lim_{x\to -1} g(x) = 0$, so we cannot use law 5.

In fact, $\lim_{x\to -1^+}\frac{f(x)}{g(x)} = -\infty$

$\lim_{x\to -1^-}\frac{f(x)}{g(x)} = +\infty$

so $\boxed{\lim_{x\to -1}\frac{f(x)}{g(x)} \text{ DNE}}$

(e) $\lim_{x\to 2}[x^3 f(x)]$

$= \lim_{x\to 2} x^3 \cdot \lim_{x\to 2} f(x)$ [law 4]

$= (2)^3\cdot(2)$ [law 9]

$= \boxed{16}$

(f) $\lim_{x\to 1}\sqrt{3 + f(x)}$

$= \sqrt{\lim_{x\to 1}(3 + f(x))}$ [law 11]

$= \sqrt{\lim_{x\to 1} 3 + \lim_{x\to 1} f(x)}$ [law 1]

[law 7] $= \sqrt{3+1} = \boxed{2}$

2. Evaluate each limit and justify each step by indicating the appropriate Limit Law(s).

(a) $\lim_{x\to 8}(1 + \sqrt[3]{x})(2 - x^2)$

$= \lim_{x\to 8}(1 + \sqrt[3]{x})\cdot\lim_{x\to 8}(2 - x^2)$ [law 4]

$= \left(\lim_{x\to 8} 1 + \lim_{x\to 8}\sqrt[3]{x}\right)\left(\lim_{x\to 8} 2 - \lim_{x\to 8} x^2\right)$ [law 1]

$= (1 + \sqrt[3]{8})(2 - (8)^2)$ [law 9+10]

$= (1+2)(2-64)$

$= 3(-62) = \boxed{-186}$

(b) $\lim_{x\to 1}\sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\lim_{x\to 1}\left(\frac{2x^2+1}{3x-2}\right)}$ [law 11]

$= \sqrt{\frac{\lim_{x\to 1}(2x^2+1)}{\lim_{x\to 1}(3x-2)}}$ [law 5 since $\lim_{x\to 1}(3x-2) \neq$]

$= \sqrt{\frac{2(1)^2+1}{3(1)-2}}$ [Direct substitution]

$= \boxed{\sqrt{3}}$

3. Find the limit by simplifying the function. Also, explain why the direct substitution property is not valid.

$$\lim_{h\to 0}\frac{(4+h)^2-16}{h}$$

Cannot use direct substitution since we have a 0 in the denominator. Let us simplify.

$$\lim_{h\to 0}\frac{(4+h)^2-16}{h}=\lim_{h\to 0}\frac{\cancel{16}+8h+h^2-\cancel{16}}{h}=\lim_{h\to 0}\frac{8h+h^2}{h}=\lim_{h\to 0}\frac{\cancel{h}(8+h)}{\cancel{h}}$$

$$=8+0$$

$$=\boxed{8}$$

Like I mentioned in class. Try substitution. If you have division by 0, then simplify until you can substitute.

4. Find the limit by rationalizing the function.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$$

Cannot use direct substitution since we have division by 0. Let us simplify by rationalizing

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}\left(\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\right)=\lim_{x\to 0}\frac{1+x-1}{x(\sqrt{1+x}+1)}=\lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}$$

$$=\frac{1}{\sqrt{1+0}+1}=\boxed{\frac{1}{2}}$$

5. It is true that

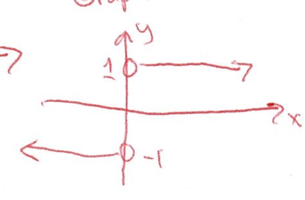
$$\lim_{x\to 0}\frac{|x|}{x}$$

does not exist. In your groups, work out and discuss why this is so.

Recall, $|x|=\begin{cases}x, & x\geq 0\\ -x, & x\leq 0\end{cases}$ so dividing by x gives

$$\frac{|x|}{x}=\underbrace{\begin{cases}\frac{x}{x} & x>0\\ \frac{-x}{x} & x<0\end{cases}}_{}=\begin{cases}1, & x>0\\ -1, & x<0\end{cases}\rightarrow$$

Graph



note: we can no longer write $\geq$ or ≤ 0 since x is in the denominater

So we have a jump discontinuity at 0, hence

$\lim_{x\to 0}\frac{|x|}{x}$ DNE.