Solutions

Math 1300-005 - Spring 2017

Linearization and Linear Approximation - 3/14/17

Guidelines: This will not be handed in, but is a study resource for Midterm 3.

1. Find the linearization, $L(x)$, of each of the following functions at the given values of a.

(a) $f(x) = x^4 + 3x^2, \quad a = -1$

$L(x) = f(a) + f'(a)(x-a)$

Here $a = -1$, $f'(x) = 4x^3 + 6x$

so $f(-1) = 1+3 = 4$

$f'(-1) = 4(-1)^3 + 6(-1)$
$= -4 - 6$
$= -10$

$L(x) = 4 + (-10)(x-(-1))$

$\hookrightarrow \boxed{L(x) = 4 - 10(x+1)}$

(b) $f(x) = \ln(x), \quad a = 1$

Here, $a=1$, $f'(x) = \frac{1}{x}$, so

$f(1) = \ln(1) = 0$

$f'(1) = \frac{1}{1} = 1$

$L(x) = f(1) + f'(1)(x-1)$

$\hookrightarrow L(x) = 0 + 1(x-1)$

$\hookrightarrow \boxed{L(x) = x-1}$

(c) $f(x) = \cos(x), \quad a = \pi/2$

Here, $a = \frac{\pi}{2}$, $f'(x) = -\sin(x)$

$f(\pi/2) = \cos(\pi/2) = 0$

$f'(\pi/2) = -\sin(\pi/2) = -1$

$L(x) = f(\pi/2) + f'(\pi/2)(x - \pi/2)$

$\hookrightarrow L(x) = 0 + (-1)(x - \pi/2)$

$\hookrightarrow \boxed{L(x) = -(x - \pi/2)}$

(d) $f(x) = x^{3/4}, \quad a = 16$

Here, $a=16$, $f'(x) = \frac{3}{4}x^{-1/4}$

$f(16) = (16)^{3/4} = (16^{1/4})^3 = 2^3 = 8$

$f'(16) = \frac{3}{4}(16)^{-1/4} = \frac{3}{4} \cdot \frac{1}{(16)^{1/4}}$
$= \frac{3}{4 \cdot 2}$
$= \frac{3}{8}$

$L(x) = f(16) + f'(16)(x-16)$

$\hookrightarrow \boxed{L(x) = 8 + \frac{3}{8}(x-16)}$

2. In this problem, we shall estimate $e^{-0.015}$ using linear approximation. The idea is that -0.015 is very close to 0 and since we know the value of $e^0 = 1$, we can use the linearization of $f(x) = e^x$ at $x = 0$ to perform the estimate.

   (a) Let $f(x) = e^x$. Find the linearization, $L(x)$, of f at $a = 0$.

   Here $a=0$, $f'(x) = e^x$, so

   $f(0) = 1$

   $f'(0) = 1$

   $L(x) = f(0) + f'(0)(x-a)$

   $\hookrightarrow L(x) = 1 + 1(x-0)$

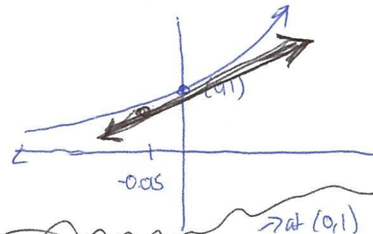
   $\hookrightarrow \boxed{L(x) = 1 + x}$

   (b) Use $L(x)$ to estimate $e^{-0.015}$. Is this an overestimate or underestimate? Please justify.

   $e^{-0.015} = f(-0.015) \approx L(-0.015)$

   $= 1 + (-0.015)$

   $= \boxed{0.985}$



   Tangent line to e^x at (0,1) is below the curve at -0.015, so this is an underestimate

3. In this problem, we shall estimate $(8.06)^{2/3}$ using linear approximation. The idea is that 8.06 is very close to 8 and since we know the value of $8^{2/3} = (8^{1/3})^2 = 4$, we can use the linearization of $f(x) = x^{2/3}$ at $x = 8$ to perform the estimate.

   (a) Let $f(x) = x^{2/3}$. Find the linearization, $L(x)$, of f at $a = 8$.

   $f'(x) = \frac{2}{3}x^{-1/3}$

   $f(8) = (8)^{2/3} = (8^{1/3})^2 = 2^2 = 4$

   $f'(8) = \frac{2}{3}(8)^{-1/3} = \frac{2}{3 \cdot 2} = \frac{1}{3}$

   $L(x) = f(8) + f'(8)(x-8)$

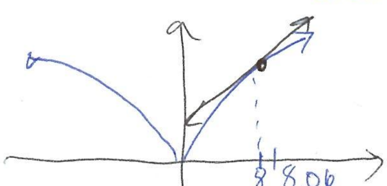
   $\hookrightarrow \boxed{L(x) = 4 + \frac{1}{3}(x-8)}$

   (b) Use $L(x)$ to estimate $(8.06)^{2/3}$. Is this an overestimate or underestimate? Please justify.

   $(8.06)^{2/3} = f(8.06) \approx L(8.06)$

   $= 4 + \frac{1}{3}(8.06 - 8)$

   $= 4 + \frac{1}{3}\left(\frac{6}{100}\right) = 4 + \frac{2}{100} = \boxed{4.02}$



   The tangent line to f at $a = 8$ is above the graph at 8.06, so this is an overestimate.