Math 1300-005 - Spring 2017

Applied Optimization, Pt. 1 - 4/5/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3.

1. Consider
$$f(x) = \frac{x}{1+x^2}$$
on the open interval $(0, \infty)$. Use the First Derivative Test for Absolute Extrema to determine whether or not f has an absolute maximum or absolute minimum on $(0, \infty)$. Be sure to include full justification.

2. Consider
$$g(x) = 12x - x^3$$
on the open interval $(-\infty, 0)$. Use the First Derivative Test for Absolute Extrema to determine whether or not g has an absolute maximum or absolute minimum on $(-\infty, 0)$. Be sure to include full justification.

Having practiced the first derivative test for absolute extrema in a general setting, let us now apply it to applied optimization problems. Today we will start off easy.

3. Find two numbers whose difference is 100 and whose product is a minimum.

   (a) Let the two numbers in question be denoted x and y. By the given info we know the difference of x and y is 100, so
   $$x - y = 100.$$
   This equation is called our *constraint equation* as it puts constraints on what x and y can be.

   (b) We are seeking to minimize the product of x and y, denoted
   $$P = xy.$$
   This equation is called our *optimizing equation*, and we will eventually do our calculus here.

   (c) Notice our expression for P involves two variables! To get around this, let us solve our constraint equation $x - y = 100$ for y, giving
   $$y = 100 - x.$$
   Substituting this into our expression for P gives
   $$P = x(100 - x) = 100x - x^2.$$

   (d) Notice P is now a function of a single variable. What is the domain of P? Well x and y can in principle be anything, so our domain is $(-\infty, \infty)$.

   (e) Now use the first derivative test for absolute extrema to find the minimum value of $P = 100x - x^2$ on the interval $(-\infty, \infty)$. Be sure to include full justification.

   (f) The step above will give the value of x the satisfies the problem; but we were told to find two numbers. To find y, substitute the value for x back into the constraint. Thus

   $x =$ $\qquad\qquad\qquad$ $y =$

4. If 27 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

   (a) Let x denote the length of the sides of the square base of the box, and let y denote the height. We know the surface area of the box is 27 cm^2, so write this as an equation involving x and y. This is our *constraint*. [Hint: a picture helps a ton!]

   (b) We are seeking to minimize the volume V of the box. Write an equation involving V, x, and y. This is our *optimizing equation*.

   (c) Your expression for V should involve x and y. To get it purely in terms of x, solve your constraint equation from (a) for y and substitute this into your optimizing equation and simplify. V should now be a function of x alone.

   (d) Since x and y are lengths of sides of a box, what is the domain of V?

   (e) Apply the first derivative test for absolute extrema to find the maximum value of V on the domain found above. Be sure to include full justification.

   (f) What is the largest possible volume?

5. Now try one on your own. Find two positive numbers whose product is 100 and whose sum is a minimum.