Math 1300-005 - Spring 2017
Derivatives Using the Power Rule - 2/14/17

Solutions

Guidelines: Please work in groups of two or three. As you finish problems, raise your hand and call me over to check your work. This will not be handed in and is a study resource for the next midterm.

1. Differentiate the function. Do whatever algebraic simplifications are necessary so that everything is written as a sum/difference of powers of x.

(a) $f(x) = 4\pi^2$

$\boxed{f'(x) = 0}$

since $4\pi^2$ is constant

(b) $F(x) = 2 - \frac{2}{3}x$

$\boxed{F'(x) = -\frac{2}{3}}$

(c) $g(x) = x^3 - 4x + 6$

$\boxed{g'(x) = 3x^2 - 4}$

(d) $h(u) = (u-2)(2u+3)$

$= 2u^2 + 3u - 4u - 6$

$= 2u^2 - u - 6$

$\boxed{h'(u) = 4u - 1}$

(e) $m(s) = -\dfrac{12}{s^5}$

$= -12s^{-5}$

$m'(s) = -12(-5)s^{-6}$

$\boxed{= \dfrac{60}{s^6}}$

(f) $G(t) = 2t^{-3/4}$

$G'(t) = 2\left(\frac{-3}{4}\right)t^{-7/4}$

$\boxed{= -\frac{3}{2}t^{-7/4}}$

(f) $y = \sqrt{x}(x-1)$

$= x^{1/2} \cdot x - x^{1/2}$

$= x^{3/2} - x^{1/2}$

$\boxed{\dfrac{dy}{dx} = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}$

(g) $u = \sqrt[5]{t} + 4\sqrt{t^5}$

$= t^{1/5} + 4t^{5/2}$

$\dfrac{du}{dt} = \frac{1}{5}t^{-4/5} + 4\left(\frac{5}{2}\right)t^{3/2}$

$= \boxed{\frac{1}{5}t^{-4/5} + 10t^{3/2}}$

2. Differentiate the following functions. Before doing so, you must simplify the function using exponent rules so that everything is written as a sum/difference of powers of x.

(a) $f(x) = \dfrac{x^2 - 3x + 1}{x^2}$

$= \dfrac{x^2}{x^2} - \dfrac{3x}{x^2} + \dfrac{1}{x^2}$

$= 1 - 3x^{-1} + x^{-2}$

$f'(x) = 0 - 3(-1)x^{-2} - 2x^{-3}$

$= \boxed{3x^{-2} - 2x^{-3}}$

(b) $g(x) = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

$= \dfrac{x^2}{x^{1/2}} + \dfrac{4x}{x^{1/2}} + \dfrac{3}{x^{1/2}}$

$= x^{3/2} + 4x^{1/2} + 3x^{-1/2}$

$g'(x) = \frac{3}{2}x^{1/2} + 4\left(\frac{1}{2}\right)x^{-1/2} + 3\left(\frac{-1}{2}\right)x^{-3/2}$

$= \boxed{\frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2}}$

3. Find an equation of the tangent line to the curve at the given x-value.

(a) $y = x^4 + 2x^2 - x$ at $x = 1$

$\dfrac{dy}{dx} = 4x^3 + 4x - 1$. At $x=1$,

$\left.\dfrac{dy}{dx}\right|_{x=1} = 4(1)^3 + 4(1) - 1$

$= 7$.

And $y(1) = (1)^4 + 2(1)^2 - 1 = 2$

$\boxed{y - 2 = 7(x-1)}$

(b) $y = (1 + 2x)^2$ at $x = 2$

$= (1+2x)(1+2x) = 1 + 4x + 4x^2$

So $\dfrac{dy}{dx} = 4 + 8x$, so at $x=2$, $\left.\dfrac{dy}{dx}\right|_{x=2} = 4 + 8(2)$

$= 20$

And $y(2) = (1 + 2(2))^2 = (5)^2 = 25$

So $\boxed{y - 25 = 20(x-2)}$

4. The equation of motion of a particle is $s(t) = t^3 - 3t$ where s is in meters and t is in seconds. Find:

(a) The velocity and acceleration as functions of t. What are the units in each case?

$v(t) = s'(t) = \underline{3t^2 - 3}$, in $\underline{m/s}$. $\quad a(t) = v'(t) = \underline{6t}$, in $\underline{m/s^2}$

(b) The acceleration after 2 s.

$a(2) = 6(2) = \boxed{12\ m/s^2}$

(c) The acceleration when the velocity is 0.

We need to find t when $0 = v(t) = 3t^2 - 3$. So $3(t^2 - 1) = 0$, so $\underline{t = \pm 1}$.
Time is positive, so we only consider $t = 1$ sec. Thus $\boxed{a(1) = 6\ m/s^2}$