Solutions

Math 1300-005 - Spring 2017

Related Rates, Pt. III - 3/3/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3. This third worksheet over related rates covers some more intermediate examples now that we are used to the process.

For ***each*** of the following related rates problems:

(a) Draw a picture of the situation and assign variables.

(b) Write down the known and unknown quantities in terms of the assigned variables.

(c) Use your picture to write an equation that relates the variables.

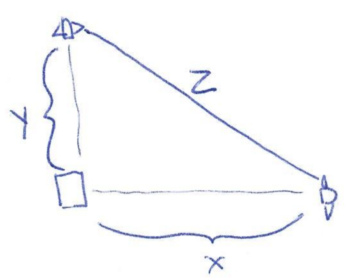
(d) Take d/dt of each side of this equation, solve for the unknown quantity, and then plug in the known quantities.

1. Two space ships leave from a docking station at the same time on perpendicular trajectories. One of the space ships travels at a speed of ~~0.2~~ 1 light-years per year, and the other at a speed of 2.4 light-years per year. How fast is the distance between the spaceships changing at 5 years of travel?

(a) Picture

Start

After Some Time



(b) Known, unknown

Known

$\frac{dx}{dt} = 2.4$ lyr/yr

$\frac{dy}{dt} = 0.2$ lyr/yr

unknown $\frac{dz}{dt}$ after 5 years.

@ 5 yrs, $x = 2.4$ lyr/yr $\cdot$ 5yr $= 12$ lyr

$y = 1$ lyr/yr $\cdot$ 5yr $= 5$ lyr

(c) Relation: $x^2 + y^2 = z^2$

(d) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

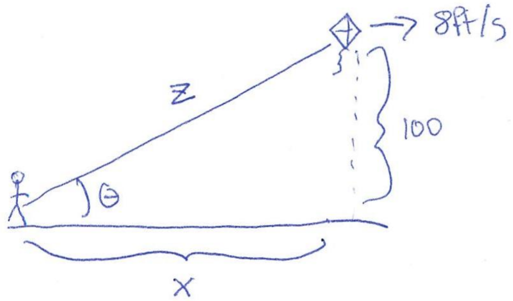
$\Rightarrow \frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$

After 5 years, $z = \sqrt{12+5} = 13$, so

$\frac{dz}{dt} = \frac{1}{13}\left(12(2.4) + 5(1)\right) = \boxed{2.6 \text{ lyr/yr}}$

2. A kite 100 ft above ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

(a) Picture



(b) Known: $\frac{dx}{dt} = 8$ ft/s

unknown: $\frac{d\theta}{dt}$ when $z = 200$ ft.

(c) Relation: $\tan(\theta) = \frac{100}{x} \Leftrightarrow x = 100\cot(\theta)$.

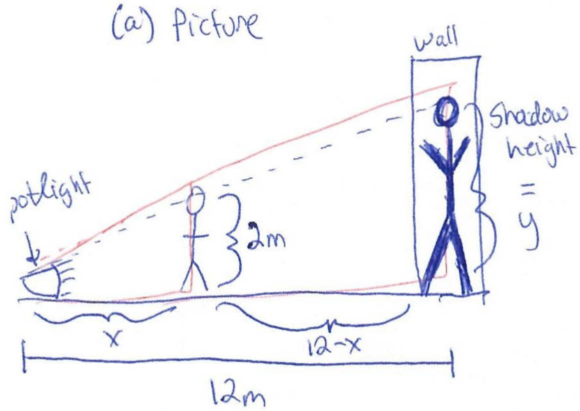
(d) $\frac{dx}{dt} = 100(-\csc^2(\theta))\frac{d\theta}{dt}$. Recall, $\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{Hyp}{Opp} = \frac{z}{100}$.

so when $z = 200$, $\csc^2(\theta) = \left(\frac{200}{100}\right)^2 = 4$

Thus $\frac{d\theta}{dt} = \frac{dx/dt}{-100\csc^2(\theta)} = \frac{8}{-100(4)} = -\frac{2}{100} = \boxed{\frac{-1}{50} \text{ rad/s}}$

3. A spotlight on the ground shines on a wall 12 m away. If a man 2m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

(a) Picture

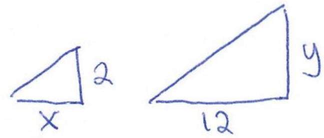


(b) Known $\frac{dx}{dt} = 1.6$ m/s

Unknown $\frac{dy}{dt}$ when he is 4 m from building

Note: If he is 4 meters from the building, he is $x = 8$m from the spotlight!

(c) Relation: Similar triangles!



so $\frac{x}{2} = \frac{12}{y}$, so $y = \frac{24}{x} = 24x^{-1}$

(d) $\frac{dy}{dt} = -24x^{-2} \cdot \frac{dx}{dt}$, when $x = 8$m, $\frac{dy}{dt} = -24(8)^{-2} \cdot (1.6) = \frac{-24}{64} \cdot (1.6) = \frac{-24}{40}$

$= \boxed{\frac{-6}{10} \text{ m/s}}$