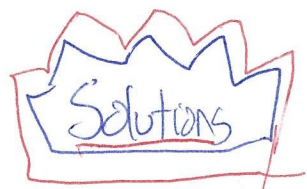


**Math 1300-005 - Spring 2017**  
The Intermediate Value Theorem - 1/31/17



*Guidelines:* Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval. Remember, in solving each problem, you must verify that each of the hypotheses of the IVT are satisfied.

(a)  $x^4 + x - 3 = 0$ ,  $(1, 2)$

Let  $f(x) = x^4 + x - 3$ , which is continuous on  $[1, 2]$  b/c it is a polynomial.

Since  $f(1) = 1 + 1 - 3 < 0$ ,  $0$  is between  $f(1)$  &  $f(2)$ , so by IVT  
 $f(2) = 16 + 2 - 3 > 0$

There exists  $c$  in  $(1, 2)$  such that  $f(c) = 0$ , i.e.,  $c^4 + c - 3 = 0$ .

(b)  $\sqrt[3]{x} = 1 - x$ ,  $(0, 1)$

Rearrange first as  $\sqrt[3]{x} + x - 1 = 0$ . Let  $f(x) = \sqrt[3]{x} + x - 1$ , which is cont. on  $[0, 1]$   
b/c it is a root function plus a polynomial. Since

$f(0) = -1 < 0$ ,  $0$  is between  $f(0)$  and  $f(1)$ . By the IVT  
 $f(1) = 1 > 0$

There exists  $c$  in  $(0, 1)$  such that  $f(c) = 0$ , i.e.,  $\sqrt[3]{c} + c - 1 = 0$ .

(c)  $e^x = 3 - 2x$ ,  $(0, 1)$

Rearrange first as  $e^x + 2x - 3 = 0$ . Let  $f(x) = e^x + 2x - 3$ , which is cont. on  $[0, 1]$  b/c it is an exponential function plus a polynomial. Since

$$f(0) = e^0 + 2(0) - 3 = 1 - 3 < 0$$

$$f(1) = e + 2 - 3 = e - 1 > 0 \text{ (since } e \approx 2.7), 0 \text{ is between } f(0) \text{ and } f(1). \text{ By}$$

the IVT, there exists  $c$  in  $(0, 1)$  such that  $f(c) = 0$ , i.e.,  $e^c + 2c - 3 = 0$ .

(d)  $\sin(x) = x^2 - x$ ,  $(1, 2)$

To get that  $\sin(1) > 0$ , note  $1$  is between  $0$  and  $\frac{\pi}{2}$  and  $\sin(x) > 0$  on  $(0, \frac{\pi}{2})$  [draw a graph]. Rearrange first as  $\sin(x) + x - x^2 = 0$ . Let  $f(x) = \sin(x) + x - x^2$ , which is continuous on  $[1, 2]$  b/c it is a trig function plus a polynomial. Since

$$f(1) = \sin(1) + 1 - (1)^2 = \sin(1) > 0$$

$$f(2) = \sin(2) + 2 - (2)^2 = \sin(2) - 2 < 0, 0 \text{ is between } f(1) \text{ and } f(2). \text{ By}$$

To get  $\sin(2) - 2 < 0$ , note

$-1 \leq \sin(2) \leq 1$ , so at most  $\sin(2) = 1$ , and  $1 < 2$ , so  $\sin(2) - 2 < 0$

the IVT, there exists  $c$  in  $(1, 2)$  such that  $f(c) = 0$ , i.e.,  $\sin(c) + c - c^2 = 0$ .

The following problems are review of the material we covered Monday 1/30 over the definition of continuity.

2. State the interval(s) where the following function is continuous.

$$f(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$$

We need only check continuity at  $a=0$ .

①  $f(0)$  is defined and  $f(0)=0$ .

$$\begin{aligned} \textcircled{2} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \cos(x) = \cos(0) = 1 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} 1 - x^2 = 1 - (0)^2 = 1 \end{aligned} \rightarrow \text{so } \lim_{x \rightarrow 0} f(x) = 1.$$

③ Since  $\lim_{x \rightarrow 0} f(x) \neq f(0)$ ,  $f$  is not continuous at 0. Hence

$$f \text{ is continuous on } (-\infty, 0) \cup (0, \infty)$$

3. For what value of the constant  $c$  is the function  $f$  continuous on  $(-\infty, \infty)$ ?

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^{2x+c} & \text{if } x \geq 0 \end{cases}$$

We need only check continuity at  $a=0$ .

①  $f(0)$  is defined and  $f(0) = e^{2(0)+c} = e^c$

$$\textcircled{2} \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x+2) = 2$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^{2x+c} = e^c$$

We need ~~RHL = LHL = f(0)~~  $RHL = LHL = f(0)$ , so we need

$$2 = e^c \text{ or }$$

$$c = \ln(2)$$