Math 1300-005 - Spring 2017

Derivatives and the Shapes of Curves, Pt. I - 3/22/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3.

1. Consider the function $g(x)$ and its first and second derivatives.

$$g(x) = \frac{x^2}{(x-2)^2}, \qquad g'(x) = \frac{-4x}{(x-2)^3}, \qquad g''(x) = \frac{8(x+1)}{(x-2)^4}$$

   (a) Find the x-intercept(s) of g, if any. Find the y-intercept(s) of g, if any.

   (b) Find the vertical asymptote(s) of g, if any. Find the horizontal asymptote(s) of g, if any

   (c) Find all values of x such that $g'(x) = 0$ ***AND*** all values of x such that the denominator of g' is zero. Which of these x-values are critical numbers?

   (d) Plot *all* values from (c) on a sign chart for g'. If an x-value is critical, place it on the sign chart with a solid dot. If an x-value is not critical, place it on the sign chart with an open dot. Fill in your sign chart using test points.

   (e) Find the intervals of increase or decrease. Justify your answer.

   (f) When finding local extrema, we only consider numbers on our sign chart for g' with a solid dot...why? Find the x-coordinates of the local maximum and minimum values. Justify your answer.

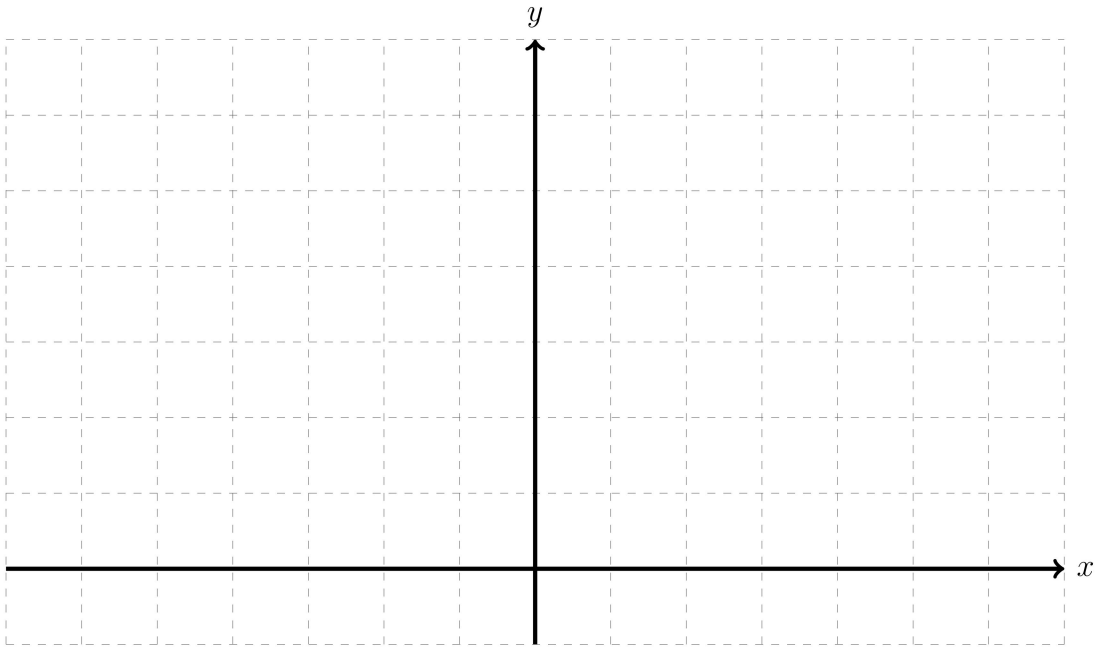
(g) Find all values of x such that $g''(x) = 0$ ***AND*** all values of x such that the denominator of g'' is zero.

(h) Plot all values from (g) on a sign chart for g''. If an x-value is in the domain of g, place it on the sign chart with a solid dot. If an x-value is not in the domain of g, place it on the sign chart with an open dot. Fill in your sign chart using test points.

(i) Find the intervals of concavity. Justify your answer.

(j) When finding inflection points, we only consider numbers on our sign chart for g'' with a solid dot...why? Find the x-coordinates of any inflection points. Justify your answer.

(k) Using all the information from parts (a) through (j), sketch a graph of $g(x)$ below.



2. Consider the function $f(x)$ and its first and second derivatives.

$$f(x) = x^{2/3}(3-x)^{1/3}, \qquad f'(x) = \frac{2-x}{x^{1/3}(3-x)^{2/3}}, \qquad f''(x) = \frac{-2}{x^{4/3}(3-x)^{5/3}}$$

(a) Find the x-intercept(s) of f, if any. Find the y-intercept(s) of f, if any.

(b) Find the vertical asymptote(s) of f, if any. Find the horizontal asymptote(s) of f, if any.

(c) Find all values of x such that $f'(x) = 0$ ***AND*** all values of x such that the denominator of f' is zero. Which of these x-values are critical numbers?

(d) Plot *all* values from (c) on a sign chart for f'. If an x-value is critical, place it on the sign chart with a solid dot. If an x-value is not critical, place it on the sign chart with an open dot. Fill in your sign chart using test points.

(e) Find the intervals of increase or decrease. Justify your answer.

(f) When finding local extrema, we only consider numbers on our sign chart for f' with a solid dot...why? Find the x-coordinates of the local maximum and minimum values. Justify your answer.

(g) Find all values of x such that $f''(x) = 0$ ***AND*** all values of x such that the denominator of f'' is zero.

(h) Plot all values from (g) on a sign chart for f''. If an x-value is in the domain of f, place it on the sign chart with a solid dot. If an x-value is not in the domain of f, place it on the sign chart with an open dot. Fill in your sign chart using test points.

(i) Find the intervals of concavity. Justify your answer.

(j) When finding inflection points, we only consider numbers on our sign chart for f'' with a solid dot...why? Find the x-coordinates of any inflection points. Justify your answer.

(k) Using all the information from parts (a) through (j), sketch a graph of $f(x)$ below.