MATH 1300: HW #11

Due on April 6, 2017 at 10:00am

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Section 4.5

42. Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's rule does not apply, explain why.

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{\frac{b}{x}}$$
$$\ln(L) = \frac{b}{x} \ln\left(1 + \frac{a}{x}\right)$$
$$\ln(L) = \frac{b}{\infty} \ln\left(1 + \frac{a}{\infty}\right)$$
$$\ln(L) = \frac{b}{\infty} \ln(1)$$
$$\ln(L) = 0 \cdot 0$$
$$= 0$$

54. Use l'Hospital's Rule to help find the asymptotes of f. Then use them, together with information from f' and f'', to sketch the graph of f. Check your work with a graphing device.

$$f(x) = xe^{-x^2}$$
$$f'(x) = e^{-x^2}(1 - 2x^2)$$
$$f''(x) = 2e^{-x^2}x(2x^2 - 3)$$
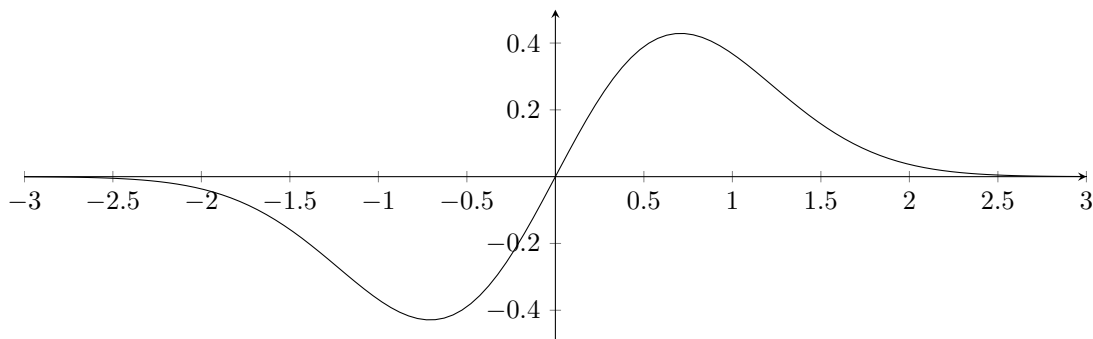
64. Prove that
$$\lim_{x \to \infty} \frac{e^x}{x^n} = \infty$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

$$\lim_{x \to \infty} \frac{e^x}{nx^{n-1}}$$
$$\frac{\infty}{n\infty^{n-1}}$$
$$\frac{\infty}{\infty}$$

66. If any object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is
$$v = \frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right)$$

where g is the acceleration due to gravity and c is a positive constant.

(a) Calculate $\lim\limits_{t\to\infty} v$. What is the meaning of this limit?

$e^{-\frac{ct}{m}}$ approaches 0 as t approaches infinity. This means that the limit is $\frac{mg}{c}$. The meaning of this limit is the maximum (or terminal) speed of an object when it is dropped. This speed is when the force of gravity is equal to the resistance of air.

(b) For fixed t, use l'Hospital's Rule to calculate the limit of $\lim\limits_{c\to 0^+} v$. What can you conclude about the velocity of a falling object in a vacuum?

$$\lim_{c\to 0^+} v$$
$$\lim_{c\to 0^+} mg(\frac{1 - e^{-\frac{ct}{m}}}{c})$$
$$\lim_{c\to 0^+} mg(\frac{1 - e^{-\frac{t}{me}}}{1}) = tg$$

The velocity of the falling object in a vacuum is simply tg.

73. If f' is continuous, $f(2) = 0$, and $f'(2) = 7$, evaluate

$$\lim_{x\to 0} \frac{f(2 + 3x) + f(2 + 5x)}{x}$$

$$\frac{f(2 + 3 \cdot 0) - f(2 + 5 \cdot 0)}{0}$$
$$= \frac{0}{0}$$
$$\frac{\frac{d}{dx}(f(2 + 3x)) - \frac{d}{dx}(f(2 + 5x))}{\frac{d}{dx}(0)}$$
$$= 3 \cdot f'(2 + 3x) + 5 \cdot f'(2 + 5x)$$
$$- 3 \cdot f'(2) + 5 \cdot f'(2)$$
$$= 3 \cdot 7 + 5 \cdot 7$$
$$= 56$$