Math 1300-005 - Spring 2017

Limit Laws and Important Properties

Purpose: This sheet collects the limit laws. Please bring this daily to class until (at least) the first exam.

A. Main Limit Laws

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$ $\quad c$ a constant.
4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $\quad$ if $\lim_{x\to a} g(x) \neq 0$

B. Further Limit Laws

6. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ $\quad$ where n is a positive integer
7. $\lim_{x\to a} c = c$ $\quad c$ a constant
8. $\lim_{x\to a} x = a$
9. $\lim_{x\to a} x^n = a^n$ $\quad$ where n is a positive integer
10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ $\quad$ where n is a positive integer, and if n is even, we assume $a > 0$.
11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ $\quad$ where n is a positive integer, and if n is even, we assume $\lim_{x\to a} f(x) > 0$.

C. ***Direct Substitution Property*** If f is a polynomial or rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a).$$

D. ***Simplification Property*** If $f(x) = g(x)$ when $x \neq a$, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x),$$

provided the limits exist. Thus we can simplify and THEN compute the limit.

E. ***Order Theorem*** If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x).$$

F. ***Squeeze Theorem*** If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then

$$\lim_{x\to a} g(x) = L.$$