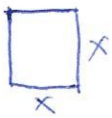
Solutions

Math 1300-010 - Fall 2016

Related Rates, Pt. I - 10/17/16

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3. This first worksheet over related rates covers some easier examples so we can get used to the process.

1. Each side of a square is increasing at a rate of 5 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2.

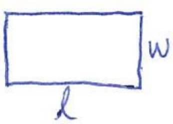


$A = x^2 \longrightarrow \frac{dA}{dt} = 2x \cdot \frac{dx}{dt}$. When $A = 16\text{cm}^2$, $x = 4\text{cm}$, so

Given:
$\frac{dx}{dt} = 5$ cm/s

$$\boxed{\frac{dA}{dt} = 2(4)(5) = 40 \text{ cm}^2/\text{s}}$$

2. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?



$A = l \cdot w \longrightarrow \frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt}$. When $w = 10$ and $l = 20$,

Given:
$\frac{dl}{dt} = 8$ cm/s
$\frac{dw}{dt} = 3$ cm/s

$$\frac{dA}{dt} = 8(10) + 20(3) = \boxed{140 \text{ cm}^2/\text{s} = \frac{dA}{dt}}$$

3. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing? For a cylinder, $V = \pi r^2 h$.

Clever Way: r is a constant!

$V = (\pi r^2) h$
constant,

Given:
$\frac{dV}{dt} = 3\text{m}^3/\text{min}$

So $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

when r = 5m

$$\boxed{\frac{dh}{dt} = \frac{dV/dt}{\pi(5)^2} = \frac{3}{25\pi} \text{ m/s}}$$

Not So Clever:

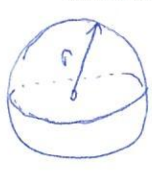
$V = \pi r^2 h \longrightarrow \frac{dV}{dt} = \pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$.

we don't know h, but $\frac{dr}{dt} = 0$ since r = 5m is constant.

So $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{dV/dt}{\pi r^2}$. When r = 5,

$$\boxed{\frac{dh}{dt} = 3/25\pi \text{ m/s}}$$

4. The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 80 mm?



$V = \frac{4}{3}\pi r^3 \longrightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. When d = 80m, $r = \frac{80}{2}$m = 40m, so

Given:
$\frac{dr}{dt} = 4$mm/s

$\frac{dV}{dt} = 4\pi(40)^2 \cdot 4$

$= 16 \cdot 1600\pi$

$$\boxed{\frac{dV}{dt} = 25{,}600\pi \text{ mm}^3/\text{s}}$$

5. Suppose $y = \sqrt{2x+1}$, where x and y are functions of t.

(a) If $dx/dt = 3$, find dy/dt when $x = 4$.

$$\frac{dy}{dt} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2\frac{dx}{dt} = \frac{dx/dt}{\sqrt{2x+1}}.$$

$$\boxed{\frac{dy}{dt} = 1 \text{ unit/time}}$$

$$\text{So } \frac{dy}{dt} = \frac{3}{\sqrt{2(4)+1}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1.$$

(b) If $dy/dt = 5$, find dx/dt when $y = 5$.

In (a), we saw $\frac{dy}{dt} = \frac{1}{\sqrt{2x+1}} \cdot \frac{dx}{dt}$, so $\frac{dx}{dt} = (\sqrt{2x+1}) \cdot \frac{dy}{dt}$. But $y = \sqrt{2x+1}$,

so $\frac{dx}{dt} = y \cdot \frac{dy}{dt}$. Thus $\frac{dx}{dt} = (5)(5) = 25$, $\boxed{\frac{dx}{dt} = 25 \text{ units/ times}}$

6. If $x^2 + y^2 = 25$ and $dy/dt = 6$, find dx/dt when $y = 4$.

Given:

$\frac{dy}{dt} = 6$ units/time

$$x^2+y^2=25 \rightarrow 2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Plug in given info: If $y=4$, $x^2=25-y^2=25-16=9$, so $x=\pm 3$

when $x=3$, $2(3)\frac{dx}{dt} + 2(4)(6)=0 \rightarrow \boxed{\frac{dx}{dt} = -\frac{48}{6} = -8 \text{ units/time}}$

when $x=-3$, $2(-3)\frac{dx}{dt} + 2(4)(6) \Rightarrow \boxed{\frac{dx}{dt} = \frac{-48}{-6} = 8 \text{ units/time}}$

7. If $x^2 + y^2 = r^2$ and if $dx/dt = 2$ and $dy/dt = 3$, find dr/dt when $x = 5$ and $y = 12$.

Given:

$\frac{dx}{dt} = 2$ units/time

$\frac{dy}{dt} = 3$ units/time

$$x^2+y^2=r^2 \rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

when $x=5, y=12$, $r^2 = 25+144=169 \rightarrow r=13$ or $r=-13$

when $r=13$, $\overset{20}{2(5)(2)} + \overset{72}{2(12)(3)} = 2(13)\frac{dr}{dt} \rightarrow \boxed{\frac{dr}{dt} = \frac{92}{26} \text{ units/time}}$

when $r=13$, $2(5)(2) + 2(12)(3) = 2(-13)\frac{dr}{dt}$ $\boxed{\frac{dr}{dt} = -\frac{92}{26} \text{ units/time}}$

8. A partical moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2, 3)$ the y-coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

Given:

$\frac{dy}{dt} = 4$cm/s

$$y=\sqrt{1+x^3} \rightarrow \frac{dy}{dt} = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2 \cdot \frac{dx}{dt} = \frac{3x^2}{2\sqrt{1+x^3}} \cdot \frac{dx}{dt}$$

$$\text{so } dx/dt = \frac{2\sqrt{1+x^3}}{3x^2} \cdot \frac{dy}{dt}.$$

$$\boxed{\frac{dx}{dt} = 2\text{cm/s}}$$

when we are at $(2,3)$, $x=2$ so

$$\frac{dx}{dt} = \frac{2\sqrt{1+8}}{3 \cdot 4}(4) = \frac{2 \cdot 3 \cdot \cancel{4}}{3 \cdot \cancel{4}} = 2$$