APPM 1345/1350 **Final Exam** **Spring 2014**

INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1345/1350/FINAL EXAM**, (3) **lecture number/instructor name** and (4) **SPRING 2014** on the front of your bluebook. Also make a **grading table** with room for 6 problems and a total score. **Start each problem on a new page.** **Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK**

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1. (a) (5 pts. each) Find the following derivatives, $f'(x)$, for the f given:

   (a) $f(x) = x^2 \sin^{-1}(x^2)$ (b) $f(x) = \dfrac{1}{1+x^2}$ (c) $f(x) = x \ln x$ (d) $f(x) = x^x$

   (b) (15 pts) Using rules for differentiation it is easy to show that $\dfrac{d}{dx}(3x^2+1) = 6x$. Show that this is true directly from the definition of differentiation. No credit will be given for quoting rules of differentiation.

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2. (5 pts each) In answering the following questions, justify each part. Given $f(x) = \dfrac{x|x|}{2+x}$, for f:

   (a) Find the vertical and horizontal asymptotes.

   (b) Find the intervals of increase or decrease.

   (c) Find the local maximum and minimum values.

   (d) Find the intervals of concavity and the inflection points.

   (e) Use parts (a) - (d) to the sketch the graph of f. LABEL your sketch (Intercepts, asymptotes, etc.).

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3. (15 points)You are designing a mural and you would like to have a margin of 1 feet on the left, right, and top of the artwork, but none on the bottom. If you allow 32 ft^2 for the area containing the artwork itself, what dimensions should the artwork have if you want to minimize the total area of the mural (i.e., of the artwork and the margins.)

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4. (6 points each)

   (a) The interval $[-1, 3]$ is partitioned into n subintervals of equal length. Let r_k denote the right-hand endpoint of the k^{th} subinterval. Express the integral $\int_{-1}^{3}(3x^2 - 2x + 5)dx$ as the limit of a Riemann sum using the right-hand endpoints of each subinterval.

   (b) Given that $a < b$, what values of a and b minimize the value of $\int_a^b (t^4 - 2t^2)dt$?

   (c) Solve the initial value problem: $\frac{dy}{dx} = x\sqrt{1+x^2}$ with $y(1) = -2$.

   (d) $\dfrac{d}{dx}\displaystyle\int_1^{10^x} t^t dt = ?$

   (e) $\displaystyle\int_0^{\sqrt{ln\pi}} 2xe^{x^2}\cos(e^{x^2})dx = ?$

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5. (6 points each) Evaluate the following:

   (a) $\lim_{t \to 3} \cos^{-1}\left(\log_3 \sqrt{t}\right)$

   (b) $\lim_{x \to \infty} x \tan(5/x)$

   (c) $\lim_{x \to \infty} (e^x + x)^{3/x}$

   (d) $\displaystyle\int \frac{\sinh x}{1 + \cosh x}\, dx$

   (e) $\displaystyle\int \frac{\sin(2x)}{\cos^2(2x) + 1}\, dx$

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6. The intensity $L(x)$ of light x feet beneath the surface of the ocean satisfies the differential equation $dL/dx = kL$.

   (a) (5 points) Use the law of exponential decay to find an expression for $L(x)$ in terms of k.

   (b) (5 points) If diving to 18 ft cuts the light intensity in half, what is the rate constant k?

   (c) (5 points) Once the intensity falls below one-tenth of the surface value, you will not be able to work without artificial light. How deep can you work without artificial light?

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