

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) **your name**, (2) **1350/EXAM 3**, (3) **lecture number/instructor name** and (4) **FALL 2013** on the front of your blue-book. Also make a **grading table** with room for 5 problems and a total score. **Start each problem on a new page.** **Box** your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK**

1. (15 pts) Given that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$, $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, and $\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$,

use the limit definition of the integral, with **left** endpoints and an equally spaced partition, to find the

exact value of the definite integral $\int_0^1 x^2 dx$. Show all work.

2. (25 pts) The following parts are **not related**:

(a) Evaluate the integrals:

(i) (6 pts) $\int \frac{\sec^2(4 + 1/x^2)}{4x^3} dx$ (ii) (6 pts) $\int_{-2}^2 |x^3 + x| dx$ (iii) (6 pts) $\int_1^{\sqrt{2}} 2x^3 \sqrt{x^2 - 1} dx$

(b) (7 pts) Find $\int_0^\pi f'(x) dx$ if the antiderivative of $f(x)$ satisfies $F'(x) + \sin(x) - 4x - 1 = 0$, $F(0) = 1$.

3. (20 pts) We wish to construct a box with a square base and an open top, let x denote the length of the edge of the square base of this box and let h denote the height. If 1200 cm^2 of material is available to make this box then:

(a) (10 pts) Find the dimensions of the box that give the largest possible volume of the box.

(b) (3 pts) Justify that your answer in part (a) does indeed yield a maximum volume.

(c) (3 pts) Suppose it costs 10 cents per cm^2 to construct the base of the box and 5 cents per cm^2 to construct the sides of the box. Write down a function, in terms of x and h , that describes how much it costs (in dollars) to construct the box.

(d) (4 pts) Now, set up, **but do not solve**, an optimization problem to minimize the cost of constructing this box. Be sure to specify the mathematical function that is being optimized, specify whether this function is being minimized or maximized and specify what condition(s) need to be satisfied.

PROBLEMS #4 AND #5 ON THE OTHER SIDE

4. (20 pts) The following parts are **not related**:

(a) Let $f(x) = x \int_0^x \cos(t) dt$, then:

(i) (5 pts) Find $f'(\pi)$.

(ii) (5 pts) Show that for nonzero x , we have $\frac{f(x)}{x} - f'(x) = -x \cos x$.

(b) (10 pts) Find $g(2)$ if $\int_0^x g(t) dt = x^2(1+x)$.

5. (20 pts, 5 ea.) Answer either **Always True** or **False**. Do **NOT** justify your answer. Do **NOT** abbreviate your answer.

(a) By the property of integrals, we have $1 \leq \int_{-1}^1 \frac{dx}{1+x^2} \leq 2$.

(b) Using Riemann sums, one can show that $\lim_{n \rightarrow \infty} \frac{1}{n^5} [1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4] = \frac{1}{5}$

(c) We can show that $\int_0^2 \sqrt{4-x^2} dx - \int_0^1 \sqrt{1-x^2} dx = \frac{3\pi}{4}$

(d) Using Newtons Method, we can derive the algorithm

$$x_{n+1} = \frac{5}{6}x_n + \frac{5}{3x_n^5}, n = 1, 2, 3, \dots$$

for approximating $\sqrt[5]{10}$.

THE LIST OF APPM 1350 LECTURE NUMBERS/INSTRUCTOR NAMES FOR THE FRONT OF YOUR BLUE BOOK:

Lecture #	Time	Location	Instructor
100	8-8:50	BESC 180	Sujeet BHAT
200	9-9:50	ECCR 200	Ryan CROKE
300	12-12:50	ECCR 200	Chris KETELSEN
400	1-1:50	ECCR 1B40	Murray COX
500	2-2:50	ECCR 265	Chris KETELSEN
600	1-1:50	ANDS N103	Sujeet BHAT
700	3-3:50	FLMG 154	Ryan CROKE

— END —