Solutions

Math 1300-005 - Spring 2017

Area Between Curves - 5/3/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

1. (a) Find where the two curves $y = 2x^2 + 2x - 6$ and $y = x^2 + 3x$ intersect.

   $2x^2+2x-6 = x^2+3x \rightarrow x^2-x-6=0$
   $(x-3)(x+2)=0 \rightarrow$ so $\boxed{x=-2,\ x=3.}$

   (b) Choose a test point between your two intersection points, plug it into each curve to determine which curve is on top.

   Choose $x=0$. $2(0)^2+2(0)-6=-6$, $0^2+3(0)=0$. $0>-6$ so $\boxed{x^2+3x \text{ is top curve.}}$

   (c) Find the area enclosed by the two curves.

   $$A=\int_{-2}^{3}(x^2+3x-(2x^2+2x-6))dx=\int_{-2}^{3}(-x^2+x+6)dx$$
   $$=-\frac{1}{3}x^3+\frac{1}{2}x^2+6x\Big]_{-2}^{3} \rightarrow = \boxed{\frac{125}{6}}$$
   $$=\boxed{\left[-\frac{1}{3}(3)^3+\frac{1}{2}(3)^2+6(3)\right]-\left[-\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2+6(-2)\right]}$$

2. (a) Find where the two curves $y = x^3 - 4x^2$ and $y = 2x^2 - 5x$ intersect.

   $x^3-4x^2=2x^2-5x \rightarrow x^3-6x^2+5x=0$
   $x(x^2-6x+5)=0 \rightarrow x(x-1)(x-5)$ so $\boxed{x=0,1,5}$

   (b) Choose a test point between your first two intersection points, plug it into each curve to determine which curve is on top in this interval. Do the same for the second two intersection points.

   Choose $\frac{1}{2}$ between 0 and 1
   $(\frac{1}{2})^3-4(\frac{1}{2})^2=\frac{1}{8}-1=\frac{-7}{8}$
   $2(\frac{1}{2})^2-5(\frac{1}{2})=\frac{1}{2}-\frac{5}{2}=-2$
   $-\frac{7}{8}>-2$ so $\boxed{x^3-4x^2 \text{ on top on } [0,1]}$

   Choose 2 between 1 and 5
   $(2)^3-4(2)^2=8-16=-8$
   $2(2)^2-5(2)=8-10=-2$
   $-2>-8$ so $\boxed{2x^2-5x \text{ on top.}}$

   (c) Find the area enclosed by the two curves.

   $$A=\int_0^1\left[x^3-4x^2-(2x^2-5x)\right]dx+\int_1^5\left[2x^2-5x-(x^3-4x^2)\right]dx$$
   $$=\int_0^1\left[x^3-6x^2+5x\right]dx+\int_1^5\left[-x^3+6x^2-5x\right]dx$$
   $$=\boxed{\frac{1}{4}x^4-2x^3+\frac{5}{2}x^2\Big]_0^1+\left(-\frac{1}{4}x^4+2x^3-\frac{5}{2}x^2\right)\Big]_1^5}$$

   $\rightarrow$ you do the rest.

$x = 5 - 5y^2$ $\quad$ $x = 5y^2 - 5$

3. (a) Find where the two curves $x + 5y^2 = 5$ and $x + 5 = 5y^2$ intersect.

Since we have y^2 terms, its easier to solve for x and integrate with y

$$5 - 5y^2 = 5y^2 - 5 \rightarrow 0 = 10y^2 - 10 = 10(y^2 - 1) \rightarrow \text{so } \boxed{\text{intersection at } y = -1,\ y = 1}$$

(b) Choose a test point between your two intersection points, plug it into each curve to determine which curve is the rightmost curve.

Choose $y = 0$. $\quad 5 - 5(0)^2 = 5 \quad 5 > -5$ so

$5(0)^2 - 5 = -5 \quad \boxed{x = 5 - 5y^2 \text{ is on top}}$ ie, rightmost

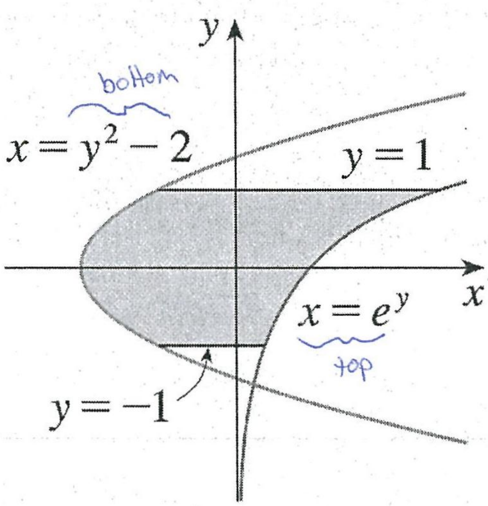
(c) Find the area enclosed by the two curves.

$$A = \int_{-1}^{1} \left[(5 - 5y^2) - (5y^2 - 5)\right] dy = \int_{-1}^{1} (10 - 10y^2)\, dy$$

$$= 10 \int_{-1}^{1} (1 - y^2)\, dy$$

$$= \boxed{10 \left(y - \tfrac{1}{3} y^3\right]_{-1}^{1} \Big)}$$

4. Find the area of the shaded region.



From the picture we see we should integrate with respect to y.

We integrate from $y = -1$ to $y = 1$ with the top (rightmost) curve as $x = e^y$. So

$$A = \int_{-1}^{1} \left[e^y - (y^2 - 2)\right] dy$$

$$= \int_{-1}^{1} \left[e^y - y^2 + 2\right] dy$$

$$= \boxed{e^y - \tfrac{1}{3} y^3 + 2y \Big]_{-1}^{1}}$$