INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/Test 2**, (3) **lecture number/instructor name** and (4) **FALL 2014** on the front of your bluebook. Also make a **grading table** with room for 4 problems and a total score. **Start each problem on a new page.** **Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK! SIMPLIFY YOUR ANSWERS AS MUCH AS POSSIBLE!**

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1. For these problems, justify and cite any theorems you have used.

   (a) (9 pts) Verify that the function $f(x) = \sqrt{x} - \frac{x}{3}$ satisfies the three hypotheses of Rolle's Theorem on the interval $[0, 9]$, then find all numbers c that satisfy the conclusion of Rolle's Theorem.

   (b) (8 pts) Let g be a differentiable odd function defined on $(-\infty, \infty)$. Show that for every positive number x there exists a number c in $(-x, x)$ such that $g(x) = xg'(c)$.

   (c) (8 pts) Find the linearization of $h(x) = \sqrt[3]{1+3x}$ at $a = 0$ and use it to give an approximate value for $\sqrt[3]{1.03}$.

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2. Some useful formulas: Volume of a sphere, $V = \frac{4}{3}\pi r^3$, and surface area of a sphere, $SA = 4\pi r^2$, where r = radius.

   (a)(10 pts) Consider a spherical raindrop falling through a layer of dry air. As the raindrop falls, it starts to evaporate and the volume decreases. Suppose the volume decreases at a rate of 1 mm^3/s, find the rate at which the diameter of the raindrop decreases when the radius is 5 mm.

   (b)(10 pts) The radius of the spherical raindrop is initially measured to be 6 mm, with a possible error of ± 0.01 mm. Approximate the maximum possible percentage error in calculating the surface area of the sphere.

   (c)(5 pts) Now consider the volume of the spherical raindrop from part (a) as the radius changes from $r = a$ to $r = b$ (where a and b are some real numbers, $0 < a < b$). (*i*) Prove that the average rate of change of the volume (with respect to the radius) as the radius changes from $r = a$ to $r = b$ is equal to the surface area of the raindrop with radius equal to some number c where $a < c < b$. (*ii*) Find c in terms of a and b. Justify your answers and cite any theorems you have used. $\big($*Hint:* $x^3 - y^3 = (x-y)(x^2+xy+y^2)\big)$

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3. (a)(9 pts) Find the local and absolute extreme values of $f(x) = \dfrac{3x-4}{x^2+1}$ on $[-2, 2]$. Justify your answer.

   (b)(9 pts) Find all local maximum and minimum values of $g(x) = x\sqrt{6-x}$. Justify your answer.

   (c)(7 pts) In your blue book clearly sketch the graph of a function $h(x)$ that satisfies the following properties (label any extrema, inflection points or asymptotes):

   - $h(0) = 0$, $h'(-2) = h'(1) = h'(9) = 0$
   - $\lim_{x\to\infty} h(x) = 0$, $\lim_{x\to 6} h(x) = -\infty$
   - $h'(x) < 0$ on the intervals $(-\infty, -2)$, $(1, 6)$ and $(9, \infty)$
   - $h'(x) > 0$ on the intervals $(-2, 1)$ and $(6, 9)$
   - $h''(x) > 0$ on the intervals $(-\infty, 0)$ and $(12, \infty)$
   - $h''(x) < 0$ on the intervals $(0, 6)$ and $(6, 12)$

PROBLEM #4 ON THE OTHER SIDE

4. (a)(8 pts) The curve $x^3y + y^3 = 1$ defines y implicitly as one or more functions of x. Find the slope of this curve at the point $(0, 1)$. Show all work.

   (b)(8 pts) Given that p is a differentiable function for all t, find $q'(t)$ in terms of $p'(t)$ if $q(t) = \dfrac{p(p(t))}{\sqrt{t}}$, for $t > 0$.

   (c)(9 pts) A function f and its first two derivatives have values as shown in Table 1 below:

Table 1

x	$f(x)$	$f'(x)$	$f''(x)$
0	0	1	2
1	1	1	1
2	3	2	1
4	6	3	0

Table 2: **Copy this table in your bluebook**

x	$g'(x)$	$g''(x)$
0		
1		
2		

Let $g(x) = xf(x^2)$. Copy the table for $g'(x)$ and $g''(x)$ above in your bluebook and fill it in with *fully simplified values*. Only the final answers in your table in your bluebook will be graded – no partial credit will be given.

THE LIST OF APPM 1350 LECTURE NUMBERS/INSTRUCTOR NAMES FOR THE FRONT OF YOUR BLUE BOOK:

Lecture #	Instructor	Class Time	Location
110	Ryan CROKE	MWF 8-8:50	BESC 180
120	Ryan CROKE	MWF 9-9:50	ECCR 200
130	Murray COX	MWF 10-10:50	ECCR 245
150	Sujeet BHAT	MWF 12-12:50	ECCR 200
160	James CURRY	MWF 1-1:50	ECCR 1B40
170	Sujeet BHAT	MWF 2-2:50	ECCR 265
180	Jonathan KISH	MWF 3-3:50	EKLC 1B20
594R	Jonathan KISH	MWF 1-1:50	ANDS N103

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