Print Name ________________________________

APPM 1350 **Exam 3** **Summer 2016**

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

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1. The following parts are not related.
   (a) (12 pts) A company wants to make a square-bottomed, open-top, cake pan that holds a volume of 108 in^3 while minimizing construction materials. Find the dimensions of the pan that minimize the materials used to make the cake pan. Justify your answer.
   (b) (9 pts) An astronaut on Mars drops a ball from a 50 meter high cliff. Given that the constant of acceleration on Mars is 4 meters per second squared, find both the time it takes for the ball to hit the ground and the velocity when the ball hits the ground. Show all work to justify your answer.
2. Suppose an object, traveling in a straight line, has a velocity of $v(t) = 3t^2 - 6t$, find the following.
   (a) (5 pts) Find the net distance traveled (i.e. displacement) by the object for $0 \le t \le 3$. Show all work to justify your answer.
   (b) (7 pts) Find the total distance traveled by the object for $0 \le t \le 3$. Show all work to justify your answer.
3. Using right end points, a definite integral is approximated by the Riemann sum:

$$\sum_{i=1}^{n} \left[2\left(\frac{3i}{n}\right)^2 - \frac{3i}{n}\right]\frac{3}{n}$$

   (a) (6 pts) Find a definite integral representing this Riemann sum. Show all work to justify your answer.
   (b) (6 pts) Evaluate $\sum_{i=1}^{n} \left[2\left(\frac{3i}{n}\right)^2 - \frac{3i}{n}\right]\frac{3}{n}$, in other words, find the sum in terms of n. Show all work to justify your answer.
   (c) (6 pts) Find the value of $\lim_{n\to\infty} \sum_{i=1}^{n} \left[2\left(\frac{3i}{n}\right)^2 - \frac{3i}{n}\right]\frac{3}{n}$. Show all work to justify your answer.

   Formulas: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

TWO MORE ON THE OTHER SIDE

4. The following are unrelated. For (a) find the general indefinite integral. For (b) and (c) evaluate the integral. For (d) compute the derivative of the integral. Show all work to justify your answer and make sure to simplify as much as possible.

   (a) (9 pts) $\displaystyle\int \left(\sec x \tan x + \frac{-\cos x \sin^2 x + \cos x}{\cos^2 x} \right) dx$

   (b) (9 pts) $\displaystyle\int_0^3 \sqrt{9 - x^2}\, dx$

   (c) (10 pts) $\displaystyle\int_0^6 \left(|t-2| - \frac{3}{2}\sqrt{t} \right) dt$

   (d) (9 pts) $\displaystyle\frac{d}{dx} \int_a^{\sqrt{x}} (t + t\sin^2 t)^3\, dt$ where a is a constant.

5. The following are unrelated. Clearly state "Always True" or "False" for each statement. No justification is required and no partial credit will be given.

   (a) (4 pts) $\displaystyle\int_{x^3}^0 f(t)\, dt - \int_{x^3}^x f(t)\, dt = -\int_0^x f(t)\, dt$ for any real number x and f a continuous function on the real numbers.

   (b) (4 pts) For a velocity function $v(t)$ continuous on an interval $[a, b]$ such that $v(t) < 0$ on $[a, b]$ then $\displaystyle\int_a^b |v(t)|\, dt = \left| \int_a^b v(t)\, dt \right|$.

   (c) (4 pts) $\displaystyle\int_0^a 3x^n\, dx = \frac{3a^{(n+1)}}{n+1} - \frac{3(0)^{(n+1)}}{n+1} = \frac{3a^{(n+1)}}{n+1}$ by the evaluation theorem where n is a real number such that $n \neq -1$ and a is any positive real number.

$\boxed{\text{END}}$