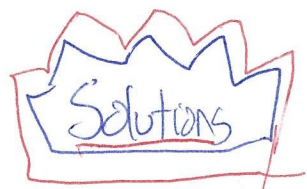


Math 1300-005 - Spring 2017
The Intermediate Value Theorem - 1/31/17



Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval. Remember, in solving each problem, you must verify that each of the hypotheses of the IVT are satisfied.

(a) $x^4 + x - 3 = 0$, $(1, 2)$

Let $f(x) = x^4 + x - 3$, which is continuous on $[1, 2]$ b/c it is a polynomial.

Since $f(1) = 1 + 1 - 3 < 0$, 0 is between $f(1)$ & $f(2)$, so by IVT
 $f(2) = 16 + 2 - 3 > 0$

There exists c in $(1, 2)$ such that $f(c) = 0$, i.e., $c^4 + c - 3 = 0$.

(b) $\sqrt[3]{x} = 1 - x$, $(0, 1)$

Rearrange first as $\sqrt[3]{x} + x - 1 = 0$. Let $f(x) = \sqrt[3]{x} + x - 1$, which is cont. on $[0, 1]$
b/c it is a root function plus a polynomial. Since

$f(0) = -1 < 0$, 0 is between $f(0)$ and $f(1)$. By the IVT
 $f(1) = 1 > 0$

There exists c in $(0, 1)$ such that $f(c) = 0$, i.e., $\sqrt[3]{c} + c - 1 = 0$.

(c) $e^x = 3 - 2x$, $(0, 1)$

Rearrange first as $e^x + 2x - 3 = 0$. Let $f(x) = e^x + 2x - 3$, which is cont. on $[0, 1]$ b/c it is an exponential function plus a polynomial. Since

$$f(0) = e^0 + 2(0) - 3 = 1 - 3 < 0$$

$$f(1) = e + 2 - 3 = e - 1 > 0 \text{ (since } e \approx 2.7), 0 \text{ is between } f(0) \text{ and } f(1). \text{ By}$$

the IVT, there exists c in $(0, 1)$ such that $f(c) = 0$, i.e., $e^c + 2c - 3 = 0$.

(d) $\sin(x) = x^2 - x$, $(1, 2)$

To get that $\sin(1) > 0$, note 1 is between 0 and $\frac{\pi}{2}$ and $\sin(x) > 0$ on $(0, \frac{\pi}{2})$ [draw a graph]. Rearrange first as $\sin(x) + x - x^2 = 0$. Let $f(x) = \sin(x) + x - x^2$, which is continuous on $[1, 2]$ b/c it is a trig function plus a polynomial. Since

$$f(1) = \sin(1) + 1 - (1)^2 = \sin(1) > 0$$

$$f(2) = \sin(2) + 2 - (2)^2 = \sin(2) - 2 < 0, 0 \text{ is between } f(1) \text{ and } f(2). \text{ By}$$

To get $\sin(2) - 2 < 0$, note

$-1 \leq \sin(2) \leq 1$, so at most $\sin(2) = 1$, and $1 < 2$, so $\sin(2) - 2 < 0$

the IVT, there exists c in $(1, 2)$ such that $f(c) = 0$, i.e., $\sin(c) + c - c^2 = 0$.

The following problems are review of the material we covered Monday 1/30 over the definition of continuity.

2. State the interval(s) where the following function is continuous.

$$f(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$$

We need only check continuity at $a=0$.

① $f(0)$ is defined and $f(0)=0$.

$$\begin{aligned} \textcircled{2} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \cos(x) = \cos(0) = 1 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} 1 - x^2 = 1 - (0)^2 = 1 \end{aligned} \quad \rightarrow \text{so } \lim_{x \rightarrow 0} f(x) = 1.$$

③ Since $\lim_{x \rightarrow 0} f(x) \neq f(0)$, f is not continuous at 0. Hence

$$f \text{ is continuous on } (-\infty, 0) \cup (0, \infty)$$

3. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^{2x+c} & \text{if } x \geq 0 \end{cases}$$

We need only check continuity at $a=0$.

① $f(0)$ is defined and $f(0) = e^{2(0)+c} = e^c$

$$\begin{aligned} \textcircled{2} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (x+2) = 2 \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} e^{2x+c} = e^c \end{aligned}$$

We need ~~RHL = LHL~~ $RHL = LHL = f(0)$, so we need

$$2 = e^c \quad \text{or}$$

$$c = \ln(2)$$