MATH 1300: HW #12

Due on April 13, 2017 at 10:00am

Professor Braden Balentine Section 005

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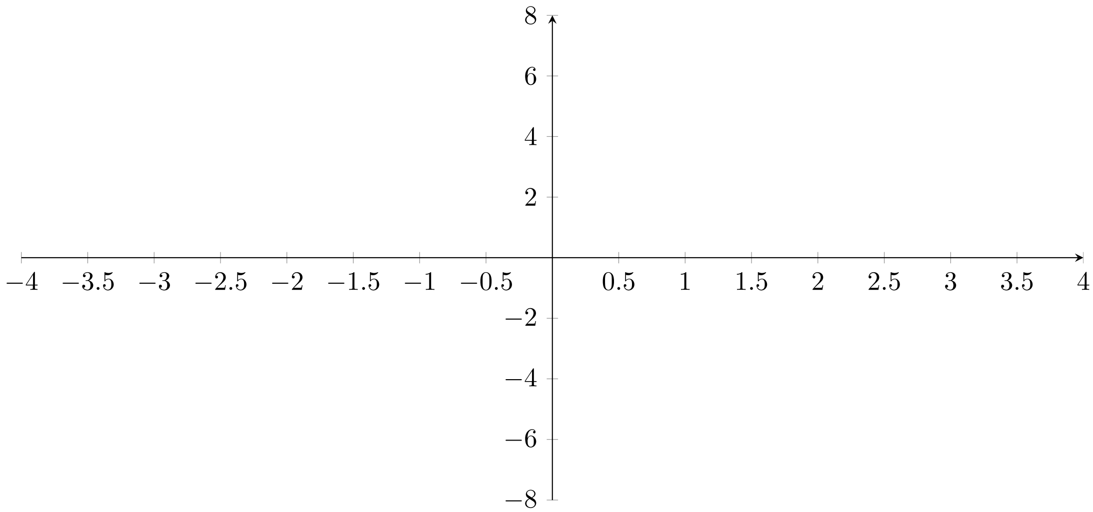
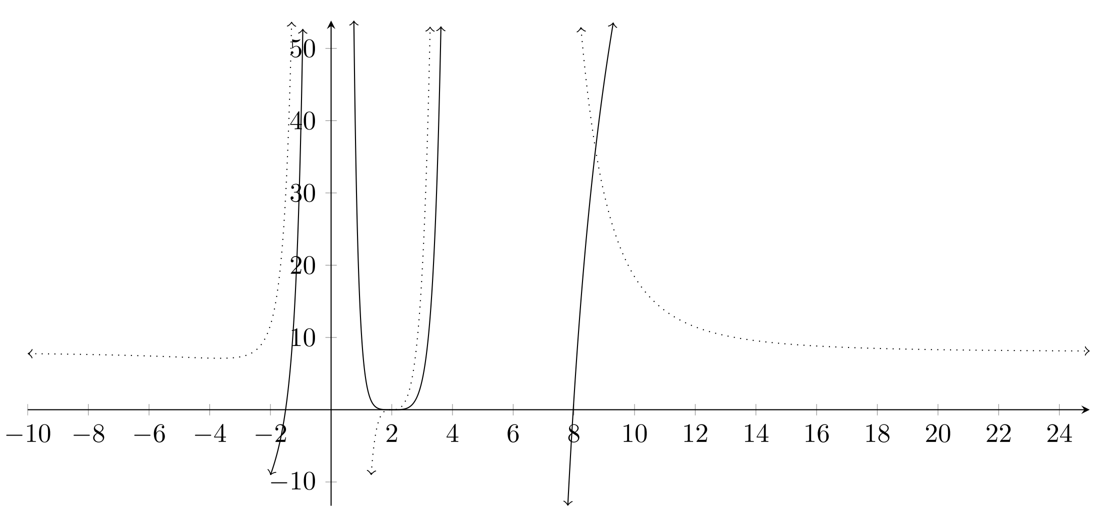
Section 4.4

12. Sketch the graph by hand using asymptotes and intercepts, but not derivatives. Then use your sketch as a guide to producing graphs (with a graphing device) that display the major features of the curve. Use these graphs to estimate the maximum and mini- mum values.

$$f(x) = \frac{(2x+3)^2(x-2)^5}{x^3(x-5)^2}$$



14. If f is the function of Exercise 12, find f' and f'' and use their graphs to estimate the intervals of increase and decrease and concavity of f.
Include graphs of f' and f'' to fully explain the behavior of f. Use technology to calculate these derivatives.
$f'(x) = \frac{2(-2+x)^4(-135-120x-62x^2-22x^3+4x^4)}{(-5+x)^3x^4}$
$f''(x) = \frac{2(-2+x)^3(5400+1710x+805x^2+460x^3+216x^4-56x^5+4x^6)}{(-5+x)^4x^5}$



Section 4.6

14. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of the base it twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

$$\begin{aligned}
Volume: \quad & V = Lwh \\
& 10 = 2w \cdot w \cdot h \\
& h = \frac{5}{w^2} \\
Cost: \quad & C = 10(Lw) + 2(6(hw)) + 2(6(hL)) \\
& C' = 40w - 180w^{-2} \\
Critical\ Points: \quad & w = 1.65\ m \\
& L = 3.3\ m \\
& h = 1.84\ m \\
Cheapest\ Cost: \quad & 10(3.3)(1.65) + 2(6(1.84)(1.65)) + 2(6(1.84)(3.3)) = \$165.75
\end{aligned}$$

34. A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat is headed due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

$$\begin{aligned}
\text{South boat:}20\ km/hr \text{ with a distance of } x &= 20t \\
\text{Other boat:}15\ km/hr \text{ with a distance of } y &= 10(1-t) \\
\text{Distance } = d^2 &= (15(1-t))^2 + (20t)^2 \\
D &= 2(15)2(1t) + 2(20)2t \\
t &= \frac{2(15)^2}{2(15^2+20^2)} = \boxed{\frac{9}{20}}
\end{aligned}$$

48. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager change to maximize revenue?

$$\begin{aligned}
p(x) &= -10x^1 800 \\
R(x) &= -10x^2 + 1800x \\
R'(x) &= -20 + 1800x \\
0 &= -20 + 1800x \\
x &= 90 \\
R(0) &= 0 \\
R(90) &= 81,000 \\
r(100) &= 80,000
\end{aligned}$$

The revenue is maximized when the manager rents 90 units, or charges \$900 per month.

22. A cylindrical can without a top is made to contain 2,000 cm^3 of liquid. Find the dimensions that will

minimize the cost of the metal to make the can.

$$\begin{aligned}
h &= \frac{V}{\pi r^2} \\
s(r) &= \frac{2V}{r} + \pi r^2 \\
s'(r) &= -\frac{2V}{r^2} + 2\pi r \\
0 &= -\frac{2V}{r^2} + 2\pi r \\
r &= \sqrt[3]{\frac{V}{\pi}} \\
r &= \boxed{\sqrt[3]{\frac{2000}{\pi}}}
\end{aligned}$$

Section 4.8

16. Find the most general antiderivative of the function (Check your answer by differentiation.)

$$\begin{aligned}
f(x) &= \frac{2 + x^2}{1 + x^2} \\
&= x + \tan^{-1}(x) + C
\end{aligned}$$

46. Two balls are thrown upward from the edge of the cliff in Example 6. The first is thrown with a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?

$$\begin{aligned}
-16t^2 + 16t + 464 &= -16t^2 + 24t + 432 \\
t &= 4
\end{aligned}$$

The first ball is traveling faster, and will pass the second ball, hitting the ground first.

Required Problem

Batman was driving the Batmobile at 90 mph (132 ft/sec), when he sees a brick wall directly ahead. When the Batmobile is 400 feet from the wall, he slams on the brakes, decelerating at a constant rate of 22ft/sec^2. Does he stop before he hits the brick wall? If so, how many feet to spare? If not, what is his impact speed? Now the Joker had been driving next to him, also at 90 mph. But the Joker did not hit his brakes as soon as Batman, continuing for 1 second longer than Batman before hitting his brakes, decelerating at a constant rate of 22ft/sec^2. How fast is he going when he hits the wall? (Don't worry about Joker - he jettisoned at the last instant, to fight for another day!)

$$\begin{aligned}
v(t) &= 132 - 22t \\
0 &= 132 - 22t \\
t &= 6 \\
s(t) &= 132t - 11t^2 \\
&= 132(6) - 11(36) = 396
\end{aligned}$$

The batmobile stops after 396 feet, which is 4 feet short of the wall.