Solutions

Math 1300-005 - Spring 2017

Product & Quotient Rule - 2/20/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the next midterm.

1. Recall the product rule:

$$\frac{d}{dx}(u(x)v(x)) = u'(x)v(x) + u(x)v'(x).$$

Use it to differentiate the following. **You do not need to simplify your answers.**

(a) $f(x) = (x^3 + 2x)e^x$

$u(x) = (x^3+2x)$ $v(x) = e^x$

$u'(x) = 3x^2+2$ $v'(x) = e^x$

$\boxed{f'(x) = (3x^2+2)e^x + (x^3+2x)e^x}$

(b) $g(x) = \sqrt{x}e^x$

$u(x) = x^{1/2}$ $v(x) = e^x$

$u'(x) = \frac{1}{2}x^{-1/2}$ $v'(x) = e^x$

$\boxed{g'(x) = \left(\frac{1}{2}x^{-1/2}\right)e^x + x^{1/2} \cdot e^x}$

(c) $R(t) = (t + e^t)(3 - \sqrt{t})$

$u(t) = t + e^t$ $v(t) = 3 - t^{\frac{1}{2}}$

$u'(t) = 1 + e^t$ $v'(t) = -\frac{1}{2}t^{-1/2}$

$\boxed{R'(t) = (1+e^t)(3-\sqrt{t}) + (t+e^t)\left(-\frac{1}{2}t^{-1/2}\right)}$

(d) $F(y) = \underbrace{\left(\frac{1}{y^2} - \frac{3}{y^4}\right)}^{(y^{-2}-3y^{-4})}(y + 5y^3)$

$u(y) = (y^{-2} - 3y^{-4})$ $v(y) = y + 5y^3$

$u'(y) = (-2y^{-3} + 12y^{-5})$ $v'(y) = 1 + 15y^2$

$\boxed{F'(y) = (12y^{-5} - 2y^{-3})(y + 5y^3) + (y^{-2} - 3y^{-4}) \cdot (1 + 15y^2)}$

2. Find $f'(x)$ and $f''(x)$ for $f(x) = x^4e^x$.

$\boxed{f'(x) = \underbrace{4x^3 \cdot e^x}_{PR} + \underbrace{x^4 \cdot e^x}_{PR}}$

$\boxed{f''(x) = \left[12x^2 \cdot e^x + 4x^3 \cdot e^x\right] + \left[4x^3 \cdot e^x + x^4 \cdot e^x\right]}$

3. Recall the quotient rule:

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}.$$

Use it to differentiate the following. **You do not need to simplify your answers.**

(a) $f(x) = \dfrac{e^x}{x^2}$

$u(x) = e^x \quad v(x) = x^2$

$u'(x) = e^x \quad v'(x) = 2x$

$$\boxed{f'(x) = \frac{e^x \cdot x^2 - e^x(2x)}{(x^2)^2}}$$

(b) $g(t) = \dfrac{2t}{4+t^2}$

$u(t) = 2t \quad v(t) = 4+t^2$

$u'(t) = 2 \quad v'(t) = 2t$

$$\boxed{g'(t) = \frac{2(4+t^2) - 2t(2t)}{(4+t^2)^2}}$$

(c) $y = \dfrac{v^3 - 2v\sqrt{v}}{v+1} = \dfrac{v^3 - 2v^{3/2}}{v+1}$

$$\boxed{\frac{dy}{dv} = \frac{(3v^2 - 3v^{1/2})(v+1) - (v^3 - 2v^{3/2})(1)}{(v+1)^2}}$$

(d) $h(x) = \dfrac{1 - xe^x}{x + e^x}$ PR!

$u(x) = 1 - xe^x \quad v(x) = x + e^x$

$u'(x) = -(e^x + xe^x) \quad v'(x) = 1 + e^x$

$$\boxed{h'(x) = \frac{-(e^x + xe^x)(x+e^x) - (1 - xe^x)(1+e^x)}{(x+e^x)^2}}$$

4. Find an equation of the tangent line to $y = \dfrac{\sqrt{x}}{x+1}$ at $x = 4$. .

$y(4) = \dfrac{\sqrt{4}}{4+1} = \dfrac{2}{5}$

$\dfrac{dy}{dx} = \dfrac{\frac{1}{2}x^{-1/2}(x+1) - \sqrt{x}(1)}{(x+1)^2} \xrightarrow[4]{\text{Plug in}}$ slope $= \dfrac{\frac{1}{2}(4)^{-1/2}(4+1) - \sqrt{4}}{(4+1)^2}$

$= \dfrac{\frac{5}{4} - 2}{25} = \dfrac{-3/4}{25} = \dfrac{-3}{100}$

$$\boxed{y - \frac{2}{5} = \frac{-3}{100}(x-4)}$$

5. A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	-6	5	7
2	-1	2	-3	-4
3	0	1	6	3

(a) If $H(x) = f(x)g(x)$, find $H'(1)$.

$$H'(1) = f'(1)g(1) + f(1)g'(1)$$
$$= 5(-6) + 4(7) = \boxed{-2}$$

(b) If $L(x) = \dfrac{f(x)}{g(x)}$, find $L'(2)$.

$$L'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2} = \frac{(-3)(2) - (-1)(-4)}{(2)^2}$$
$$= \frac{-6-4}{4} = \boxed{\frac{-5}{2}}$$

(c) If $B(x) = [x + f(x)]g(x)$, find $B'(1)$.

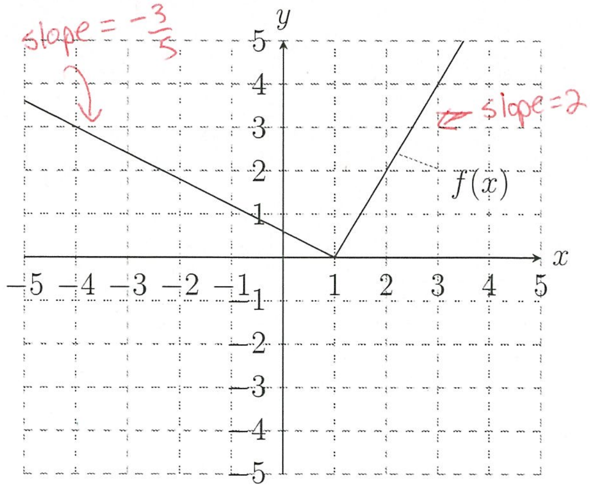
$$B'(x) = [1 + f'(x)]g(x) + [x + f(x)]g'(x)$$
$$\hookrightarrow B'(1) = [1 + f'(1)]g(1) + [1 + f(1)]g'(1)$$
$$= [1 + 5](-6) + [1 + 4](7)$$
$$= -36 + 35 = \boxed{-1}$$

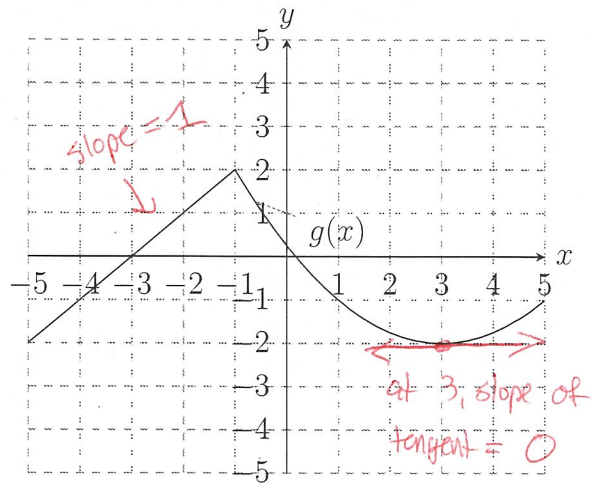
(d) If $F(x) = \dfrac{g(x)}{f(x)}$, find $F'(3)$.

$$F'(3) = \frac{g'(3)f(3) - g(3)f'(3)}{(f(3))^2}, \text{ but } f(3) = 0$$

so $\boxed{F'(3) \text{ DNE}}$

6. Consider the piecewise functions f and g whose graphs are shown below.





(a) If $P(x) = f(x)g(x)$, find $P'(-4)$.

$$P'(-4) = f'(-4)g(-4) + f(-4)g'(-4)$$

$$= -\frac{3}{5}(-1) + 3(1) = \frac{3}{5} + 3 = \boxed{\frac{18}{5}}$$

(b) If $Q(x) = \dfrac{f(x)}{g(x)}$, find $Q'(3)$.

$$Q'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{(g(3))^2} = \frac{2(-2) - 4(0)}{(-2)^2} = \frac{-4}{4} = \boxed{-1}$$

(c) If $C(x) = \dfrac{g(x)}{f(x)}$, find $C'(-1)$.

$$C'(-1) = \frac{g'(-1)f(-1) - g(-1)f'(-1)}{(f(-1))^2}$$, but $g'(-1)$ DNE b/c of the cusp,

so $\boxed{C'(-1) \text{ DNE}}$

(d) If $N(x) = \underbrace{x^2 f(x)}$, find $N'(2)$. Product!

$$N'(x) = 2x f(x) + x^2 f'(x)$$

$$N'(2) = 2(2)f(2) + (2)^2 f'(2)$$

$$= 4(2) + 4(2) = \boxed{16}$$