Solutions

# Math 1300-005 - Spring 2017

Quiz 15/Final Review - 5/5/17

*Guidelines:* This quiz serves as a review for the final exam, covering material from the third midterm onwards. If you either a) stay the full class time working on the review, or b) complete the review and show me (the work must be good if not absolutely correct), then you recieve 7 out of 7 for today's quiz grade.

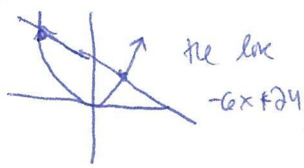
1. Find the area of the region enclosed by the graphs of $y = 3x^2$ and $y = -6x + 24$.

① Intersection: $3x^2 = -6x + 24 \to 3x^2 + 6x - 24 = 0$

$3(x^2 + 2x - 8) = 0$

so $x = -2, 4$. $\quad 3(x-2)(x+4)$.

② which curve on top?




or

choose 0 as test point

$3(0)^2 = 0$

$-6(0) + 24 = 24 \to y = -6x + 24$ on top

$$A = \int_{-2}^{4} [(-6x+24) - 3x^2]\,dx = \int_{-2}^{4} (-3x^2 - 6x + 24)\,dx = \boxed{-x^3 - 3x^2 + 24x \Big]_{-2}^{4}}$$

2. (a) Compute $\lim_{x \to 2\pi} \dfrac{\int_{2\pi}^{x} \sqrt{10 - \sec(t)}\, dt}{x - 2\pi}$ $= \lim_{x \to 2\pi} \dfrac{\int_0^x \sqrt{10-\sec(t)}\,dt - \int_0^{2\pi} \sqrt{10-\sec(t)}\,dt}{x - 2\pi}$

$= F'(2\pi)$, where $F(x) = \int_0^x \sqrt{10 - \sec(t)}\, dt$.

So $F'(x) = \sqrt{10 - \sec(x)}$ by FTC, Hence

$F'(2\pi) = \sqrt{10 - \sec(2\pi)} = \sqrt{9} = \boxed{3}$

note: should be 0

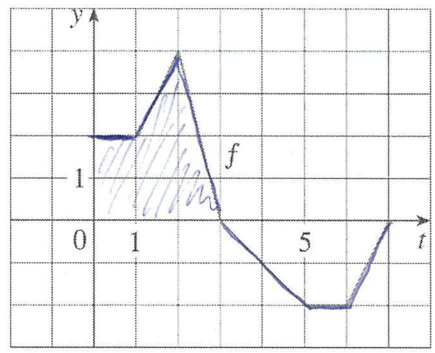
(b) Compute $\lim_{h \to 0} \dfrac{\int_0^{7+h} \arctan(t)\, dt - \int_0^7 \arctan(t)\, dt}{h}$

$= \lim_{h \to 0} \dfrac{F(7+h) - F(7)}{h}$ where $F(x) = \int_0^x \arctan(t)\,dt$.

$= F'(7)$. Since $F'(x) = \arctan(x)$ by FTC,

the original limit equals $\boxed{\arctan(7)}$

3. Suppose a bug starts walking along the top of a fence and its velocity is given by the curve below, where $t$ is measured in minutes and $v = f(t)$ is measured in meters per minute.




(a) At what time is the bug farthest from where it started? How far away from its starting point is the bug at this time?

$t = 3$ minutes. Distance $= \int_0^3 f(t)\,dt = \boxed{7 \text{ meters.}}$

(b) On what interval(s) is the bug heading back towards where it started?

We want where $v = f(t) < 0$. So $\boxed{(3,7).}$

(c) What is the change in the bug's displacement from minute 1 to minute 5?

Displacement $= S(5) - S(1) = \int_1^5 f(t)\,dt =$

$= 5 - 2 = \boxed{3 \text{ meters}}$

(d) What is the total distance travelled by the bug over the full 7 minutes?.

Distance $= \int_0^7 |f(t)|\,dt = \int_0^3 f(t)\,dt - \int_3^7 f(t)\,dt = 7 - (-5) = \boxed{12 \text{ meters}}$ since $v = f(t)$

4. True or false? Explain your answer and include a picture.

$$\frac{d}{dx}\left(\int_0^4 \sqrt{t}\,dt\right) = \sqrt{x}$$

This is false. $\int_0^4 \sqrt{t}\,dt =$  this area, which is a number.

So $\boxed{\frac{d}{dx}\left(\int_0^4 \sqrt{t}\,dt\right) = 0.}$

5. Below is a table representing the speed of a car in $ft/s$ during the first 30 seconds of a race.

| Time (s) | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| Velocity (ft/s) | 25 | 31 | 35 | 43 | 47 | 46 | 41 |

Using a Riemann sum with 6 subintervals and taking the sample points to be left endpoints, approximate the distance the car traveled over 30 seconds. Include units in your final answer. $\Delta t = 5$ sec. Using left end points

$$\boxed{L_6 = 5(25+31+35+43+47+46) \text{ ft.}}$$

6. Write down a definite integral that represents the following limit of a Riemann sum:

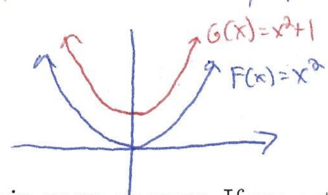
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(2+\frac{3i}{n}\right)^2. = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \, f(x_i).$$

Several correct answers! $\Delta x = \frac{3}{n} = \frac{b-a}{n}$. Since $x_i = a + i\Delta x$, we can take $a=2$, so $b=5$. Since $x_i$ is being squared, $f(x)=x^2$.

So $\boxed{\int_2^5 x^2\,dx}$ Could also say $a=0$, $b=3$, $f(x)=(2+x)^2 \to \int_0^3 (2+x)^2\,dx$.

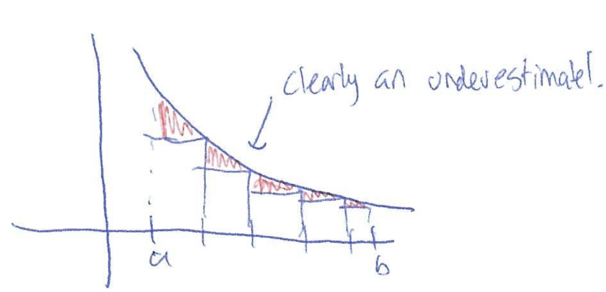
7. True or false? Explain your answer and include a picture. If $F'(x) = 2x$ and $G'(x) = 2x$, then $F(x) = G(x)$.

False. If $F(x)=x^2$, $G(x)=x^2+1$, then $F'(x)=2x$, $G'(x)=2x$, but $F(x) \neq G(x)$.




8. True or False? Please explain your answer: If we estimate the area under a curve using right endpoints, then our approximation will be an overestimate.

False. Choose a decreasing function.



9. Find the most general antiderivative.

(a) $\int \frac{e^x}{1+e^x}\,dx$ $\quad u=1+e^x,\ du=e^x dx$

back sub

$$= \int \underbrace{\frac{1}{1+e^x}}_{\frac{1}{u}} \underbrace{e^x dx}_{du} = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|1+e^x| + C}$$

10. Evaluate the definite integral.

(a) $\cancel{\int_{-\pi/6}^{\pi/6} \cot(x)\,dx}$ → bad example, so sorry. Cot(x) is not continuous on $\left[\frac{\pi}{6}, \frac{\pi}{6}\right]$ since cot(0) DNE (vertical asymptote)

Try instead $\int_{-\pi/6}^{\pi/6} \tan(x)\,dx$.

1st method: tan(x) is odd so $\boxed{\int_{-\pi/6}^{\pi/6} \tan(x)\,dx = 0}$

2nd method: substitution.

$$\int_{-\pi/6}^{\pi/6} \tan(x)dx = \int_{-\pi/6}^{\pi/6} \frac{\sin(x)}{\cos(x)}\,dx. \quad u=\cos(x),\ du=-\sin(x) \rightarrow = \int_{\cos(-\pi/6)}^{\cos(\pi/6)} \frac{1}{u}(-du) = -\int_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{u}\,du = \boxed{0}$$

(same upper & lower bound)

(b) $\int_0^{\sqrt{\pi/4}} x\sec^2(x^2)\,dx = \int_0^{\sqrt{\pi/4}} \underbrace{\sec^2(x^2)}_{\sec^2(u)} \underbrace{x dx}_{\frac{1}{2}du}$

Set $u = x^2$

$du = 2x\,dx$

$$= \int_{0^2}^{(\sqrt{\pi/4})^2} \sec^2(u)\left(\tfrac{1}{2}du\right) = \frac{1}{2}\int_0^{\pi/4} \sec^2(u)\,du = \frac{1}{2}\tan(u)\Big]_0^{\pi/4} = \frac{1}{2}\tan(\pi/4)$$

$$= \boxed{\frac{1}{2}}$$

11. Find the first and second derivative of

$$F(x) = \int_x^{-5} \sqrt{1-e^{t^2}}\,dt. = -\int_{-5}^{x} \sqrt{1-e^{t^2}}\,dt.$$

So by FTC, $\boxed{F'(x) = -\sqrt{1-e^{x^2}}}$

By chain rule, $F''(x) = \frac{-1}{2\sqrt{1-e^{x^2}}} \cdot \frac{d}{dx}(1-e^{x^2}) = \frac{-1}{2\sqrt{1-e^{x^2}}}(-e^{x^2}\cdot 2x) = \boxed{\frac{2xe^{x^2}}{2\sqrt{1-e^{x^2}}}}$

12. Find the derivative of

$$G(x) = \int_{x^3}^{x^4} \arccos(t)\,dt.$$

$$= \boxed{\arccos(x^4)\cdot 4x^3 - \arccos(x^3)\cdot 3x^2}$$