MATH 1300: HW #1

Due on January 19, 2017 at 10:00am

Professor Braden Balentine Section 005

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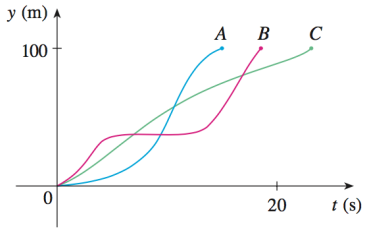
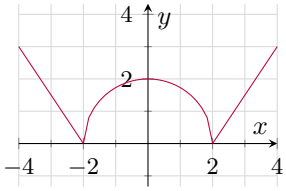
Section 1.1

12. Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race?



The winner of the race is runner A, with runners B and C following closely behind. All three runners did finish the race, which can be determined by all three runners stopping at the same maximum distance (100m).

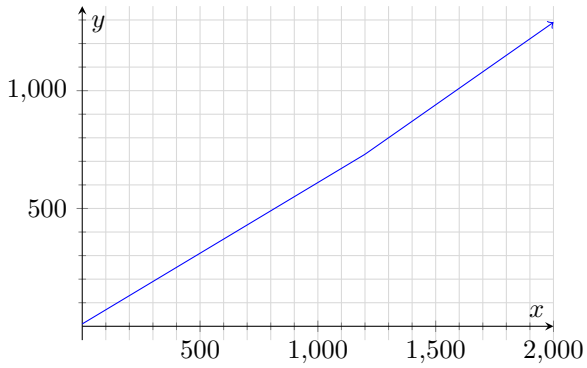
52. Find an expression for the function whose graph is the given curve:



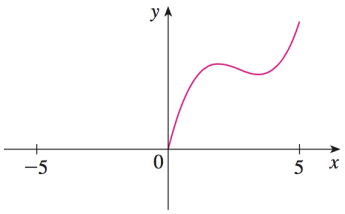
$$f(x) = \begin{cases} -\frac{3}{2}x - 3, & -4 \le x < -2 \\ \sqrt{2^2 - x^2}, & -2 \le x \le 2 \\ \frac{3}{2}x - 3, & 2 < x \le 4. \end{cases}$$

60. An electricity company charges its customers a base rate of \$10 a month, plus 6 cents per kilowatt-hour (kWh) for the first 1200 kWh and 7 cents per kWh for all usage over 1200 kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 \le x \le 2000$.

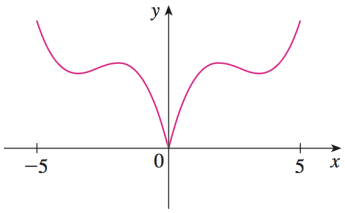
$$E(x) = \begin{cases} 10 + 0.6x, & 0 \le x \le 1200 \\ 10 + 0.6 \times 1200 + 0.07(x - 1200), & x > 1200. \end{cases}$$



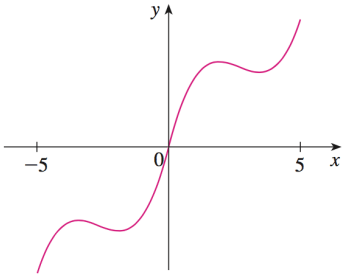
66. A function f has domain [-5,5] and a portion of its graph is shown.



(a) Complete the graph of f if it is known that f is even.



(b) Complete the graph of f if it is known that f is odd.



74. If f and g are both even functions, is the product fg even? If f and g are both odd functions, is fg odd? What if f is even and g is odd? Justify your answers.

$f(x)$ (even)		$g(x)$ (even)		$f \cdot g$ (even)	
-2	3	-2	10	-2	30
-1	1	-1	5	-1	5
0	0	0	0	0	0
1	1	1	5	1	5
2	3	2	10	2	30

$f(x)$ (even)		$g(x)$ (odd)		$f \cdot g$ (odd)	
-2	3	-2	-10	-2	-30
-1	1	-1	-5	-1	-5
0	0	0	0	0	0
1	1	1	5	1	5
2	3	2	10	2	30

$f(x)$ (odd)		$g(x)$ (odd)		$f \cdot g$ (even)	
-2	-3	-2	-10	-2	30
-1	-1	-1	-5	-1	5
0	0	0	0	0	0
1	1	1	5	1	5
2	3	2	10	2	30

As demonstrated by the three tables above, when $f(x)$ and $g(x)$ are both even, $f \cdot g$ is even. When $f(x)$ is even and $g(x)$ is odd, $f \cdot g$ is odd. When $f(x)$ and $g(x)$ are both odd, $f \cdot g$ is even.

Section 1.2

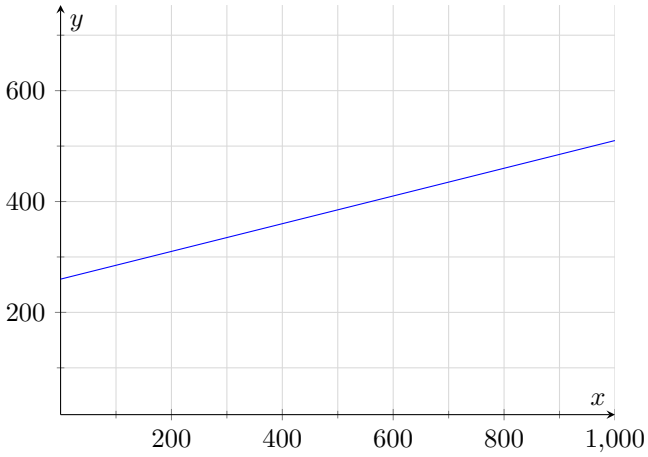
18. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May it cost her \$380 to drive 480 mi and in June it cost her \$460 to drive 800 mi.

(a) Express the monthly cost C as a function of the distance driven d, assuming that a linear relationship gives a suitable model.

- Calculating the slope:
$$\frac{460-380}{800-480}=\frac{80}{320}=\frac{1}{4}$$
- Point-slope formula:
$$C-380=\frac{1}{4}(d-180)=\frac{d}{4}-120$$
- Simplifying:
$$C(d)=\frac{d}{4}+260$$

(b) Use part (a) to predict the cost of driving 1500 miles per month.
$$C(1500)=\frac{1500}{4}+260=375+260=635$$

(c) Draw the graph of the linear function. What does the slope represent?



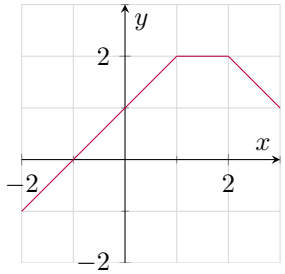
The slope reflects the cost per mile driven.

(d) What does the C-intercept represent?
The C-intercept represents the starting value, which in this case is how much it costs to own a car ($260).

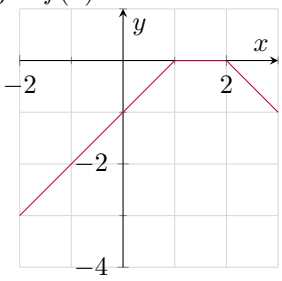
(e) Why does a linear function give a suitable model in this situation?
This is a suitable model because it is expected that the cost per mile driven is proportional across a car's lifespan.

Section 1.3

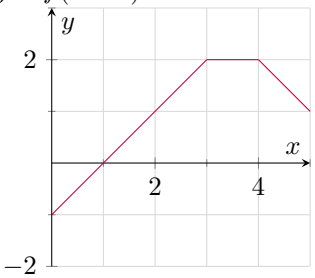
4. The graph of f is given. Draw the graphs of the following functions.



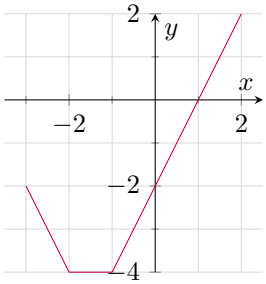
(a) $y = f(x) - 2$



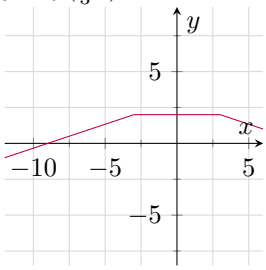
(b) $y = f(x - 2)$



(c) $y = -2f(-x)$



(d) $y = f(\frac{1}{3}x) + 1$



54. A spherical balloon is being inflated and the radius of the balloon is increasing at a rate of 2 cm/s.

(a) Express the radius r of the balloon as a function of the time t in seconds.

$$r(t) = t2$$

(b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

$$(V \circ r)(t) = \frac{4}{3}\pi(2t)^3$$

This equation represents how the volume of the balloon changes over time. This was obtained by using the equation for volume of a sphere ($\frac{4}{3}\pi r^3$) and inserting the equation from (A) for r.

60. If you invest x dollars at 4% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.04x$. Find $A \circ A$, $A \circ A \circ A$, and $A \circ A \circ A \circ A$. What do these compositions represent? Find a formula for the composition of n copies of A.

(a) $A \circ A = 1.04 \cdot 1.04 = 1.0816$

(b) $A \circ A \circ A = 1.04 \cdot 1.04 \cdot 1.04 = 1.124864$

(c) $A \circ A \circ A \circ A = 1.04 \cdot 1.04 \cdot 1.04 \cdot 1.04 = 1.16985856$

The above three operations represent compounding of interest over time.

$$A(n) = 1.04^n$$

62. If $f(x) = x + 4$ and $h(x) = 4x - 1$, find a function g such that $g \circ f = h$.

$$g(x) = 4x - 17$$

Challenge Question

Find a formula for a function $f(x)$ such that:

- $f(3) = 0$
- $f(x)$ is even
- f has a horizontal asymptote at $y = 2$
- f has a vertical asymptotes at $x = 4$ and $x = -4$
- $f(0) = 1$ (Meeting this requirement is the trickiest part!)

$$f(x) = \frac{x^2 - 2}{x^2 - 16} + 1$$

The above equation meets all requirements except for the last one, $f(0) = 1$.

$$f(x) = \frac{x^2}{x^2 - 16} + 1$$

This equation meets all requirements except for the first one, $f(3) = 0$.

$$f(x) = \begin{cases} \frac{x^2-2}{x^2-16} + 1, & x < 0 \\ 1, & x = 0 \\ \frac{x^2-2}{x^2-16} + 1, & x > 0. \end{cases}$$

This equation meets all requirements, no exceptions.