Solutions

Math 1300-005 - Spring 2017
Midterm 3 Review, Part I - 4/7/17

Guidelines: Please work in groups of two or three.

1. Find the absolute maximum value and the absolute minimum value of

$$f(x) = 2^{(x^2-4x)}$$

on the interval $[0, 3]$.

Closed interval method: $f'(x) = 2^{x^2-4x} \cdot \ln(2) \cdot \frac{d}{dx}(x^2-4x)$

$= \ln(2) \cdot 2^{x^2-4x}(2x-4)$.

$2^{x^2-4x} \neq 0$ (its always positive), so $f'(x)=0$ when $2x-4=0$, so $x=2$ is the critical number. Check the value of f at $0, 2, 3$.

$f(0) = 2^{(0^2-4(0))} = 2^0 = 1$

$f(2) = 2^{2^2-4(2)} = 2^{4-8} = 2^{-4} = \frac{1}{16}$

$f(3) = 2^{3^2-4(3)} = 2^{9-12} = 2^{-3} = \frac{1}{8}$

$\boxed{\text{So the absolute maximum value is } 1 \text{ at } x=0, \text{ the absolute minimum value is } \tfrac{1}{16} \text{ at } x=2}$

2. Use logarithmic differentiation to find the derivative of

$$y = (x^2 \tan(x))^x.$$

$\ln(y) = x \ln(x^2 \cdot \tan(x))$

Take $\frac{d}{dx}$ of each side,

$\frac{1}{y} y' = \ln(x^2 \cdot \tan(x)) + \underbrace{x \cdot \frac{1}{x^2 \cdot \tan(x)}}_{\text{cancel}} \cdot \underbrace{\frac{d}{dx}(x^2 \cdot \tan(x))}_{\text{Product}}$

$= \ln(x^2 \tan(x)) + \frac{1}{x \tan(x)} \cdot (2x\tan(x) + x^2\sec^2(x))$

$y' = y\left[\ln(x^2\tan(x)) + \frac{2x\tan(x) + x^2\sec^2(x)}{x\tan(x)}\right]$

$$\boxed{y' = (x^2\tan(x))^x\left[\ln(x^2\tan(x)) + \frac{2x\tan(x)+x^2\sec^2(x)}{x\tan(x)}\right]}$$

3. (a) Use the linearization of the function

$$f(x) = \tan\left(\frac{1}{4}x\right)$$

at the x value $a = 0$ to find an estimate for

$$\tan\left(\frac{1}{4}(-0.08)\right).$$

$f(0) = \tan\left(\frac{1}{4}0\right) = \tan(0) = 0$

$f'(x) = \frac{1}{4}\sec^2\left(\frac{1}{4}x\right)$, so

$f'(0) = \frac{1}{4}\sec^2\left(\frac{1}{4}(0)\right) = \frac{1}{4}$

At $a=0$, $L(x) = f(0) + f'(0)(x-0)$.

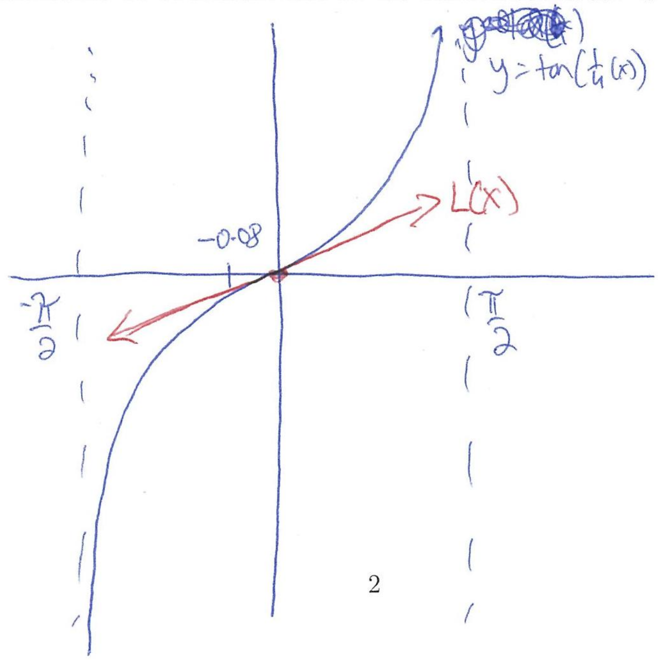
Hence $L(x) = 0 + \frac{1}{4}(x-0) = \boxed{\frac{1}{4}x = L(x)}$

So $\tan\left(\frac{1}{4}(-0.08)\right) = f(-0.08) \approx L(-0.08)$

$= \frac{1}{4}(-0.08)$

$\boxed{= -0.02}$

(b) Is this estimation of overestimate or an underestimate? Justify your answer.



At $x = -0.08$, the tangent line $L(x)$ is above the graph of $\tan\left(\frac{1}{4}x\right)$, so

$\boxed{-0.02 \text{ is an overestimate.}}$