Print Name _____

APPM 1350   **Exam 1**   **Summer 2016**

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 5 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.

- **Show all work and simplify your answers!** Answers with no justification will receive no points.

- Please begin each problem on a new page.

- No notes or papers, calculators, cell phones, or electronic devices are permitted.

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1. (28 pts, 7 pts each) Evaluate the following limits and show all supporting work (please do not use l'Hospital's Rule). If a limit does not exist, clearly state that fact and explain your reasoning. Make sure to simplify your answer completely.

   (a) $\displaystyle\lim_{\theta \to 0} \cos\left(3\sqrt{\frac{\pi}{4}\left(\pi + \theta\pi\right)}\right)$

   (b) $\displaystyle\lim_{x \to 0} \frac{x^2}{\tan^2(2x)}$

   (c) $\displaystyle\lim_{x \to 7} \frac{2x - 14}{|x - 7|}$

   (d) $\displaystyle\lim_{x \to -\infty} \frac{12x}{\sqrt{2x^2 - 7}}$

2. (19 pts) Let $f(x) = \sqrt{-x} - 2$, $k(x) = \sin(x)$

   (a) (4 pts) State the domain of f.

   (b) (5 pts) Sketch the graph of f.

   (c) (4 pts) State the range of f.

   (d) (6 pts) Does the function $\dfrac{k(x)}{f(x)}$ have vertical asymptotes? Use appropriate limits to justify.

3. (14 pts) Consider the function $g(x) = \begin{cases} bx^2 + \frac{7\sqrt{2}}{8}, & x > -1, \\ \sqrt{1-x}, & x \le -1 \end{cases}$.

   (a) (10 pts) For what value of b is $g(x)$ <u>differentiable</u> at $x = -1$? Use the <u>limit definition of the derivative</u> for this problem. Justify your answer.

   (b) (4 pts) Use your answer to part (a) to find the tangent line of $g(x)$ at $x = -1$.

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TWO MORE ON THE OTHER SIDE

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4. (18 pts) Consider the function $s(x) = \dfrac{|5x|}{x^2 + 2x}$

   (a) (8 pts) For what value(s) of x is this function discontinuous? Justify your answer(s) by showing how the <u>definition of continuity</u> fails for each value.

   (b) (4 pts) Label each discontinuity in part (a) as: *removable*, *jump*, or *infinite* discontinuity. Justification is not necessary for this part.

   (c) (6 pts) Is the function $s(x)$ odd, even, or neither? Justify your answer.

5. (21 pts, 7 pts each) Some unrelated short answer questions:

   (a) The limit, $\displaystyle\lim_{h \to 0} \dfrac{\sqrt[4]{16 + h} - 2}{h}$, represents the derivative of some function f at some number a. State both the function f and the number a.

   (b) Does $\sqrt[3]{x} = x^2 - \dfrac{5}{2}$ have a solution? Justify your answer.

   (c) Either sketch or formulate a function, f, with the following properties: f is an even function, $\displaystyle\lim_{x \to 3^+} f(x) = a$ where a is a nonzero value, and the domain of f is $(-\infty, -3) \cup (3, \infty)$.