MATH 1300: HW #14

Due on April 27, 2017 at 10:00am

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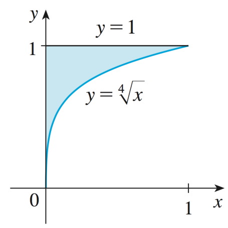
Section 5.3

50. The boundaries of the shaded region are the y-axis, the line $y = 1$, and the curve $y = \sqrt[3]{x}$. Find the area of this region by writing x as a function of y and integrating with respect to y (as in Exercise 49).



$$x = y^4 \text{ on } [0, 1]$$

$$\int_0^1 (y^4)dy$$

$$y\frac{5}{5}\Bigg]_0^1 = \boxed{\frac{1}{5}}$$

56. If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_3^5 f(x)dx$ represent?

$\int_3^5 f(x)dx$ represents the change in height (or elevation) between miles 3 and 5 of the trail.

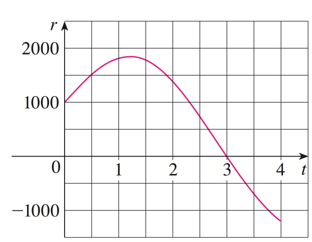
58. If the units for x are feet and the units for $a(x)$ are pounds per foot, what are the units for da/dx? What units does $\int_2^8 f(x)ax$ have?

$\int_2^8 f(x)ax$ has units of (lb/ft)/ft.

63. The linear density of a rod of length 4 m is given by $p(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

$$\begin{aligned} \text{total mass} &= \int_0^4 (9 + 2\sqrt{x})dx \\ &= \left[9x + \frac{4}{3}x^{\frac{3}{2}}\right]_0^4 \\ &= 36 + \frac{32}{3} - (0 + 0) \\ &= \frac{140}{3} \text{ kg} \end{aligned}$$

68. Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown, If the amount of water in the tank at time $t = 0$ is 25,000 L, use the Midpoint Rule to estimate the amount of water four days later.



$$\begin{aligned}
\int_0^4 r(t)dt &= v(4) - v(10) \\
3250L &= v(4) - 25000 \\
v(4) &= 28250L \\
\Delta t &= 1 \\
\int_0^\Delta r(t)dt &= r(0.5)\Delta t + r(1.5)\Delta t + r(2.5)\Delta t + r(3.5)\Delta t \\
&= 1(1500 + 1750 + 750 - 750) \\
&= 1 \text{ day}(3250 \text{ L/day}) \\
&= \boxed{3,250 \text{ L}}
\end{aligned}$$

Additional Problem

Suppose h is a function such that $h(2) = -4$, $h'(2) = -7$, $h''(2) = 6$, $h(5) = 8$, $h'(5) = 10$, and $h''(5) = 20$, and h'' is continuous everywhere. Evaluate $\int_2^5 h''(u)\,du$.

$$\begin{aligned}
h''(v) &= (h'(v))' \\
\int_2^5 h''(v)dv& \\
h'(5) - h'(2) &= 10 - (-7) \\
&= \boxed{17}
\end{aligned}$$