

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) **your name**, (2) **1350/EXAM 1**, and (3) **SUMMER 2014** on the front of your bluebook. Also make a grading table with room for 6 problems and a total score. **Work all problems. Start each problem on a new page. Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK**

1. (24 pts) Evaluate the following limits:

$$(a) \lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3} \quad (b) \lim_{x \rightarrow a^-} \left(1 + \frac{x - a}{|x - a|} \right) \quad (c) \lim_{x \rightarrow -\infty} \sqrt{9x^2 + 1} - 3x$$

For (a) and (b) only, locate and classify any discontinuities as either *jump*, *removable*, or *infinite*.

2. (16 pts) Let $f(x) = \cos x$

- (a) State the limit definition of the derivative of a function.
(b) Use your definition from (a) to show that $\frac{d}{dx}(\cos x) = -\sin x$.

$$\text{Hint: } \cos(a + b) = \cos a \cos b - \sin a \sin b, \text{ and } \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0.$$

3. (24 pts) Consider the function

$$g(x) = \begin{cases} x^2 + cx + 2, & x \leq 0 \\ 7x + d, & x > 0, \end{cases}$$

where c and d are real constants.

- (a) What is the domain of $g(x)$?
(b) What does it mean for a function to be continuous at $x = a$? Your definition must include limits.
(c) Find the value of d that makes $g(x)$ continuous at $x = 0$, **using your definition from (b)**. Are there any restrictions on the value of c ?
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4. (16 pts) Let $f(x) = -3x^5 + 2 \sin x + 3$.

- (a) Find the equation of the tangent line to $f(x)$ at $x = 0$. You may use rules of differentiation.
(b) Show that there is at least one solution to $f(x) = 4$ on $(-\pi, \pi)$.
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5. (14 pts) Let the displacement of an object traveling along a straight line be given by

$$s(t) = t^3 - 12t^2 + 45t + 2.$$

- (a) Find the velocity and acceleration functions of the object. You may use rules of differentiation.
(b) When is the object moving in the positive direction? When is the object at rest?
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6. (6pts) True or False: (Write the word **True** or **False**, do not write T/F.)

(a) If $f(x)$ is odd, then $f(x+1)$ is odd.

(b) If a function is continuous at $x = a$, then it is differentiable at $x = a$.

(c) $\lim_{x \rightarrow 2} \left[(x-2)^2 \sin \left(\frac{2}{x-2} \right) \right] = 0$.

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