INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/Test 2**, (3) **lecture number/instructor name** and (4) **SUMMER 2015** on the front of your bluebook. Also make a **grading table** with room for 5 problems and a total score. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK! JUSTIFY ALL YOUR ANSWERS!**

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1. Consider the equation $f(x) = \sqrt{x}$

   (a) (6 pts) Find dy/dx and d^2y/dx^2 at the point $(4, 2)$

   (b) (6 pts) Find the linearization of the equation at the point $(4, 2)$. Use the linearization to estimate the y-value when $x = 4.1$

   (c) (4 pts) Does the approximation in part (b) overestimate or underestimate the actual value of y at $x = 4.1$? Explain.

   (d) (4 pts) What is dy/dx at $x = 0$?

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2. (25 pts) A rocket is launched vertically and is tracked by a radar station located on the ground 4 miles from the launch site. What is the vertical speed of the rocket at the instant when its distance from the radar station is 5 miles and this distance is increasing at the rate of 3600 mi/hr ?

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3. Calculate dy/dx for each of the following: (After finding dy/dx, do not simplify. In the case of implicit differentiation, solve for dy/dx but you do not need to simplify further.)

   (a) (5 pts) $y = \frac{\tan x}{1+\cos x}$

   (b) (5 pts) $y = \left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$

   (c) (5 pts) $\sin(xy) = x^2 - y$

   (d) (5 pts) $y = \tan^2(\sin\theta)$

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4. Consider the function

$$f(x) = \frac{2 + x - x^2}{(x-1)^2}, \quad f'(x) = \frac{x-5}{(x-1)^3}, \quad f''(x) = \frac{2(7-x)}{(x-1)^4}$$

   (a) (3 pts) Find any vertical, horizontal, or slant asymptotes of f. Use appropriate limits to justify your answer.

   (b) (3 pts) On what intervals is f increasing? decreasing?

   (c) (4 pts) Find all local maximum and minimum values of f.

   (d) (4 pts) On what intervals is f concave up? concave down?

   (e) (3 pts) Find all inflection points of f.

   (f) (3 pts) Using the information from (a)-(e), sketch a graph of f. Clearly label any points of interest, including any asymptotes, local extrema, and inflection points.

PROBLEM #5 ON THE OTHER SIDE

5. (a) (7 pts) State the Mean Value Theorem

   (b) (8 pts) Suppose that f and g are continuous on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$. [Hint: Apply the Mean Value Theorem to the function $h = f - g$.]