Math 1300, Midterm 2

March 6, 2017

PRINT YOUR NAME: John Keller

PRINT INSTRUCTOR'S NAME: Braden Balentine

Mark your section/instructor:

☐	Section 001	Brendt Gerics	8:00 - 8:50
☐	Section 002	Leo Herr	9:00 - 9:50
☐	Section 003	Tyler Schrock	9:00 - 9:50
☐	Section 004	Lee Roberson	10:00 - 10:50
☒	Section 005	Braden Balentine	10:00 - 10:50
☐	Section 006	Xingzhou Yang	10:00 - 10:50
☐	Section 007	Lee Roberson	11:00 - 11:50
☐	Section 008	Shen Lu	11:00 - 11:50
☐	Section 009	Suzanne Craig	12:00 - 12:50
☐	Section 010	Carlos Pinilla-Suarez	12:00 - 12:50
☐	Section 011	Nathan Davidoff	1:00 - 1:50
☐	Section 012	Sion Ledbetter	1:00 - 1:50
☐	Section 013	Ruofan Li	2:00 - 2:50
☐	Section 014	Daniel Martin	2:00 - 2:50
☐	Section 015	Isabel Corona	3:00 - 3:50
☐	Section 016	Ira Becker	3:00 - 3:50
☐	Section 017	Ira Becker	4:00 - 4:50
☐	Section 0430R	Patrick Newberry	11:00 - 11:50

Question	Points	Score
1	6	5
2	6	6
3	12	9
4	12	5
5	10	8
6	5	5
7	10	9
8	10	~~6~~ 8
9	10	6
10	9	9
11	10	8
Total:	100	78

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (6 points) Let $f(x) = \frac{1}{3}x^3 + x^2 - x + 2$. Find all value(s) of a such that $y = \boxed{2}x + 11$ is the tangent line to the graph of $f(x)$ at $x = a$.

$f'(x) = 1x^2 + 2x - 1$ 1

$2 = 1x^2 + 2x - 1$ 2

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$ 1

$x = -3, 1$

$\frac{1}{3}(-3)^3 + (-3)^2 - (-3) + 2$

$-9 + 9 + 3 + 2$

5 1

$y = 2(-3) + 11$

$y = -6 + 11$

$y = 5$

$5 = 5$ ✓

$\frac{1}{3}(1)^3 + (1)^2 - (1) + 2$

$\frac{1}{3} \cancel{+ 1 - 1} + 2$

$\frac{7}{3}$ 1

$y = 2(1) + 11$

$y = 13$

$13 \neq \frac{7}{3}$

$\boxed{\cancel{a = 1}}$ 1

$\boxed{a = -3}$

5/6

$+ \to -$ max

$- \to +$ min

2. (6 points) Let $f(x) = \frac{2}{3}x^3 - 2x + 4$.

$f'(x) = 2x^2 - 2$

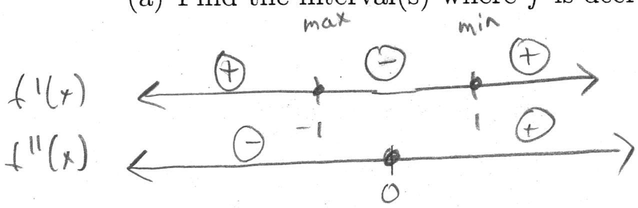
$0 = 2x^2 - 2$

$2 = \frac{2x^2}{2}$

$1 = x^2$

c-points: $x = 1, -1$

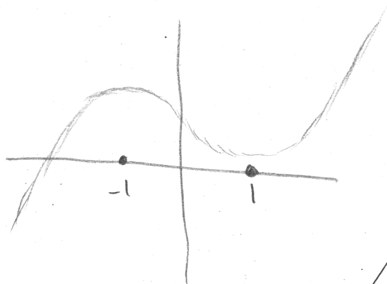
(a) Find the interval(s) where f is decreasing.



$f''(x) = 4x$

$\frac{0}{4} = x$

Decreasing @ $(-1, 1)$



(b) Find the interval(s) where f is concave up.

$f''(x) = 4x$

$0 = 4x$

$\frac{0}{4} = x$

Concave up @ $(0, \infty)$

3. (12 points) **Multiple choice** – circle the derivative of the given function. You DO NOT need to show your work.

$-\cos(x)\sec(x)$

(a) $g(t) = \sec\left(8t^2\right)$ $\quad -\cos(8t^2)\sec(8t^2) +$

A. $8t^2 \sec(8t^2)\tan(8t^2)$

B. $16t\sec(8t^2)\tan(8t^2)$

C. $16t\,(\tan(t))^2$

D. $-8t^2\cos(8t^2)\cot(8t^2)$

+9

(b) $f(x) = \sin(1+\pi^2)$ $\quad \cos(1+\pi^2)(0)$

A. $\cos(1+\pi^2)$

B. $\cos(1+\pi^2)(2\pi)$

C. $-\cos(1+\pi^2)(2\pi)$

D. 0

(c) $v(t) = \sqrt{t^2+1}$ $\quad \frac{1}{2}(t^2+1)^{-1/2}(2t) \quad \frac{1}{2}\left(\frac{2}{t^2+1}\right)(2t)$

A. $\dfrac{t}{\sqrt{t^2+1}}$

B. $-2\sqrt{2t}+1$ $\quad \left(\frac{1}{t^2+1}\right)2t$

C. $-\dfrac{t}{\sqrt{t^2+1}}$

D. $\sqrt{2t}(t^2+1)$

(d) $P(q) = \sin(q^3) + (\cos(q))^3$

A. $3q^2\cos(q^3) - 3q^2\sin(q)$

B. $3q^2\cos(q^3) - 3\sin(q)\,(\cos(q))^2$

C. $3q^2\cos(q^3) - 3\,(\sin(q))^2$

D. $3\left(\sin(q^3)+(\cos(q))^3\right)^2\left(\cos(q^3)-(\sin(q))^2\right)$

$\cos(q^3)(3q^2) + 3(\cos(q))^2(-\sin(q))$

$3q^2\cos(q^3)$

4. (12 points) Compute the following derivatives. $\boxed{\text{DO NOT simplify your answers.}}$

(a) $\frac{d}{dx}\left[5x^3 - \frac{3}{4x} + 7^x\right] =$ anything^x stays

power quotient rule

$$\boxed{\frac{d}{dx} = 15x^2 - \left(\frac{0 \cdot 4x - 3 \cdot 4}{(4x)^2}\right) + 7^x \ln(7)}$$

2

③

(b) $\frac{d}{dx}\left[\frac{\tan x}{7x^3 - 4x}\right] =$

quotient rule!

$\sec^2(x)$

$$\frac{d}{dx} = \frac{\boxed{\cancel{\csc(x)}(7x^3 - 4x) - \tan(x)(21x^2 - 4)}}{(7x^3 - 4x)}$$

?

2

(c) $\frac{d}{dx}\left[\sqrt{x}\boxed{e^{-x^2}}\right]$

$a = \sqrt{x}$

$b = e^x$

$c = -x^2$

$(\sqrt{x})(e^{(-x^2)} \cdot (-2x))$

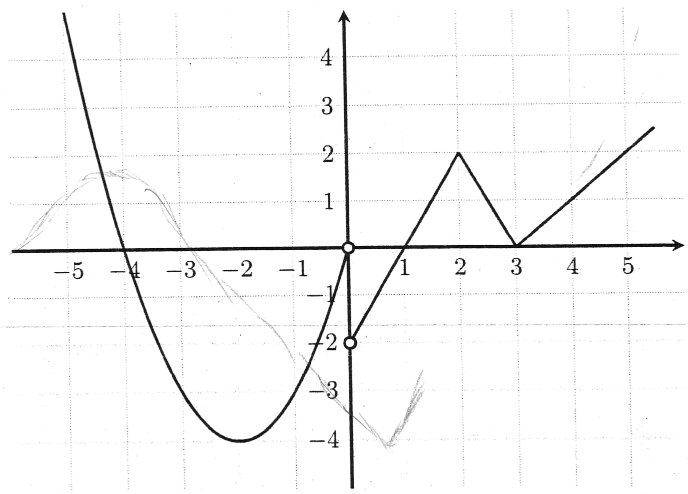
$$\boxed{\frac{d}{dx} = \sqrt{x}(e^{-x^2} \cdot (-2x))}$$

Product Rule

1

5. (10 points) **Multiple choice** – circle the correct answer. The graph below shows the **derivative** of some function $f(x)$.

Graph of $f'(x)$



(a) On which of the following intervals is $f(x)$ concave up?

A. $(-5, -4)$ B. $(-5, 0)$ C. $(-2, 0)$ D. $(0, 5)$ E. $(2, 3)$

(b) At which of the following x-values does $f(x)$ have a local minimum?

A. -4 B. -2 C. 0 D. 1 E. 2

(c) On which of the following intervals is $f(x)$ decreasing?

A. $(-5, -2)$ B. $(-4, -2)$ C. $(0, 2)$ D. $(2, 3)$ E. $(3, 5)$

8

(d) At which of the following x-values is $f''(x) = 0$?

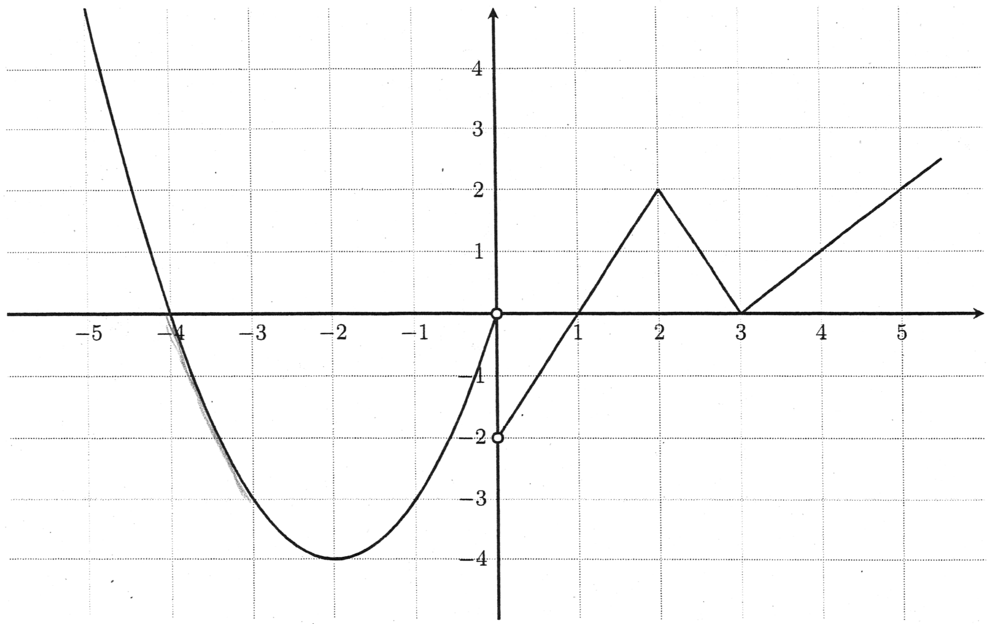
A. -4 B. -2 C. 0 E. 2 E. 3

(e) What is $f''(2.5)$?

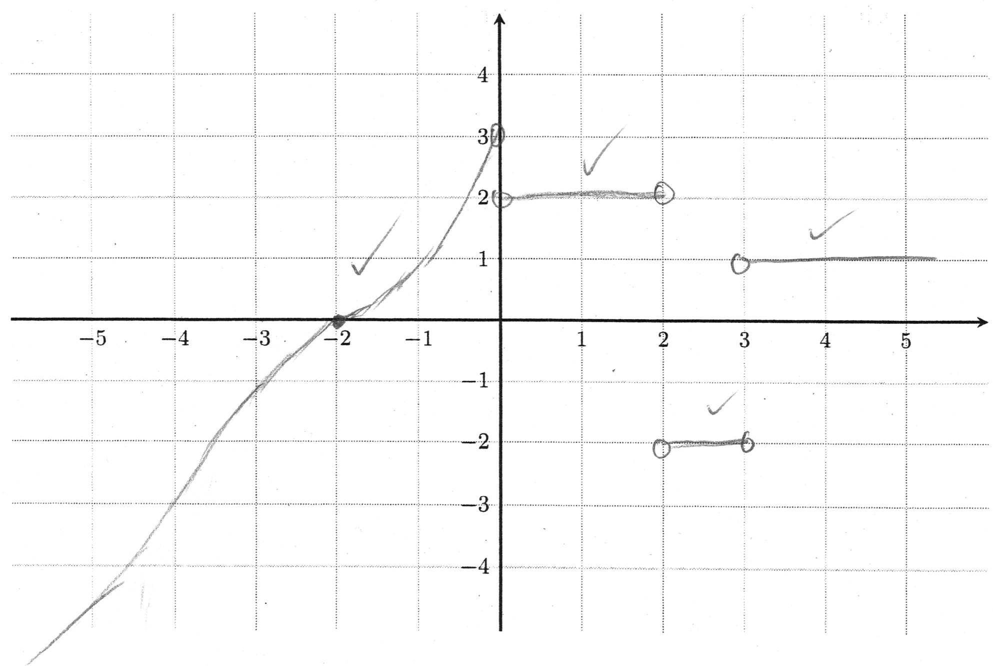
A. -2 B. -1 C. 0 D. 1 E. 2

6. (5 points) The graph below shows the **derivative** of some function $f(x)$. Sketch a graph for **the second derivative** of $f(x)$.

Graph of $f'(x)$



Graph of $f''(x)$



+5

7. (10 points) Consider the equation $y^2 - xy = 3 - x^2$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}(y^2 - xy) = \frac{d}{dx}(3 - x^2)$$

$$2y\frac{dy}{dx} - 1y + x\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}(2y + x) = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y - 2x}{2y + x}}$$

4

(b) Find an equation of the tangent line to $y^2 - xy = 3 - x^2$ at $(-1, 1)$.

$$m = \frac{(1) - 2(-1)}{2(1) + (-1)} = \frac{3}{1} = 3$$

$$1 = 3(-1) + b$$

$$\boxed{y = 3x + 4}$$

5

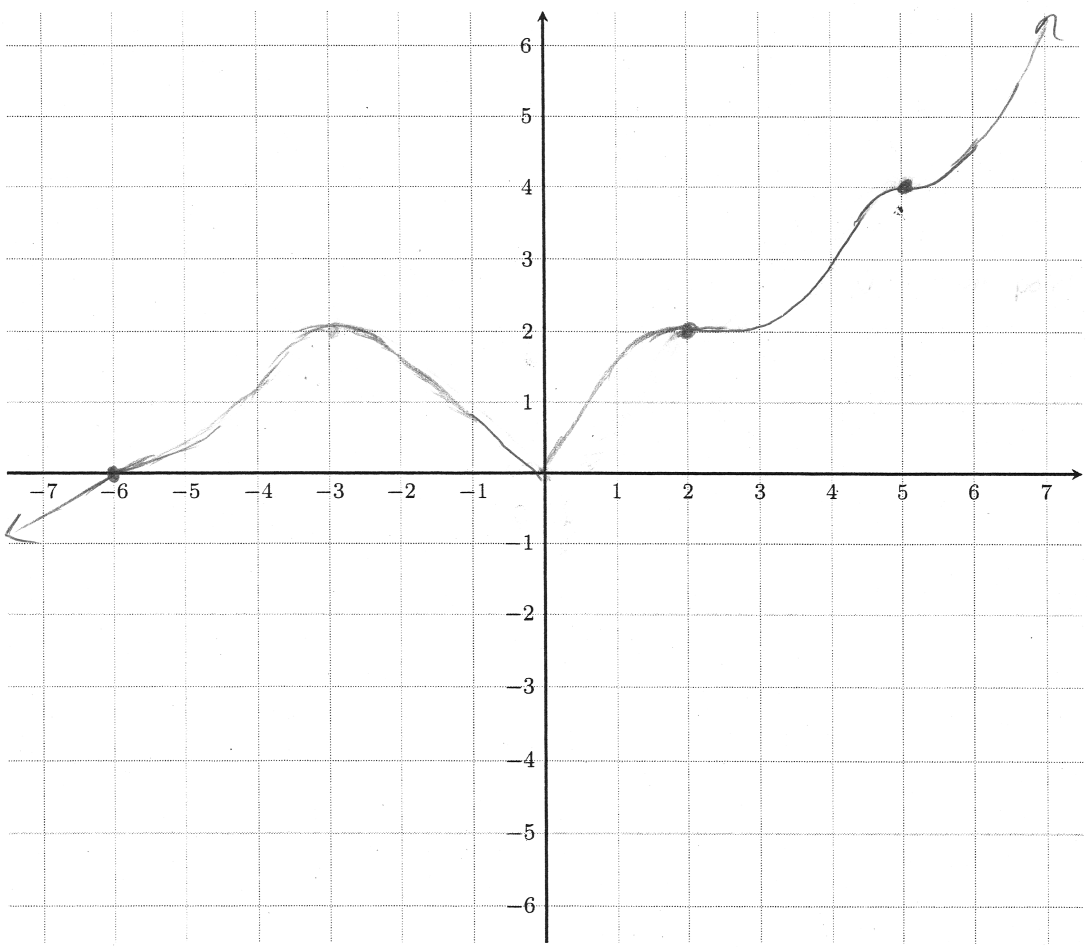
$$1 - 3(-1) = b$$

$$4 = b$$

8. (10 points) Sketch the graph of a **function** $f(x)$ that satisfies **ALL** the conditions below:

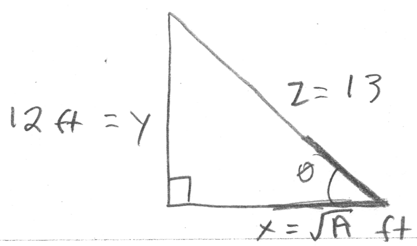
- $f(x)$ is **continuous** on $(-\infty, \infty)$;
- $f(-6) = f(0) = 0$;
- $f'(-6)$ does not exist; -2
- $f'(-3) = f'(2) = f'(5) = 0$;
- $f'(x) > 0$ on $(2, 5)$ and $(5, \infty)$;
- $f''(x) > 0$ on $(0, 4)$ and $(5, \infty)$.

Graph of $f(x)$



SOH CAH TOA

9. (10 points) A ladder 13 feet long is leaning against the side of a building. If the bottom of the ladder is pulled away from the building at a constant rate of 2 feet per second, how fast is the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 feet above the ground? **Include units in your answer.**



2

$x = \cancel{\sqrt{A}}$

$z = 13$ ft

$y = 12$ ft

$\sin(\theta) = \frac{y}{z}$

$\frac{dx}{dt} = 2$ ft/sec

$\boxed{\frac{d\theta}{dt} = ?}$

$\frac{dz}{dt} =$

$\theta = \sin(12/13)$

$$\frac{d}{dx}\left(\cos(\theta)\right) = \frac{d}{dx}\left(\frac{x}{z}\right) \quad 2$$

$$-\sin(\theta)\frac{d\theta}{dt} = \frac{\frac{dx}{dt}z - x\frac{dz}{dt}}{z^2}$$

$$= \frac{(2)(13) - (\sqrt{A})(0)}{13^2}$$

$$= \frac{26 - 0}{13^2}$$

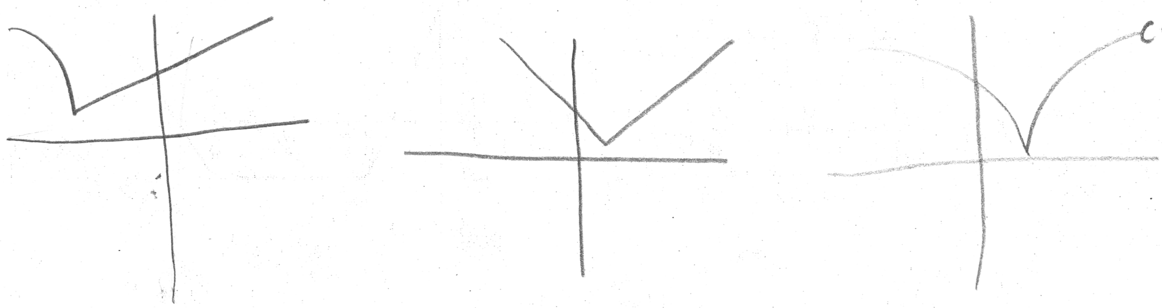
$$-\sin(\theta)\frac{d\theta}{dt} = \frac{26}{169} \quad 2$$

$$\boxed{\frac{d\theta}{dt} = \frac{\frac{26}{169}}{-\sin(12/13)}} \quad \text{rad/sec.}$$

10. (9 points) The following statements are all false. For **each** statement, justify why it is false by providing an explanation that includes a picture.

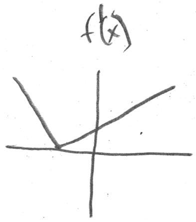
(a) If $f(x)$ is continuous on $(-\infty, \infty)$, then $f(x)$ is differentiable on $(-\infty, \infty)$.

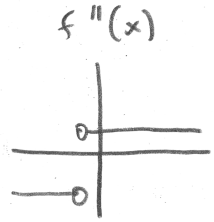
+3 This statement is false because even though a function is contimous, the derivative fails at cusps and corners.



(b) If $f(x)$ is differentiable at $x = a$, then $f'(x)$ is also differentiable at $x = a$.

+3 This statement is false because when a cusp or corner is located at "a", then $f''(a)$ is undefined, making $f'(a)$ not differentiable

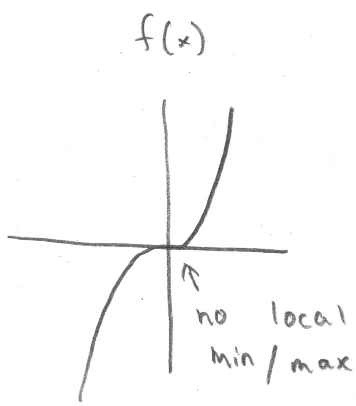


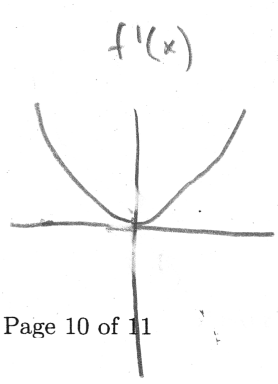


(c) If $f'(a) = 0$, then $f(x)$ either has a local minimum or a local maximum at $x = a$.

+3

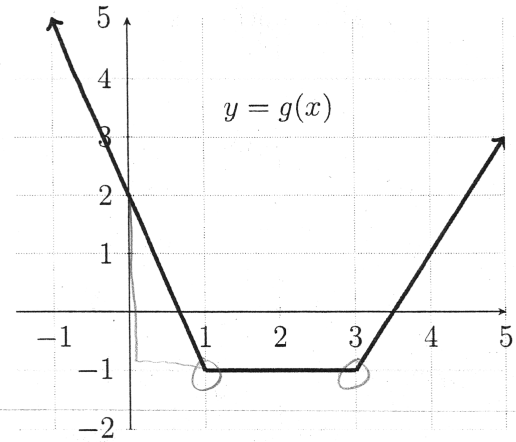
This statement is false because even though the $f'(a)$ slope is zero, the function can still increase after.





11. (10 points) Suppose f and is a differentiable functions with values and derivatives given in the table below, and $y = g(x)$ be represented by the graph below.

x	$f(x)$	$f'(x)$
0	2	−2
1	−2	5
2	9	3
3	0	4
4	1	6



(a) Let $p(x) = f(x) + g(x)$. Then $p'(1) =$

A. −7 B. −2 C. 0 D. 3 E. 8 F. DNE

5 + DNE

(b) Let $h(x) = f(x) \cdot g(x)$. Then $h'(2) =$

A. −5 B. −3 C. 0 D. 2 E. 4 F. DNE

$3(-1) + 9(0)$

(c) Let $k(x) = g(f(x))$. Then $k'(3) =$

A. −12 B. −3 C. 0 D. 1 E. 2 F. DNE

$g'(0) = (-3)(4)$

8

(d) Let $R(x) = \sqrt{f(x)}$. Then $R'(2) =$

A. $\frac{1}{12}$ B. $\frac{1}{6}$ C. $\frac{1}{2}$ D. $\frac{3}{2}$ E. $\frac{9}{2}$ F. DNE

$\frac{1}{2}(f(2))^{-1/2} \quad f'(2)$ $\quad \frac{1}{2}(9)^{-1/2}(3) \quad \frac{1}{2}\left(\frac{1}{18}\right)(3)$

$\frac{1}{36} \quad \frac{3}{36} = \frac{1}{12}$

(e) Let $L(x) = g(x^2)$. Then $L'(2) =$

A. 0 B. 1 C. 2 D. 4 E. 8 F. DNE

$g'(4) \cdot 4$

$2 \cdot 4$