MATH 1300: HW #3

Due on February 2, 2017 at 10:00am

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Section 2.3

8. (a) What is wrong with the following equation?

$$\frac{x^2+x-6}{x-2} = x+3$$

$$\frac{x^2+x-6}{x-2} = \frac{(x+3)(x-2)}{x-2} = x+3$$

The issue with this equation is that through factoring process, $x+3$ is only equal if $x \neq 2$.

(b) In lieu of part (a). explain why the equation

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2}(x+3)$$

is correct.

$$\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \frac{(x-2)(x+3)}{x-2} = x+3$$

This equation is true because the limit is not a set number, but rather the trend towards a given number. This means that x does not necessarily need to be equal to 2. (for the given equality to be true $x \neq 2$)

22. Evaluate the limit of

$$\lim_{x\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right)$$

$\frac{1}{x} - \frac{1}{x^2+x} = \frac{1}{x+1}$
$\lim_{x\to 0} \frac{1}{x+1}$
$\frac{1}{0+1} = \boxed{1}$

31. Prove that $\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0$

$$\lim_{x\to 0} x^4 \lim_{x\to 0} \cos\left(\frac{2}{x}\right) =$$

$$\lim_{x\to 0} x^4 = 0^4 = 0$$

$$0 \times \cos\left(\lim_{x\to 0}\frac{2}{x}\right) = \boxed{0}$$

49. Is there a number a such that

$$\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$$

exists? If so, find the value of a and the value of the limit.

Plugging in -2.1 (LHL)

$$\frac{3(-2.1)^2+a(-2.1)+a+3}{(-2.1)^2+(-2.1)-2} =$$

$$\frac{13.23-2.1a+a+3}{4.41-4.1} =$$

$$\frac{16.23-1.1a}{0.31} =$$

Plugging in -1.9 (RHL)

$$= \frac{3(-1.9)^2+a(-1.9)+a+3}{(-1.9)^2+(-1.9)-2}$$

$$= \frac{3.61-1.9a+a+3}{3.61-3.9}$$

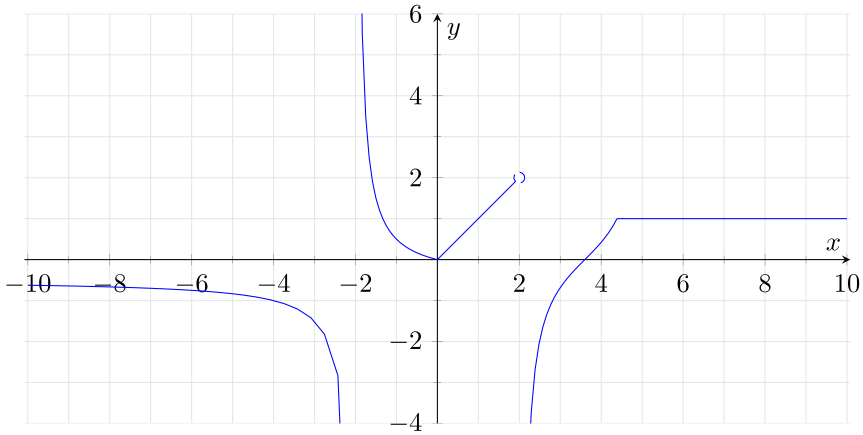
$$= \frac{6.61-0.9a}{-0.29}$$

Now, combining the two sides.

$$\begin{aligned}
\frac{6.61 - 0.9a}{-0.29} &= frac16.23 - 1.1a0.31 \\
(6.61 - 0.9a)0.31 &= (16.23 - 1.1a) - 0.29 \\
2.0491 - 0.279a &= -4.7067 + 0.319a \\
6.7558 &= 0.598a \\
a &\approx \boxed{11.297}
\end{aligned}$$

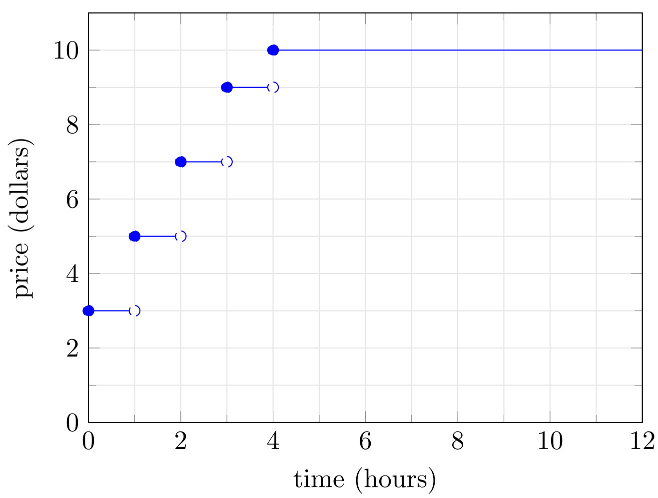
Section 2.4

8. Sketch the graph of a function f that is continuous except of the stated discontinuity: Neither left nor right continuous at -2, continuous only from the left at 2.



9. A parking lot charges \$3 for the first hour (or only part of an hour) and \$2 for each succeeding hour (or part), up to a daily maximum of \$10.

   (a) Sketch the graph of the cost of parking at this lot as a function of the time parked there.

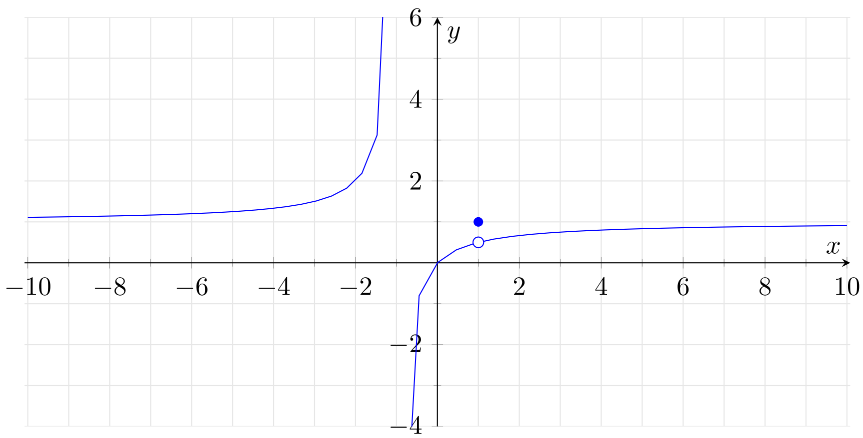


   (b) Discuss the discontinuities of this function and their significance to someone who parks in this lot.

The discontinuities of this function are very significant to a person parking in the lot looking to make their money go the furthest possible. This means, it would be most efficient if you want to stay for 4 hours to really only stay for 3:59, because you would save \$2 in the end, just before the function jumps to a higher price.

16. Explain why the function is discontinuous at the given number a. Sketch the graph of the function:

$$f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \qquad a = 1$$



The function is discontinous at number a simply because a is not located along the graph line for the function $\frac{x^2-x}{x^2-1}$. This causes the graph above, with a discontinuity at $x = 1$.

42. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval: $\sqrt[2]{x} = 1 - x, \quad (0, 1)$.

$$\sqrt[2]{x} = 1 - x$$
$$\sqrt[2]{x} + x - 1 = 0$$

$x = 0$
$\sqrt[2]{0} + 0 - 1 = -1$ The curve is below 0!

$x = 1$
$\sqrt[2]{1} + 1 - 1 = 2 - 1 = 1$ The curve is above 0!

Because square root is continuous, there is a root to the equation $\sqrt[2]{x} = 1 - x$ in the interval $(0, 1)$.