Name: Solutions

Math 1300-005 - Spring 2017

Quiz 11 - 4/7/17

> *On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.*
>
> Signature: ______________________________

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. Compute the following limits. If you use l'Hospital's Rule, indicate at which step this occurs. And don't drop your limits!

   (a) $\lim\limits_{x\to\infty} \left[\sqrt{x^2-1} - \sqrt{x}\right]$ $\approx \infty - \infty$ ... indeterminate but can't use L'Hospital yet.

$$\lim_{x\to\infty} \sqrt{x^2-1} - \sqrt{x} = \lim_{x\to\infty} \left(\sqrt{x^2-1} - \sqrt{x}\right) \cdot \frac{\left(\sqrt{x^2-1} + \sqrt{x}\right)}{\sqrt{x^2-1} + \sqrt{x}} = \lim_{x\to\infty} \frac{(x^2-1) - x}{\sqrt{x^2-1} + \sqrt{x}} \approx \frac{\infty}{\infty}$$

$$\overset{\text{l'Hospital}}{=} \lim_{x\to\infty} \frac{2x-1}{\frac{2x}{2\sqrt{x^2-1}} + \frac{1}{2\sqrt{x}}} = \frac{\lim\limits_{x\to\infty}(2x-1) \to \infty}{\lim\limits_{x\to\infty}\left(\frac{2x}{2\sqrt{x^2-1}}\right) \to 1 + \lim\limits_{x\to\infty}\left(\frac{1}{2\sqrt{x}}\right) \to 0}$$

$$= \frac{\infty}{1+0} = \boxed{\infty}$$

   (b) $\lim\limits_{x\to 0^+} (x)^{\sqrt{x}}$

Let $L = \lim\limits_{x\to 0^+} (x)^{\sqrt{x}}$. Then $\ln(L) = \lim\limits_{x\to 0^+} \ln\left((x)^{\sqrt{x}}\right) = \lim\limits_{x\to 0^+} \sqrt{x} \cdot \ln(x) \approx 0 \cdot (-\infty)$.

$$= \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{\sqrt{x}}} \rightarrow \text{write as } x^{-1/2}$$

$$= \lim_{x\to 0^+} \frac{1/x}{\frac{1}{-2x^{3/2}}} \quad \text{(simplify)}$$

$$= \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{-2x^{3/2}}{1}$$

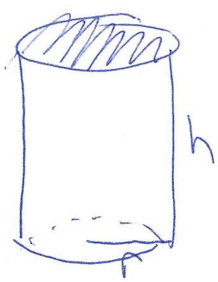
$$= \lim_{x\to 0^+} -2x^{1/2} = 0.$$

So $\ln(L) = 0$, hence

$$\boxed{L = e^0 = 1}$$

2. Suppose we want to construct a cylinder, open on one end, and we have 48π cm^2 worth of material to do so. What radius will give the maximum volume? Please justify your answer using the first derivative test for absolute extrema.

You will need the following formulas: $SA = \pi r^2 + 2\pi r h$ and $V = \pi r^2 h$ where h is the height of the cylinder.



constraint: $48\cancel{\pi} = \cancel{\pi} r^2 + 2\cancel{\pi} rh \Rightarrow 48 = r^2 + 2rh.$

optimizing: $V = \pi r^2 h$

solve constraint for h; substitute result in to V.

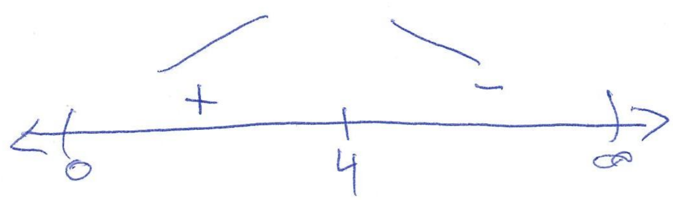
$$48 = r^2 + 2rh \Leftrightarrow 2rh = 48 - r^2 \Leftrightarrow h = \frac{48 - r^2}{2r}$$

So $$V = \pi r^{\cancel{2}}\left(\frac{48 - r^2}{2\cancel{r}}\right) = \frac{\pi}{2} r(48 - r^2) = \frac{\pi}{2}(48r - r^3).$$

Domain: $(0, \infty)$ → don't forget to mention this!

$$V' = \frac{\pi}{2}(48 - 3r^2), \text{ so } V' = 0 \text{ when } 48 = 3r^2 \Leftrightarrow 16 = r^2 \Leftrightarrow \pm 4 = r.$$

Since $(0, \infty)$ is our domain, we only work with $r = 4$.



Since $r = 4$ is the only critical # on $(0, \infty)$ and since $V(4)$ is a local max, $V(4)$ must be the absolute maximum volume.

Hence $r = 4$cm maximizes the volume.