Solutions

Math 1300-005 - Spring 2017

Chain Rule Activity, Part I - 2/22/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the next midterm.

Recall ***the chain rule***, which states

$$\frac{d}{dx}q(r(x)) = q'(r(x))\frac{d}{dx}r(x)$$

1. Use the chain rule to differentiate the following.

(a) $f(x) = \sqrt{1+\cos(x)}$ outside: $(\ )^{1/2}$ inside: $1+\cos(x)$

$f'(x) = \frac{1}{2}(1+\cos(x))^{-1/2} \cdot \frac{d}{dx}(1+\cos(x))$

$= \boxed{\frac{1}{2}(1+\cos(x))^{-1/2} \cdot (-\sin(x))}$

(b) $g(x) = e^{(x^2-\cot(x))}$ outside: $e^{(\ )}$ inside: $x^2-\cot(x)$

$g'(x) = e^{(x^2-\cot(x))} \cdot \frac{d}{dx}(x^2-\cot(x))$

$= \boxed{e^{(x^2-\cot(x))} \cdot (2x+\csc^2(x))}$

(c) $h(x) = \sec(2x^3 - 9x^2 + 4)$ outside: $\sec(\ )$ inside: $2x^3-9x^2+4$

$h'(x) = \underbrace{\sec(2x^3-9x^2+4)\tan(2x^3-9x^2+4)}_{\text{this is all deriv of outside composed w/ inside}} \cdot \frac{d}{dx}(2x^3-9x^2+4)$

$= \boxed{\sec(2x^3-9x^2+4)\tan(2x^3-9x^2+4) \cdot (6x^2-18x)}$

(d) $\ell(x) = \sin(2^x - \tan(x))$ outside: $\sin(\ )$ inside: $2^x-\tan(x)$

$\ell'(x) = \cos(2^x-\tan(x)) \cdot \frac{d}{dx}(2^x-\tan(x))$

$= \boxed{\cos(2^x-\tan(x)) \cdot (2^x\ln(2) - \sec^2(x))}$

2. It often happens that you have to do the chain rule within the product and quotient rules. Keep this in mind to differentiate the following.

(a) $F(x) = (2x-5)^4(8x^2-3x)^{-3}$ → Product rule first

$F'(x) = \underbrace{\left[\frac{d}{dx}(2x-5)^4\right]}_{\text{chain; outside: } (\ )^4,\ \text{inside: } 2x-5} \cdot (8x^2-3x)^{-3} + (2x-5)^4 \cdot \underbrace{\frac{d}{dx}(8x^2-3x)^{-3}}_{\text{chain; outside: } (\ )^{-3},\ \text{inside: } 8x^2-3x}$

$= \left[4(2x-5)^3 \cdot \frac{d}{dx}(2x-5)\right](8x^2-3x)^{-3} + (2x-5)^4 \cdot \left[-3(8x^2-3x)^{-4}\frac{d}{dx}(8x^2-3x)\right]$

$= \boxed{\left[4(2x-5)^3 \cdot 2\right](8x^2-3x)^{-3} + (2x-5)^4 \cdot \left[-3(8x^2-3x)^{-4}(16x-3)\right]}$

(b) $G(x) = \dfrac{x}{\sqrt{x^2+1}}$ → quotient rule first

$G'(x) = \dfrac{\left(\frac{d}{dx}x\right)\sqrt{x^2+1} - x \cdot \frac{d}{dx}\sqrt{x^2+1}}{\left(\sqrt{x^2+1}\right)^2}$ → chain: outside: $(\ )^{1/2}$ inside: x^2+1

$= \dfrac{(1)\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot \frac{d}{dx}(x^2+1)\right]}{\left(\sqrt{x^2+1}\right)^2}$

$= \boxed{\dfrac{\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right]}{x^2+1}}$

3. As well, it often happens that you must do the product or quotient rule within the chain rule. Differentiate the following.

(a) $H(x) = \left(\dfrac{1+x^2}{2-x^4}\right)^{-1/3}$

outside: $(\quad)^{-1/3}$

inside: $\dfrac{1+x^2}{2-x^4}$ } quotient rule!

$$H'(x) = -\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{d}{dx}\left(\frac{1+x^2}{2-x^4}\right)$$

$\hookrightarrow$ QR

$$= -\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{\left[\frac{d}{dx}(1+x^2)\right]\cdot(2-x^4) - (1+x^2)\frac{d}{dx}(2-x^4)}{(2-x^4)^2}$$

$$= \boxed{-\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{2x(2-x^4) - (1+x^2)(-4x^3)}{(2-x^4)^2}}$$

(b) $L(x) = e^{(x^2 \csc(x))}$

outside: $e^{(\quad)}$

inside: $\underbrace{x^2 \csc(x)}_{\text{product}}$

$$L'(x) = e^{(x^2\csc(x))} \cdot \frac{d}{dx}\left(x^2 \csc(x)\right)$$

$\hookrightarrow$ product rule

$$= \boxed{e^{(x^2\csc(x))} \cdot \left(2x\csc(x) + x^2(-\csc(x)\cot(x))\right)}$$

4. Finally, it is possible that multiple iterations of the chain rule will be necessary. Differentiate the following.

(a) $m(x) = \sin(\cos(\tan(x)))$

outside: $\sin(\quad)$

inside: $\cos(\tan(x))$

$$m'(x) = \cos(\cos(\tan(x))) \cdot \underbrace{\frac{d}{dx}\cos(\tan(x))}_{\text{chain again}}$$

chain again

outside: $\cos(\quad)$

inside: $\tan(x)$

$$= -\cos(\cos(\tan(x))) \cdot (-\sin(\tan(x)) \cdot \frac{d}{dx}\tan(x)$$

$$= \boxed{-\cos(\cos(\tan(x))) \cdot \sin(\tan(x)) \cdot \sec^2(x)}$$

(b) $b(x) = \sqrt{x + e^{\cos(x)}}$

outside: $(\quad)^{1/2}$

inside: $x + e^{\cos(x)}$

$$b'(x) = \frac{1}{2}\left(x + e^{\cos(x)}\right)^{-1/2} \cdot \frac{d}{dx}\left(x + e^{\cos(x)}\right)$$

$$= \frac{1}{2}\left(x + e^{\cos(x)}\right)^{-1/2} \cdot \left[1 + \underbrace{\frac{d}{dx}e^{\cos(x)}}_{\text{chain again}}\right]$$

chain again

outside: $e^{(\quad)}$

inside: $\cos(x)$

$$= \frac{1}{2}\left(x + e^{\cos(x)}\right)^{-1/2} \cdot \left[1 + e^{\cos(x)} \cdot \frac{d}{dx}(\cos(x$$

$$= \boxed{\frac{1}{2}\left(x + e^{\cos(x)}\right)^{-1/2} \cdot \left[1 - e^{\cos(x)} \cdot \sin(x)\right]}$$

$-$ since $\frac{d}{dx}\cos x = -\sin(x)$