Intermediate Value Theorem:

Suppose that $f(x)$ is continuous on $[a, b]$ and let M be any number between $f(a)$ and $f(b)$. Then there exists a number c such that $a < c < b$ and $f(c) = M$.

1. Is the function continuous?
2. Plug in both interval values and solve
3. If one is (+) and one is (-) then 0 is between
4. "By the <u>IVT</u>, there exists c in (x,y) with $f(c)=0$ ~ {write eq. with c}."

Limits at Infinity:

$$\lim_{x\to\infty} \frac{\boxed{3x^2} - x - 2}{\boxed{5x^2} + 4x + 1} = \frac{3x^2}{5x^2} = \boxed{\frac{3}{5}}$$

1. Take two biggest powers in equation
2. If equal exponents, x values cancel, otherwise think which value is larger when x goes to infinity!

Continuous Functions:

"don't pick up the pencil"

Polynomials, Rational Functions, Root Functions, Exponentials, Logarithmic, Trigonometric

If f and g are <u>continuous</u> then so are $f+g$, $f*g$, f/g, $f\circ g$

mathmatical definition:
$$\lim_{x\to a} f(x) = f(a)$$

Derivative & Tangent Line:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Law of Exponents

1. $a^{x+y} = a^x a^y$ **2.** $a^{x-y} = \dfrac{a^x}{a^y}$

3. $(a^x)^y = a^{xy}$ **4.** $(ab)^x = a^x b^x$

Rate of Change:

$$\text{Average velocity} = \frac{\text{change in position}}{\text{time elapsed}} = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

Important Limit Laws:

3. $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$

5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

6. $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$

7. $\lim_{x\to a} c = c$ c a constant

8. $\lim_{x\to a} x = a$

9. $\lim_{x\to a} x^n = a^n$

10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ if n is even, we assume $a > 0$.

11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$
if n is even, we assume $\lim_{x\to a} f(x) > 0$.

Tangent Line Equation:

(after finding slope)

$$\boxed{y - y_1 = m(x - x_1)}$$

no simplification needed

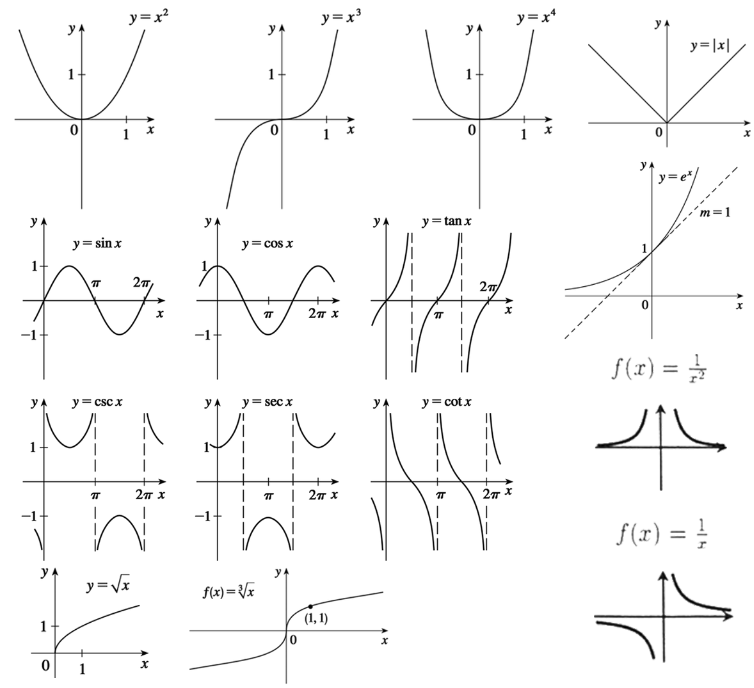
Steps for Squeeze Theorem:

1. Start with -1 ≤ *second* ≤ 1
2. The value of *first* is ≥ 0, so we multiply
3. Set -(*first*) ≤ *whole* ≤ *first*
4. Apply the limit values! (just add in front of the previous step)
5. Because both sides are 0, {*whole including limit*} must be 0 too.

Note: Look for two seperate parts after the limit!

Copy this onto exam first thing before you forget!

$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$
θ radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	←				
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und.
$\cot\theta$	←				
$\sec\theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	und.
$\csc\theta$	←				



$f(x) = \frac{1}{x^2}$

$f(x) = \frac{1}{x}$

Log Rules:

(1) $\log_a m + \log_a n = \log_a mn$

(2) $\log_a m - \log_a n = \log_a\left(\dfrac{m}{n}\right)$

(3) $\log_a m^n = n\log_a m$

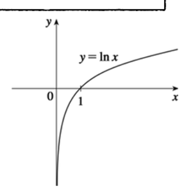
(4) $\log_a 1 = 0$

(5) $\log_a a = 1$

(6) $a^{\log_a x} = x$

(7) $\log_a x = \dfrac{\log_b x}{\log_b a}$

Important Rules:
$\log_e x = \ln x$
$\ln e = 1$
$\ln 1 = 0$
$\ln 0 = -\infty$



Simplification Property If $f(x) = g(x)$ when $x \neq a$, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x),$$

provided the limits exist. Thus we can simplify and THEN compute the limit.

Order Theorem If $f(x) \le g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x\to a} f(x) \le \lim_{x\to a} g(x).$$

Squeeze Theorem If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then

$$\lim_{x\to a} g(x) = L.$$

Trnsformations:

y=f(x ± ↔) ± ↕

Random Review Notes:

- When writing "lim" there must be a function following!
- The last part of every theorem problem <u>must</u> end in a sentence
- With absolute values, turn them into piecewise functions!

Steps for Limits:

1. Is it infinity?
2. Can I plug in the limit directly?
3. Can I factor it?
4. If neither:
   a. Conjugate → $\lim_{h\to 0} \dfrac{\sqrt{4+h}-2}{h} \cdot \left(\dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2}\right)$
   b. things cancel
   c. then plug in the limit!