Solutions

Math 1300-005 - Spring 2017

Introduction to Continuity - 1/30/17

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Find the numbers, if any, at which the following functions are discontinuous. Explain your answer by showing which part of the definition of continuity the function fails to satisfy.

(a) $f(x) = \begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

We need only check continuity at ~~x~~ $a=3$:

① $f(3)$ is defined and $f(3)=6$

② $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{2x^2-5x-3}{x-3} = \lim\limits_{x\to 3} \dfrac{(2x+1)\cancel{(x-3)}}{\cancel{x-3}} = 7$

③ since $f(3) \neq \lim\limits_{x\to 3} f(x)$, $\boxed{f \text{ is discontinuous at } a=3.}$

(b) $g(x) = \begin{cases} x+1 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$

We need only only check continuity at $\underbrace{a=0 \text{ and } a=1}_{\text{places where } f \text{ switches}}$

$\boxed{a=0}$

① $f(0)=e^0=1$, so defined at 0.

② $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x+1) = 1$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} e^x = 1$

③ $f(0) = \lim\limits_{x\to 0} f(x)$, so

f is continuous at 0.

$\boxed{a=1}$

① $f(1)=e^1=e$, so defined at $a=1$

② $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} e^x = e$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (2-x) = 2-1 = 1$

since $e \neq 1$, we have LHL $\neq$ RHL so $\lim\limits_{x\to 1} f(x)$ DNE. Hence

$\boxed{f \text{ is discontinuous at } a=1 \text{ only.}}$

2. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

We need only verify continuity at $a=2$.

① $f(2)$ is defined and $f(2) = 2^3 - c(2) = 8-2c$ ✓

② $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (cx^2+2x) = 4c+4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^3 - cx) = 8-2c$

we need RHL = LHL so $8-2c = 4c+4 \Leftrightarrow 4 = 6c$, so $c = \frac{2}{3}$.

$\boxed{\text{Thus if } c = \frac{2}{3}, f \text{ is cont. on } (-\infty, \infty)}$

3. Find the values of a and b that make g continuous on $(-\infty, \infty)$.

too hard for Midterm 1 but good to practice understanding.

$$g(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

we must verify g is cont. at $x=2$ and $x=3$.

$\boxed{x=2}$

① $g(2) = a(2)^2 - b(2) + 3 = 4a - 2b + 3$

② $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim_{x \to 2^-} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= 4$

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$.

so we need $\underset{x \to 2}{LHL} = \underset{x \to 2}{RHL} \Leftrightarrow 4 = 4a - 2b + 3$

$\boxed{x=3}$

① $g(3) = 2(3) - a + b = 6 - a + b$

② $\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

$\lim_{x \to 3^+} g(x) = \lim_{x \to 3^+} (2x - a + b) = 6 - a + b$

so for $\underset{x \to 3}{LHL} = \underset{x \to 3}{RHL}$ we need $9a - 3b + 3 = 6 - a + b$

To get continuity at both $x=2$ and $x=3$, we have to simultaneously solve 2 equations for 2 unknowns

$4 = 4a - 2b + 3$ (1)

$9a - 3b + 3 = 6 - a + b$ (2)

Rearranging (2) gives $10a - 4b = 3$, so

solve

Rearrange 1 gives $4a - 2b = 1$, so solve

(1) $1 = 4a - 2b$

(2) $3 = 10a - 4b$

you should know how to solve this from high school

Solving 1 for a gives $a = \frac{1+2b}{4}$, plug this into (2)

$3 = 10\left(\frac{1+2b}{4}\right) - 2b \rightarrow \boxed{b = \frac{1}{6} \text{ so } a = \frac{1}{3}}$