MATH 1300: HW #2

Due on January 26, 2017 at 10:00am

Professor Braden Balentine Section 005

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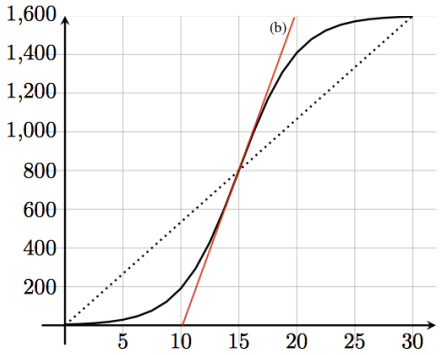
Graphical Problems

2. The solid curve in the graph below gives the position s of a car along a straight roadway (measured in meters), as a function of time t (measured in seconds).



(a) Find the slope of the dotted line in the graph above. Explain (including units), what this slope represents.

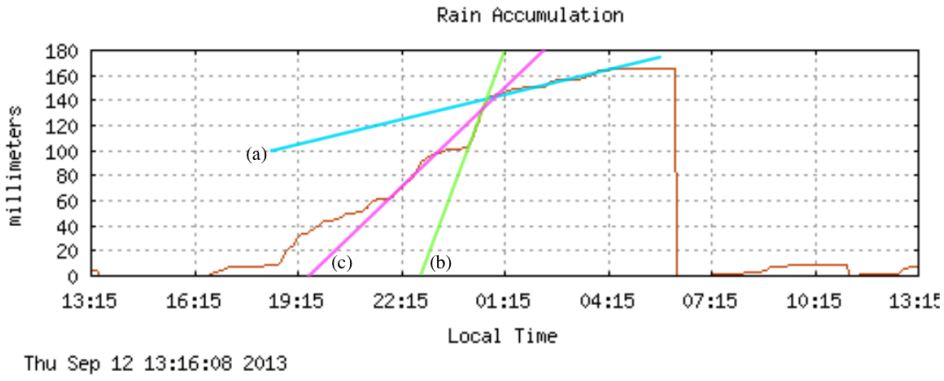
$$\text{Slope: } \frac{1600}{30} = \frac{160}{3} \approx 53.3 \text{ m/s}$$

In this problem, the slope represents the average meters per second the car travels in the test.

(b) Estimate the instantaneous velocity at $t = 15$. Include Units. Draw and label the line you used to estimate this.

$$\frac{1600 - 0}{19 - 11} = \frac{1600}{8} = 200 \text{ m/s}$$

3. Below is plot of the rainfall accumulation from the 2013 Boulder flood taken from the Foothills Lab Weather Station. The rainfall is measured in millimeters.



(a) Use the graph to estimate the average rainfall rate at 04:15 pm (marked as 16:15 on the graph) and 4:15 am the next morning (marked as 05:14 on the graph). Show all work including units. Draw the line that you are finding the slope of.

$$\text{4:14pm: } 0 \text{ mm of rain}$$

$$\text{4:15am: } \frac{78 - 44}{4 - 1} = 11.34 \text{ mm of rain per hour}$$

(b) When is it raining hardest? Explain how you know.
It appears that the time with the greatest rainfall is just after midnight (0:15), when the graph has the greatest slope.

(c) Estimate the rainfall rate at 22:15 (including units). Draw the line that you are finding the slope of.

$$\frac{145}{7} \approx 20.72 \text{ mm/s}$$

(d) What does the graph indicate is happening to the rainfall during the hour after 4:15 am?
Because the line has no slope and remains at the same y position, the graph is telling us that the rain stopped at 4:15am.

(e) Explain the precipitous drop between 04:15 and 07:15.
This is when all the rain accumulation vanished in a short period of time, which can also be thought of as a flood.

Section 2.1

4. The point $P(0.5, 0)$ lies on the curve $y = \cos \pi x$.

(a) If Q is the point $(x, \cos \pi x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

i. 0 Slope: 0
ii. 0.4 Slope: 4
iii. 0.49 Slope: 49
iv. 0.499 Slope: 499
v. 1 Slope: -2
vi. 0.6 Slope: -6
vii. 0.51 Slope -51
viii. 0.501 Slope: -501

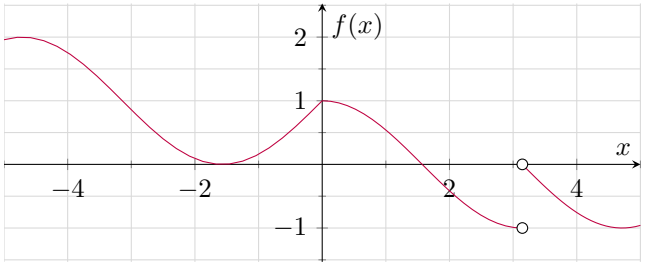
(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

Slope: 500

Section 2.2

8. Sketch the graph of the function and use it to determine the values for a for which $\lim_{x \to a} f(x)$ exists.

$$f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x < \pi \\ \sin x & \text{if } x > \pi \end{cases}$$

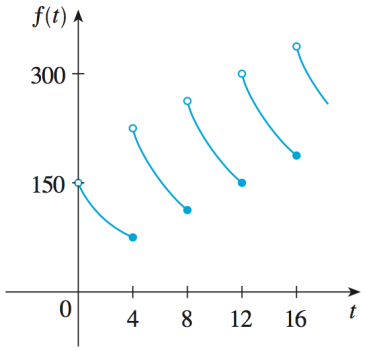


$$a = (-\infty, \pi] \cup [\pi, \infty)$$

12. A patient recieves a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \qquad \text{and} \qquad \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.

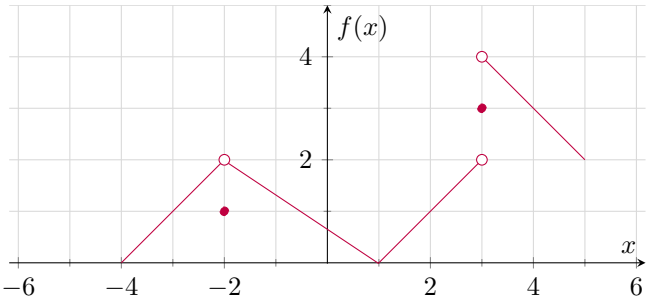


$$\lim_{t\to 12^-} f(t) = 150 \qquad \lim_{t\to 12^+} f(t) = 300$$

One-sided limits are almost more crucial than any other kind, as they have real world application. The limits in this problem demonstrate that just before 12, the patient has a very small dosage in their system, but after a booster injection, that amount is doubled to 300mg.

15. Sketch the graph of an example of a function f that satisfies all of the given conditions.

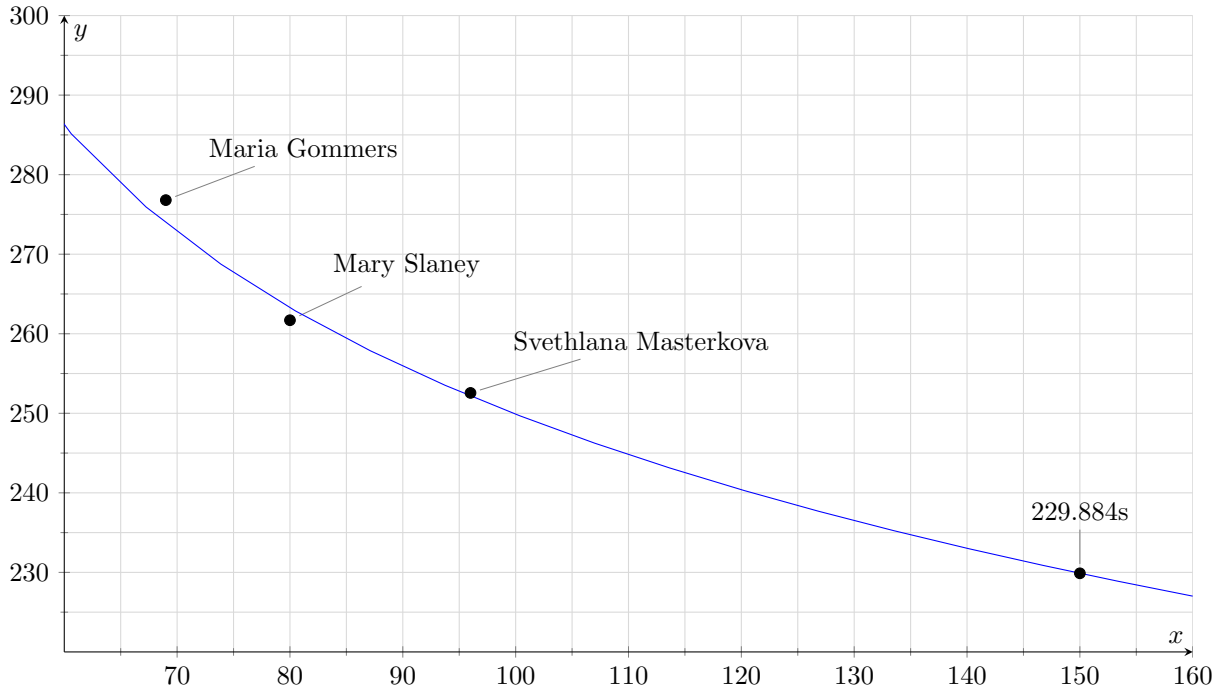
- $\lim_{x\to 3^+} f(x) = 4$
- $\lim_{x\to 3^-} f(x) = 2$
- $\lim_{x\to -2} f(x) = 2$
- $f(3) = 3$
- $f(-2) = 1$



Challenge Problem

- In 1969, the world record time for the mile was 4:36.8, held by Maria Gommers.
- In 1980, the world record was held by Mary Slaney, with a time of 4:21.7 (data from Wikipedia).
- In 1996, the world record was set by Svethlana Masterkova, in a time of 4:12.56

Using all the data given, find a shifted exponential model and use it to predict the record in 2050. What is the end-behavior of this model, and what does it represent?



$$-33\ln(x-40)+385$$

The end result of this model is that it gets increasingly harder and harder to break the world record, taking nearly 5,500 years to get below 100s. This means that the record will continue to be broken, but less and less often, or by smaller and smaller amounts.