Solutions

1. The purpose of this problem is to see how to construct a derivative function one point at a time by looking at a graph.
   Background review: estimating derivatives, one point at a time:

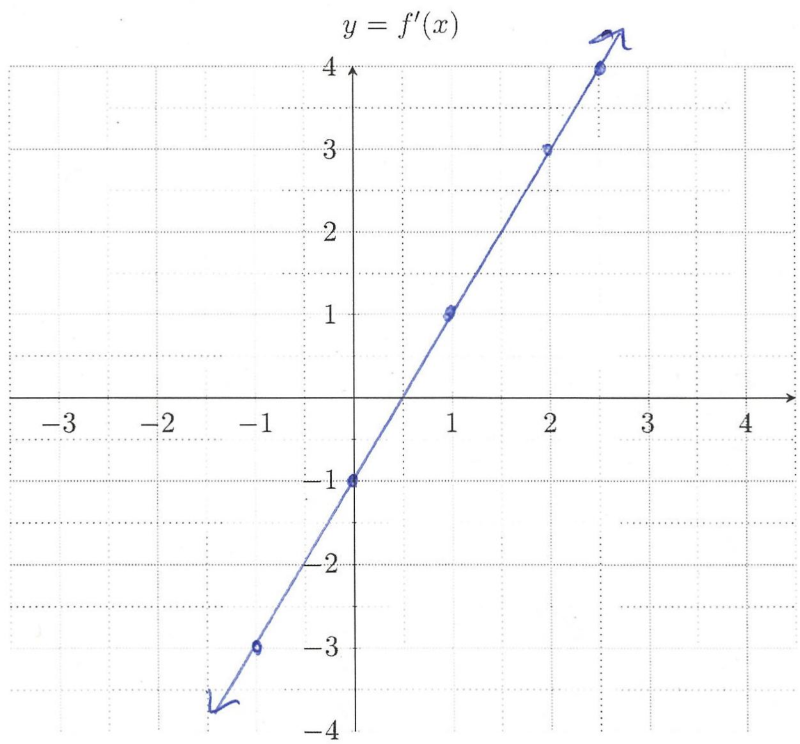
   - The derivative of a function at a point represents the slope (or rate of change) of a function at that point.
   - If you have a graph, you can estimate the derivative one point at a time by drawing the tangent line at that point, then calculating the slope of that tangent line (remember, slope is rise over run).

   (a) Go to the website `http://www.shodor.org/interactivate/activities/Derivate/`

   (b) Enter the function $y = x^2 - x - 2$. Use the tool to calculate the slope of the graph at each of the points x = -1, 0, 1, 2 and 2.5. Enter the values of the slope in the following table:

x	-1	0	1	2	2.5
slope of f at x	-3	-1	1	3	4

   (c) Now plot these points and connect them smoothly to see a graph of $f'(x)$



   (d) What do you think the formula for this graph is?

From the graph, m=2 and b=-1, so y=2x-1 is the formula.

2. In this problem, you'll calculate the derivative of the same function as the previous problem, but this time you'll do it analytically (with formulas)

   (a) Calculate the derivative of $f(x) = x^2 - x - 2$.

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - (x+h) - 2 - (x^2 - x - 2)}{h}$$

   $$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-x} - h \cancel{-2} \cancel{-x^2} \cancel{+x} \cancel{+2}}{h}$$

   [notice: everything w/o an h cancels!]

   $$= \lim_{h\to 0} \frac{2xh + h^2 - h}{h}$$

   [factor h from numerator]

   $$= \lim_{h\to 0} \frac{\cancel{h}(2x + h - 1)}{\cancel{h}}$$

   $$= \lim_{h\to 0} (2x + h - 1)$$

   $$= \boxed{2x - 1}$$

   (b) Do your results from this problem match your results from the last problem?

   Yes, duh!

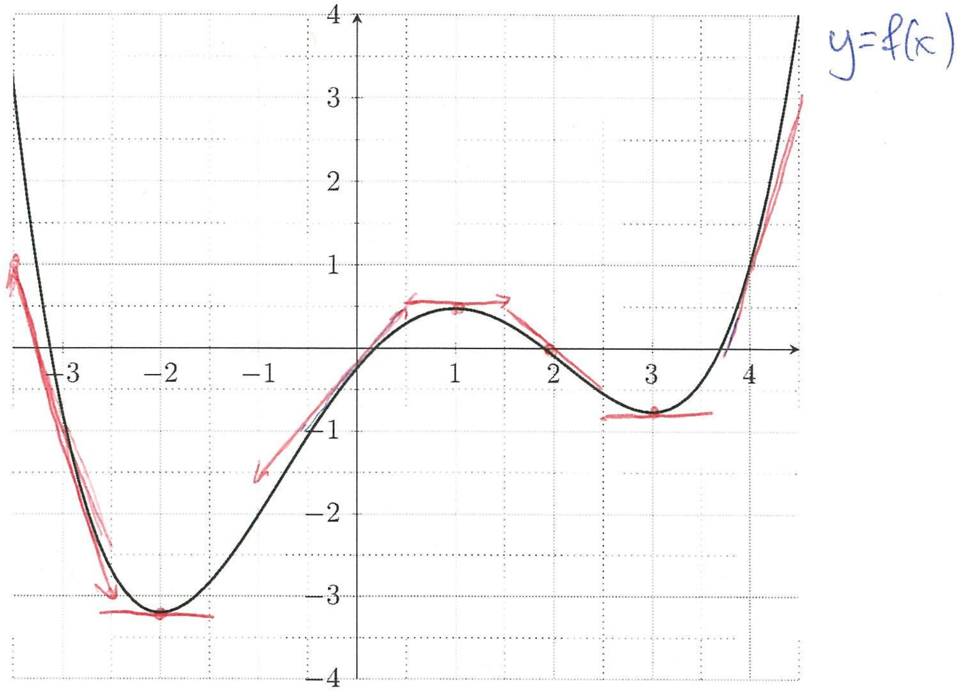
3. Khan academy has an exercise that gives a good visual and tactile experience of producing the derivative function point by point. Do at least one exercise on this website:

   `http://www.tinyurl.com/math1300-week4`

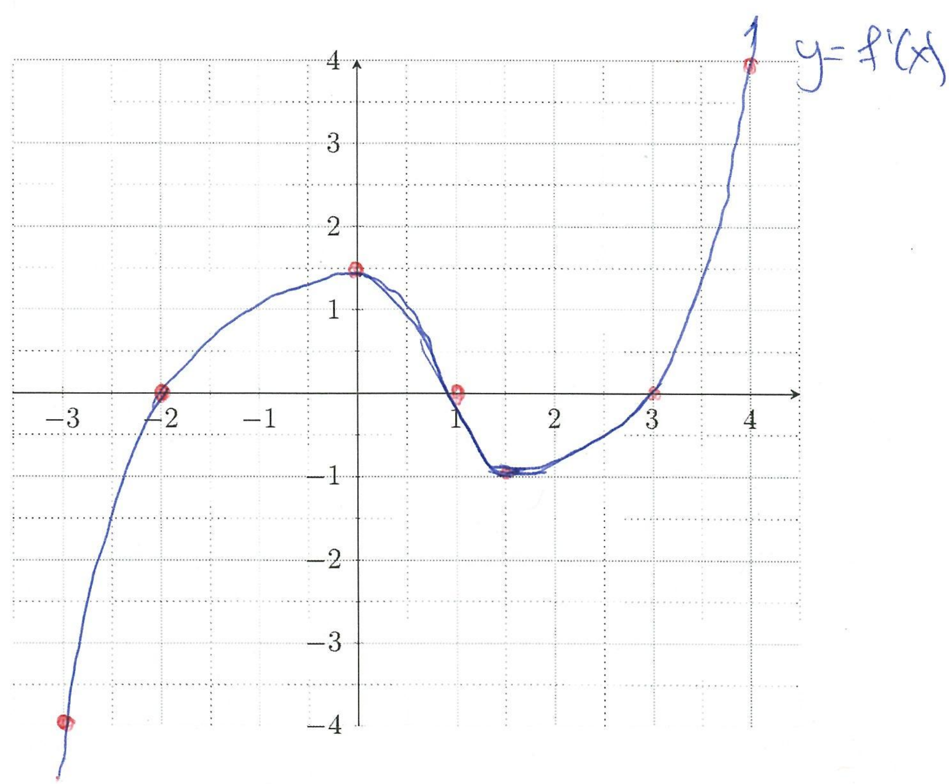
   In the space below, sketch the graph of $f(x)$ and $f'(x)$ from one of the exercises you did.

   Do on your own if you want.

4. Below is a graph of a function.



Without the aid of technology, use the graph of the function above to sketch a graph of its derivative function on the axes below.



1) Find x-values on the graph above where the tangent line has zero slope. These are the roots for the derivative

{-2, 1, 3}

2) Between these values, analyze how slopes of tangent lines are changing to make a qualitative sketch. To be more specific, notice:

at x=-3, slope=-4 ; x=2, slope = -1 ; x=0, slope = 3/2 ; x=4, slope = 4 } plot these as points for derivative graph → then you can fill in the shape!

5. Based on your experience above, what seems to be true about the relationship between $f(x)$ and $f'(x)$?

   (a) Where $f(x)$ is increasing, $f'(x)$ is positive.

   (b) Where $f(x)$ is decreasing, $f'(x)$ is negative.

6. Below are some more involved questions. We will be addressing these in the coming sections. Do you have any guesses for these?

   (a) Where $f(x)$ is concave up, $f'(x)$ is increasing.

   (b) Where $f(x)$ is concave down, $f'(x)$ is decreasing.

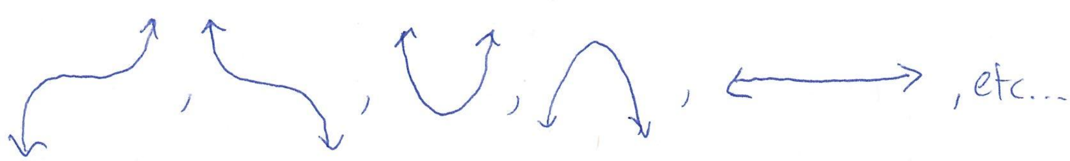
   (c) Where $f(x)$ is non-existant, has a corner or cusp, or is discontinuous $f'(x)$ is undefined.

Remember how this can happen!

   (d) What will the derivative be when $f(x)$ has a relative high point (maximum) or relative low point (minimum)?

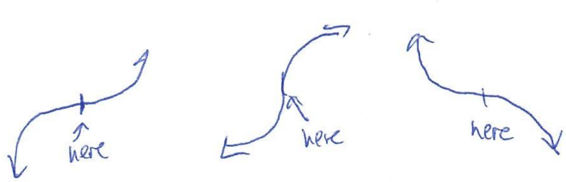
Either 0 or non-existant in the case of a cusp or corner

   (e) If the derivative is 0 at a point, what are all the ways the original function could look?



   (f) What about if the derivative has a relative maximum or minimum?

This means $f''(x)=0$ or f'' non-existant. Could look like



, etc... basically these are points where concavity changes.