MATH 1300: HW #15

Due on May 4, 2017 at 10:00am

Professor Braden Balentine Section 005

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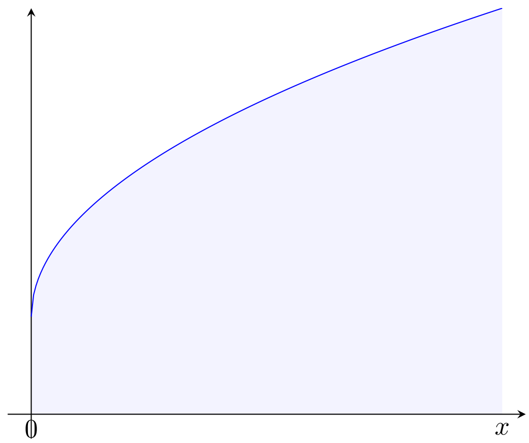
Section 5.4

6. Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using the Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

$$g(x) = \int_0^x (1+\sqrt{t})dt$$



(a)

$$\begin{aligned} g(x) &= -\int_0^x (1+\sqrt{t})dt \\ &= -\int_x^0 (1+\sqrt{t})dt \\ g'(x) &= -1 \cdot \frac{d}{dt}\left[\int_0^x (1+\sqrt{t})dt\right] \\ g'(x) &= \boxed{\frac{2x^{\frac{2}{3}}}{3} + x} \end{aligned}$$

(b)

$$\begin{aligned} g(x) &= \int_0^x \left(1+\sqrt{t}\right) dt \\ &= \left(t + \frac{2t^{\frac{3}{2}}}{3}\right)\Bigg]_0^x \\ &= \left(x + \frac{2(x)^{\frac{3}{2}}}{3}\right) - \left(0 + \frac{2(0)^{\frac{3}{2}}}{3}\right) \\ &= \boxed{\frac{2x^{\frac{2}{3}}}{3} + x} \end{aligned}$$

18. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_{\sin x}^{\cos x} (1+v^2)^{10} dv$$

$$\begin{aligned}
y &= -\int_{\cos x}^{\sin x} (1+v^2)^{10} dv \\
&= -\int_{\cos x}^{\sin x} 1+v^{20} dv \\
\frac{d}{dy}(y) &= -1 \cdot \frac{d}{dy}\left[\int_{\cos x}^{\sin x} 1+v^{20} dv\right] \\
y' &= -1 \cdot \left(1+y^{20}\right) \\
&= \boxed{-1-y^{20}}
\end{aligned}$$

23. On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2+t+2} dt$$

$$= t - \frac{3\arctan\left(\frac{2\left(t+\frac{1}{2}\right)}{\sqrt{7}}\right)}{\sqrt{7}} - \frac{1}{2}\ln\left|\left(1+\frac{1}{2}\right)^2 + \frac{7}{4}\right| + C$$

$$= x + \frac{1}{14}\left(-7\ln\left(x^2+x+2\right) - 6\sqrt{7}\arctan\left(\frac{2x+1}{\sqrt{7}}\right) + \ln(128) + 6\sqrt{7}\text{arccot}\left(\sqrt{7}\right)\right)$$

30. Let

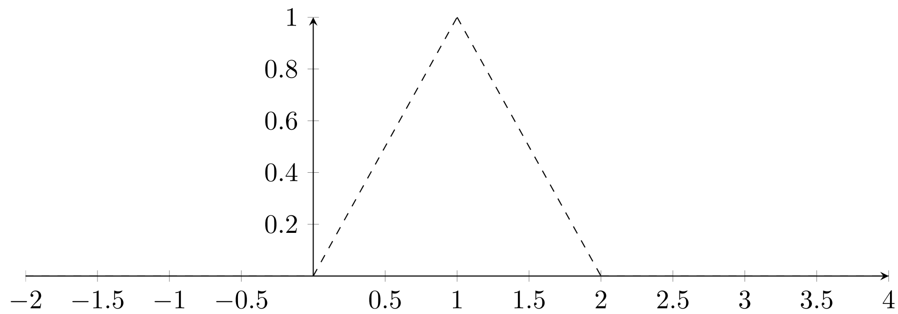
$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } 1 < x < 2 \\ 0 & \text{if } x > 2 \end{cases}$$

and

$$g(x) = \int_0^x f(t) dt$$

(a) Find an expression for $g(x)$ similar to the one for $f(x)$.

(b) Sketch the graphs of f and g.



(c) Where is f differentiable? Where is g differentiable?

Section 5.5

15. Evaluate the indefinite integral.

$$\int \frac{dx}{5-3x}$$

$$\int \frac{1}{5-3x}dx$$

$$\int -\frac{1}{3u}du$$

$$-\frac{1}{3}\cdot\int \frac{1}{u}du$$

$$-\frac{1}{3}\ln|u| - \frac{1}{3}\ln|5-3x|$$

$$-\frac{1}{3}\ln|5-3x| + C$$

68. If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.

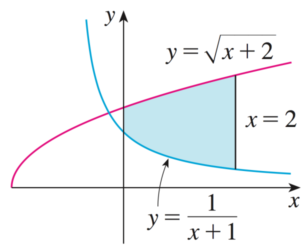
70. If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(x+c)dx = \int_{a+c}^{b+c} f(x)dx$$

$$\int_a^b f(x)dx + \int_a^b f(c)dx$$

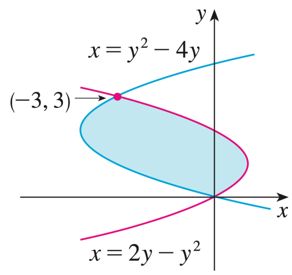
Section 6.1

2. Find the area of the shaded region.



$$\int_0^2 \left(-\frac{1}{1+x} + \sqrt{2+x}\right) dx = \frac{1}{3}\left(16 - 4\sqrt{2}\right) - \ln(3) \approx 2.3491$$

4. Find the area of the shaded region.



$$\int_0^3 (y^2 - 4y) - (2y - y^2)$$

$$\int_0^3 2y^2 - 6y = \frac{2y^3}{3} - \frac{6y^2}{2}\Big]_0^3 = \left(\frac{2(3)^3}{3} - \frac{6(3)^2}{2}\right) - \left(\frac{2(0)^3}{3} - \frac{6(0)^2}{2}\right) = \boxed{-9}$$

10. Sketch the region enclosed by the given circles. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

$$4x + y^2 = 12, \qquad x = y$$