

Math 1300, Midterm 2

March 6, 2017

PRINT YOUR NAME: John Keller

PRINT INSTRUCTOR'S NAME: Braden Balentine

Mark your section/instructor:

<input type="checkbox"/>	Section 001	Brendt Gerics	8:00 - 8:50
<input type="checkbox"/>	Section 002	Leo Herr	9:00 - 9:50
<input type="checkbox"/>	Section 003	Tyler Schrock	9:00 - 9:50
<input type="checkbox"/>	Section 004	Lee Roberson	10:00 - 10:50
<input checked="" type="checkbox"/>	Section 005	Braden Balentine	10:00 - 10:50
<input type="checkbox"/>	Section 006	Xingzhou Yang	10:00 - 10:50
<input type="checkbox"/>	Section 007	Lee Roberson	11:00 - 11:50
<input type="checkbox"/>	Section 008	Shen Lu	11:00 - 11:50
<input type="checkbox"/>	Section 009	Suzanne Craig	12:00 - 12:50
<input type="checkbox"/>	Section 010	Carlos Pinilla-Suarez	12:00 - 12:50
<input type="checkbox"/>	Section 011	Nathan Davidoff	1:00 - 1:50
<input type="checkbox"/>	Section 012	Sion Ledbetter	1:00 - 1:50
<input type="checkbox"/>	Section 013	Ruofan Li	2:00 - 2:50
<input type="checkbox"/>	Section 014	Daniel Martin	2:00 - 2:50
<input type="checkbox"/>	Section 015	Isabel Corona	3:00 - 3:50
<input type="checkbox"/>	Section 016	Ira Becker	3:00 - 3:50
<input type="checkbox"/>	Section 017	Ira Becker	4:00 - 4:50
<input type="checkbox"/>	Section 0430R	Patrick Newberry	11:00 - 11:50

Question	Points	Score
1	6	5
2	6	6
3	12	9
4	12	5
5	10	8
6	5	5
7	10	9
8	10	6.8
9	10	6
10	9	9
11	10	8
Total:	100	78

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $100/7$ or expressions like $\ln(3)/2$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (6 points) Let $f(x) = \frac{1}{3}x^3 + x^2 - x + 2$. Find all value(s) of a such that $y = 2x + 11$ is the tangent line to the graph of $f(x)$ at $x = a$.

$$\begin{aligned} f'(x) &= 1x^2 + 2x - 1 & \frac{1}{3}(-3)^3 + (-3)^2 - (-3) + 2 \\ 2 &= 1x^2 + 2x - 1 & -9 + 9 + 3 + 2 \\ 0 &= x^2 + 2x - 3 & 5 \\ 0 &= (x+3)(x-1) & \\ x &= -3, 1 \end{aligned}$$

$$\begin{aligned} y &= 2(-3) + 11 & \frac{1}{3}(1)^3 + (1)^2 - (1) + 2 \\ y &= -6 + 11 & 1 + 1 - 1 + 2 \\ y &= 5 & 7 \\ y &= 2(1) + 11 & 13 \neq 7 \\ y &= 13 \end{aligned}$$

$$\boxed{\begin{array}{l} a = 1 \\ a = -3 \end{array}}$$

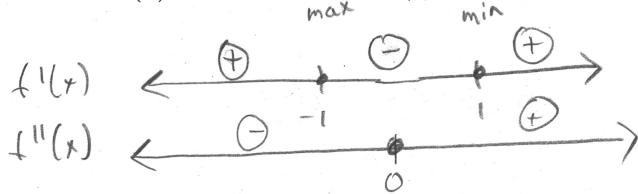
5/6

$+ \rightarrow -$ max
 $- \rightarrow +$ min

2. (6 points) Let $f(x) = \frac{2}{3}x^3 - 2x + 4$.

$$f'(x) = 2x^2 - 2$$

(a) Find the interval(s) where f is decreasing.



$$0 = 2x^2 - 2$$

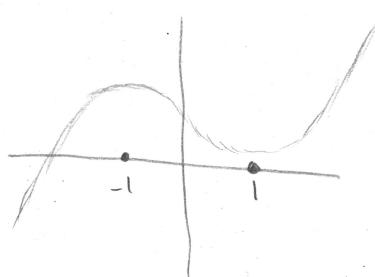
$$2 = \frac{2x^2}{2}$$

$$1 = x^2$$

c-points:
 $x = 1, -1$
 $f''(x) = 4x$

$$\frac{0}{4} = x$$

Decreasing @ $(-1, 1)$



(b) Find the interval(s) where f is concave up.



$$f''(x) = 4x$$

$$0 = 4x$$

$$\frac{0}{4} = x$$

Concave up @ $(0, \infty)$

3. (12 points) **Multiple choice** – circle the derivative of the given function. You DO NOT need to show your work.

$$-\cos(x) \sec(x)$$

(a) $g(t) = \sec(8t^2)$ $-\cos(8t^2) \sec(8t^2) +$

A. $8t^2 \sec(8t^2) \tan(8t^2)$ B. $16t \sec(8t^2) \tan(8t^2)$

C. $16t(\tan(t))^2$

D. $-8t^2 \cos(8t^2) \cot(8t^2)$

(b) $f(x) = \sin(1 + \pi^2)$ $\cos(1 + \pi^2)(0)$

A. $\cos(1 + \pi^2)$

B. $\cos(1 + \pi^2)(2\pi)$

C. $-\cos(1 + \pi^2)(2\pi)$

D. 0

(c) $v(t) = \sqrt{t^2 + 1}$ $\frac{1}{2} (t^2 + 1)^{-1/2} (2t)$ $\frac{1}{2} \left(\frac{2}{t^2 + 1} \right) (2t)$

A. $\frac{t}{\sqrt{t^2 + 1}}$

B. $-2\sqrt{2t} + 1$ $\left(\frac{1}{\sqrt{2t}} \right) 2t$

C. $-\frac{t}{\sqrt{t^2 + 1}}$

D. $\sqrt{2t}(t^2 + 1)$

(d) $P(q) = \sin(q^3) + (\cos(q))^3$

A. $3q^2 \cos(q^3) - 3q^2 \sin(q)$

B. $3q^2 \cos(q^3) - 3 \sin(q) (\cos(q))^2$

C. $3q^2 \cos(q^3) - 3(\sin(q))^2$

D. $3(\sin(q^3) + (\cos(q))^3)^2 (\cos(q^3) - (\sin(q))^2)$

$$\cos(q^3)(3q^2) + 3(\cos(q))^2(-\sin(q))$$

$$3q^2 \cos(q^3)$$

4. (12 points) Compute the following derivatives. DO NOT simplify your answers.

(a) $\frac{d}{dx} \left[5x^3 - \frac{3}{4x} + 7^x \right] =$
 anything stays
 ↑ ↑
 power quotient rule

$$\frac{d}{dx} = 15x^2 - \left(\frac{0 \cdot 4x - 3 \cdot 4}{(4x)^2} \right) + 7^x \ln(7)$$

2

②

(b) $\frac{d}{dx} \left[\frac{\tan x}{7x^3 - 4x} \right] =$ quotient rule!

sin(x)

$$\frac{(7x^3 - 4x) \cancel{(\sec(x))} - \tan(x)(21x^2 - 4)}{(7x^3 - 4x)}$$

2

(c) $\frac{d}{dx} \left[\sqrt{x} e^{-x^2} \right]$

a = \sqrt{x}

b = e^x

c = $-x^2$

$(\sqrt{x}) \left(e^{-x^2} \circ (-2x) \right)$

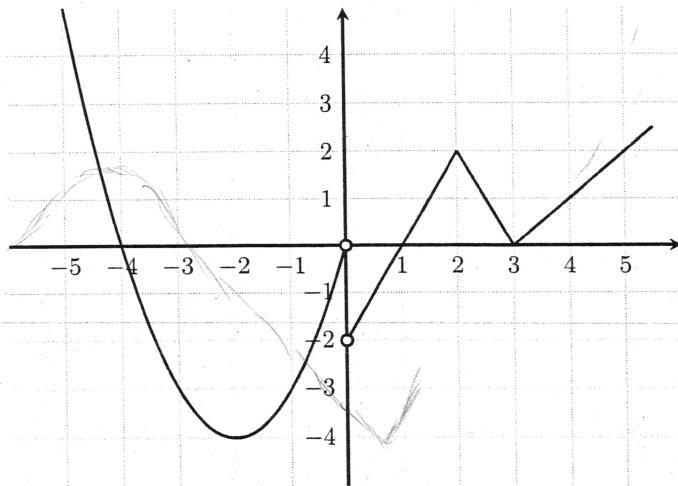
$$\frac{d}{dx} = \sqrt{x} \left(e^{-x^2} \circ (-2x) \right)$$

product rule

1

5. (10 points) **Multiple choice** – circle the correct answer. The graph below shows the derivative of some function $f(x)$.

Graph of $f'(x)$



- (a) On which of the following intervals is $f(x)$ concave up?

- A. $(-5, -4)$ B. $(-5, 0)$ C. $(-2, 0)$ D. $(0, 5)$ E. $(2, 3)$

$$\begin{array}{c} \leftarrow \textcircled{+} \\ -4 \end{array} \quad \begin{array}{c} \textcircled{-} \\ 0 \end{array} \quad \begin{array}{c} \textcircled{-} \\ 1 \end{array} \quad \begin{array}{c} \textcircled{+} \\ 2 \end{array} \quad \begin{array}{c} \textcircled{+} \\ 3 \end{array}$$

- (b) At which of the following x -values does $f(x)$ have a local minimum?

- A. -4 B. -2 C. 0 D. 1 E. 2

- (c) On which of the following intervals is $f(x)$ decreasing?

- A. $(-5, -2)$ B. $(-4, -2)$ C. $(0, 2)$ D. $(2, 3)$ E. $(3, 5)$

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- (d) At which of the following x -values is $f''(x) = 0$?

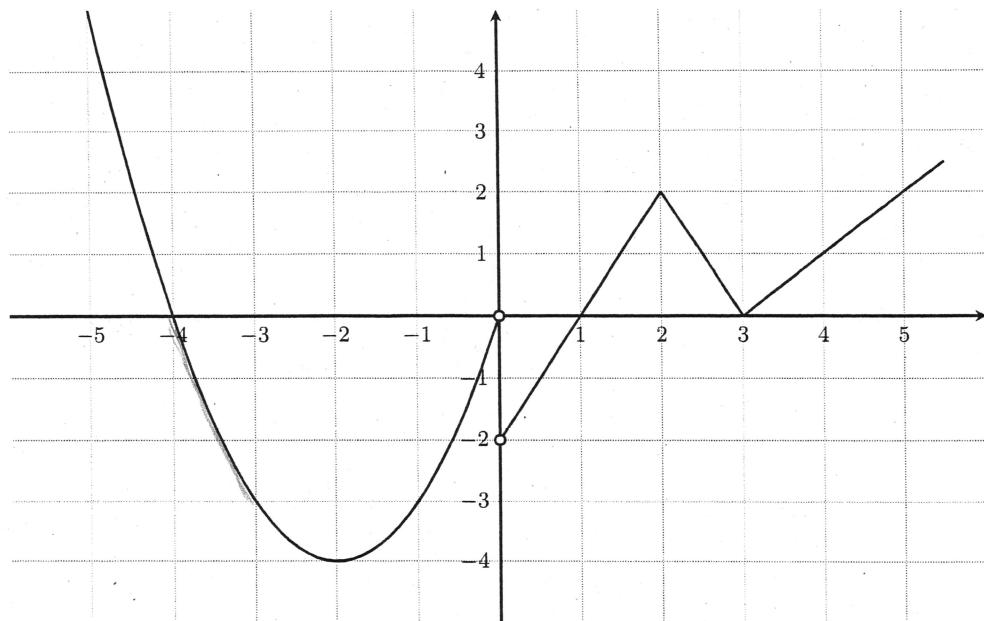
- A. -4 B. -2 C. 0 D. 2 E. 3

- (e) What is $f''(2.5)$?

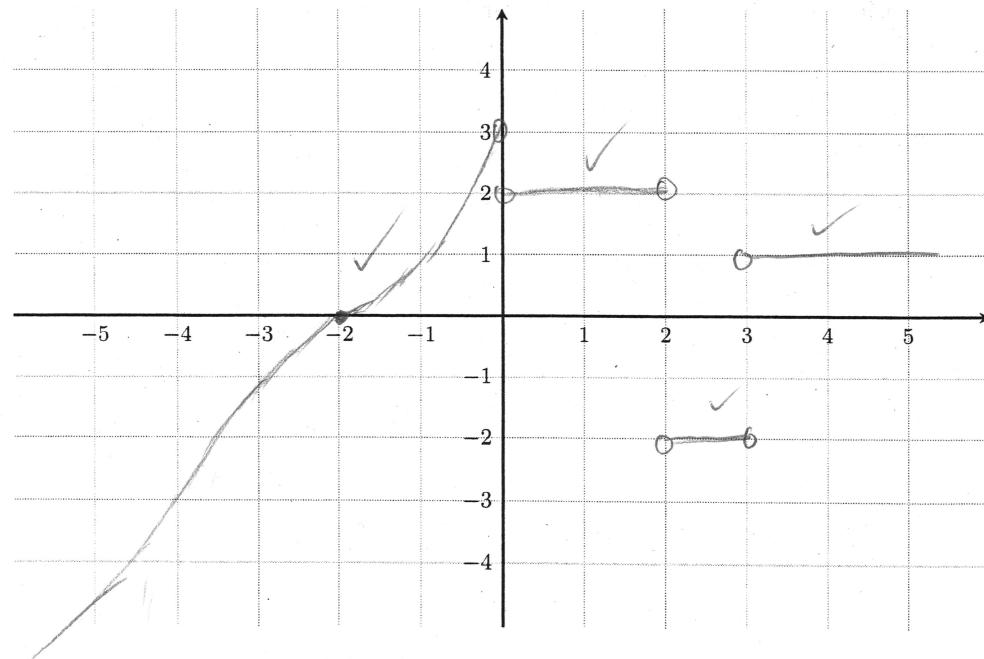
- A. -2 B. -1 C. 0 D. 1 E. 2

6. (5 points) The graph below shows the derivative of some function $f(x)$. Sketch a graph for the second derivative of $f(x)$.

Graph of $f'(x)$



Graph of $f''(x)$



7. (10 points) Consider the equation $y^2 - xy = 3 - x^2$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}(y^2 - xy) = \frac{d}{dx}(3 - x^2)$$

$$2y\frac{dy}{dx} - 1y + x\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}(2y + x) = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y - 2x}{2y + x}}$$

4

(b) Find an equation of the tangent line to $y^2 - xy = 3 - x^2$ at $(-1, 1)$.

$$m = \frac{(1) - 2(-1)}{2(1) + (-1)} = \frac{3}{1} = 3$$

$$1 = 3(-1) + b$$

$$\boxed{y = 3x + 4}$$

5

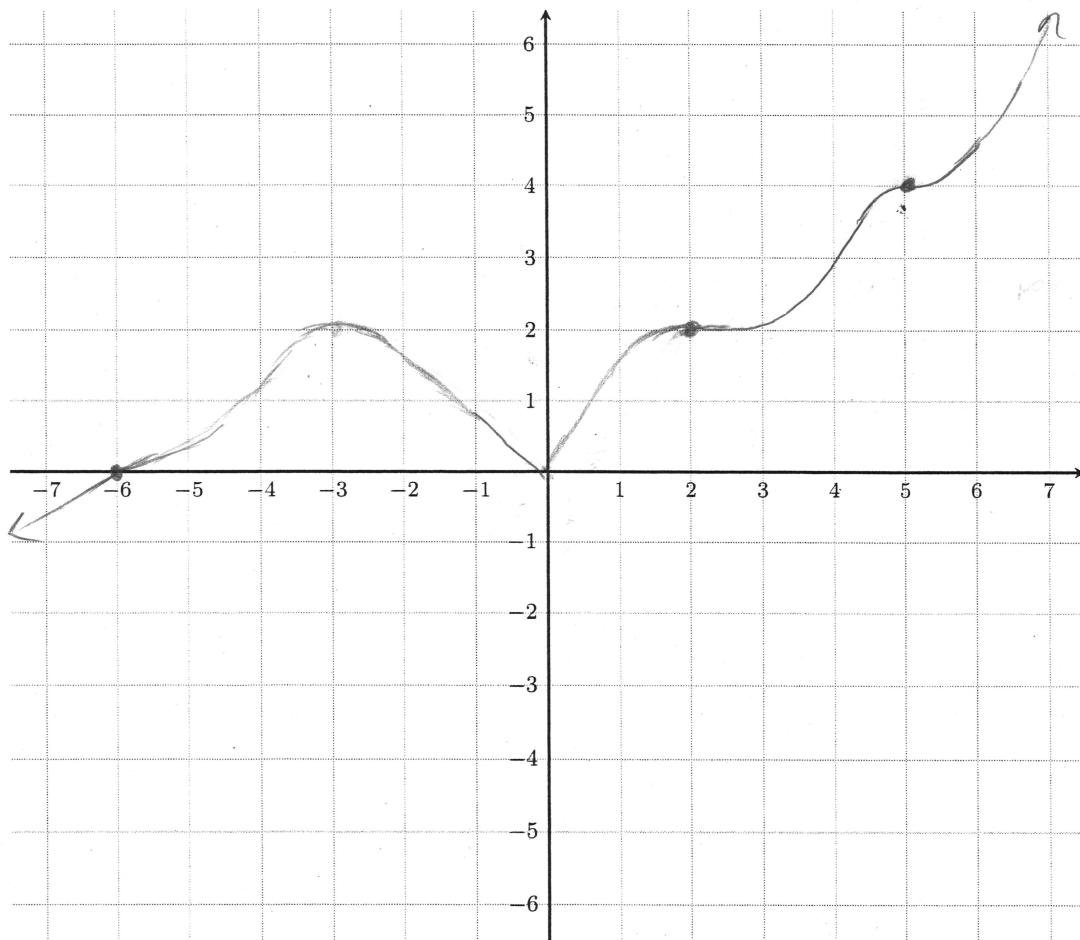
$$1 - 3(-1) = b$$

$$4 = b$$

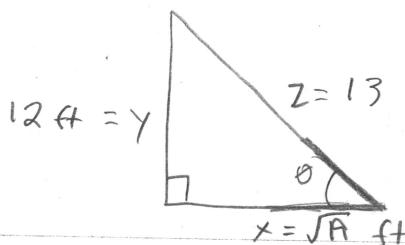
8. (10 points) Sketch the graph of a **function** $f(x)$ that satisfies **ALL** the conditions below:

- $f(x)$ is **continuous** on $(-\infty, \infty)$; ✓
- $f(-6) = f(0) = 0$; ✓
- $f'(-6)$ does not exist; -2
- $f'(-3) = f'(2) = f'(5) = 0$; ✓
- $f'(x) > 0$ on $(2, 5)$ and $(5, \infty)$; ✓
- $f''(x) > 0$ on $(0, 4)$ and $(5, \infty)$. ✓

Graph of $f(x)$



9. (10 points) A ladder 13 feet long is leaning against the side of a building. If the bottom of the ladder is pulled away from the building at a constant rate of 2 feet per second, how fast is the angle formed by the ladder and the ground changing at the instant when the top of the ladder is 12 feet above the ground? **Include units in your answer.**



$$x = \sqrt{z^2 - y^2}$$

$$z = 13 \text{ ft}$$

$$y = 12 \text{ ft}$$

$$\frac{dx}{dt} = 2 \text{ ft/sec}$$

$$\boxed{\frac{d\theta}{dt} = ?}$$

$$\frac{dy}{dt} =$$

$$\sin(\theta) = \frac{y}{z}$$

$$\theta = \sin^{-1}(12/13)$$

$$\frac{d}{dx} (\cos(\theta)) = \frac{d}{dx} \left(\frac{x}{z} \right) \quad 2$$

$$-\sin(\theta) \frac{d\theta}{dt} = \frac{dx}{dt} z - x \frac{dz}{dt} \quad \frac{z^2}{z^2}$$

$$\frac{(2)(13) - (\sqrt{13})(0)}{13^2}$$

$$\frac{26 - 0}{13^2}$$

$$-\sin(\theta) \frac{d\theta}{dt} = \frac{26}{169} \quad 2$$

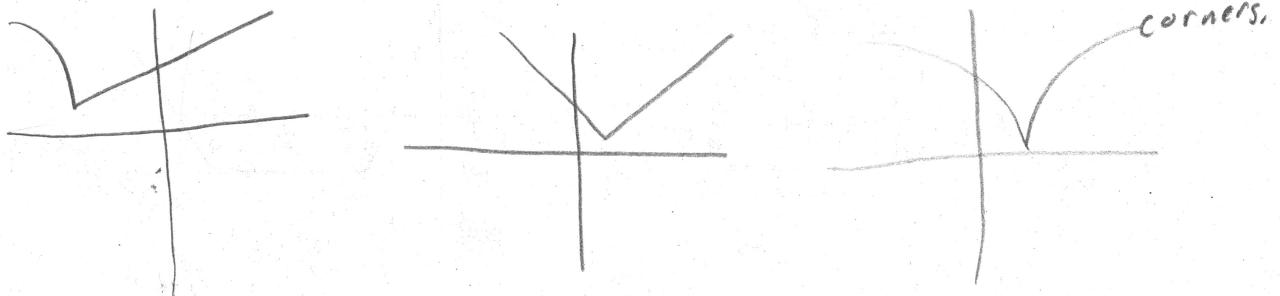
$$\boxed{\frac{d\theta}{dt} = \frac{\frac{26}{169}}{-\sin(\theta)}}$$

rad/sec.

10. (9 points) The following statements are all false. For each statement, justify why it is false by providing an explanation that includes a picture.

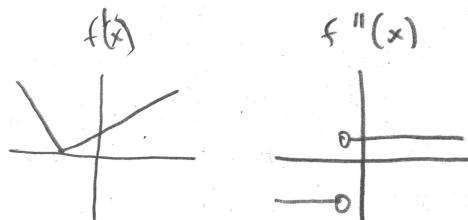
(a) If $f(x)$ is continuous on $(-\infty, \infty)$, then $f(x)$ is differentiable on $(-\infty, \infty)$.

$\times 3$ This statement is false because even though a function is continuous, the derivative fails at cusps and corners.



(b) If $f(x)$ is differentiable at $x = a$, then $f'(x)$ is also differentiable at $x = a$.

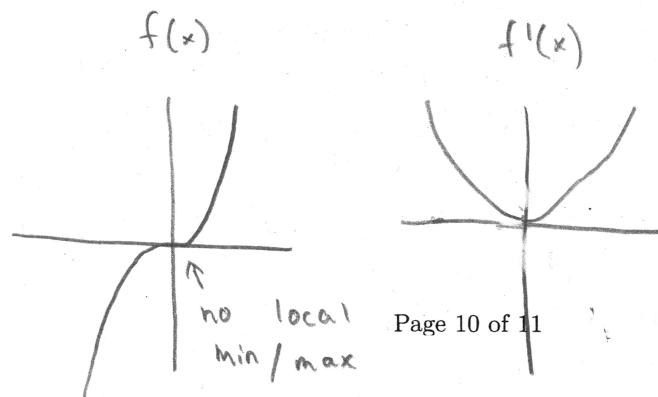
$\times 3$ This statement is false because when a cusp or corner is located at "a", then $f''(a)$ is undefined, making $f'(a)$ not differentiable.



(c) If $f'(a) = 0$, then $f(x)$ either has a local minimum or a local maximum at

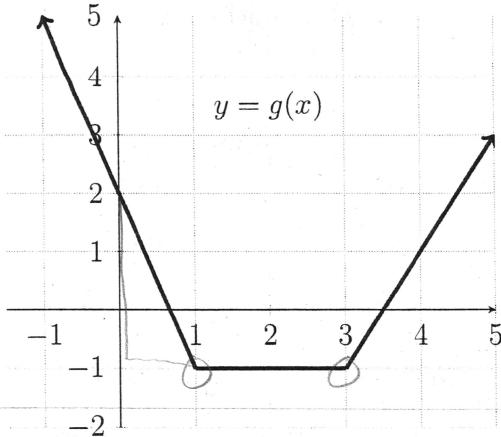
$x = a$.

$\times 3$ This statement is false because even though the $f'(a)$ slope is zero, the function can still increase after.



11. (10 points) Suppose f and g are differentiable functions with values and derivatives given in the table below, and $y = g(x)$ be represented by the graph below.

x	$f(x)$	$f'(x)$
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	6



(a) Let $p(x) = f(x) + g(x)$. Then $p'(1) =$

- A. -7 B. -2 C. 0 D. 3 E. 8 F. DNE

$5 + \text{DNE}$

(b) Let $h(x) = f(x) \cdot g(x)$. Then $h'(2) =$

- A. -5 B. -3 C. 0 D. 2 E. 4 F. DNE

$3(-1) + 9(4)$

(c) Let $k(x) = g(f(x))$. Then $k'(3) =$

- A. -12 B. -3 C. 0 D. 1 E. 2 F. DNE

$g'(0) = (-3)(4)$

8

(d) Let $R(x) = \sqrt{f(x)}$. Then $R'(2) =$

- A. $\frac{1}{12}$ B. $\frac{1}{6}$ C. $\frac{1}{2}$ D. $\frac{3}{2}$ E. $\frac{9}{2}$ F. DNE

$\frac{1}{2}(f(2))^{-1/2} f'(2)$

$\frac{1}{2}(9)^{-1/2}(3)$

$\frac{1}{2}(\frac{1}{18})(3)$

(e) Let $L(x) = g(x^2)$. Then $L'(2) =$

- A. 0 B. 1 C. 2 D. 4 E. 8 F. DNE

$\frac{1}{36} \cdot \frac{3}{36} = \frac{1}{12}$

$g'(4) \cdot 4$

$2 \cdot 4$