Math 1300-005 - Spring 2017

Solutions

Chain Rule Activity, Part I - 2/22/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the next midterm.

Recall ***the chain rule***, which states

$$\frac{d}{dx}q(r(x)) = q'(r(x))\frac{d}{dx}r(x)$$

1. Use the chain rule to differentiate the following.

(a) $f(x) = \sqrt{1+\cos(x)}$

outside: $(\quad)^{1/2}$
inside: $1+\cos(x)$

$$f'(x) = \frac{1}{2}(1+\cos(x))^{-1/2} \cdot \frac{d}{dx}(1+\cos(x))$$

$$= \boxed{\frac{1}{2}(1+\cos(x))^{-1/2} \cdot (-\sin(x))}$$

(b) $g(x) = e^{(x^2-\cot(x))}$

outside: $e^{(\quad)}$
inside: $x^2-\cot(x)$

$$g'(x) = e^{(x^2-\cot(x))} \cdot \frac{d}{dx}(x^2-\cot(x))$$

$$= \boxed{e^{(x^2-\cot(x))} \cdot (2x+\csc^2(x))}$$

(c) $h(x) = \sec(2x^3 - 9x^2 + 4)$

outside: $\sec(\quad)$
inside: $2x^3-9x^2+4$

this is all deriv of outside composed w/ inside

$$h'(x) = \underbrace{\sec(2x^3-9x^2+4)\tan(2x^3-9x^2+4)}_{} \cdot \frac{d}{dx}(2x^3-9x^2+4)$$

$$= \boxed{\sec(2x^3-9x^2+4)\tan(2x^3-9x^2+4) \cdot (6x^2-18x)}$$

(d) $\ell(x) = \sin(2^x - \tan(x))$

outside: $\sin(\quad)$
inside: $2^x - \tan(x)$

$$\ell'(x) = \cos(2^x-\tan(x)) \cdot \frac{d}{dx}(2^x-\tan(x))$$

$$= \boxed{\cos(2^x-\tan(x)) \cdot (2^x\ln(2) - \sec^2(x))}$$

2. It often happens that you have to do the chain rule within the product and quotient rules. Keep this in mind to differentiate the following.

(a) $F(x) = (2x-5)^4(8x^2-3x)^{-3}$ → Product rule first

chain ↑   chain ↑

$$F'(x) = \underbrace{\left[\frac{d}{dx}(2x-5)^4\right]}_{} \cdot (8x^2-3x)^{-3} + (2x-5)^4 \cdot \frac{d}{dx}(8x^2-3x)^{-3}$$

outside: $(\quad)^4$
inside: $2x-5$

outside: $(\quad)^{-3}$
inside: $8x^2-3x$

$$= \left[4(2x-5)^3 \cdot \frac{d}{dx}(2x-5)\right](8x^2-3x)^{-3} + (2x-5)^4 \cdot \left[-3(8x^2-3x)^{-4}\frac{d}{dx}(8x^2-3x)\right]$$

$$= \boxed{\left[4(2x-5)^3 \cdot 2\right](8x^2-3x)^{-3} + (2x-5)^4 \cdot \left[-3(8x^2-3x)^{-4}(16x-3)\right]}$$

(b) $G(x) = \dfrac{x}{\sqrt{x^2+1}}$ → quotient rule first

→ chain:
outside: $(\quad)^{1/2}$
inside: x^2+1

$$G'(x) = \frac{\left(\frac{d}{dx}x\right)\sqrt{x^2+1} - x \cdot \frac{d}{dx}\sqrt{x^2+1}}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{(1)\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot \frac{d}{dx}(x^2+1)\right]}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \boxed{\frac{\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right]}{x^2+1}}$$

3. As well, it often happens that you must do the product or quotient rule within the chain rule. Differentiate the following.

(a) $H(x) = \left(\frac{1+x^2}{2-x^4}\right)^{-1/3}$

outside: $(\quad)^{-1/3}$

inside: $\frac{1+x^2}{2-x^4}$ } quotient rule!

$$H'(x) = -\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{d}{dx}\left(\frac{1+x^2}{2-x^4}\right) \rightarrow QR$$

$$= -\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{\left[\frac{d}{dx}(1+x^2)\right]\cdot(2-x^4) - (1+x^2)\cdot\frac{d}{dx}(2-x^4)}{(2-x^4)^2}$$

$$= \boxed{-\frac{1}{3}\left(\frac{1+x^2}{2-x^4}\right)^{-4/3} \cdot \frac{2x(2-x^4) - (1+x^2)(-4x^3)}{(2-x^4)^2}}$$

(b) $L(x) = e^{(x^2 \csc(x))}$

outside: $e^{(\quad)}$

inside: $\underbrace{x^2 \csc(x)}_{\text{product}}$

$$L'(x) = e^{(x^2\csc(x))} \cdot \frac{d}{dx}(x^2\csc(x)) \rightarrow \text{product rule}$$

$$= \boxed{e^{(x^2\csc(x))} \cdot \left(2x\csc(x) + x^2(-\csc(x)\cot(x))\right)}$$

4. Finally, it is possible that multiple iterations of the chain rule will be necessary. Differentiate the following.

(a) $m(x) = \sin(\cos(\tan(x)))$

outside: $\sin(\quad)$

inside: $\cos(\tan(x))$

$$m'(x) = \cos(\cos(\tan(x))) \cdot \underbrace{\frac{d}{dx}\cos(\tan(x))}_{\text{chain again}}$$

chain again

outside: $\cos(\quad)$

inside: $\tan(x)$

$$= \cos(\cos(\tan(x))) \cdot (-\sin(\tan(x))) \cdot \frac{d}{dx}\tan(x)$$

$$= \boxed{-\cos(\cos(\tan(x))) \cdot \sin(\tan(x)) \cdot \sec^2(x)}$$

(b) $b(x) = \sqrt{x + e^{\cos(x)}}$

outside: $(\quad)^{1/2}$

inside: $x + e^{\cos(x)}$

$$b'(x) = \frac{1}{2}(x+e^{\cos(x)})^{-1/2} \cdot \frac{d}{dx}(x+e^{\cos(x)})$$

$$= \frac{1}{2}(x+e^{\cos(x)})^{-1/2} \cdot \left[1 + \underbrace{\frac{d}{dx}e^{\cos(x)}}_{\text{chain again}}\right]$$

chain again

outside: $e^{(\quad)}$

inside: $\cos(x)$

$$= \frac{1}{2}(x+e^{\cos(x)})^{-1/2} \cdot \left[1 + e^{\cos(x)} \cdot \frac{d}{dx}(\cos(x))\right]$$

$$= \boxed{\frac{1}{2}(x+e^{\cos(x)})^{-1/2} \cdot \left[1 - e^{\cos(x)} \cdot \sin(x)\right]}$$

– since $\frac{d}{dx}\cos x = -\sin(x)$