On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

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1. (28 pts, 7pts each) Evaluate each of the following.

   (a) $\displaystyle\int_0^{2\pi/3} \left(\sin\theta\cos^2\theta + \sin^3\theta\right) d\theta$

   (b) $\displaystyle\int \frac{(2y-1)^{3/2}-1}{\sqrt{2y-1}}\, dy$

   (c) $\displaystyle\int_e^5 \frac{1}{x\ln x}\, dx$

   (d) Use logarithmic differentiation to find dy/dx for $y = \dfrac{\ln x}{(x+1)(x^2+2)}$.
   Leave your answer unsimplified.

2. (15 pts) Using right hand endpoints, a definite integral is approximated by the Riemann sum :

$$\sum_{i=1}^{n}\left[\left(\frac{4i}{n}\right)^2 - 3\right]\frac{4}{n}.$$

   (a) Find a definite integral represented by this Riemann sum.

   (b) Evaluate $\displaystyle\sum_{i=1}^{n}\left[\left(\frac{4i}{n}\right)^2 - 3\right]\frac{4}{n}$, that is, find the sum in terms of n. Simplify your answer.

   (c) Use either part (a) or part (b) to find the value of $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{4i}{n}\right)^2 - 3\right]\frac{4}{n}$.

TURN OVER - More problems on the back!

3. The following problems are not related.

   (a) (5 pts) Write the sum in sigma notation. (Note: Do not try to find the value.)

$$\frac{3}{7} - \frac{4}{8} + \frac{5}{9} - \frac{6}{10} + \cdots + \frac{23}{27}.$$

   (b) (7 pts) Water is flowing into a tub at $3t + \dfrac{1}{t+1}$ gallons per minute. Assuming the tub started with 10 gallons of water at time $t = 0$, how much water is in the tub after 2 minutes?

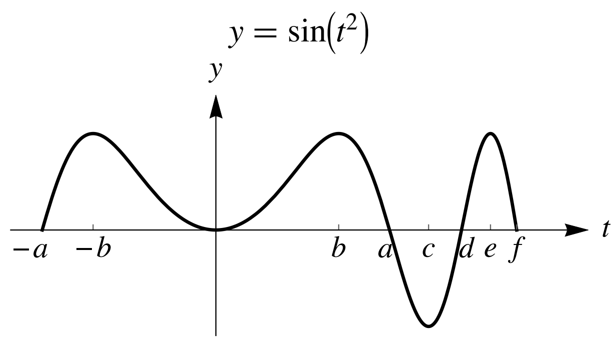
   (c) (7 pts) Use Newton's Method to find a root of the equation $x^3 - 7x - 6 = 0$. Start with an initial guess of $x_1 = 1$ and find x_2 and x_{100}.

   (d) (7 pts) Let $f(x) = \displaystyle\int_2^x \sqrt{1+t^3}\, dt$. Show that f is one-to-one (i.e. so it has an inverse) and find $\left(f^{-1}\right)'(0)$.

   (e) (7 pts) Find the average value of the function $f(x) = x\left(\sqrt[3]{x} + \sqrt[5]{x}\right)$ on $[-1, 1]$.

4. (24 pts, 4 pts each) Consider the function $y = \sin\left(t^2\right)$, shown below.

   Let $g(x) = \displaystyle\int_{-a}^{x} \sin\left(t^2\right) dt$, $-a \le x \le f$. Answer the following questions about $g(x)$. Your answers to parts (iii), (iv), and (v) will be in terms of a, b, c, d, e, and f. No justification is needed for this problem.



   (i) Find $g'(x)$.

   (ii) Find $g''(x)$.

   (iii) On which interval(s) is g decreasing?

   (iv) At what value(s) of x does g have local minimum values?

   (v) Suppose we wish to estimate the value of $g(f)$. Calculate the lower and upper sums using $n = 1$ subinterval.

   (vi) Now find the numerical value of a and use it to find the numerical value of $g''(a)$.

Formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$