MATH 1300: HW #10

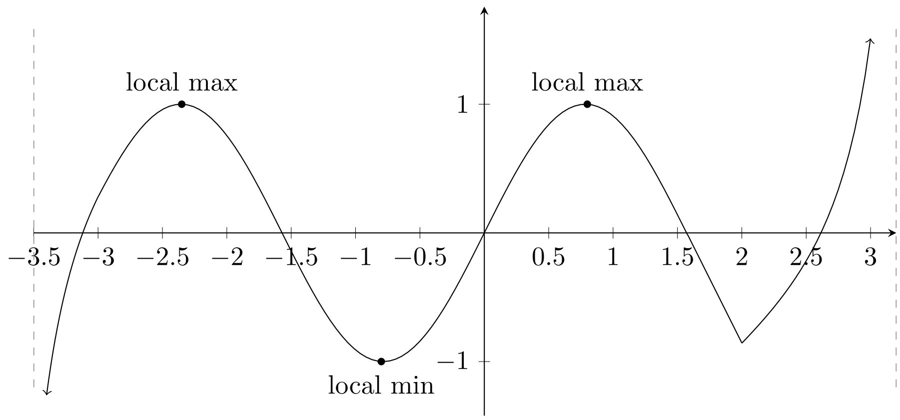
Due on March 23, 2017 at 10:00am

Professor Braden Balentine Section 005

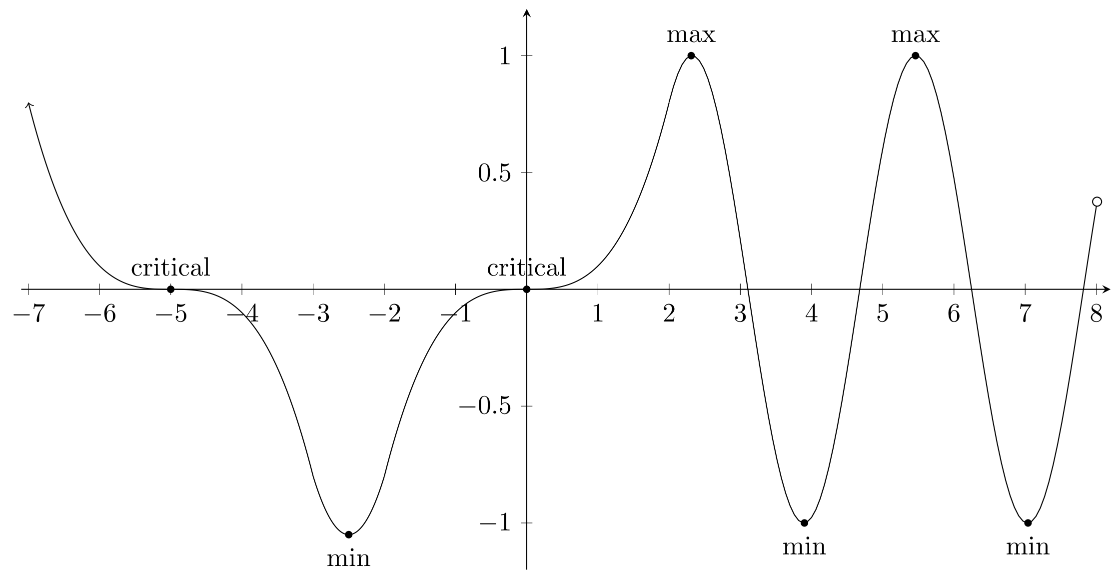
John Keller

Section 4.2

14. (a) Sketch the graph of a function that has two local maxima, one local minimum, and no absolute minimum.



(b) Sketch the graph of a function that has three local minima, two local maxima, and seven critical numbers.



62. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \leq \theta \leq \frac{\pi}{2}$. Show that F is minimized when $\tan\theta = \mu$.

$$\begin{aligned}
F' &= \frac{\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} \\
0 &= \sin\theta - \mu\cos\theta \\
\mu\cos\theta &= \sin\theta \\
\mu &= \tan\theta \\
F(0) &= \frac{\mu W}{\mu\sin 0 + \cos 0} \\
F(0) &= \mu W \\
F(\frac{\pi}{2}) &= \frac{\mu W}{\mu\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})} \\
F(\frac{\pi}{2}) &= \frac{\mu W}{\mu} \\
F(\frac{\pi}{2}) &= W
\end{aligned}$$

66. A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

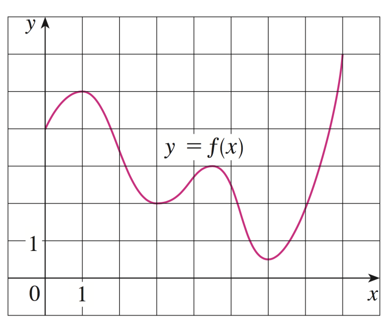
   (a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

   - If $f(x) = x^3 + x^2 - x + 1$, $f'(x) = 3x^2 + 2x - 1$, and since it is zero at $x = \frac{1}{3}, -1$, then there are two critical points
   - If $f(x) = x^3$, $f'(x) = 3x^2$, and since it is zero at $x = 0$, then there is one critical point
   - If $f(x) = x^3 + x$, $f'(x) = 3x^2 + 1$, and because $f'(x)$ is never negative, there are no critical points

   (b) How many local extreme values can a cubic function have?
   A cubic function can have 0 or 2 local extreme values because it must retain the shape of a cubic function, but adding x^2 simply puts a small dip in the curve, making it have 2 local extremes.

Section 4.3

1. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0,8].



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
$$f'(c) = \frac{6 - 4}{8 - 0}$$
$$f'(c) = \frac{2}{8}$$
$$f'(c) = \frac{1}{4}$$

20. (a) Find the critical numbers of $f(x) = x^4(x-1)^3$.

$$f'(x) = -\frac{2x}{(-1+x^2)^2}$$
$$0 = (-1+x)^2 x^3(-4+7x)$$
$$x = 1, 0, \frac{4}{7}$$

(b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?
The Second Derivative Test tells us if there is a local maximum or minimum at each critical point.

(c) What does the First Derivative Test tell you?
The First Derivative Test tells us if there is a local maximum or minimum, as well as if there is neither, at the critical point.

34.

$$f(x) = \frac{x^2}{(x-2)^2}$$

(a) Find the vertical and horizontal asymptotes.

Finding vertical:

$$0 = (x-2)^2$$
$$x = \boxed{2}$$

Finding horizontal:

$$\frac{x^2}{x^2} = \boxed{1}$$

(b) Find the intervals of increase or decrease.

$$f'(x) = -\frac{4x}{(x-2)^3}$$
$$0 = -\frac{4x}{(x-2)^3}$$
$$x = 0$$

Increase: $(0, 2)$ Decrease: $(-\infty, 0) \cup (2, \infty)$

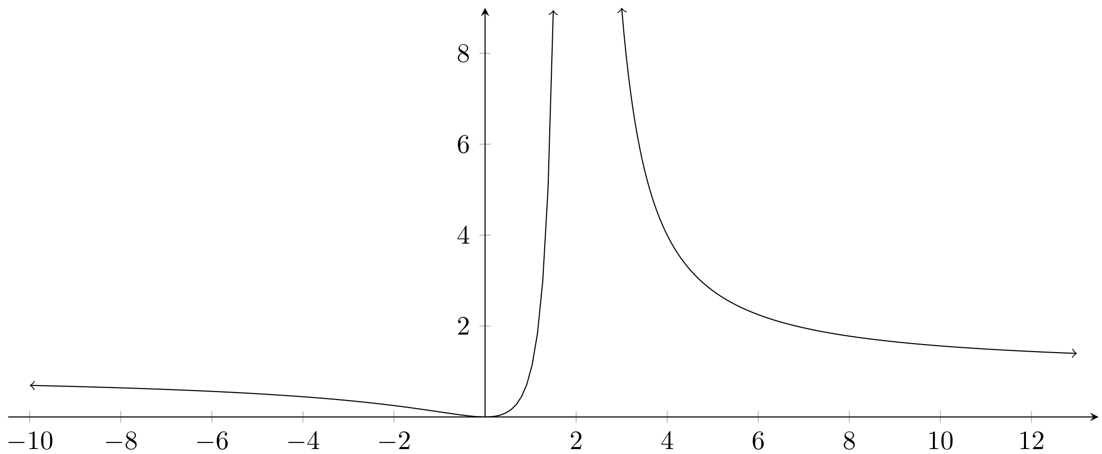
(c) Find the local maximum and minimum values.
Local max: None Local min: $(0, 0)$

(d) Find the intervals of concavity and the inflection points.

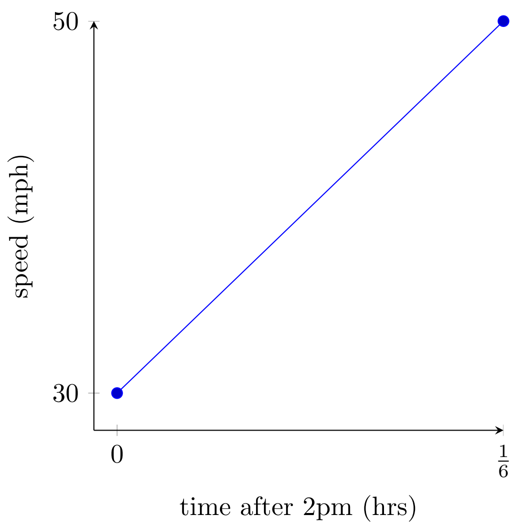
$$f''(x) = \frac{8 + 8x}{-2^4 + x^4}$$
$$0 = \frac{8 + 8x}{-16 + x^4}$$
$$x = -1$$

Concave up: $(-1, 2)$ Concave down: $(-\infty, 1)$

(e) Use the information from parts (a)-(d) to sketch the graph of f.



66. At 2:00 PM a car's speedometer reads 30 mi/h. At 2:10 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2.



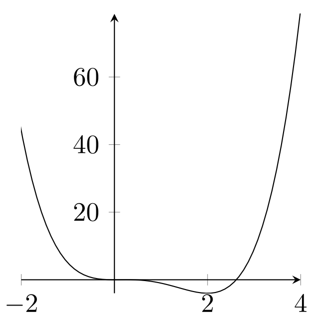
Determining the slope for the graph above:

$$y = mx + b$$
$$m = \frac{20}{\frac{1}{6}}m \qquad = 120x$$

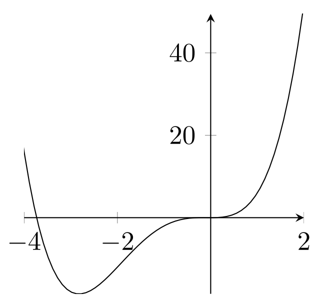
As long as x starts at 0 (or 2pm), the acceleration is going to start off at 120 mi/h^2.

70. For what values of c does the polynomial $P(x) = x^4 + cx^3 + x^2$ have two inflection points? One inflection point? None? Illustrate by graphing P for several values of c. How does the graph change as c decreases?

$$
\begin{aligned}
P'(x) &= 4x^3 + 3cx^2 + 2x \\
P''(x) &= 12x^2 + 6cx + 2 \\
0 &= 12x^2 + 6cx + 1 \\
2 &= 12x^2 + 6cx \\
2 &= 6x(2x + c) \\
2 &= 6x \\
x &= \frac{1}{3} \\
2 &= 2x + c \\
x &= \frac{2-c}{2} \\
\frac{1}{3} &= \frac{2-c}{2} \\
\frac{2}{2} &= 2 - c \\
c &= 2 - \frac{2}{3} \\
c &= \frac{4}{3}
\end{aligned}
$$



$P(x) = x^4 + (-3)x^3 + x^2$



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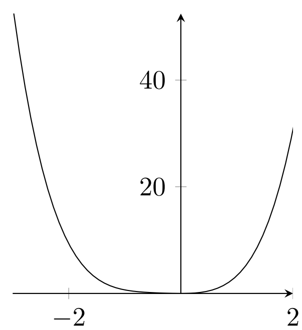

$P(x) = x^4 + (\frac{4}{3})x^3 + x^2$

Additional Problem

Find the absolute extrema of the function $f(x) = xe^{-x^2/18}$ on the interval $[-2, 4]$.

$$f'(x) = e^{\frac{-x^2}{18}}(1 - \frac{1}{9}x^2)$$
$$0 = e^{\frac{-x^2}{18}}$$
$$\ln(0) = \frac{-x^2}{18}$$
$$18 = -x^2$$
$$x = \sqrt{-18} \text{ DNE}$$
$$0 = 1 - \frac{1}{9}x^2$$
$$x = \pm 3$$
$$f(3) = 3e^{\frac{1}{2}} \text{ relative max}$$
$$f(-2) = -2e^{\frac{2}{9}} \text{ local min}$$
$$f(4) = 4e^{\frac{8}{9}} \text{ local max}$$