

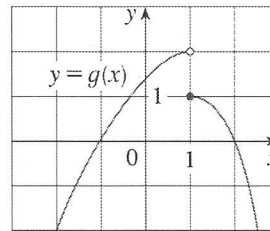
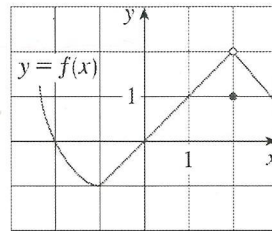


Math 1300-005 - Spring 2017

Using the Limit Laws - 1/25/17

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

- The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



$$(a) \lim_{x \rightarrow 2} [f(x) + 3g(x)]$$

$$= \lim_{x \rightarrow 2} f(x) + 3 \lim_{x \rightarrow 2} g(x) \quad [\text{Law } 1+3]$$

$$= 2 + 3 \cdot (0)$$

$$= \boxed{2}$$

$$(c) \lim_{x \rightarrow 0} [f(x)g(x)]$$

$$= \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) \quad [\text{Law } 4]$$

$$= 0 \cdot \frac{1}{3} = \boxed{0}$$

$$(e) \lim_{x \rightarrow 2} [x^3 f(x)]$$

$$= \lim_{x \rightarrow 2} x^3 \cdot \lim_{x \rightarrow 2} f(x) \quad [\text{Law } 4]$$

$$= (2)^3 \cdot (2) \quad [\text{Law } 9]$$

$$= \boxed{16}$$

$$(b) \lim_{x \rightarrow 1} [2f(x) + g(x)] \quad \rightarrow [\text{Law } 1+3]$$

$$2 \cdot \lim_{x \rightarrow 1^+} f(x) + \lim_{x \rightarrow 1^+} g(x) = 2 \cdot 1 + 1 = 3$$

$$2 \cdot \lim_{x \rightarrow 1^-} f(x) + \lim_{x \rightarrow 1^-} g(x) = 2 \cdot 1 + 2 = 4$$

$$\text{RHL} \neq \text{LHL} \text{ so } \boxed{\text{DNE}}$$

$$(d) \lim_{x \rightarrow -1} \frac{f(x)}{g(x)}$$

$$\lim_{x \rightarrow -1} g(x) = 0, \text{ so we cannot use Law 5.}$$

$$\text{In fact, } \lim_{x \rightarrow -1^+} \frac{f(x)}{g(x)} = -\infty \text{ so } \lim_{x \rightarrow -1} \frac{f(x)}{g(x)} \text{ DNE}$$

$$(f) \lim_{x \rightarrow 1} \sqrt{3 + f(x)}$$

$$= \sqrt{\lim_{x \rightarrow 1} (3 + f(x))} \quad [\text{Law } 11]$$

$$= \sqrt{\lim_{x \rightarrow 1} 3 + \lim_{x \rightarrow 1} f(x)} \quad [\text{Law } 1]$$

$$[\text{Law } 7] = \sqrt{3 + 1} = \boxed{2}$$

- Evaluate each limit and justify each step by indicating the appropriate Limit Law(s).

$$(a) \lim_{x \rightarrow 8} (1 + \sqrt[3]{x})(2 - x^2)$$

$$= \lim_{x \rightarrow 8} (1 + \sqrt[3]{x}) \cdot \lim_{x \rightarrow 8} (2 - x^2) \quad [\text{Law } 4]$$

$$= \left(\lim_{x \rightarrow 8} 1 + \lim_{x \rightarrow 8} \sqrt[3]{x} \right) \left(\lim_{x \rightarrow 8} 2 - \lim_{x \rightarrow 8} x^2 \right) \quad [\text{Law } 1]$$

$$= (1 + \sqrt[3]{8})(2 - (8)^2) \quad [\text{Law } 9+10]$$

$$= (1+2)(2-64)$$

$$= 3(-62) = \boxed{-186}$$

$$(b) \lim_{x \rightarrow 1} \sqrt{\frac{2x^2 + 1}{3x - 2}} = \sqrt{\lim_{x \rightarrow 1} \left(\frac{2x^2 + 1}{3x - 2} \right)} \quad [\text{Law } 11]$$

$$= \sqrt{\frac{\lim_{x \rightarrow 1} (2x^2 + 1)}{\lim_{x \rightarrow 1} (3x - 2)}} \quad [\text{Law } 5 \text{ since } \lim_{x \rightarrow 1} (3x - 2) \neq 0]$$

$$= \sqrt{\frac{2(1)^2 + 1}{3(1) - 2}} \quad [\text{Direct substitution}]$$

$$= \sqrt{\frac{3}{1}} = \boxed{\sqrt{3}}$$

3. Find the limit by simplifying the function. Also, explain why the direct substitution property is not valid.

$$\lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h}$$

Cannot use direct substitution since we have a 0 in the denominator. Let us simplify.

$$\lim_{h \rightarrow 0} \frac{(4+h)^2 - 16}{h} = \lim_{h \rightarrow 0} \frac{16 + 8h + h^2 - 16}{h} = \lim_{h \rightarrow 0} \frac{8h + h^2}{h} = \lim_{h \rightarrow 0} \cancel{h} \frac{8+h}{\cancel{h}}$$

$$= 8 + 0$$

$$= \boxed{8}$$

Like I mentioned in class. Try substitution. If you have division by 0, then simplify until you can substitute.

4. Find the limit by rationalizing the function.

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Cannot use direct substitution since we have division by 0. Let us simplify by rationalizing

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} \cdot \frac{(\sqrt{1+x} + 1)}{(\sqrt{1+x} + 1)} = \lim_{x \rightarrow 0} \frac{1+x - 1}{x(\sqrt{1+x} + 1)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1}$$

$$= \frac{1}{\sqrt{1+0} + 1} = \boxed{\frac{1}{2}}$$

5. It is true that

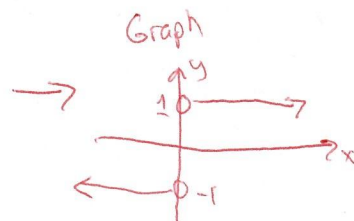
$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

does not exist. In your groups, work out and discuss why this is so.

Recall, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ so dividing by x gives

$$\frac{|x|}{x} = \begin{cases} \frac{x}{x} & x > 0 \\ \frac{-x}{x} & x < 0 \end{cases} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

note: we can no longer write \geq or ≤ 0 since x is in the denominator



So we have a jump discontinuity at 0, hence

$$\lim_{x \rightarrow 0} \frac{|x|}{x} \text{ DNE.}$$