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Math 1300, Midterm 1

February 6, 2017

PRINT YOUR NAME: John Keller

PRINT INSTRUCTOR'S NAME: Braden Balentine

Mark your section/instructor:

☐	Section 001	Brendt Gerics	8:00 - 8:50
☐	Section 002	Leo Herr	9:00 - 9:50
☐	Section 003	Tyler Schrock	9:00 - 9:50
☐	Section 004	Lee Roberson	10:00 - 10:50
☒	Section 005	Braden Balentine	10:00 - 10:50
☐	Section 006	Xingzhou Yang	10:00 - 10:50
☐	Section 007	Lee Roberson	11:00 - 11:50
☐	Section 008	Shen Lu	11:00 - 11:50
☐	Section 009	Suzanne Craig	12:00 - 12:50
☐	Section 010	Carlos Pinilla-Suarez	12:00 - 12:50
☐	Section 011	Nathan Davidoff	1:00 - 1:50
☐	Section 012	Sion Ledbetter	1:00 - 1:50
☐	Section 013	Ruofan Li	2:00 - 2:50
☐	Section 014	Daniel Martin	2:00 - 2:50
☐	Section 015	Isabel Corona	3:00 - 3:50
☐	Section 016	Ira Becker	3:00 - 3:50
☐	Section 017	Ira Becker	4:00 - 4:50

Question	Points	Score
1	9	8
2	18	15
3	11	10
4	12	12
5	8	7
6	8	8
7	12	10
8	10	10
9	12	9
Total:	100	89

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

$\frac{\text{dist trav}}{\text{time elap.}}$

8/9

1. The position, $s(t)$, of a banana slug from the end of a log is checked at various times t. The collected information is given in the table below.

t (in minutes)	10	20	60	110	300
$s(t)$ (in feet)	3.25	2.60	1.00	0.25	0.22

(a) (3 points) Find the slope of the secant line intersecting $s(t)$ at $t = 10$ and $t = 110$.

(110, 10)

$$m = \frac{s(10)-s(110)}{10-110} = \frac{3.25-0.25}{10-110} = \frac{3}{-100}$$

$$\boxed{m = -\frac{3}{100}}$$

3/3

(b) (3 points) Estimate $s'(20)$.

40
12
2

$[10, 20]$: $m = \frac{s(10)-s(20)}{10-20} = \frac{3.25-2.60}{-10} = \frac{1.35}{-10} = -\frac{1.35}{10} = -\frac{13.5}{100} = \boxed{-\frac{54}{400}}$

$[20, 60]$: $m = \frac{s(20)-s(60)}{20-60} = \frac{2.6-1}{-40} = \frac{1.6}{-40} = -\frac{1.6}{40} = -\frac{160}{400}$

$\frac{54+166}{2} = \frac{214}{2} = 107$

$$\boxed{y - 2.6 = -\frac{107}{400}(x-20)}$$

3/3

$\text{Avg} = -\frac{107}{400}$

(c) (3 points) Suppose $s'(50) = -0.3$. What does the value -0.3 represent in the context of the problem? Include units.

Within the context of this problem, $s'(50) = -0.3$ simply means the banana slug went backwards 0.3 feet, units of time?

2/3

2. Determine the value of the following limits. You do NOT need to show work.

(a) (3 points) $\lim_{x \to 0} \frac{x-1}{\cos x}$

I) -1 II) 0 III) 1 IV) ∞ V) DNE

$\cos 0 = 1$

$\frac{-1}{1} = -1$

(b) (3 points) $\lim_{x \to \infty} \frac{5-x^2}{x+1}$

I) $-\infty$ II) -1 III) 0 IV) 1 V) ∞

(c) (3 points) $\lim_{x \to \frac{7}{2}} \frac{4x^2-49}{2x-7}$

$\frac{7}{2} = 3.5$

I) 0 II) $\frac{7}{2}$ III) 14 IV) ∞ V) DNE

$\frac{\cancel{(2x-7)}(2x+7)}{\cancel{2x-7}} = 2x+7$

$\frac{14}{2} + 7$

$7 + 7 = 14$

$\frac{-2.5}{0.25}$ $\frac{172 - 81}{11}$

(d) (3 points) $\lim_{x\to 9} \frac{6-x}{(9-x)^2}$ $= \frac{-4}{1} = \frac{5}{3}$

I) $-\infty$ II) 0 III) 6 IV) 15 V) ∞

$\lim_{x\to 9} \frac{6-x}{(9-x)^2} \cdot \frac{6+x}{6+x}$

$\lim_{x\to 9} \frac{36-x^2}{(9-x)^2(6+x)}$

$7 = -1/4$
$8 = -2$
$10 = -4$
$11 = 5-3$

$9.5 = -10$

(e) (3 points) $\lim_{x\to -3^-} \frac{|x+3|}{x+3}$

I) -3 II) -1 III) 0 IV) 1 V) DNE

$|x+3| = \begin{cases} x+3 & \geq 0 \\ -(x+3) & < 0 \end{cases}$

$\frac{|x+3|}{x+3} = \frac{x+3}{x+3} = 1 \quad x > -3$

$\frac{-(x+3)}{x+3} = \boxed{-1} \; x < -3$

(f) (3 points) $\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}$

I) 0 II) $\frac{1}{12}$ III) $\frac{1}{6}$ IV) ∞ V) DNE

$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$

$\lim_{x\to 5} \frac{x+4-9}{(x-5)(\sqrt{x+4}+3)}$

$\lim_{x\to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x+4}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

2

3. (a) (2 points) State the mathematical definition of what it means for the function $f(x)$ to be continuous at $x = a$.

$$\boxed{\lim_{x \to a} f(x) = f(a)}$$

(b) (9 points) Find the value of k such that the following function is continuous.

8

$$g(x) = \begin{cases} x + k, & \text{if } x \geq 2 \\ kx^2, & \text{if } x < 2 \end{cases}$$

$$\lim_{x \to 2^+} x + k = \lim_{x \to 2^-} kx^2$$

$$2 + k = k(2)^2$$

$$2 + k = 4k$$

$$2 = 3k$$

$$\boxed{k = \frac{3}{2}}$$

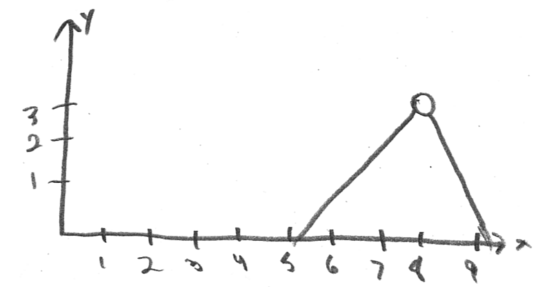
4. The following statements are all false. Justify why each statement is false by providing an explanation that includes a picture.

(a) (4 points) If $\lim_{x\to 8^+} f(x) = 51 = \lim_{x\to 8^-} f(x)$, then $f(x)$ is continuous at $x = 8$.

f(8) = und.

4/4

This statement is incorrect because ① it is unknown what type of function f(x) is and ② f(8) can be undefined while still having limits (see graph)



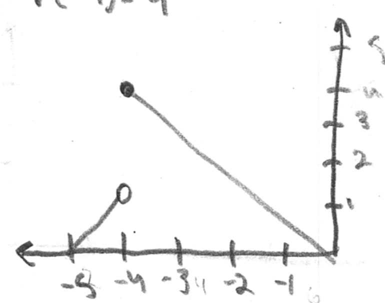
(b) (4 points) If $\lim_{x\to -4} r(x)$ does not exists, then $r(-4)$ does not exist.

4/4

This statement is not true because there could be a jump continuity at r(-4).

(see graph)

r(-4) = 4



4-1 3/3

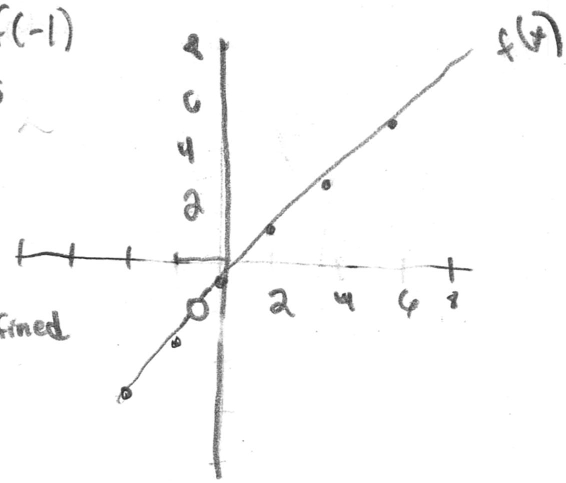
(c) (4 points) If $f(x) = \dfrac{x^2-1}{x+1}$ and $g(x) = x - 1$, then $f(x)$ is equal to $g(x)$.

3

4/4

This statement is not true because f(-1) is NOT equal to g(-1). f(-1) is undefined because the equation has 0 in the denominator.

$f(-1) = \dfrac{(-1)^2 - 1}{-1+1} = \dfrac{1-1}{0} = \dfrac{0}{0}$ = undefined



5. (8 points) Use the Squeeze Theorem to determine

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$$\lim_{x\to 5} |x-5| \cos\left(\frac{2}{x-5}\right).$$

① $-1 \le \cos\left(\frac{2}{x-5}\right) \le 1$ +2

② $-|-x+5| \le \cos\left(\frac{2}{x-5}\right) \le |x-5|$ +1

③ Let's apply the limits!

$$\lim_{x\to 5} |-x+5| \le \lim_{x\to 5} |x-5| \cos\left(\frac{2}{x-5}\right) \le \lim_{x\to 5} |x-5|$$

④ Now inputting limit values as $x \to 5$ makes: +2

$|-5+5| = 0$ and $|5-5| = 0$

⑤ Therefore, by the squeeze theorem, because

$$0 \le \lim_{x\to 5} |x-5| \cos\left(\frac{2}{x-5}\right) \le 0,$$

it can be determined that

$$\boxed{\lim_{x\to 5} |x-5| \cos\left(\frac{2}{x-5}\right) = 0}$$ +2

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6. (8 points) Let $f(x) = x^5 + 2x^3 + 2x^2 - 2$. Use the Intermediate Value Theorem to show $f(x)$ crosses the x-axis in the interval $[-1, 1]$. You must check that the hypotheses of the Intermediate Value Theorem are satisfied to receive full credit.

① Because $f(x)$ is a polynomial, $f(x)$ is continuous.

② $f(-1) = (-1)^5 + 2(-1)^3 + 2(-1)^2 - 2 = -1 - 2 + 2 - 2 = -3$

$f(1) = (1)^5 + 2(1)^3 + 2(1)^2 - 2 = 1 + 2 + 2 - 2 = 3$

0 is between these numbers

③ Therefore, by the IVT, there is a "c" value in the interval $(-1, 1)$ where $f(c) = 0$.

7. Consider the function $f(x) = \frac{3}{x+2}$.

(a) (9 points) Use the limit definition of the derivative to find $f'(1)$.

$$f'(1) = \lim_{h \to 0} \frac{\frac{3}{(1+h)+2} - \left(\frac{3}{1+2}\right)}{h}$$

$$\lim_{h \to 0} \frac{\frac{3}{(1+h)+2} - \left(\frac{3+h}{1+2+h}\right)}{h}$$

$$\lim_{h \to 0} \frac{\frac{\ominus h}{h+3}}{h}$$

$$\lim_{h \to 0} \frac{1}{h} \cdot \frac{-h}{h+3}$$

$$\lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(h+3)}$$

$$\lim_{h \to 0} \frac{-1}{h+3} = \boxed{\cancel{\frac{1}{4}}}$$

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(b) (3 points) Write the equation of the tangent line of the function at $x = 1$.

$(1, 1)$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 1 = \frac{1}{4}(x - 1)}$$

3

or

$$y = \frac{1}{4}x + \frac{3}{4}$$

8. (10 points) Draw a function which meets the following requirements:

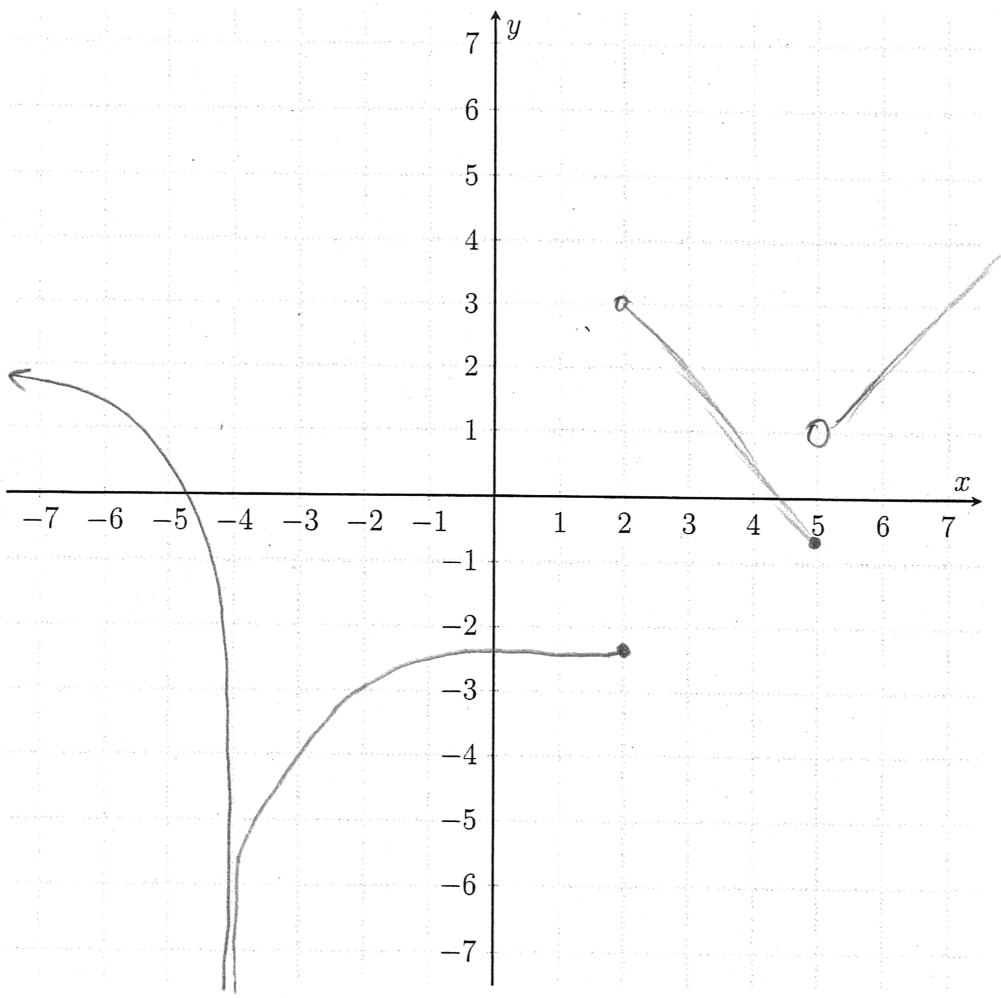
(a) $\lim_{x \to -\infty} f(x) = 2$ ✓

(b) $\lim_{x \to -4} f(x) = -\infty$ ✓

(c) $\lim_{x \to -1} f(x) = -3$ ✓

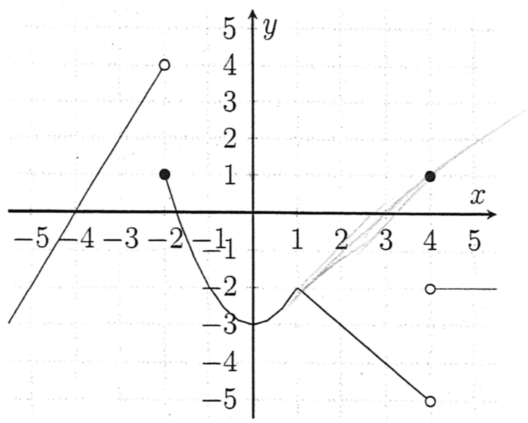
(d) $\lim_{x \to 2^+} f(x) = 3$ ✓

(e) $\lim_{x \to 5} f(x)$ DNE ✓



9. Answer the questions about the following functions. You do NOT need to show work.

f(x)



(-3, -18)

$$g(x) = \begin{cases} -x^2+1 & x \le -3 \\ x+1 & x > -3 \end{cases}$$

(a) (3 points) $\dfrac{f(4)}{g(4)}$ $\frac{1}{5}$

I) $\frac{-4}{15}$ II) $\frac{-2}{5}$ III) $\frac{-1}{15}$ IV) $\frac{1}{5}$ V) $\frac{1}{3}$

$-5 - 5$

(b) (3 points) $\lim_{x \to 4^-} (f(x) - g(x))$

I) -22 II) -20 III) -10 IV) 13 V) DNE

3

(c) (3 points) $\lim_{x \to 2} f(g(x))$

I) -4 II) -2 III) 0 IV) 2 V) DNE

-3

(d) (3 points) $\lim_{x \to 0} g(f(x))$

I) -10 II) -8 III) -2 IV) 10 V) DNE