APPM 1345/1350 **Final Exam** **Spring 2013**

On the front of your bluebook, please write: a grading key, your name, and instructor's name (Chang or Rubio). This exam is worth 150 points and has 10 questions. **Show all work! Simplify all answers.** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

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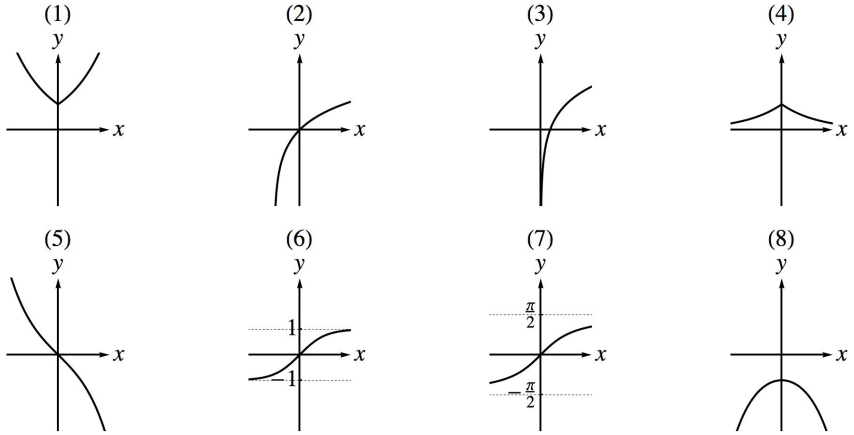
1. (8 points) Match the following functions to their graphs below. No explanation is necessary.

   (a) $y = \ln(x+1)$ (b) $y = 2^{|x|}$ (c) $y = \tan^{-1} x$ (d) $y = -\sinh(x)$.



2. (38 points) Evaluate the following expressions.

   (a) $\displaystyle\int \frac{x}{\sqrt{1-4x^2}}\,dx$ (b) $\displaystyle\int e^{2013\ln x}\,dx$ (c) $\displaystyle\int_{-1}^{1} 2^{|x|}\,dx$

   (d) $\displaystyle\int_0^{\sqrt{3}} \frac{e^{\arctan x}}{1+x^2}\,dx$ (e) $\displaystyle\frac{d}{dx}\left(x^{\cos 3x}\right)$ (f) $\displaystyle\lim_{h\to 0}\frac{\sec^3(x+h)-\sec^3(x)}{h}$

3. (15 points) Consider the function $f(x)$. Is there a value of c that will make f continuous at $x = 0$? Use the definition of continuity to justify your answer.

$$f(x) = \begin{cases} \dfrac{\tan x}{2x} & x < 0 \\ c & x = 0 \\ x^{2x} & x > 0 \end{cases}$$

4. (14 points) Find the area of the largest rectangle that can be inscribed in the first quadrant of the unit circle if one side of the rectangle lies along the x-axis and another lies along the y-axis.
   (*Hint:* The equation of a unit circle centered at the origin is $x^2 + y^2 = 1$.)

5. (14 points) Sketch a graph of a single function $y = g(x)$ that satisfies all of the following conditions. No explanation is necessary.

   (a) $g(-x) = -g(x)$ (b) $\lim_{x\to\infty} g(x) = 1$ (c) $\lim_{x\to 2} g(x) = \infty$

   (d) $\lim_{x\to 4} g(x) = -2$ (e) $g'' > 0$ for x in $(2, 5)$ (f) $g'' < 0$ for x in $(5, \infty)$

   (g) $\lim_{h\to 0} \dfrac{g(4+h) - g(4)}{h} = 0$

6. (12 points) Let $f(x) = \dfrac{\sinh x}{e^x}$.

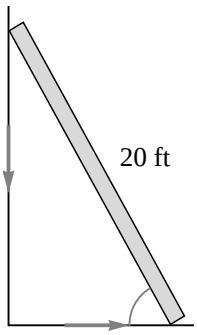
   (a) Simplify $f(x)$ using the definition of $\sinh x$.

   (b) Find the value of $f(-\ln 2)$.

   (c) Is f increasing or decreasing at $x = -\ln 2$?

   (d) Is f concave up or down at $x = -\ln 2$?

7. (14 points) A 20-ft ladder is leaning against a wall when its base starts to slide away. At the moment when the angle between the ladder and the ground is $\pi/3$ radians, the top of the ladder is sliding down the wall at a rate of 1/4 ft/sec. How fast is the base of the ladder moving away from the wall then?



8. (10 points) Let $g(x) = \displaystyle\int_0^x \frac{dt}{\sqrt{1-t^2}}$. Find the linearization of g at $x = -1/2$.

9. (10 points) Suppose $h(x)$ is an even, continuous function with roots at $x = \pm 2$. Given $\displaystyle\int_0^2 h(x)\,dx = 7$ and $\displaystyle\int_{-5}^0 h(x)\,dx = -4$, find the values of the following expressions.

   (a) $\displaystyle\int_{-5}^{-2} h(x)\,dx$ (b) h_{ave} on $[-5, 5]$ (c) $\displaystyle\int_{-5}^{5} |h(x)|\,dx$

10. (15 points) Zach walks into a casino with \$1000 and steadily loses 20% of his money each hour. Meredith enters the casino with \$500 and steadily loses 15% of her money each hour.

    (a) How long will it take before Zach is left with just \$50?

    (b) Who will reach \$50 first: Zach or Meredith?

    Use the law of exponential decay and the following approximations to compute your answers.

    $\ln 0.15 \approx -1.9$ $\ln 0.2 \approx -1.6$ $\ln 0.8 \approx -.22$

    $\ln 0.85 \approx -.16$ $\ln 10 \approx 2.3$ $\ln 20 \approx 3.0$