Name: ____________________

Math 1300-005 - Spring 2017

Quiz 10 - 3/24/17

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: ____________________

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. Consider the function $f(x) = x^2 - 5x + 7$ on $[-1, 3]$.

   (a) Does f satisfy the hypotheses of the MVT on $[-1, 3]$? Explain your answer.

   f is a polynomial so is cont. on $[-1,3]$.
   $f'(x) = 2x - 5$ is a line, so f' is differentiable on $(-1,3)$.

   (b) Find all numbers c that satisfy the conclusion of the MVT for f on $[-1, 3]$.

   By MVT, there exists c in $(-1,3)$ with $f'(c) = \frac{f(3)-f(-1)}{3-(-1)}$

   $f(3) = 3^2 - 5(3) + 7 = 9 - 15 + 7 = 1$

   $f(-1) = (-1)^2 - 5(-1) + 7 = 1 + 5 + 7 = 13$

   $f'(c) = 2c - 5$

   $\rightarrow 2c - 5 = \frac{1-13}{4}$

   $2c - 5 = -3$

   $2c = 2$

   $\boxed{c = 1}$

2. Using the same function f as in question 1, find the absolute maximum and absolute minimum values of f on $[-1, 3]$. At what x-value(s) do the max and min occur?

   $f'(x) = 0 \Leftrightarrow 2x - 5 = 0$, so $x = \frac{5}{2}$ is the critical number.

   $f(-1) = 13$

   $f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 7 = \frac{25}{4} - \frac{25}{2} + 7 = \frac{25}{4} - \frac{50}{4} + \frac{28}{4} = \frac{3}{4}$

   $f(3) = 1$.

   So the abs max is 13 at $x = -1$
   the abs min is $\frac{3}{4}$ at $x = 5/2$

3. Consider the function $f(x)$ and its first and second derivatives.

$$f(x) = \frac{3x(x-4)}{(x+2)^2}, \qquad f'(x) = \frac{24(x-1)}{(x+2)^3}, \qquad f''(x) = \frac{-24(2x-5)}{(x+2)^4}$$

(a) Find the x-intercept(s) of f, if any. Find the y-intercept(s) of f, if any.

x-int: set $f(x)=0$,

$\frac{3x(x-4)}{(x+2)^2} = 0$

$x=0$, $x=4$. The points are $\boxed{(0,0),\ (4,0)}$

y-int: set $x=0$.

$y=f(x)= \frac{3(0)(0-4)}{(0+2)^2} = 0$.

$\boxed{\text{so the y-int is } (0,0)}$

(b) Find the vertical asymptote(s) of f, if any. Find the horizontal asymptote(s) of f, if any.

VA: when denom. of $f(x)$ is 0.

so $\boxed{x=-2}$

HA: $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x(x-4)}{(x+2)^2} \approx \lim_{x\to\infty} \frac{3x^2}{x^2} = 3$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{3x(x-4)}{(x+2)^2} \approx \lim_{x\to-\infty} \frac{3x^2}{x^2} = 3$

so $\boxed{y=3}$

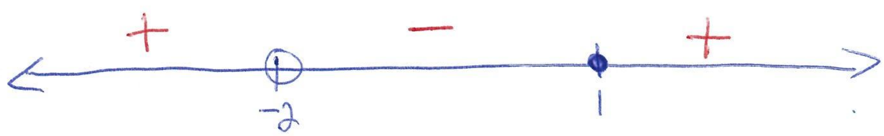
(c) Find all values of x such that $f'(x) = 0$ ***AND*** all values of x such that the denominator of f' is zero. Which of these x-values are critical numbers?

$f'(x)=0$ at $\boxed{x=1}$

Denom of $f'=0$ at $\boxed{x=-2}$

$\boxed{x=1 \text{ is critical b/c } x=1 \text{ is in domain of } f.}$

(d) Plot *all* values from (c) on a sign chart for f'. If an x-value is critical, place it on the sign chart with a solid dot. If an x-value is not critical, place it on the sign chart with an open dot. Fill in your sign chart using test points.



(e) Find the intervals of increase or decrease for f. Justify your answer.

f increase $(-\infty,-2)\cup(1,\infty)$ since $f'>0$.

f decreases $(-2,1)$ since $f'<0$.

(f) Find the x-coordinates and y-coordinates of the local maximum and minimum values of f. Justify your answer.

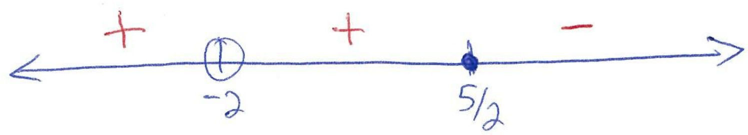
local min at $(1, f(1)) = (1,-1)$ since f' goes $(-)$ to $(+)$

no local max.

(g) Find all values of x such that $f''(x) = 0$ ***AND*** all values of x such that the denominator of f'' is zero.

$f''(x)=0$ at $x=\frac{5}{2}$. Denom of $f''=0$ at $x=-2$

(h) Plot all values from (g) on a sign chart for f''. If an x-value is in the domain of f, place it on the sign chart with a solid dot. If an x-value is not in the domain of f, place it on the sign chart with an open dot. Fill in your sign chart using test points.



(i) Find the intervals of concavity for f. Justify your answer.

f is concave up $(-\infty,-2)\cup(-2,5/2)$ since $f''>0$

f is concave down $(5/2,\infty)$ since $f''<0$.

(j) Find the x-coordinates and y-coordinates of any inflection points of f. To save time, $f(5/2) = -5/9$. Justify your answer.

Inflection at $(5/2, f(5/2)) = (5/2, -5/9)$ since f'' switches sign there.

(k) Using all the information from parts (a) through (j), sketch a graph of $f(x)$ below.