Name: Solutions

Math 1300-005 - Spring 2017

Quiz 9 - 3/17/17

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: ______________________________

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. In this problem, we shall estimate $(3.996)^{1/2}$.

   (a) Let $f(x) = x^{1/2}$. Find the linearization, $L(x)$, of f at $a = 4$.

   $L(x) = f(4) + f'(4)(x-4)$.

   $f(4) = 4^{1/2} = 2$

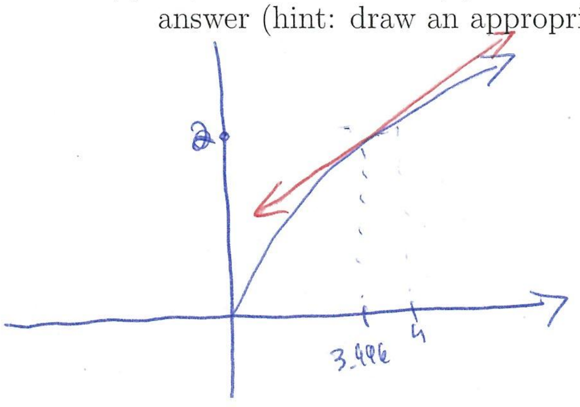
   $f'(x) = \frac{1}{2\sqrt{x}}$, so $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$.

   $\boxed{L(x) = 2 + \frac{1}{4}(x-4)}$

   (b) Use $L(x)$ from part (a) to estimate $(3.996)^{1/2}$.

   $$\begin{aligned}(3.996)^{1/2} = f(3.996) &\approx L(3.996) \\ &= 2 + \frac{1}{4}(3.996-4) \\ &= 2 + \frac{1}{4}(-.004) \\ &= 2 - 0.001 \\ &= \boxed{1.999}\end{aligned}$$

   (c) Is your answer from (b) and overestimate or underestimate? You must justify your answer (hint: draw an appropriate tangent line).



at $x = 3.996$, the tangent line $L(x)$ is above the curve $\sqrt{x}$, so 1.999 is an overestimate.

2. Find the following derivatives.

(a) $f(x) = \log_5(xe^x)$

$$f'(x) = \frac{1}{xe^x \ln(5)} \cdot \frac{d}{dx}(xe^x)$$

$$= \boxed{\frac{e^x + xe^x}{xe^x \ln(5)}}$$

or cancelling the e^x

$$= \boxed{\frac{1+x}{x\ln(5)}}$$

(b) $g(x) = \arctan(\ln(2x))$.

$$g'(x) = \frac{1}{1+(\ln(2x))^2} \cdot \frac{1}{2x} \cdot 2$$

$$= \boxed{\frac{1}{x(1+(\ln(2x))^2)}}$$

3. Use logarithmic differentiation to find the derivative of

$$y = (\cos(x))^x$$

$\ln(y) = x\ln(\cos(x))$. $\rightarrow$ implicitly differentiate.

$$\frac{1}{y} \cdot y' = \ln(\cos(x)) + \frac{x}{\cos(x)} \cdot (-\sin(x))$$

so $y' = y\left(\ln(\cos(x)) - \frac{x\sin(x)}{\cos(x)}\right)$

$$= \boxed{(\cos(x))^x \left(\ln(\cos(x)) - \frac{x\sin(x)}{\cos(x)}\right)}$$

or $\boxed{(\cos(x))^x(\ln(\cos(x)) - x\tan(x))}$