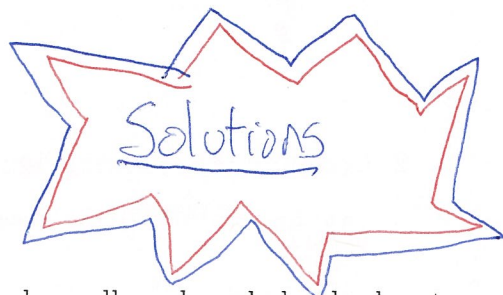


# Math 1300-005 - Spring 2017

Limits Involving Infinity - 2/1/17



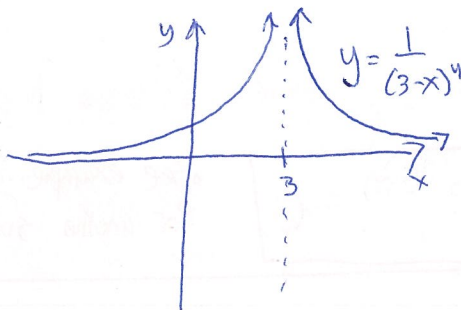
*Guidelines:* Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Compute the following limits.

$$(a) \lim_{x \rightarrow 3} \frac{7-x}{(3-x)^4} = \lim_{x \rightarrow 3} \left[ (7-x) \cdot \frac{1}{(3-x)^4} \right]$$

• We cannot immediately use a limit law. However, note that  $\lim_{x \rightarrow 3} (7-x) = 4$ .

• Now, let's graph  $y = \frac{1}{(3-x)^4} = \frac{1}{(-(x-3))^4} = \frac{1}{(x-3)^4}$ .



so  $\lim_{x \rightarrow 3} \frac{1}{(3-x)^4} = \infty$  from the picture.

• Remember,  $\infty$  DNE, but since  $RHL = LHL$ , we can, in an abuse of notation, write

$$\lim_{x \rightarrow 3} \left[ (7-x) \cdot \frac{1}{(3-x)^4} \right] = \lim_{x \rightarrow 3} (7-x) \cdot \lim_{x \rightarrow 3} \left[ \frac{1}{(3-x)^4} \right]$$

$$= 4 \cdot \infty$$

$$= \boxed{\infty}$$

$$(b) \lim_{x \rightarrow \infty} \frac{x^3 - 5x}{2x^3 - x^2 + 4}$$

• Quick Solution: Since the largest powers in the numerator and denominator are equal, we can simply take the ratio of coefficients. } you must include a sentence like this

$$\lim_{x \rightarrow \infty} \frac{x^3 - 5x}{2x^3 - x^2 + 4} = \boxed{\frac{1}{2}}$$

• Long Solution: The highest power in the denominator is  $x^3$ , so

$$\lim_{x \rightarrow \infty} \frac{x^3 - 5x}{2x^3 - x^2 + 4} = \lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^3} - \frac{5}{x^2}}{\frac{2x^3}{x^3} - \frac{x^2}{x^3} + \frac{4}{x^3}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{5}{x^2}}{2 - \frac{1}{x} + \frac{4}{x^3}}$$

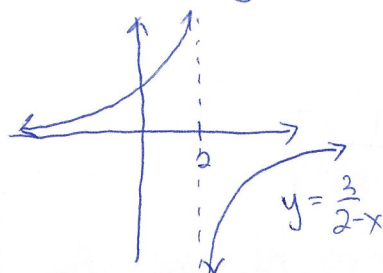
$$= \boxed{\frac{1}{2}}$$

You may use either method

2. Compute the following limits.

(a)  $\lim_{x \rightarrow 2^+} e^{3/(2-x)}$  [Hint: first consider  $\lim_{x \rightarrow 2^+} 3/(2-x)$ ]

• Let's first draw  $y = \frac{3}{2-x}$ .



so  $\lim_{x \rightarrow 2^+} \frac{3}{2-x} = -\infty$ .

• Using the above, let  $t = \frac{3}{2-x}$ . Then as  $x \rightarrow 2^+$ ,  $t \rightarrow -\infty$ , so

$$\lim_{x \rightarrow 2^+} e^{3/(2-x)} = \lim_{t \rightarrow -\infty} e^t = 0, \text{ since } 0 \text{ is a h.a. for } e^x.$$

Thus  $\boxed{\lim_{x \rightarrow 2^+} e^{3/(2-x)} = 0}$

• see example 7 in the book for another such example.

(b)  $\lim_{x \rightarrow \infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4}$

• Quick Solution: since the largest power in the numerator is greater than the largest power in the denominator,

$$\lim_{x \rightarrow \infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4} = \boxed{\infty}$$

• Long Solution:  $x^4$  is the largest power of  $x$  in the denominator, so

$$\lim_{x \rightarrow \infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4} = \lim_{x \rightarrow \infty} \frac{\frac{x}{x^4} + \frac{x^3}{x^4} + \frac{x^5}{x^4}}{\frac{1}{x^4} - \frac{x^2}{x^4} + \frac{x^4}{x^4}} = \lim_{x \rightarrow \infty} \frac{\cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x}} + x}{\cancel{\frac{1}{x^4}} - \cancel{\frac{1}{x^2}} + 1}$$

$$= \lim_{x \rightarrow \infty} x$$

$$= \boxed{\infty}$$