MATH 1300: HW #7

Due on March 9, 2017 at 10:00am

Professor Braden Balentine Section 005

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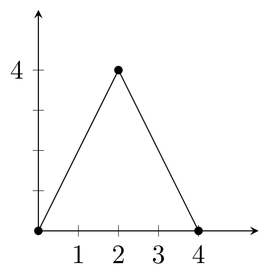
Additional Problems

1. Determine where $f(x) = \arctan(x^2 - 2x)$ is increasing.

$$\frac{1}{1+(x^2-2x)}(2x-2) > 0$$
$$\frac{2}{x-1} > 0$$

$$(3, \infty)$$

2. The graph of $f(x)$ is shown and the table gives values of (x) and $g'(x)$.



x	0	1	2	3
$g(x)$	4	3	2	1
$g'(x)$	-1.1	-0.9	-1.2	-0.8

(The function $f(x)$ is piecewise linear)

(a) Given $h(x) = f(g(x))$, find $h'(1)$.
-1.8

(b) Given $k(x) = g(f(x))$, find $k'(3)$.
1.44

(c) Given $l(x) = g(g(x))$, find $l'(x)$.
$g'(g(x)) \cdot g'(x)$

(d) Given $m(x) = \sqrt{f(x)}$, find $m'(1)$.
$\frac{1}{\sqrt{2}}$

3. The length of the day in Boulder (Latitude 40 N) can be modeled approximately by

$$l(t) = -3\cos\left(\frac{2\pi}{365}(t+10)\right) + 12$$

where l is given in hours and t is the day of the year.

(a) Evaluate $l(355)$; fully interpret the result in the context of this problem, including units.

$$\begin{aligned} l(355) &= -3\cos\left(\frac{2\pi}{365}(355+10)\right) + 12 \\ &= 9 \text{ hours} \end{aligned}$$

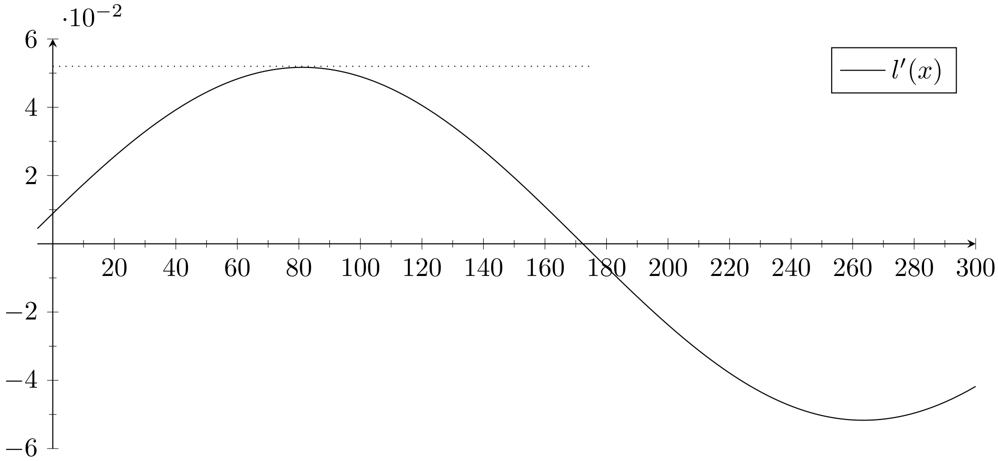
This answer simply means that on day number 355, the length of the day will be approximately 9 hours long.

(b) Evaluate $l'(265)$; fully interpret the result in the context of this problem, including units.

$$\begin{aligned} l'(t) &= \frac{6}{365}\pi \sin\left(\frac{2}{365}\pi(265+10)\right) \\ &= -\frac{6}{365}\pi\cos\left(\frac{\pi}{146}\right) \\ &\approx -0.05 \text{ hours/day} \end{aligned}$$

This answer means that on day 265, the length of the day is changing at a rate of -0.05 hours per day.

(c) Calculate when $l'(t)$ is largest. Explain.



The largest value for $l'(t)$ is approximately 0.052 at 80 days.

4. The U.S. gross domestic product can be modeled by

$$P(t) = 4.351e^{0.0368t}$$

where P is given in billions of dollars and t is years since 1970.

(a) Find $P(244)$; fully interpret the result in the context of this problem, including units.

$$\begin{aligned} P(244) &= 4.351e^{0.0368(244)} \\ &= 34,530.8 \text{ billion dollars} \end{aligned}$$

This answer means that in the year 1970+244, the GDP can be estimated to be \$34,530.8 billion using the model $P(t) = 4.351e^{0.0368t}$.

(b) When was the GDP one trillion dollars?

$$\begin{aligned} 1000 &= 4.351e^{0.0368t} \\ t &= 147.754 \text{ years after 1970} \end{aligned}$$

(c) How many years does it take for the GDP to double?

$$\begin{aligned} P(0) &= 4.351e^{0.0368(0)} = \$4.351b4.351 * 2 \quad = 4.351e^{0.0368t} \\ x &= 18.8355 \text{ years past 1970} \end{aligned}$$

(d) What is $P'(224)$? Again, fully interpret (including units).

$$\begin{aligned} P'(224) &= 0.160117e^{0.0368(224)} \\ &= 608.715 \text{ billion dollars} \end{aligned}$$

This answer means that at 1970+224, the GDP is changing at \$608.715 b per year.

Section 3.6

32. Find y' if $\tan^{-1}(xy) = 1 + x^2y$.

$$\begin{aligned}
\arctan(xy) &= 1 + x^2y \\
\frac{1}{1+(xy)^2}(y + \frac{dy}{dx}x) &= 2xy + \frac{dy}{dx}x^2 \\
\frac{y + \frac{dy}{dx}x}{1+(xy)^2} &= (2xy + \frac{dy}{dx}x^2) \\
\frac{y + \frac{dy}{dx}x}{2y + \frac{dy}{dx}x} &= x(1+(xy)^2) \\
y + \frac{dy}{dx}x &= 2xy + 2x^3y^2 + \frac{dy}{dx}x^2 + \frac{dy}{dx}x^4y^2 \\
\frac{dy}{dx}x - \frac{dy}{dx}x^2 - \frac{dy}{dx}x^4y^2 &= 2xy + 2x^3y^2 - y \\
\frac{dy}{dx}(x - x^2 - x^4y^2) &= 2xy + 2x^3y^2 - y \\
\frac{dy}{dx} &= \frac{2xy + 2x^3y^2 - y}{x - x^2 - x^4y^2}
\end{aligned}$$

41. (a) Suppose f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. Use implicit differentiation to show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

provided that the denominator is not 0.

$$\begin{aligned}
f \circ f^{-1}(x) &= x \\
f'(f^{-1}(x)) \cdot (f^{-1})'(x) &= 1 \\
(f^{-1})'(x) &= \frac{1}{f'(f^{-1}(x))}
\end{aligned}$$

(b) If $f(4) = 5$ and $f'(4) = \frac{2}{3}$, find $(f^{-1})'(5)$.

$$\begin{aligned}
(f^{-1})'(5) &= \frac{1}{f'(f^{-1}(5))} \\
&= \frac{1}{f'(4)} \\
&= \frac{3}{2}
\end{aligned}$$

42. (a) Show that $f(x) = 2x + \cos x$ is one-to-one.

$f'(x) = 2 - \sin(x)$ Because f'(x) is a constant sign, f(x) is one-to-one

(b) What is the value of $f^{-1}(1)$?

$$f^{-1}(1) =$$

(c) Use the formula from Exercise 41(a) to find $(f^{-1})'(1)$.

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

22. Find the derivative of $f(x) = x\ln(\arctan x)$

$$
\begin{aligned}
f(x) &= x\ln(\arctan x) \\
u &= \arctan(x) \\
u' &= \frac{1}{1+x^2} \\
f'(x) &= \ln(u) + x\frac{1}{u}u' \\
f'(x) &= \ln(\arctan x) + x\frac{1}{\arctan x}\left(\frac{1}{1+x^2}\right) \\
f'(x) &= \ln(\arctan x) + \frac{x}{(\arctan x)(1+x^2)}
\end{aligned}
$$