Math 1300-010 - Fall 2016

The Substitution Rule, Part II - 12/6/16

$\boxed{\text{Solutions}}$

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

1. Evaluate the definite integral.

(a) $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$ $\quad u=x^2,\ du=2x\,dx$

$$=\frac{1}{2}\int_{u(0)}^{u(\sqrt{\pi})}\cos(u)\,du=\frac{1}{2}\int_0^{\pi}\cos(u)\,du=\frac{1}{2}\sin(u)\Big]_0^{\pi}=\boxed{0}$$

(b) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ $\quad u=\sqrt{x},\ du=\frac{1}{2\sqrt{x}}dx$

$$=2\int_{u(1)}^{u(4)} e^u\,du=2\int_1^2 e^u\,du=2e^u\Big]_1^2=\boxed{2e^2-2e}$$

(c) $\int_0^1 (3t-1)^{50}\,dt$ $\quad u=3t-1,\ du=3\,dt$

$$=\frac{1}{3}\int_{u(0)}^{u(1)} u^{50}\,du=\frac{1}{3}\int_{-1}^{2} u^{50}\,du=\frac{1}{3}\,\frac{u^{51}}{51}\Big]_{-1}^{2}=\boxed{\frac{1}{153}\left(2^{51}-(-1)^{51}\right)}$$

(d) $\int_0^{\pi/2} \cos(x)\sin(\sin(x))\,dx$ $\quad u=\sin(x),\ du=\cos(x)dx$

$$=\int_{u(0)}^{u(\pi/2)}\sin(u)\,du=\int_0^1\sin(u)\,du=-\cos(u)\Big]_0^1=-\cos(1)-(-\cos(0))$$

$$=\boxed{1-\cos(1)}$$

(e) $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln(x)}}\,dx$ $u = \ln(x)$, $du = \frac{1}{x}dx$

$$= \int_{u(e)}^{u(e^4)} \frac{1}{\sqrt{u}}\,du = \int_1^4 u^{-\frac{1}{2}}du = 2u^{\frac{1}{2}}\Big]_1^4$$

$$= 2(4^{\frac{1}{2}}) - 2(1)^{1/2}$$

$$= 4 - 2 = \boxed{2}$$

(f) $\displaystyle\int_0^{1/2} \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx$ $u = \arcsin(x)$, $du = \frac{1}{\sqrt{1-x^2}}dx$

$$= \int_{u(0)}^{u(1/2)} u\,du = \int_0^{\pi/6} u\,du$$

$$= \frac{1}{2}u^2\Big]_0^{\pi/6} = \frac{1}{2}\frac{\pi^2}{36} - 0 = \boxed{\frac{\pi^2}{72}}$$

(g) $\displaystyle\int_0^1 \frac{e^{2x}}{1+e^{2x}}\,dx$ $u = 1+e^{2x}$, $du = 2e^{2x}dx$

$$= \frac{1}{2}\int_{u(0)}^{u(1)} \frac{du}{u} = \frac{1}{2}\int_2^{1+e^2} \frac{1}{u}du = \frac{1}{2}\ln|u|\Big]_2^{1+e^2}$$

$$= \boxed{\frac{1}{2}\ln(1+e^2) - \frac{1}{2}\ln(2)}$$

(h) $\displaystyle\int_0^{\ln(3)/4} \frac{e^{2x}}{1+e^{4x}}\,dx$ [Hint: $e^{4x} = (e^{2x})^2$] $u = e^{2x}$, $du = 2e^{2x}dx$

$e^{2(\ln(3)/4)}$

$= e^{\ln(3)/2}$

$= e^{\ln(\sqrt{3})}$

$= \sqrt{3}$

$$= \frac{1}{2}\int_{u(0)}^{u(\ln(3)/4)} \frac{du}{1+u^2} = \frac{1}{2}\int_1^{\sqrt{3}} \frac{du}{1+u^2} = \frac{1}{2}\arctan(u)\Big]_1^{\sqrt{3}}$$

$$= \frac{1}{2}\arctan(\sqrt{3}) - \frac{1}{2}\arctan(1)$$

$\frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \to \theta = \frac{\pi}{3}$

$$= \frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right) = \boxed{\frac{\pi}{6} - \frac{\pi}{8}}$$