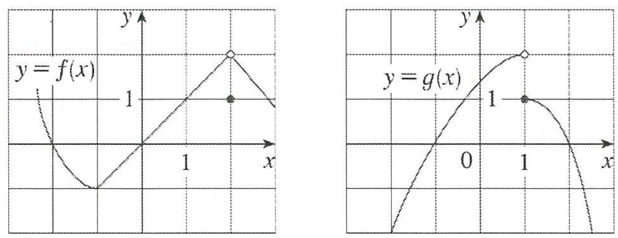
Solutions

Math 1300-005 - Spring 2017

Using the Limit Laws - 1/25/17

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



(a) $\lim_{x\to 2}[f(x)+3g(x)]$

$= \lim_{x\to2} f(x) + 3\lim_{x\to2} g(x)$ [Law 1+3]

$= 2 + 3\cdot(0)$

$= \boxed{2}$

(b) $\lim_{x\to 1}[2f(x)+g(x)]$ → [Law 1+3]

$2\cdot\lim_{x\to1^+} f(x) + \lim_{x\to1^+} g(x) = 2\cdot1+1 = 3$

$2\cdot\lim_{x\to1^-} f(x) + \lim_{x\to1^-} g(x) = 2\cdot1+2 = 4$

RHL ≠ LHL so $\boxed{\text{DNE}}$

(c) $\lim_{x\to 0}[f(x)g(x)]$

$= \lim_{x\to0} f(x)\cdot\lim_{x\to0} g(x)$ [law 4]

$= 0\cdot\frac{1}{3} = \boxed{0}$

(d) $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$

$\lim_{x\to-1} g(x) = 0$, so we cannot use law 5.

In fact, $\lim_{x\to-1^+}\frac{f(x)}{g(x)} = -\infty$ so $\boxed{\lim_{x\to-1}\frac{f(x)}{g(x)}\ \text{DNE}}$

$\lim_{x\to-1^-}\frac{f(x)}{g(x)} = +\infty$

(e) $\lim_{x\to 2}[x^3 f(x)]$

$= \lim_{x\to2} x^3\cdot\lim_{x\to2} f(x)$ [law 4]

$= (2)^3\cdot(2)$ [law 9]

$= \boxed{16}$

(f) $\lim_{x\to 1}\sqrt{3+f(x)}$

$= \sqrt{\lim_{x\to1}(3+f(x))}$ [law 11]

$= \sqrt{\lim_{x\to1} 3 + \lim_{x\to1} f(x)}$ [law 1]

[law 7] $= \sqrt{3+1} = \boxed{2}$

2. Evaluate each limit and justify each step by indicating the appropriate Limit Law(s).

(a) $\lim_{x\to 8}(1+\sqrt[3]{x})(2-x^2)$

$= \lim_{x\to8}(1+\sqrt[3]{x})\cdot\lim_{x\to8}(2-x^2)$ [law 4]

$= \left(\lim_{x\to8}1 + \lim_{x\to8}\sqrt[3]{x}\right)\left(\lim_{x\to8}2 - \lim_{x\to8}x^2\right)$ [law 1]

$= (1+\sqrt[3]{8})(2-(8)^2)$ [law 9+10]

$= (1+2)(2-64)$

$= 3(-62) = \boxed{-186}$

(b) $\lim_{x\to1}\sqrt{\dfrac{2x^2+1}{3x-2}}$ $= \sqrt{\lim_{x\to1}\left(\frac{2x^2+1}{3x-2}\right)}$ [law 11]

$= \sqrt{\dfrac{\lim_{x\to1}(2x^2+1)}{\lim_{x\to1}(3x-2)}}$ [law 5 since $\lim_{x\to1}(3x-2)\neq$]

$= \sqrt{\dfrac{2(1)^2+1}{3(1)-2}}$ [Direct substitution]

$= \boxed{\sqrt{3}}$

3. Find the limit by simplifying the function. Also, explain why the direct substitution property is not valid.

$$\lim_{h \to 0} \frac{(4+h)^2 - 16}{h}$$

Cannot use direct substitution since we have a 0 in the denominator. Let us simplify.

$$\lim_{h \to 0} \frac{(4+h)^2-16}{h} = \lim_{h \to 0} \frac{\cancel{16} + 8h + h^2 - \cancel{16}}{h} = \lim_{h \to 0} \frac{8h+h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(8+h)}{\cancel{h}}$$

$$= 8+0$$

$$= \boxed{8}$$

Like I mentioned in class. Try substitution. If you have division by 0, then simplify until you can substitute.

4. Find the limit by rationalizing the function.

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

Cannot use direct substitution since we have division by 0. Let us simplify by rationalizing

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}\left(\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}\right) = \lim_{x \to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{1+x}+1}$$

$$= \frac{1}{\sqrt{1+0}+1} = \boxed{\frac{1}{2}}$$

5. It is true that

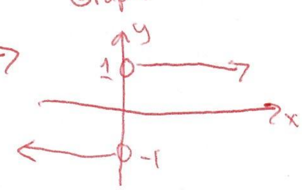
$$\lim_{x \to 0} \frac{|x|}{x}$$

does not exist. In your groups, work out and discuss why this is so.

Recall, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$ so dividing by x gives

$$\frac{|x|}{x} = \underbrace{\begin{cases} \frac{x}{x} & x > 0 \\ \frac{-x}{x} & x < 0 \end{cases}}_{\text{note: we can no longer write } \geq \text{ or } \leq 0 \text{ since } x \text{ is in the denominator}} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases} \rightarrow$$

Graph



So we have a jump discontinuity at 0, hence $\lim_{x \to 0} \frac{|x|}{x}$ DNE.