Additional Related Rate Problems

1) **Problem.** *A baseball diamond is a square with sides* 90 *feet long. A runner travels from first base to second base at* 30*ft/sec. How fast is the runner's distance from home plate changing when the runner is* 50 *feet from first base?*

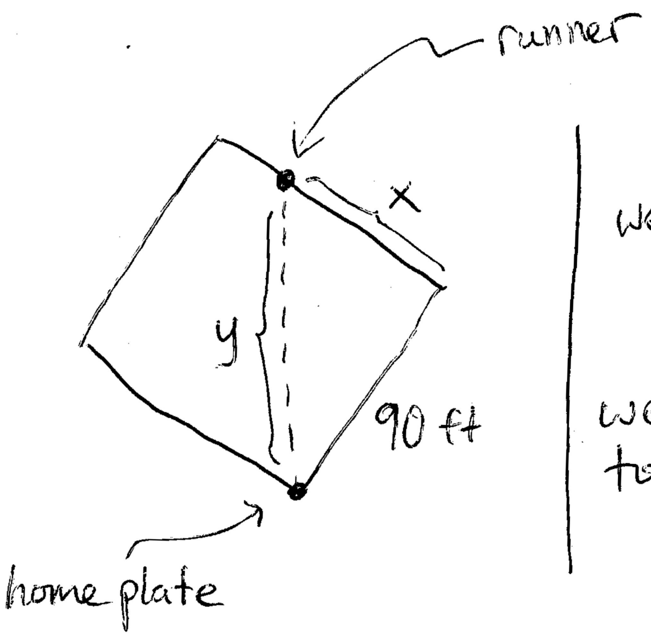
2) **Problem.** *Two planes leave an airport simultaneously. One travels north at* 300 *mph and the other travels west at* 400 *mph. How fast is the distance between them changing after* 10 *minutes?*

3) **Problem.** *If the temperature of a gas is held constant, then Boyle's Law guarantees that the pressure* P *and the volume* V *of the gas satisfy the equation* $PV = c$*, where* c *is a constant. Suppose the volume is increasing at a rate of* 15 *cubic inches per second. How fast is the pressure decreasing when the pressure is* 125 *pounds per square inch and the volume is* 25 *inches?*

4) **Problem.** *A balloon rises vertically from a point that is* 150 *feet from an observer at ground level. The observer notes that the angle of elevation is increasing at the rate of* 15 *degrees per second when the angle of elevation is* $60°$*. Find the speed of the balloon at this instant.*

5) **Problem.** *Ethan is cleaning the exterior of a glass building with a squeegee. The base of the* 10 *foot long squeegee handle, which is resting on the ground, makes an angle of* A *with the horizontal. The top of the squeegee is* x *feet above the ground on the building. Ethan pushes the bottom of the squeegee handle toward the wall. Find the rate at which* x *changes with respect to* A *when* $A = 60°$*. Express the answer in feet per degree.*

1)



we know: $\frac{dx}{dt} = 30 \text{ ft/sec}$

we want to know: $\frac{dy}{dt}$ when $x = 50$ ft.

STEP 1: Setup equation.

$$90^2 + x^2 = y^2$$

STEP 2: Implicit Differentiation.

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

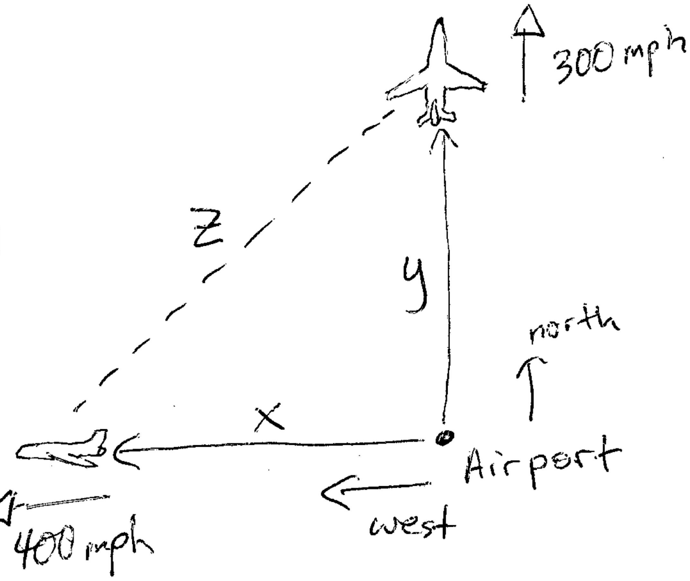
STEP 3: Plug in known values and solve for dy/dt.

$$2(50 \text{ ft.})(30 \text{ ft/sec}) = 2(y)\frac{dy}{dt}$$

$\left\{ \begin{array}{l} y = \sqrt{50^2 + 90^2} \\ \approx 103 \text{ ft} \end{array} \right.$

$$\boxed{\frac{dy}{dt} \approx 14.6 \text{ ft/sec}}$$

2)



we know: $\frac{dx}{dt} = 300$ mph

$\frac{dy}{dt} = 400$ mph

we want to know: $\frac{dz}{dt}$ after 10 min.

STEP 1: $x^2 + y^2 = z^2$

STEP 2: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

10 min. is 1/6 of an hour

STEP 3: At $t = 10$, we have: $x = \underset{3}{\cancel{4}}00 \cdot \frac{1}{6} = \overset{50}{\cancel{}}$ m

and $y = 400 \cdot \frac{1}{6} = \frac{200}{3}$ m. Now plug these into related rates equation from STEP 2:

$$2(50\text{ m})(300\text{ mph}) + 2\left(\frac{200}{3}\text{ m}\right)(400\text{ mph}) = 2(z)\frac{dz}{dt}$$

$$z = \sqrt{(50)^2 + \left(\frac{200}{3}\right)^2} = \frac{250}{3}$$

$$\boxed{\frac{dz}{dt} = 500\text{ mph}}$$

3)

$PV = c$

$\frac{dV}{dt} = 15 \text{ in}^3/\text{sec}$

$\frac{dP}{dt} = ?$ when $P = 125 \text{ lb}/\text{in}^2$ and $V = 25 \text{ in}^3$

(use product rule when doing implicit diff.)

$$P \cdot \frac{dV}{dt} + V \cdot \frac{dP}{dt} = 0$$

because c is a constant

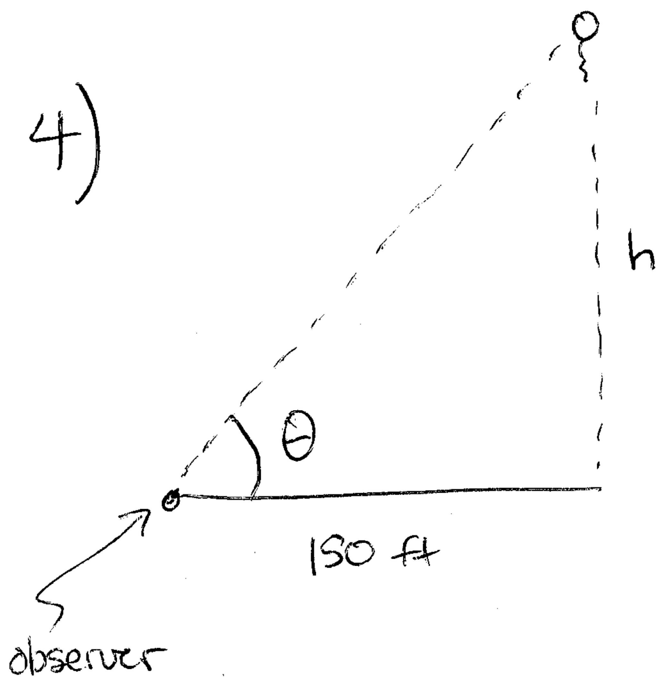
$$(125 \text{ lb}/\text{in}^2)(15 \text{ in}^3/\text{sec}) + (25 \text{ in}^3)\left(\frac{dP}{dt}\right) = 0$$

$$\boxed{\frac{dP}{dt} = -75 \text{ lb}/\text{in}^2\cdot\text{sec}}$$

NOTE: make sure the units agree with your equation:

$$\left[\frac{\text{lb}}{\text{in}^2}\right]\left[\frac{\text{in}^3}{\text{sec}}\right] + \left[\text{in}^3\right]\left[\begin{array}{c}\text{units}\\ \text{of}\\ dP/dt\end{array}\right] = \left[\frac{\text{lb}\cdot\text{in}}{\text{sec}}\right]$$

4)



$\frac{d\theta}{dt} = 15^\circ/\text{sec} = \frac{\pi}{12}\ \frac{\text{rad}}{\text{sec}}$ when

$\theta = 60^\circ = \frac{\pi}{3}$ rad.

$\frac{dh}{dt} = ?$ ft/sec when

$\theta = \pi/3$ rad.

$$\tan(\theta) = \frac{h}{150}$$

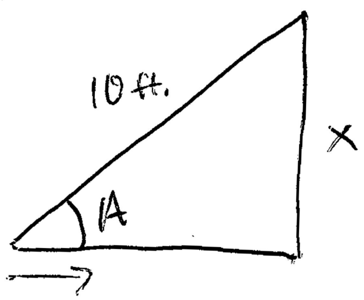
$\Downarrow$ Implicit diff.

$$\sec^2(\theta)\frac{d\theta}{dt} = \frac{dh}{dt}\cdot\frac{1}{150}$$

$$\sec^2(\pi/3)(\pi/12\ \text{rad/sec}) = \frac{dh}{dt}\cdot\frac{1}{150}$$

$$\boxed{\frac{dh}{dt} = 50\pi\ \text{ft./sec}}$$

5)



Find $\frac{dx}{dA}$ when $A = 60°$.

(This is tricky since we are not interested in the change of x with respect to time. We want change of x with respect to A.)

$$\sin(A) = \frac{x}{10}$$

$\Downarrow$ Implicit diff. with respect to A.

$$\cos(A) = \frac{dx}{dA} \cdot \frac{1}{10}$$

$$\boxed{\frac{dx}{dA} = 10\cos(60°) = 5 \text{ ft./deg.}}$$