INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/EXAM 1**, (3) **lecture number/instructor name** and (4) **FALL 2013** on the front of your blue-book. Also make a **grading table** with room for 4 problems and a total score. **Start each problem on a new page.** **Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK**

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1. In this problem, *if you need to find a derivative you must do so using only the limit definition of the derivative*, you may not use the rules of differentiation. Justify your answers.

   In economics, the function $C(x)$ is called a *cost function*, it measures the cost of production. For example, suppose the cost (in dollars) of producing x units of an energy efficient lightbulb is $C(x) = 50 + 10x + 5x^2$.

   (a) (10 pts) What is the average rate of change of the cost when productions levels are changed from $x = 1$ to $x = 2$.

   (b) (10 pts) Although $C(x)$ is not differentiable, economists define the *marginal cost* of production as the instantaneous rate of change of the cost with respect to the number of units produced. Marginal costs can be used to predict future production costs. Find the marginal cost of producing energy efficient lightbulbs when $x = 2$.

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2. All parts of this problem refer to the function $f(x)$ given below. In this problem, *if you need to find a derivative you must do so using only the limit definition of the derivative* and, for all parts of the problem, be sure to show all work and justify your answers. Consider the function

$$f(x) = \begin{cases} \dfrac{x+2}{x-1}, & \text{if } x \leq -2 \\ -x, & \text{if } -2 < x \leq 4 \\ \sqrt{x} - 6, & \text{if } x > 4 \end{cases}$$

   (a) (5 pts) Find all horizontal and vertical asymptotes of $f(x)$.

   (b) (5 pts) Find and classify all discontinuities of $f(x)$ as *jump*, *removable* and/or *infinite*.

   (c) (10 pts) Find the slope of the curve $f(x)$ exactly when $x = 5$.

   (d) (10 pts) Use the limit definition of the derivative to find $f'(4)$ if possible.

   (e) (5 pts) Is $f(x)$ differentiable for all x in its domain? If not, where is it *non-differentiable* and explain why it fails to have a derivative there.

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3. (a) (10 pts) Show that $\lim_{x\to\infty} 2^{-x}\sin(2\pi x) = 0$. Hint: Use the Squeeze Theorem. Show all work.

   (b) (10 pts) Is there a solution to the equation $x2^x = 1$ on the interval $[0, 2]$? Justify your response.

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4. (a) Evaluate the following limits, you may not use l'Hospital's Rule, justify your answers:

   (i) (7 pts) $\lim_{t\to 0} \dfrac{\tan(10t)}{\sin(6t)}$ (ii) (7 pts) $\lim_{x\to 3} \dfrac{x-3}{|x-3|}$ (iii) (7 pts) $\lim_{x\to -1^-} \dfrac{x-1}{x^4-1}$

   (b) (4 pts) Find a number c so that the following limit exists, justify your answer: $\lim_{x\to -2} \dfrac{2x^2 + 2cx + c - 6}{x^2 + x - 2}$.

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