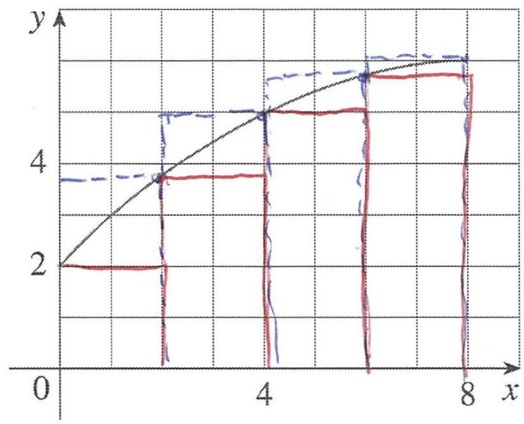
Solutions

# Math 1300-005 - Spring 2017

Areas and Distances - 4/17/17

*Guidelines:* Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

1. Consider the function $y = f(x)$ graphed below. We shall estimate the area $A$ under the graph of $f$ for $0 \le x \le 8$ using four approximating rectangles of equal length.




- - - → Right endpoints

—— → Left endpoints

(a) Find $L_4$. Does this over-approximate or under-approximate the true area $A$?

$$L_4 = 2f(0) + 2f(2) + 2f(4) + 2f(6)$$
$$= 2(2 + 3.8 + 5 + 5.7) = \text{whatever}$$

This is an under-approx because the rectangles all lie below the curve.

(b) Find $R_4$. Does this over-approximate or under-approximate the true area $A$?

$$R_4 = 2f(2) + 2f(4) + 2f(6) + 2f(8)$$
$$= 2(3.8 + 5 + 5.7 + 6) = \text{whatever}$$

This is an over-approx because the rectangles all lie above the curve

(c) Name at least two ways we could make these approximations better.

(1) Use midpoints

(2) More Rectangles

(3) Average together $L_4$ and $R_4 \to \dfrac{L_4 + R_4}{2}$

2. Let $A$ be the area under the graph of $f(x) = \frac{2x}{x^2+1}$ from $1 \le x \le 3$. Use Definition 2 to find an expression for the area under the graph of $f$ as a limit. Do not evaluate this limit.

Recall: $A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i)$ where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

Here $\Delta x = \frac{3-1}{n} = \frac{2}{n}$ and $x_i = 1 + i\left(\frac{2}{n}\right) = 1 + \frac{2i}{n}$.

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left[\frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1}\right]}$$

3. Determine a region whose area is equal to the given limit.

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\left(\frac{i\pi}{4n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f(a+i\Delta x)$

Here $\Delta x = \frac{\pi}{4n} = \frac{(\pi/4)}{n}$

$x_i = \underbrace{\frac{i\pi}{4n}}_{i\Delta x} \to$ so $a = 0$ and $\Delta x = \frac{b-0}{n} = \frac{(\pi/4)}{n}$

so $b = \frac{\pi}{4}$

$\boxed{\text{This limit is the area under } f(x) = \tan(x) \text{ on } [0, \pi/4]}$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \ln\left(1 + i\frac{3}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f(a+i\Delta x)$

Here $\Delta x = \frac{3}{n}$ so $a = 1$, and $\Delta x = \frac{b-1}{n} = \frac{3}{n}$, so

$x_i = 1 + \underbrace{i\frac{3}{n}}_{i\Delta x}$ $b = 4$

$\boxed{\text{This limit is the area under } f(x) = \ln(x) \text{ on } [1, 4]}$

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1. The speed of a runner *increased* steadily during the first three seconds of race. Her speed at half-second intervals is given in the table.

$\Delta t = 0.5$

| $t$ (s) | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|---|---|---|---|---|---|---|---|
| $v$ (ft/s) | 0 | 6.2 | 10.8 | 14.9 | 18.1 | 19.4 | 20.2 |

→ 7 data points but 6 time intervals, so we need $R_6$, $L_6$, etc... → Not $R_7$ or $L_7$

include units!

(a) Find a lower estimate for the distance she travelled during these three seconds.

Since the speed (velocity) increases, left endpoints give lower estimate (see the notes I gave out)

Lower estimate Distance $= L_6 = \left(0(0.5) + 6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5)\right)$
= "whatever" ft.

(b) Find an upper estimate for the distance she travelled during these three seconds.

Since the speed (velocity) increase, right endpoints give upper estimate

Upper estimate Distance $= R_6 = \left(6.2(0.5) + 10.8(0.5) + 14.9(0.5) + 18.1(0.5) + 19.4(0.5) + 20.2(0.5)\right)$ ft
= "whatever" ft.

2. Oil leaked from a tank at a rate of $r(t)$ liters per hour. The rate *decreased* as time passed and values of the rate at two-hour time intervals are shown in the table.

$\Delta t = 2$

| t (h) | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| r(t) (L/h) | 8.7 | 7.6 | 6.8 | 6.2 | 5.7 | 5.3 |

→ 6 data points but 5 time intervals so we need $R_5$, $L_5$, etc... → not $R_6$ or $L_6$

(a) Find a lower estimate for the total amount of oil that leaked out.

Since the rate decreases, right endpoints give lower estimate.

lower estimate $= R_5 = \left(7.6(2) + 6.8(2) + 6.2(2) + 5.7(2) + 5.3(2)\right)$ liters
= "whatever" liters.

(b) Find an upper estimate for the total amount of oil that leaked out.

Since the rate decreases, left endpoints give upper estimate

upper estimate $= L_5 = \left(8.7(2) + 7.6(2) + 6.8(2) + 6.2(2) + 5.7(2)\right)$ liters
= "whatever" liters.