Solutions

Math 1300-005 - Spring 2017

Implicit Differentiation Intro - 2/27/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 2.

1. Our goal in this problem is to use implicit differentiation to find an equation of the tangent line to the curve
$$x^2 + xy = 3 - y^2$$
at the point $(1, 1)$.

   (a) First, apply d/dx to both sides of $x^2 + xy = 3 - y^2$ and use the chain rule and the guidelines from lecture. Notice the second term on the left-hand side is a product.

   $$\frac{d}{dx}(x^2+xy) = \frac{d}{dx}(3-y^2)$$

   PR

   $$2x + y + xy' = -2yy'$$

   (b) Move all terms that have a y' to the left-hand side of the equation from (a) and move terms that do NOT have a y' to the right-hand side of the equation from (a). Solve for y'.

   $$xy' + 2yy' = -2x - y$$

   $$y'(x+2y) = -2x - y \rightarrow \boxed{y' = \frac{-2x-y}{x+2y}}$$

   (c) Plug in $x = 1$ and $y = 1$ to your expression for y' to get the slope of the tangent line to the curve at $(1, 1)$.

   $$y' \text{ at } (1,1): \quad y' = \frac{-2(1)-(1)}{1+2(1)} = \frac{-3}{3} = \boxed{-1}$$

   (d) Write an equation of the tangent line based on your work in (a), (b), and (c).

   $$\boxed{y - 1 = -1(x-1)}$$

2. Following the same procedure outline in problem 1, find an equation of the tangent line to the curve

$$y\sin(2x) = x\cos(2y)$$

at the point $(\pi/2, \pi/4)$. Notice that each side of the equation above involves a product.

(a) $\frac{d}{dx}(y\sin(2x)) = \frac{d}{dx}(x\cos(2y))$ [Product rule to each side, then chain rule as needed]

$y'\sin(2x) + y\cos(2x)\cdot 2 = \cos(2y) - x\sin(2y)\cdot 2y'$

(b) Rearranging: $y'\sin(2x) + 2xy'\sin(2y) = \cos(2y) - 2y\cos(2x)$

$y'(\sin(2x) + 2x\sin(2y)) = \cos(2y) - 2y\cos(2x)$

$\boxed{y' = \dfrac{\cos(2y) - 2y(\cos(2x))}{\sin(2x) + 2x\sin(2y)}}$

(c) Plug in $(\pi/2, \pi/4)$:

$y' = \dfrac{\cos(\frac{2\pi}{4}) - 2(\frac{\pi}{4})\cos(2(\frac{\pi}{2}))}{\sin(2(\pi/2)) + 2(\pi/2)\sin(2(\pi/4))} = \dfrac{0 - \frac{\pi}{2}(-1)}{0 + \pi(1)} = \dfrac{\pi/2}{\pi} = \boxed{\dfrac{1}{2}}$

(d) $\boxed{y - \pi/4 = \frac{1}{2}(x - \pi/2)}$

3. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

$$\frac{d}{dx}\underbrace{\tan(x-y)}_{\text{Chain}} = \frac{d}{dx}\frac{y}{1+x^2} \quad QR$$

$\sec^2(x-y)\cdot\frac{d}{dx}(x-y) = \dfrac{y'(1+x^2) - y\cdot 2x}{(1+x^2)^2}$

$\sec^2(x-y)\cdot(1-y') = \dfrac{y'(1+x^2) - 2xy}{(1+x^2)^2}$

$\sec^2(x-y) - y'\sec^2(x-y) = \dfrac{y'(1+x^2) - 2xy}{(1+x^2)^2}$ → multiply both sides

$(1+x^2)^2[\sec^2(x-y)] - y'(1+x^2)^2\sec^2(x-y) = y'(1+x^2) - 2xy$

$-y'(1+x^2)^2\sec^2(x-y) - y'(1+x^2) = -(1+x^2)^2\sec^2(x-y) - 2xy$

$y'(-(1+x^2)^2\sec^2(x-y) - (1+x^2)) = -(1+x^2)^2\sec^2(x-y) - 2xy$

$\boxed{y' = \dfrac{-(1+x^2)^2\sec^2(x-y) - 2xy}{-(1+x^2)^2\sec^2(x-y) - (1+x^2)}}$ or $\boxed{y' = \dfrac{(1+x^2)^2\sec^2(x-y) + 2xy}{(1+x^2)^2\sec^2(x-y) + (1+x^2)}}$

4. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

$$\overbrace{e^y\cos(x)}^{PR} = 1 + \sin(\overbrace{xy}^{PR})$$

chain

$\frac{d}{dx}(e^y\cos(x)) = \frac{d}{dx}(1+\sin(xy))$

$e^y\cdot y'\cos(x) + e^y(-\sin(x)) = \cos(xy)\cdot\frac{d}{dx}(xy)$ PR

$e^y\cdot y'\cos(x) - e^y\sin(x) = \cos(xy)(y + xy')$

$e^y\cdot y'\cos(x) - e^y\sin(x) = y\cos(xy) + xy'\cos(xy)$

$e^y\cdot y'\cos(x) - xy'\cos(xy) = y\cos(xy) + e^y\sin(x)$

$y'(e^y\cos(x) - x\cos(xy)) = y\cos(xy) + e^y\sin(x)$

$\boxed{y' = \dfrac{y\cos(xy) + e^y\sin(x)}{e^y\cos(x) - x\cos(xy)}}$