Math 1300-010 - Fall 2016
Antiderivatives - 11/9/16

$\boxed{\text{Solutions}}$

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

1. Find the most general antiderivative F of the function.

   (a) $f(x) = 8x^9 - 3x^6 + 12x^3$

   $F(x) = \boxed{\frac{8}{10}x^{10} - \frac{3}{7}x^7 + \frac{12}{4}x^4 + C}$

   (b) $f(x) = 6\sqrt{x} - \sqrt[6]{x} = 6x^{1/2} - x^{1/6}$

   $\boxed{F(x) = 6\left(\frac{2}{3}x^{3/2}\right) - \left(\frac{6}{7}x^{7/6}\right) + C}$

   (c) $f(x) = 3e^x + 7\sec^2(x)$

   $\boxed{F(x) = 3e^x + 7\tan(x) + C}$

   (d) $f(x) = \cos(x) - \dfrac{2}{x} =$

   $\boxed{F(x) = \sin(x) - 2\ln|x| + C}$

2. Find f. That is, find the most general antiderivative of the given function, then use the given data to solve for C.

   (a) $f'(x) = 1 - 6x, \quad f(0) = 8$

   $\hookrightarrow$ so $f(x) = x - 3x^2 + C \Leftrightarrow 8 = f(0) = 0 - 0 + C$

   $C = 8$ $\qquad \boxed{f(x) = x - 3x^2 + 8}$

   $x^{-4} \to \frac{x^{-3}}{-3}$

   (b) $f'(x) = 2x - \dfrac{3}{x^4}, \quad f(1) = 3$

   $f(x) = x^2 - \frac{3x^{-3}}{-3} + C$

   $= x^2 + x^{-3} + C$

   $\to 3 = f(1) = 1^2 + 1^{-3} + C$

   $3 = 2 + C$

   $C = 1$

   $\boxed{f(x) = x^2 + x^{-3} + 1}$

(c) $f'(x) = \dfrac{4}{\sqrt{1-x^2}}$, $\quad f(1/2) = 1$

$f(x) = 4\arcsin(x) + C$

$\Leftrightarrow 1 = f(1/2) = 4\arcsin(\tfrac{1}{2}) + C$

$1 = 4\left(\frac{\pi}{6}\right) + C$

$C = 1 - \frac{2}{3}\pi$

$\boxed{f(x) = 4\arcsin(x) + 1 - \frac{2}{3}\pi}$

(d) $f''(x) = 8x^3 + 5, \quad f(1) = 0, \quad f'(1) = 8$

$f'(x) = 2x^4 + 5x + C \Leftrightarrow 8 = f'(1) = 2 + 5 + C \Leftrightarrow C = 1$

$f'(x) = 2x^4 + 5x + 1$

$\Leftrightarrow f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + C$

$\Rightarrow 0 = \frac{2}{5} + \frac{5}{2} + 1 + C \Leftrightarrow$ solve for C!

3. Find the most general antiderivative of

$$f(x) = (x+1)(2x-1). = 2x^2 - x + 2x - 1 = 2x^2 + x - 1$$

CAUTION: Just as the derivative of a product is NOT the product of derivatives, the antiderivative of a product is NOT the product of antiderivatives. To solve this, foil.

$$\boxed{F(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C}$$

4. Find the most general antiderivative of

$$f(x) = \frac{x^5 - x^3 + 2\sqrt{x}}{x^4}.$$

CAUTION: The antiderivative of a quotient is NOT the quotient of antiderivatives. To solve this, use exponent rules and the fact that

$$\frac{x^5 - x^3 + 2\sqrt{x}}{x^4} = \frac{x^5}{x^4} - \frac{x^3}{x^4} + 2\frac{\sqrt{x}}{x^4}. = x - x^{-1} + 2x^{-\frac{7}{2}}$$

$$\boxed{F(x) = \frac{1}{2}x^2 - \ln|x| + 2\left(-\frac{2}{5}x^{-\frac{5}{2}}\right) + C}$$