On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 150 points and has 8 questions on both sides of this paper.

- **Show all work and simplify your answers!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes or papers, calculators, cell phones, or electronic devices are permitted.

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1. Evaluate the following limits.

(a) (6 pts) $\lim_{\theta \to \pi/2} \frac{\sin(3\theta)}{\theta}$ (b) (6 pts) $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$

(c) (6 pts) $\lim_{x \to 0} |x| \cos(1/x)$ (d) (8 pts) $\lim_{x \to 0} (1 - 2x)^{1/x}$

2. (16 pts, 8 pts each) Evaluate the following integrals.

(a) $\int_1^e \frac{1}{x(1 + (\ln x)^2)}\, dx$ (b) $\int_0^1 2e^{-x} \cosh x \, dx$

3. (12 pts) Let $h(x) = x^{3/2} + \int_1^{x^3} \frac{1}{1 + t^3}\, dt$

(a) Find the linearization of $h(x)$ at $a = 1$.

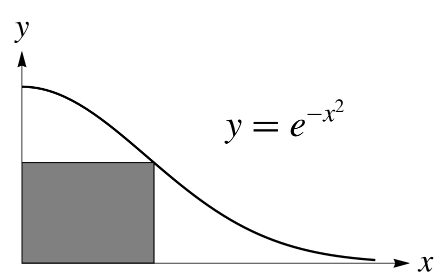
(b) Use the linearization to approximate $h(1.1)$.

4. (15 pts) Sketch a graph of a single function $y = g(x)$ that satisfies all of the following conditions. No explanation is necessary. Clearly label all important features of the graph.

(a) $g(-x) = -g(x)$ (b) $g(-1) = 1$ (c) $\lim_{h \to 0} \frac{g(4+h) - g(4)}{h} < 0$

(d) $\lim_{x \to 2} g(x) = -\infty$ (e) $\lim_{x \to -1} g(x) = 3$

5. (15 pts) The rectangle shown has sides along the positive x and y axes and its upper right vertex on the curve $y = e^{-x^2}$. What dimensions give the rectangle its largest area?



6. The following questions are unrelated.

   (a) (10 pts) Write $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{(3i/n+2)^2} \frac{3}{n}$ as a definite integral and evaluate.

   (b) (10 pts) Find dy/dx for $xy = \tan(y+3)$ at the point $(x, y) = (0, -3)$.

   (c) (6 pts) Simplify $\sum_{k=1}^{5} \arcsin\left((-1)^k\right)$.

   (d) (10 pts) Let $g(x) = x^{(1/\ln x)}$. (i) What is the domain of $g(x)$? (ii) Find $g'(x)$

7. (15 pts) The graph of a function $f(x)$ is shown below. Suppose $f(x)$ is the **derivative** of $F(x)$. Assume that $F(x)$ is continuous on the interval $[-2, 8]$. No justification is required for the following questions. If the answer to any question is "none", write "none".

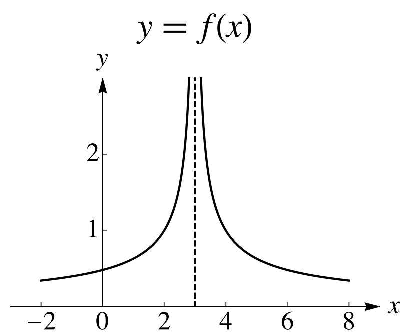
   (a) On what intervals is F increasing?

   (b) On what intervals is F concave up?

   (c) What are the x-coordinates of the absolute maximum and minimum values of F?

   (d) What are the x-coordinates of the inflection points of F?

   (e) Suppose we restrict the domain of f to $(3, 8]$ so that it is one-to-one. Then what is the value of $f^{-1}(1)$?



8. (15 pts) A skier glides on flat terrain. His motion is slowed only by friction with the snow. His velocity $v(t)$ obeys the equation:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

where k is a constant. His initial velocity is 10 meters per second; after $50\,\mathrm{s}$, his velocity is 5 meters per second.

   (a) Find the velocity of the skier at an arbitrary time t.

   (b) Find the velocity of the skier after 25 seconds. Simplify your answer.

   (c) Let $s(t)$ represent the distance traveled by the skier by time t, where t is measured in seconds. Find an equation for $s(t)$.