Math 1300-005 - Spring 2017

The Mean Value Theorem - 3/21/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3.

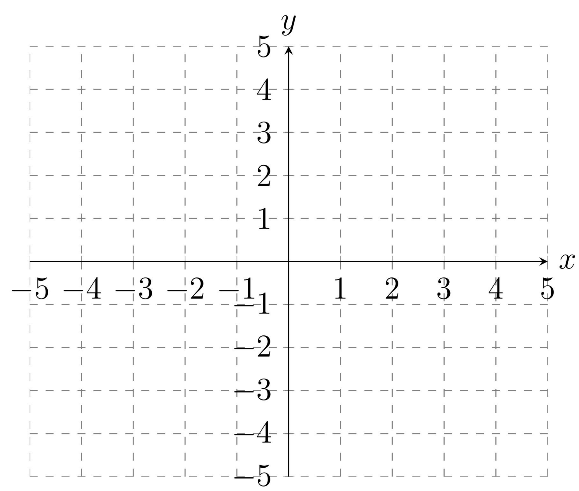
The purpose of this worksheet is to explore the ***Mean Value Theorem***, which states that if f is continuous on $[a, b]$ and differentiable on $(a.b)$, then there exists a number c between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \tag{1}$$

or equivalently,

$$f(b) - f(a) = f'(c)(b - a). \tag{2}$$

1. The MVT requires f to be continuous on $[a, b]$ *and* differentiable on (a, b). On the axes below, sketch an example of a function f, continuous on $[a, b]$, but *not* differentiable on (a, b) such that the conclusion of the MVT is false. That is, such that there is no c between a and b such that (1) is true.



2. Let $f(x) = (x - 4)^2 - 1$ on $[3, 6]$.

   (a) Does f satisfy the hypotheses of the MVT on $[3, 6]$? That is, is f continuous on $[3, 6]$ and differentiable on $(3, 6)$? Please explain.

   (b) Determine all numbers c which satisfy the conclusion of the MVT for f on $[3, 6]$.

3. A corollary of the Mean Value Theorem is known as **Rolle's Theorem**. In this problem, we will derive the result of this theorem. Suppose f is continuous on $[a, b]$ and differentiable on (a, b). Suppose as well that $f(a) = f(b)$.

   (a) Write down the conclusion of the MVT for f (this is (1) on the first page).

   (b) What is the value of the right hand side of (1) in this case?

   (c) Finish the statement of **Rolle's Theorem**: If f is continuous on $[a, b]$, differentiable on (a, b) and if $f(a) = f(b)$, then there exists a number c between a and b such that

$$f'(c) = \underline{\qquad}$$

4. Let $f(x) = x^2 - 2x - 8$ on $[-1, 3]$.

   (a) Does f satisfy the hypotheses of Rolle's theorem on $[-1, 3]$? That is, is f continuous on $[-1, 3]$ and differentiable on $(-1, 3)$ and does $f(-1) = f(3)$? Please explain.

   (b) Determine all numbers c which satisfy the conclusion of Rolle's theorem for f on $[-1, 3]$.

5. Let $f(x) = x^3 + 6x^2 + 6x$ on $[-6, 0]$.

   (a) Does f satisfy the hypotheses of the MVT on $[-6, 0]$? That is, is f continuous on $[-6, 0]$ and differentiable on $(-6, 0)$? Please explain.

   (b) Determine all numbers c which satisfy the conclusion of the MVT for f on $[-6, 0]$.

6. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. The inequality gives a restriction on the rate of growth of f, which then imposes a restriction on the possible values of f. Use the MVT to determine how large $f(4)$ can possible be. [Hint: setup the MVT using (2) on the first page, and solve for $f(4)$.]