INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/Final Exam**, (3) **lecture number/instructor name** and (4) **SUMMER 2015** on the front of your bluebook. Also make a **grading table** with room for 5 problems and a total score. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK! JUSTIFY ALL YOUR ANSWERS!**

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1. (10 pts) Sketch a graph of a function $y = g(x)$ that satisfies all of the following conditions. No explanation is necessary.

   (a) $\lim_{x\to\infty} g(x) = 3$ (c) $\lim_{x\to 4^+} g(x) = \infty$ (e) g is an odd function.

   (b) $g(-2) = g(2) = 0$ (d) $\lim_{h\to 0} \dfrac{g(0+h) - g(0)}{h} = 1$ (f) $\lim_{x\to 4^-} g(x) = -\infty$

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2. (10 pts)

   (a) What is the definition of continuity? In other words, what conditions must a function obey in order to be considered continuous at some point a.

   (b) Find the numbers at which the function below is discontinuous. Use the definition of continuity from part (a) to justify your answers.

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

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3. (10 pts) Find y'' by implicit differentiation:

$$x^3 + y^3 = 1$$

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4. (10 pts) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is $600cm^3$, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant?

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5. (10 pts) Consider the number: $(1.999)^4$

   (a) What function, $f(x)$, could be used to create a linear approximation to estimate this number? What a value would be used?

   (b) What is the linear approximation, $L(x)$?

   (c) Use this linear approximation to estimate $(1.999)^4$. [You do not need to simplify all the way.]
