

Math 1300-005 - Spring 2017

Related Rates, Pt. III - 3/3/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3. This third worksheet over related rates covers some more intermediate examples now that we are used to the process.

For **each** of the following related rates problems:

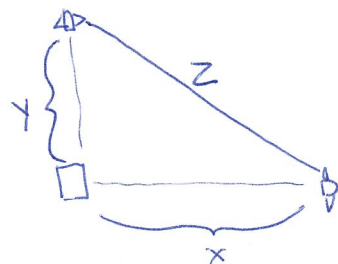
- Draw a picture of the situation and assign variables.
- Write down the known and unknown quantities in terms of the assigned variables.
- Use your picture to write an equation that relates the variables.
- Take d/dt of each side of this equation, solve for the unknown quantity, and then plug in the known quantities.

- Two space ships leave from a docking station at the same time on perpendicular trajectories. One of the space ships travels at a speed of ~~0.2~~ 2.4 light-years per year, and the other at a speed of 2.4 light-years per year. How fast is the distance between the spaceships changing at 5 years of travel?

~~Q11~~ (a) Picture

Start

After Some Time



(b) Known, unknown

Known

$$\frac{dx}{dt} = 2.4 \text{ lyr/yr}$$

$$\frac{dy}{dt} = 0.2 \text{ lyr/yr}$$

unknown $\frac{dz}{dt}$ after 5 years.

@ 5 yrs, $x = 2.4 \text{ lyr/yr} \cdot 5 \text{ yr} = 12 \text{ lyr}$
 $y = \frac{1}{10} \text{ lyr/yr} \cdot 5 \text{ yr} = 5 \text{ lyr}$

(c) Relation: $x^2 + y^2 = z^2$

(d) $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

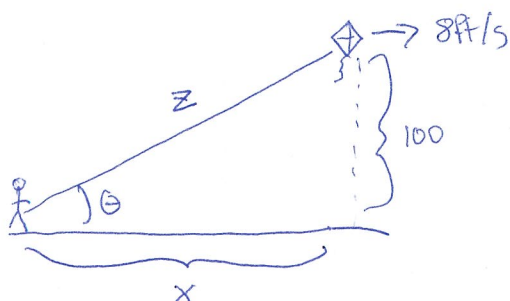
$\hookrightarrow \frac{dz}{dt} = \frac{1}{z} \left(x \frac{dx}{dt} + y \frac{dy}{dt} \right)$

After 5 years, $z = \sqrt{12^2 + 5^2} = 13$, so

$\frac{dz}{dt} = \frac{1}{13} (12(2.4) + 5(1)) = \boxed{2.6 \text{ lyr/yr}}$

2. A kite 100 ft above ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

(a) Picture



(b) known: $\frac{dx}{dt} = 8 \text{ ft/s}$

unknown: $\frac{d\theta}{dt}$ when $z = 200 \text{ ft}$.

(c) Relation: $\tan(\theta) = \frac{100}{x} \Leftrightarrow x = 100 \cot(\theta)$.

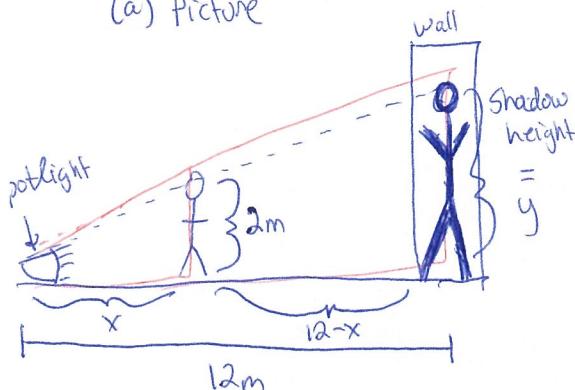
(d) $\frac{dx}{dt} = 100 (-\csc^2(\theta)) \frac{d\theta}{dt}$. Recall, $\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{\text{Hyp}}{\text{opp}} = \frac{z}{100}$.

so when $z = 200$, $\csc^2(\theta) = \left(\frac{200}{100}\right)^2 = 4$

Thus $\frac{d\theta}{dt} = \frac{dx/dt}{-100 \csc^2(\theta)} = \frac{8}{-100(4)} = -\frac{2}{100} = \boxed{-\frac{1}{50} \text{ rad/s}}$

3. A spotlight on the ground shines on a wall 12 m away. If a man 2m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

(a) Picture

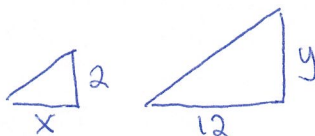


(b) Known $\frac{dx}{dt} = 1.6 \text{ m/s}$

unknown $\frac{dy}{dt}$ when he is 4 m from building

Note: If he is 4 meters from the building, he is $x = 8 \text{ m}$ from the spotlight!

(c) Relation: Similar triangles!



so $\frac{x}{2} = \frac{12}{y}$, so $y = \frac{24}{x} = 24x^{-1}$

(d) $\frac{dy}{dt} = -24x^{-2} \cdot \frac{dx}{dt}$, when $x = 8 \text{ m}$, $\frac{dy}{dt} = -24(8)^{-2} \cdot (1.6) = \frac{-24}{64} \cdot (1.6) = \frac{-24}{40}$

$\boxed{-\frac{6}{10} \text{ m/s}}$