Math 1300-005 - Spring 2017

The Evaluation Theorem - 4/24/17

Solutions

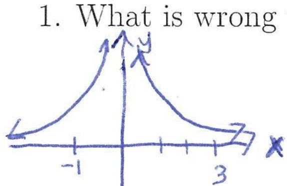
Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

Recall the ***evaluation theorem***: if f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

where F is any antiderivative of f, that is, $F' = f$.

1. What is wrong with the equation $\displaystyle\int_{-1}^{3} \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Big]_{-1}^{3} = -\frac{4}{3}$?



$\frac{1}{x^2}$ is not continuous on $[-1,3]$ so the evaluation theorem does not apply. In fact $\int_{-1}^{3} \frac{1}{x^2}\,dx = \infty$.

2. Evaluate the integral.

(a) $\displaystyle\int_{-2}^{3} (x^2 - 3)\,dx = \frac{1}{3}x^3 - 3x\Big]_{-2}^{3}$

$= \left(\frac{1}{3}(3)^3 - 3(3)\right) - \left(\frac{1}{3}(-2)^3 - 3(-2)\right) = \frac{27}{3} - 9 - \left(\frac{-8}{3} + 6\right)$

$= \boxed{\frac{8}{3} - 6}$

(b) $\displaystyle\int_0^1 \sqrt[5]{x^4}\,dx = \frac{5}{9}x^{9/5}\Big]_0^1 = \frac{5}{9}(1)^{9/5} - \frac{5}{9}(0)^{9/5}$

$\sqrt[5]{x^4} = x^{4/5}$

$= \boxed{\frac{5}{9}}$

(c) $\displaystyle\int_1^9 \frac{1}{2x}\,dx = \frac{1}{2}\ln(|x|)\Big]_1^9 = \frac{1}{2}\ln(|9|) - \frac{1}{2}\ln(|1|)$

$\frac{1}{2x} = \frac{1}{2}\left(\frac{1}{x}\right)$

$= \frac{1}{2}\ln(9) - 0$

$= \boxed{\frac{1}{2}\ln(9)}$

(d) $\displaystyle\int_0^2 \underbrace{(y-1)(2y+1)}_{2y^2+y-2y-1 \,=\, 2y^2-y-1}\,dy = \frac{2}{3}y^3 - \frac{1}{2}y^2 - y\Big]_0^2 = \left(\frac{2}{3}(2)^3 - \frac{1}{2}(2)^2 - 2\right) - \left(\frac{2}{3}(0)^3 - \frac{1}{2}(0)^2 - 0\right)$

$= \left(\frac{16}{3} - \frac{4}{2} - 2\right) - 0$

$= \boxed{\frac{16}{3} - 4}$

3. Find the general indefinite integral (also known as the general antiderivative). On (e) and (f), trig identities are required.

(a) $\displaystyle\int \frac{x-1}{\sqrt{x}}\,dx = \int\left(\frac{x}{x^{1/2}} - \frac{1}{x^{1/2}}\right)dx = \int\left(x^{1/2} - x^{-1/2}\right)dx$

$\displaystyle = \boxed{\frac{2}{3}x^{3/2} - 2x^{1/2} + C}$

(b) $\displaystyle\int e^{x+1}\,dx$

Note: $\frac{d}{dx}e^{x+1} = e^{x+1}\cdot\frac{d}{dx}(x+1)$

$= e^{x+1}$, so $\displaystyle\int e^{x+1}dx = \boxed{e^{x+1} + C}$

(c) $\displaystyle\int \frac{4}{t^2+1}\,dt = \boxed{4\arctan(t) + C}$

(d) $\displaystyle\int \frac{t^2-1}{t^4-1}\,dt = \int \frac{(t+1)(t-1)}{(t^2+1)(t^2-1)}dt = \int \frac{\cancel{(t+1)}\cancel{(t-1)}}{(t^2+1)\cancel{(t+1)}\cancel{(t-1)}}dt$

$\displaystyle = \int \frac{1}{t^2+1}\,dt = \boxed{\arctan(t) + C}$

(e) $\displaystyle\int (1+\tan^2(x))\,dx = \int \sec^2(x)\,dx$

$= \boxed{\tan(x) + C}$

(f) $\displaystyle\int \frac{\sin(x)}{1-\sin^2(x)}\,dx = \int \frac{\sin(x)}{\cos^2(x)}\,dx = \int \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}\,dx$

$\displaystyle = \int \tan(x)\sec(x)\,dx$

$= \boxed{\sec(x) + C}$

4. Evaluate the integral.

(a) $\displaystyle\int_0^{\pi/4} \sec(\theta)\tan(\theta)\,d\theta = \sec(\theta)\Big]_0^{\pi/4}$

$= \sec(\pi/4) - \sec(0)$

$= \boxed{\frac{2}{\sqrt{2}} - 1}$

(b) $\displaystyle\int_{-1}^{0} (2x - e^x)\,dx = x^2 - e^x\Big]_{-1}^{0}$

$= [(0)^2 - e^0] - [(-1)^2 - e^{-1}]$

$= -1 - [1 - e^{-1}] = \boxed{e^{-1} - 2}$

(c) $\displaystyle\int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x})\,dx = \int_0^1 (x^{4/3} + x^{5/4})\,dx$

$x(x^{1/3} + x^{1/4})$

$= \frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4}\Big]_0^1 = \left(\frac{3}{7}(1)^{7/3} + \frac{4}{9}(1)^{9/4}\right) - \left(\frac{3}{7}(0)^{7/3} + \frac{4}{9}(0)^{9/4}\right)$

$= \boxed{\frac{3}{7} + \frac{4}{9}}$

(d) $\displaystyle\int_{-5}^{5} e\,dx = ex\Big]_{-5}^{5} = [e(5) - e(-5)]$

constant, so antiderivative is ex (not e^x)

$= 5e + 5e$

$= \boxed{10e}$

(e) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}}\,dt = 6\arcsin(t)\Big]_{1/2}^{\sqrt{3}/2} = 6\arcsin\left(\frac{\sqrt{3}}{2}\right) - 6\arcsin\left(\frac{1}{2}\right)$

$= 6\left(\frac{\pi}{3}\right) - 6\left(\frac{\pi}{6}\right)$

$= 2\pi - \pi = \boxed{\pi}$

(f) $\displaystyle\int_1^{18} \sqrt{\frac{3}{z}}\,dz = \int_1^{18} \frac{\sqrt{3}}{\sqrt{z}}\,dz = \sqrt{3}\int_1^{18} \frac{1}{\sqrt{z}}\,dz = \sqrt{3}\int_1^{18} z^{-1/2}\,dz$

$= (\sqrt{3})\,2z^{1/2}\Big]_1^{18}$

$= \sqrt{3}\left(2\sqrt{18} - 2\sqrt{1}\right)$

$= \boxed{\sqrt{3}\left(2\sqrt{18} - 2\right)}$