Math 1300-005 - Spring 2017

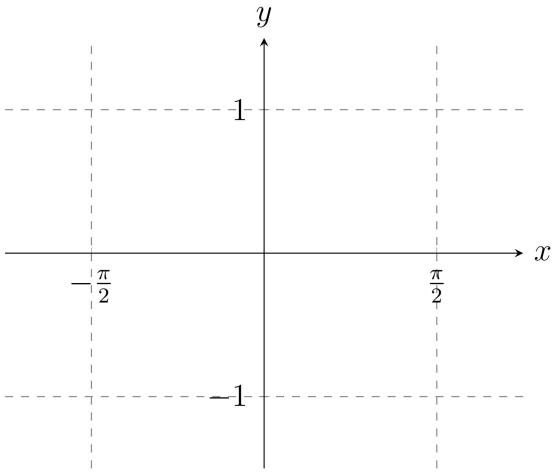
Inverse Trig Derivatives - 3/8/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 2.

The goal of this worksheet is to discover the derivatives of arctangent, arcsine, and arccosine using implicit differentiation.

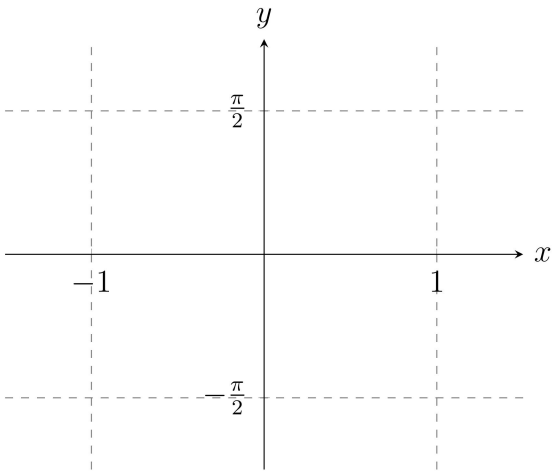
1. **The Derivative of Arcsine**

   (a) On the axes below, sketch a graph of $y = \sin(x)$ from $-\pi/2 < x < \pi/2$.



   (b) Why does $f(x) = \sin(x)$, when restricted to $-\pi/2 < x < \pi/2$, have an inverse?

   (c) Sketch the graph of the inverse $y = f^{-1}(x) = \arcsin(x)$ on the axes below.

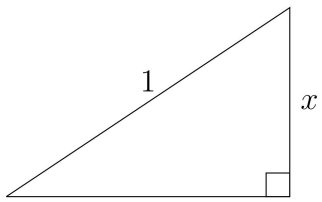


(d) By properties of inverse functions, we have the following identity.

$$\sin(\arcsin(x)) = x$$

Differentiate both sides of this equation to find a formula for the derivative of $\arcsin(x)$. Express your answer in terms of $\cos(\arcsin(x))$.

(e) Referring to the triangle below, explain why $\cos(\arcsin(x)) = \sqrt{1-x^2}$.



(f) Combining the results of part (d) and part (e), we conclude

$$\frac{d}{dx}\arcsin(x) =$$

2. **The Derivative of Arccosine**

   Here I will just state the result. You can arrive at this by a method very similar to that for arctangent and arcsine.

$$\frac{d}{dx}\arccos(x) = -\frac{1}{\sqrt{1-x^2}}.$$

   What is the difference between the derivatives of $y = \arcsin(x)$ and $y = \arccos(x)$?

3. Derivative practice using inverse trig. Find dy/dx for the following.

   (a) $y = (\arctan(x))^2$

   (b) $y = \arcsin(2x + e^x)$

   (c) $y = \arccos(\arcsin(x))$

(d) $\arccos(xy) = 1 + x^2 y$ [This involves implicit differentiation]

(e) $y = x \arcsin(x) + \sqrt{1 - x^2}$

4. Compute the following limit.

$$\lim_{x \to 4^-} \arctan\left(\frac{1}{4 - x}\right)$$