Solutions

## Math 1300-005 - Spring 2017

Midterm 3 Review, Part III - 4/10/17

*Guidelines:* Please work in groups of two or three. ~~Combining this review with quizzes 8 and 9, in-class worksheets since Midterm 2, and the review we shall do on Monday gives a good approximation of what appears on the exam.~~

1. Consider the function $f(x)$ and its first and second derivatives.

$$f(x) = \frac{36x}{(x+2)^2}, \qquad f'(x) = \frac{-36(x-2)}{(x+2)^3}, \qquad f''(x) = \frac{72(x-4)}{(x+2)^4}$$

(a) Find the $x$-intercept(s) of $f$, if any. Find the $y$-intercept(s) of $f$, if any.

X-int: set $f(x)=0$

$0 = \frac{36x}{(x+2)^2}$

so $x=0$. The point is $(0, f(0)) = \boxed{(0,0)}$

y-int: set $x=0$

$y = \frac{36(0)}{(0+2)^2} = 0$.

The point is $\boxed{(0,0)}$

(b) Find the vertical asymptote(s) of $f$, if any. Find the horizontal asymptote(s) of $f$, if any.

VA: $x+2=0$, so $\boxed{x=-2}$

HA: $\lim_{x\to\infty} \frac{36x}{(x+2)^2} \approx \lim_{x\to\infty} \frac{36x}{x^2} = 0$ (bottom heavy).

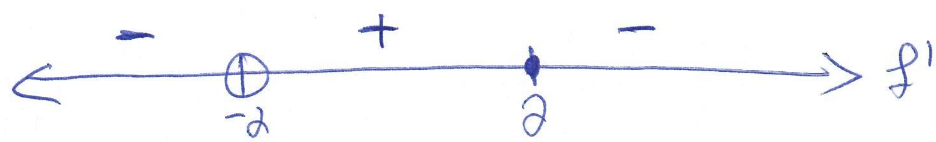
$\lim_{x\to-\infty} \frac{36}{(x+2)^2} \approx \lim_{x\to-\infty}$

(c) Find all values of $x$ such that $f'(x) = 0$ ***AND*** all values of $x$ such that the denominator of $f'$ is zero. Which of these $x$-values are critical numbers?

$f'(x)=0$ when $-36(x-2)=0$. So $\boxed{x=2. \text{ This is critical}}$

Denominator $=0$ when $x+2=0$ so $\boxed{x=-2. \text{ Not critical}}$

(d) Plot *all* values from (c) on a sign chart for $f'$. If an $x$-value is critical, place it on the sign chart with a solid dot. If an $x$-value is not critical, place it on the sign chart with an open dot. Fill in your sign chart using test points.




(e) Find the intervals of increase or decrease for $f$. Justify your answer.

Decreasing $(-\infty,-2) \cup (2,\infty)$ since $f'<0$

Increasing $(-2,2)$ since $f'>0$.

$$f(x) = \frac{36x}{(x+2)^2}, \qquad f'(x) = \frac{-36(x-2)}{(x+2)^3}, \qquad f''(x) = \frac{72(x-4)}{(x+2)^4}$$

(f) Find the $x$-coordinates and $y$-coordinates of the local maximum and minimum values of $f$. Justify your answer.

Local max at $x=2$ since $f'$ goes $(+)$ to $(-)$. $\boxed{\text{Local max } (2, 4.5)}$

y-coordinat $f(2) = \frac{36(2)}{(2+2)^2} = \frac{36(2)}{16} = \frac{36}{8} = 4.5$

No local min.

(g) Find all values of $x$ such that $f''(x) = 0$ ***AND*** all values of $x$ such that the denominator of $f''$ is zero.

$f''(x)=0$ when $72(x-4)=0$, so $x=4$ (solid dot)

Denominator $=0$ at $x=-2$ (open dot)

(h) Plot all values from (g) on a sign chart for $f''$. If an $x$-value is in the domain of $f$, place it on the sign chart with a solid dot. If an $x$-value is not in the domain of $f$, place it on the sign chart with an open dot. Fill in your sign chart using test points.


(i) Find the intervals of concavity for $f$. Justify your answer.

Concave down $(-\infty, -2) \cup (-2, 4)$ since $f'' < 0$.

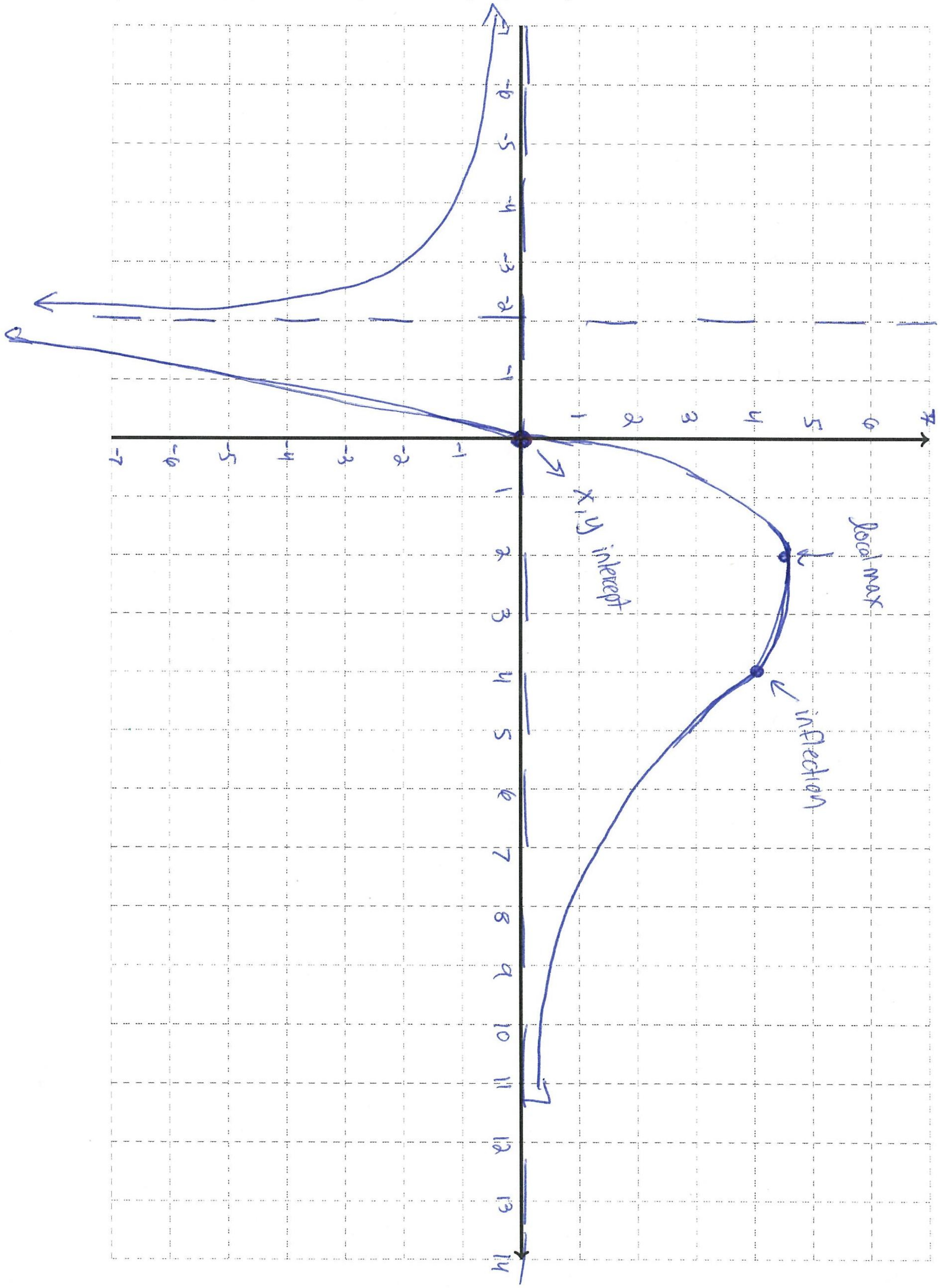
Concave up $(4, \infty)$ since $f'' > 0$.

(j) Find the $x$-coordinates and $y$-coordinates of any inflection points of $f$. Justify your answer.

Inflection point at $x=4$ since $f''$ goes $(-)$ to $(+)$.

y-coordinate $f(4) = \frac{36(4)}{(4+2)^2} = \frac{36(4)}{36} = 4$. $\boxed{\text{Point } (4,4)}$

(k) Using all the information from parts (a) through (j), sketch a graph of $f(x)$ below. (Turn the paper sideways for the proper fit)



2. Sketch a graph of a function $g(x)$ that has a vertical asymptote at $x = 3$, a horizontal asymptote at $y = -1$ and satisfies the following sign charts for $g$, $g'$ and $g''$.