AP Calculus AB (Vahsen) Name: ____________________

Average Velocity & Instantaneous Velocity

Using a table of values for a position function

The table shown below represents the position of an object as a function of time. Use the table to answer the questions that follow.

Time (sec)	2.8	2.9	3.0	3.1	3.2	3.3
Position (m)	7.84	8.41	9.00	9.61	10.24	10.89

1. What is the object's position at time $t = 3$ sec? 9 m At time 3.3 sec? 10.89 m

2. What is the *total change* in the object's position over the time interval from 3 to 3.3 seconds?

   1.89m

3. Find the *average rate of change* in the object's position over the time interval from 3 to 3.3 seconds. Show an appropriate set-up. Be sure to include UNITS in your answer.

   $$\frac{1.89}{3.3-3} = 6.3 \text{ m/s}$$

4. By what familiar name do we refer to **average rate of change in position**?

   average velocity

5. Estimate the *instantaneous rate of change* in the object's position at time $t = 3$ sec. Show an appropriate set-up. Be sure to include UNITS in your answer.

   $$\frac{9.61-9}{3.1-3} = 6.1 \text{ m/s}$$

6. By what familiar name do we refer to **instantaneous rate of change of position**?

   instantaneous velocity

7. Find two other reasonable estimates for the object's velocity at time $t = 3$ seconds. Show your set-ups.

   $$\frac{9-8.41}{3-2.9} = 5.9 \text{ m/s} \qquad \frac{9.61-8.41}{3.1-2.9} = 6 \text{ m/s}$$

8. Of your three estimates for velocity at time $t = 3$, which one do you prefer? Justify.

   one done in #5
   shortest time int

STOP: Check your answers and get Ms. V to initial your half sheet (5 points)

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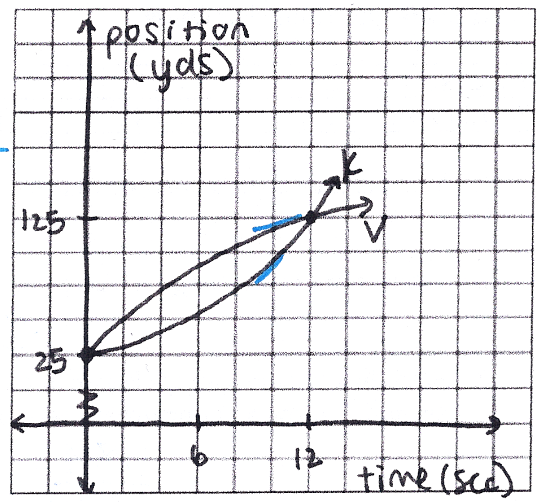
Total change vs. average rate of change vs. instantaneous rate of change

Ms. V told Ms. K that she could beat her in a 100-yard go-kart race with her eyes closed. Ms. K took Ms. V up on the challenge. You can see below a graph of their positions during the race (position is given in yards; time is in seconds). Let $V(t)$ represent Ms. V's position graph and let $K(t)$ represent Ms. K's position graph.



1. Who won the race? Explain.

   tied - got to 125 at same time

2. What is the "total change" in the *V* function over the first 12 seconds of the race? Give the numerical value, including units, and explain its real-world meaning.

   100 yds

3. Repeat question #2 for the *K* function.

   100 yds

4. What is the "average rate of change" of *V* over the first 12 seconds? Give a numerical value, including units, and a real-world meaning.

   $\frac{100}{12} = 8.3$ yds/sec

5. Repeat question #4 for the *K* function.

   $\frac{100}{12} = 8.3$ yds/sec

6. a) What does the "instantaneous rate of change" at $t = 10$ represent in real-world terms?

   instantaneous velocity

   b) Who has the greater instantaneous rate at $t = 10$? How can you tell?

   Ms K - greater slope

   c) Use symbols instead of words to express you answer to the first part of question 6b.

   $K'(10) > V'(10)$

7. Given the shapes of Ms. V's and Ms. K's position functions are so very different, how do you account for the fact that their average velocities are equal over the first 12 seconds? Say something simple and obvious if you can.

   V started fast and slowed down
   K started slow and speed up

STOP: Check your answers and get Ms. V to initial your half sheet (5 points)

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Local linearization

Consider the radical function, $f(x)=\sqrt{x}$. The slope of this function at any point in its domain is given by the expression, $\dfrac{1}{2\sqrt{x}}$.

1. Find the slope of $f(x)=\sqrt{x}$ at x = 4. $\frac{1}{2\sqrt{x}} = f'(x)$ $f'(4)=\frac{1}{4}$

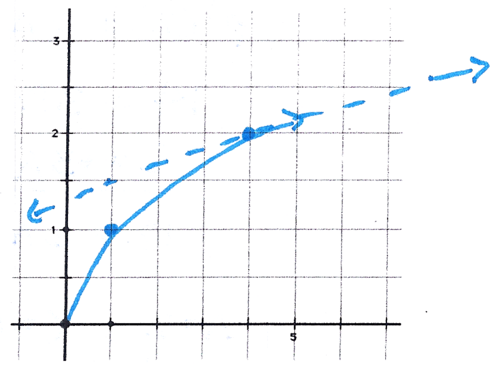
2. Write an expression for the local linearization (AKA tangent line) of *f* near *x* = 4.

   $y-2=\frac{1}{4}(x-4)$

3. Use your answer from question #2 to find an approximate value for $\sqrt{4.1}$. *Do not use a calculator.* Show your set-up.

   $y=\frac{1}{4}(4.1-4)+2 = 2.025$

4. Graph the function $f(x)=\sqrt{x}$ and its tangent line at x = 4.



5. Do you think your answer to question #3 is an over-estimate or an under-estimate for the actual value of $\sqrt{4.1}$? Justify your answer by reference to the behavior of the graph of $f(x)=\sqrt{x}$.

   over - tangent line over the graph ($\sqrt{x}$ concave down)

6. Use your calculator to find the true value of $\sqrt{4.1}$. = 2.024845673

7. What was the error in your estimate?

   0.000143268...

8. Explain why the local linearization produced a good approximation for $\sqrt{4.1}$ but would produce a poor approximation for $\sqrt{16}$.

   tangent slope = derive at that point but not elsewhere

STOP: Check your answers and get Ms. V to initial your half sheet (5 points)

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BONUS PROBLEMS: Earn up to 4 bonus points for today! You might need extra paper.

1. (+1) Find k such that the line $y = -\frac{3}{4}x + 3$ is tangent to the graph of the function $f(x) = \frac{k}{x}$.

$f'(x) = -\frac{k}{x^2}$

$+\frac{k}{x^2} = +\frac{3}{4}$

$4k = 3x^2$

$k = \frac{3}{4}x^2$

$-\frac{3}{4}x + 3 = \frac{k}{x}$

$-\frac{3}{4}x + 3 = \frac{\frac{3}{4}x^2}{x}$

$-3x + 12 = 3x$

$12 = 6x$

$x = 2$

$k = \frac{3}{4}(2)^2$

$= 3$

2. (+1) Find a third-degree polynomial ($p(x) = Ax^3 + Bx^2 + Cx + D$) that is tangent to the line $y = 14x - 13$ at the point (1, 1), and tangent to the line $y = -2x - 5$ at the point (-1, -3).

$p'(x) = 3Ax^2 + 2Bx + C$

$14 = 3A + 2B + C$

$-2 = +3A - 2B + C$

$12 = 6A + 2C$

$1 = \cancel{A} + B + \cancel{C} + D$

$-3 = -A + B - C + D$

$-2 = 2B + 2D$

$A = 2$ $\quad C = 0$

$B = 4$ $\quad D = -5$

$p(x) = 2x^3 + 4x^2 - 5$

3. (+2) CHALLENGE: Graph the two parabolas $y = x^2$ and $y = -x^2 + 2x - 5$ in the same coordinate plane. Find equations of the two lines simultaneously tangent to both parabolas.

$\frac{dy}{dx} = 2x$ $\quad \frac{dy}{dx} = -2x + 2$

STOP: Check your answers and get Ms. V to initial your half sheet (up to 4 points)

BONUS #3

define two arbitrary x values a and b

a = touches x^2

b = touches $-x^2+2x-5$

$\frac{dy}{dx} = 2x$ slope $\frac{dy}{dx} = -2x+2$

$= 2a$ $= -2b+2$

(a, a^2) point $(b, -b^2+2b-5)$

slope with two points = deriv

$$\frac{-b^2+2b-5-a^2}{b-a} = 2a$$

deriv = deriv

$2a = -2b+2$

$a = 1-b$

$-b^2+2b-5-a^2 = 2ab-2a^2$

$-b^2+2b-5-(1-b)^2 = 2(1-b)b - 2(1-b)^2$

$-b^2+2b-5-1+2b-b^2 = 2b-2b^2-2+4b-2b^2$

$-2b^2+4b-6 = -4b^2+6b-2$

$2b^2-2b-4=0$

$b^2-b-2=0$

$(b+1)(b-2)=0$

$b=-1$	$b=2$
$a=2$	$a=-1 \leftarrow a=1-b$
$m=2(2)=4$	$m=2(-1)=-2 \leftarrow$ deriv $= 2a$
$y-4=4(x-2)$	$y-1=-2(x+1)$
$y=4x-4$	$y=-2x-1$