Solutions

Math 1300-010 - Fall 2016

The Closed Interval Method - 10/21/16

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3.

The purpose of this worksheet is to explore the ***closed interval method*** for finding the absolute extrema of a continuous function f on a closed interval $[a, b]$.

1. Let us find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 6x^2 + 9x + 2$$

on the closed interval $[-1.4]$.

(a) Find the critical numbers of f. Recall these are numbers c *in the domain of* f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$

so the critical numbers are $x = 1$ and $x = 3$.

(b) Find the values of f at the critical numbers from (a) that are within the open interval $(-1, 4)$.

$f(1) = 1 - 6 + 9 + 2 = 6$

$f(3) = 27 - 54 + 27 + 2 = 2$

(c) Find the values of $f(-1)$ and $f(4)$. Note that -1 and 4 are the endpoints of our closed interval $[-1, 4]$.

$f(-1) = -1 - 6 - 9 + 2 = -14$

$f(4) = 64 - 6(16) + 36 + 2$
$= 102 - 36 = 66$

(d) The largest value found in parts (b) and (c) will be the absolute maximum of f on $[-1.4]$. The smallest value found in parts (b) and (c) will alternatively be the absolute minimum of f on $[-1.4]$. What is the absolute maximum and at what x-value does it occur? What is the absolute minimum and at what x-value does it occur?

Absolute maximum of 66 at $x = 4$

Absolute minimum of -14 at $x = -1$

The steps outlined in the previous problem are known as the closed interval method and can be summarized as follows:

The Closed Interval Method: To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

(i) Find the values of f at the critical numbers of f in the open interval (a, b).

(ii) Find the values of f at the endpoints of the interval. That is, find $f(a)$ and $f(b)$.

(iii) The largest of the values from Steps (i) and (ii) is the absolute maximum value; the smallest of these values is the absolute minimum value.

2. Use the closed interval method to find the absolute maximum and absolute minimum values of

$$f(x) = 12 + 4x - x^2$$, $f'(x) = 4 - 2x = 2(2-x)$

on the closed interval $[0, 5]$.

$f'(x) = 0$ at $x = 2$

(i) Critical #'s are $x=2$,

$f(2) = 12 + 8 - 4 = 16 \to$ max

(ii) $f(0) = 12$

$f(5) = 12 + 20 - 25 = 7 \to$ min

(iii) $\boxed{\text{Absolute max of } 16 \text{ at } x=2 \quad \text{Absolute min of } 7 \text{ at } x=5}$

3. Use the closed interval method to find the absolute maximum and absolute minimum values of

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the closed interval $[-3, 1]$. $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$

so $f'(x) = 0$ at $x = -1, 2$

(i) $f(-1) = -2 - 3 + 12 + 1 = 8 \to$ max

$f(2) \to$ we don't care, because 2 is not in $(-3,1)$

(ii) $f(-3) = 2(-3)^3 - 3(-3)^2 - 12(-3) + 1 = -54 - 27 + 36 + 1 = 37 - 81 = -44 \to$ min

$f(1) = 2 - 3 - 12 + 1 = -12$

(iii) $\boxed{\text{Absolute max of } 8 \text{ at } x=-1 \quad \text{Absolute min of } -44 \text{ at } x=-3}$