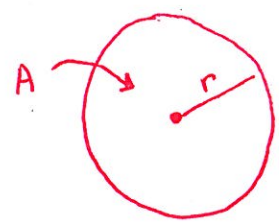
1. Bacteria are growing in a circular colony one bacterium thick. The bacteria are growing at a constant rate, thus making the area of the colony increase at a constant rate of 12 mm^2/hr.

(a) Draw a picture of the situation.



(b) Find an equation expressing the rate of change of area as a function of the radius, r, of the colony.

$$A = \pi r^2$$

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

(c) How fast is r changing when r equals 3 mm?

$$\frac{dr}{dt} = \frac{dA}{dt}\Big/2\pi r = \left(12 \text{ mm}^2/\text{hr}\right)\Big/2\pi(3\text{ mm}) = \frac{2}{\pi} \text{ mm/hr}$$

$$\approx 0.6366 \text{ mm/hr}$$

(d) Describe the way dr/dt changes as r increases.

$\frac{dr}{dt} = \frac{12}{2\pi r}$ As r increases, $\frac{dr}{dt}$ decreases.

2. Joe blows up a spherical balloon. He recalls that the volume is $(4/3)\pi r^3$. Find dV/dt as a function of r and dr/dt. In order for the radius to increase at 2 cm/sec, how fast must Joe blow air into the balloon when $r = 3$? When $r = 6$?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

When $r=3$: $\frac{dV}{dt} = 4\pi\,(3\text{ cm})^2\left(2\ \frac{\text{cm}}{\text{sec}}\right) = 72\pi\ \text{cm}^3/\text{sec} \approx 226\ \text{cm}^3/\text{sec}$

When $r=6$: $\frac{dV}{dt} = 4\pi\,(6\text{ cm})^2\left(2\ \frac{\text{cm}}{\text{sec}}\right) = 288\pi\ \text{cm}^3/\text{sec} \approx 905\ \text{cm}^3/\text{sec}$

3. You recall that the area of an ellipse is $A = \pi ab$, where a and b are the lengths of the semiaxes (like radii but for an ellipse). Suppose that an ellipse is changing size but always keeps the same proportions, $a = 2b$. At what rate is the length a of the major axis changing when $b = 12$ cm and the area is decreasing at 144 cm^2/sec?

$$A = \pi ab \qquad a = 2b \rightarrow b = \tfrac{1}{2}a$$

$$A = \pi a\left(\tfrac{1}{2}a\right) \qquad \text{When } b = 12 \text{ cm}, \ a = 24 \text{ cm}$$

$$A = \frac{\pi}{2}a^2$$

$$\frac{d}{dt}[A] = \frac{d}{dt}\left[\frac{\pi}{2}a^2\right]$$

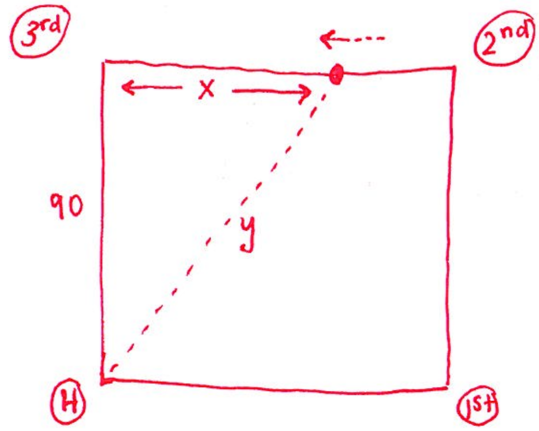
$$\frac{dA}{dt} = \pi a \frac{da}{dt}$$

$$\frac{da}{dt} = \frac{dA}{dt}\Big/\pi a = \left(-144 \text{ cm}^2/\text{sec}\right)\Big/\pi(24 \text{ cm}) = \frac{-6}{\pi} \text{ cm/sec}$$

$$\approx -1.9 \text{ cm/sec}$$

4. Chris hits a line drive to center field. As he rounds second base, he heads directly for third, running at 20 ft/sec. Assume that the baseball diamond is square and the length between any two bases is 90 ft.

(a) Draw a picture of the situation.



(b) Write an equation expressing the rate of change of his distance from home plate as a function of his displacement from third base.

$$x^2 + 90^2 = y^2$$

$$\frac{d}{dt}\left[x^2 + 90^2\right] = \frac{d}{dt}\left[y^2\right]$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y} \frac{dx}{dt}$$

$$\frac{dx}{dt} = -20 \text{ ft/sec}$$

$$y = \sqrt{x^2 + 90^2}$$

$$\frac{dy}{dt} = \frac{x}{\sqrt{x^2 + 90^2}}(-20)$$

(c) How fast is his distance from home plate changing when he is halfway to third? At third? Is the latter answer reasonable? Explain.

When $x = 45$ ft, $\frac{dy}{dt} = \frac{45}{\sqrt{45^2 + 90^2}}(-20) = -8.9$ ft/sec

When $x = 0$ ft, $\frac{dy}{dt} = \frac{0}{\sqrt{0^2 + 90^2}}(-20) = 0$ ft/sec

(He is running perpendicular to the line toward home plate)