Math 1300-005 - Spring 2017

Midterm 3 Review, Part II - 4/9/17

Solutions

Guidelines: Please work in groups of two or three.

1. (a) The speed of sound is roughly 340 meters per second. Suppose two people are standing 24,000 meters apart and a jet is flying overhead. If the jet passes over the first person at 1:00PM and then passes over the second person at 1:02PM, can we use the Mean Value Theorem to conclude that the jet broke the sound barrier?

   Jet travels 24,000 meters in 2min = 120 seconds, so the average speed from 1:00 − 1:02 pm is $\frac{24{,}000\text{m}}{120\text{sec}} = 200\text{m/s}$. MVT says the jet must have had instant speed = 200m/s between 1:00 and 1:02.

   So we cannot conclude the jet broke the sound barrier

   (b) If your answer is yes, by how much did the jet break the sound barrier? If your answer is no, can we conclude the jet never broke the sound barrier? Please explain.

   Our answer was no, but it is possible that the jet travelled 340m/s or faster so long as it travelled slow enough other times to average out to 200m/s.

2. At 12:00 PM Rebecca leaves her house. At 4:00 PM, Rebecca is now 16 miles from home. Explain why at some point between 12:00 PM and 4:00 PM, Rebecca must have been traveling at a velocity of 4 mph.

   Rebecca's average speed is $\frac{16\text{ miles}}{4\text{ hours}} = 4\text{ mph}$. MVT says there is some time between 12 and 4pm where she was travelling at exactly 4mph (instant speed).

3. Compute the following limits.

   (a) $\lim_{x\to\infty}[x - \sqrt{x^2 - x}]\left(\frac{x+\sqrt{x^2-x}}{x+\sqrt{x^2-x}}\right)$

   $$= \lim_{x\to\infty} \frac{x^2-(x^2-x)}{x+\sqrt{x^2-x}} = \lim_{x\to\infty} \frac{x}{x+\sqrt{x^2-x}} \approx \frac{\infty}{\infty}$$

   $$\overset{l'H}{=} \lim_{x\to\infty} \frac{1}{1+\frac{2x-1}{2\sqrt{x^2-x}}} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

   $$\frac{2x-1}{2\sqrt{x^2-x}} \approx \frac{2x}{2\sqrt{x^2}} = \frac{2x}{2x} = 1 \text{ as } x\to\infty$$

   (b) $\lim_{x\to\infty}(x)^{1/x}$. $L = \lim_{x\to\infty}(x)^{1/x} \approx \infty^0$

   $$\ln(L) = \lim_{x\to\infty} \frac{1}{x}\ln(x) = \lim_{x\to\infty}\frac{\ln(x)}{x} \approx \frac{\infty}{\infty}$$

   $$\overset{l'H}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0.$$

   So $\ln(L)=0$, hence $\boxed{L=e^0=1}$

4. Find the derivative of the following.

   (a) $f(x) = \log_{10}(4^x - 8x^2)$

   $$\boxed{f'(x) = \frac{1}{(4^x-8x^2)\ln(10)}\cdot(4^x\ln(4) - 16x)}$$

   (b) $g(x) = \sqrt{x^5}\cdot\arccos(2x) = x^{\frac{5}{2}}\arccos(2x)$

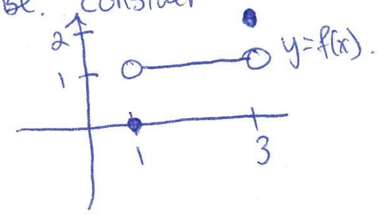
   $$\boxed{g'(x) = \frac{5}{2}x^{\frac{3}{2}}\arccos(2x) + x^{\frac{5}{2}}\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\cdot 2}$$

5. Determine whether the following statements are true or false. In either case, **explain your reasoning with a picture**.

(a) If f is continuous and differentiable on (a, b), then there must exist some c in (a, b) with $f'(c) = \frac{f(b)-f(a)}{b-a}$.

False. Consider

$$f(x) = \begin{cases} 0 & x = 1 \\ 1 & 1 < x < 3 \\ 2 & x = 3 \end{cases}$$



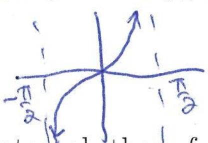
f is cont. and diff on $(1,3)$ and $f'(c) = 0$ for all c in $(1,3)$

But $\frac{f(3)-f(1)}{3-1} = \frac{2-0}{3-1} = 1$.

so $f'(c) \neq \frac{f(3)-f(1)}{3-1}$.

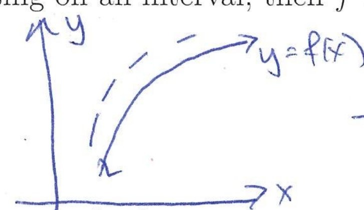
(b) If f is continuous on (a, b), then there must exist c and d in (a, b) such that $f(c)$ is an absolute max and $f(d)$ is an absolute min.

Let $f(x) = \tan(x)$ on $(\frac{-\pi}{2}, \frac{\pi}{2})$. f is cont. but has no absolute max or absolute min.



(c) If f' is decreasing on an interval, then f must also be decreasing on that interval.

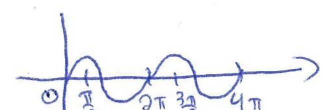
False.



→ here f' is decreasing but f is increasing

(d) If f is continuous on $[a, b]$, then f can have an absolute maximum that occurs more than once.

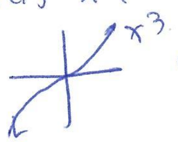
True. Take $f(x) = \sin(x)$ on $[0, 4\pi]$



max $y=1$ occurs at $x = \frac{\pi}{2}, \frac{3\pi}{2}$

(e) If $f'(c) = 0$, then f must have a local max or a local min at $x = c$.

False. Let $f(x) = x^3$. At $c=0$, $f'(0) = 3(0)^2 = 0$.



but $f(c)$ is neither max nor min (local).

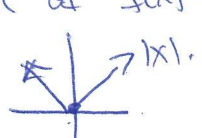
(f) If f is increasing on an interval, then f' must also be increasing on that interval.

False. Again as in (c),

f is increasing but f' decreasing.

(g) If f has a local maximum (or minimum) at $x = c$, then $f'(c) = 0$.

False. Think of $f(x) = |x|$.



0 is location of local min but $f'(0)$ DNE.