

On the front of your bluebook, please write: a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 6 questions on both sides of this paper.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
 - **Show all work and simplify your answers!** Answers with no justification will receive no points.
 - Please begin each problem on a new page.
 - No notes or papers, calculators, cell phones, or electronic devices are permitted.
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1. Consider the curve $y = \frac{x-2}{x+2}$.

- (5 pts) Find dy/dx . Simplify your answer.
 - (8 pts) Find the (x, y) point(s) on the curve where the tangent line is perpendicular to $x + 4y + 12 = 0$.
 - (8 pts) Use a linearization of y to estimate the value of $-\frac{2.03}{1.97}$.
 - (7 pts) The Mean Value Theorem can be applied to this function on the interval $[a, 2]$.
 - For what values of a will the hypotheses be met?
 - The theorem states that there will be a number c in $(a, 2)$ such that $y'(c)$ equals some expression. Write the expression in terms of a and simplify your answer.
2. (12 pts) Find the (x, y) points where the function $y = 2 \sin x + \sin^2 x$ on the interval $[0, 2\pi]$ attains its absolute maximum and minimum values.
3. (12 pts) Find all asymptotes (horizontal, vertical and slant) for $y = \frac{3x^2}{x+2}$. Justify your answer with the appropriate limits.
4. (15 pts) A spotlight on the ground shines on a wall 10 m away. If a young girl, 1 meter tall, walks from the spotlight toward the building at a speed of $\frac{4}{5}$ m/s, how fast is the length of her shadow on the building decreasing when she is 2 m from the building? Simplify your answer. (Be sure to draw a diagram and clearly label all quantities.)
5. (15 pts) A rectangular storage container with an open top and square base is to have a volume of $\frac{9}{4}$ m³. Material for the base costs \$4 per square meter. Material for the sides costs \$3 per square meter. Find the dimensions of the cheapest such container. (Be sure to draw a diagram and clearly label all quantities.)

TURN OVER - More problems on the back!
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6. (18 pts) The graph of the **derivative** $f'(x)$ of some function $f(x)$ is shown below. Assume that $f(x)$ is continuous on the interval $[0, 9]$. Answer the following questions concerning the function $f(x)$. If the answer to any question is “none”, write “none”. For parts (a) through (d), no justification is required.

- (a) What are the critical numbers of f on $(0, 9)$?
- (b) What are the x -coordinates of the local maximum values of f ?
- (c) On what intervals is f concave up?
- (d) What are the x -coordinates of the inflection points of f ?
- (e) If $f(0) = -3$ and $f(9) = -5$, is there a value of c in $(0, 9)$ such that $f'(c) = \frac{f(9) - f(0)}{9}$? Justify your answer.
- (f) If the linearization of f at $a = 4.5$ is used to approximate the value of $f(4.4)$, would the approximation be an underestimate, overestimate, or neither? Explain.

