Solutions

Math 1300-005 - Spring 2017

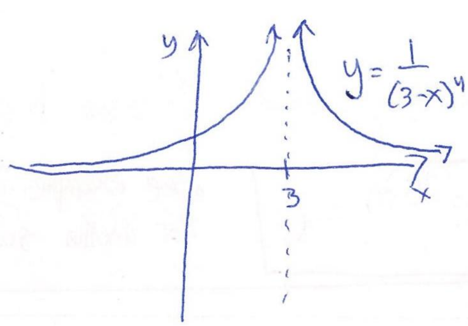
Limits Involving Infinity - 2/1/17

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Compute the following limits.

(a) $\lim_{x\to 3} \frac{7-x}{(3-x)^4} = \lim_{x\to 3} \left[(7-x)\cdot\frac{1}{(3-x)^4}\right]$.

- We cannot immediatley use a limit law. However, note that $\lim_{x\to 3}(7-x) = 4$.
- Now, lets graph $y = \frac{1}{(3-x)^4} = \frac{1}{(-(x-3))^4} = \frac{1}{(x-3)^4}$.



so $\lim_{x\to 3}\frac{1}{(3-x)^4} = \infty$ from the picture.

- Remember, ∞ DNE, but since RHL = LHL, we can, in an abuse of notation, write

$$\lim_{x\to 3}\left[(7-x)\cdot\frac{1}{(3-x)^4}\right] = \lim_{x\to 3}(7-x)\cdot\lim_{x\to 3}\left[\frac{1}{(3-x)^4}\right]$$
$$= 4\cdot\infty$$
$$= \boxed{\infty}$$

(b) $\lim_{x\to\infty} \frac{x^3-5x}{2x^3-x^2+4}$

- Quick Solution: Since the largest powers in the numerator and denominator are equal, we can simply take the ration of coefficients

you must include a sentence like this

$$\lim_{x\to\infty}\frac{x^3-5x}{2x^3-x^2+4} = \boxed{\frac{1}{2}}$$

- Long Solution: The highest power in the denominater is x^3, so

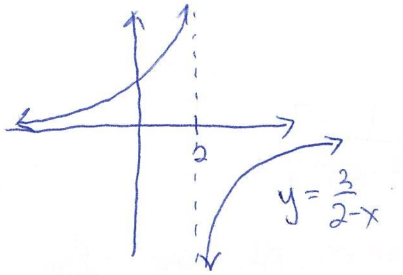
$$\lim_{x\to\infty}\frac{x^3-5x}{2x^3-x^2+4} = \lim_{x\to\infty}\frac{\frac{x^3}{x^3}-\frac{5}{x^3}}{\frac{2x^3}{x^3}-\frac{x^2}{x^3}+\frac{4}{x^3}} = \lim_{x\to\infty}\frac{1-\cancel{\frac{5}{x^3}}\to 0}{2-\cancel{\frac{1}{x}}\to 0+\cancel{\frac{4}{x^3}}\to 0}$$
$$= \boxed{\frac{1}{2}}$$

You may use either method

2. Compute the following limits.

(a) $\lim_{x\to 2^+} e^{3/(2-x)}$ [Hint: first consider $\lim_{x\to 2^+} 3/(2-x)$]

• Lets first draw $y = \frac{3}{2-x}$,



, so $\lim_{x\to 2^+} \frac{3}{2-x} = -\infty$.

• Using the above, let $t = \frac{3}{2-x}$. Then as $x \to 2^+$, $t \to -\infty$, so

$$\lim_{x\to 2^+} e^{3/(2-x)} = \lim_{t\to -\infty} e^t = 0,$$ since 0 is a h.a. for e^x.

Thus $\boxed{\lim_{x\to 2^+} e^{3/(2-x)} = 0}$

• See example 7 in the book for another such example.

(b) $\lim_{x\to\infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4}$

• <u>Quick Solution</u>: since the largest power in the numerator is greater than the largest power in the denominator,

$$\lim_{x\to\infty} \frac{x+x^3+x^5}{1-x^2+x^4} = \boxed{\boxed{\infty}}$$

• <u>Long Solution</u>: x^4 is the largest power of x in the denominator, so

$$\lim_{x\to\infty} \frac{x+x^3+x^5}{1-x^2+x^4} = \lim_{x\to\infty} \frac{\frac{x}{x^4}+\frac{x^3}{x^4}+\frac{x^5}{x^4}}{\frac{1}{x^4}-\frac{x^2}{x^4}+\frac{x^4}{x^4}} = \lim_{x\to\infty} \frac{\cancelto{0}{\frac{1}{x^3}}+\cancelto{0}{\frac{1}{x}}+x}{\cancelto{0}{\frac{1}{x^4}}-\cancelto{0}{\frac{1}{x^2}}+1}$$

$$= \lim_{x\to\infty} x$$

$$= \boxed{\boxed{\infty}}$$