## Solutions

## Math 1300-005 - Spring 2017

Implicit Differentiation Intro - 2/27/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 2.

1. Our goal in this problem is to use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + xy = 3 - y^2$$

at the point (1,1).

(a) First, apply d/dx to both sides of  $x^2 + xy = 3 - y^2$  and use the chain rule and the guidelines from lecture. Notice the second term on the left-hand side is a product.

$$\frac{1}{2}(x^{3} + xy) = \frac{1}{2}(3-y^{3})$$
 $2x + y + xy' = -2yy'$ 

(b) Move all terms that have a y' to the left-hand side of the equation from (a) and move terms that do NOT have a y' to the right-hand side of the equation from (a). Solve for y'.

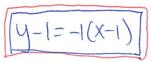
$$y'(x+\partial y) = -\partial x - y \qquad \longrightarrow \qquad \qquad \boxed{y' = \frac{-\partial x - y}{x + \partial y}}$$

(c) Plug in x = 1 and y = 1 to your expression for y' to get the slope of the tangent line to the curve at (1,1).

$$y'$$
 at  $(1,1)$ :  $y' = \frac{-2(1)-(1)}{1+2(1)} = \frac{-3}{3} = \boxed{-1}$ 

(d) Write an equation of the tangent line based on your work in (a), (b), and (c).

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2. Following the same procedure outline in problem 1, find an equation of the tangent line to the curve

$$y\sin(2x) = x\cos(2y)$$

at the point  $(\pi/2, \pi/4)$ . Notice that each side of the equation above involves a product.

(a) 
$$\frac{1}{2}(y\sin(2x)) = \frac{1}{2}(x\cos(2y))$$
 [Product rule to each side, then chain rule as needed]  
 $y\sin(2x) + y\cos(2x) \cdot \lambda = \cos(2y) - x\sin(2y) \cdot 2y'$ 

(b) Rearranging: 
$$y'\sin(2x) + 2xy'\sin(3y) = \cos(3y) - 3y\cos(2x)$$
  
 $y'(\sin(2x) + 2x\sin(3y)) = \cos(3y) - 3y\cos(2x)$   
 $y' = \cos(3y) - 3y\cos(2x)$   
 $y' = \cos(3y) - 3y\cos(2x)$ 

(c) Plly in (
$$\Pi_{2} \pi/4$$
):

$$y' = \frac{(3(2\pi) - 2\pi)}{\sin(2(\pi))} + 2(\pi) \sin(2(\pi))} = \frac{0 - \pi(-1)}{0 + \pi(-1)} = \frac{\pi}{2}$$

3. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

$$\frac{1}{2} \frac{1}{2} \frac{1$$

4. Find dy/dx by implicit differentiation according to steps (a) and (b) in problem 1.

$$e^y \cos(x) = 1 + \sin(xy)$$

$$\frac{1}{2} \left( \frac{1}{2} \cos(x) \right) = \frac{1}{2} \left( \frac{1}{2} \sin(x) \right)$$

$$e^{y} \cdot y' \cos(x) + e^{y} \left( -\sin(x) \right) = \cos(xy) \cdot \frac{1}{2} (xy)$$

$$e^{y} \cdot y' \cos(x) - e^{y} \sin(x) = (\cos(xy) \cdot (y + xy')$$

$$e^{y} \cdot y' \cos(x) - e^{y} \sin(x) = y \cos(xy) + xy' \cos(xy)$$

$$e^{y} \cdot y' \cos(x) - xy' \cos(xy) = y \cos(xy) + e^{y} \sin(x)$$

$$y'(e^{9}\cos(x) - x\cos(xy)) = y\cos(xy) + e^{9}\sin(x)$$

$$y' = \frac{y\cos(xy) + e^{9}\sin(x)}{e^{9}\cos(x) - x\cos(xy)}$$