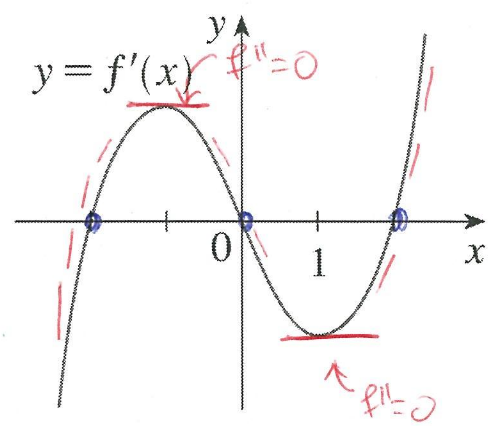
Name: Solutions

Math 1300-005 - Spring 2017

Section 2.8 Activity - 2/13/17

Guidelines: Guidelines: Please work in your groups of two or three. As you finish problems, raise your hand and call me over to check your work. This will not be handed in and is a study resource for the next midterm.

1. The graph of the *derivative* f' of a function f is given.



(a) Draw and label a sign chart for f'. On what intervals is f increasing? Decreasing? Justify your answers in either case.

You need to write everything I write on these questions

f': (−) | −2 | (+) | 0 | (−) | +2 | (+)

label your chart

f is increasing $(-2,0)\cup(2,\infty)$ since $f'>0$ there

f is decreasing $(-\infty,-2)\cup(0,2)$ since $f'<0$ there.

(b) At what values of x does f have a local maximum or minimum? Justify your answer.

local maximum at $x=0$ since f' goes (+) to (−) there.

local minimums at $x=-2, 2$ since f' goes (−) to (+) there.

(c) Draw and label a sign chart for f''. Where is f concave upward or downward? Justify your answer. State the x-coordinates of any inflection points of f.

f'': (+) | −1 | (−) | 1 | (+)

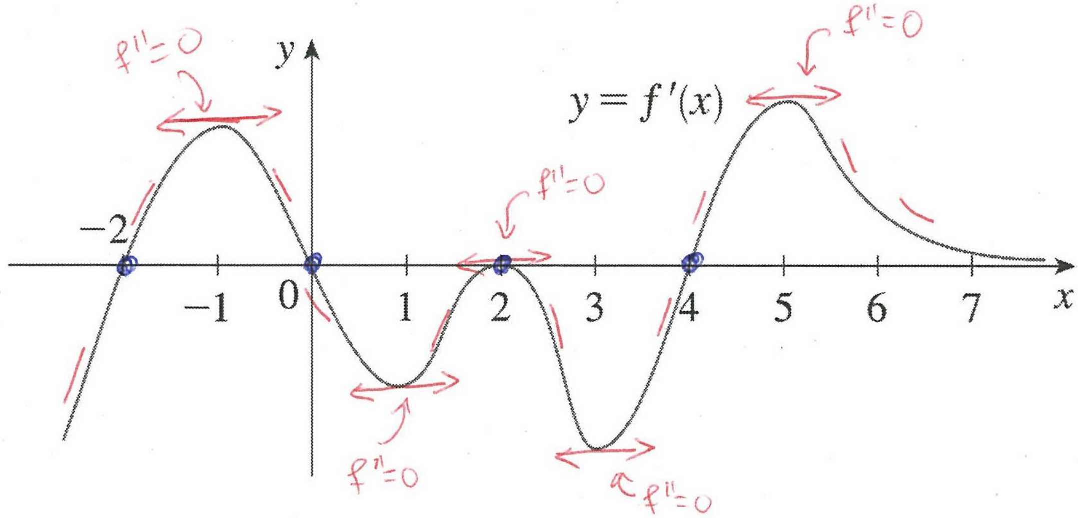
label your chart!

concave up $(-\infty,-1)\cup(1,\infty)$ since $f''>0$ there

concave down $(-1,1)$ since $f''<0$ there

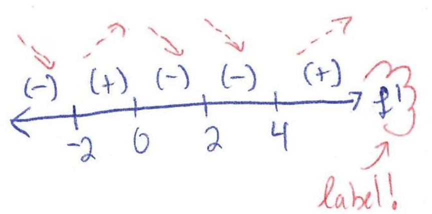
Inflection points located at $x=\pm 1$ since f'' changes sign at these points

2. The graph of the *derivative* f' of a function f is given.



(a) Draw and label a sign chart for f'. On what intervals is f increasing? Decreasing? Justify your answers in either case.

You need to write everything I write



f increases on $(-2,0) \cup (4,\infty)$ since $f'>0$ there.

f decreases on $(-\infty,-2) \cup (0,2) \cup (2,4)$ since $f'<0$ there.

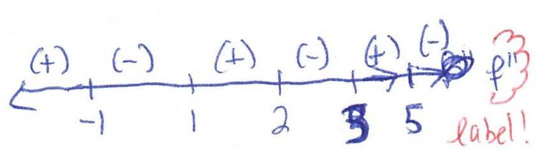
(b) At what values of x does f have a local maximum or minimum? Justify your answer.

local max at $x=0$ since f' goes (+) to (−) there.

local mins at $x=-2, 4$ since f' goes (−) to (+) there.

$x=2$ is neither a local max nor local min.

(c) Draw and label a sign chart for f''. Where is f concave upward or downward? Justify your answer. State the x-coordinates of any inflection points of f.



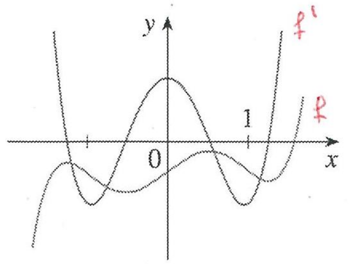
f is concave up $(-\infty,-1) \cup (1,2) \cup (3,5)$ since $f''>0$ there.

f is concave down $(-1,1) \cup (2,3) \cup (5,\infty)$ since $f''<0$ there.

Inflection points located at $x=-1, 1, 2, 3, 5$ since f'' changes sign at these points.

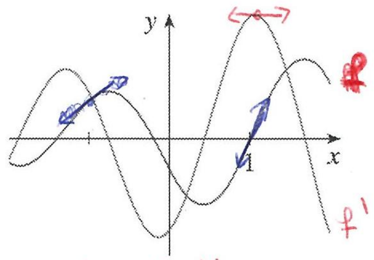
3. In each figure, the graph of a function f and its derivative f' are shown. Which is bigger, $f'(-1)$ or $f''(1)$?

(a)



First we need to determine which is f and which is f'!
I have labeled them and leave it to you to figure out why...
Now, f' has a local min at 1, so $f''(1) = 0$
And we can see $f'(-1) < 0$. so $\boxed{f''(1) > f'(-1)}$

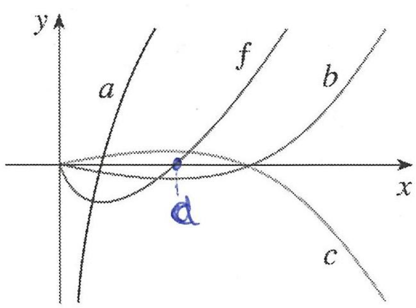
(b)



This one is tricky! After correctly identifying f and f',
we see $f''(1) = 0$ since f' has a local max there
And we see $f'(-1) > 0$, so
$\boxed{f'(-1) > f''(1)}$.

4. In each figure, the graph of a function f is shown. Which graph is an antiderivative of f and why?

(a)

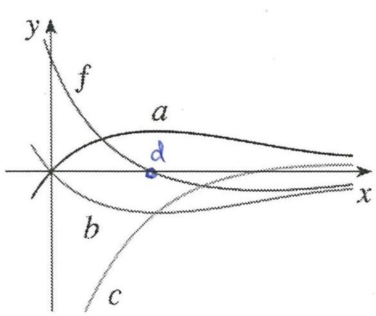


We know $F' = f$ where $F = a, b,$ or c. Using a sign chart

$$\xleftarrow{\quad (-) \quad} d \xrightarrow{\quad (+) \quad} \; f = F'$$

So F must increase (d, ∞) since $f = F' > 0$ there.
F must decrease $(0, d)$ since $f = F' < 0$ there.
And F has a local min at d. The only graph which fits these criteria is b, so
$\boxed{F = b\text{'s graph.}}$

(b)



Again, $F' = f$ where $F = a, b,$ or c. Using a sign chart,

$$\xleftarrow{\quad (+) \quad} d \xrightarrow{\quad (-) \quad} \; f = F'$$

So F must increase $(-\infty, d)$ since $F' = f > 0$
F must decrease (d, ∞) since $F' = f < 0$.
And F has a local max at d. The only choice that fits is $\boxed{\text{curve } a}$.

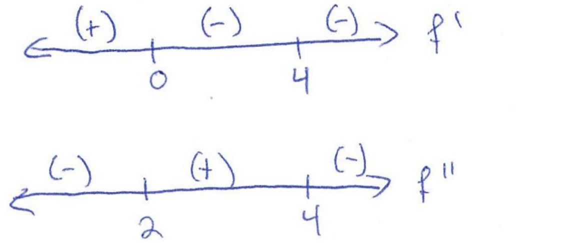
5. Sketch the graph of a function that satisfies all of the given conditions.

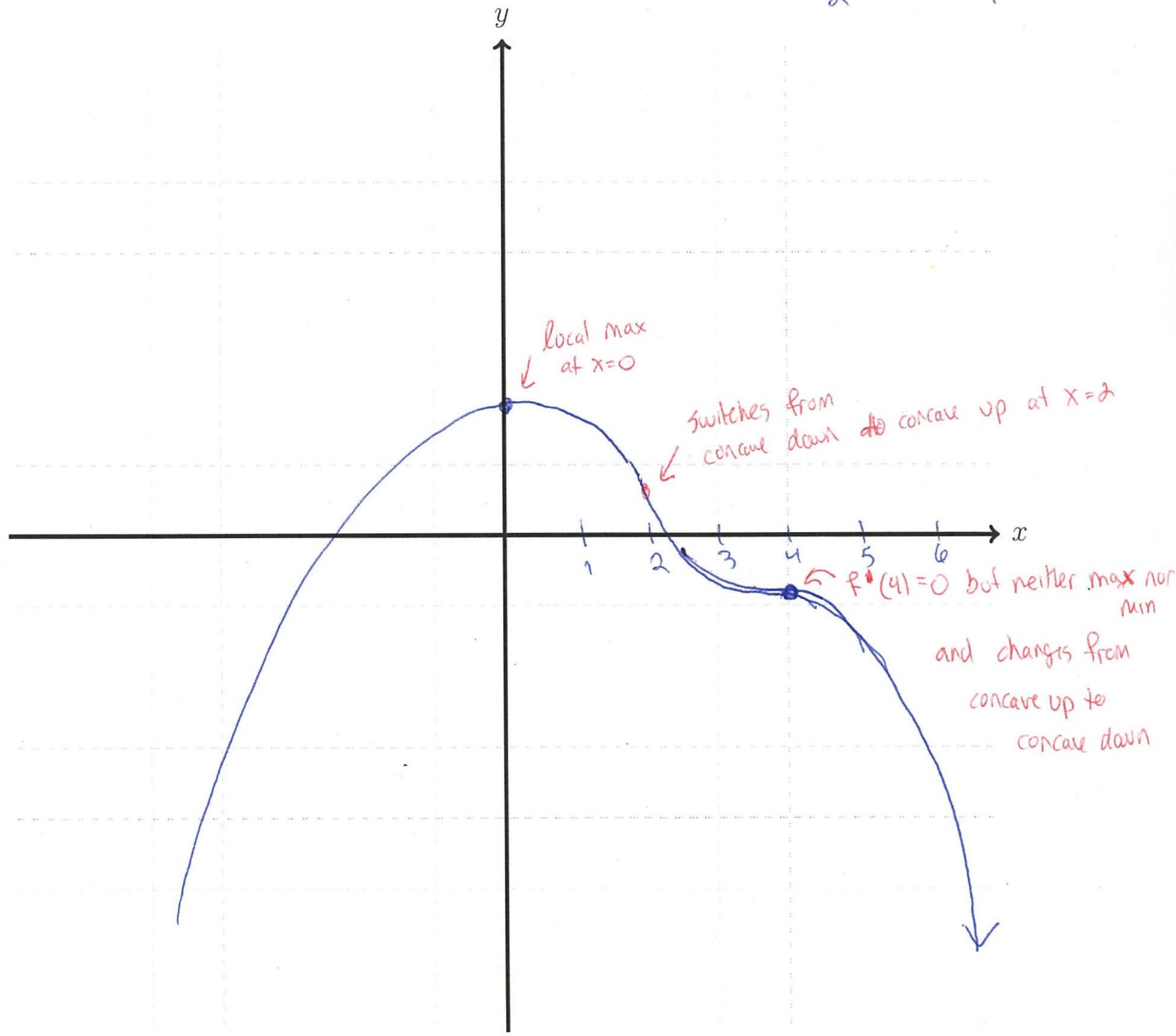
(a) $f'(0) = f'(4) = 0$,
(b) $f'(x) > 0$ if $x < 0$,
(c) $f'(x) < 0$ if $0 < x < 4$ or if $x > 4$,

$\}$ make a sign chart for f'

(d) $f''(x) > 0$ if $2 < x < 4$,
(e) $f''(x) < 0$ if $x < 2$ or $x > 4$.

$\}$ make a sign chart for f''





Based on the charts, f should increase $(-\infty, 0)$, decrease $(0,4) \cup (4,\infty)$ and have a local max at $x=0$.

As well, f should be concave down $(-\infty, 2) \cup (4,\infty)$, concave up $(2,4)$, and switch concavity at $x=2,4$.