Math 1300-005 - Spring 2017

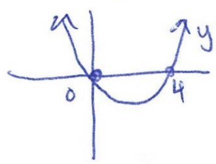
Midterm 2 Review - 3/6/17

Guidelines: Please work in groups of two or three.

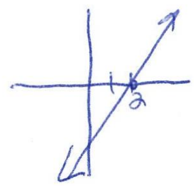
1. Let $f(x) = 2x^3 - 12x^2 + 3$. Please answer the following questions and remember to *fully justify your responses.*

   (a) Construct sign charts for f' and f''. The easiest way is to draw rough sketches of the graphs of f', which is a quadratic, and f'', is a line.

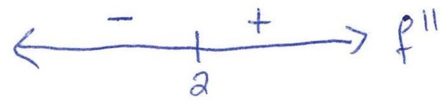
   $f'(x) = 6x^2 - 24x = 6x(x-4)$

   $f''(x) = 12x - 24$
   $= 12(x-2)$

   (b) On what intervals is f increasing? On what interval is f decreasing?

   $\boxed{\text{Increasing } (-\infty, 0) \cup (4, \infty) \text{ since } f' > 0 \quad \text{Decreasing } (0, 4) \text{ since } f' < 0}$

   (c) On what intervals is f concave up? On what intervals is f concave down?

   $\boxed{\text{Concave up } (2, \infty) \text{ since } f'' > 0 \quad \text{concave down } (-\infty, 2) \text{ since } f'' < 0}$

2. Find the point(s) a such that $y = 4x + 10$ is the tangent line to $f(x) = x^3 - 6x^2 - 11x + 2$ at $x = a$.

   We need $f'(x) = 4$ first of all, so

   $3x^2 - 12x - 11 = 4$

   $\hookrightarrow 3x^2 - 12x - 15 = 0$

   $3(x^2 - 4x - 5) = 0$

   $3(x-5)(x+1)$

   so our slope is 4 at $a = 5$
   $a = -1$

   $a = 5$

   $f(5) = 125 - 6(25) - 55 + 2$

   $= 125 - 150 - 55 + 2$

   $= -25 - 55 + 2 = -78$, so

   $y - 78 = 4(x-5) \to y = 4x - 20 + 78$
   $= 4x + 58$ (nope!)

   $a = -1$

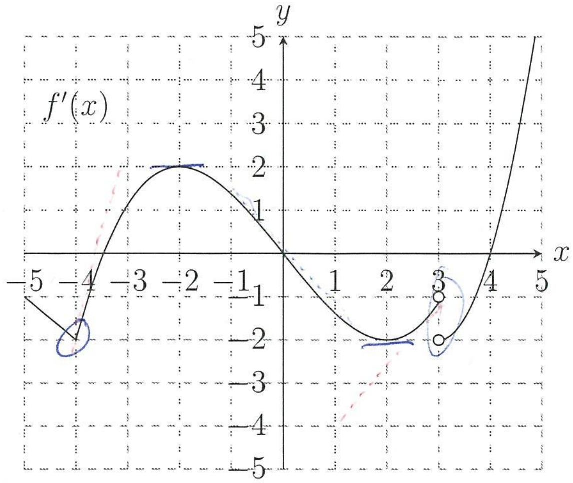
   $f(-1) = -1 - 6 + 11 + 2 = 6$

   $y - 6 = 4(x+1) \to y = 4x + 10$

   so $\boxed{a = -1}$

3. The graph below is the **derivative**, f', of some function f. Construct sign charts for f' and f'' in the space to the right of the graph.

Your sign chart for f' should include any points where $f' = 0$ as well as any points where f' DNE (like $x = 3$). Your sign chart for f'' should include any points where $f'' = 0$ as well as points where f'' DNE (corners and discontinuities on the graph of f').



$-$ $+$ $-$ $-$ $+$ f'
-3.5 0 3 4

$-$ $+$ $-$ $+$ $+$ f''
-4 -2 2 3

(a) On what intervals is f increasing? Decreasing? At values of x, if any, does f have a local maximum? A local minimum? Justify your answer.

f increasing $(-3.5, 0) \cup (4, \infty)$ since $f' > 0$
f decreasing $(-\infty, -3.5) \cup (0, 3) \cup (3, 4)$ since $f' < 0$

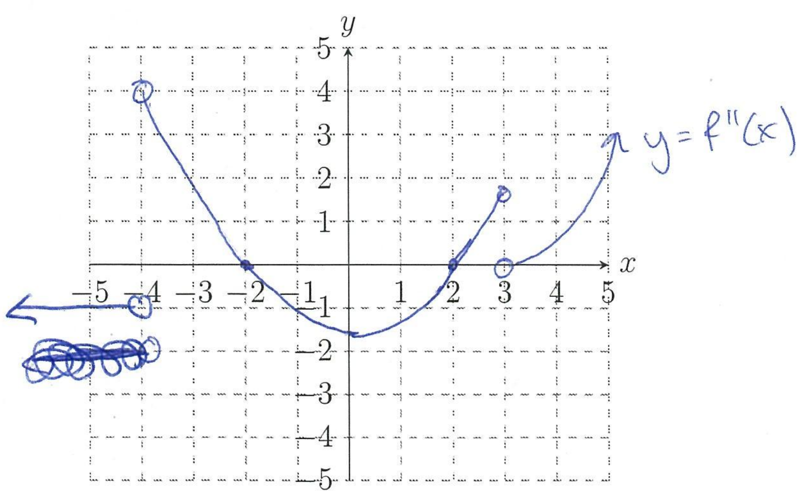
local min $x = -3.5$ and $x = 4$ since f' goes $(-)$ to $(+)$
local max $x = 0$ since f' goes $(+)$ to $(-)$

(b) On what intervals is f concave up? Concave down? At values of x, if any, does f have inflection points? Justify your answer.

f cc up on $(-4, 2) \cup (2, 3) \cup (3, \infty)$ since $f'' > 0$
f cc down on $(-\infty, -4) \cup (-2, 2)$ since $f'' < 0$

Inflection at $x = -4, -2, 2$
since f'' switches sign.

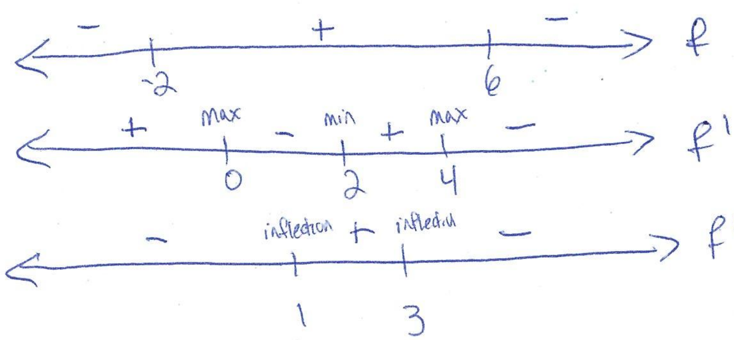
(c) Sketch a graph of $f''(x)$, which is the derivative of the graph shown above.



4. Consider the function f satisfying all of the following conditions.

- $f(x)$ is continuous on $(-\infty, \infty)$
- $f(-2) = f(6) = 0$
- $f(x) > 0$ on $(-2, 6)$ **and** $f(x) < 0$ on $(-\infty, -2) \cup (6, \infty)$
- $f'(0) = f'(2) = f'(4) = 0$
- $f'(x) > 0$ on $(-\infty, 0) \cup (2, 4)$ **and** $f'(x) < 0$ on $(0, 2) \cup (4, \infty)$
- $f''(1) = f''(3) = 0$
- $f''(x) > 0$ on $(1, 3)$ **and** $f''(x) < 0$ on $(-\infty, 1) \cup (3, \infty)$

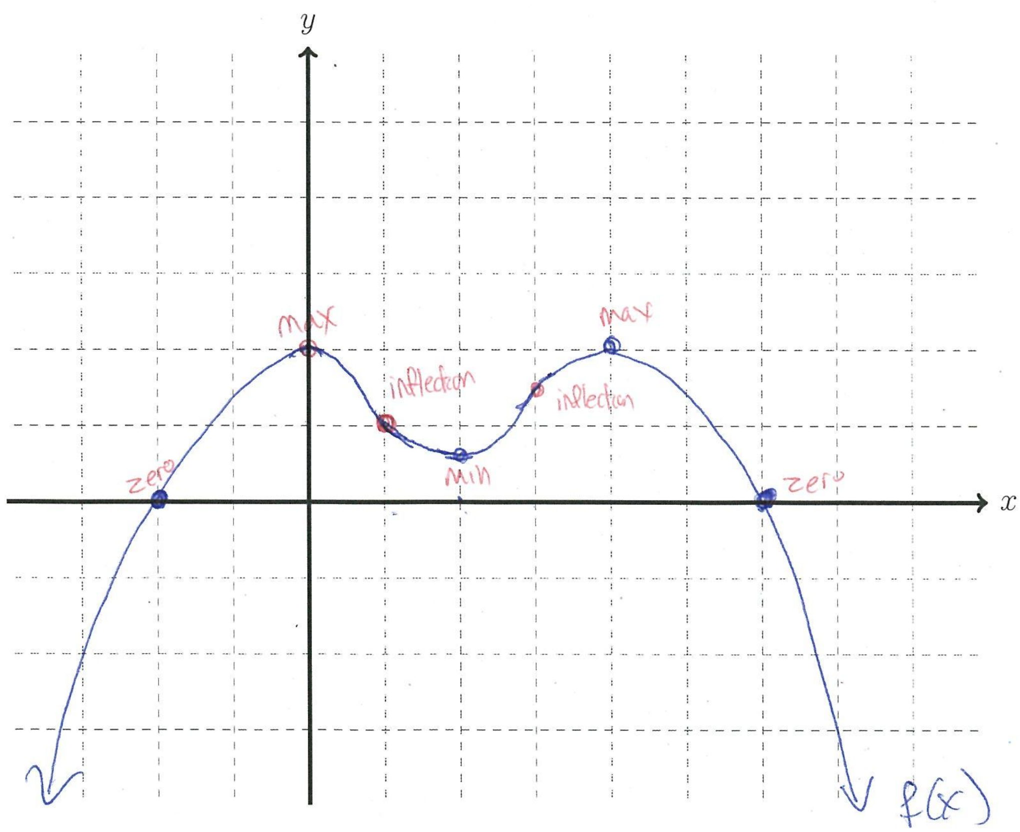
(a) Construct and label sign charts for f, f', and f'' based on the given information.



inc: $(-\infty, 0) \cup (2,4)$
dec: $(0,2) \cup (4,\infty)$

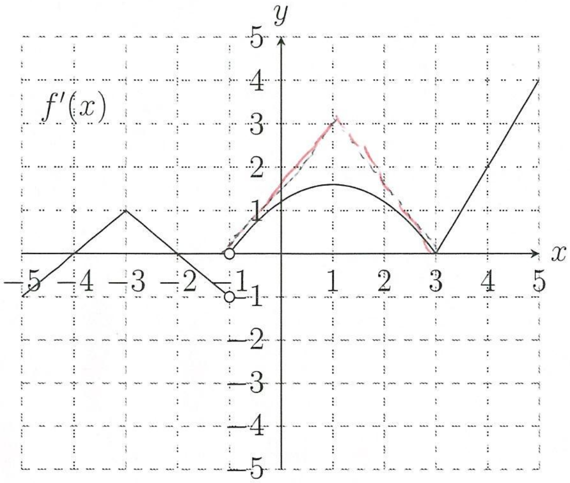
cc down: $(-\infty, 1) \cup (3, \infty)$
cc up: $(1,3)$

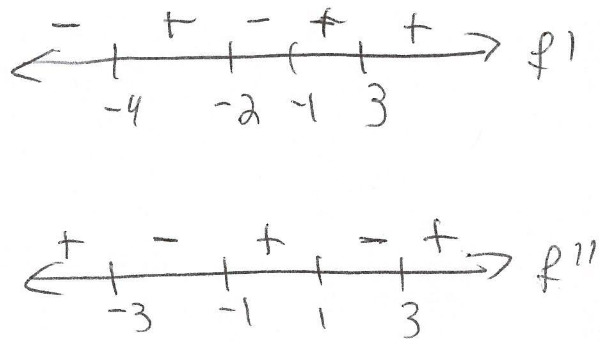
(b) Using your sign charts, sketch a graph of $y = f(x)$.



5. The graph below is the **derivative**, f', of some function f. Construct sign charts for f' and f'' in the space to the right of the graph.

Your sign chart for f' should include any points where $f' = 0$ as well as any points where f' DNE (like $x = -1$). Your sign chart for f'' should include any points where $f'' = 0$ as well as points where f'' DNE (corners and discontinuities on the graph of f').





(a) On what intervals is f increasing? Decreasing? At values of x, if any, does f have a local maximum? A local minimum? Justify your answer.

f inc $(-4,-2) \cup (-1,3) \cup (3,\infty)$ since $f'>0$ local max $x=-2$ f' goes (+) to (−)

f dec $(-\infty,-4) \cup (-2,-1)$ since $f'<0$ local min $x=-4,-1$ f' goes (−) to (+)

(b) On what intervals is f concave up? Concave down? At values of x, if any, does f inflection points? Justify your answer.

f cc up $(-\infty,-3) \cup (-1,1) \cup (3,\infty)$ b/c $f''>0$ inflection at $x=-3,-1,1,3$

f cc down $(-3,-1) \cup (1,3)$ b/c $f''<0$ since f'' changes sign.

(c) Sketch a graph of $f''(x)$, which is the derivative of the graph shown above.