INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/Test 1**, (3) **lecture number/instructor name** and (4) **SUMMER 2015** on the front of your bluebook. Also make a **grading table** with room for 5 problems and a total score. **Start each problem on a new page.** **Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK! JUSTIFY ALL YOUR ANSWERS!**

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1. For this problem, suppose $f(x) = 2\cos x$ and $g(x) = \frac{1}{x^2-1}$.

   (a) (6 pts) Find $(g \circ f)(x)$.

   (b) (6 pts) What is the domain of $(g \circ f)(x)$?

   (c) (8 pts) Suppose we let $h(x) = \begin{cases} f(x), \text{if } x > 2\pi \\ g(x), \text{if } x \leq 2\pi \end{cases}$, are there any values of x for which $h(x)$ is *not* continuous? Justify your answer. What type of discontinuities does $h(x)$ have (i.e. *jump*, *removable*, or *infinite*), if any?

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2. Evaluate the following limits and show all supporting work. If a limit does not exist, clearly state that fact and explain your reasoning. (Note: You may not use l'Hopital's Rule.)

   (a) (4 pts) $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2-4x+3}$

   (b) (4 pts) $\lim\limits_{x\to -\infty} 2x - \sqrt{4x^2-5x}$

   (c) (4 pts) $\lim\limits_{x\to 0^+} \sqrt{x}\cos\dfrac{\pi}{x}$

   (d) (4 pts) $\lim\limits_{x\to 0^-} \dfrac{x}{x-|x|}$

   (e) (4 pts) $\lim\limits_{x\to \infty} \sqrt{\dfrac{4x^2-x}{x^2+9}}$

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3. (a) (5 pts) Given the function $f(x) = 3^{-x}\cos(10x)$. Is f a continuous function of x? Justify why or why not.

   (b) (5 pts) Does $f(x) = 3^{-x}\cos(10x)$ have a real root? Justify why or why not.

   (c) (5 pts) Use continuity to evaluate: $\lim_{x\to\pi} \sin(x + \sin x)$.

   (d) (5 pts) Use the definition of the derivative to show that $b(x) = \sqrt{x} + x - 1$ is an increasing function.

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4. (a) (7 pts) Use the limit definition of the derivative to find the slope of $f(x) = 3x^2 - 10x - 7$ at any point x.

   (b) (7 pts) Find an equation of the tangent line to the parabola $f(x) = 3x^2 - 10x - 7$ whose slope is $m = -8$.

   (c) (6 pts) If $s(t) = 3t^2 - 10t - 7$ for $t \geq 0$ describes the position of an object (in feet) at time t, find the average velocity of the object from $t = 1$ second to $t = 2$ seconds.

PROBLEM #5 ON THE OTHER SIDE

5. The following parts are *not* related:

   (a) (6 pts) For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?

   (b) (6 pts) Find the first and second derivatives of: $G(r) = \sqrt{r} + \sqrt[3]{r}$.

   (c) (8 pts) Find the n^{th} derivative of each function by calculating the first few derivatives and observing the pattern that occurs:

      i. $f(x) = x^n$

      ii. $f(x) = \frac{1}{x}$