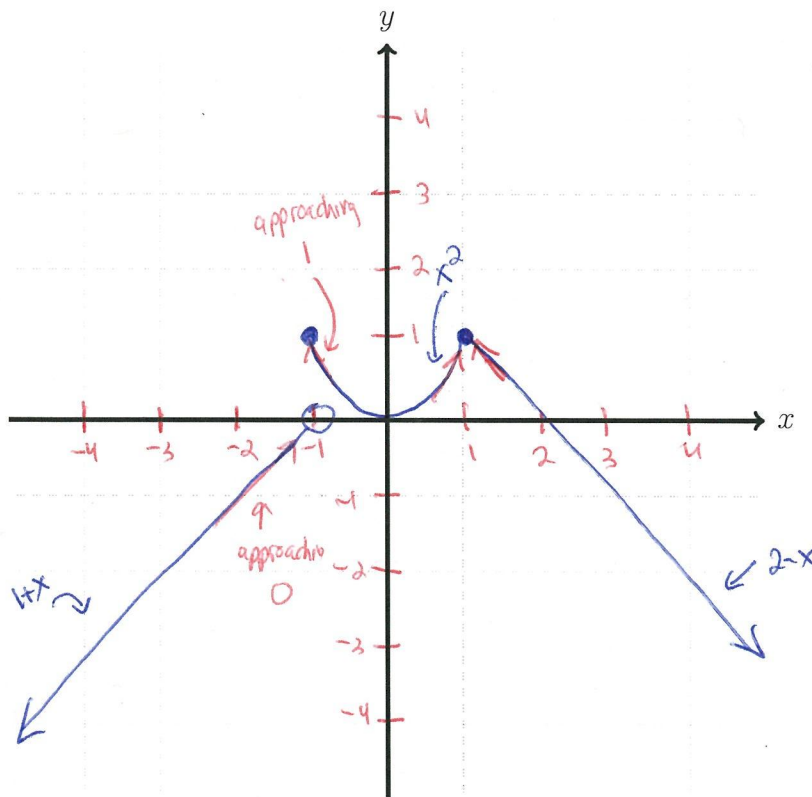*Solutions*

Guidelines: Please work in groups of two or three. Please show all work and clearly denote your answer.

1. Sketch the graph of the function and use it to determine the values of a for which

$$\lim_{x \to a} f(x)$$

   exists. Please use interval notation for your answer.

$$f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2 - x & \text{if } x \ge 1. \end{cases}$$
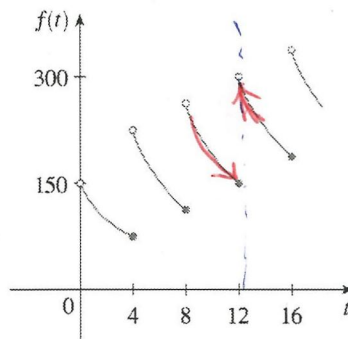

At $a = -1$, $\lim\limits_{x \to -1^-} f(x) \ne 0$ and $\lim\limits_{x \to -1^+} f(x) = 1$.

Since RHL $\ne$ LHL, $\lim\limits_{x \to -1} f(x)$ DNE. The limit exists everywhere else, so the answer is

$$\boxed{(-\infty, -1) \cup (-1, \infty)}$$

2. A patient receives a 150-mg injection of a drug every 4 hours. The graph below shows the amount $f(t)$ of drug in the bloodstream after t hours. Find

$$\lim_{t \to 12^-} f(t) \quad \text{and} \quad \lim_{t \to 12^+} f(t)$$

and explain the significance of these one-sided using complete sentences.

to the left
$\neq$ 12, the drug
level has fallen
o 150mg

$$\lim_{t \to 12^-} f(t) = 150 \text{ mg}$$

$$\lim_{t \to 12^+} f(t) = 300 \text{ mg}$$

right after
2 we recieve
n injection so
x level rises
o 300 mg

Every four hours, the patient recieves an injection. As time passes, the body metabolizes the drug and its level falls until the next injection.

3. Sketch the graph of an example of a function f that satisfies

$$\lim_{x \to 0} f(x) = 1, \quad \lim_{x \to 3^-} f(x) = -2, \quad \lim_{x \to 3^+} f(x) = 2,$$
$$f(0) = -1, \quad f(3) = 1.$$

There are many correct answers
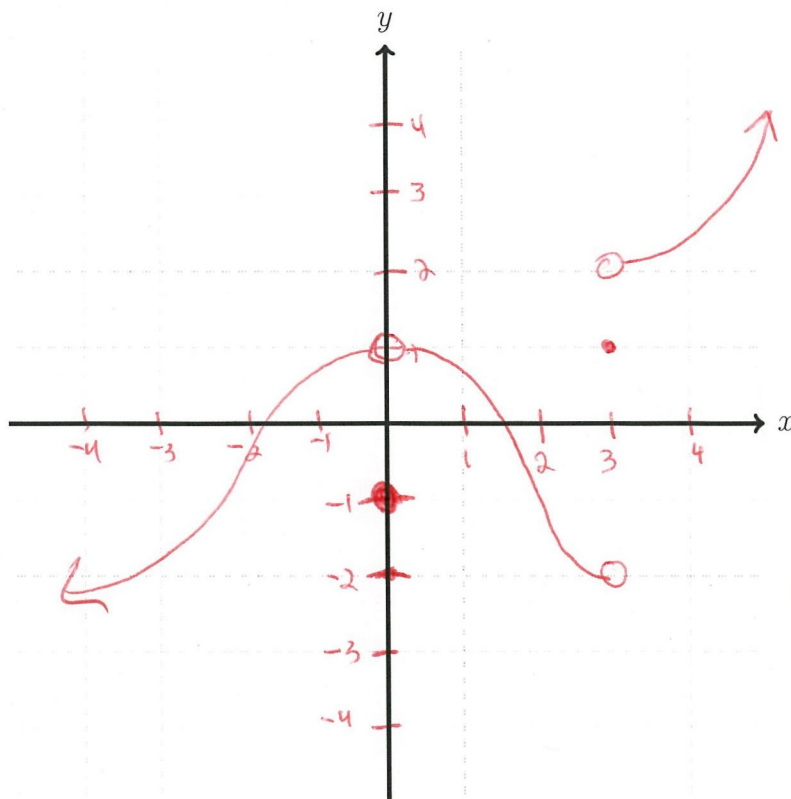
Check that all my features are correct!

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