APPM 1350 **Midterm 2** **Spring 2013**

On the front of your bluebook, please write: a grading key, your name, and instructor's name (Chang or Rubio). This exam is worth 100 points and has 6 questions. **Show all work! Simplify all answers.** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

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1. (15 points)

   (a) Find the linearization of $f(x) = \sqrt[4]{1-x}$ at $x = 0$.

   (b) Use the linearization to approximate the value of $\sqrt[4]{0.92}$.

2. (30 points) Consider the function $f(x) = \dfrac{-2x}{x^2-3}$, $f'(x) = \dfrac{2(x^2+3)}{(x^2-3)^2}$, $f''(x) = \dfrac{-4x(x^2+9)}{(x^2-3)^3}$.

   (a) Find any vertical, horizontal, or slant asymptotes of f. Use appropriate limits to justify your answer.

   (b) On what intervals is f increasing? decreasing?

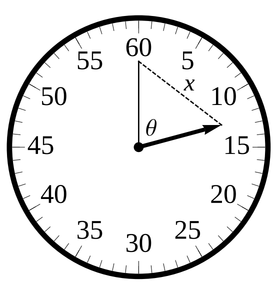
   (c) Find all local maximum and minimum values of f.

   (d) On what intervals is f concave up? concave down?

   (e) Find all inflection points of f.

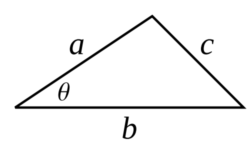
   (f) Using the information from (a) to (e), sketch a graph of f. Clearly label any asymptotes, local extrema, and inflection points.

3. (15 points) The second hand on a stopwatch, 5 centimeters in length, makes a full revolution every minute. Let x represent the distance between the tip of the hand and its starting position at the 60-second mark. At what rate is x increasing when the hand reaches the 15-second mark? Express your answer in centimeters per second.



Hint: Use the Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos\theta.$



4. (12 points) Let $f(x) = \dfrac{1}{x}$ on $[a, b]$, where $0 < a < b$.

   (a) Verify that f satisfies the hypotheses of the Mean Value Theorem.

   (b) Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem. Express your answer in terms of a and b.

5. (12 points) For the following statements, answer TRUE if the statement is always true and justify your answer. Otherwise provide a sketch of a COUNTEREXAMPLE to show that the statement may be false.

   (a) If f is differentiable for all x, then f has an absolute minimum value on $[-5, 5]$.

   (b) If g is decreasing for $x < -2$ and increasing for $x > -2$, then g has a local minimum value at $x = -2$.

   (c) If h is continuous and $h(-3) = h(7)$, then there is a number c in $(-3, 7)$ such that $h'(c) = 0$.

6. (16 points) Hank Hill is designing a propane tank with a volume of 64π cubic meters. The tank is cylindrical with spherical endcaps. The spherical endcaps cost $8/3$ as much per square meter as the cylindrical body. What dimensions will minimize the cost of materials for the tank?