Solutions

Math 1300-010 - Fall 2016

The Substitution Rule - 12/5/16

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for the final exam.

1. Evaluate the integral by making the given subsitution.

(a) $\int x^3(2+x^4)^4\,dx, \quad u = 2+x^4$

$du = 4x^3dx \rightarrow x^3dx = \frac{1}{4}du$

$= \frac{1}{4}\int u^4 du$

$= \frac{1}{4}\frac{u^5}{5} + C = \boxed{\frac{1}{20}(2+x^4)^5 + C}$

(b) $\int x^2\sqrt{x^3+1}\,dx, \quad u = x^3+1$

$du = 3x^2dx \rightarrow \frac{1}{3}du = x^2dx$

$= \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$

(c) $\int \frac{dt}{(1-6t)^4}, \quad u = 1-6t$

$du = -6dt \rightarrow dt = \frac{-1}{6}du$

$\frac{-1}{6}\int \frac{du}{u^4} = \frac{1}{6}\cdot\frac{-1}{3}u^{-3} + C = \boxed{\frac{-1}{18}(1-6t)^{-3} + C}$

(d) $\int \cos^3(\theta)\sin(\theta)\,d\theta, \quad u = \cos(\theta)$

$du = -\sin(\theta)d\theta \rightarrow -du = \sin(\theta)d\theta$

$= -\int u^3 du = -\frac{1}{4}u^4 + C = \boxed{-\frac{1}{4}\cos^4(\theta) + C}$

(e) $\int \frac{\sec^2(1/x)}{x^2}\,dx, \quad u = 1/x$

$du = \frac{-1}{x^2}dx$

$= -\int \sec^2(u)du = -\tan(u) + C = \boxed{-\tan(\frac{1}{x}) + C}$

2. Evaluate the indefinite integral.

(a) $\int e^x \cos(e^x)\, dx$ $\quad u = e^x$

$du = e^x dx$

$= \int \cos(u)\, du = \sin(u) + C = \boxed{\sin(e^x) + C}$

(b) $\int \frac{dx}{5-3x} \cancel{dx}$ $\quad u = 5-3x,\ du = -3dx$

$-\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = \boxed{-\frac{1}{3}\ln(5-3x) + C}$

(c) $\int \frac{(\ln x)^2}{x}\, dx$ $\quad u = \ln(x),\ du = \frac{1}{x}dx$

$= \int u^2 du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}(\ln(x))^3 + C}$

(d) $\int \frac{\sin(x)}{1+\cos^2(x)}\, dx$ $\quad u = \cos(x)$

$du = -\sin(x)dx$

$= -\int \frac{du}{1+u^2} = -\arctan(u) + C = \boxed{-\arctan(\cos(x)) + C}$

(e) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$ $\quad u = \sqrt{x},\ du = \frac{1}{2\sqrt{x}}dx$

$2\int \sin(u)du = -2\cos(u) + C = \boxed{-2\cos(\sqrt{x}) + C}$

(f) $\int (x^2+1)(x^3+3x)^4\, dx$ $\quad u = x^3+3x,\ du = 3x^2 + \cancel{3x} = 3(x^2+1)dx$

$= \frac{1}{3}\int u^4 du = \frac{1}{3}\frac{u^5}{5} + C = \frac{1}{15}u^5 + C$

$= \boxed{\frac{1}{15}(x^3+3x)^5 + C}$