Math 1300-005 - Spring 2017

The Closed Interval Method - 3/20/17

Guidelines: Please work in groups of two or three. This will not be handed in, but is a study resource for Midterm 3.

The purpose of this worksheet is to explore the ***closed interval method*** for finding the absolute extrema of a continuous function f on a closed interval $[a, b]$.

1. Let us find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 6x^2 + 9x + 2$$

on the closed interval $[-1.4]$.

(a) Find the critical numbers of f. Recall these are numbers c *in the domain of* f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

(b) Find the values of f at the critical numbers from (a) that are within the open interval $(-1, 4)$.

(c) Find the values of $f(-1)$ and $f(4)$. Note that -1 and 4 are the endpoints of our closed interval $[-1, 4]$.

(d) The largest value found in parts (b) and (c) will be the absolute maximum of f on $[-1.4]$. The smallest value found in parts (b) and (c) will alternatively be the absolute minimum of f on $[-1.4]$. What is the absolute maximum and at what x-value does it occur? What is the absolute minimum and at what x-value does it occur?

The steps outlined in the previous problem are known as the closed interval method and can be summarized as follows:

The Closed Interval Method: To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

(i) Find the values of f at the critical numbers of f in the open interval (a, b).

(ii) Find the values of f at the endpoints of the interval. That is, find $f(a)$ and $f(b)$.

(iii) The largest of the values from Steps (i) and (ii) is the absolute maximum value; the smallest of these values is the absolute minimum value.

2. Use the closed interval method to find the absolute maximum and absolute minimum values of

$$f(x) = 12 + 4x - x^2$$

on the closed interval $[0, 5]$.

3. Use the closed interval method to find the absolute maximum and absolute minimum values of

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

on the closed interval $[-3, 1]$.