Name: Solutions

Math 1300-005 - Spring 2017

Quiz 8 - 3/10/17

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

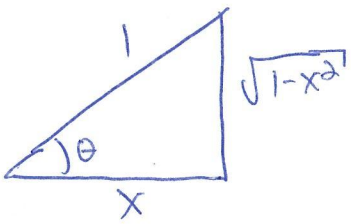
Signature: _____

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. Simplify the following. In each case, you must draw a right triangle, label it appropriately, and then use the picture to simplify the expression.

   (a) $\csc(\arccos(x))$

Let $\theta = \arccos(x)$. Then $\cos(\theta) = x = \frac{A}{H}$.



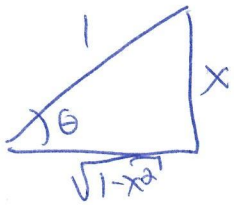
So $\csc(\arccos(x)) = \csc(\theta)$

$= \frac{Hyp}{opp}$

$= \boxed{\frac{1}{\sqrt{1-x^2}}}$

   (b) $\cot(\arcsin(x))$

Let $\theta = \arcsin(x)$. Then $\sin(\theta) = x = \frac{opp}{Hyp}$



So $\cot(\arcsin(x)) = \cot(\theta)$

$= \frac{adj}{opp} = \boxed{\frac{\sqrt{1-x^2}}{x}}$

2. Compute the following limit.

$$\lim_{x\to-\infty} \arctan\left(\frac{1+x^3}{2x+x^2}\right)$$

let $t = \frac{1+x^3}{2x+x^2}$. as $x \to -\infty$, $t \approx \frac{x^3}{x^2} = x \to -\infty$.

so $\lim_{x\to-\infty} \arctan\left(\frac{1+x^3}{2x+x^2}\right) = \lim_{t\to-\infty} \arctan(t) = \boxed{-\frac{\pi}{2}}$

3. Find an equation of the tangent line to the curve $y = 3\arccos(x/2)$ at $x = 1$.

$\frac{dy}{dx} = \frac{-3}{\sqrt{1-(\frac{x}{2})^2}} \cdot \frac{1}{2}$. at $x=1$, $\frac{dy}{dx} = \frac{-3}{\sqrt{1-\frac{1}{4}}} \cdot \frac{1}{2} = \frac{-3}{\sqrt{\frac{3}{4}}} \cdot \frac{1}{2}$

$= \frac{-3}{\frac{\sqrt{3}}{2}} \cdot \frac{1}{2}$

$= \frac{-3}{\sqrt{3}}$.

and at $x=1$, $3\arccos(\frac{1}{2}) = 3(\frac{\pi}{3}) = \pi$.

so $\boxed{y - \pi = \frac{-3}{\sqrt{3}}(x-1)}$ is the tangent line.

4. Find the derivative of

$$g(x) = x\arcsin\left(\frac{x}{4}\right) + \sqrt{16-x^2}.$$

$$g'(x) = \arcsin\left(\frac{x}{4}\right) + \frac{x}{\sqrt{1-(\frac{x}{4})^2}} \cdot \frac{1}{4} + \frac{1}{2\sqrt{16-x^2}} \cdot (-2x)$$