Name: Solutions

Math 1300-005 - Spring 2017

Quiz 3 - 2/3/17

> *On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.*
>
> Signature: ______________________________

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. Use the squeeze theorem to evaluate the following limits. Remember, there is a step-by-step process to answering these, so please include all steps that are necessary.

(a) $\lim_{x\to 0} |x| \sin\left(\frac{4}{x}\right)$ [Your ~~section~~ answer should have everything mine does]

① $-1 \le \sin\left(\frac{4}{x}\right) \le 1$

② since $|x| \geq 0$, $-|x| \le |x|\sin\left(\frac{4}{x}\right) \le |x|$

③ Then $\lim_{x\to 0}(-|x|) \le \lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) \le \lim_{x\to 0}|x|$

④ $0 \le \lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) \le 0$. So by the Squeeze Theorem,

$\lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) = 0.$

(b) $\lim_{x\to\infty} \left(\frac{1}{x^4}\right)\cos(x)$ [Your answer should everything mine does]

① $-1 \le \cos(x) \le 1$

② since $\frac{1}{x^4} \geq 0$, $-\frac{1}{x^4} \le \left(\frac{1}{x^4}\right)\cos(x) \le \frac{1}{x^4}$

③ Then $\lim_{x\to\infty}\left(-\frac{1}{x^4}\right) \le \lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) \le \lim_{x\to\infty}\frac{1}{x^4}$

④ $0 \le \lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) \le 0$. So by the Squeeze Theorem,

$\lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) = 0.$

2. (a) Let $f(x) = x^4 + 5x^3 - 2x^2 - 7$. Use the Intermediate Value Theorem to show $f(x)$ crosses the x-axis in the interval $[-1, 2]$. You must justify your use of the IVT to receive credit.

[Your answer should include everything that mine does.]

f is a polynomial and is therefore continuous on $[-1,2]$. Since

$$f(-1) = 1 - 5 - 2 - 7 = -13$$
$$f(2) = 16 + 40 - 8 - 7 = 41,$$

0 is between $f(-1)$ and $f(2)$. By the IVT, there exists c in $(-1, 2)$ with

$$f(c) = 0 \iff c^4 + 5c^3 - 2c^2 - 7 = 0.$$

(b) Let $g(x) = \ln(x) + 2x - 3$. Use the Intermediate Value Theorem to show $g(x)$ crosses the x-axis in the interval $[1, e]$. You must justify your use of the IVT to receive credit.

[Your answer should include everything mine does]

g is a polynomial plus a logarithm and is therefore continuous on $[1,e]$. Since

$$g(1) = \ln(1) + 2(1) - 3 = 0 + 2 - 3 = -1$$
$$g(e) = \ln(e) + 2e - 3 = 1 + 2e - 3 = 2e - 2 > 0,$$

0 is between $g(1)$ and $g(e)$. By the IVT, there exists c in $(1, e)$ with

$$g(c) = 0 \iff \ln(c) + 2c - 3 = 0.$$

3. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx + 5 & \text{if } x \leq 2 \\ 7x - c & \text{if } x > 2 \end{cases}$$

[If you don't use limits on this problem, you are doing it wrong!!]

We must check continuity at $a=2$. So we need

$$f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x), \text{ so}$$

$$\lim_{x \to 2^-} (cx+5) = \lim_{x \to 2^+} (7x - c)$$

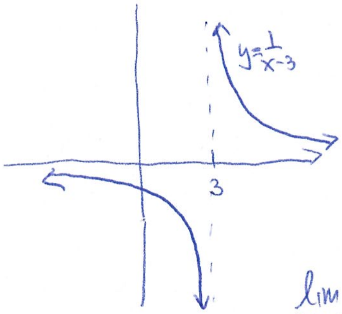
$$\iff 2c + 5 = 14 - c$$

$$3c = 9 \longrightarrow \boxed{c = 3}$$

If you start from this step you will lose a lot of points.

4. Compute the following limits. Show all work, and if necessary, explain your reasoning to receive full credit.

(a) $\displaystyle\lim_{x \to 3^-} \frac{x+1}{x-3}$ Note: $\dfrac{(x+1)}{x-3} = (x+1) \cdot \dfrac{1}{x-3}$



$$\lim_{x \to 3^-} (x+1) = 4$$

$$\lim_{x \to 3^-} \frac{1}{x-3} = -\infty. \text{ So}$$

$$\lim_{x \to 3^-} \left(\frac{x+1}{x-3}\right) = \lim_{x \to 3^-} (x+1) \cdot \lim_{x \to 3^-} \frac{1}{x-3}$$
$$= 4 \cdot (-\infty)$$
$$= \boxed{-\infty}$$

(b) $\displaystyle\lim_{x \to -\infty} \frac{2x^3 + x - 1}{x^2 + x + 2}$

Quick Solution: The largest power on top exceeds the largest power on bottom, so

$$\lim_{x \to -\infty} \frac{2x^3+x-1}{x^2+x+2} = \boxed{-\infty}$$

[think about why its $-\infty$, not $+\infty$]

Long Solution: Since x^2 is the largest power in the denom,

$$\lim_{x \to -\infty} \frac{2x^3+x-1}{x^2+x+2} = \lim_{x \to -\infty} \frac{\frac{2x^3}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{2x + \cancel{\frac{1}{x}}^{\to 0} - \cancel{\frac{1}{x^2}}^{\to 0}}{1 + \cancel{\frac{1}{x}}^{\to 0} + \cancel{\frac{2}{x^2}}^{\to 0}} = \boxed{-\infty}, \text{ since } 2x \to -\infty \text{ as } x \to -\infty$$