INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your name**, (2) **1350/FINAL**, (3) **lecture number/instructor name** and (4) **FALL 2013** on the front of your bluebook. Also make a **grading table** with room for 5 problems and a total score. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK**

---

1. The following problems are not related:

   (a) (15 pts, 5 pts ea.) Find the following limits or show that they do not exist:

   $$(i)\ \lim_{x\to\infty} e^{-x}\sqrt{x} \qquad (ii)\ \lim_{x\to 0}\frac{\arcsin(x)}{x} \qquad (iii)\ \lim_{x\to 0^+}(\sin x)^x$$

   (b) (5 pts) Find and classify all relative extrema of $f(x)=\dfrac{x^5-5x}{5}$.

   (c) (10 pts) A right triangle is formed in the first quadrant by both the x-axis and y-axis and a line through the point $(1,2)$. Find the vertices of the triangle so that its area is a minimized.

   [*Hint:* the slope of the hypotenuse should be the same whether slope is found using the y-intercept and the point (1,2) or the x-intercept and the point (1,2).]

---

2. The following problems are not related:

   (a) (15 pts, 5 pts ea.) Find dy/dx if:

   $$(i)\ y=e^{x+\tan^{-1}(x)} \qquad (ii)\ y=x^2\cosh(\sqrt{x}) \qquad (iii)\ xe^y-\tanh(y)=10,690$$

   (b) (7 pts) Consider the function $f(x)=2x^2-4x+1$ on the interval $[0,2]$.

   (i) (3 pts) Does the function $f(x)$ satisfy the hypotheses of the Mean Value Theorem on the given interval?

   (ii) (4 pts) If it satisfies the hypotheses, find all numbers c that satisfy the conclusion of the Mean Value Theorem. If it does not satisfy the hypotheses, write DNE.

   (c) (8 pts) A kite 100 feet above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

---

3. The following problems are not related:

   (a) (15 pts, 5 pts ea.) Evaluate the integrals:

   $$(i)\ \int_1^2 \frac{e^{1/x^3}}{x^4}\,dx \qquad (ii)\ \int \frac{1}{x^2+4}\,dx \qquad (iii)\ \int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$$

   (b) (5 pts) Find h' in terms of f' and g' if $h(x)=\dfrac{f(x)g(x)}{f(x)+g(x)}$.

   (c) (10 pts) The Ideal Gas Law relating the *pressure*, *volume*, and *temperature* of a gas is given by $PV=nRT$ where R is the ideal gas constant. Assuming that the current pressure of the gas, P, is 10 atmospheric pressure units (atm) and assuming that n, R, and T are held fixed, use differentials to estimate the **relative change** in the volume, V, if the pressure increases by 0.1 atms.

PROBLEMS #4 & #5 ON THE OTHER SIDE

4. The following problems are not related:

(a) (15 pts, 5 pts ea.) Find dy/dx given:

$$(i)\ y = (\sin x)^x \qquad (ii)\ y = \frac{2^x}{1+2^x} \qquad (iii)\ y = e^{u(x)} \text{ where } u(x) = \int_0^{g(x)} \cos(f(t))\,dt$$

(b) (5 pts) In your blue book clearly sketch the graph of a function $f(x)$ that satisfies the following properties (label any extrema, inflection points or asymptotes):

- $f(0) = 0$, $f(-x) = f(x)$
- $\lim_{x\to 1^-} f(x) = -\infty$, $\lim_{x\to -1^-} f(x) = +\infty$
- $\lim_{x\to\infty} f(x) = 2$
- $f'(x) > 0$ if $x < 0$
- $f''(x) < 0$ if $|x| < 1$

(c) (10 pts) A bacteria culture initially contains 106 cells and its population grows at rate proportional to its size. After an hour the population has increased to 420. Find an expression for the number of bacteria after t hours.

---

5. The following problems are not related:

(a) (15 pts, 5 pts ea.) Evaluate the integrals:

$$(i)\ \int_{-4}^{4} \left(\sqrt{16-x^2} + \frac{\sin(x)}{1+x^2}\right) dx \qquad (ii)\ \int \frac{1+x}{1+x^2}\,dx \qquad (iii)\ \int \frac{dx}{x\ln(x^e)}$$

(b) (5 pts) Use the Squeeze Theorem to show $\lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x}\right) = 0$.

(c) (10 pts) The equation of motion of a particle is $s(t) = \frac{t^3}{3} - \frac{t^2}{2} - 6t$ ft

(i) Find the velocity at time t.

(ii) When is the particle moving in the positive direction?

(iii) Find the acceleration after 3 seconds.

(iv) Find the total distance traveled during the first 4 seconds.

— END —