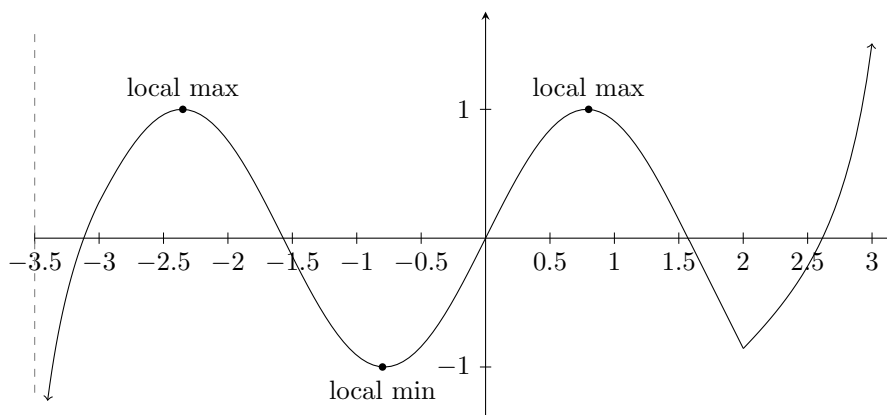# MATH 1300: HW #10

Due on March 23, 2017 at 10:00am

Professor Braden Balentine Section 005
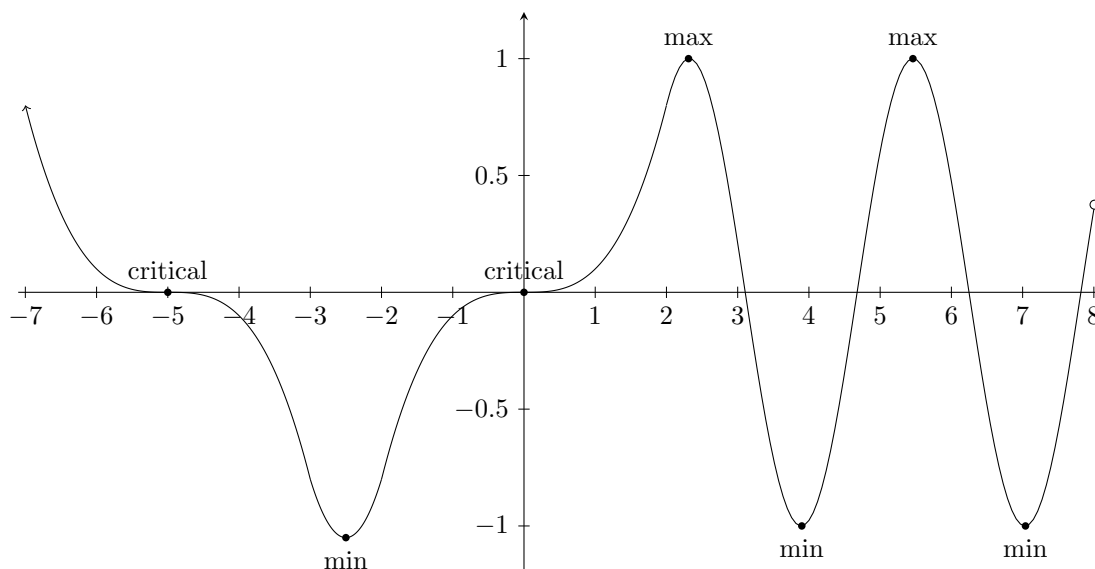
John Keller

Section 4.2

14. (a) Sketch the graph of a function that has two local maxima, one local minimum, and no absolute minimum.

(b) Sketch the graph of a function that has three local minima, two local maxima, and seven critical numbers.

62. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

where μ is a positive constant called the *coefficient of friction* and where $0 \leq \theta \leq \frac{\pi}{2}$. Show that F is

minimized when $\tan\theta = \mu$.

$$F' = \frac{\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$$

$$0 = \sin\theta - \mu\cos\theta$$

$$\mu\cos\theta = \sin\theta$$

$$\mu = \tan\theta$$

$$F(0) = \frac{\mu W}{\mu\sin 0 + \cos 0}$$

$$F(0) = \mu W$$

$$F(\frac{\pi}{2}) = \frac{\mu W}{\mu\sin(\frac{\pi}{2}) + \cos(\frac{\pi}{2})}$$

$$F(\frac{\pi}{2}) = \frac{\mu W}{\mu}$$

$$F(\frac{\pi}{2}) = W$$

66. A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

    (a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

- If $f(x) = x^3 + x^2 - x + 1$, $f'(x) = 3x^2 + 2x - 1$, and since it is zero at $x = \frac{1}{3}, -1$, then there are two critical points
- If $f(x) = x^3$, $f'(x) = 3x^2$, and since it is zero at $x = 0$, then there is one critical point
- If $f(x) = x^3 + x$, $f'(x) = 3x^2 + 1$, and because $f'(x)$ is never negative, there are no critical points

    (b) How many local extreme values can a cubic function have?

    A cubic function can have 0 or 2 local extreme values because it must retain the shape of a cubic function, but adding x^2 simply puts a small dip in the curve, making it have 2 local extremes.

Section 4.3

1. Use the graph of f to estimate the values of c that satisfy the conclusion of the Mean Value Theorem for the interval [0,8].
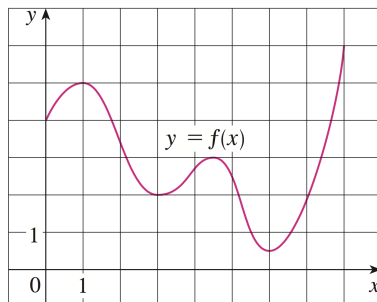
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{6 - 4}{8 - 0}$$

$$f'(c) = \frac{2}{8}$$

$$f'(c) = \frac{1}{4}$$

20. (a) Find the critical numbers of $f(x) = x^4(x - 1)^3$.

$$f'(x) = -\frac{2x}{(-1 + x^2)^2}$$

$$0 = (-1 + x)^2 x^3(-4 + 7x)$$

$$x = 1, 0, \frac{4}{7}$$

(b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?
The Second Derivative Test tells us if there is a local maximum or minimum at each critical point.

(c) What does the First Derivative Test tell you?
The First Derivative Test tells us if there is a local maximum or minimum, as well as if there is neither, at the critical point.

34.

$$f(x) = \frac{x^2}{(x - 2)^2}$$

(a) Find the vertical and horizontal asymptotes.

Finding vertical: Finding horizontal:

$$0 = (x - 2)^2$$

$$x = \boxed{2}$$

$$\frac{x^2}{x^2} = \boxed{1}$$

(b) Find the intervals of increase or decrease.

$$f'(x) = -\frac{4x}{(x - 2)^3}$$

$$0 = -\frac{4x}{(x - 2)^3}$$

$$x = 0$$

Increase: $(0, 2)$ Decrease: $(-\infty, 0) \cup (2, \infty)$

(c) Find the local maximum and minimum values.
Local max: None   Local min: $(0, 0)$

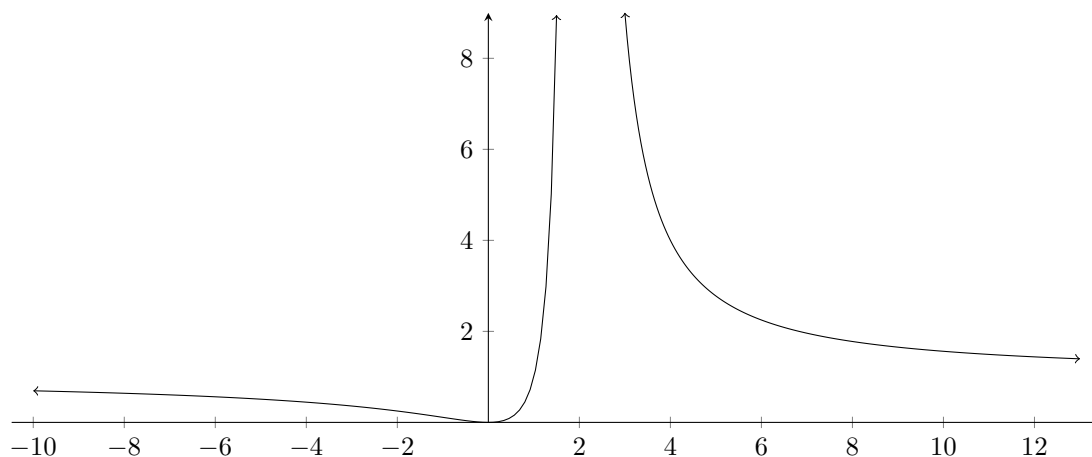(d) Find the intervals of concavity and the inflection points.

$$f''(x) = \frac{8 + 8x}{-2^4 + x^4}$$

$$0 = \frac{8 + 8x}{-16 + x^4}$$
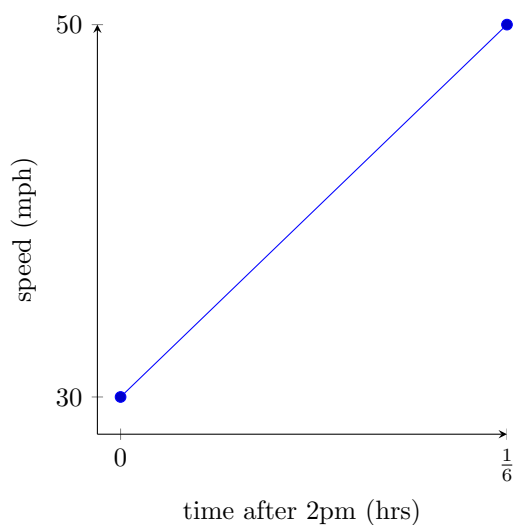
$$x = -1$$

Concave up: $(-1, 2)$ Concave down: $(-\infty, 1)$

(e) Use the information from parts (a)-(d) to sketch the graph of f.

66. At 2:00 PM a car's speedometer reads 30 mi/h. At 2:10 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2.

Determining the slope for the graph above:

$$y = mx + b$$
$$m = \frac{20}{\frac{1}{6}}m \qquad = 120x$$

As long as x starts at 0 (or 2pm), the acceleration is going to start off at 120 mi/h^2.

70. For what values of c does the polynomial $P(x) = x^4 + cx^3 + x^2$ have two inflection points? One inflection point? None? Illustrate by graphing P for several values of c. How does the graph change as c decreases?

$$P'(x) = 4x^3 + 3cx^2 + 2x$$
$$P''(x) = 12x^2 + 6cx + 2$$
$$0 = 12x^2 + 6cx + 1$$
$$2 = 12x^2 + 6cx$$
$$2 = 6x(2x + c)$$
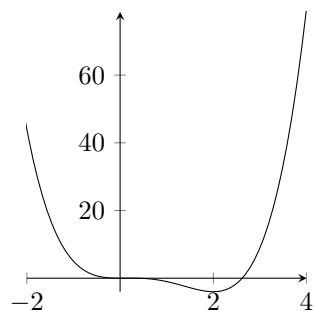$$2 = 6x$$
$$x = \frac{1}{3}$$
$$2 = 2x + c$$
$$x = \frac{2 - c}{2}$$
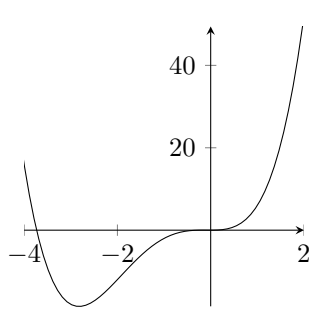$$\frac{1}{3} = \frac{2 - c}{2}$$
$$\frac{2}{2} = 2 - c$$
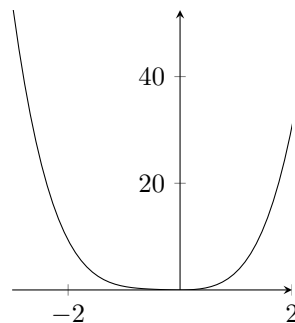$$c = 2 - \frac{2}{3}$$
$$c = \frac{4}{3}$$

$P(x) = x^4 + (-3)x^3 + x^2$

$P(x) = x^4 + (4)x^3 + x^2$

$P(x) = x^4 + (\frac{4}{3})x^3 + x^2$

Additional Problem

Find the absolute extrema of the function $f(x) = xe^{-x^2/18}$ on the interval $[-2, 4]$.

$$f'(x) = e^{\frac{-x^2}{18}}\left(1 - \frac{1}{9}x^2\right)$$

$$0 = e^{\frac{-x^2}{18}}$$

$$\ln(0) = \frac{-x^2}{18}$$

$$18 = -x^2$$

$$x = \sqrt{-18} \text{ DNE}$$

$$0 = 1 - \frac{1}{9}x^2$$

$$x = \pm 3$$

$$f(3) = 3e^{\frac{1}{2}} \text{ relative max}$$

$$f(-2) = -2e^{\frac{2}{9}} \text{ local min}$$

$$f(4) = 4e^{\frac{8}{9}} \text{ local max}$$