Name: Solutions

Math 1300-005 - Spring 2017

Quiz 3 - 2/3/17

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: ____________________

Guidelines: You are permitted to use notes, the book, in-class worksheets/solutions, and your classmates on this quiz. Computers and graphing technology of any kind, including calculators, are not allowed (exceptions made for those who have an e-book). Please show all work and clearly denote your answer.

1. Use the squeeze theorem to evaluate the following limits. Remember, there is a step-by-step process to answering these, so please include all steps that are necessary.

(a) $\lim_{x\to 0} |x| \sin\left(\frac{4}{x}\right)$ [Your ~~section~~ answer should have everything mine does]

① $-1 \leq \sin\left(\frac{4}{x}\right) \leq 1$

② since $|x| \geq 0$, $-|x| \leq |x|\sin\left(\frac{4}{x}\right) \leq |x|$

③ Then $\lim_{x\to 0}(-|x|) \leq \lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) \leq \lim_{x\to 0}|x|$

④ $0 \leq \lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) \leq 0$. So by the Squeeze Theorem,

$$\lim_{x\to 0} |x|\sin\left(\frac{4}{x}\right) = 0.$$

(b) $\lim_{x\to\infty} \left(\frac{1}{x^4}\right)\cos(x)$ [Your answer should everything mine does]

① $-1 \leq \cos(x) \leq 1$

② since $\frac{1}{x^4} > 0$, $-\frac{1}{x^4} \leq \left(\frac{1}{x^4}\right)\cos(x) \leq \frac{1}{x^4}$

③ Then $\lim_{x\to\infty}\left(-\frac{1}{x^4}\right) \leq \lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) \leq \lim_{x\to\infty}\frac{1}{x^4}$

④ $0 \leq \lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) \leq 0$. So by the Squeeze Theorem,

$$\lim_{x\to\infty}\left(\frac{1}{x^4}\right)\cos(x) = 0.$$

2. (a) Let $f(x) = x^4 + 5x^3 - 2x^2 - 7$. Use the Intermediate Value Theorem to show $f(x)$ crosses the x-axis in the interval $[-1, 2]$. You must justify your use of the IVT to receive credit.

[Your answer should include everything that mine does.]

f is a polynomial and is therefore continuous on $[-1,2]$. Since

$$f(-1) = 1-5-2-7 = -13$$
$$f(2) = 16+40-8-7 = 41,$$

0 is between $f(-1)$ and $f(2)$. By the IVT, there exists c in $(-1,2)$ with

$$f(c) = 0 \iff c^4+5c^3-2c^2-7=0.$$

(b) Let $g(x) = \ln(x) + 2x - 3$. Use the Intermediate Value Theorem to show $g(x)$ crosses the x-axis in the interval $[1, e]$. You must justify your use of the IVT to receive credit.

[Your answer should include everything mine does]

g is a polynomial plus a logarithm and is therefore continuous on $[1,e]$. Since

$$g(1) = \ln(1)+2(1)-3 = 0+2-3 = -1$$
$$g(e) = \ln(e)+2e-3 = 1+2e-3 = 2e-2>0,$$

0 is between $g(1)$ and $g(e)$. By the IVT, there exists c in $(1,e)$ with

$$g(c)=0 \iff \ln(c)+2c-3=0.$$

3. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx+5 & \text{if } x \le 2 \\ 7x-c & \text{if } x > 2 \end{cases}$$

[If you don't use limits on this problem, you are doing it wrong!!]

We must check continuity at $a=2$. So we need

$$f(2) = \lim_{x\to 2^-} f(x) = \lim_{x \to 2^+} f(x), \text{ so}$$

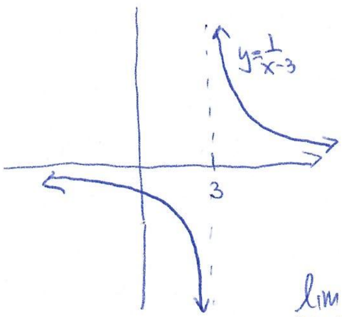
$$\lim_{x\to 2^-}(cx+5) = \lim_{x\to 2^+}(7x-c)$$

If you start from this step you will lose a lot of points.

$$\iff 2c+5 = 14-c$$
$$3c = 9 \longrightarrow \boxed{c=3}$$

4. Compute the following limits. Show all work, and if necessary, explain your reasoning to receive full credit.

(a) $\displaystyle\lim_{x\to 3^-} \frac{x+1}{x-3}$ Note: $\dfrac{(x+1)}{x-3} = (x+1)\cdot\dfrac{1}{x-3}$



$$\lim_{x\to 3^-}(x+1) = 4$$

$$\lim_{x\to 3^-}\frac{1}{x-3} = -\infty. \text{ So}$$

$$\lim_{x\to 3^-}\left(\frac{x+1}{x-3}\right) = \lim_{x\to 3^-}(x+1)\cdot\lim_{x\to 3^-}\frac{1}{x-3}$$
$$= 4\cdot(-\infty)$$
$$= \boxed{-\infty}$$

(b) $\displaystyle\lim_{x\to -\infty} \frac{2x^3+x-1}{x^2+x+2}$

Quick Solution: The largest power on top exceeds the largest power on bottom, so

$$\lim_{x\to -\infty}\frac{2x^3+x-1}{x^2+x+2} = \boxed{-\infty}$$

[think about why it's $-\infty$, not $+\infty$]

Long Solution: Since x^2 is the largest power in the denom,

$$\lim_{x\to -\infty}\frac{2x^3+x-1}{x^2+x+2} = \lim_{x\to -\infty}\frac{\frac{2x^3}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{2}{x^2}}$$

$$= \lim_{x\to -\infty}\frac{2x+\cancel{\frac{1}{x}}^{\to 0}-\cancel{\frac{1}{x^2}}^{\to 0}}{1+\cancel{\frac{1}{x}}^{\to 0}+\cancel{\frac{2}{x^2}}^{\to 0}} = \boxed{-\infty}, \text{ since } 2x\to-\infty \text{ as } x\to-\infty$$