

1.)

	a	b	c	d	e	f	g	h
A	4	5	5	1			3	2
B		3	4	3	1	2	1	
C	2		1	3		4	5	3

(a) utility Matrix as Boolean

	a	b	c	d	e	f	g	h
A	1	1	0	1	1	0	1	1
B	0	1	1	1	1	1	1	0
C	1	0	1	1	0	1	1	1

Jaccard Similarity b/w A and B = $\frac{4}{8} = \frac{1}{2}$

Jaccard distance b/w A and B = $1 - \frac{1}{2}$

$= \frac{1}{2}$

Jaccard Similarity b/w B and C = $\frac{4}{8} = \frac{1}{2}$

Jaccard Distance b/w B and C = $1 - \frac{1}{2} = \frac{1}{2}$

Jaccard similarity b/w A and C = $\frac{4}{8} = \frac{1}{2}$

Jaccard distance b/w A and C = $1 - \frac{1}{2} = \frac{1}{2}$

Cosine Distance:

$$\text{b/w A and B} = \frac{|1 \times 1| + |1 \times 1| + |1 \times 1|}{\sqrt{6} \cdot \sqrt{6}}$$

$$= \frac{4}{6} = \frac{2}{3} \Rightarrow \angle AB = 48.18^\circ$$

$$\text{b/w B and C} = \frac{2}{3} \Rightarrow \angle BC = 48.18^\circ$$

$$\text{b/w A and C} = \frac{2}{3} \Rightarrow \angle AC = 48.18^\circ$$

(b) Discretization:

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

$$\text{Jaccard similarity b/w } A \text{ \& } B = \frac{2}{5}$$

$$\text{Jaccard Distance b/w } A \text{ \& } B = 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$\text{Jaccard similarity b/w } B \text{ \& } C = \frac{1}{6}$$

$$\text{Jaccard Distance b/w } B \text{ \& } C = 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

$$\text{Jaccard similarity b/w } A \text{ \& } C = \frac{2}{6} = \frac{1}{3}$$

$$\text{Jaccard Distance b/w } A \text{ \& } C = 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Cosine distance:

$$A \text{ and } B = \frac{1 \times 1 + 1 \times 1}{\sqrt{4} \cdot \sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\angle A B = 54.73^\circ$$

$$B \text{ and } C = \frac{1 \times 1}{\sqrt{3} \cdot \sqrt{4}} = \frac{1}{\sqrt{12}}$$

$$\angle BC = 73.22^\circ$$

$$A \text{ and } C = \frac{1 \times 1 + 1 \times 1}{\sqrt{4} \cdot \sqrt{4}} = \frac{1}{2}$$

$$\angle AC = 60^\circ$$

Comparing this with Part "a."

We can Infer.

Discretization gives better result than Treating the Matrix as a boolean.

Because in part "a" we got all the Jaccard distances to be equal and also the cosine distances are all equal.

Whereas in part "b" we can clearly see that distance b/w AB < distance b/w BC < distance b/w AC.

Using cosine rule.

(c) Normalize the Matrix:

$$\text{Average of } A = \frac{10}{3}$$

$$\text{Average of } B = \frac{7}{3}$$

$$\text{Average of } C = 3.$$

	a	b	c	d	e	f	g	h
A	$\frac{2}{3}$	$\frac{5}{3}$	0	$\frac{5}{3}$	$-\frac{7}{3}$	0	$-\frac{1}{3}$	$-\frac{4}{3}$
B	0	$\frac{2}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$	0
C	-1	0	-2	0	0	1	2	0

Cosine Distances:

$$\text{Between A and B} = \frac{52}{9}$$

$$\sqrt{\frac{120}{9}} \cdot \sqrt{\frac{66}{9}}$$

$$= \frac{52}{88.99438}$$

$$\angle AB = 54.24^\circ$$

$$\text{Between B and C} = \frac{-19}{3} \pm \frac{\sqrt{660}}{3}$$

$$= -0.73957$$

$$\angle BC = 137.694^\circ$$

$$\text{Between A and C} = \frac{-4}{3} \pm \frac{\sqrt{1200}}{3}$$

$$= -0.11547$$

$$\angle AC = 96.63^\circ$$

By this method we can clearly observe that

Distance b/w AB < Distance b/w AC < Distance b/w BC.