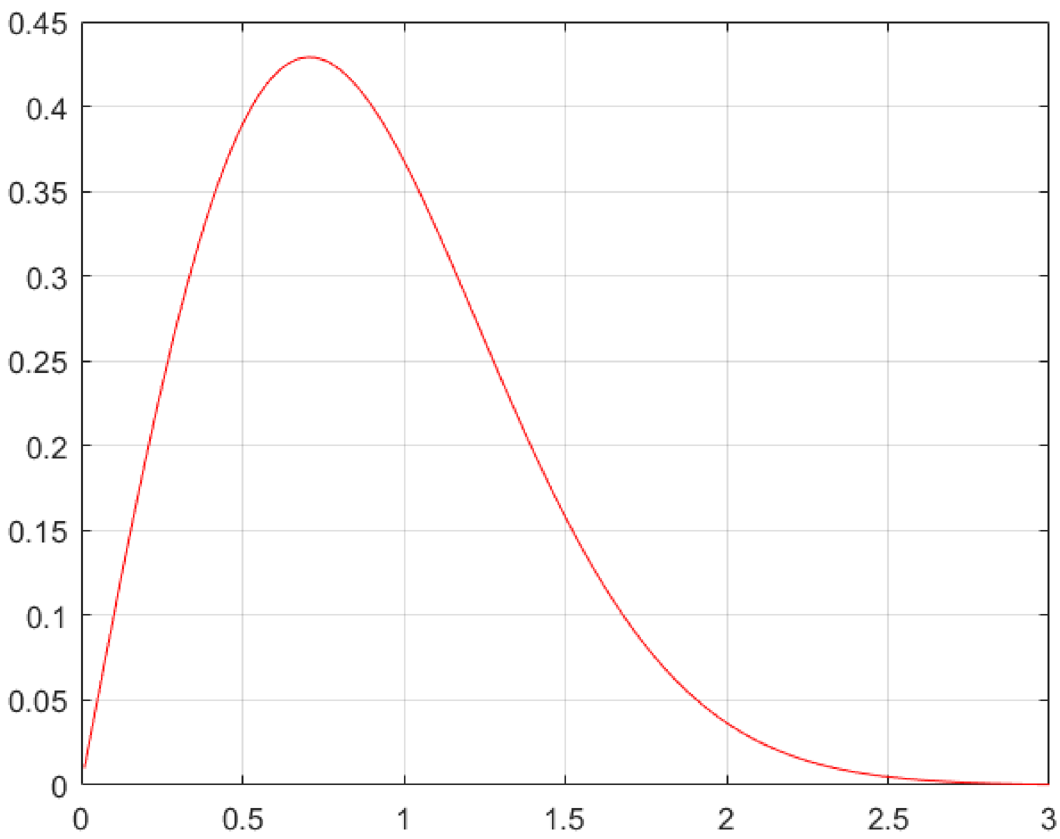
1.)

(a.) Approximate the solution of the initial value problem using a fourth order Runge-Kunga method:

By using ode45 function in matlab, solved the given function.

Below are the plots obtained

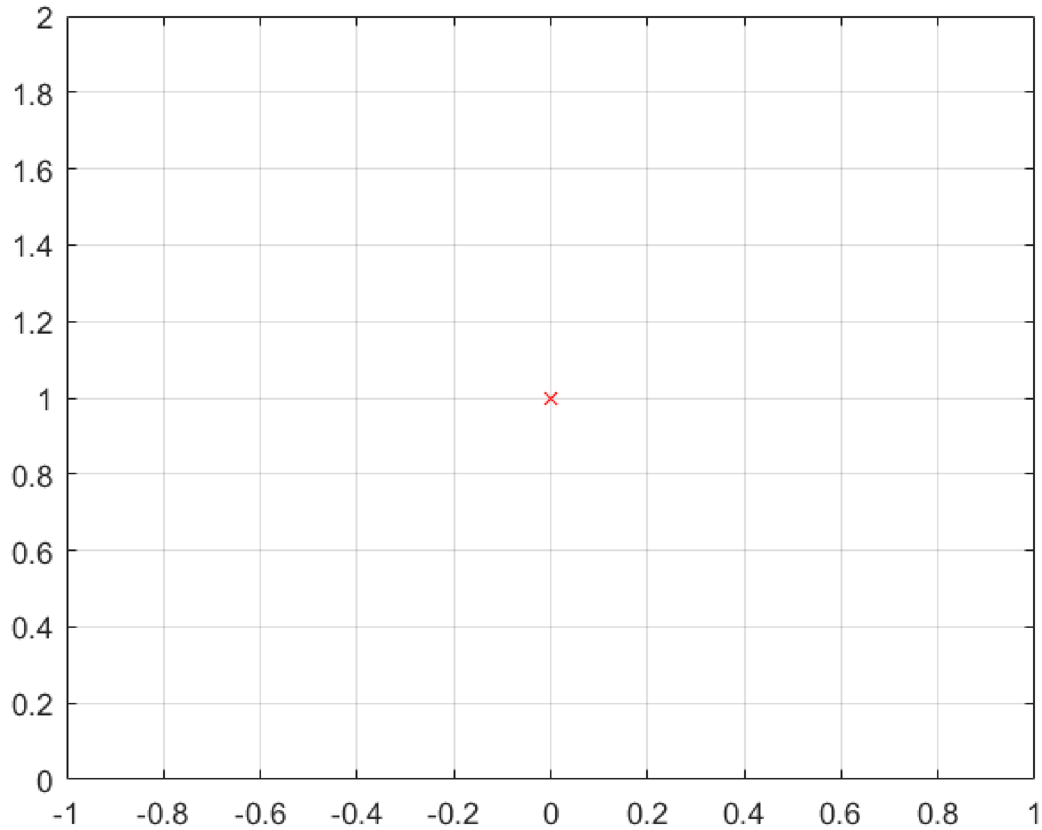
<u>When ‘t’ is not equal to zero:</u>



From the above plot it can be observed that the approximate value of the function when t=1 is 0.3679 (matlab result). Using integrals, the solution obtained is 1/e which is 0.36787.

Therefore, the solution obtained by Runge-Kunga method is very close to the solution obtained by integral method. The step size used for this method is 300.

When ‘t’ is zero (t=0):



This is the plot obtained for t=0.

Given differential of y=1 at t=0.

Therefore, only a single point is obtained on the plot.

1.) $$\dot{y} = \begin{cases} y\left(-2t + \frac{1}{t}\right) & , t \neq 0 \\ 1 & , t = 0. \end{cases}$$

(b.) For $t \neq 0$:,

$$\dot{y} = y\left(-2t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = y\left(-2t + \frac{1}{t}\right)$$

$$\frac{dy}{y} = \left(-2t + \frac{1}{t}\right) \cdot dt.$$

Integrating on both sides for the above equation.

$$\int \frac{dy}{y} = \int \left(-2t + \frac{1}{t}\right) \cdot dt.$$

$$\log y = -2 \cdot \frac{t^2}{2} + \log t + C_1$$

where C_1 – constant.

$$\log y - \log t = -t^2 + C_1.$$

$$\boxed{\log\left(\frac{y}{t}\right) = -t^2 + C_1.}$$

$$y = t\left(e^{-t^2} \times e^{C_1}\right) \Rightarrow \boxed{y = k \cdot t \cdot e^{-t^2}}$$ where $k = e^{C_1}$ i.e constant.

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For $t = 0$:

$$\dot{y} = 1$$

$$\frac{dy}{dt} = 1$$

$$dy = dt$$

Integrating on both sides.

$$\int dy = \int dt$$

$$y = t + C_2$$

where C_2 = constant.

Given

$$y(0) = 1$$

$\therefore$ $C_2 = y(0) = 1$ //.

(c) $y(1) \rightarrow$ compute the exact solution.

This is the case of $t \neq 0$

for which we know

$$\log\left(\frac{y}{t}\right) = -t^2 + C_1$$

$$\Rightarrow \boxed{y = k \cdot t \cdot e^{-t^2}}$$ where k is constant.

$$y(1) = k \cdot 1 \cdot e^{-1}$$

$$\boxed{y(1) = \frac{k}{e}}$$

Here we cannot solve for k value

as we donot have sufficient information for $y(t)$. Values.

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1.)

(d.) In this problem the quadrature rule that I used is composite trapezoidal rule.

The approximate value of the integral at t = 1 using composite trapezoidal rule is 0.36396.

Using integrals, the solution obtained is 1/e which is 0.36787

Therefore, Error % = (0. 36787 – 0. 36396)/0.36787

= 0.0106

Therefore, the error % is less than 1.

The number of points we need to get within 1% accuracy of the exact solution is obtained as 5 from matlab.

From 1a, the number of time steps = 300 (Runge-Kunga method)

From above, the number of points/ steps = 5 (composite trapezoidal rule).

Therefore using composite trapezoidal rule, the accuracy can be obtained in less number of step size. Whereas for Runge-Kunga method we need higher step size to obtain the precision.