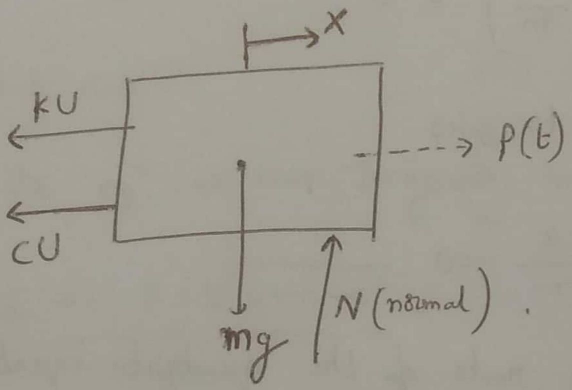
1.) Assume $P(t)$ is zero.

Parameters $m$, $k$ and $c$ are all Positive.

(a.) <u>F.B.D of the system:</u>

Given Force from the damper, $F_d = -c \cdot \dot{u}$.




Applying Newton's second Law of Motion,

$$-kU - c\dot{U} + P(t) = m\ddot{U}$$

given $P(t)$ is zero.

$$\therefore -kU - c\dot{U} = m\ddot{U}$$

$$\Rightarrow m\ddot{U} + c\dot{U} + kU = 0$$

(b.) General solution to the second order differential equation.

$$m\left(\ddot{U} + c\dot{U} + kU\right) = 0$$

$$\boxed{\ddot{U} + \frac{c}{m}\dot{U} + \frac{k}{m} \cdot U = 0} \quad \text{——— } (1)$$

Let the solution be of the form, $U = e^{\lambda t}$.

$$\frac{dU}{dt} = \dot{U} = \lambda e^{\lambda t} \quad ; \quad \frac{d^2U}{dt^2} = \ddot{U} = \lambda^2 \cdot e^{\lambda t}.$$

Now substituting the values of $u$, $\dot{u}$ and $\ddot{u}$ in ①

$$\ddot{u} + \frac{c}{m} \cdot \dot{u} + \frac{k}{m} u = 0.$$

$$\lambda^2 \cdot e^{\lambda t} + \frac{c}{m}\left(\lambda \cdot e^{\lambda t}\right) + \frac{k}{m}\left(e^{\lambda t}\right) = 0$$

$$e^{\lambda t}\left[\lambda^2 + \frac{\lambda c}{m} + \frac{k}{m}\right] = 0$$

$e^{\lambda t}$ tends to zero

$$\Rightarrow \lambda^2 + \frac{\lambda c}{m} + \frac{k}{m} = 0$$

using formula for the roots of the quadratic equation.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4 \times \frac{k}{m}}}{2 \cdot 1}$$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}.$$

The three cases that we need to consider are

$c^2 - 4mk > 0$, $c^2 - 4mk < 0$ and $c^2 - 4mk = 0$.

Case I: $c^2 - 4mk > 0$.

Then the eq$^n$ has two real roots.

$$\lambda_1 = \frac{-c + \sqrt{c^2 - 4mk}}{2m}$$

$$\lambda_2 = \frac{-c - \sqrt{c^2 - 4mk}}{2m}$$

Case II: $c^2 - 4mk < 0$.

Then the eq$^n$ has two Imaginary roots.

$$\lambda_1 = \frac{-c + i\sqrt{4mk - c^2}}{2m}$$

$$\lambda_2 = \frac{-c - i\sqrt{4mk - c^2}}{2m}$$

Case III: $c^2 - 4mk = 0$.

Then the eq$^n$ has single root.

$$\therefore \lambda = \frac{-c}{2m}.$$

(c.) solution for each of the three Cases.

Case I: $c^2 - 4mk > 0$.

We are having two real roots.

$\therefore$ solution of $U = c_1 \cdot e^{\lambda_1 t} + c_2 \cdot e^{\lambda_2 t}$

where $c_1$ and $c_2$ are constants.

Case II: $c^2 - 4mk < 0$.

We are having two Imaginary roots

Solution, $U = e^{-c/2m}\left(a\cos\left(\frac{\sqrt{4mk-c^2}}{2m}.t\right) + b\sin\left(\frac{\sqrt{4mk-c^2}}{2m}\right)t\right)$

Case III: $c^2 - 4mk = 0$.

We are having a single root.

solution, $U = (c_1 + c_2 t)e^{\lambda t}$.

(d.) The General solution to the second order differential equation is

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0.$$

Let $U = x$ and $V = \dot{x}$

$$\dot{U} = \dot{x} = V \quad \text{and} \quad \dot{V} = \ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x.$$

$$= -\frac{c}{m}V - \frac{k}{m}x$$

$$= -\frac{c}{m}V - \frac{k}{m}U.$$

$\therefore$ $\dot{U} = V$ and $\dot{V} = -\frac{c}{m}V - \frac{k}{m}U$

$$\dot{U} = 0 \cdot U + 1 \cdot V$$

$$\dot{V} = \left(-\frac{k}{m}\right)U + \left(-\frac{c}{m}\right).V.$$

e.) given $m = 1kg$ , $c = 0.1 kg/s$, $k = 1000 N/m$.

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\frac{-k}{m} = \frac{-1000}{1} = -1000$$

$$\frac{-c}{m} = \frac{-0.1}{1} = -0.1$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1000 & -0.1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\boxed{\dot{\vec{u}} = A \cdot \vec{u}}$$

where $A = \begin{bmatrix} 0 & 1 \\ -1000 & -0.1 \end{bmatrix}$, $\vec{u} = [x, v]^T$ with $V = \dot{x}$

f) Done in Matlab.

g.) The eigen values of A are $-0.0500 + 31.6227i$

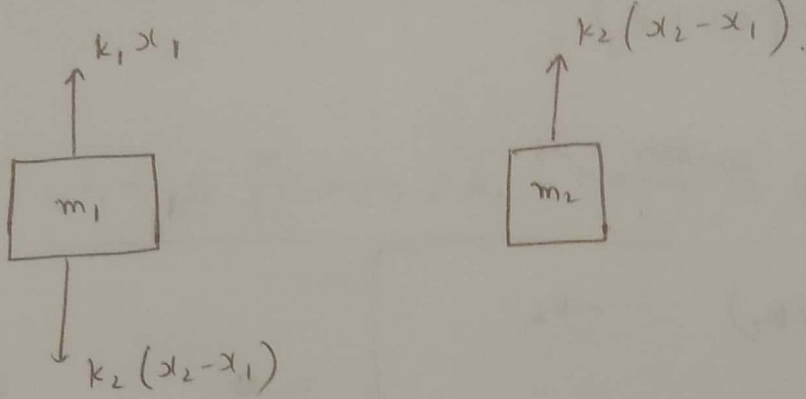
and $-0.0500 - 31.6227i$

vectors are $V = \begin{bmatrix} -0.0000 - 0.0316i & -0.000 + 0.0316i \\ 0.9995 + 0.0i & 0.9995 + 0.00i \end{bmatrix}$

$W = \begin{bmatrix} 0.9995 + 0.0i & 0.9995 + 0.00i \\ 0.0 + 0.0316i & 0.00 - 0.0316i \end{bmatrix}$

2) F.B.D of the system:

(a)




equations of motion of the system:

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0 \quad \text{--- (1)}$$

$$m_2 \ddot{x}_2 + k_2(x_1 - x_2) = 0 \quad \text{--- (2)}$$

(b) $m_1 = 2$, $m_2 = 1$

$k_1 = 1000$, $k_2 = 2000$ :

The equation of motion of the system.

$$z(\omega) = k_1 k_2 - m_1 k_2 \cdot \frac{k_2}{m_2} - k_1 \cdot m_2 \cdot \frac{k_2}{m_2} - k_2 \cdot m_2 \cdot \frac{k_2}{m_2} + m_1 m_2 \cdot \frac{k_2}{m_2} \cdot \frac{k_2}{m_2}$$

$$z(\omega) = k_1 k_2 - m_1 \cdot \frac{k_2}{m_2} - k_1 k_2 - k_2^2 + \frac{m_1 \cdot k_2^2}{m_2}.$$

Hence the equation of motion of the system is

$$\boxed{z(\omega) = k_1 k_2 - m_1 \cdot \frac{k_2}{m_2} - k_1 k_2 - k_2^2 + \frac{m_1 \cdot k_2^2}{m_2}.}$$

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$$x_1(t) = X_1 \cos(\omega t + \phi)$$

$$x_2(t) = X_2 \cos(\omega t + \phi).$$

The frequency equation is.

$$\begin{bmatrix} (-m_1\omega^2 + 2k_1) & -k_2 \\ -k_1 & -m_2\omega^2 + 2k_2 \end{bmatrix} = 0.$$

$$m_1 m_2 \omega^4 - 2k_1(2k_2) \cdot m_2 + 3k_1 k_2 = 0.$$

The solution of the above eq$^n$ is.

$$\omega_i = \left\{ \frac{4k_1 m_1 \pm [16 m_1 m_2 k_1 k_2 - 12 m_1 m_2 k_1 k_2]^{1/2}}{2m_1^2} \right\}^{1/2}.$$

where $i = 1, 2.$

$$\therefore \omega_1 = \left[ \frac{8000 - 4000}{8} \right]^{1/2}$$

$$\omega_1 = (500)^{1/2}$$

$$\omega_1 = 22.36\,Hz.$$

$$\omega_2 = \left[ \frac{8000 + 4000}{8} \right]^{1/2}$$

$$\omega_2 = 38.73\,Hz.$$

(c) The complete solution is already computed which is.

$$\omega_i = \left\{ \frac{4k_1m_1 \pm \left[16 m_1 m_2 k_1 k_2 - 12 m_1 m_2 k_1 k_2\right]^{1/2}}{2m_1^2} \right\}^{1/2}.$$

where $i = 1, 2$

(d) The solution Implies two Masses oscillate about the rest Position with a frequency of $\omega_1 = 22.36\,Hz$ and $\omega_2 = 38.73\,Hz$.

$$\omega_1 = 22.36\,Hz = 140.63\ rad/s. \left(1\ rad/s \simeq 0.159\,Hz\right).$$

$$\omega_2 = 38.73\,Hz = 243.584\ rad/s.$$

~~As the~~

given $x_1(0) = 1$, $x_2(0) = 1$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$.

$$\therefore\ x_1(t) = \cos 140.63\,t.$$

$$x_2(t) = \cos 243.58\,t.$$

3.) ODE: $\frac{d^2 u}{dx^2} + 10u = 2$ , $0 < x < 1$

BC: $u(0) = 10$ , $\frac{du}{dx}(1) = 0$.

(a) Homogenious solution

$$\frac{d^2 u}{dx^2} + 10u = 0.$$

substitute $u = e^{at}$.

$\therefore a^2 + 10 = 0$

$\Rightarrow a = \pm i\sqrt{10}$.

$$u_h = A \cdot e^{i\sqrt{10}\,x} + B e^{-i\sqrt{10}\,x}$$

$$\therefore \boxed{u_h = A \sin\sqrt{10}\,x + B \cos\sqrt{10}\,x}.$$

(b) Particular solution.

Since $g(t) = 2$,

The Particular solution is $U_p = A$

$\therefore U_p' = 0$ and $U_p'' = 0$.

$$\therefore 10 \cdot (A) = 2 \Rightarrow A = \frac{2}{10} = \frac{1}{5}$$

The required Particular solution is $U_p = \frac{1}{5} = 0.2$

(c) Complete Solution:

The complete solution = $U_h + U_p$

$$\Rightarrow u = A \sin\sqrt{10}\, x + B\cos\sqrt{10}\, x + \frac{1}{5}$$

(or)

$$U = A\sin\sqrt{10}\, x + B\cos\sqrt{10}\, x + 0.2$$

(d) Integral constants By applying the Boundary conditions.

$$u(0) = 10 \quad \text{and} \quad \frac{du}{dx}(1) = 0.$$

$$u(0) = 10.$$

$$A\sin 0 + B\cos 0 + \frac{1}{5} = 10$$

$$B + \frac{1}{5} = 10$$

$$B = 10 - \frac{1}{5}$$

$$B = \frac{49}{5}$$

given $u'(1) = 0$

$$u'(x) = A\sqrt{10}\cos\sqrt{10}\, x - \sqrt{10}\, B\cdot\sin\sqrt{10}\, x.$$

$$u'(1) = A\sqrt{10}\cos\sqrt{10} - B\sqrt{10}\sin\sqrt{10} = 0.$$

4) (a) $\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin(\theta) = 0$

For small angles, $\sin\theta \simeq \theta$.

∴ The equation is

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

This equation is a non-linear differential equation.

(b) Let $\theta = \frac{\pi}{6}$.

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = 0.5$$

$$\theta = \frac{\pi}{6} = 0.52.$$

$$\text{Error} = \frac{(0.52 - 0.50)}{0.50} \times 100$$

$$= \frac{0.02}{0.50} \times 100$$

$$= \frac{2}{50} \times 100$$

$$= 4\%.$$

Therefore if $\theta = \frac{\pi}{6}$ then error in angle approximation $\sin(\theta) \simeq \theta$.

is less than 5%.

$$A \cdot \sqrt{10} \cos\sqrt{10} - B\sqrt{10} \sin\sqrt{10} = 0 .$$

$$A \cos\sqrt{10} = B \sin\sqrt{10}$$

we got $B = \frac{49}{5}$

$$A = B \frac{\sin\sqrt{10}}{\cos\sqrt{10}} .$$

$$A = \frac{49}{5} \tan\sqrt{10} .$$

Final solution is

$$U = A \sin\sqrt{10}\, x + B \cos\sqrt{10}\, x + \frac{1}{5} .$$

$$U = \frac{49}{5} \cdot \tan\sqrt{10} \cdot \sin\sqrt{10}\, x + \frac{49}{5} \cos\sqrt{10}\, x + \frac{1}{5}$$

4) (c) $$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0.$$

Auxillary equation for this will be,

$$m^2 + \frac{g}{l} = 0.$$

$$m^2 = -\frac{g}{l}.$$

$$m = \pm\sqrt{\frac{g}{l}}\, i$$

So, the solution of the equation will be.

$$\theta(t) = c_1 \cdot \cos\left(\sqrt{\frac{g}{l}}\, t\right) + c_2 \cdot \sin\left(\sqrt{\frac{g}{l}}\, t\right).$$

where $c_1$ and $c_2$ are constants.

(d.) $l = 0.5\ m.$

$\theta(0) = 3°$, $\theta'(0) = 0.$

$g = 9.8\ m/s^2$

$$\theta(t) = c_1 \cos\left(\sqrt{\frac{9.8}{0.5}}\, t\right) + c_2 \cdot \sin\left(\sqrt{\frac{9.8}{0.5}}\, t\right)$$

$$\theta(t) = c_1 \cos\left(\sqrt{19.6}\, t\right) + c_2 \sin\left(\sqrt{19.6}\, t\right).$$

given $\theta(0) = 3^\circ$.

$$3 = C_1 \cos(0) + C_2 \sin(0).$$

$$3 = C_1 + 0.$$

$$\boxed{C_1 = 3.}$$

Also given $\theta'(0) = 0$

$$\theta'(t) = -C_1 \sqrt{19.6} \sin(\sqrt{19.6}\, t) + C_2 \cdot \sqrt{19.6} \cos(\sqrt{19.6}\, t).$$

$$\theta'(0) = 0$$

$$\therefore 0 = -C_1 \sqrt{19.6} \sin(0) + C_2 \cdot \sqrt{19.6} \cos(0).$$

$$0 = 0 + C_2 \cdot \sqrt{19.6}.$$

$$\boxed{C_2 = 0}$$

$$\therefore \theta(t) = C_1 \cos(\sqrt{19.6}\, t) + C_2 \sin(\sqrt{19.6}\, t)$$

$$\boxed{\theta(t) = 3 \cos(\sqrt{19.6}\, t)}$$ – Solution for the Motion of the Pendulum under the small angle approximation.

5) $f_1(x) = \sin^2 x$

$$f_2(x) = \begin{cases} x+\pi, & -\pi \le x < 0 \\ x, & 0 \le x < \pi \end{cases}$$

(a) Write Down the Integral formula, to find the coefficients of the fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$ — Fourier series.

→ $f_1(x) = \sin^2 x$ :

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \cos(nx)\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin^2 x \sin(nx)\, dx = 0 \text{ (odd Function)}$$

→ $$f_2(x) = \begin{cases} x+\pi, & -\pi \le x < 0 \\ x, & 0 \le x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{0} (x+\pi)\,dx + \frac{1}{\pi}\int_{0}^{\pi} x \cdot dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\cos(nx)\cdot dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos(nx)\,dx.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\sin(nx) + \frac{1}{\pi}\int_{0}^{\pi} x\sin(nx)\cdot dx.$$

(b) done in Matlab.

6.) $\nabla^2 \cdot u = 0 \Rightarrow U_{xx} + U_{yy} = 0$ — ①

$u(x,0) = 0 \qquad u(x,b) = 5$

$u(0,y) = 0 \qquad u(a,y) = 0.$

(a) $u(x,y) = X(x) \cdot Y(y)$, Develop ODES for X and y.

$\Rightarrow U_x = Y \cdot X'$, $U_{xx} = YX''$ and $U_{yy} = XY''$.

equation ① yields $Y \cdot X'' + X \cdot Y'' = 0$.

$$\therefore \frac{X''}{X} = \frac{-Y''}{Y} = \lambda \text{ (say)}$$

$\therefore \Rightarrow \boxed{X'' - \lambda X = 0, \quad Y'' + \lambda Y = 0.} \Rightarrow$ $X = A\cosh \lambda x + B\sinh \lambda x$

$Y = C\cos \lambda y + D\sin \lambda y$.

(b) Boundary conditions.

$u(x,0) = 0 \qquad u(x,b) = 5$

$u(0,y) = 0 \qquad u(a,y) = 0.$

$u(x,0) = 0 \Rightarrow X(x) \cdot Y(0) \Rightarrow Y(0) = 0 \Rightarrow C = 0.$

$u(0,y) = 0 \Rightarrow X(0) \cdot Y(y) \Rightarrow X(0) = 0 \Rightarrow A = 0.$

$u(a,y) = 0 \Rightarrow X(a) \cdot Y(y) \Rightarrow X(a) = 0 \Rightarrow B\sinh \lambda a = 0.$

$\Rightarrow \boxed{\lambda_n = \frac{n\pi}{a}}$, $n = 1, 2, 3, \ldots$

$\therefore \boxed{X_n = \sinh\left(\frac{n\pi x}{a}\right)}$

$$Y_n = D \sin\left(\frac{n\pi y}{a}\right)$$

$$u(x, y) = C_1 \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

c) $$u(x,y) = \sum_{n=1}^{\infty} C_n \cdot \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

$$u(x,y) = \sum_{n=1}^{\infty} C_n \cdot \sinh\left(\frac{n\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{a}\right)$$

$$u(x,b) = 5$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n \cdot \sinh\left(\frac{n\pi x}{a}\right) \cdot \sin\left(\frac{n\pi b}{a}\right) = 5$$

where $$C_n \sin\left(\frac{n\pi b}{a}\right) = \frac{2}{b}\int_0^b 5 \sinh\left(\frac{n\pi x}{a}\right) dx$$

$$C_n \sin\left(\frac{n\pi b}{a}\right) = \frac{10}{n\pi}\left[\cosh n\pi\right], \quad n = 1, 3, 5, \ldots$$

$$\therefore u(x,y) = \sum_{n=1,3,5,\ldots} \frac{10}{n\pi \sin\left(\frac{n\pi b}{a}\right)} \cdot \sinh\left(\frac{n\pi x}{a}\right) \cdot (\cosh n\pi)$$

7)

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

(a) Perform 3 Iterations of the Power Method.

Initialize non-zero approximation of

$$X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

$$X_1 = A \cdot X_0 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} \simeq 3 \begin{bmatrix} 1 \\ 1.33 \\ 1 \end{bmatrix}$$

$$X_2 = A \cdot X_1 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \\ 10 \end{bmatrix} = 10 \begin{bmatrix} 1 \\ 1.4 \\ 1 \end{bmatrix}$$

$$X_3 = A \cdot X_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 14 \\ 10 \end{bmatrix} = \begin{bmatrix} 34 \\ 48 \\ 34 \end{bmatrix} \simeq 34 \begin{bmatrix} 1 \\ 1.4 \\ 1 \end{bmatrix}.$$

Now, $X_2 \simeq X_3 = \begin{bmatrix} 1 \\ 1.4 \\ 1 \end{bmatrix}.$

Therefore we can say that

$$(|\lambda_3| = 34) > (|\lambda_2| = 10)$$

We can say that $\lambda_3 = 34$ is the largest eigen value of $A$.

(b.) Perform 3 Iterations of Inverse Power Method.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$|A| = 2(4-1) - 1(2) + 0 = 4$$

$$\text{Adj } A = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= B$$

Initialize non-zero approximation of $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

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$$X_1 = B \cdot X_0 = \frac{1}{4}\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

$$X_2 = B \cdot X_1 = \frac{1}{4}\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$X_3 = B \cdot X_2 = \frac{1}{4}\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 6 \\ -8 \\ 6 \end{bmatrix} \simeq \frac{6}{8}\begin{bmatrix} 1 \\ -1.3 \\ 1 \end{bmatrix}$$

$$X_4 = B \cdot X_3 = \frac{1}{4}\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix} \cdot \frac{6}{8}\begin{bmatrix} 1 \\ -1.3 \\ 1 \end{bmatrix} = \frac{3}{16}\begin{bmatrix} 6.6 \\ -9.2 \\ 6.6 \end{bmatrix} \simeq \frac{99}{80}\begin{bmatrix} 1 \\ -1.4 \\ 1 \end{bmatrix}$$

So $X_3 \simeq X_4$

Largest Eigen value of $B$: $\lambda_4 \simeq 1.24$.

So eigen value of $A = \frac{1}{\lambda}$

$\therefore$ Smallest Value of $A$ is $\frac{1}{\lambda_4} \simeq \tilde{\lambda}_4 \simeq \frac{1}{1.24} \simeq 0.8$

$$\tilde{\lambda}_4 = 0.8$$

c) ratio of your estimates of the largest and smallest Eigen values.

$$\frac{\lambda_3}{\lambda_4} = \frac{34}{0.8} \simeq 42.5.$$

14) Using Linear Regression: [linear Fitting]

$$\mu = a \cdot e^{-b/T}$$

$$\ln u = \ln a - \frac{b}{T}$$

$$\ln u - \ln a = -\frac{b}{T}$$

$$\ln u = -b\left(\frac{1}{T}\right) + \ln a$$

$$y \qquad mx \qquad c$$

$$\boxed{b = -4.4059}$$

$$\ln a = -10.28$$

$$a = e^{-10.28}$$

$$a = \frac{1}{e^{10.28}}$$

$$\boxed{a = 0.000035}$$

17) a) $m\dot{u}_x(t) = -\rho A C V(t) u_x(t)/2$

$$m\dot{u}_y(t) = -\rho A C V(t) \cdot u_y(t)/2 - mg.$$

$$V(t) = \sqrt{u_x^2 + u_y^2}.$$

20.) Given

$$f(x) = \frac{1}{1+25x^2} \quad \forall\ x \in [-1,1].$$

5 Points let them be $-1, -0.5, 0, 0.5, 1.$

calculating Lagranges Independent.

$$L_0(x) = \frac{(x-0.5)(x-0)}{(-1+0.5)(-0.5)} = \frac{(x-0.5)}{0.25}x$$

$$= 4x(x-0.5)$$

$$L_1(x) = \frac{(x+1)(x)}{(-0.5+1)(-0.5)} = -4x(x+1)$$

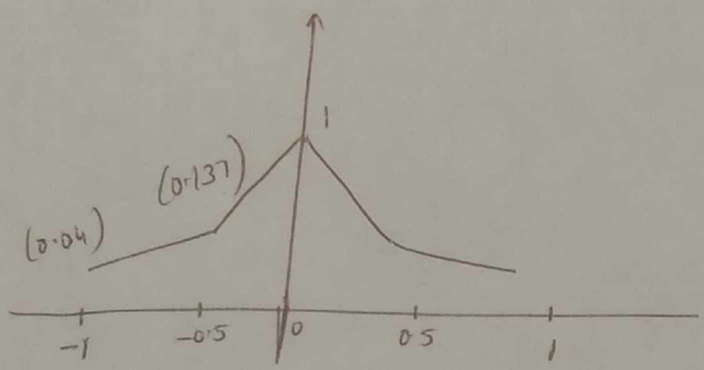
$$L_2(x) = \frac{(x)(x+0.5)}{(0+1)(0+0.5)} = 2x(x+0.5)$$

$$L_3(x) = \frac{(x-0.5)(x-1)}{(-0.5)(-1)} = 2(x-0.5)(x-1)$$

$$L_4(x) = \frac{(x-1)(x-0)}{(1-0)(1-0.5)} = 2x(x-1).$$

$f(-1) = \frac{1}{26}$ ; $f(-0.5) = \frac{4}{25}$ , $f(0) = 1$

$f(0.5) = \frac{4}{29}$ , $f(1) = \frac{1}{26}$ .




11 points over $[-1, 1]$

$-1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1.$

$f(-1) = \frac{1}{26} = f(1).$

$f(-0.8) = 0.06 = f(0.8).$

$f(-0.6) = 0.1 = f(0.6)$

$f(-0.4) = 0.2 = f(0.4).$

$f(-0.2) = 0.5 = f(0.2).$

$f(0) = 1$