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ELECTROSTATICS - I
ELECTRIC CHARGES AND FIELDS

ELECTROSTATICS - I

Electricity is the branch of physics which deals with study of charges. The studies are classified into two types

(i) **Electrostatics** : This deals with study of charges at rest

(ii) **Electrodynamics or current electricity** : This deals with study of charges in motion

Frictional Electricity

Consider two neutral bodies. Electrical neutrality is due to the equality in the number of electrons and protons. When they are rubbed together, heat energy is developed at their contact surface due to friction. Using this energy, some electrons are transfered from one body to another. Body loosing the electrons gain a positive charge due to deficiency of electrons and the body gaining these electrons gain an equal negative charge due to excess of electrons. These charges are known as frictional electricity.

e.g : When glass rod is rubbed with silk cloth, electrons are transferred from glass to silk. So glass gets a positive charge and silk an equal negative charge.

Usually, electrons are removed from the body in which they are less tightly bound to the nucleus and are transferred to the body in which they are more tightly bound to the nucleus. The minimum energy required to remove an electron from a body is known as work function. So electrons are removed from body with lesser work function and transferred to the body with more work function. So work function is less for glass and more for silk.

The least value of charge that can be transferred between two bodies when they are rubbed together is the charge of a single electron, $e = 1.6 \times 10^{-19}$ C. So it is treated as the basic unit of charge. So the positive or negative charge that can appear on a body will be an integer multiple of electronic charge. This fact is called quantisation of charge.

$$Q = \pm ne \,;\; n = 1, 2, 3, ...$$

Here n is the number of electrons transferred when two bodies are rubbed or the excess number of electrons in a negative body or deficient number of electrons in a positive body.

eg : Consider a body with 1C positive charge. Then deficient number of electrons are

$$n = \frac{Q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

If a charge Q flows for a time t, then current established; $I = \frac{Q}{t} \quad \therefore Q = It$

$$\therefore \; ne = Qt$$

Here n is the number of free electrons move through the wire.

Unit of charge : Coulomb (SI)
esu of charge (cgs)

IC = 3 × 10^9 esu

Dimensions of charge = [AT]

Properties of charge

(i) Charge is quantised

(ii) For an isolated system, total charge is conserved

(iii) Charge is a scalar

(iv) Charges can flow and the direction of charge flow between two bodies is determined by their potential values.

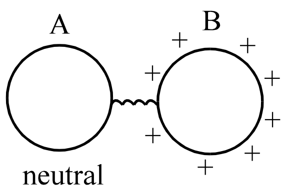
(v) When two identical conducting bodies are made in contact, total charges are equally shared between them. But this is not true for non identical bodies.

(vi) Charges accumulate at the sharp edges of a conducting body.

(vii) Like charges repel while unlike charges attract. But attraction is possible between a charged body and a neutral body. Repulsion happens only between charged bodies. So only repulsion characterise a charge.

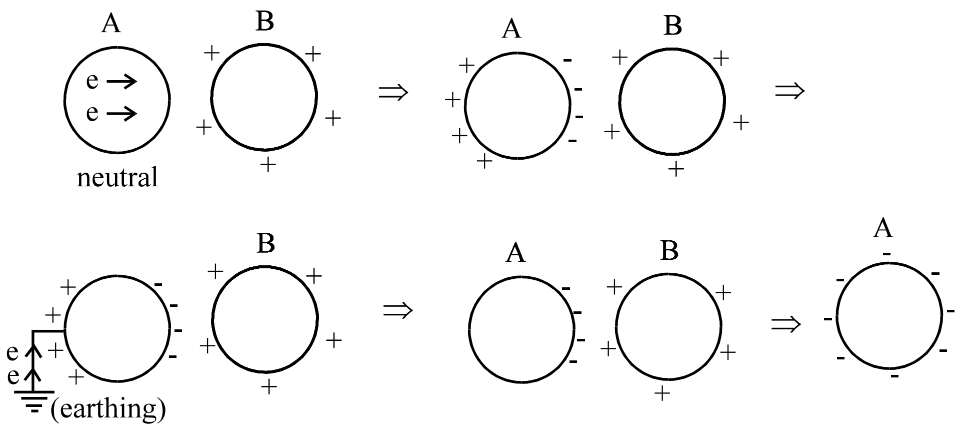
Charging by conduction

It is a method of charging in a neutral conductor by using a direct contact with a charged body. When a contact is made, free electrons travel from A to B. Hence positive charge on B reduces but due to electron loss an equal positive charge appears on A. Here loss of charge in one body results a gain in charge on the other. Also only similar type of charge can be produced.



Charging by induction

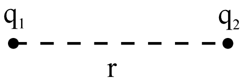
This is a charging method which does not require a direct contact between bodies.



So induction followed by proper earthing is required for charging by induction.

Coulomb's law

It gives the electrostatic force between two point charges.



$F \propto \frac{q_1 q_2}{r^2}$ or $\boxed{F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}}$

$\varepsilon \rightarrow$ absolute electrical permittivity of the medium between the charges. If free space or vacuum is present between the charges, ε is used as ε_0, the permittivity of free space. $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / Nm^2$.

In such a case; $\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$ $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

Unit of ε is C^2/Nm^2.

Dimensions of $\varepsilon = \frac{A^2T^2}{MLT^{-2}L^2} = \left[M^{-1}L^{-3}T^4A^2\right]$

ε and ε_0 are related as; $\varepsilon = K\varepsilon_0$ or $\varepsilon = \varepsilon_r\varepsilon_0$

$K \rightarrow$ dielectric constant or $\varepsilon_r \rightarrow$ relative permittivity of the medium. It is the property of a medium by which it opposes the passage of electric interaction through it. K values are ;

Vacuum	**air**	**good conductor (metal)**	**Perfect insulator**
K = 1	$K \approx 1$	$K = \infty$	K = 0

$\therefore F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$. If a dielectric medium K is filled in the entire spacing between charges.

$F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$

$\boxed{F_{med} = \frac{F_{air}}{K}}$

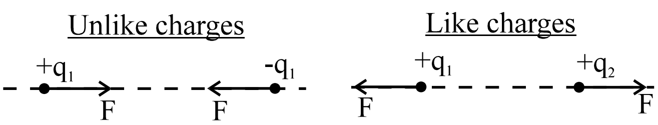
Characteristics of electric force

(i) It is a conservative force

(ii) It is either repulsive or attractive

(iii) It is a central force. So the direction of electric force between charges always acts along the line joining their centres

(iv) Direction of attractive force on a charge is always towards the other charge and the direction of repulsive force on a charge is always away from the other charge along the line joining their centres.



(v) Force between two charges is independent of presence of other charges but depends on the medium between charges.

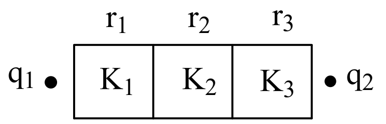
(vi) Electric force is a strong force

(viii) If a medium of dielectric constant K is filled between the charges, force is

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left(r\sqrt{K}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2}$$

$d = r\sqrt{K}$, is the interactive distance between charges. This idea is used to find the force between charges when more than one dielectrics are present between the charges.

eg :



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2}$$

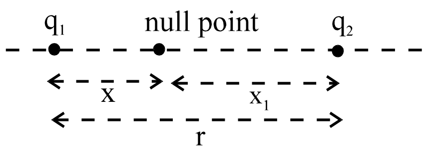
$$d = r_1\sqrt{K_1} + r_2\sqrt{K_2} + r_3\sqrt{K_3}$$

Null point (Neutral point)

It is a point in a region of charges where the net electric force is zero.

e.g. 1 : Two like point charges

Here null point lies between the charges on the line joining them. Null point will be more close to the charge with lesser magnitude.

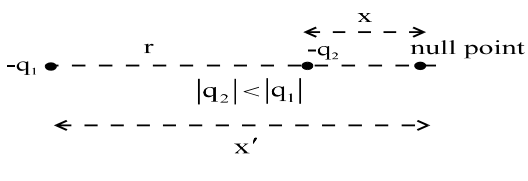


$$x = \frac{r}{1+\sqrt{\frac{q_2}{q_1}}}$$ from q_1.

$$x' = \frac{r}{1+\sqrt{\frac{q_1}{q_2}}}$$ from q_2

e.g. 2 : Two unlike point charges

Here null point lies outside the system of charges on the line joining them, more close to the charge with lesser magnitude.



$$x = \frac{r}{1-\sqrt{\frac{q_2}{q_1}}}$$ from q_1

$$x' = \frac{r}{1+\sqrt{\frac{q_1}{q_2}}}$$

e.g. 3 : If equal charges are placed at all the corners of a polygon of equal sides, its geometrical centre is a null point

ELECTRIC FIELD

It is a region surrounding a charge where the influence due to that charge exist. Charge producing the field is called a source charge and all other charges are test charges. Any point outside this region is called in finity. The word influence means any test charge entering into this region will experience a force of attraction or repulsion.

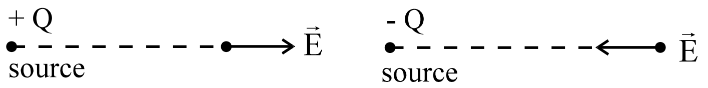
The strength of influence at a point in the field is given by a vector called electric field intensity ($\vec{E}$). At a point it is defined as the force experienced by a unit positive charge placed at that point.

$$E = \frac{F}{q_0} \qquad \text{Unit} = \frac{N}{C} \text{ or } \frac{V}{m} \qquad [E] = \left[MLT^{-3}A^{-1}\right]$$

Theoretically this test charge (q_0) must be selected infinitesimally small so that its field at its position can be neglected.

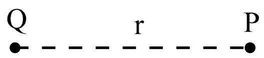
$$\therefore \; E = \lim_{q_0 \to 0}\left(\frac{F}{q_0}\right)$$

Direction of electric field depends on the sign of source charge. If source charge is (+), direction of $\vec{E}$ at a point is directed away from source charge and if source charge is negative, it is directed towards the source charge along the line joining the point to the centre of the source charge.



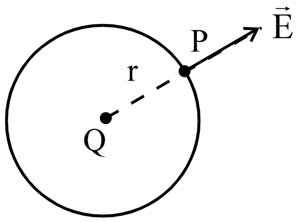
Electric field due to a point charge

Consider a point P at a distance r from a point charge Q.



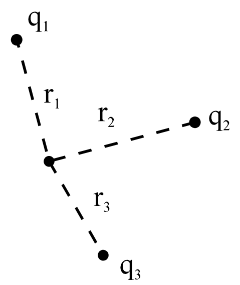
Then field at P, $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$

Consider a sphere of radius r with P on the surface and Q at centre. Then $\vec{E}$ acts radially outwards. If Q is negative, $\vec{E}$ is directed radially inwards. That is why the electric field produced by a point charge is called radial electric field.



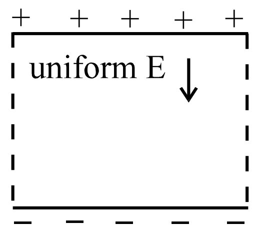
Consider a number of point charges q_1, q_2, q_3...... in a region. P is a point in the region at respective distances r_1, r_2,....... from the charges.

If $\vec{E}_1, \vec{E}_2, \vec{E}_3, ...$ are the electric fields at P due to these charges, then net field at P can be calculated using superposition principle as $\vec{E} = \vec{E}_1 + \vec{E}_2 + ...$



Uniform Electric field

It is an electric field where both value and direction of field is same at every point. To produce it, we need to take two identical metallic plates and place them close and parallel to each other. Give a positive charge to one plate and equal negative to the other. Then in the region between the plates field will be uniform, directed from positive to negative plate.

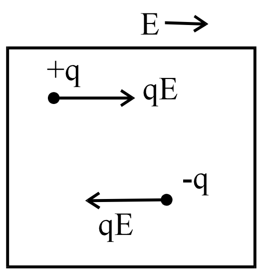


If field varies by value or direction or both from point to point, it is a non uniform $\vec{E}$ field.

Behaviour of a charge placed in a uniform $\vec{E}$

Consider a charged particle of mass m and charge q placed at a point in a uniform $\vec{E}$.

When field is switched ON, at once charge experience a force F = qE. If charge is positive, direction of force is same as the direction of field and if charge is negative, direction of force is opposite to the direction of field. Due to this force, charge starts moving with an acceleration a.



F = ma

$$\therefore \; ma = qE \Rightarrow \boxed{a = \frac{qE}{m}} \text{(uniform)}$$

So after time t ;

Velocity V = u + at

$$\Rightarrow \; V = \frac{qEt}{m}$$

displacement, $S = ut + \frac{1}{2}at^2 \;\; \Rightarrow \; S = \frac{qEt^2}{2m}$

KE gained = $\frac{1}{2}mV^2 = \frac{q^2E^2t^2}{2m} = qES$

Here initial KE is zero for particle. But after a time t, it gains a KE. So KE of a charged particle changes in an electric field.

Then using work-energy theorem we can conclude that, the electric field can do work on a charge given by ;

$$W = \vec{F}.\vec{S} \qquad\qquad \vec{F} = q\vec{E}$$

$$\boxed{W = q\left(\vec{E}.\vec{S}\right)} \qquad\qquad \vec{S} \rightarrow \text{displacement}$$

So using work-energy theorem,

work done = change in KE

$$\boxed{q\left(\vec{E}.\vec{S}\right) = \frac{1}{2}m\left(V^2 - u^2\right)}$$

u $\rightarrow$ initial speed

V $\rightarrow$ final speed

Power delivered by $\vec{E}$ to change at any instant

$$P = \vec{F}.\vec{V}$$

$$\boxed{P = q\left(\vec{E}.\vec{V}\right)}$$

Hence an $\vec{E}$ can do work and deliver power to a charge. It can change the speed and also KE of charge. But magnetic field cannot do any of these things on a charged particle. Both electric field and magnetic field can deflect a charge.

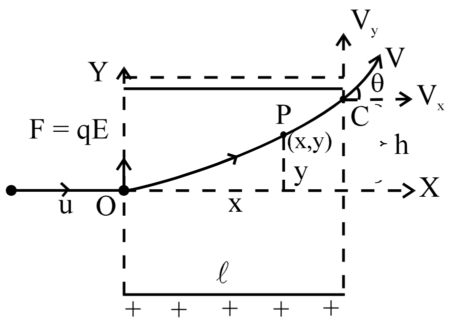
Charge Entering a uniform $\vec{E}$

Consider a charged particle of charge q and mass m entering a uniform electric field E with a speed u

(ii) Antiparallel Entry

Here the above cases are just reversed. That is when a (+) charge enter antiparallel to the field, work done by field on the charge is either negative or zero. When a (-) charge enter antiparallel to the field, work done by field on the charge is always positive. During parallel and antiparallel entry, charge follows straight line paths.

(iii) Perpendicular Entry



Due to F, charge deflects into a curved path as shown. Let it reaches a point P after time t.

X - direction

$U_x = U, a_x = 0$

$S_x = x$

$S_x = U_x t + \frac{1}{2} a_x t^2$

$x = Ut$

$t = \frac{x}{U} \rightarrow (1)$

Y- direction

$U_y = 0,\ a_y = \frac{qE}{m},\ S_y = y$

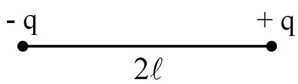
$S_y = U_y t + \frac{1}{2} a_y t^2$

$y = 0 + \frac{1}{2} \frac{qE}{m} \frac{x^2}{U^2}$

$\boxed{y = \frac{qEx^2}{2mU^2}}$ $\quad y \propto x^2$

Electric Dipole

A pair of equal and opposite point charges separated by a short vector distance is called an electric dipole.



Vector separation is known as dipole length. Its direction is from - q to + q.

A dipole is characterised by its electric dipole moment $(\vec{P})$. It is the product of magnitude of one of the charges and dipole length

$$\vec{P} = q \times 2\vec{\ell} \qquad \text{Unit : (cm or Debye (D))}$$

It is a vector whose direction is from - q to + q.

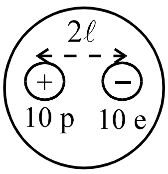
Dipoles are usually found in the molecular world. Consider an HCl molecule. An electron pair is shared between them. Since Cl atom is more electronegative, the shared electron pair will be more shifted towards Cl atom. So Cl atom gets a partial negative charge and H atom an equal partial positive charge.

$$\overset{\delta^+}{\mathrm{H}}\ :\overset{\delta^-}{\mathrm{Cl}}$$

So it behaves like a molecule of permanent dipole moment. Such molecules with permanent electric dipole moment are known as polar molecules.

eg : HCl, HBr, HF, H_2O, NH_3 etc.

If we consider a water molecule, the ten protons in it generates a (+) charge centre and ten electrons a (-) charge centre. At normal state, these charge centres lie at a small separation, giving a net dipole moment to the water molecule.



P = Charge of 10 protons × 2 ℓ

But there are certain molecules which do not posess permanent electric dipole moment. Such molecules are called non polar molecules.

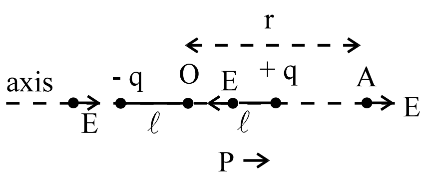
eg : CO_2, BeF_2, CH_4 etc.

If we consider a non polar molecule, the (+) and (-) charge centres coincide so that dipole length is zero. So dipole moment is zero. But if we apply a strong $\vec{E}$ to such a molecule, (+) and (-) charge centres tend to seperate and hence they gain a electric dipole moment. Such a dipole is called an induced dipole.

Electric field due to a dipole

Consider a dipole of charge q, dipole length 2ℓ and dipole moment p.

(i) At an axial point



Field at axial point A; $E_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{\left(r^2-\ell^2\right)^2}$

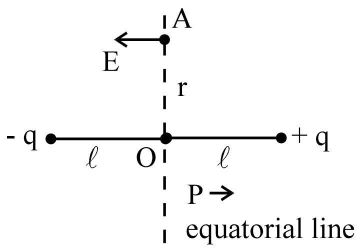
For a short dipole $\ell << r.$ Then

$$\boxed{E_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

Direction of electric field at an axial point is same as the direction of dipole moment. But if the point lies between the charges, direction of electric field will be opposite to that of dipole moment.

(ii) <u>At an equatorial point</u>

An equatorial line is a perpendicular bisector to the dipole.



At equatorial point A;

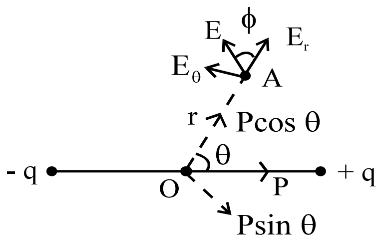
$$E_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{P}{\left(r^2+\ell^2\right)^{3/2}}$$

For a short dipole; $\ell << r.$ Then

$$\boxed{E_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}}$$

Direction of electric field at an equatorial point is opposite to the direction of dipole moment

(iii) <u>At any point surrounding dipole</u>



For $\vec{p}\cos\theta$ component, A is an axial point and for $\vec{p}\sin\theta$ component, A is an equatorial point. Let they produce fields E_r (radial component) and E_θ (angular component) at point A respectively.

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{2P\cos\theta}{r^3}; \quad E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{P\sin\theta}{r^3}$$

net field at A

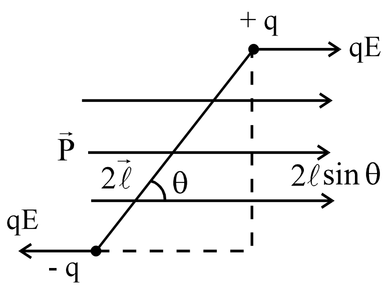
$$E = \sqrt{E_r^2 + E_\theta^2} \quad \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}\sqrt{3\cos^2\theta + 1}}$$

Let, net field at $A\left(\vec{E}\right)$ makes an angle θ with $\vec{r}$. Then $E_r = E\cos\phi$ and $E_\theta = E\sin\phi$.

$$\therefore \frac{E_\theta}{E_r} = \tan\phi \quad \Rightarrow \frac{\left(\frac{1}{4\pi\varepsilon_0}\frac{P\sin\theta}{r^3}\right)}{\left(\frac{1}{4\pi\varepsilon_0}\frac{2P\cos\theta}{r^3}\right)} = \frac{\tan\theta}{2}$$

$$\Rightarrow \quad \boxed{\tan\phi = \frac{1}{2}\tan\theta}$$

<u>Dipole placed in a uniform</u> $\vec{E}$



Since equal and opposite forces are acting on the dipole, net force on the dipole is zero. But these equal and opposite forces constitute a couple and produce a torque.

$\tau = qE \times$ perpendicular distance between forces

$$= qE \times 2\ell\sin\theta = \left(q \times 2\ell\right)E\sin\theta$$

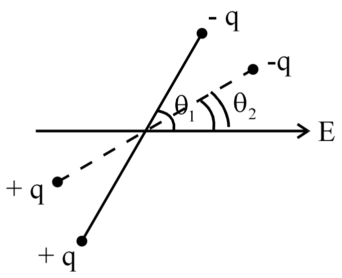
$$\boxed{\tau = PE\sin\theta} \quad \text{or } \vec{\tau} = \vec{P}\times\vec{E}$$

If dipole is perpendicular to field $\left(\theta = 90^0\right)$, torque is maximum. $\tau_{max} = PE$

If dipole is either parallel or perpendicular to field, torque will be zero.

Work done to rotate a dipole

When a dipole is placed at an angle θ to a uniform electric field, it experience a torque. Work done by an external agent to rotate the dipole through a small angle $d\theta$ against this torque



$dW = \tau d\theta = PE\sin\theta d\theta$

Then total workdone to rotate the dipole from an initial angle θ_1 to final angle θ_2 with the field

$$W = \int dW = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta$$

$= -PE(\cos\theta)\theta_1$

$$\boxed{W = PE(\cos\theta_1 - \cos\theta_2)}$$

Since $\vec{E}$ is conservative this work done appears as change in potential energy stored in the dipole.

$\Delta U = PE(\cos\theta_1 - \cos\theta_2)$

To find the absolute potential energy of a dipole at a particular position with the field, let us assume that, potential energy is zero when dipole is placed perpendicular to field. From there, let dipole is rotated to a position where it makes an angle θ with the field.

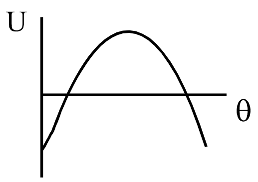
$U_2 - U_1 = PE(\cos\theta_1 - \cos\theta_2)$

When $\theta_1 = 90^0,\ U_1 = 0,\ \theta_2 = \theta,\ U_2 = U$

$U - 0 = PE(\cos 90 - \cos\theta)$

$\boxed{U = -PE\cos\theta}$ Or $U = -\vec{P}\cdot\vec{E}$

So potential energy - θ graph is a cosine curve



Equilibrium of a dipole in a uniform field

Consider different positions of a dipole rotating in a uniform electric field

(i) When dipole is parallel to field ($\theta = 0^0$)

$\tau = PE\sin 0 = 0$, U = - PEcos0 = - PE(minimum)

So here dipole is in equilibrium with least potential energy and hence is in stable equilibrium.

(ii) When dipole is perpendicular to field ($\theta = 90^0$)

$\tau = PE\sin 90$ = PE(maximum), U = - PEcos90 = 0

Here dipole is not in equilibrium

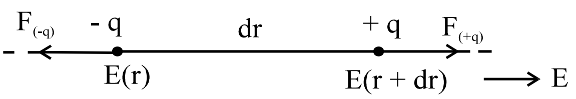
(iii) When dipole is antiparallel to field ($\theta = 180^0$)

$\tau = PE\sin 80 = 0$, U = - PEcos180 = + PE (maximum)

So here also dipole is in equilibrium but with maximum potential energy and hence is in unstable equilibrium

So in a uniform electric field, dipole is always in translational equilibrium. In two positions, dipole is in rotational equilibrium also. So when a dipole rotates in a uniform electric field, there are two positions of equilibrium, one is stable and the other is unstable.

Force on a dipole in a non uniform field



Consider a dipole of charge q and dipole length dr placed in a non uniform field as shown. Let E_r and E(r + dr) are the electric fields at positions of - q and + q. So forces are ;

F - q = qE_r; $\quad$ F_{+q} = q E(r + dr)

net force; $\quad$ F = F_{-q} - F_{+q} = q[E_r - E(r + dr)]

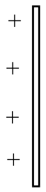
F = q $\quad$ $dE \times \frac{dr}{dr} = q \times dr\left(\frac{dE}{dr}\right)$ $\quad$ q × dr = P

$$\boxed{F = P\left(\frac{dE}{dr}\right)}$$

Charged bodies with continuous charge distribution

Coulomb’s law can be used only to find electric field due to point charges. But there are bodies with larger size like sheet, ring, sphere etc. over which charges are continuously distributed. To find electric field due to such bodies, we cannot use coulomb’s law. For such bodies we can define three type of charge densities.

(i) Linear charge density (λ)



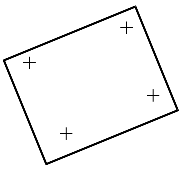
Let a charge Q is distributed along a line of length L. Then its charge per unit length is the linear charge densities.

$$\lambda = \frac{Q}{L} \text{ or } \lambda = \frac{dq}{d\ell}$$

For bodies like thin conductor, circular arc, ring etc we define λ

(ii) Surface charge density (σ)

Let a charge Q is distributed on the surface of body having area A. Then its charge per unit area is called surface charge density (σ)

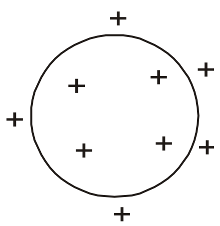


$$\sigma = \frac{Q}{A}$$

For bodies like thin sheet, thin disc etc, we define σ

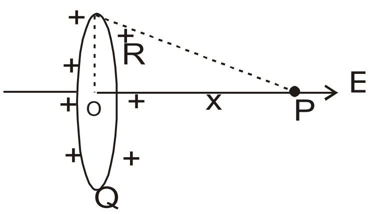
(iii) Volume charge density (ρ)

Let a charge Q is distributed over the entire volume of a body V. Then its charge per unit volume is called volume charge density (ρ)



$$\rho = Q/V$$

IV. Field due to charged circular ring



At axial point (P)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2 + x^2\right)^{3/2}}$$

At centre (O)

X = 0 $\boxed{E = 0}$

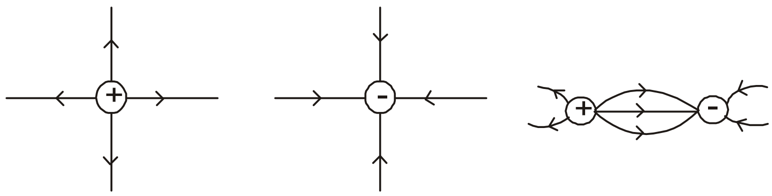
Electric field lines

If we place a (+) charge at a point, large number of invisible lines are sprayed out of it with the help of these lines charge produces an electric field in its surrounding. If we draw a tangent at any point on this line it gives the direction of $\vec{E}$ at that point.

An electric field line is a line or a curve surrounding a charge such that tangent drawn at any point gives the direction of electric field at that point.

Properties

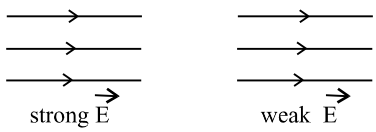
(i) Field lines start from a(+) charge and terminate on (-) charge.



(ii) Field lines are continuous but they never form closed loops

(iii) Number of field lines surrounding a charge is directly proportional to the magnitude of charge

(iv) In a strong field, lines are closely spaced and in a weak field, they are far separated



If d is the separation between field lines, then $E \propto \frac{1}{d}$

(v) In a uniform $\vec{E}$, lines are parallel and equally spaced

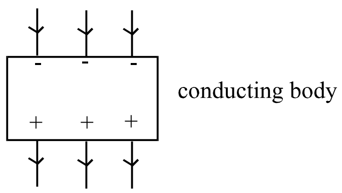
(vi) Two field lines never intersect

(vii) Lines between two unlike charges contract longitudinal causing an attraction

(viii) Lines from two like charges exert lateral pressure on one another causing a repulsion

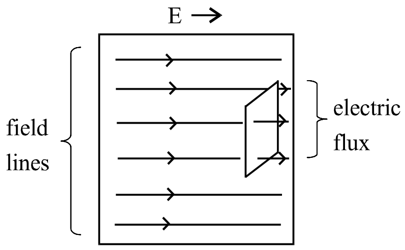
(ix) Electric field lines always start or terminate perpendicular to the surface of a conducting body

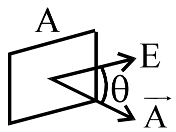
(x) Field lines never penetrate into the inside regions of a conducting body



Electric flux (ϕ)

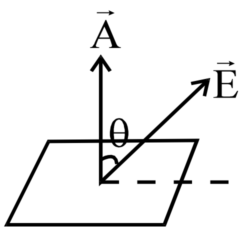
Flux is generally an amount of a vector field passing normally through a surface. Consider a plane surface of area A placed in a uniform field $\vec{E}$. Then electric flux of that surface is the total number of field lines passing normally through that surface





$$\boxed{\begin{array}{l}\phi = EA\cos\theta \\ \phi = \vec{E}.\vec{A}\end{array}}$$

For calculating flux, area is taken as a vector, whose direction is perpendicular to the plane of surface outwards. Here θ is the angle between $\vec{E}$ and $\vec{A}$



$\theta = 90^0$ - angle between $\vec{E}$ and plane of surface.

Case 1 : If plane of surface is perpendicular to field

$$\theta = 90 - 90 = 0^0$$

$$\phi = EA\cos 0 = EA \text{ (maximum)}$$

Case 2 : If plane of surface is parallel to field

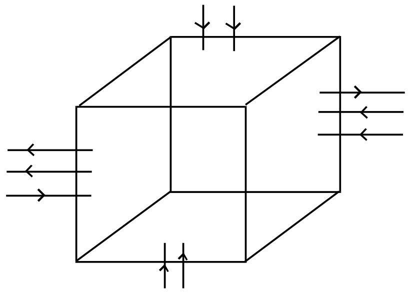
$$\theta = 90 - 0 = 90^0$$

$$\phi = EA\cos 90 = 0 \text{ (minimum)}$$

Electric flux for a volume enclosing surface

For a two dimensional planar surface, flux is the total number of field lines passing through it. But for a three dimensional volume enclosing surface, flux can be defined in three ways.

(i) flux entering (ii) flux leaving (iii) net flux

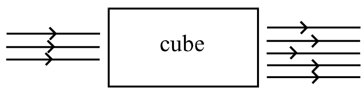


consider a cubical surface in an electric field. Lines can enter and leave the cube through all six faces. Flux entering and flux leaving gives the total number of lines entering and leaving the surface. Then net flux is; $\phi_{net} = \phi_{leaving} + \phi_{entering}$

$\phi_{leaving} = (+), \phi_{entering} = -$

$\therefore \boxed{\phi_{net} = \phi_{leaving} - \phi_{entering}}$

Case 1 :

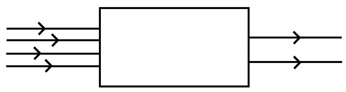


Here $\phi_{leaving} > \phi_{entering}$

$\therefore \phi_{net} = (+)$

This indicates that a net positive charge is present inside the cube.

Case 2 :



Here $\phi_{leaving} < \phi_{entering}$

$\therefore \phi_{net} = (-)$

This indicates that a net negative charge is present inside the cube.

Case 3 :

Here $\phi_{leaving} = \phi_{entering}$

$\therefore \phi_{net} = 0$

So net charge inside is zero. If a volume enclosing surface is placed in a uniform $\vec{E}$, then $\phi_{net} = 0$ for it.

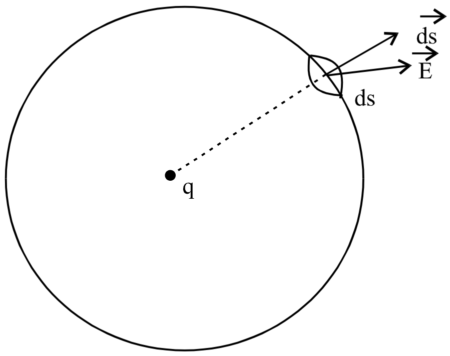
Gauss's law

Consider a charge (q) placed at a point. Let it is placed inside a closed spherical surface. Consider a small elementary area ds on its surface. If $\vec{E}$ is the electric field over the element, then flux through the element

$d\phi = \vec{E}.\vec{ds}$

Net outward flux through the surface

$\phi = \int_S \vec{E}.\vec{ds}$

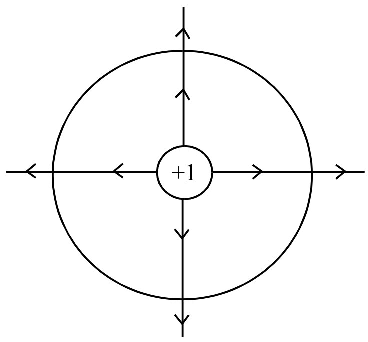


According to Gauss, the net outward flux from a charge enclosing surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\boxed{\phi = \frac{1}{\varepsilon_0} q_{net}}$$

$$\boxed{\oint \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0} q_{net}}$$

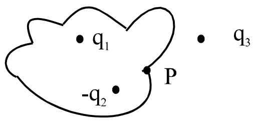
To find the number of field lines coming from a (+ 1C) charge, we can use this law. For that let us first enclose this (+ 1C) inside a closed surface.



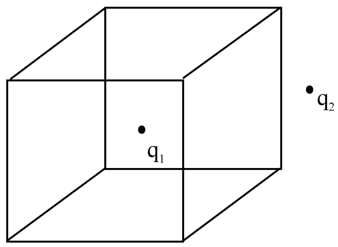
Then it is equal to net outward flux from that closed surface.

$$\phi = \frac{1}{\varepsilon_0} q_{net} = \frac{1}{8.85 \times 10^{-12}} \times 1 = 1.13 \times 10^{11}$$

Case 1 : Flux of a closed surface depends only on the charges enclosed. But if we consider any point on the surface, the electric field will be due to all the charges in the region. Flux of this surface depends only on q_1 and $-q_2$. But if we find field at P, it is due to all the charges q_1, $-q_2$ and q_3.

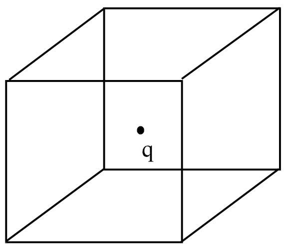


Case 2 :



Here total flux of cube is due to q_1 only. But if we consider flux of right face, it depends on both q_1 and q_2

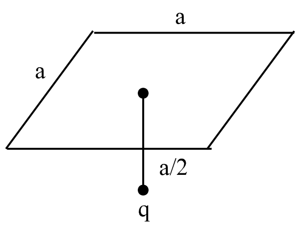
Case 3 : Let a point charge is placed at the centre of a cube.



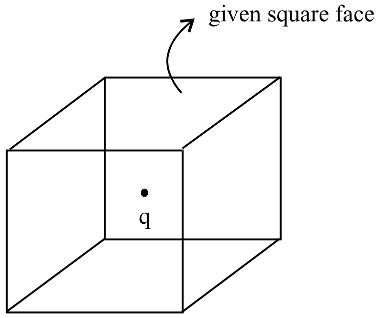
Total flux of cube = $\frac{1}{\varepsilon_0}q$. Since q is at centre, this flux will be equally shared by all 6 faces. $\therefore$ flux through one face = $\frac{q}{6\varepsilon_0}$

Case 4 : Let a dipole is placed inside a closed surface. Since $q_{net} = 0,\ \phi_{net} = 0$

Case 5 : Let a point charge is placed at a distance of a/2 below the centre of a square face of side a.



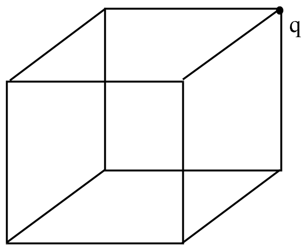
Here we cannot directly apply Gauss's law because q is not enclosed inside a closed surface. So let us consider five other similar faces to complete a cube such that q will be at the centre of cube. Then out of the total flux q/ε_0 through entire face, $\frac{1}{6}$ will come through the given square face.



Flux through given square face $= \dfrac{q}{6\varepsilon_0}$

Case 6 : Let a point charge q is placed at one corner of a cube. Consider 7 other similar cubes sharing that corner. Then q will be at the centre of a closed surface formed by 8 cubes. So the net charge enclosed by one cube q_{net} = q/8

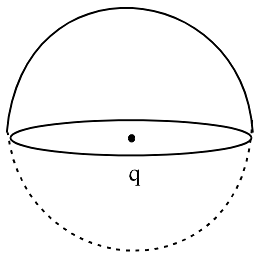
$\therefore$ Net flux from cube = $\dfrac{1}{\varepsilon_0}\left(\dfrac{q}{8}\right) = \dfrac{q}{8\varepsilon_0}$



If we consider the 3 faces which contain the corner in which charge is placed, no flux will be associated. So the total flux will be equally shared by 3 other faces.

$\therefore \phi_{\text{each face}} = 0$ OR $\phi_{\text{each face}} = \dfrac{1}{3}\left(\dfrac{q}{8\varepsilon_0}\right) = \dfrac{q}{24\varepsilon_0}$

Case 7 : Let a point charge q is placed at the centre of the base of a hemisphere.



Imagine a similar hemisphere to complete a sphere with q at centre. Then out of total flux q/ε_0 through entire sphere, half $(q/2\varepsilon_0)$ will come through given hemisphere.

Applications of Gauss's theorem

Coulomb's theorem can be used to find the electric field due to point charges only. To find the field produced by charged bodies of larger size, we use Gauss's theorem. This theorem is applied in two steps.

Step 1 : Imagine a closed surface called Gaussian surface enclosing the large. For this we can use the points

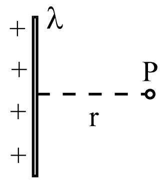
(i) The point at which field is to be calculated must appear on the surface of the Gaussian surface

(ii) It should not pas through discrete charges but can pass through continuous charges

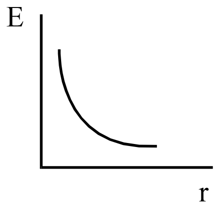
(iii) It is better to use symmetric Gaussian surface. It is a surface over which field value is the same at every point

Step 2 : Apply the equation $\oint \vec{E}.\overrightarrow{ds} = \frac{1}{\varepsilon_0}$ q_{net} over the surface and find E

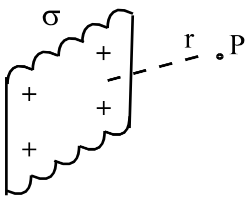
I. Field due to infinite line of charge



Field at P; $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$ $E \propto \frac{1}{r}$

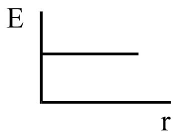


II. Field due to infinite thin conducting sheet



field at P $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

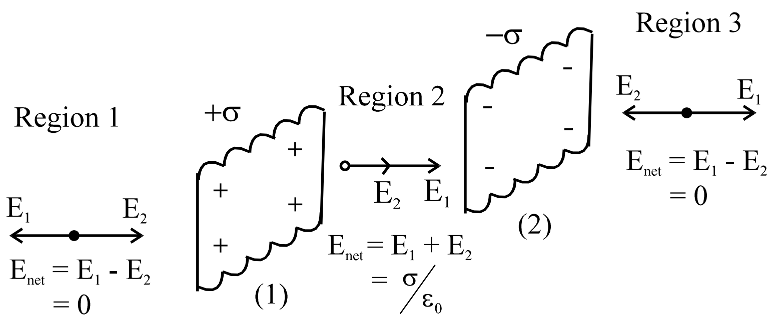
E is independent of r



For a thick sheet; $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

For a non conducting sheet; $E = \frac{\sigma}{2\varepsilon_0}$

PRODUCTION OF UNIFORM $\vec{E}$



Consider two infinite thin conducting sheet placed parallel and are given with equal and opposite surface charge densities. Field due to the sheet are, $E_1 = E_2 = \dfrac{\sigma}{2\varepsilon_0}$

So in the region between the plates, field is uniform with a value $E = \dfrac{\sigma}{\varepsilon_0}$ and direction from the positive plate to negative plate. Beyond the plates field is zero.

Charge distribution over conducting bodies

A conducting body contains a large number of free electrons inside. So if a charge is given to any inside point of a conducting body, free electrons will take these charges to its outer surface.

(i) Charges do not stay inside a conducting body

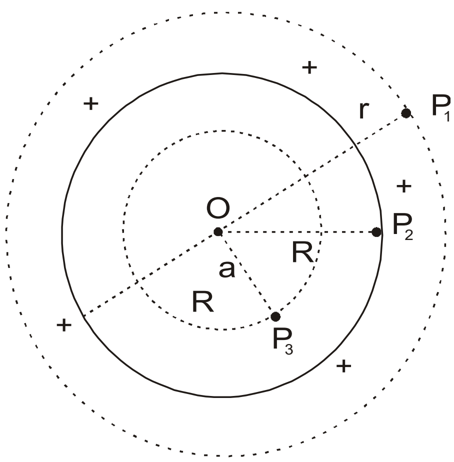
(ii) If a charge is given to a conducting body, these charges will be distributed only over its outer surface

(iii) For an isolated regular conducting sphere, such a charge will be uniformly distributed over its outer surface. But if surface is irregular or any surrounding charges are present, distribution will become nonuniform

III. Electric field due to conducting sphere/hollow sphere

For a spherical charge distribution, Gaussian surface is a concentric sphere with the point where the field is to be calculated on its surface.

Let a charge Q is given to an isolated conducting sphere of radius R. It will be uniformly distributed only over its outer surface. So we can define only surface charge density (σ) and not volume charge density (ρ).



$$\sigma = \frac{Q}{4\pi R^2}$$

To find the field at a point, imagine a Gaussian sphere through that point. Then only the charges enclosed by that sphere will produce field at that point.

Outside point (P$_1$)

$$\boxed{E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

$$E_{out} \propto 1/r^2$$

Surface point (P$_2$)

$r = R$

$$\boxed{E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$$

$$Q = 4\pi R^2\sigma$$

$$\boxed{E_{surface} = \frac{\sigma}{\varepsilon_0}}$$

Inside point (P$_3$)

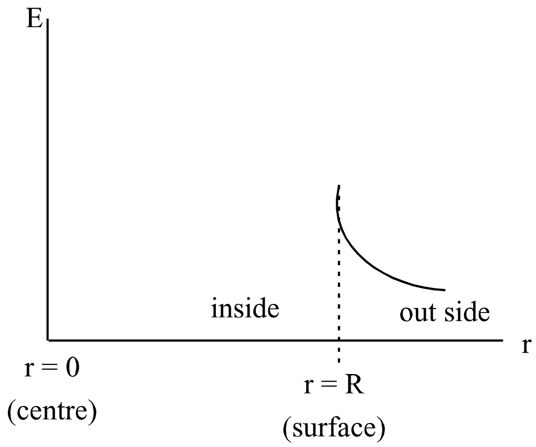
Gaussian sphere through P$_3$ encloses no charge. So

$$\boxed{E_{in} = 0}$$

Centre (0)

$E = 0$

Not only for a conducting sphere, for every conducting body, net electric field at an inside point is always zero



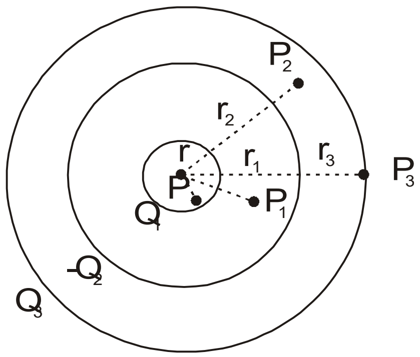
Field due to concentric thin conducting spheres

To find field at a point, imagine a concentric Gaussian sphere through that point only the charges enclosed by that sphere constitute field at that point and is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{net}}{R^2}$$

$Q_{net} \rightarrow$ net charge enclosed by the gaussian sphere

$R \rightarrow$ distance to that point from centre



Point P

Gaussian sphere through P encloses no charge

$$\therefore Q_{net} = 0$$

$$\therefore E_P = 0$$

Point P_1

Gaussian sphere through P_1 enclose a charge Q_1

$$\therefore \; Q_{net} = Q_1, \; R = r_1$$

$$E_{P_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2}$$

Point P_2

Gaussian sphere through P_2 encloses charges Q_1 and - Q_2

$$\therefore \; Q_{net} = Q_1 - Q_2, \; R = r_2$$

$$E_{P_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 - Q_2}{r_2^2}\right)$$

Point P_3

Gaussian sphere through P_3 encloses all charges

$$\therefore \; Q_{net} = Q_1 - Q_2 + Q_3$$

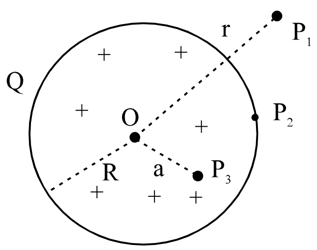
$$R = r_3$$

$$E_{P_3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 - Q_2 + Q_3}{r_3^2}\right)$$

IV Field due to nonconducting sphere of uniform density

A nonconducting body lacks free electrons. So if a charge is given to any inside point of such a body, that charge will bound to that point. Let a charge Q is uniformly distributed over the entire volume (V). Then its volume charge density

$$P = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$



Outside point (P_1)

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

$$E_{out} \propto 1/r^2$$

Surface point (P_2)

$$\boxed{E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$$

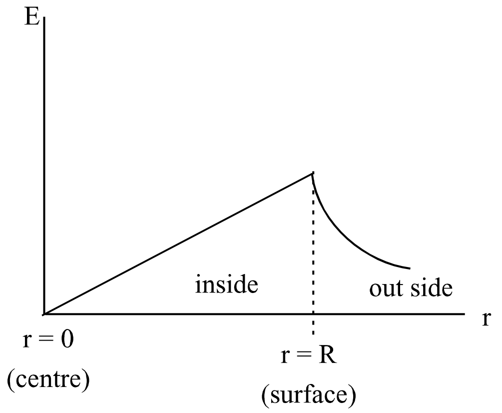
Inside point (P_3)

$$\boxed{E_{in} = \frac{\rho a}{3\varepsilon_0}}$$

$$Ein \propto a$$

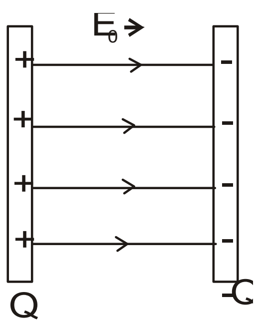
Centre (0)

E = 0

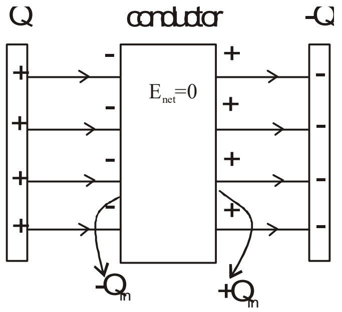


Electrostatic shielding

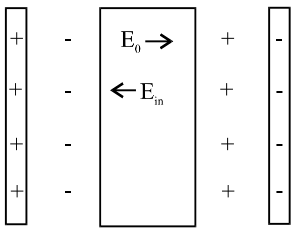
Consider a uniform electric field (E_0) applied between two identical metallic plates. If we consider the field lines between the plates, all the field lines starting from the (+) plate will terminate all the (-) plate.



Now a conducting sheet $(K = \infty)$ is inserted in the region between the plates. Then induction happens and induced charges are accumulated as shown. Then all the electric field lines starting from the (+) plate will terminate at the (-) ve induced charge. But an equal number of field lines start from the (+) induced charge on the other side of the conductor and terminate on the (-) plate. So no field lines penetrate into the inside regions of the conducting sheet. So net electric field at the inside region of the conducting sheet is zero. Hence we can conclude that the inside regions of the conducting sheet is free from the electric effects outside. This is called electrostatic shielding and such a conducting cavity is called a Faraday's cage.



In actual practice E_0 will surely enter the region inside the conductor. Then an another electric field is developed inside the conductor due to the induced charges accumulated at the surface.



Direction of this field $\left(\vec{E}_{in}\right)$ is opposite to the direction of $\vec{E}_0$ so that net field in the region,

$E_{net} = E_0 - E_{in}$, where $E_{in} = E_0\left(1-\frac{1}{K}\right)$

for a conductor, $K = \infty$ $\quad\therefore E_{in} = E_0 \;\; \therefore E_{net} = E_0 - E_0 = 0$

Hence E_{in} cancels E_0 before it make any effects.

ELECTROSTATICS - 2
ELECTRIC POTENTIAL AND CAPACITANCE

ELECTRIC POTENTIAL (V)

Just like electric field intensity ($\vec{E}$), electric potential is a scalar physical quantity which is used to characterise an electric field. It referes to an ability to do work to move a charge in an electric field. It has many roles in electricity.

(i) The concept of potential helps us to find the work to be done to move a charge in an electric field.

(ii) The direction of charge flow between two connected bodies is determined by their potential values. Positive charge always flow from higher to lower and negative charge from lower to higher potential and this charge flow will continue untill both the bodies attain a common potential (Electrostatic Equilibrium).

(iii) If two connected bodies are in electrostatic equilibrium, their potentials will be equal.

(iv) Electric potential can be used to accelerate or decelerate charges.

Electric Potential at a Point

Consider a point (P) in an electric field. To find the potential at this point, place a test charge q at infinity and then an external agent has to bring this charge from infinity to that point P, slowly. If W is the work done for this, then potential at that point; then $\boxed{V = \frac{W}{q}}$ If q = IC, then V = W.

Hence electric potential at a point in an electric field is defined as the work done to bring a unit positive charge from infinity to that point without an acceleration.

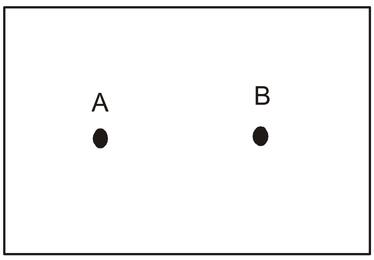
Unit : Volt (SI) or esu of potential (cgs)

1esu = 300V

$[V] = [ML^2T^{-3}A^{-1}]$

Electric Potential difference between the points

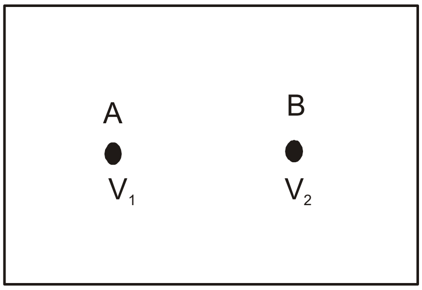
Consider two points in an electric field A and B. Let a charge q is moved from point A to point B without an acceleration. If W is the work done for this process, then potential difference between these points;



$\boxed{dV = \frac{W}{q}}$ dV = V_{final} - $V_{initial}$ [here dV = V_B - V_A]

If q = IC, dV = W. Hence potential difference between two points can be defined as the work done to move a unit positive charge between the two points without an acceleration.

Work done to move a charged particle in an E field



Consider two points in an electric field A and B where potentials are V_1 and V_2. Let a charged particle of charge q is moved from point A to point B very slowly without an acceleration. Then work done by the external agent for this motion;

W = charge to be moved × p.d. between the two points here

$V_{initial} = V_1$

$V_{final} = V_2$

$$\boxed{W = q\left(V_{final} - V_{initial}\right)}$$

Since this work is done in a conservative electric field, this is stored as change in potential energy.

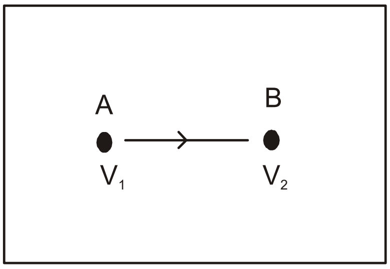
$$\boxed{\Delta U = q\left(V_{final} - V_{initial}\right)}$$

Using the idea, work done by conservative field = - change in PE

$\therefore$ Work done by electric field = - Δ PE

$$\boxed{W E.field = -q\left(V_{final} - V_{initial}\right)}$$

Work-energy theorem applied in an E field while a charge is moved



Let a charged particle of charge q and mass m is moved by an external agent from point A to point B in an electric field. Then both external agent and electric field performs work on charge. So total work done on charge

$W = W_{ext} + W_{E\text{-field}}$

Using work - energy theorem; W = change in KE

$$W_{ext} + W_{E.field} = \Delta KE$$

but $W_{E.field} = -\Delta PE$

$$W_{ext} - \Delta PE = \Delta KE \quad +W_{ext} = \Delta PE + \Delta KE$$

but $\Delta PE = q\left(V_{final} - V_{initial}\right)$

$$\boxed{W_{ext} = q\left(V_{final} - V_{initial}\right) + \Delta KE}$$

When charged particle is moved from one point to another, we can consider two situations.

(i) If charge is moved without an acceleration speed does not change. $\Delta KE = 0$

$$W_{ext} = q\left(V_{final} - V_{initial}\right)$$

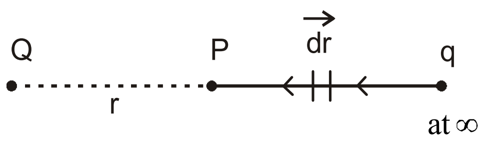
(ii) If charge is moved with an acceleration speed changes and $\Delta KE \neq 0$. Then

$$W_{ext} = q\left(V_{final} - V_{initial}\right) + \Delta KE$$

<u>Relation between electric field (E) and electric potentials (V)</u>

Consider a source charge Q producing an electric field in the surrounding. P is a point at a distance r from Q.

Let a charged particle



q is moved by an external agent from infinity to point P without an acceleration. Consider a small element $\overrightarrow{dr}$ in the path where the electric field is $\vec{E}$. Small work done to move the particle across the element

$$dW = -\vec{F}.\overrightarrow{dr} \qquad F = q\vec{E}$$

$$dW = -q\left(\vec{E}.\overrightarrow{dr}\right) \Rightarrow \frac{dW}{q} = -\vec{E}.\overrightarrow{dr}$$

But $\frac{dW}{q} = dV$, the p.d. across the element

$$\boxed{dV = -\vec{E}.\overrightarrow{dr}}$$

between two points of position vectors r_1 and r_2

$$\boxed{dV = -\int_{r_1}^{r_2} \vec{E}.\overrightarrow{dr}}$$

Let initial point is at infinity where potential is zero. Let potential is V at point, P

i.e., $r_1 = \infty, V_1 = 0; r_2 = r, V_2 = V$

$$V_2 - V_1 = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$$

$$V = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$$

So potential difference between two points is the negative line integral of electric field. Practically, let

$$\vec{E} = Ex\,\hat{i} + Ey\,\hat{j} + Ez\,\hat{k}$$
$$\overrightarrow{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\vec{E}.\overrightarrow{dr} = Exdx + Eydy + Ezdz$$

Let $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}; \quad \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\therefore \boxed{V_2 - V_1 = -\left[\int_{x_1}^{x_2} Ex\,dx + \int_{y_1}^{y_2} Ey\,dy + \int_{z_1}^{z_2} Ez\,dz\right]}$$

Now electric field can be written as

$$\boxed{\vec{E} = -\frac{dV}{dr}}$$

Hence electric field is the negative gradient of potential.

In three dimensions, gradient is a vector differential operator given by $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

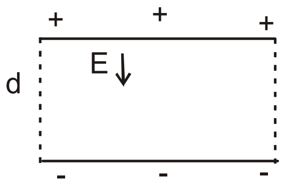
$\therefore E = -\nabla V$ or

$$\boxed{\therefore E = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]}$$

Negative sign in the relation between $\vec{E}$ and $\vec{V}$ shows that, the direction of electric field is same as the direction in which potential is reducing.

Consider a uniform electric field E between two parallel plates, separated by a distance d. Then potential difference between the plates

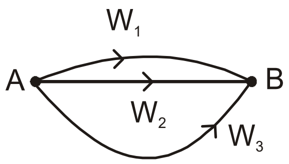
$$\boxed{V = Ed}$$



Conservative nature of electrostatic field

Electrostatic field, since conservative, posses the following properties.

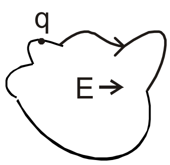
(i) Work done to move a charge between two points in an electrostatic field depend on initial and final positions only and is independent of path through which it is moved.



Let a charge q is moved from point A to B through three different paths as shown. Then work done

$W_1 = W_2 = W_3$

(ii) Work done to move a charge once around closed loop in an electrostatic field is zero.



$W = q\,dV = 0$

(iii) Potential difference over a closed loop in an electrostatic field is zero.

dV = 0 but $dV = -\oint \vec{E}.\overrightarrow{dr}$

$\therefore \oint \vec{E}.\overrightarrow{dr} = 0$

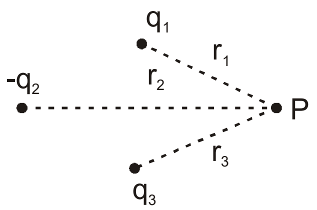
So the line integral of electrostatic field over a closed loop is zero, which indicates the conservative nature of electrostatic field.

Potential due to a point charge

Consider a point P at a distance r from point charge Q.

Potential at P; $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$

Since potential is a scalar, sign of charge must be substituted in all the equations in which potential are calculated. Consider a system of charges,

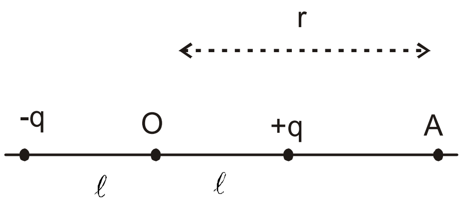


Potential at P; $V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q_2}{r_2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$

Potential due to a dipole

Consider a dipole of charge q, dipole length 2ℓ and moment P

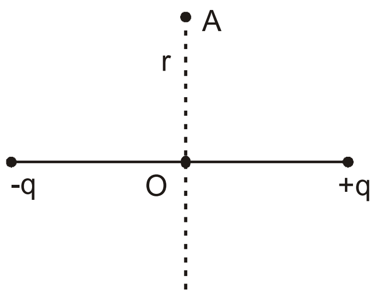
(i) **Axial point**



Potential at A $V = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^2 - \ell^2}$ for a short dipole $\ell^2 << r^2$

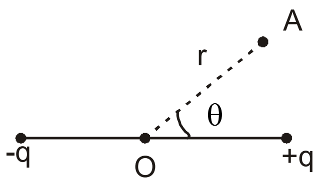
$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}}$$

(ii) **Equatorial Point**



Potential at A; $\boxed{V = 0}$

(iii) **Any general point**

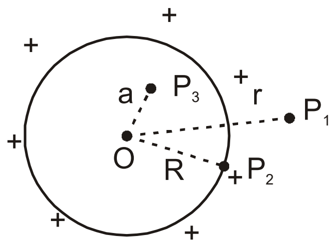


Potential at A $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2 - \ell^2\cos^2\theta}$ for a short dipole $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}$ or

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}.\vec{r}}{r^3}$$

Potential due to a conducting sphere

Consider a conducting sphere of charge Q and radius R. Charge is uniformly distributed over its outer surface. So its surface charge density $\sigma = \frac{Q}{4\pi R^2}$



P_1 (outside point)

$$\boxed{V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}} \quad Q = 4\pi R^2\sigma$$

or $\boxed{V_{out} = \frac{\sigma R^2}{\varepsilon_0 r}}$

P_2 (surface point)

$$\boxed{V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$ or

$$\boxed{V_{surface} = \frac{\sigma R}{\varepsilon_0}}$$

P_3 (inside point)

$$\boxed{V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

or

$$\boxed{V_{in} = \frac{\sigma R}{\varepsilon_0}}$$

O (centre)

$$\boxed{V_{centre} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

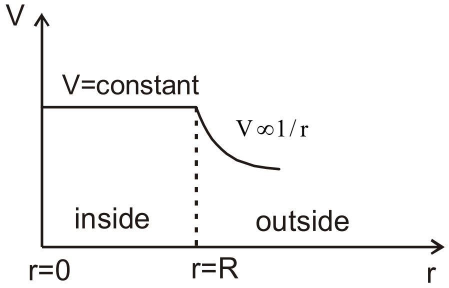
or

$$\boxed{V_{centre} = \frac{\sigma R}{\varepsilon_0}}$$

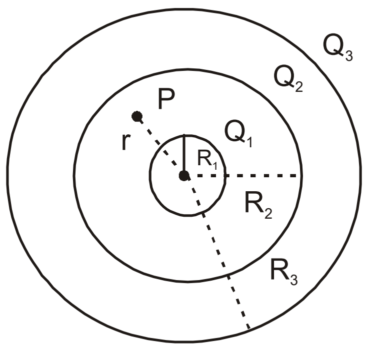
So for a conducting sphere;

$$\boxed{V_{centre} = V_{in} = V_{surface}}$$

This is because, electric field inside a conducting sphere is zero. So to bring IC positive charge from infinite to inside point, work is need to be done up to surface only



Potential due to Concentric thin conducting spheres



When we calculate, potential due to concentric thin conducting spheres, there is no need to consider the charges produced by induction. We will get same answers even if we take or donot take induction. Consider a point P at a distance r from common centre. To find potential at P, find potential due to each sphere at P and take their sum. When we calculate potential due to a sphere, take that sphere only and identify the position of the point P for that sphere, outside, surface, inside or centre. Use the corresponding equation. Remember that, if the point lies inside for the sphere, potential value at the surface of sphere must be taken.

V_P = V due to inner sphere + V due to middle sphere + V due to outer sphere.

$$=\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r}+\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}+\frac{1}{4\pi\varepsilon_0}\frac{Q_3}{R_3}$$

To find the potential of a particular sphere, take a point on its surface and find potential of that point, using the method discussed above. For example let us find potential of middle sphere. For that mark a point on the surface of middle sphere and find potential there.

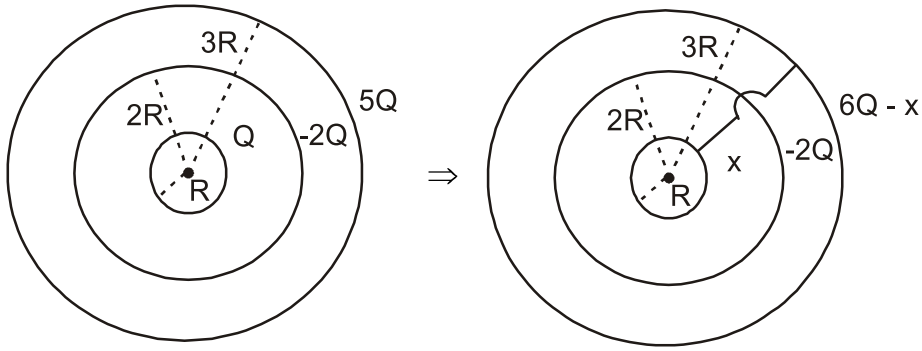
$$V_{\text{middle sphere}}=\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_2}+\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}+\frac{1}{4\pi\varepsilon_0}\frac{Q_3}{R_3}$$

Charge sharing between two connected bodies

When two bodies are connected together, a charge flow happens between them. The direction of charge flow between two bodies is decided by their potential values. Positive charge always flow from body with higher potential to the body with lower potential. But negative charge flow from body with lower potential to the body with higher potential.

Charge flow will stop when both attain common potential. So when two bodies are connected together, a charge flow will happen between them untill both attain a common potential. So if two bodies are found to be connected together, their final potentials can be equated.

eg: Consider three concentric conducting thin spherical shells. Of radius R, 2R and 3R given with respective charges Q, -2Q and 5Q. Now the inner and outer spheres are connected using a wire. Find the final charges appearing on all spheres.



Charges given

Final charges after distribution

When inner and outer spheres are inter connected, a charge flow happen between them and soon the potentials of inner and outer spheres become equal. Let x be the final charge on inner sphere (unknown). Then using conservation of charge between inner and outer spheres, final charge on outer sphere is 6Q-x. No changes will happen on the middle sphere.

Total potential on inner sphere (final) = Total potential on outer sphere (final)

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{x}{R}+\frac{-2Q}{2R}+\frac{6Q-x}{3R}\right]=\frac{1}{4\pi\varepsilon_0}\left[\frac{x}{3R}+\frac{-2Q}{3R}+\frac{6Q-x}{3R}\right]$$

On solving; $x=\frac{Q}{2}$

final charge are;

(i) inner sphere = $x=\frac{Q}{2}$

(ii) middle sphere = -2Q

(iii) outer sphere = $6Q-x=\frac{11Q}{2}$

Charge flown through wire = Q/2, from inner to outer sphere.

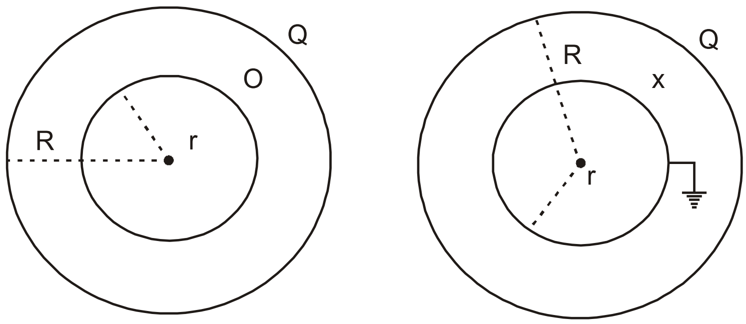
Earthing

In electricity, earth is always postulated to be at zero potential. So if a body is connected to earth, a charge flow will happen between that body and earth untill potential of that body becomes zero. So final potential of a earthed body is always zero.

It is a misconcept that, if a charged body is earthed its entire charge flows to earth. Actually, when a body is earthed, it is not the charge, but it is the potential which becomes zero. Its final charge may or may not become zero. If an isolated charged body is earthed, the entire charge on the body will flow to earth. But non isolated body is earthed, final charge on it will not become zero. When a non isolated body is earthed, only the unwanted charges on it will loss to earth. If we consider parallel conducting plates, charges appearing at the two extreme surfaces of the arrangement are unwanted. If we consider concentric conducting spherical shells, charges appearing at the outer surface of the outer most shell are unwanted. If such a body is earthed, that unwanted charge will flow to earth.

eg: Consider a solid conducting sphere of radius r, surrounded by a hollow conducting sphere of radius R. The outer sphere is given a charge Q and inner sphere is earthed. Find the final charge on inner sphere.

Ans:



Initial charges

final charges after earthing

x $\rightarrow$ charge gained by inner sphere due to earthing.

Since inner sphere is earthed, its final potential is zero (V = 0)

V due to inner sphere + V due to outer sphere = 0

$$\frac{1}{4\pi\varepsilon_0}\frac{x}{r}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}=0 \Rightarrow \frac{x}{r}=\frac{-Q}{R}$$

$$x=-\left(\frac{r}{R}\right)Q$$

Equipotential Surfaces

It is a surface over which potential is the same at every point on the surface. Consider a spherical surface with a point charge at the centre. Then every point on its surface has same potential value. So it is an equipotential surface.

Properties

1. Potential difference between two points is zero.
2. Work done to move a charge between two points is zero
3. Electric field is always perpendicular to every point on the surface of a equipotential surface
4. Tangential component of electric field is zero
5. Field lines are always perpendicular to such a surface
6. Two equipotential surfaces never intersect

7. In a uniform electric field, equipotential surfaces are parallel planes with equal separations and equal potential differences.
8. In a strong electric field, separation between neighbouring equipotential surfaces are very small. But in a weak field, they are far separated.

The most important point, we need to consider about an equipotential surface is that, all conducting surfaces are equipotential. If we consider a conducting body of any shape and size, every point on its surface is at the same potential. So all conducting surfaces obey all the properties of equipotential surfaces. So $\vec{E}$ is always perpendicular to a conducting surface. Field lines are also always perpendicular to conducting surface.

The value of this common potential at every point on a conducting surface is given by $V = \frac{\sigma R}{\varepsilon_0}$

R $\rightarrow$ radius of curvature at that point

$\sigma \rightarrow$ surface charge density of the region surrounding that point

Since V = constant, for a conducting surface

$\frac{\sigma R}{\varepsilon_0} = \text{Constant}$ or σR = Constant or $\boxed{\sigma \propto \frac{1}{R}}$

So at points where radius of curvature is less (sharp points), σ will be high and charges accumulate more. That is why it is said that, charges accumulate more at the sharp edges of a conducting surface. This fact is utilized in the working of instruments like lightning arrester and Van De Graaff generator.

Electric Potential Energy

It is the potential energy developed due to the interaction between charged bodies. Since electrostatic field is conservative, if a work is done by an external agent to move a charged particle slowly in an electric field, that work does not loss but is stored as electrostatic potential energy. If we release this energy, this potential energy converts to kinetic energy and make the charges to move.

Case 1: Potential energy of a single charged particle placed at a point

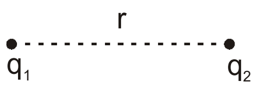
Let a charged particle of charge q is placed at a point. Then its potential energy is defined as the work done to bring that particle from infinity to that point without an acceleration. It is given by U = q V.

So

$$\boxed{\textbf{Potential energy of a charged particle placed at a point} = \textbf{that charge} \times \textbf{Potential of the point where charge is placed}}$$

Case 2: Potential energy of a system of two point charges

Consider two point charges q_1 and q_2 separated by a distance r. Potential energy of this system is defined as the work done to bring these charges from infinite separation to r separation.



$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

For like charges; U is positive and hence force is repulsive. But for unlike charges, U is negative and force is attractive.

Case 3: Potential energy of a system of more than two point charges

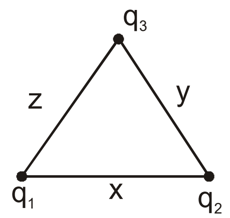
It is also defined as the work done to assemble the system from infinity. It can be calculated using pairing method in 3 steps.

Step 1: Split the given charges into all possible pairs

Step 2: Find PE of each pair using $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

Step 3: To find the PE of total system, take the sum of PE of all pairs. To find the PE of a single charge, take the sum of potential energy of pairs containing that charge only.

eg: Let three point charges are placed at the 3 corners of a triangle as shown.



Step 1: Pairs are (q_1, q_2), (q_1, q_3), (q_2, q_3)

Step 2: PE of each pair are $U_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{x}$

$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{y}$ $\qquad$ $U_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{z}$

Step 3: $U_{total} = U_1 + U_2 + U_3$

PE of $q_1 = U_1 + U_3$

PE of $q_2 = U_1 + U_2$

PE of $q_3 = U_2 + U_3$

PE of q_1 + PE of q_2 + PE of q_3 = $2(U_1 + U_2 + U_3)$

$\therefore$ Total PE of system $= \frac{1}{2}$[sum of PE of individual charges]

Note 1: To find the work done to assemble the system, find PE of system

Note 2: To find the work done to break the system to infinity, find the negative of PE of system.

Note 3: To find the work done to bring a single charge to a system from infinity, find the PE of that charge in the system.

Note 4: To find the work done to escape a charge from a system to infinity, find PE of that charge in the system and take its negative.

Accelerating Potential

Let a charged particle q and mass m is placed at rest. Let a potential V is applied to it. Then work done by potential on it is W = qV. Due to this, the particle starts moving with entire work done converting into KE of the particle.

KE gained by particle = qV

If u is the speed of the particle, then

$$\frac{1}{2}mu^2 = qV \qquad u = \sqrt{\frac{2qV}{m}}$$

Let 1 V potential difference is applied to an electron. Then KE gained by it

$$KE = qV = 1.6\times10^{-19}\times1 = 1.6\times10^{-19}\,J$$

It is called 1 eV of energy.

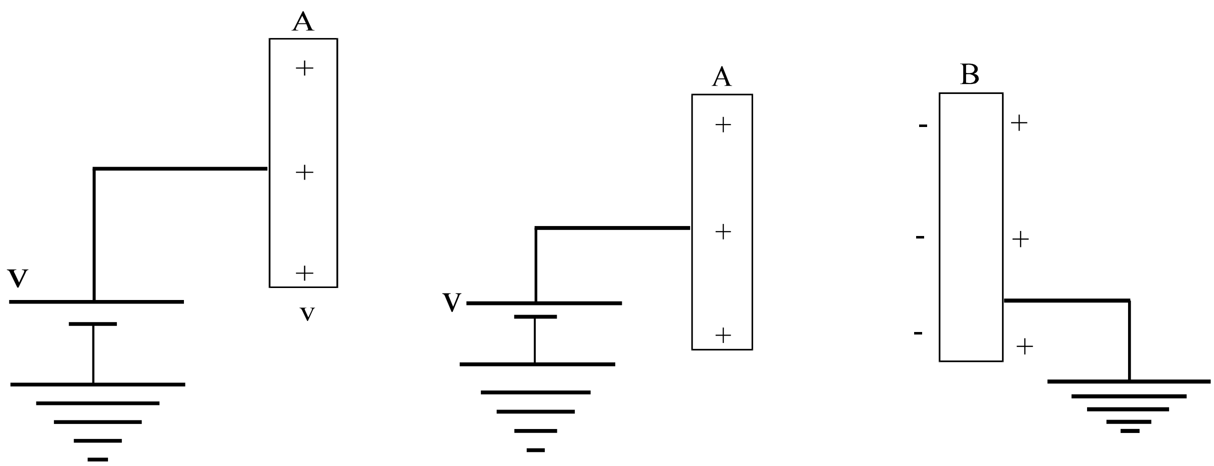
$$1\ eV = 1.6\times10^{-19}\,J$$

CAPACITOR

Capacitor is a tiny device used to store charge. There are many type of capacitors like parallel plate capacitor, spherical capacitor, cylindrical capacitor etc.

Principle

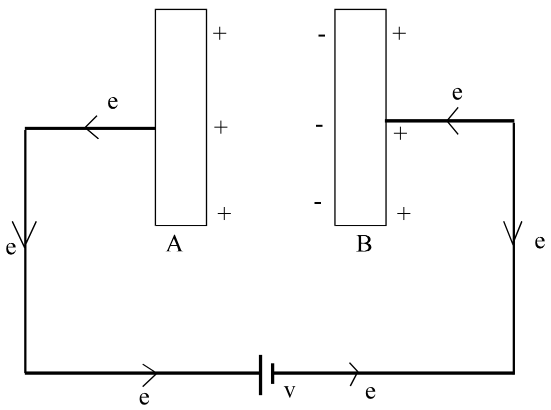
Consider a conductions plate (A) connected to a supply of potential V. Then plate gets charged due to potential V and charging stops when the potential of plate becomes equal to that of supply V. Now an another similar plate (B) is brought near this plate. Then induction happens in the second plate and induced charges are accumulated as shown.



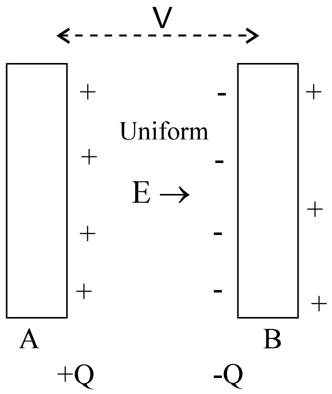
Now right surface of B is earthed. Then all positive induced charges there flows to earth. Due to the remaining negative induced charges on B, a negative potential develops on A. So total potential of A reduce. So to maintain potential at V, more charge is send to plate A. Then charge on plate A increases. This again increases the amount of negative induced charge on B. Hence total potential on A again reduce due to the increased (-) potential on it. So again battery send more charge on it. This process repeats and charge on system goes on increasing. So, an arrangement of two parallel plates can store infinite amount of charge.

Charging of Capacitor using a battery

Consider two parallel plates A and B, connected to a supply of potential V as shown.



Then free electrons are attracted from plate A towards (+) of battery. Due to electron loss, (+) charges are accumulated over A at right surface as shown. This causes induction over B. (-) induced charges are accumulated at the left surface of B and (+) at right. Now electrons flow from (-) of battery to B and nullifies the positive induced charges on the right surface. This cycle repeats untill capacitor is fully charged and at that instant, potential difference between the capacitor plates become equal to V.



1. Charging completes when potential difference between capacitor plates becomes equal to the applied potential.

2. If a capacitor has a charge Q, one of its plates has charge +Q and other plate -Q. Usually the plate connected to (+) of battery gets (+) charge and other negative.

3. The equal but opposite charges appearing at the nearby surfaces of the capacitor plates only are considered as the charge stored in the capacitor.

4. A uniform electric field exist between plates.

5. If a charged capacitor is connected to an another battery of higher potential, capacitor further charges. If a charged capacitor is connected to an another battery of lower potential, capacitor discharges. Charging and discharging stops when potential difference between the plates become equal to the supply potential.

6. During charging, electrons are removed from one plate and transferred to the other. The plate losing

electrons get a (+) charge and plate gaining electrons get a (-) charge. So battery does not give any charge to capacitor. Instead, it performs the amount of work required to transfer electrons from one plate to the other. Work done by battery is given by

$$\boxed{W_{bat} = \Delta Q V}$$

$\Delta Q \rightarrow$ amount of charge flown through battery.

$V \rightarrow$ Potential of battery

The amount of charge stored in a capacitor is directly proportional to the potential difference between its plates. That is

$Q \propto V$ or $Q = CV$

$\boxed{C = \frac{Q}{V}}$ $C \rightarrow$ Capacitance of capacitor. It is the ability to store charge

Unit : Farad (F) (SI) $[C] = M^{-1}L^{-2}T^{4}A^{2}$

egs $\rightarrow$ esu of capacitance

$IF = 9 \times 10^{11}$ esu of capacitance.

eg. Consider a spherical conductor of radius R and charge Q. Then its potential

$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R}$. It can be treated as a capacitor of capacitance.

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{R}\right)}$$

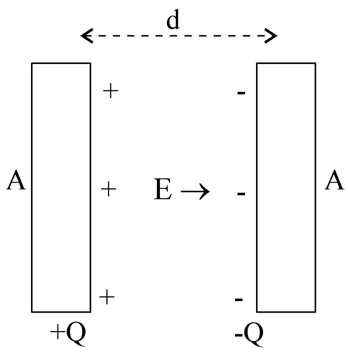
$$\boxed{C = 4\pi\varepsilon_o R} \qquad C \propto R$$

If we consider earth as a spherical capacitor of radius R = 6380 km

$$\boxed{C = 711\,\mu F}$$

Practically $V = 0$ for earth. So C is infinity.

Parallel plate capacitor



It consists of two identical metallic plates placed parallel and close to each other Let A be the area of each plate and d is the separation between the plates. Then uniform electric field between the plates,

$E=\frac{\sigma}{\varepsilon_o}$ $\sigma=\frac{Q}{A}$ $E=\frac{Q}{\varepsilon_o A}$. So potential difference between the plates; V = Ed

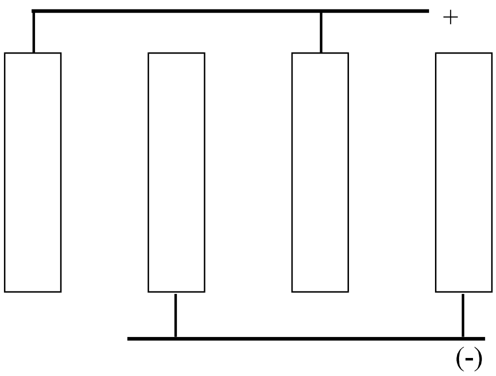
$$V=\frac{Q}{\varepsilon_o A}d$$

$$C=\frac{Q}{V}\Rightarrow \boxed{C=\frac{\varepsilon_o A}{d}} \rightarrow \text{air core capacitor}$$

If entire spacing between plates is filled with a dielectric of constant K, then

$$C=\frac{\varepsilon A}{d}=\frac{K\varepsilon_o A}{d} \quad \therefore \boxed{C^{l}=KC}$$

Consider a number of identical plates placed in parallel. Let alternate plates are given with like polarity such an arrangement is called a capacitor with N plates.

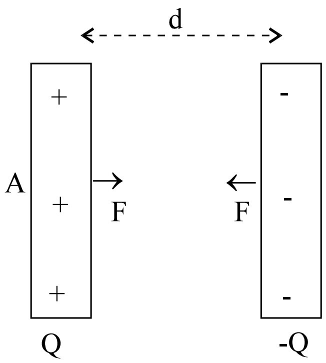


$$\boxed{C=(N-1)\frac{\varepsilon_o A}{d}}$$

Force between Capacitor plates

An attractive force always exist between the capacitor plates.

Here each plate is placed in the electric field produced by the other, given by



$$E=\frac{\sigma}{2\varepsilon_o}$$

Force on the plates; $F = QE = \frac{Q\sigma}{2\varepsilon_o}\left(\sigma = \frac{Q}{A}\right)$

$$\boxed{F = \frac{Q^2}{2\varepsilon_o A}}$$

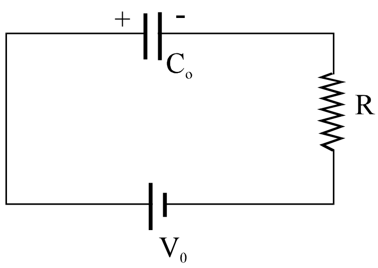
Diving both sides with area A

$$\frac{F}{A} = \frac{Q^2}{2\varepsilon_o A^2} \qquad \frac{F}{A} = Pe; \qquad \frac{Q}{A} = \sigma$$

$\boxed{Pe = \frac{\sigma^2}{2\varepsilon_o}}$ $Pe \rightarrow$ Electrostatic pressure on the plates.

Energy stored in a capacitor

Consider a capacitor C_o connected to a battery of potential V_o for charging.



Then, when capacitor is fully charged (Q_o), amount of energy stored in the electric field between the plates of the capacitor; given by

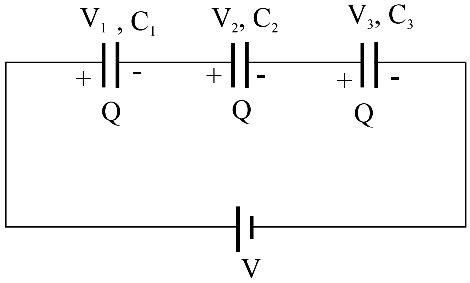
$$U = \frac{Q_o^2}{2C} \quad [Q_o = C_0 V_o] \quad U = \frac{1}{2} C_o V_o^2 \quad U = \frac{1}{2} Q_o V_o$$

work done by the battery; $W_{bat} = Q_o V_o$ of this only $\frac{1}{2} Q_o V_o$ is stored in the capacitor. So the remaining half is lost as heat or em radiations during charging. In actual practice, energy loss during charging is given by

$\Delta H = W_{bat}$ - energy stored in capacitor

Grouping of capacitors

(i) Series:



C_1, C_2, C_3 are initially uncharged. Since charging currents are equal, the amount of charge gained by all series capacitor are equal. Final charges may or may not be equal. Final charges will be equal only if their initial charges were equal. Here all the three capacitors were initially uncharged. So their final charges are also equal. But voltage drops are different. Using KVL

$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_{eff}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \boxed{\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

multiplying throughout with $\frac{Q^2}{2}$

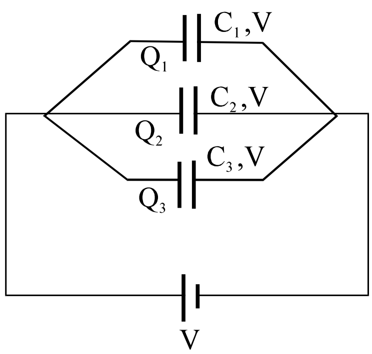
$$\frac{Q^2}{2C_{eff}} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$\boxed{U_{total} = U_1 + U_2 + U_3} \rightarrow \text{energy stored}$$

To find the common charge obtained by series connection; $Q = C_{eff} \times \text{applied voltage}$

Individual voltage drops; $V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$

Parallel grouping



Voltages are some in all but charges are different

$$Q_1 = C_1V$$

$$Q_2 = C_2V$$

$$Q_3 = C_3V$$

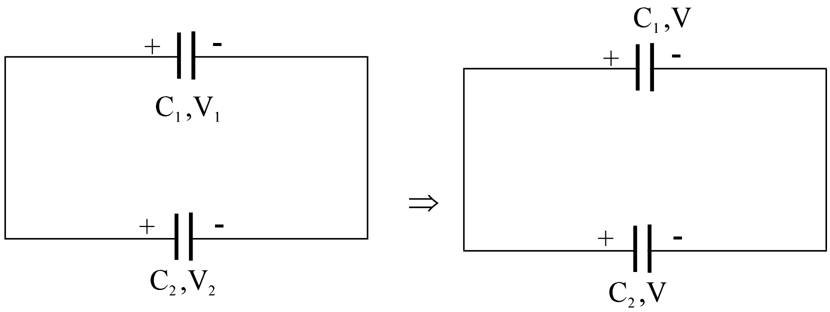
$$Q = Q_1 + Q_2 + Q_3$$

$$C_{eff}V = C_1V + C_2V + C_3V \Rightarrow \boxed{C_{eff} = C_1 + C_2 + C_3}$$

Redistribution of charges between capacitors

Consider two capacitors C_1 and C_2 charged by potential V_1 and V_2. Now they are interconnected in two ways as shown

(i) Like plates connected together



Then a charge flow happen between them until both attain a common potential (V). Using charge conservation; $C_1V_1 + C_2V_2 = C_1V + C_2V$

$$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

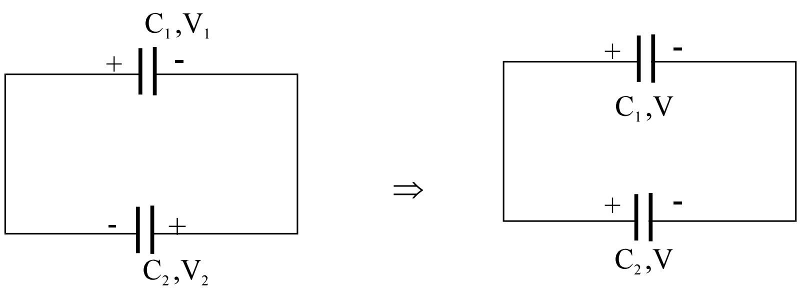
During this redistribution some energy is lost as heat and em radiations.

$$\Delta H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right)$$

on solving

$$\boxed{\Delta H = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2}$$

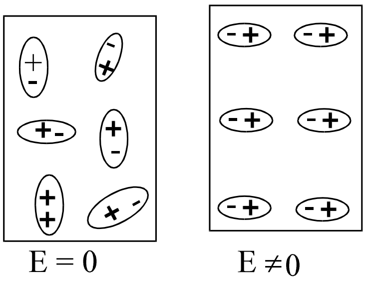
(ii) Unlike plates connected together



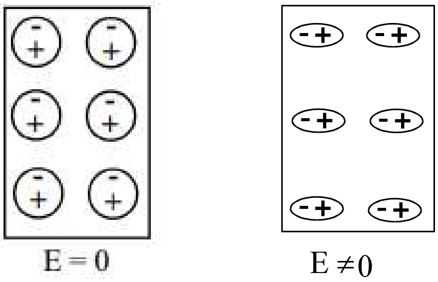
$$\boxed{V = \frac{|C_1V_1 - C_2V_2|}{C_1 + C_2}} \qquad \boxed{\Delta H = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 + V_2)^2}$$

Dielectric Polarisation

If we consider a polar dielectric, each molecule posess a permanant electric dipole moment. But under the absence of an external field, all the molecular dipoles are randomly oriented so that net dipole moment is zero. But when an external electric field is applied, most of the dipoles are alligned in the direction of field so that dielectric gain a net dipole moment.



For a nonpolar dielectric, individual molecules do not posess dipole moment so that net dipole moment is zero. This is due to the overlapping of (+) and (-) charge centres. But when a strong electric field is applied, charge centres separate to give a dipole moment to each molecule. Also they allign in the direction of field.



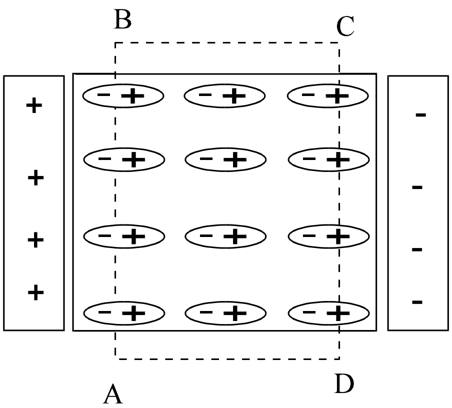
Thus in either case, polar or nonpolar, a dielectric develops a net dipole moment in the presence of external electric field. The dipole moment per unit volume is called polarisation (P).

$P \propto E$ or $\boxed{P = \varepsilon_0 \chi E}$

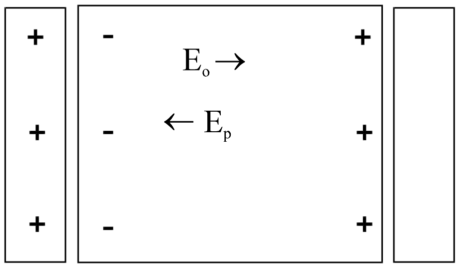
Where x is a constant characterising the dielectric called electric susceptibility. Dielectric constant can be written as $K = 1 + x$

Introduction of a dielectric between capacitor plates

Consider a parallel plate capacitor with a uniform electric field E_o between the plates. Now a dielectric is inserted between the plates. Then dielectric polarisation happens.



If we consider the volume element ABCD, equal and opposite charges of the neighbouring dipoles cancel. So polarised charges exist only at the end surfaces.



Due to the polarised charges, a new electric field E_P is developed in the region between the plates in a direction opposite to E. So net electric field in the region

$$E = E_o - E_p$$

So due to the introduction of a dielectric between the capacitor plates, net electric field between the plates reduces. $E_o = \frac{\sigma}{\varepsilon_o}$ $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_o K} = \frac{E_o}{K}$

$$\frac{E_o}{K} = E_o - E_p$$

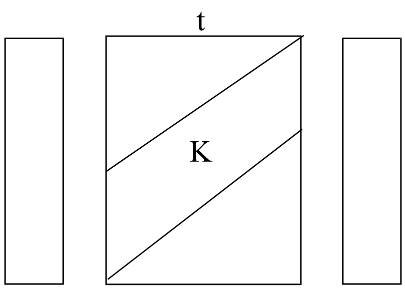
$$E_p = E_o - \frac{E_o}{K} \Rightarrow \boxed{E_p = E_o\left(1 - \frac{1}{K}\right)}$$

Polarised charges; $\boxed{Q_p = Q_o\left(1 - \frac{1}{K}\right)}$

Let a capacitor C is connected to a battery of potential V_o. The potential difference between the plates is V_o when charging completes. Now a dielectric is inserted. Then field between the plates reduces to $\frac{E_o}{K}$. Then potential difference also reduces to $\frac{V_o}{K}$. Since battery is still connected, battery will send more charge to the capacitor to take its potential back to V_o. Hence capacitor gains more charge and hence capacitance can be said to be increased. So capacitance is said to be increased by the introduction of a dielectric between its plates.

Equation for Capacitance with dielectric in between

Consider a parallel plate capacitor with plate area A and plate separation d. Let a dielectric of constant K and thickness t is introduced between the plates.



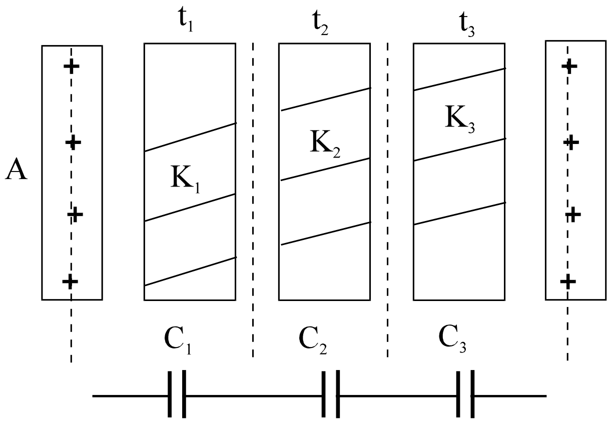
If the entire spacing between the plates is filled with dielectric; then t = d. Then capacitance becomes

$$\boxed{C = K\frac{\varepsilon_o A}{d}}$$

Introduction of more than one dielectrics

(i) Series arrangement of dielectrics

In this arrangement, plate area will became for the dielectric but plate separation is shared by the dielectrics. The arrangement can be given as below.



Region of each dielectric behaves as a capacitor

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{K_{eff}\frac{\varepsilon_o A}{d}} = \frac{1}{K_1\frac{\varepsilon_o A}{d_1}} + \frac{1}{K_2\frac{\varepsilon_o A}{d_2}} + \frac{1}{K_3\frac{\varepsilon_o A}{d_3}}$$

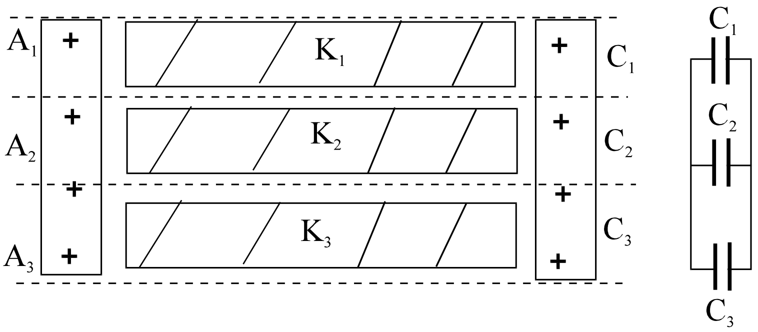
$$\frac{d}{K_{eff}} = \frac{d_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}$$

$$K_{eff} = \frac{d}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}} \Rightarrow \boxed{K_{eff} = \frac{d}{\Sigma \, t/K}}$$

$$C_{eff} = K_{eff}\left(\frac{\varepsilon_o A}{d}\right)$$

(ii) Parallel arrangement of dielectrics

In this all dielectrics have same thickness but they share plate area as shown.



Here also region of each dielectric behaves like an individual capacitor

$$C_{eff} = C_1 + C_2 + C_3$$

$$K_{eff}\frac{\varepsilon_o A}{d} = \frac{K_1\varepsilon_o A_1}{d} + \frac{K_2\varepsilon_o A_2}{d} + \frac{K_3\varepsilon_o A_3}{d}$$

$$K_{eff} = \frac{K_1A_1 + K_2A_2 + K_3A_3}{A}$$

$$\boxed{K_{eff} = \frac{\Sigma KA}{A}} \quad \boxed{C_{eff} = K_{eff}\frac{\varepsilon_o A}{d}}$$

Methods of changing the capacitance value

Practically, we can use two methods to change the capacitance value of a parallel plate capacitor.

(i) By changing plate separation d

(ii) By changing the medium between the plates by introducing a dielectric

When we do this, we can approach the system in two ways

(i) With battery still connected

(ii) After disconnecting the battery

If battery is still connected, potential difference between plates (V) will remain constant. But if battery is disconnected, the charge in the capacitor will remain a constant.

In such situations, total work done on the capacitor is; $W_{total} = W_{bat} + W_{ext}$

$W_{bat} \rightarrow$ Work done by battery

$W_{ext} \rightarrow$ Work done by external agent

A part of the total work done is appears as change in energy stored in the capacitor (dU) and remaining part lost as heat and em radiations (dH)

$$\boxed{W_{bat} + W_{ext} = dU + dH}$$

Work done by battery (W_{bat})

Battery performs work either to charge or discharge the capacitor. It is given by the equations.

$$\boxed{W_{bat} = \Delta Q \times V}$$

$\Delta Q \rightarrow$ amount of charge flown through battery

$V \rightarrow$ potential of battery

Usually, if battery is connected to a capacitor directly, then ΔQ can be taken as the change in charge stored in the capacitor.

If capacitor is charging, then $\Delta Q = +$, then $W_{bat} = (+)$. Then work is said to be done by battery. If capacitor is discharging, then $\Delta Q_2 = (-)$, then $W_{bat} = -$. Then work is said to be done on the battery.

Work done by external agent (W_{ext})

External agent performs the required work to do the mechanical process for changing the capacitance value, like changing the plate separation, inserting dielectric etc.

Change in energy stored in the capacitor (dU)

dU = Final energy stored - initial energy stored

Energy loss (dH)

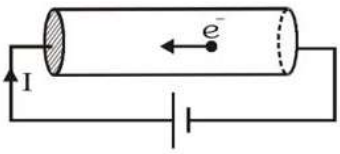
Energy loss will appear either as joule's heat across resistors or em radiations if charges are accelerated. If resistors are absent heat loss will be zero. If processes are done slowly, charges will not be accelerated, then em radiations will be zero.

CURRENT ELECTRICITY

In previous chapter we deal largely with electrostatics that is, with charges at rest. With this chapter we begin to focus on electric currents, that is, charges in motion.

ELECTRIC CURRENT

Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.



Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors. When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (I). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time as

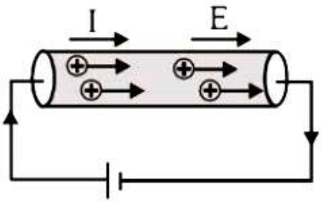
- Average current $I_{av} = \dfrac{\Delta Q}{\Delta t}$
- Instantaneous current $I = \lim\limits_{\Delta t \to 0} \dfrac{\Delta Q}{\Delta t} = \dfrac{dQ}{dt}$

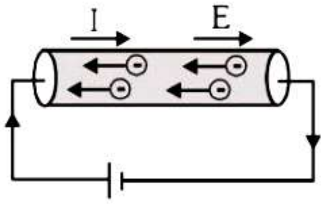
dQ = Idt $\therefore$ Total charge flows in a time 't'

$$Q = \int_0^t dQ = \int_0^t Idt$$

- Current is a fundamental quantity with dimension $\left[M^0L^0T^0A\right]$
- Current is a scalar quantity with its SI unit ampere

  Ampere : The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross-section of wire
- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons





- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second
- For a given conductor current does not change with change in its cross-section because current is simply rate of flow of charge
- If n particles each having a charge q pass per second per unit area then current associated with cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross-sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$

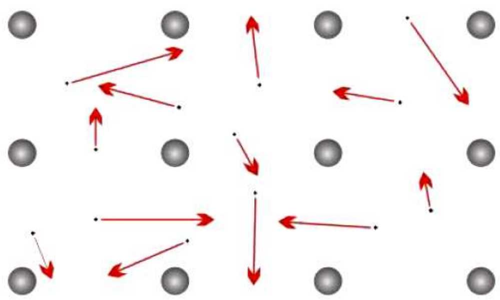
Behaviour of conductor in the absence of potential difference

The randomly moving free electrons inside the metal collide with the lattice and follow a zig-zag path. In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of curent due to thermal motion of free electrons in a conductor. The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron

At room temperature T = 300K, $v_{rms} = 10^5 m/s$



The small dots represent the free electrons and the red arrows represent the random thermal motion of the electrons

Thermal velocity : All electrons in the atom are not capable of motion. Only a few which have little higher level of energy leave their orbit and are capable of moving around. These electrons are called "free electrons". These free electrons are in very large quantity $\approx 10^{29} m^{-3}$ in free metals. Due to temperature and thermal energy they have a thermal velocity $\approx 10^{5} m^{-1}$. This velocity is in all directions and of magnitudes varying from zero to maximum. Due to large number of electrons we can assume that vector sum of thermal velocities at any instant is zero.

i.e. $\overrightarrow{u_1} + \overrightarrow{u_2} + \overrightarrow{u_3} + ... + u_n = 0$

- Even when rms speed due to thermal motion is very high, there is no current flow through the conductor in the absence of electric field, because thermal motion is random and average thermal velocity of electrons is zero.

Behaviour of a conductor in the presence of external potential difference

When two ends of a conductors are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of higher to lower potential

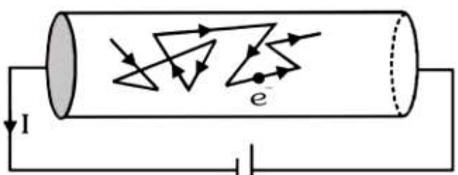
$E = \frac{V}{L}$ where V = emf of the battery, L = length of the conductor.

The field exerts an electric force on free electrons causing acceleration of each electron.

Acceleration of electron $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$

DRIFT VELOCITY

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field : Random motion of an electron with superimposed drift

Mean Free path : The fast moving electrons keep striking other atoms / ions in the conductor. They are reflected and move in other direction. They keep moving till they strike another ion/atom.

The path between two consecutive collisions is called free path. The average length of these free paths is called "Mean Free Path"

Mean free path $\lambda : \left(\lambda = 10\,\overset{o}{A}\right) \lambda = \dfrac{\text{total distance travelled}}{\text{number of collisions}}$

Relaxation Time : The time to travel mean free path is called Relaxation Period or Relaxation Time, denoted by Greek letter Taue "τ"

Relaxation time : $\tau = \dfrac{\text{total time taken}}{\text{number of collisions}}$

If t_1, t_2, ...t_n are the time period for n collisions then Relaxation Time $\tau = \dfrac{1}{n}\left(t_1 + t_2 + ...t_n\right)$

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ = the thermal velocity and $\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$\vec{v}_1 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, ...\vec{v}_N = \vec{u}_N + \vec{a}\tau_N$

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ...\vec{v}_N}{N} = \frac{(u_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N} = \frac{(\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N)}{N} + \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right)$$

$$\because \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0 \;\therefore\; \vec{v}_d = \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right) \Rightarrow \boxed{\vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau}$$

Note : Order of drift velocity is 10^{-4} m/s.

Relation between current and drift velocity

Let n = number density of free electrons and A = area of cross-section of conductor.

Number of free electrons in conductor of length L = nAL, total charge on these free electrons $\Delta q = neAL$

Time taken by drifting electrons to cross conductor $\Delta t = \dfrac{L}{v_d}$ $\therefore$ current $I = \dfrac{\Delta q}{\Delta t} = neAL\left(\dfrac{v_d}{L}\right) = neAv_d$

$\boxed{I = neAV_d}$ but $V_d = \dfrac{eE}{m}\tau$

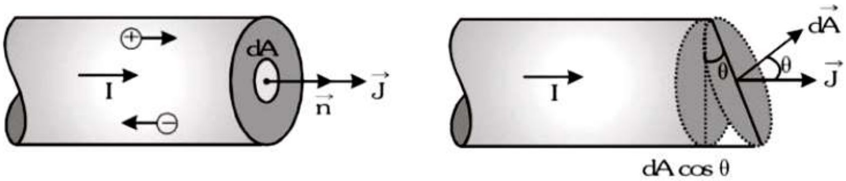
$\therefore I = neA\dfrac{eE\tau}{m}$

$$\boxed{I = \frac{ne^2AE\tau}{m}}$$

CURRENT DENSITY (J)

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the micriscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

- Current density at point P is given by $\vec{J} = \frac{dI}{dA}\vec{n}$



- If the cross-sectional area is not normal to the current, but makes an angle θ with the direction of current then $J = \frac{dI}{dA\cos\theta} \Rightarrow dI = JdA\cos\theta = \vec{J}.d\vec{A} \Rightarrow \boxed{I = \int \vec{J}.d\vec{A}}$

If current density is uniform, then $\boxed{\begin{array}{l} I = \vec{J}.\vec{A} \\ I = JA\cos\theta \end{array}}$

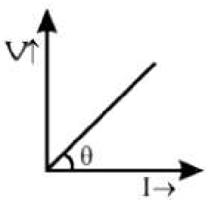
If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then : $i = \int \vec{J}.\overrightarrow{ds} = \vec{J}.\int \overrightarrow{ds}$ [as $\vec{J}$ = constant]

or $i = \vec{J} \cdot \vec{A} = JA\cos 0 = JA \Rightarrow J = \frac{i}{A}$ [as $\int \overrightarrow{dA} = \vec{A}$ and $\theta = 0°$]

Ohm's Law

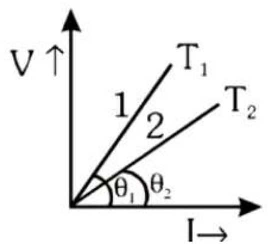
If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e. $I \propto V \Rightarrow V = IR$ where R is a proportionality constant, known as electric resistance. Ohm's law (at microscopic level)

- Ohm's law is not a universal law. The substances, which obey ohm's law are known as ohmic.
- Graph between V and I for a metallic conductor is a straight line as shown



Slope of the line = $\tan\theta = \frac{V}{I} = R$

- At different temperatures V-I curves are different.



Here $\tan\theta_1 > \tan\theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$

Resistance of a Conductor :

Definition : The property of substance by virtue of which it opposes the flow of curent through it, is known as the resistance.

Cause of resistance of a conductor : It is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor

Resistance R of a conductor depends on the following factors:

a) Length 'ℓ' of the conductor : $R\alpha\ell$, That is , resistance of conductor varies directly as its length.

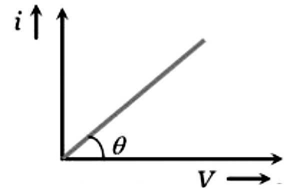
b) Area of cross-section 'A' : $R\alpha \frac{1}{A}$. Resistance of a conductor varies inversely as its area of cross-section.

For a conductor having greater area of cross-section, more free electrons cross that section of conductor in one second, thereby giving a large current. A large current means a lesser resistance.

Combining the two factors together, we get $R\,\alpha\frac{\ell}{A}$ or $\boxed{R=\frac{\rho\ell}{A}}$

where ρ is called resistivity of the conductor

Conductance (G): Reciprocal of resistance is known as conductance. $G=\frac{1}{R}$ It's unit is $\frac{1}{\Omega}$ or Ω^{-1} or "Siemen".



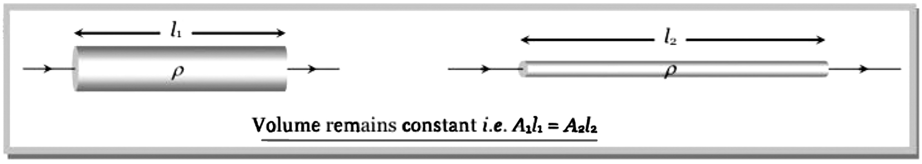
$$\text{Slope} = \tan\theta = \frac{i}{V} = \frac{1}{R} = G$$

Stretching of Wire: If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length $=\ell_1$, area of cross-section = A_1, radius =

r_1, diameter = d_1, and resistance $R_1 = \rho \frac{\ell_1}{A_1}$

Before stretching ....   After stretching



After stretching length $= \ell_2$, area of cross-section = A_2, radius = r_2, diameter = d_2 and resistance $= R_2 = \rho \frac{\ell_2}{A_2}$

Ratio of resistance $\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2} \times \frac{A_2}{A_1} = \left(\frac{\ell_1}{\ell_2}\right)^2 = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{d_2}{d_1}\right)^4$ since $A_1\ell_1 = A_2\ell_2$

1) If length is given then $R \propto \ell^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

2) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$

Note: After stretching if length increases by n times then resistance will increase by n^2 times i.e. $R_2 = n^2R_1$. Similarly if radius be reduced to $\frac{1}{n}$ times then area of cross-section decreases $\frac{1}{n^2}$ times so the resistance becomes n^4 times i.e. $R_2 = n^4R_1$.

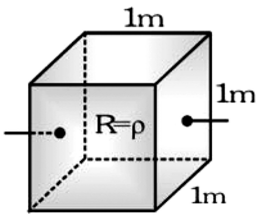
• After stretching if length of a conductor increases by x% then resistance will increases by 2x% (valid only if x < 10%) % change in resistance, if there is small change in length $\frac{\Delta R}{R} \times 100 = 2\frac{\Delta \ell}{\ell} \times 100$

RESISTIVITY

Resistivity: $\rho = RA/\ell$ if $\ell = 1\text{m}$, A = 1 m² then $\rho = R$

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross-section area and unit length.

Resistivity depends on (i) Nature of material (ii) Temperature of material, ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.

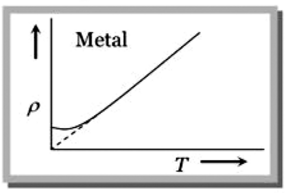


Unit and dimension: It's S.I. units is ohm × m and dimension is $\left[ML^3T^{-3}A^{-2}\right]$

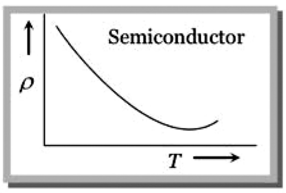
It's dependence: Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (i.e., ℓ and A). it depends on the followings:

(i) Nature of the body: For different substances their resistivity also different e.g. $\rho_{silver} =$ minimum = 1.6 × 10^{-8} $\Omega - m$ and $\rho_{fused\,quartz}$ = minimum $\approx 10^{16}\,\Omega - m$

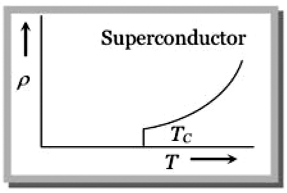
(ii) Temperature: Resistivity depends on the temperature. For metals $\rho_t = \rho_0\left(1+\alpha\Delta t\right)$ i.e. resistivity increases with temperature.



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

(iii) Impurity and mechanical stress: Resistivity increases with impurity and mechanical stress.

(iv) Effect of magnetic field: Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

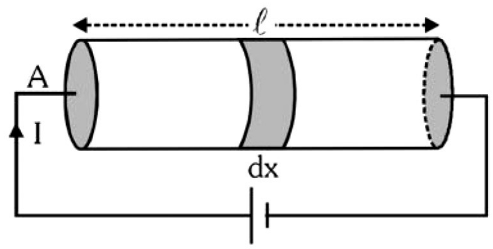
(v) Effect of light: Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

Resistivity of some electrical material: $\rho_{insulator} > \rho_{alloy} > \rho_{semi-conductor} > \rho_{conductor}$

Reciprocal of resistivity is called conductivity (σ) i.e. $\boxed{\sigma = \frac{1}{\rho}}$ with unit mho/ m and dimensions $\left[M^{-1}L^{-3}T^3A^2\right]$.

RELATION BETWEEN CURRENT DENSITY, CONDUCTIVITY AND ELECTRIC FIELD

Let the number of free electrons per unit volume in a conductor = n



Total number of electrons in dx distance = n (Adx)

Total charge dQ = n (Adx)e

Current $I = \frac{dQ}{dt} = nAe\frac{dx}{dt} = neAV_d$, Current density $J = \frac{1}{A} = neV_d$

$$= ne\left(\frac{eE}{m}\right)\tau \quad \because V_d = \left(\frac{eE}{m}\right)\tau \Rightarrow J = \left(\frac{ne^2\tau}{m}\right)E \Rightarrow J = \sigma E$$ where conductivity $\boxed{\sigma = \frac{ne^2\tau}{m}}$

σ depends only on the material of the conductor and its temperature.

In vector form $\boxed{\vec{J} = \sigma\vec{E}}$ Ohm's law (at microscopic level)

MOBILITY

Conductivity arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionised gas, they are electrons and positive charged ions; in an electrolyte, these can be both positive and negative ions.

An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field $\boxed{\mu = \frac{|V_d|}{E}}$

The SI unit of mobility is m^2/Vs

$\boxed{V_d = \frac{e\tau E}{m}}$ Hence, $\boxed{\mu = \frac{V_d}{E} = \frac{e\tau}{m}}$ where τ is the average collision time for electrons.

We have, $I = neAV_d$, thus $V_d = \frac{I}{neA}$

So, $\boxed{\mu = \frac{V_d}{E} = \frac{I}{neAE}}$

EFFECT OF TEMPERATURE ON RESISTANCE

The effect of rise in temperature is:

i) to increase the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line. As would be presently clarified, metals have a positive temperature co-efficient of resistance.

ii) to increase the resistance of alloys, though in their case, the increase is relatively small and irregular. For some high-resistance alloys like Eureka (60% Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.

iii) to decrease the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a negative temperature-coefficient of resistance.

TEMPERATURE COEFFICIENT OF RESISTANCE

Let a metallic conductor having a resistance of R_0 at 0°C be heated of t°C and let its resistance at this temperature be R_r. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

i) directly on its initial resistance

ii) directly on the rise in temperature

iii) on the nature of the material of the conductor.

or $R_t - R_0 \propto R \times t$ or $R_t - R_0 = \alpha R_0 t$ ...(i)

where α (alpha) is a constant and is known as the temperature coefficient of resistance of the conductor.

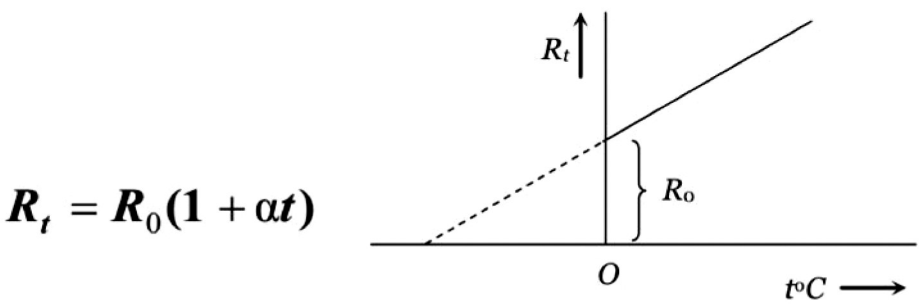
Rearranging Eq (ii), we get $\boxed{\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\Delta R}{R_0 \times t}}$

If $R_0 = 1\Omega, t = 1^\circ C$, then $\alpha = \Delta R = R_t - R_0$

Hence, the temperature-coefficient of a material may be defined as:

the increase in resistance per ohm original resistance per °C rise in temperature.

From Eq. (i), we find that $\boxed{R_t = R_0(1 + \alpha t)}$ ...(ii)



If R_1 and R_2 are the resistances at $t_1{}^\circ C$ and $t_2{}^\circ C$ respectively then $\frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$

$$\boxed{\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}}$$

The value of α is different at different temperature. Temperature coefficient of resistance averaged over the temperature range $t_1{}^\circ C$ to $t_2{}^\circ C$ is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1 [1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.

generally $\boxed{\alpha = \frac{1}{R}\frac{dR}{dt}}$ $\therefore \frac{dR}{R} = \alpha\, dt$

On integrating $\int_{R_0}^{R} \frac{dR}{R} = \alpha \int_0^t dt$

$\ell n(R)_{R_0}^{R} = \alpha t$ or $\ell n\left(\frac{R}{R_0}\right) = \alpha t$

$\frac{R}{R_0} = e^{\alpha t}$ or $\boxed{R = R_0 e^{\alpha t}}$

Variation of resistance of some electrical material with temperature:

i) Metals: For metals their temperature coefficient of resistance $\alpha > 0$. So resistance increases with temperature.

ii) Semi-conductors: For semi-conductor $\alpha < 0$ i.e. resistance decreases with temperature rise.

Physical explanation: covalent bonds breaks, liberating more free electron and conduction increases.

iii) Electrolyte: For electrolyte $\alpha < 0$ i.e. resistance decreases with temperature rise

Physical explanation: The degree of ionisation increases and solution becomes less viscous.

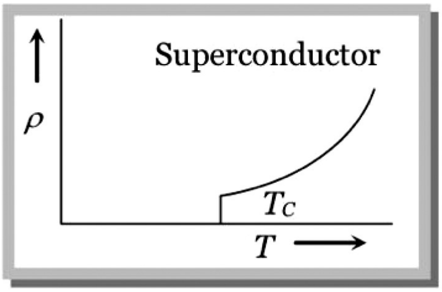
iv) Alloys: For alloys α has a small positive values. So with rise in temperature resistance of alloys is alomost constant. Further alloy resistances are slightly higher than the pure metals resistance.

Alloys are used to made standard resistance, wires of resistacne box, potentiometer wire, meter bridge wire etc.

Commonly used alloys are: constantan, mangnin, Nichrome etc.

v) Super conductors: At low temperature, the resistance of certain substances becomes exactly zero. (e.g. Hg below 4.2 K or Pb below 7.2 K)

These substances are called super conductors and phenomenon super conductivity. The temperature at which resistance becomes zero is called critical temperature and depends upon the nature of substance.



ρ decreases with temperature and becomes zero at a certain temperature

ELECTRIC POWER AND ENERGY

Consider a conductor with end points A and B, in which a current I is flowing from A to B. The electric potential at A and B are denoted by V(A) and V(B) respectively. Since current is flowing from A to B, V(A) > V(B) and the potential difference across AB is V = V(A) - V(B) > 0.

In a time interval Δt, an amount of charge $\Delta Q = I\,\Delta t$ travels from A to B.

The potential energy of the charge at A, by definition, was QV(A) and similarly at B, it is Q V(B). Thus, change in its potential energy ΔU is

ΔU = Final potential energy - Initial potential energy

$$= \Delta Q\left[V(B) - V(A)\right] = -\Delta QV$$

$$= -I\,V\Delta t < 0$$

If charges moved without collisions through the conductor, their kinetic energy would also change so that the total energy is unchanged. Conservation of total energy would then imply that,

$\Delta K = -\Delta U_{pot}$ that is, $\Delta K = IV\Delta t > 0$

Thus, in case charges were moving freely through the conductor under the action of electric field, their kinetic energy would increase as they move. We have, however, seen earlier that on the average, charge carriers do not move with acceleration but with a steady drift velocity. This is because of the collisions with ions and atoms during transit. During collisions, the energy gained by the charges thus is shared with the atoms. The atoms vibrate more vigorously, i.e., the conductor heats up. Thus, in an actual conductor, an amount of energy dissipated as heat in the conductor during the time interval Δt is,

$$\Delta W = IV\Delta t$$

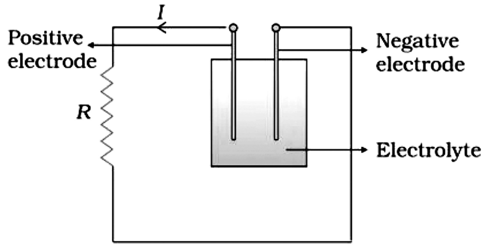
The energy dissipated per unit time is the power dissipated

$P = \Delta W / \Delta t$ and we have, P = IV

using ohm's law V = IR, we get

$$P = I^2R = \frac{V^2}{R}$$

as the power loss ("ohmic loss") in a conductor of resistance R carrying a current I. It is the power which heats up, for example, the coil of an electric bulb to incandescence, radiating out heat and light.



Heat is produced in the resistor *R* which is connected across the terminals of a cell. The energy dissipated in the resistor *R* comes from the chemical energy of the electrolyte.

We need an external source to keep a steady current through the conductor. It is clearly this source which must supply this power. In the simple circuit shown with a cell it is the chemical energy of the cell which supplies this power for as long as it can.

Consider a device R, to which a power P is to be delivered via transmission cables having a resistance Rc to be dissipated by it finally.

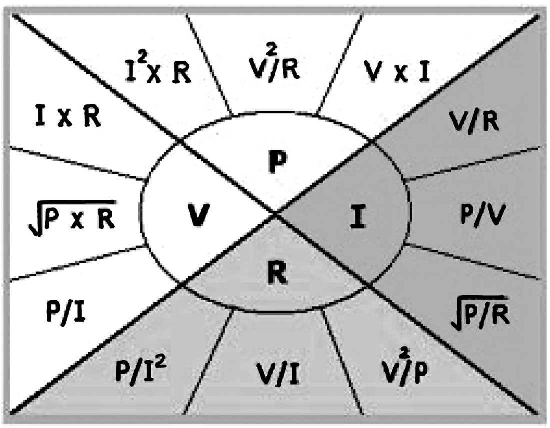
If V is the voltage across R and I the current through it, then P = VI

The connecting wires from the power station to the device has a finite resistance Rc.

The power dissipated in the connecting wires, which is wasted is Pc with,

$$P_c = I^2R_c = \frac{P^2R_c}{V^2}$$

Thus, to drive a device of power P, the power wasted in the connecting wires is inversely proportional to V^2. The transmission cables from power stations are hundreds of miles long and their resistance Rc is considerable. To reduce Pc, these wires carry current at enormous values of V and this is the reason for the high voltage danger signs on transmission



Power Relations

The SI unit of electric power is same as for any kind of power, the watt (W). 1W = 1J/s.

HEATING EFFECT OF CURRENT

Cause of Heating

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

- **Joules's Law of Heating**

When a current I is made to flow through a passive or ohmic resistance R for time t, heat Q is produced such that $Q = I^2 R\,t = P\ \ t = V\,I\,t = \frac{V^2}{R} t$

Heat produced in conductor does not depend upon the direction of current.

•**SI unit:** joule; **Practical Units:** 1 kilowatt hour (kWh)

1kWh = 3.6×10^6 joule = 1 unit

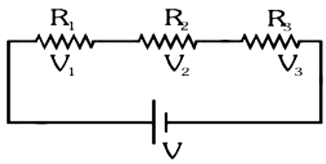
1 BTU (British Thermal Unit) = 1055 J

• Power: $P = V\,I = \frac{V^2}{R} = I^2 R$ • SI unit: Watt

The watt-hour meter placed on the premises of every consumer records the electrical energy consumed.

COMBINATION OF RESISTORS

Series Combination



- Same current passes through each resistance
- Voltage across each resistance is directly proportional to it's value

$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$

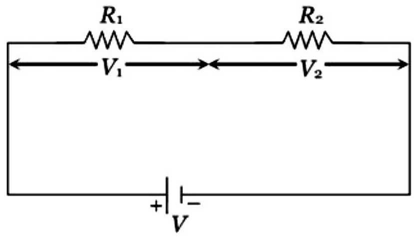
- Sum of the voltage across resistance is equal to the voltage applied across the circuit.

$V = V_1 + V_2 + V_3 \Rightarrow IR_S = IR_1 + IR_2 + IR_3 \Rightarrow \boxed{R_S = R_1 + R_2 + R_3}$ Where R_S = equivalent resistance

Equivalent resistance is greater than the maximum value of resistance in the combination

- Potential difference across any resistance $V' = \left(\frac{R'}{R_{eq}}\right).V$

Where R' = Resistance across which potential difference is to be calculated, R_{eq} = equivalent resistance of that line in which R' is connected, V = p.d. across that line in which R' is connected e.g.

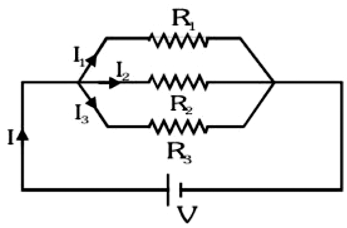


$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) \cdot V \text{ and } V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V$$

- If n identical resistance are connected in series R_{eq} = nR and p.d. across each resistance $V' = \frac{V}{n}$
- Series combination is called voltage divider circuit
- Power consumed are in the ratio of their resistance i.e., $P \propto R \Rightarrow P_1 : P_2 : P_3 = R_1 : R_2 : R_3$

  effective conductance
- $\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + ... + \frac{1}{G_n}$

Parallel Combination



- There is same drop of potential across each resistance.
- Current in each resistance is inversely proportional to the value of resistance. $I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}. I_3 = \frac{V}{R_3}$
- Current flowing in the circuit is sum of the currents in individual resistance.

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

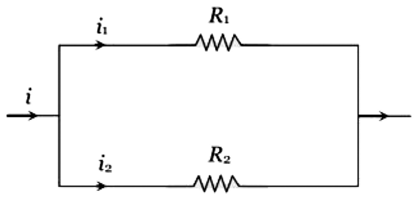
Where R_p = equivalent resistance

- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $R_{eq} = \left(R_1^{-1} + R_2^{-1} + R_3^{-1}\right)^{-1}$ or $R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_2R_1}$ equivalent resistance is smaller than the minimum value of resistance in the combination.
- **effective conductance**

  $G_{eq} = G_1 + G_2 + ... + G_n$
- Current through any resistance

  $$i' = i \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$$

  Where $i' =$ required current (branch current)
  i = main current

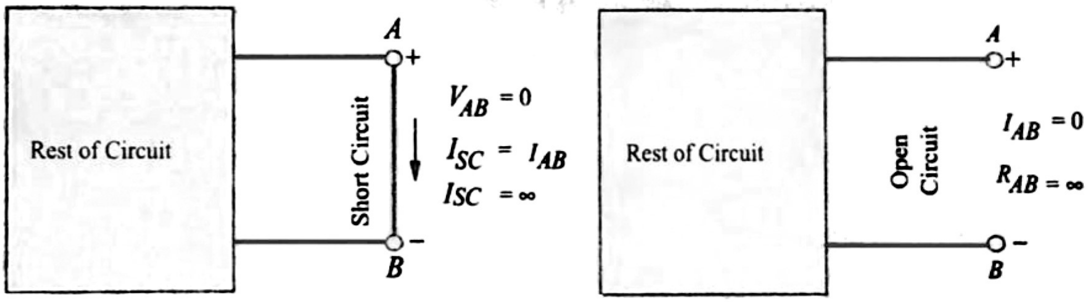


$$i_1 = i\left(\frac{R_2}{R_1 + R_2}\right) \text{ and } i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$$

- Parallel combination is called current divider circuit
- In n identical resistance are connected in parallel $R_{eq} = \frac{R}{n}$ and current through each resistance $i' = \frac{i}{n}$
- Power consumed are in the reverse ratio of resistance i.e. $P \propto \frac{1}{R} \Rightarrow P_1 : P_2 : P_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$

SHORT AND OPEN CIRCUITS

When two points of circuit are connected together by a thick metallic wire (fig.) they are said to be short-circuited. Since 'short' has practically zero resistance, it gives rise to two important facts:
i) no voltage can exist across it because V = IR = I × 0 = 0
ii) current through it (called short-circuit current) is very large (theoretically, infinity)



Two points are said to be open-circuited when there is no direct connection between them (fig.). Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

i) resistance between the two points is infinite

ii) there is no flow of current between the two points.

cell

The device which converts chemical energy into electrical energy is known as electric cell.

A cell neither creates nor destroys enarge but maintains the flow of charge present at various parts of the circuit by supplying energy needed for their organised motion.

Cell is source of constant emf but not constant current.

Mainly cells are of two types:

(i) Primary cell : Cannot be recharged

(ii) Secondary cell: Can be recharged

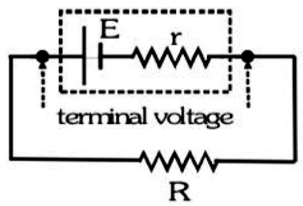
Emf of cell (E): The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called it's electromotive force (emf) i.e. emf of cell $E = \frac{W}{q}$. It's unit is volt

The potential difference across the terminals of a cell when it is not given any current is called it's emf.

• emf depends on: (i) nature of electrolyte (ii) metal of electrodes

• emf does not depend on: (i) area of plates (ii) distance between the electrode (iii) quantity of electrolyte (iv) size of cell

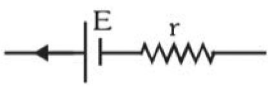
TERMINAL VOLTAGE (V)



- When current is drawn through the cell or current is supplied to cell then, the potential difference across its terminals called terminal voltage.
- When I current is drawn from cell, then terminal voltage is less than it's e.m.f $V = E - Ir$

INTERNAL RESISTANCE

Offered by the electrolyte of the cell when the electric current flows through it is known as internal resistance. Distance between two electrodes increases $\Rightarrow$ r increases



For an ideal cell internal resistance is zero

Area dipped in electrolyte increases $\Rightarrow r$ decreases

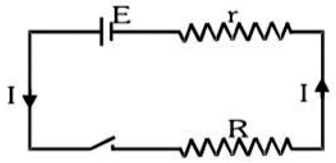
Concentration of electrolyte increases $\Rightarrow r$ increases

Temperature increases $\Rightarrow r$ decreases

Terminal Potential Difference: The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

a) **When cell is discharging:**

Current inside the cell is from cathode to anode.



Current $I = \dfrac{E}{r+R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V = E - Ir$

When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage. When a large current is drawn from a cell its terminal voltage is reduced.

Internal resistance of the cell $\boxed{r = \left(\dfrac{E}{V} - 1\right).R}$

Power dissipated in external resistance (load) $\boxed{P = Vi = i^2R = \dfrac{V^2}{R} = \left(\dfrac{E}{R+r}\right)^2.R}$

$$P = \frac{E^2R}{(R+r)^2} = \frac{E^2R}{R^2 + r^2 + 2Rr}$$

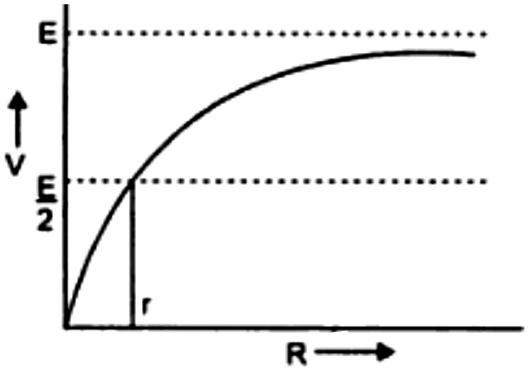
Power is maximum when, $\dfrac{R^2 + r^2 + 2Rr}{R}$ is minimum, therefore

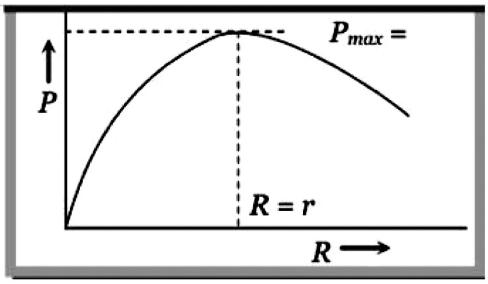
$$\frac{d}{dR}\left(R + 2r + \frac{r^2}{R}\right) = 0 \quad \text{Thus,} \quad 1 + 0 - \frac{r^2}{R^2} = 0 \; therefore \; R = r$$

$\therefore$ Power delivered will be maximum when R=r

So $P_{max} = \dfrac{E^2}{4r} = \dfrac{E^2}{4R}$

Thus generally power transferred to the external circuit is maximum, when external resistance is equal to the internal resistance. This is called "maximum power transfer theorem"

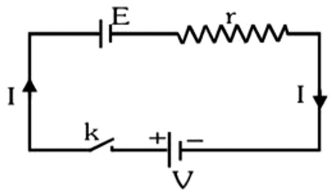




b) **When cell is charging:**

Current inside the cell is from anode to cathode.

Current $I = \frac{V-E}{r} \Rightarrow V = E + Ir$



During charging terminal potential difference is greater than emf of cell.

c) **When cell is in open circuit:**

In open circuit $R = \infty$ $\quad \therefore I = \frac{E}{R+r} = 0 \Rightarrow V = E$

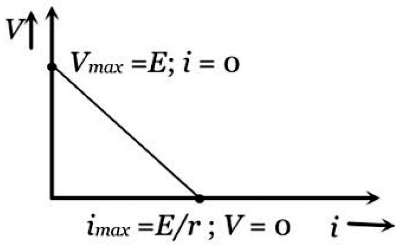
In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

d) **When cell is short circuited:**

In short circuit R = 0 $\quad \Rightarrow I = \frac{E}{R+r} = \frac{E}{r}$ and $V = IR = 0$

In short circuit current from cell is maximum and terminal potential difference is zero.

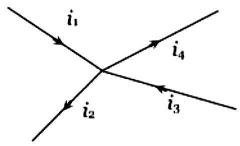
Above information/s can be summarized by the following graph



KIRCHOFF'S LAWS

1) **Kirchoff's first law:** This law is also known as junction rule or current law (KCL). According to it the algebraic sum of currents meeting at a junction is zero i.e. $\sum i = 0$.

In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$



Here it is worthy to note that:

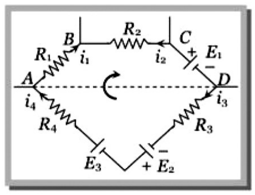
(i) If a current comes out to be negative, actual direction of current at the junction is opposite to that assumed, $i + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, i.e., leaving the junction.

(ii) This law is simply a statement of "conservation of charge" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

2) **Kirchoff's second law:** This law is also known as loop the rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", i.e., $\sum V = 0$

e.g. In the following closed loop. $\sum V_i + I_i R_i = 0$

$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$



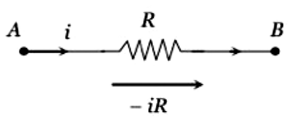
Here it is worthy to note that:

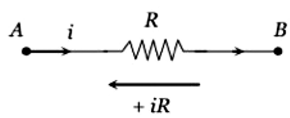
(i) This law represents "conservation of energy" as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

(ii) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be (n-1)

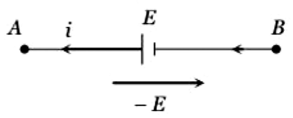
3) **Sign convention for the application of Kirchoff's law:** For the application of Kirchoff's laws following sign convention are to be considered

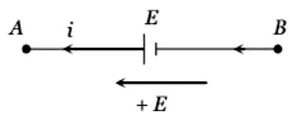
(i) The change in potential in traversing a resistance in the direction of current is -iR while in the opposite direction +iR



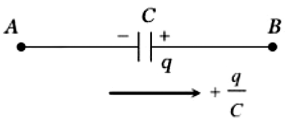


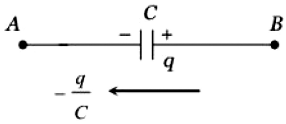
(ii) The change in potential in traversing an emf source from negative to positive terminal is +E while in the opposite direction -E irrespective of the direction of current in the circuit.



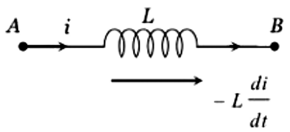


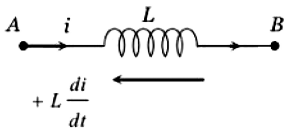
(iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminals $+\frac{q}{C}$ while in opposite direction $-\frac{q}{C}$.





(iv) The change in voltage is traversing in inductor in the direction of current is $-L\frac{di}{dt}$ while in opposite direction it is $+L\frac{di}{dt}$.





4) **Guidelines to apply Kirchoff's law**

(i) Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with 'junction rule'. It is not always easy to correctly guess the direction of current, no problem if one assumes the wrong direction.

(ii) After assuming current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor or battery we must write down, the voltage change for that element according to the above sign convention.

(iii) By applying KVL we get one equation but in order to solve the circuit we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.

(iv) After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values come out to be negative, it indicates that particular current is in the opposite direction from the assumed one.

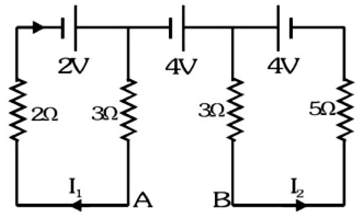
- The number of loops must be selected so that every element of the circuit must be included in at least one of the loops
- While traversing through a capacitor or battery we do not consider the direction of current
- While considering the voltage drop or gain across as inductor we always assume current to be in increasing function.

Example

In the given circuit calculate potential difference between A and B.

Solution

First applying KVL on left mesh $2 - 3\ I_1 - 2\ I_1 = 0 \Rightarrow I_1 = 0.4$ amp.



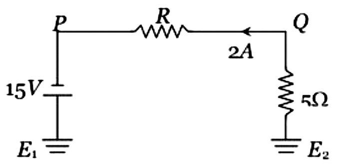
Now applying KVL on right mesh. $4 - 5\ I_2 - 3\ I_2 = 0 \Rightarrow I_2 = 0.5$ amp.

Potential difference between points A and B

$V_A - V_B = -3 \quad 0.4 - 4 + 3 \quad 0.5 = -3.7$ volt.

Example

In the following circuit the potential difference between P and Q is



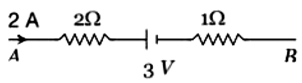
1) 15V 2) 10V 3) 5V 4) 2.5V

Sol: 3

By using KVL $-5 \times 2 - V_{PQ} + 15 = 0 \Rightarrow X = 100\,\Omega$

Example

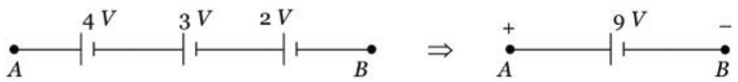
Figure represents a part of the closed circuit. The potential difference between points A and B $(V_A - V_B)$ is



1) +9V 2) -9V 3) +3V 4) +6V

Sol: 1

The given part of a closed circuit can be redrawn as follows. It should be remember that product of current and resistance can be treated as an imaginary cell having emf = iR.

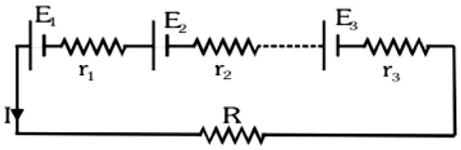


$\Rightarrow 7$ Hence $V_A - V_B = +9\ V$

COMBINATION OF CELLS

- **Series combination**

  When cells are connected in series the total emf of the series combination is equal to the sum of the emf's of the individual cells and internal resistance of the cells also come in series.



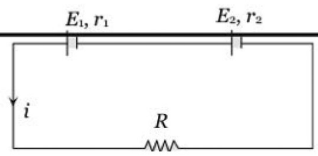
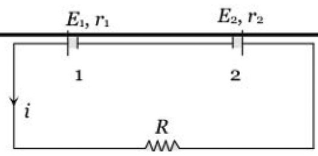
Equivalent internal resistance $r = r_1 + r_2 + r_3 + ...$ Equivalent $\text{emf} = E = E_1 + E_2 + E_3 + ...$

Current $I = \dfrac{E_{net}}{r_{net} + R}$. If all n cell are identical then $I = \dfrac{nE}{nr + R}$

If nr >> R, $I = \dfrac{E}{r} =$ current from one cell. If nr << R, $I = \dfrac{nE}{R} = n$ current from one cell

- Potential difference across each cell $V' = \dfrac{V}{n}$
- Power dissipated in the external circuit $P = \left(\dfrac{nE}{R + nr}\right)^2 .R$
- Condition for maximum power R = nr and $P_{max} = n\left(\dfrac{E^2}{4r}\right)$
- This type of combination is used when nr << R
- When m cells are wrongly connected, equivalent EMF = (n - 2m)E

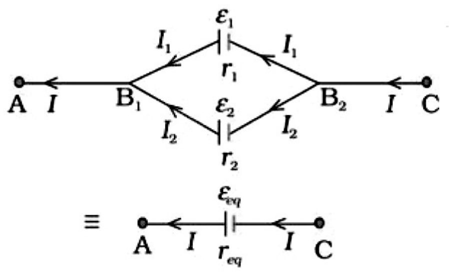
If non-identical cell are connected in series

Cells are connected in right order	Cells are wrongly connected
	$(E_1 > E_2)$ 
(a) Equivalent emf $E_{eq} = E_1 + E_2$ (b) Current $i = \dfrac{E_{eq}}{R + r_{eq}}$ (c) Potential difference across each cell $V_1 = E_1 - ir_1$ and $V_2 = E_2 - ir_2$	(a) Equivalent emf $E_{eq} = E_1 - E_2$ (b) Current $i = \dfrac{E_1 - E_2}{R + r_{eq}}$ (c) in the above circuit cell 1 is discharging so it's equation is $E_1 = V_1 + ir_1 \Rightarrow \boldsymbol{V_1 = E_1 - ir_1}$ and cell 2 is charging so it's equation $E_2 = V_2 - ir_2 \Rightarrow \boldsymbol{V_2 = E_2 + ir_2}$

PARALLEL COMBINATION OF CELLS

Consider a parallel combination of the cells I_1 and I_2 are the currents leaving the positive electrodes of the cells. At the point B_1. I_1 and I_2 flow in whereas the current I flows out. Since as much charge flows in as out, we have

$I = I_1 + I_2$



Let $V(B_1)$ and $V(B_2)$ be the potentials at B_1 and B_2 respectively. Then, considering the first cell, the potential difference across the terminals is $V(B_1) - V(B_2)$. Hence,

$$V \equiv V(B_1) - V(B_2) = \varepsilon_1 - I_1 r_1$$

Points B_1 and B_2 are connected exactly similarly to the second cell. Hence considering the second cell, we also have $V \equiv V(B_1) - V(B_2) = \varepsilon_2 - I_2 r_2$

Combining the last three equations

$I = I_1 + I_2$

$$= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Hence, V is given by, $V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\frac{r_1 r_2}{r_1 + r_2}$

if we want to replace the combination by a single cell, between B_1 and B_2, of emf ε_{eq} and internal resistance r_{eq}, we would have

$$\boxed{V = \varepsilon_{eq} - I\, r_{eq}}$$

The last two equations should be the same and hence

$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}; \qquad \boxed{r_{eq} = \frac{r_1 r_2}{r_1 + r_2}}$$

We can put these equations in a simpler way,

$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

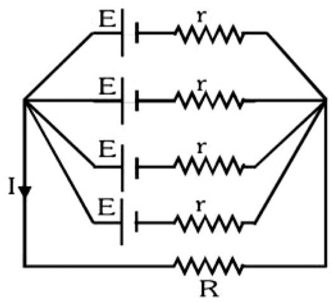
$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}$$

For n cells in parallel combination $\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \ldots + \frac{1}{r_n}}$

$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \ldots + \frac{\varepsilon_n}{r_n}}$$

m IDENTICAL CELLS IN PARALLEL COMBINATION

When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single cell and internal resistance of the cell also come in parallel. If m identical cell connected in parallel then total internal resistance of this combination $\boxed{r_{net} = \frac{r}{m}}$. Total e.m.f of this combination = E



Current in the circuit $\boxed{I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}}$

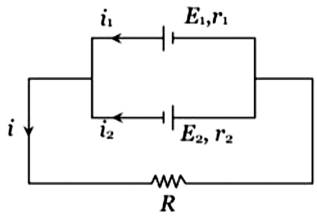
If r << mR $\quad I = E / R =$ Current from one cell

If r >> mR $\quad I = \frac{mE}{r} = m$ current from one cell

Current from each cell $i = \frac{I}{m}$ (f) Power dissipated in the circuit $P = \left(\frac{E}{R + r/m}\right)^2 .R$ (in external circuit)

Condition for max power R = r/m and $P_{max} = m\left(\frac{E^2}{4r}\right)$. This type of combination is used when mr >> R

Note: In this combination if cell's are connected with reversed polarity as shown in figure then:



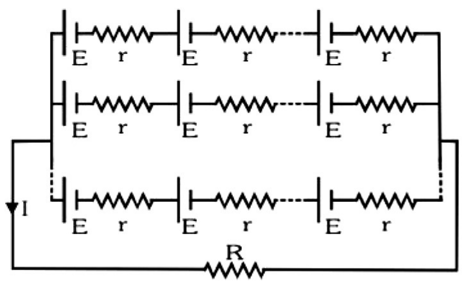
Equivalent emf $\boxed{E_{eq} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}}$

$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}}$$

$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} - \frac{\varepsilon_2}{r_2}}$$

MIXED GROUPING OF CELLS

If n cells connected in series and their are m such branches in the circuit then total number of identical cell in this circuit is nm. The internal resistance of the cells connected in a row = nr. Since there are such m rows,



Total internal resistance of the circuit $r_{net} = \frac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row E_{net} = nE

Current in the circuit $I = \dfrac{E_{net}}{R + r_{net}} = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \dfrac{nr}{m}$

- Potential difference across load V = iR
- Potential difference across each cell $V' = \dfrac{V}{n}$
- Current from each cell $i' = \dfrac{i}{n}$
- Condition for maximum power $R = \dfrac{nr}{m}$ and $P_{max} = (mn)\dfrac{E^2}{4r}$
- Total number of cell = mn

Example

In a mixed grouping of identical cells 5 rows are connected in parallel by each row contains 10 cell. This combination send a current i through an external resistance of 20Ω. If the emf and internal resistance of each cell is 1.5 volt and 1Ω respectively then the value of i is

1) 0.14 2) 0.25 3) 0.75 4) 0.68

Solution

No. of cells in a row $n = 10$; No. of such rows $m = 5$

$$i = \frac{nE}{\left(R + \frac{nr}{m}\right)} = \frac{10 \times 1.5}{\left(20 + \frac{10 \times 1}{5}\right)} = \frac{15}{22} = 0.68\ amp$$

Example

To get maximum current in a resistance of 3Ω one can use n rows of m cells connected in parallel. If the total no. of cells is 24 and the internal resistance of a cell is 0.5 then

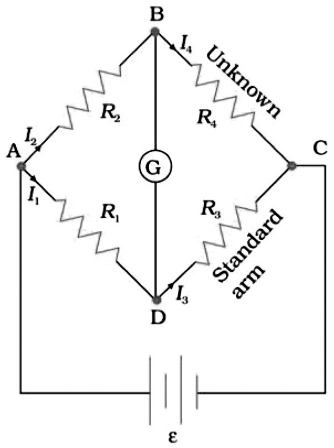
1) m = 12, n = 2 2) m = 8, n = 4 3) m = 2, n = 12 4) m = 6, n = 4

Sol: 1

In this question $R = 3\Omega, mn = 24, r = 0.5\Omega$ and $R = \dfrac{nr}{m}$. On putting the values we get n = 2 and m = 12

WHEATSTONE BRIDGE

The arrangement of four resistors in the circuit shown is called the Wheatstone bridge. The bridge has four resistors R_1, R_2 , R_3 and R_4. Across one pair of diagonally opposite points (A and C in the figure) a source is connected. This (i.e., AC) is called the battery arm. Between the other two vertices, B and D, a galvanometer G (Which is device to detect currents) is connected. This line shown as BD in the figure, is called the galvanometer arm. For simplicity, we assume that the cell has no internal resistance.



Generally a current will flow through the galvanometer, but in a balanced Wheatstone-Bridge current through galvanometer is zero.

When current through galvanometer (Ig) is zero, the currents I $I_2 = I_4$ and $I_1 = I_3$

From Kirchhoff's loop rule to closed loops ADBA and CBDC. For first loop,

$$-I_1 R_1 + 0 + I_2 R_2 = 0 \qquad (I_g = 0) \qquad thus \; \frac{I_1}{I_2} = \frac{R_2}{R_1} \qquad ...(1)$$

For the second loop $I_4 R_4 + 0 - I_3 R_3 = 0$ $\quad but \quad I_2 = I_4 \; and \; I_1 = I_3$

Therefore thus $\dfrac{I_1}{I_2} = \dfrac{R_4}{R_3}$ ...(2)

From the above equations, the ratio of the resistor is

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}} \quad or \quad \boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3}}$$

This last equation relating the four resistors is called the balance condition for the galvanometer to give zero or null deflection.

The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance.

In the given situation the unknown resistance $R_4 = R_3 \frac{R_2}{R_1}$

At balancing condition potentials at points B and D are equal i.e, $V_D = V_B$

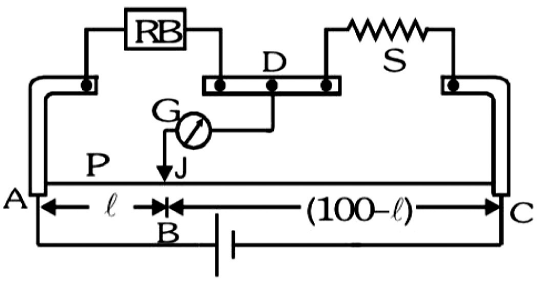
$$If\ \frac{R_2}{R_4} < \frac{R_1}{R_3},$$ then $V_B > V_D$ and current will flow from B to D.

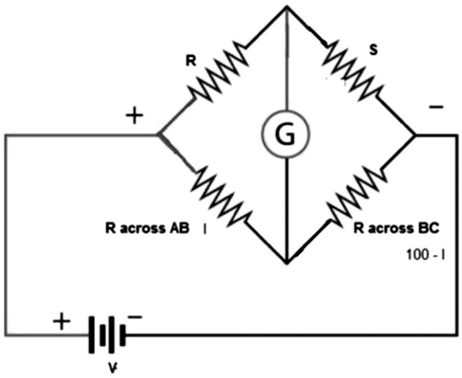
$$If\ \frac{R_2}{R_4} > \frac{R_1}{R_3},$$ then $V_B < V_D$ and current will flow from D to B.

- On mutually changing the position of cell and galvanometer balancing condition will not change.
- The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.
- The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.

METRE BRIDGE

It is based on principle of Wheatstone bridge. It is used to find out unknown resistance of wire. AC is 1m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.





$AB = \ell\,\text{cm}$ $\qquad P = r\ell$

$BC = (100-\ell)\,\text{cm}$ $\qquad Q = r(100-\ell)$ Where r = resistance per unit length on wire.

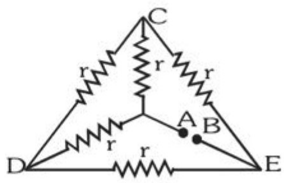
At balance condition: $\frac{P}{Q}=\frac{R}{S} \Rightarrow \frac{r\ell}{r(100-\ell)}=\frac{R}{S} \Rightarrow \boxed{S=\frac{(100-\ell)}{\ell}R}$

If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.

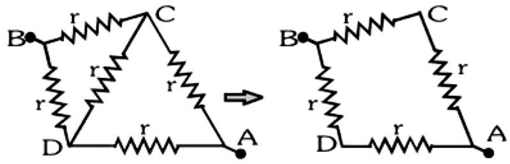
The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge.

Example

In the adjoining network of resistors each is of resistance $r\,\Omega$. Find the equivalent resistance between point A and B



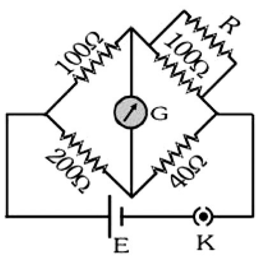
Sol: Given circuit is balanced Wheat stone bridge $\therefore \frac{1}{R_{AB}}=\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r}$ $\quad R_{AB}=r$



Example

For the following diagram the galvanometer shows zero deflection then what is the value of R?

Sol: For balanced Wheatstone bridge $\frac{100}{\frac{100R}{(100+R)}}=\frac{200}{40}$



$\Rightarrow \frac{100+R}{R}=5 \Rightarrow 100+R=5R \Rightarrow R=\frac{100}{4}=25\Omega$

ELECTRICAL APPLIANCES

Rated or Design Values:

Some of the values like; wattage, voltage etc. are printed on the electrical appliances are called rated or design values.

The maximum potential difference can be applied to a device for its safe working is called rated voltage and the corresponding maximum current is called rated current.

If V_r, I_r and P_r are the rated voltage, current and power of a device respectively, then resistance of the device

$$\boxed{R = \frac{V_r^2}{P_r} = \frac{V_r}{I_r}}$$

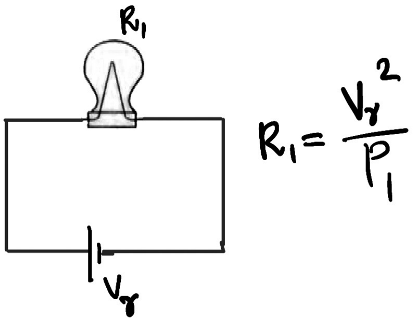
Now let a potential difference of V is applied across this device, then the power consumed is

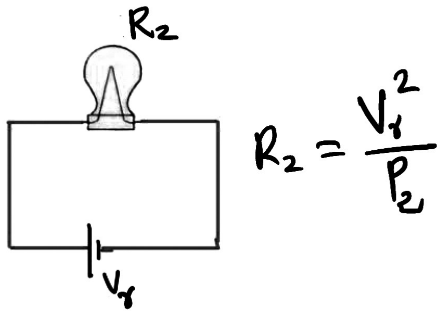
$$\boxed{P = \frac{V^2}{R} = \left(\frac{V}{V_r}\right)^2 P_r}$$

If $V = V_0$, then $P = P_0$. $\therefore$ An electric appliance consumes the rated power only when applied voltage is rated voltage.

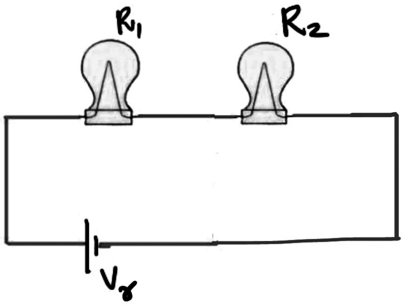
Two bulbs connected in series

Consider two bulbs of some voltage ratings V_r and rated powers P_1 and P_2 respectively. When they are connected as shown the power consumed are $P_1 = \frac{V_r^2}{R_1}$ and $P_2 = \frac{V_r^2}{R_2}$





When these two bulbs are connected in series to the same source voltage V_0, total power consumed



$$P = \frac{V_r^2}{R_1 + R_2}$$

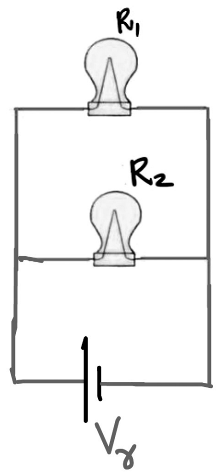
$$P = \frac{V_r^2}{\frac{V_r^2}{P_1} + \frac{V_r^2}{P_2}}$$

$\boxed{\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}}$ or $\boxed{P = \frac{P_1 P_2}{P_1 + P_2}}$

For n bulbs $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + ... + \frac{1}{P_n}$

Two bulbs are connected in Parallel

From the above situation $R_1 = \frac{V_r^2}{P_1}$ and $R_2 = \frac{V_r^2}{P_2}$



In parallel combination

$$P = \frac{V_r^2}{R_1} + \frac{V_r^2}{R_2}$$

$\boxed{P = P_1 + P_2}$

SOME APPLICATIONS

(a) Fusing of bulb when it is switched on:

Usually filament bulbs get fused when they are switched on. This is because with the rise in temperature, the resistance of the bulb increases and becomes constant in steady state. So the power consumed by the bulb $\left(V^2/R\right)$ initially is more than that in steady state and hence the bulb glows more brightly in the beginning and may get fused.

Two wires made of tinned copper having identical cross section (=10^{-6} m^2) and lengths 10 and 15cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.

Sol: The temperature of the wire rises to a certain steady temperature when the heat produced per second by the current just becomes equal to the rate of loss of heat from its surface.

Heat produced per second by the current $= I^2R = I^2\frac{\rho\ell}{\pi r^2}$, where ℓ is the length, r is radius of the wire and ρ is the specific resistance. Let H be heat lost per second per unit surface area of the wire. If we neglect the loss of heat from the end faces of the wire, then heat lost per second by the wire is H × surface area of wire $H\times 2\pi r\ell$

At steady state temperature,

$$H\times 2\pi r\ell = \frac{\ell^2\rho\ell}{\pi r^2} \text{ or } H = \frac{\ell^2\rho}{2\pi^2 r^3} \text{ ---(i)}$$

$$\boxed{I \alpha r^{3/2}}$$

From Eq. (i) we not that the rate of loss of heat (H) which, in turn, depends upon the temperature of the wire is independent of length of the wire. Hence, the fuses of two wires of the same values of r and ρ but of different lengths will melt for the same value of current in each case.

(b) Decreases in the brightness of bulb after long use:

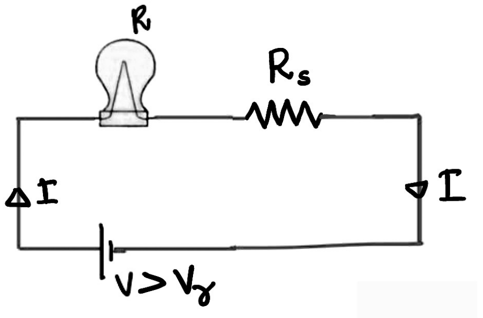
Also due to evaporation of metal from the filament (which deposits as black substance on the inner side of glass wall), the filament of the bulb becomes thinner and thinner with use. This increases the resistance $\left[R = \rho L/\pi r^2\right]$ of the bulb and as $P = V^2/R$ the brightness of light emitted by a bulb decreases gradually with time.

Note: If the source voltage is greater than the rated voltage of a device, then a resistance is connected in series to the device to reduce the voltage drop across the device to its rated value or to reduce the current to its rated value.

R = resistance of the device

R_s = series resistance

V = applied voltage



The current through the circuit cannot be greater than the rated current $\therefore I \le I_r$

$$\frac{V}{R+R_s} \le \frac{V_r}{R} \qquad VR \le V_rR + V_rR_s$$

$$\therefore \quad \boxed{R_s \ge \frac{(V-V_r)R}{V_r}} \quad \boxed{(R_s)_{min} = \frac{(V-V_r)R}{V_r}}$$

If $V = 2V_r$, then $(R_s)_{min} = \frac{(2V_r - V_r)R}{V_r} = R$

- Some of the energy lost as heat energy across the series resistance.

Nodal Method of Circuit Analysis

It is based on Kirchhoff's junction law. At any node or junction in an electrical circuit $\sum I = 0$. This equation is referred to as nodal equation.

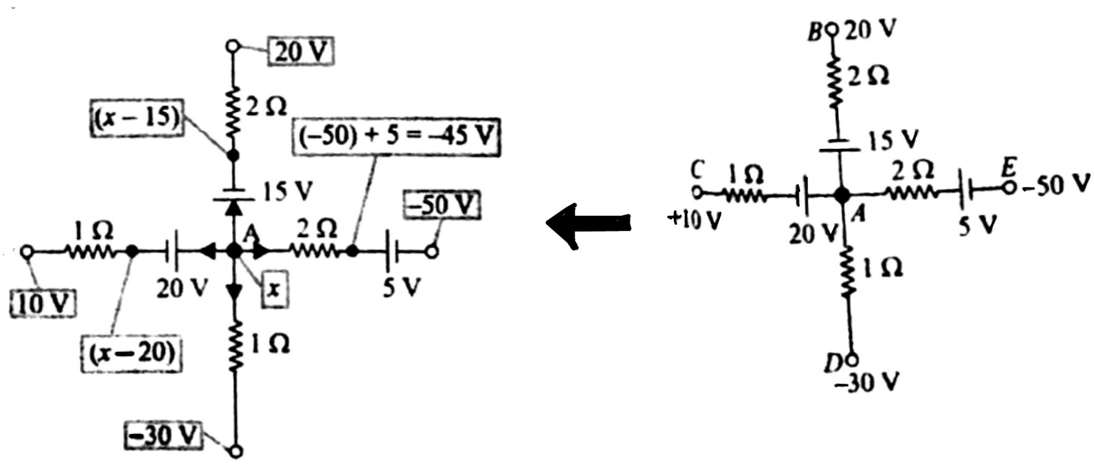
Assign potential of every junction of circuit taking potential of any one of the junctions of the circuit zero. Apply nodal equation to solve for unknown potential introduced in the circuit.

Note: In electrical circuits potential at the points where earthed are taken as zero.

Example

The resistance and batteries are connected as shown in the figure. Find out the potential at point A.

Answer



Let the potential at A, be x. Applying Kirchhoff's current law (nodal equation) at junction A,

$$\frac{(x-20)-10}{1} + \frac{(x-15)-20}{2} + \frac{x+45}{2} + \frac{x-(-30)}{1} = 0$$

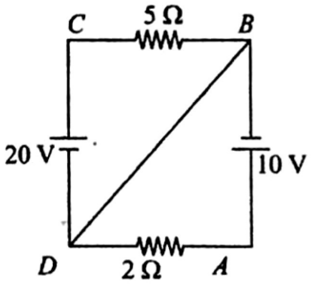
$$\frac{2x - 60 + x - 35 + x + 45 + 2x + 60}{2} = 0$$

6x + 10 = 0 or $\boxed{x = \frac{-5}{3} V}$

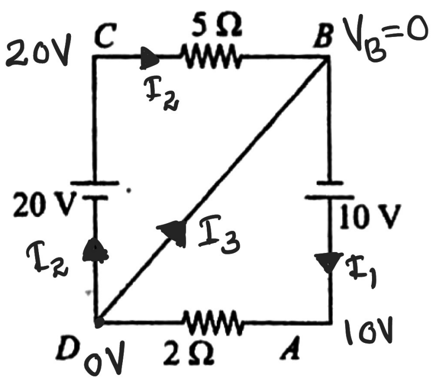
- Currents through resistances can be found out by using the potential at A.

Example

Find out the current in the wire BD



Answer



Let potential at 'B' be zero, then the potentials at other points are as shown.

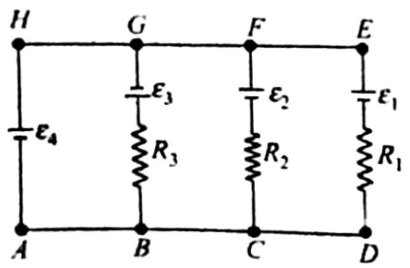
$$I_2 = \frac{20 - 0}{5} = 4A$$

$$I_1 = \frac{10 - 0}{2} = 5A$$

$I_1 = I_2 + I_3$ $\therefore 5 = 4 + I_3$ $\boxed{I_3 = 1A}$

Example

Find out the current in each wire:



Given.
$\varepsilon_1 = 50$ V, $\varepsilon_2 = 40$ V
$\varepsilon_3 = 30$ V, $\varepsilon_4 = 10$ V
$R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$,
$R_3 = 1\ \Omega$

Sol: $I_1 = \dfrac{40}{2} = 20\text{A}$

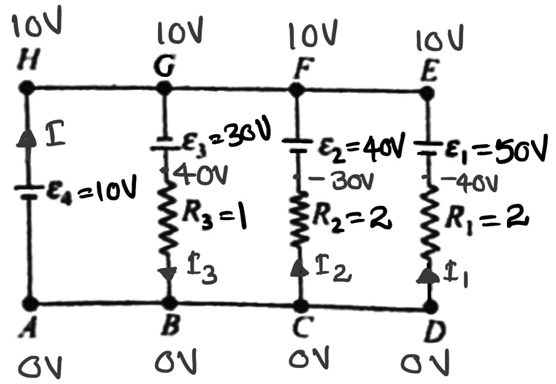
$I_2 = \dfrac{30}{2} = 15\,\text{A}$

$I_3 = \dfrac{40}{1} = 40\,\text{A}$

$I = I_3 - I_2 - I_1$

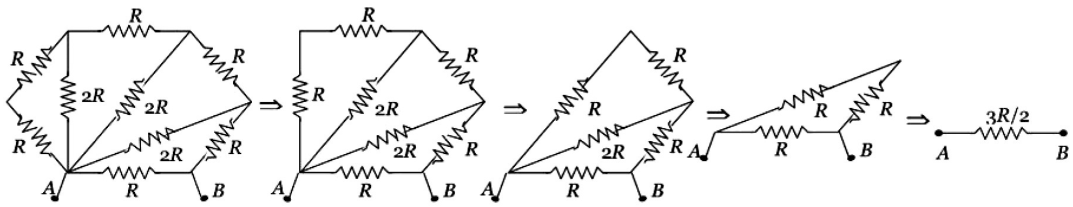
= 40 - 15 - 20

$I = 5\,\text{A}$



EQUIVALENT RESISTANCE OF COMPLEX NETWORKS

(1) **Method of successive reduction:** It is the most common technique to determine the equivalent resistance. So far, we have been using this method to find out the equivalent resistances. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations. For example to calculate the equivalent resistance between the point A and B the network shown below successively reduced.



(2) **Method of equipotential points:** This method is based on identifying the points of same potential and joining them. The basic rule to identify the points of same potential is the symmetry of the network.

The points of same potential can be found out by symmetry techniques. The most common symmetry are

- Parallel axis of symmetry
- Perpendicular axis of symmetry
- Shifted symmetry or as symmetry
- Path symmetry

Parallel axis of Symmetry

This symmetry is along the direction of current flow.

About this axis, all characteristics such as current and potential should be symmetrical. Points lying along the parallel axis of symmetry can never have same potential.

The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential.

Perpendicular Axis of Symmetry

It is perpendicular to the direction of current. Here circuit diagram is symmetric except for the fact that the input and output are reversed.

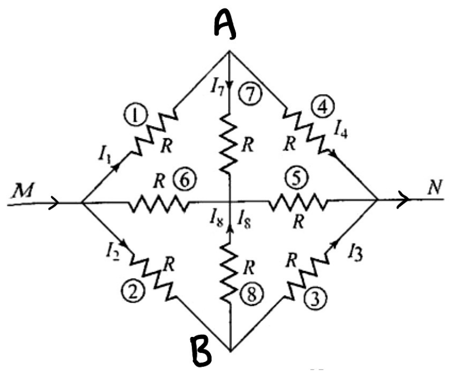
Points lying on the perpendicular axis of symmetry may have same potential.

Path Symmetry

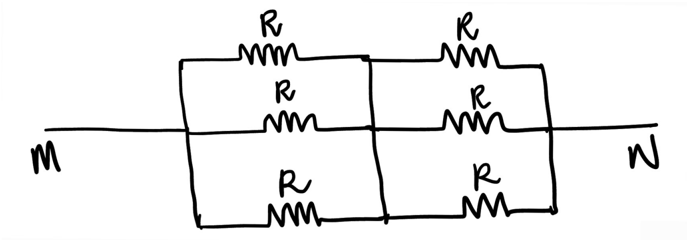
According to path symmetry, if all the paths from one point to another point have the same configuration of resistance or capacitance, then the charge or current into the beginning of the path must be same.

Example

Find out the effective resistance between M and N

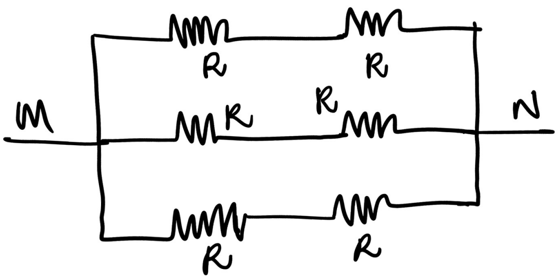


The axis MN is the parallel axis of symmetry. From parallel axis of symmetry $I_1 = I_2, I_3 = I_4$ and $I_7 = I_8$. Also potentials $V_A = V_B$ $\therefore$ A and B can be connected together. Thus the circuit can be redrawn as.



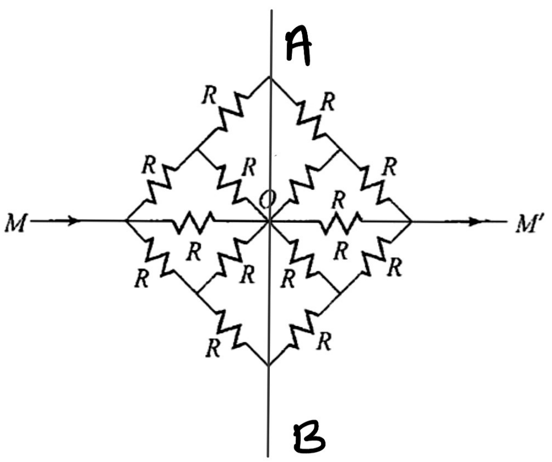
So effective resistance $R_{MN} = \frac{2R}{3}$

The above circuit has perpendicular axis of symmetry. From this $I_1 = I_4, I_6 = I_5$ and $I_2 = I_3$. thus currents through resistors (7) and (8) are zero and they can be removed from the circuit.

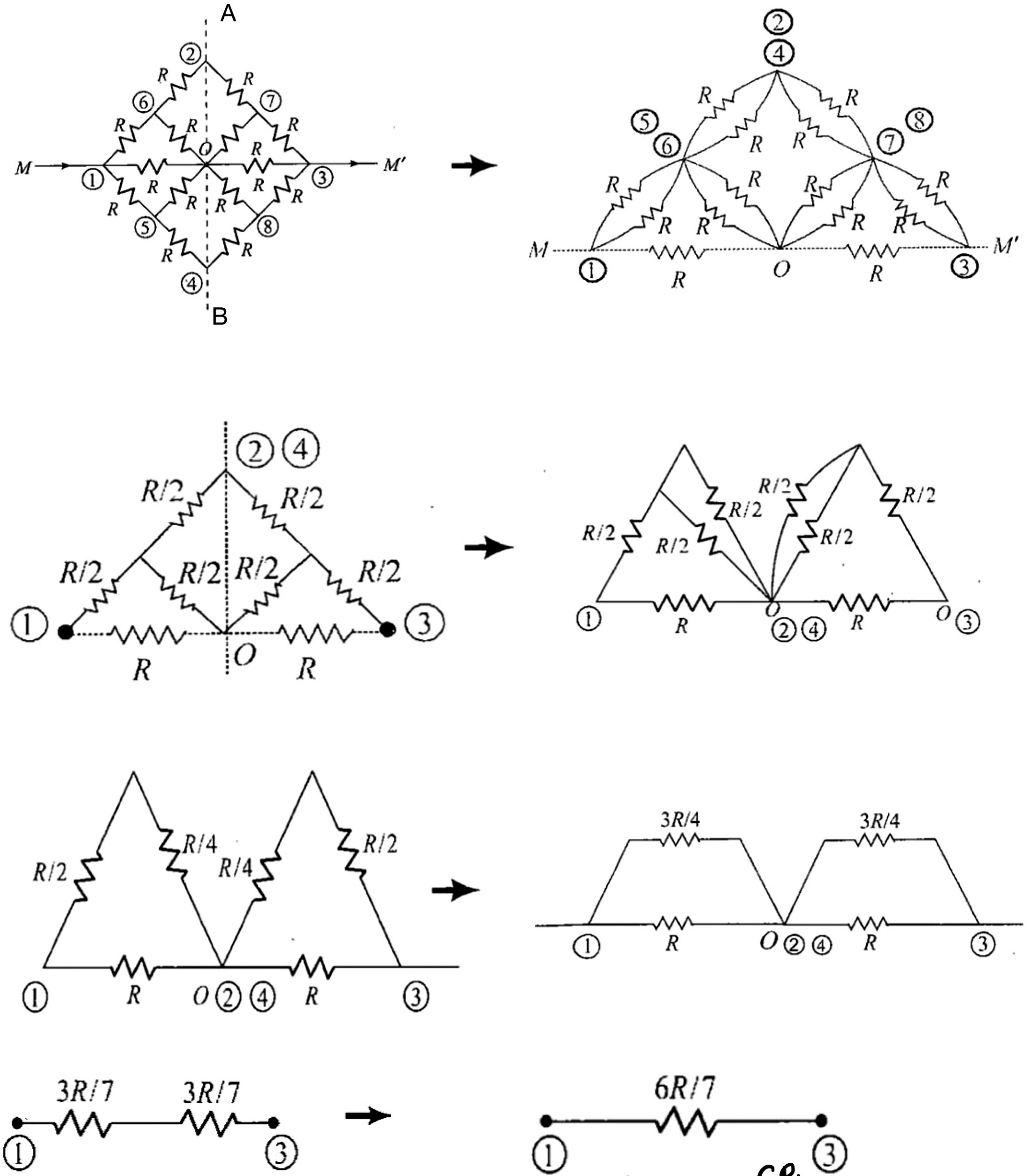


Thus $R_{MN} = \frac{2R}{3}$

- Find out Effective resistance between M and M'



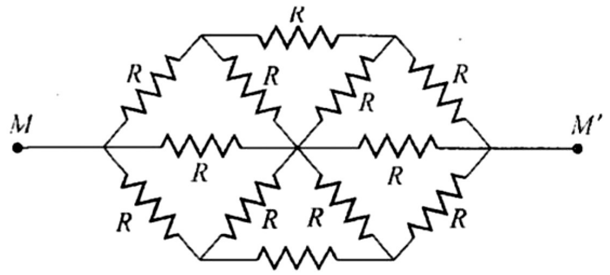
The circuit has parallel axis of symmetry $(\mathrm{MM'})$. Therefore the circuit can be folded about the axis $\mathrm{MM'}$.



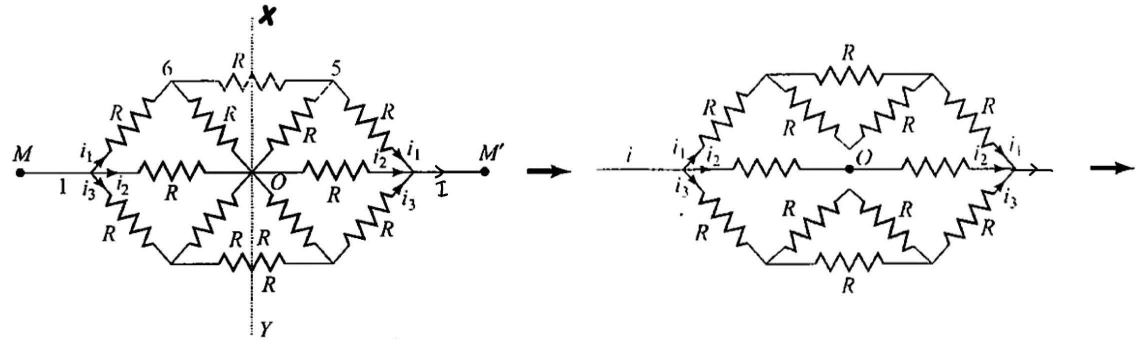
$R_{mm'} = \frac{6R}{7}$

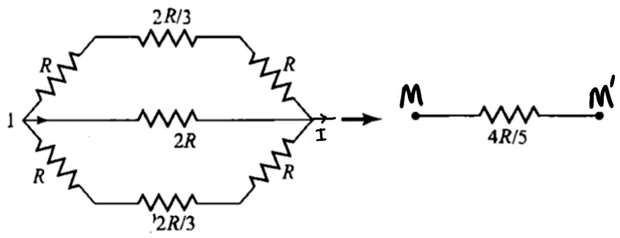
- The above circuit has perpendicular axis of symmetry about the axis AB. Therefore A, O and B can be connected together.

Example:



The circuit has perpendicular axis of symmetry.
Thus the circuit can be redrawn as shown

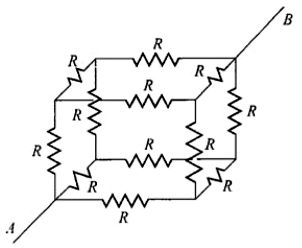




$R_{MM'} = \frac{4R}{5}$

Example

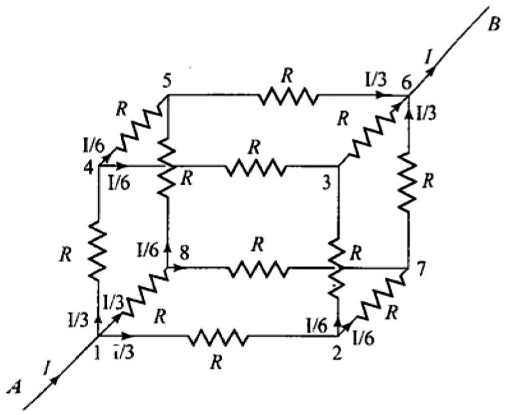
Consider a more complex example where you have resistors an all edges of a cube. The resistors are all the same. Then find the equivalent resistance between the corners A and B as shown in figure.



From symmetry $I_{12} = I_{14} = I_{18} = \frac{I}{3}$

$$I_{27} = I_{23} = \frac{1}{2}\left(\frac{1}{3}\right) = \frac{I}{6}$$

Similarly, we can say that $I_{43} = I_{45} = I_{85} = I_{87} = \frac{I}{6}$



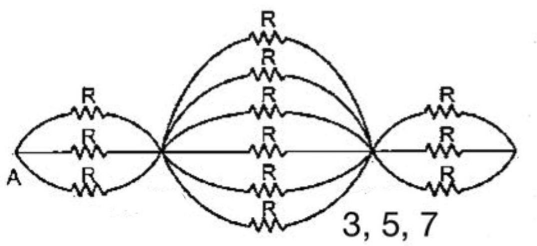
Therefore, to calculate the potential difference across 1-6 (see figure) the path A_{1276}, we have

$$V_1 - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = V_6 \Rightarrow V_1 - V_6 = I\left[\frac{R}{3} + \frac{R}{6} + \frac{R}{3}\right] = \frac{5}{6}IR$$

Hence, $\frac{(V_1 - V_6)}{I} = \frac{5}{6}R$; Therefore, the equivalent resistance is $\frac{5}{6}R$

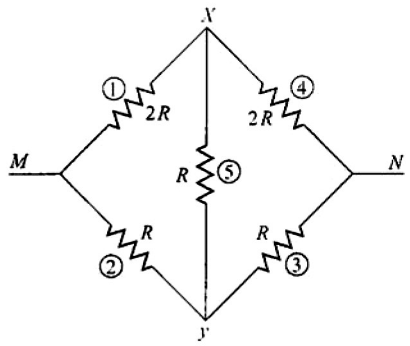
Method 2: In between A and B, symmetry of the circuit indicates that 2,8,4 are at equal potential and similarly 3,5,7. So the cube may be redrawn as

$R_{eq} = \frac{R}{3}\text{'+'}\frac{R}{6}\text{'+'}\frac{R}{3}$ where '+' stands for the series $= \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}R$

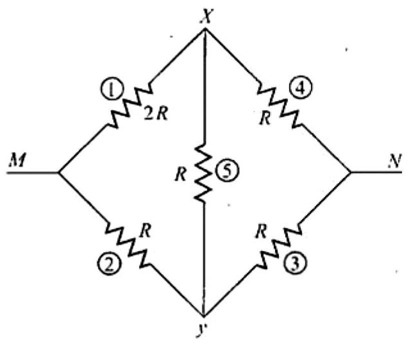


Shifted Symmetry

Shifted symmetry is the same as the parallel axis of symmetry and the perpendicular axis of symmetry principles, except that the symmetry is shifted.



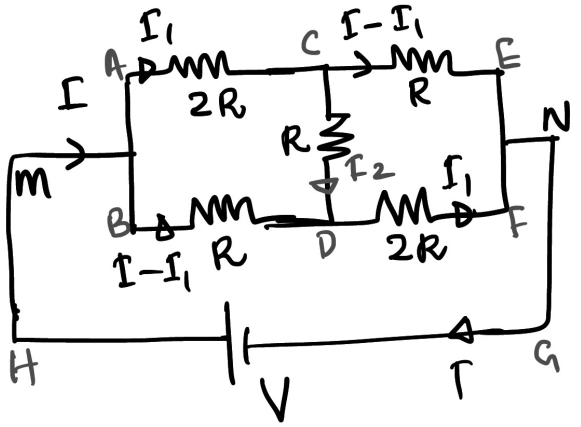
(a)



(b)

The first circuit has perpendicular axis of symmetry. $\therefore$ $I_1 = I_4$ and $I_2 = I_3$.

If we interchange the positions of (2) and (4) the circuit (b) becomes symmetric. This is called shifted symmetry. If the circuit has shifted symmetry then the currents $I_1 = I_3$ and $I_2 = I_4$



For the loop ACDBA

$$-I_1 2R - I_2 R + (I - I_1)R = 0$$

$$-3I_1 R - I_2 R + IR = 0$$

$$I = 3I_1 + I_2 \qquad ...(1)$$

For loop MBDFNGHM

$$-(I - I_1)R - 2RI_1 + V = 0$$

$$-IR - I_1 R + V = 0 \qquad ...(2)$$

From Kirchoff's first law at junction C,

$$I_1 = I_2 + I - I_1$$

$$I_2 = 2I_1 - I \qquad ...(3)$$

Put (3) in (1)

$$I = 3I_1 + 2I_1 - I$$

$$\boxed{I_1 = \frac{2I}{5}}$$

$$\boxed{I - I_1 = \frac{3I}{5}}$$

$$I_2 = \frac{-I}{5}$$

$$\boxed{I_2 = \frac{I}{5} \text{ from D ro C}}$$

From equation (2) $V = IR + \frac{2I}{5}R$

$\frac{V}{I} = \frac{7R}{5}$ But $R_{eq} = \frac{V}{I}$ $\therefore \boxed{R_{eq} = \frac{7R}{5}}$

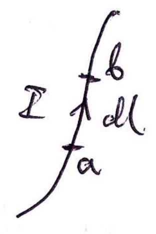
MOVING CHARGES & MAGNETISM

The branch of physics which deals with the magnetism due to electric current or moving charge is called electromagnetism.

The relation between electricity and magnetism was discovered by Oersted in 1820.

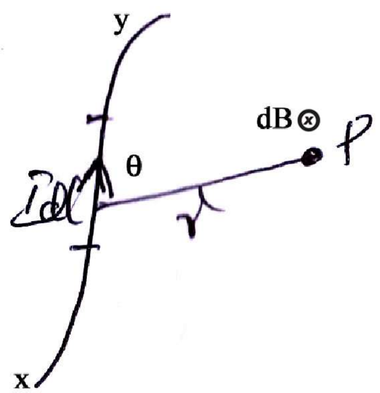
Current Element:

A very small element of length 'dl' of a thin conductor carrying current I called current element. Current element is a vector quantity whose magnitude is equal to the product of current and length of small element having the direction of the flow of current



Biot-Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to current carrying conductor.



According to Biot-Savart's Law, magnetic field at any point 'P' due to the current element Idl is dB.

dB α I, dB α dl, $dB\,\alpha\,\sin\theta$, $dB\,\alpha\,\frac{1}{r^2}$

i.e. $dB\,\alpha\,\frac{Idl\sin\theta}{r^2}$ or $dB = \frac{KIdl\sin\theta}{r^2}$

$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$ In S.I. unit

In C.G.S. unit k = 1 and In S.I. unit $k = \frac{\mu_0}{4\pi}$

S.I. unit of $\overline{B}$ is Weber / m^2 or Tesla.

In C.G.S unit is Gauss (G) or Maxwell / cm^2

IT = 10^4 gauss

where μ_0 = absolute permeability of air

or vacuum = $4\pi\times10^{-7}\frac{Wb}{A-m}$ its other unit are

$\frac{henry}{metre}$ or $\frac{N}{A^2}$ or $\frac{Tesla-metre}{Ampere}$

<u>Vector form of Biot-Savart's Law</u>

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(\vec{dl}\times\vec{r}\right)}{r^3} = \frac{\mu_0}{4\pi}\times\frac{I\left(\vec{dl}\times\hat{r}\right)}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}$$

where $\hat{n}$ is a unit vector which is perpendicular to both $I\vec{dl}$ and $\vec{r}$ i.e.B is perpendicular to both Idl and r for $\theta = 0$ or 180 $\sin\theta = 0$ i.e. magnetic field on the axis of a current carrying conductor is always zero.

In terms of current density,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{\left(\vec{J}\times\vec{r}\right)dv}{r^3}$$

J $\rightarrow$ current density

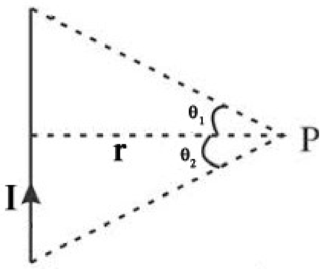
dv $\rightarrow$ small volume element

<u>Direction of magnetic field :</u>

<u>Right hand thumb rule :</u> According to this rule if a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow then the direction of folding fingers will represent the direction of magnetic lines of force.

Application of Biot-Savart's Law

(1) Field due to finite length straight conductor carrying current



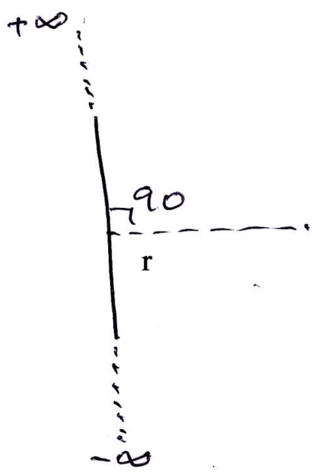
field at point P,

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\left[\sin\theta_1 + \sin\theta_2\right]$$

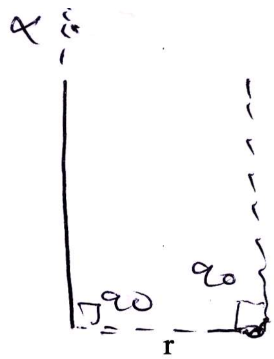
$r \rightarrow$ perpendicular distance to point P from the conductor

2) If the conductor is infinitely long, then $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$

$$B = \frac{\mu_0 I \times 2}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$



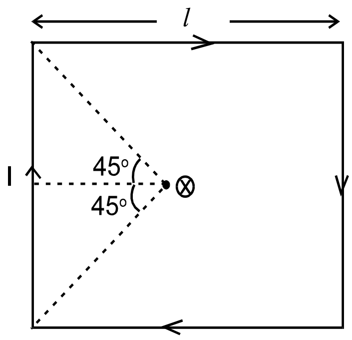
3) Magnetic field due to special semi infinite length wire at point P



$\theta_1 = 90^{\circ}, \theta_2 = 0$

$$B = \frac{\mu_0 I}{4\pi r}$$

Magnetic field at the centre of a square loop



Field due to one side at centre

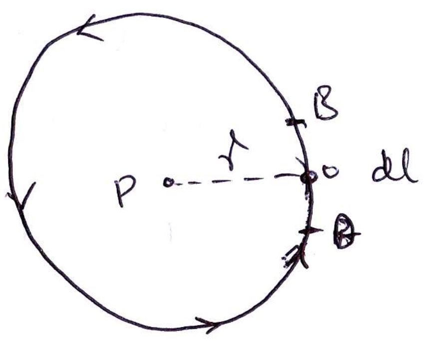
$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{\ell/2}\left[\sin 45^\circ + \sin 45^\circ\right]$$

$$= \frac{2\mu_0 I}{4\pi\ell}\,\frac{2}{\sqrt{2}}$$

Net field at centre

$$B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi\ell}$$

Magnetic field at the centre of a circular current carrying loop



$$dB = \frac{\mu_0}{4\pi}\times\frac{Idl\sin 90}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$\therefore B = \int_0^{2\pi r} \frac{\mu_0}{4\pi}\frac{I}{r^2} dl$$

$$\boxed{B = \frac{\mu_0}{2}\frac{I}{r}}$$

For a coil of N turns, $B = \frac{\mu_0}{2}\frac{NI}{r}$

The direction of the magnetic field at the centre of the circular coil can be obtained by using right hand thumb rule. If the fingers are curled along the current, then the stretched thumb will point towards the magnetic field.

Current loop as a 'magnetic dipole'

The face of the coil in which current appears to flow anticlockwise acts as magnetic north pole.

The face of the coil in which current appears to flow clockwise acts as magnetic south pole.

Magnetic field due to current carrying circular segment subtending an angle θ at the centre.

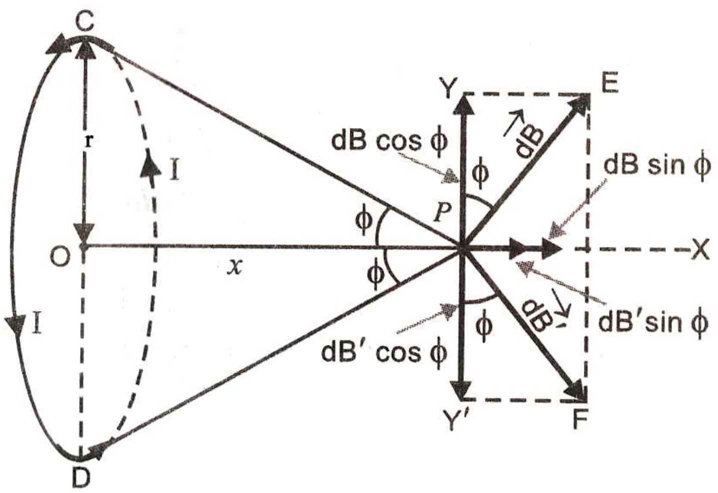
$$dB = \frac{\mu_0}{4\pi}\frac{I\ell}{r^2}$$

$$d\ell = rd\theta$$

$$dB = \frac{\mu_0}{4\pi}\frac{Ird\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\theta}{r}$$

$$B = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{Id\theta}{r} = \frac{\mu_0 I}{4\pi r}\theta$$

Magnetic field on the axis of a circular coil



$$dB = \frac{\mu_0}{4\pi} \frac{Id\ell \sin\theta}{a^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{Id\ell}{a^2} \qquad \left(\theta = 90^{o}\right)$$

$$\therefore B = \oint dB \sin\phi = \oint \frac{\mu_0}{4\pi} \frac{Id\ell}{a^2} \sin\phi = \frac{\mu_0 I}{4\pi a^2} \sin\phi \times 2\pi r$$

$$B = \frac{\mu_0 Ir}{2a^2} \times \frac{r}{a} = \frac{\mu_0 Ir^2}{2a^3} = \frac{\mu_0 Ir^2}{2\left(r^2 + x^2\right)^{3/2}}$$

It act along the axis of a circular coil. If the coil consist of N turns then

$$B = \frac{\mu_0 NIr^2}{2\left(r^2 + x^2\right)^{3/2}}$$

Special Cases

1) at the centre of the loop x = 0

$$B = \frac{\mu_0 NIr^2}{2r^2} \text{ or } \frac{\mu_0 NI}{2r}$$

2) If the observation point is far away from the coil i.e. r < < x

$$B=\frac{\mu_0 NIr^2}{2x^3}=\frac{\mu_0}{2\pi}\frac{NIA}{x^3}=\frac{\mu_0}{4\pi}\times\frac{2NIA}{x^3}=\frac{\mu_0}{4\pi}\times\frac{2m}{x^3}$$

The quantity NIA is known as the magnetic dipole moment M of the current loop.

Current loop as a magnetic dipole

Based on the fact that the magnetic field of a current loop is identical with that of a magnetic dipole, it was speculated by ampere in 1820 that all magnetism is due to current loops and this speculation is indeed correct.

Ampere's circuital law

The line integral $\oint B.d\ell$ for a closed curve is equal to μ_0 times the net current I through the area bounded by the curve.

$$\boxed{\text{i.e.} \oint B.d\ell=\mu_0 I}$$

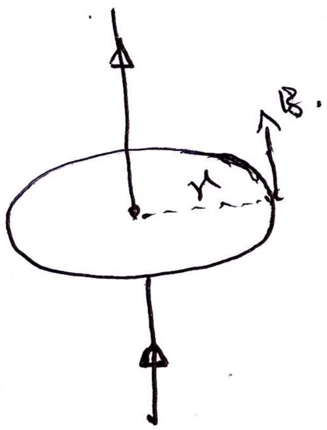
Applications of Ampere's Law

Magnetic field due to an infinite current carrying conductor

$$\oint B.d\ell=\mu_0 I$$

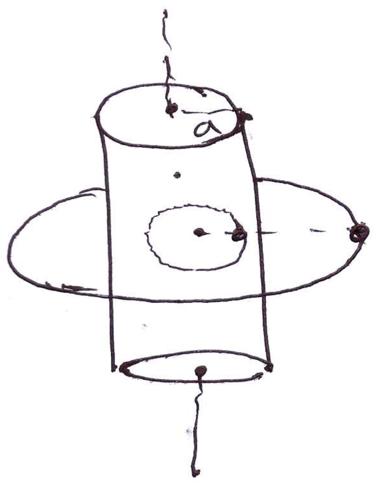
$$B\times 2\pi r=\mu_0 I$$

$$B=\frac{\mu_0 I}{2\pi r}$$



Magnetic field produced by a current along a circular cylinder of infinite length

1. When observation point is outside the cylinder



i.e. r > a

$$\oint B.d\ell = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{i.e. } B \,\alpha\, \frac{1}{r}$$

2. When observation point is on the surface of the cylinder $(r = a)$

$$B = \frac{\mu_0 I}{2\pi a}$$

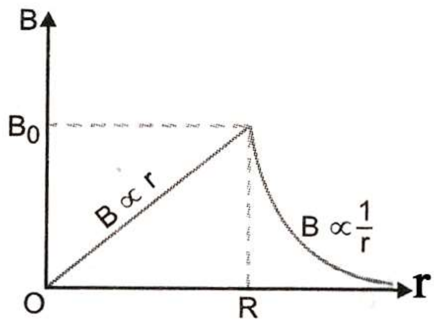
3. When observation point is inside the cylinder

$$\oint B.d\ell = \mu_0 I^1 \qquad \text{where } I^1 = \frac{I}{\pi a^2} \times \pi r^2$$

$$B \times 2\pi r = \mu_0 \frac{I}{\pi a^2} \times \pi r^2$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

Variation of B with r



Solenoid

A solenoid is used to generate magnetic field. A long solenoid is one whose length is very large, compared to its radius.

A solenoid consists of a long metallic insulated wire wound in the form of a helix, where the neighbouring, turns are closely spaced. Each turn can be regarded as a cicular loop.

Field inside the solenoid, at centre.

$$B = \mu_0 n I$$

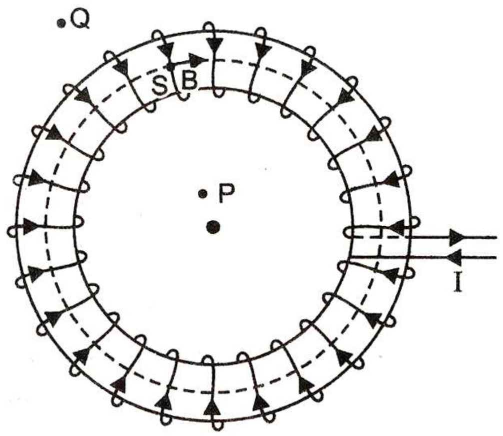
where, $n = \frac{N}{\ell}$ (number of turns per unit length)

at end point

$$B = \frac{\mu_0 nI}{2}$$

For a long solenoid the inside field is almost uniform, and out side field is near to zero.

Toroid



i) The field at a point such as P is zero. This is because the circle through P does not encloses any current

ii) The field at a point such as 'Q' is also zero. This is because each turn of the winding passes twice through the area enclosed by the circle through r, carrying equal currents in opposite directions. So the net current enclosed by this circle is zero.

iii) Inside the solenoid point such as S

$$\oint B.d\ell = \mu_0 I_0$$

$$B \times 2\pi r = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI$$

LORENTZ MAGNETIC FORCE

Consider a positive change 'q' moving in a uniform magnetic field $\vec{B}$ with a constant velocity v. The change 'q' will experience a force 'F_m' known as Lorentz magnetic force. It is given by

$$F_m = q\left(\vec{V} \times \vec{B}\right) = Bqv\sin\theta\hat{n}$$

$$\left|F_m\right| = Bqv\sin\theta$$

The direction of magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$

$\therefore$ Work done by magnetic force = 0

K.E. of the particle remains constant and magnitude of velocity remains constant

Note

1. If the charge is at rest (v = 0) then $F_m = 0$ So, a stationary charge in a magnetic field experiences no magnetic force.
2. If $\theta = 0^\circ$ or 180° i.e. If the change moves parallel to the direction of the magnetic field then $F_m = 0$
3. If the charge moves perpendicular to the direction of the magnetic field i.e. $\theta = 90^\circ$ then $F_m = Bqv$

The direction of F_m can be determined by Fleming's Left hand rule.

Statement : Stretch the middle finger, fore finger and thumb of the left hand in mutually perpendicular directions. If the fore finger points in the direction of the magnetic field, the middle finger points in the direction of motion of the +ve charge, then the thumb gives the direction of the force.

- Force on a moving charge in uniform electric and magnetic fields

$$\vec{F} = q\vec{E} + q\left(\vec{V} \times \vec{B}\right) = q\left(\vec{E} + \vec{V} \times \vec{B}\right)$$

- S.I unit of magnetic induction 'B' is Tesla and its c.g.s and is gauss.

IT = 10^4 gauss

Charged particle moving in a uniform magnetic field

Force on a charged particle moving in a uniform magnetic field

$$\vec{F}_m = q\left(\vec{V} \times \vec{B}\right) = BqV \sin\theta\, \hat{n}$$

Case I : When the charged particle is moving parallel or antiparallel to the magnetic field

i.e. $\theta = 0^\circ$ or 180°

Particle moves along a straight line path

$$F_m = BqV \sin 0 \text{ or } BqV \sin 180 \text{ or } F_m = 0$$

Case II : When changed particle enter perpendicular to the magnetic field

$$F_m = q\left(V \times B\right) = BqV \sin 90 = BqV$$

The direction of force is given by Fleming's left hand rule. In this case the centripetal force is provided by the Lorentz magnetic force. So, the changed particle follows a circular path

$$BqV = \frac{mv^2}{r} \quad \therefore r = \frac{mV}{Bq}$$

and time period of revolution $T = \dfrac{2\pi r}{V}$

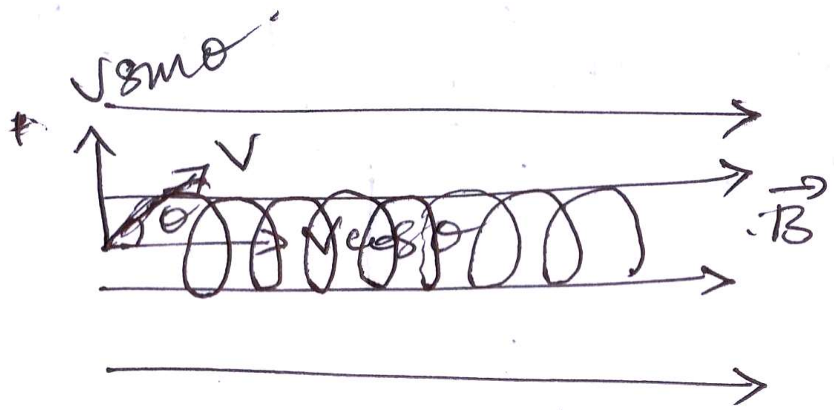
$= \frac{2\pi}{V}\frac{mv}{Bq} = \frac{2\pi m}{Bq} = \frac{2\pi}{B\left(q/m\right)}$

frequency $\nu = \frac{Bq}{2\pi m}$

Angular frequency $\omega = 2\pi\nu = 2\pi \times \frac{Bq}{2\pi m}$

or $\omega = \frac{Bq}{m}$

Case III : When a changed particle moves at an angle θ to a uniform magnetic field B such that $\theta \neq 0, \neq 90$ and $\theta \neq 180^{\circ}$



Then the charged particle will follow a helical path

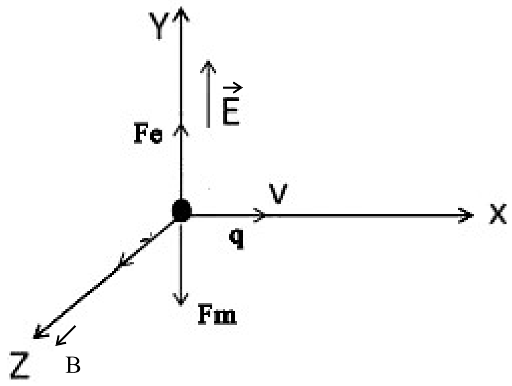
Pitch of the helical path. It is the linear distance covered by the changed particle in the magnetic field in a time during which the changed particle covers one revolution of its circular path

i.e. pitch of the helix = $V\cos\theta \times T$

$= V\cos\theta \times \frac{2\pi m}{Bq}$	Time period $T = \frac{2\pi m}{Bq}$
radius $r = \frac{mV\sin\theta}{Bq}$	and frerquency $\nu = \frac{Bq}{2\pi m}$

Motion of a charge particle in combined electric and magnetic fields

(1) Velocity selector



The charge particle experiences both electric and magnetic forces

The electrostatic force

$$\vec{F}_e = qE\hat{j}$$

The magnetic force

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right) = q\left(v\hat{i} \times B\hat{k}\right)$$

$$\vec{F}_m = -qvB\hat{j}$$

$\therefore$ Net force

$$\vec{F} = \vec{F}_e = +\vec{F}_m$$

$$\vec{F} = q\left(E - vB\right)\hat{j}$$

The electric and magnetic forces are in opposite directions

When, $F_e = F_m$

Net fofce = 0

qE = qvB

$$v = \frac{E}{B}$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds

- **Force on a current-carrying conductor in a uniform magnetic field**

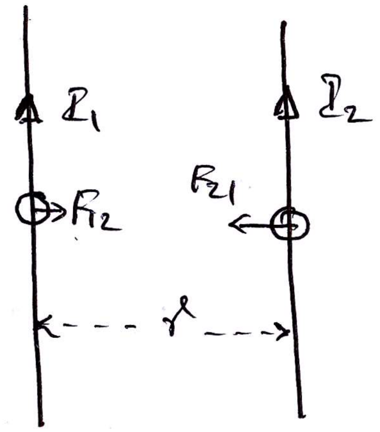
$$\vec{F} = I\left(\vec{\ell} \times \vec{B}\right)$$

The magnitude of force F = $BI\ell\sin\theta$

- The direction of the force is given by Fleming's Left hand Rule

  Statement : If the forefinger, middle finger and thumb of the left hand are held in the mutually perpendicular directions such that the forefinger shows the direction of the magnetic field, the middle finger shows the direction of the current, then the thumb will points in the direction of the force on the current carrying conductor.

  Forces between two parallel current carrying conductors



Force on first conduction due to

Second one $= F_{12} = B_2 I_1 \ell \sin 90$

$$= \frac{\mu_0 I_2}{2\pi r} I_1 \times \ell$$

$$F_{12}/\ell = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Force on second due to first one

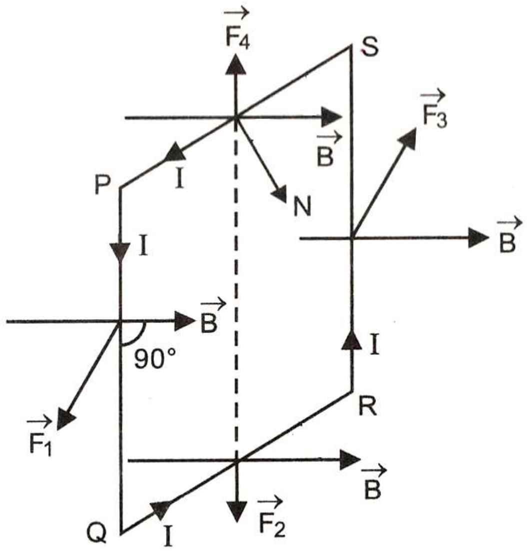
$$= F_{21} = B_1 I_2 \ell \sin 90 = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$$

$$F_{21}/\ell = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Note :

1. Two parallel current carrying conductors attract each other if the currents are in the same direction and repel each other if the currents are in the opposite direction.
2. The force between two parallel current carrying conductor is proportional to the product of the current strengths and the length of the conductor under consideration and varies inversely as the distance between them.

Torque on a current carrying coil in a magnetic field



The net magnetic force on a current loop in a uniform magnetic field is zero but a torque may acting on the loop.

$\vec{\tau} = \vec{M} \times \vec{B}$, the magnitude of torque is given by,

$\tau = MB\sin\theta$

θ is the angle between normal to the loop and direction of field.

Potential energy of the loop

$u = -MB\cos\theta$	if $\theta = 0^\circ$	if $\theta = 180^\circ$
$u = -\vec{M}.\vec{B}$	$\tau = 0$	$\tau = 0$
	$u = -MB$	$u = MB$
	stable equilibrium	unstable equilibrium

Moving coil Galvanometer

Principle : When a current carrying coil is placed in magnetic field, it experience a torque

Theory

Moment of reflecting couple = NBIA

$C \rightarrow$ torsional constant of spring

Moment of restoring couple = $C\theta$

For equilibrium of the coil

NBIA = $C\theta$

or $I = \frac{C\theta}{NBA}$ where $\frac{C}{NBA}$ is the Galvanometer constant.

Current sensitivity of a Galvanometer

The current sensitivity of a galvanometer is the deflection of the meter per unit current

$$I_S = \frac{\theta}{I} = \frac{NBA}{C}$$

Voltage sensitivity of a Galvanometer

It is defined as the deflection of the meter per unit voltage

$$V_S = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBA}{CR}$$

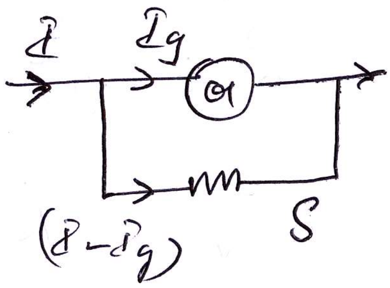
Conversion of Galvanometer to Ammeter

Ammeter is an instrument used specifically for measuring electric current.

Galvanometer can convert to ammeter by connecting a small resistance (s) in parallel with it.

$$I_g \times G = \left(I - I_g\right)S$$

$$S = \frac{I_g \times G}{\left(I - I_g\right)}$$



$G \rightarrow$ resistance of the galvanometer

The resistance of the ammeter Ra

$\rightarrow$ Resistance of ideal ammeter is zero

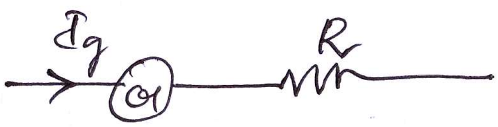
$$R_a = \frac{GS}{G+S}$$

The ammeter is always connected in series in the circuit because it doesn't alter the current due to its small resistance.

Conversion of Galvanometer to voltmeter

Voltmeter is an instrument for measuring potential difference

A galvanometer can be converted in to a voltmeter by connecting a high resistance in series with it.



$$V = I_g\left(G + R\right)$$

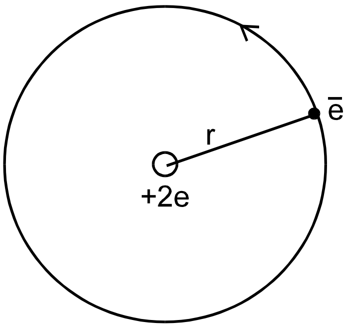
$$R = \frac{V}{I_g} - G$$

$$R_v = R + G$$

$\rightarrow$ Resistance of ideal voltmeter infinity

Voltmeter is always connected in parallel with the circuit because of its high resistance.

The magnetic dipole moment of a revolving electron



The electron performs uniform circular motion around a stationary nucleus.

The current

$$I = \frac{e}{T}$$

time period, $T = \frac{2\pi r}{v}$

$$I = \frac{ev}{2\pi r}$$

the magnetic moment,

$$\mu = IA = \frac{ev}{2\pi r} \times \pi r^2$$

$$\mu = \frac{evr}{2}$$

The angular momentum of the revolving electron

l = mvr

Gyromagnetic ratio = $\frac{\text{magnetic momet}}{\text{angular momentum}}$

$$\frac{\mu}{\ell} = \frac{e}{2m} = 8.8 \times 10^{10} \, C/kg$$

Bohr Magneton

According of Bohr's theory, angular momentum of orbital electron is given by

$L = \frac{nh}{2\pi}$. where n = 1,2,3,... and h is plank's constant

Magnetic moment of orbital electron is given by by $M = \frac{eL}{2m} = n\frac{eh}{4\pi m}$

- If n = 1 then $M = \frac{eh}{4\pi m}$, which is Bohr magneton denoted by μ_3

Definition of (μ_B)

Bohr magneton can be defined as the magnetic moment of orbital electron which revolves in first orbit of an atom.

$$\mu_B = \frac{eh}{4\pi m} = \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 0.923 \times 10^{-23} \text{ A.m.}^2$$

MAGNETISM AND MATTER

Magnetic field line

It is an imaginary curve, the tangent to which at any point give us the direction of magnetic field at that point.

Properties

✶ The magnetic field lines of a magnet (or a solenoid) from a continuous closed loops.

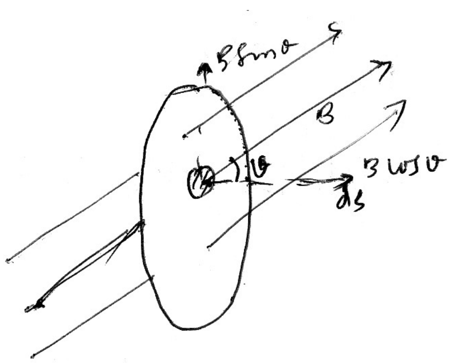
✶ The tangent to the field line at a given point represents the direction of the net magnetic field at that point.

✶ The larger the number of field lines crossing per unit normal area, the larger is the magnitude of magnetic field.

✶ The magnetic field lines do not intersect. This is so since the direction of the magnetic field would not be unique at the point of intersection.

Magnetic flux

It is the total no of magnetic field lines passing normally through unit area.



flux through the small area 'ds' is given by

$$d\phi = B\cos\theta.ds = \overline{B}.\overline{ds}$$

$\therefore$ Total flux through the entire surface is $\phi = \int_s \overline{B}.d\overline{s}$

Gauss's Theorem for Magnetism

The net magnetic flux through any closed surface is always zero.

$$\boxed{\phi_B = \oint_s \overline{B}.d\overline{s} = 0}$$

ie magnetic monopoles does not exist.

Magnetic dipole

A magnetic dipole consists of two unlike poles of equal strength and seperated by a small distance.

eg: Bar magnet, compass needle

Magnetic dipole moment of Bar magnet

It is the product of strength of either pole (m) and the magnetic length (2ℓ) of the magnet.

$$\boxed{\vec{M} = m \times 2\ell}$$

Magnetic dipole moment is a vector quantity directed from South to North pole of the magnet.

S.I unit $\rightarrow$ Am^2 or J/T

S.I unit of pole strength $\rightarrow$ Am.

Force of attraction or repulsion between two magnetic poles.

Force of attraction or repulsion between two magnetic poles of strength m_1 and m_2 separated by a distance r is directly proportional to the product of the strength and inversely proportional to square of the distance between them.

$$F \, \alpha \frac{m_1 m_2}{r^2}$$

$$F = \frac{k m_1 m_2}{r^2}$$

$$k = \frac{\mu_0}{4\pi}$$

$$\boxed{F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}}$$

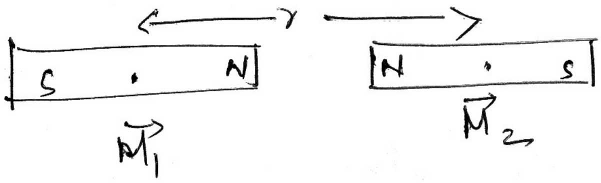
$\mu_0 \rightarrow$ magnetic permeability of free space or air.

$$\mu_0 = 4\pi \times 10^{-7} \, wb \, A^{-1} m^{-1}$$

Force between short Bar magnets

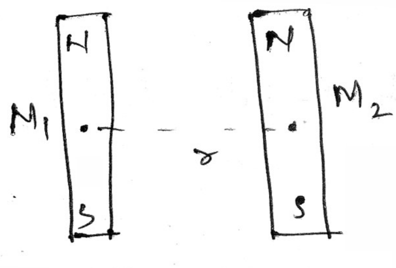
Consider two short bar magnets of magnetic moments of M_1 and M_2 where centres are separated by a small distance 'r'.

1) When they are in Axial position



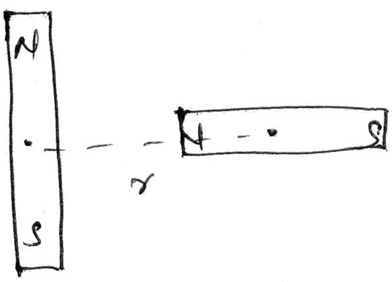
$$\boxed{F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4}}$$

2) When they are in equatorial position



$$\boxed{F = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4}}$$

3) When the two magnets are $\perp$ to each other



$$\boxed{F = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4}}$$

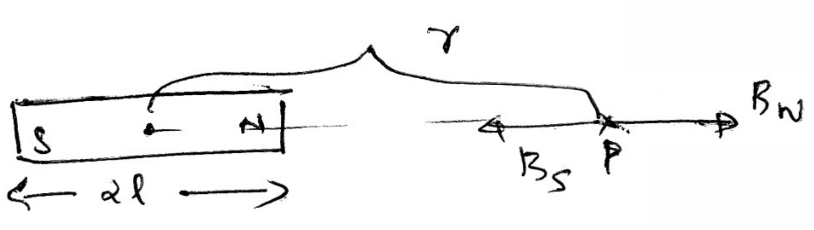
<u>Magnetic field intensity due to Bar magnet</u>

The magnetic field at any point is defined as the force experienced by a hypothetical unit, north pole placed at that point.

If a magnetic north pole of pole strength 'm' is placed in field of intensity $\vec{B}$, it experiences a force $\vec{F} = m\vec{B}$ directed along the direction of field.

If a magnetic South pole of pole strength 'm' placed in field of intensity $\vec{B}$, it experiences the force of same magnitude, but in opposite direction of the field.

Axial field



Field at point P due to N-pole $B_N = \frac{\mu_0}{4\pi}\frac{m}{(r-\ell)^2}$ (directed away from pole)

Field at point P due to R-pole $B_S = \frac{\mu_0}{4\pi}\frac{m}{(r+\ell)^2}$ (directed towards the pole)

Net field intensity, $B_{axial} = B_N - B_S$

$$= \frac{\mu_0}{4\pi} m\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

$$= \frac{\mu_0}{4\pi} m \times \frac{4r\ell}{(r^2-\ell^2)^2}$$

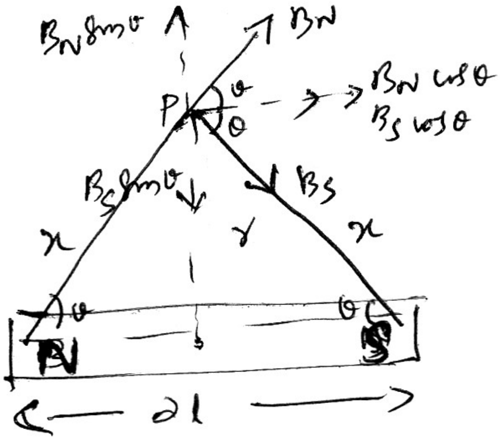
$$\boxed{B_{axial} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-\ell^2)^2}}$$

For short magnet $r^2 >> \ell^2 \therefore\ \ell^2$ can be neglected

$$\boxed{B_{axial} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}}$$

direction is in the direction of dipole moment vector.

Equatorial field



$$B_S = B_N = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

But they are inclined at angle 2θ

$\therefore$ Net field intensity

$$B_{equitorial} = B_N \cos\theta + B_S \cos\theta = 2B_N \cos\theta$$

$$= 2\times\frac{\mu_0}{4\pi}\frac{m}{r^2}\times\frac{\ell}{r} = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

$$x = \left(r^2+\ell^2\right)^{½}$$

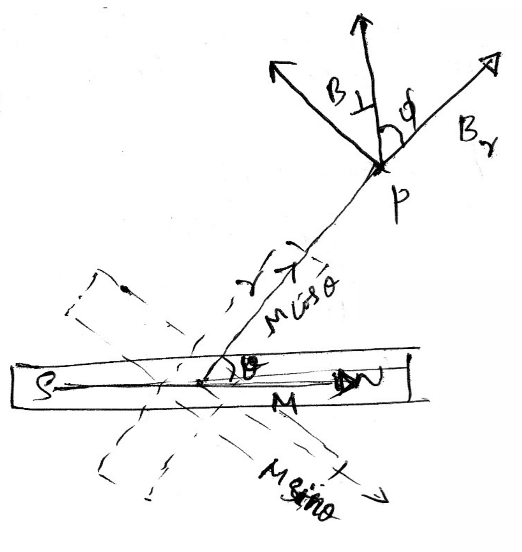
$$\therefore \boxed{B_{equitorial} = \frac{\mu_0}{4\pi}\;\frac{M}{\left(r^2+\ell^2\right)^{3/2}}}$$

For short Bar magnet $r^2 >> \ell^2, \therefore \ell^2$ can be neglected.

hence $\boxed{B_{equitorial} = \frac{\mu_0}{4\pi}\frac{M}{r^3}}$

Direction is opposite to the direction of dipole moment.

3) **At any point**



$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}$$

$$B_\perp = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

$\therefore$ Net field $B = \sqrt{B_r^2 + B_\perp^2}$

$$= \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{3\cos^2\theta + 1}}$$

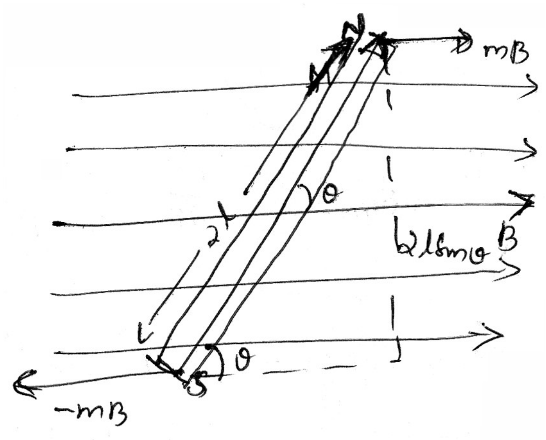
$$\tan\phi = \frac{B_\perp}{B_r} = \frac{\sin\theta}{2\cos\theta}$$

$$\therefore \boxed{\tan\phi = \frac{1}{2}\tan\theta}$$

ϕ is the angle between net field and the radius vector joining the point P and centre of dipole.

θ is the angle between dipole moment vector and the radius vector to the point P.

Torque on a bar magnet in a magnetic field



A bar magnet is held at an angle θ with uniform magnetic field B, then $F_{net} = -mB + mB = \theta$

ie magnet is in translational equilibrium.

But the two equal and opposite forces constitute a couple.

Torque, $\tau = \text{Force} \times \perp$ distance between the forces $= mB \times 2\ell \sin\theta$

$\tau = MB\sin\theta$

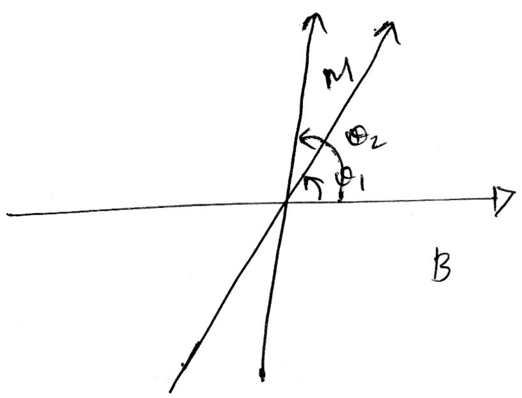
Invector form, $\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$

when $\theta = 0^\circ, \tau = 0$ stable equilibrium

when $\theta = 180^\circ, \tau = 0$ unstable equilibrium

when $\theta = 90^\circ, \tau = MB$ maximum torque

Work done in rotating a dipole



The small amount of work done in rotating the dipole through a small angle $d\theta$ against the restoring is

$$dW = \tau d\theta = MB\sin\theta\, d\theta$$

Total work done in rotating the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB(-\cos\theta]_{\theta_1}^{\theta_2} = -MB\cos\theta_2 -^{-} MB\cos\theta_1$$

$$\boxed{W = MB(\cos\theta_1 - \cos\theta_2]}$$

According to work - Energy theorem

$$W = \Delta U = U_f - U_i$$

$$U_f = -MB\cos\theta_2 \qquad U_i = -MB\cos\theta_1$$

$\therefore$ In general $\boxed{U = -MB\cos\theta = -\bar{M}.\vec{B}}$

When $\theta = 0$, U = -MB P.E is minimum. This is the position of stable equilibrium.

When $\theta = 180^o, U = +MB$ P.E is maximum. This is the position of unstable equilibrium.

Magnetic Materials

Properties of Magnetic Materials

1. Magnetic permeability (μ)

It is the ability of material to permit passage of magnetic lines of force through it.

Relative magnetic permeability is the ratio of number of magnetic field lines per unit area (then density B) in that material to the number of magnetic field lines per unit area that would be present, if the medium were replaced by vacuum (B_0).

$$\boxed{\mu_r = \frac{B}{B_0}}$$

It is also defined as the ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0).

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

Magnetic force/magnetising Intensity $\left(\vec{H}\right)$

Consider a solenoid with ‘n’ turns per unit length carrying a current $\overset{\circ}{\ell}$ would round a magnetic material. The magnetic induction of the field produced in the material.

$B_0 = \mu_0 ni$

The product nI is called magnetising force or magnetizing intensity H.

$\boxed{H = ni}$

If inside solenoid, there is free space, the magnetic induction $\boxed{B_0 = \mu_0 H}$

SI unit of $H \rightarrow$ A/m

Intensity of magnitisation (I)

It is the magnetic moment unit volume of the material.

$$\boxed{I = \frac{M}{V}}$$

unit $\rightarrow A/m$

Magnetic susceptibility (X)

It is the ratio of intensity of magnetisation (I) to the magnetising force (H) applied on it.

$\boxed{\chi = \frac{I}{H}}$ It has no units and no dimensions.

Relation between relative permeability and susceptibility

When a magnetic material is placed in a magnetising field H, the material gets magnetised. The total magnetic induction B in the material is the sum of magnetic induction B_0 in vacuum produced by magnetising intensity and magnetic B_m, due to magnetisation of material,

$$\therefore B = B_0 + B_m$$

$B_0 = \mu_0 H$, $\qquad B_m = \mu_0 I$

$$\boxed{B = \mu_0 \left(H + I\right)}$$

$$\chi = \frac{I}{H}$$

$$B = \mu_0 H\left(1 + \frac{I}{H}\right) = \mu_0 H\left(1 + X\right)$$

$$B = \mu H$$

$$\mu H = \mu_0 H(1 + X)$$

$$\frac{\mu}{\mu_0} = 1 + X$$

$$\boxed{\mu_r = 1 + X}$$

Diamagnetic Materials

- ✶ The diamagnetic substances are those in which individual atoms/molecules do not possess any net magnetic moment on their own.
- ✶ When they are placed in an external magnetic field they get fully magnetized is a direction opposite to the magnetising field.
- ✶ When placed in a non-uniform magnetic field, those substances have a tendency to move from stronger parts of the field to the weaker parts.
- ✶ When a specimen of diamagnetic material is placed in magnetising field, the magnetic field lines prefer not to pass through the specimen.

  i.e. $\mu_r < 1$
- ✶ Since $\mu_r < 1$, x is a small negative value.

  Susceptibility of diamagnetic materials does not change with temperature.

Paramagnetic Materials

- ✶ Paramagnetic substances are those is which each individual atoms/molecule has a not non-zero magnetic moment of its own.
- ✶ When placed in a non-uniform magnetic field, they tend to move from weaker parts of the field to the stronger parts.
- ✶ When a specimen of paramagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the specimen rather than through air.

  Relative permeability is always more than unity.

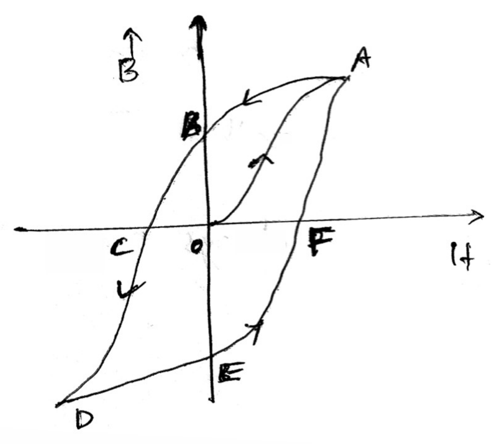
  $\mu_r < 1$
- ✶ X is a small +ve value.
- ✶ χ of paramagnetic subtances various inversity as the temperature of substance $\chi \;\alpha \frac{1}{T}$

$\boxed{\chi = \frac{C}{T}}$ Curie's law

Ferromagnetic substances

- ✶ Ferromagnetic substances are those in which each individual atom/molecule has a non-zero magnetic moment as in a paramagnetic substances.
- ✶ When they are placed in an external magnetising field, they gets strongly magnetized in the direction of the field.
- ✶ They have a tendency to move from a region of weak magnetic field to the region of strong magnetic field
- ✶ μ_r is very large
- ✶ X is also a large +ve value
- ✶ With rise in temperature, susceptibility of ferromagnetic substances decreases. At a certain temperature, ferromagnetic change over to paramagnetic substances. This transition temperature is called curie temperature.

Hysteresis



OB, OE → Retentivity

OC, OF → coercivity

Hysteresis represents the relation between magnetic induction $\vec{B}$ of a ferromagnetic material with magnetising force $\vec{H}$.

The phenomenon of lagging of magnetic induction B behind magnetising field H is called hysteresis.

The area of B-H loop represents energy dissipated per unit volume of the material.

$\therefore$ Energy loss due to hysteresis is given by

$\boxed{E = VAft}$

V $\rightarrow$ volume of specimen

A $\rightarrow$ Area of B -H loop

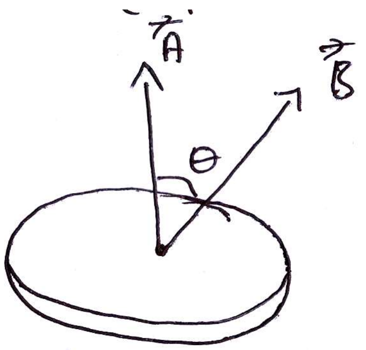
f $\rightarrow$ frequency of magnetisation

t $\rightarrow$ time

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Magnetic flux : Flux is a latin word which means 'flow'. Here flow means flow of a vector field through an area which is inside that field and it is numerically equal to the number of lines of forces passing through given area which is perpendicular to that area

$\phi = \vec{B}.\vec{A} = BA\cos\theta$



If the coil has N number of turns then $\phi = NBA\cos\theta$. Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$\phi = B \times \text{area} = \frac{F}{IL} \times L^2 = \frac{MLT^{-2}L^2}{AL}$$

= $ML^2T^{-2}A^{-1}$

So unit of flux is Joule / Ampere

= $\frac{\text{Joule} \times \text{second}}{\text{coulomb}} = \text{Wb or } Tm^2$

C.g.s unit of magnetic flux is Maxwell (Mx)

1 Wb = 10^8 Mx

If $\theta = 0^o$ then $\phi_{max} = BA$, if $\theta = 90^o$ then $\phi_{min} = 0$

If $0 \le \theta < 90^o$ flux ϕ is +ve and

If $\theta = 90^o$ flux ϕ is zero

If $90^o < \theta \le 180^o$ flux ϕ is –ve

Electromagnetic Induction : The phenomenon of producing induced emf and hence current in a closed circuit due to the change in manetic flux associated with it.

Faraday's Laws of Electromagnetic Induction

First Law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second Law

The magnitude of emf induced in a closed circuit is directly proportional to rate of change of magnetic flux linked with the circuit. i.e. $e \, \alpha \, \frac{d\phi}{dt}$. Second law does not give polarity of induced emf.

Lenz's Law : This law gives the direction of induced emf / induced current. According to this law the direction of induced emf or current in a circuit is such as to oppose the cause that produces it. This law is based on the law of conservation of energy. i.e. $e = -\frac{d\phi}{dt}$

Induced current = $-\frac{d\phi}{dt \times R}$. where R is the resistance of the coil.

Induced change = $-\frac{d\phi}{R}$

Note

1) Induced emf does not depends on nature of the coil and its resistance

2) Magnitude of induced emf is directly proportional to the relative speed of coil magnet system $(e \, \alpha \, v)$

3) Induced current is also depends on resistance of coil

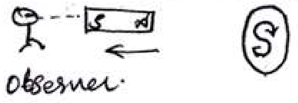
4) Induced emf does not depends on resistance of circuit, its exist in open circuit also

5) In all M.I. phenomenon induced emf is nonzero induced parameter

6) Induced charge in any coil or circuit does not depends on time in which change in flux occurs

7) Induced charge depends on change in flux through the coil and nature of the coil

The various positions of relative motion between the magnet and the coil.

Position magnet				
Direction of induced current	Anticlockwise direction	clockwise direction	Clockwise direction	Anticlockwise direction.
Behaviour of face of the coil	As a north pole	As a South pole	As a South pole	As a north pole
Type of magnetic force developed	Repulsive force	Attractive force	Repulsive force	Attractive force
magnetic field linked with the coil and its progress as viewed from left.	Cross (X), Increases	Cross(X) Decreases	Dot(•) Increases	Dot(•) Decreases.

Induced electric field

When the magnetic field changes with time there is an induced electric field in the conductor caused by the changing magnetic flux.

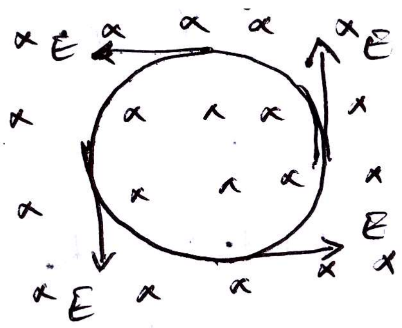
Important properties of induced electric field

1. It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero.

   Hence $\oint \vec{E}.\overrightarrow{d\ell} = e = -\frac{d\phi}{dt}$

2. Due to symmetry, the electric field E has the same magnitude at every point on the circle and it is tangential at each point.

3. Being nonconservative field, the concept of potential has no meaning for such a field



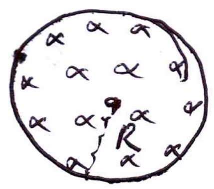
4. The relation F = qE is still valid for this field

5. This field can very with time. For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\frac{dB}{dt}$.

   $\ell \Rightarrow$ the length of the closed loop in which electric field is to be calculated

   $A \Rightarrow$ Area in which magnetic field is changing. Direction of induced electric field is same as the direction of induced current.

Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $kT/\text{sec ond}$. Find the magnitude of electric as a function of r, the distance from the geometric centre of the region

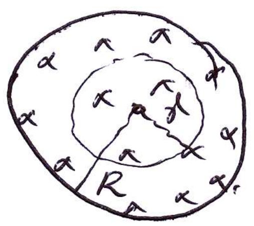


Solution for $r \le R$

$$E\ell = A\left|\frac{dB}{dt}\right|$$

$$E \times 2\pi r = \pi r^2 \times k$$

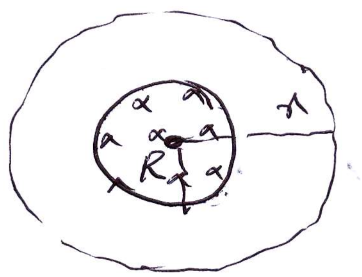
$$\therefore E = \frac{rk}{2} \quad \therefore E\,\alpha\, r$$



E - r - graph is a straight line passing through origin

If r = R, $E = \frac{Rk}{2}$

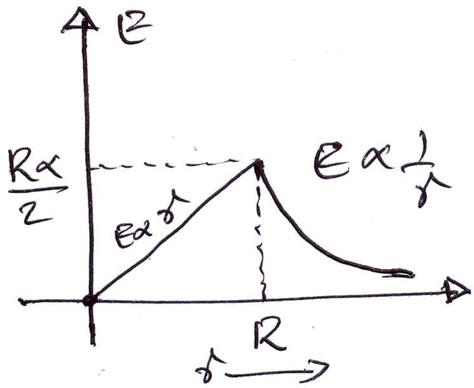
For $r \ge R$



$$E\ell = A\frac{dB}{dt}$$

$$E \times 2\pi r = \pi R^2 \times k$$

$$E = \frac{kR^2}{2r} \quad \text{or} \quad E\alpha\frac{1}{r}$$



Methods of producing induced emf

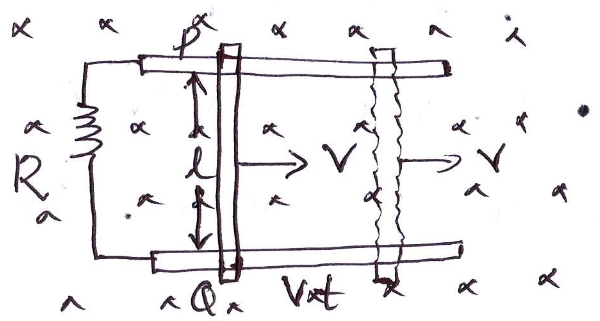
1) Changing the magnetic field B (static EMI)

2) Changing the area A of the coil (dynamic EMI)

3) Changing the relative orientation θ of $\vec{B}$ and $\vec{A}$ (periodic EMI)

Motional EMI in loop by Generated Area

If conducting rod moves on two parallel conducting rails as shown in following figure the phenomenon of induced emf can also be understood by the concept of generated area



Induced emf $|e| = \frac{d\phi}{dt} = B\ell v$

Induced current i = $\frac{e}{R} = \frac{B\ell v}{R}$

Magnetic force $F_m = BI\ell = B \times \left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{R}$

Power dissipated in moving the conductor

$$P_{mech} = P_{ext.} = \frac{dw}{dt} = F_{ext.}v = \frac{B^2\ell^2 v}{R} \times v = \frac{B^2\ell^2 v^2}{R}$$

4) **Electrical power**

$$P_{thermal} = \frac{H}{t} = I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2 v^2}{R}$$

It is clear that $P_{mech} = P_{thermal}$ which is consistent with the principle of conservatory of energy

<u>Motional emf from Lorentz Force</u>

A conductor rod PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity.

The electron experiences a magnetic force $F_m = e(v \times B)$. An electric field 'E' is set up in the conductor from P to Q

$F_e = eE$

$$F_m + F_e = 0 \qquad E = -(v \times B)$$

Potential difference between the ends P and Q is $V = E.\ell = (V \times B).\ell$

$$\in = B\ell v\left(\text{for } \overline{B} \perp \overline{v} \perp \overline{\ell}\right)$$

This this emf is produced due to the motion of the conductor, so it is called a motional emf.

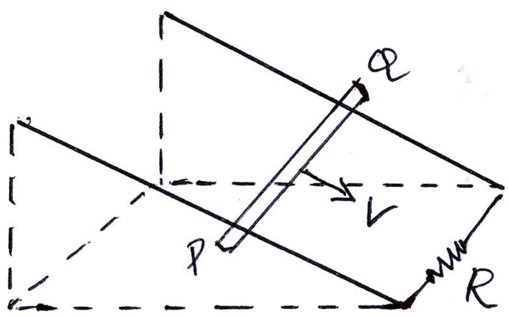
Direction of the induced current or Higher Potential (H.P.) end of the rod can be found out with the help of Fleming right hand rule.

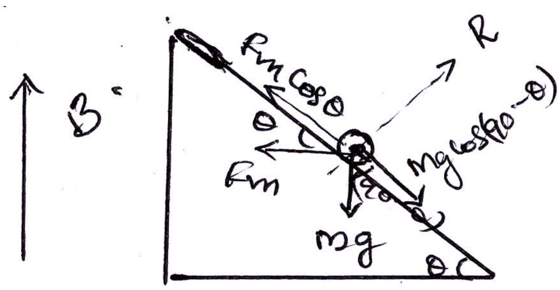
Fore finger $\Rightarrow$ In external field direction

Thumb $\Rightarrow$ In the direction of motion $(\overline{v})$ of conductor

Middle finger $\Rightarrow$ it indicates H.P. end of conductor or direction of induced current in the conductor

<u>Motion of a conducting rod in an inclined plane</u>





Induced emf across the ends of the conductor, $e = B\ell v \sin(90 - \theta) = B\ell v \cos\theta$

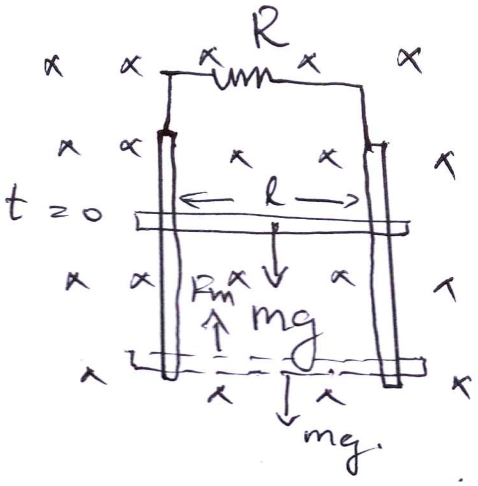
Induced current $i = \dfrac{B\ell v\cos\theta}{R}$

The rod will move down with constant velocity only if $F_m \cos\theta = Mg\cos(90-\theta) = mg\sin\theta$

$$Bi\ell\cos\theta = mg\sin\theta \Rightarrow \frac{B(BV_T\ell\cos\theta)}{R}\ell\cos\theta = Mg\sin\theta$$

$$V_T = {MgR\sin\theta}\Big/{B^2\ell^2\cos^2\theta}$$

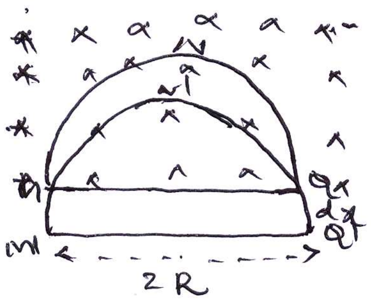
Motion of a conductor rod in a vertical plane



As the speed increases induced emf$_{(e)}$, inducced current (i), magnetic force (F_m) increases but its weight remains constant. Rod will attain a constant maximum (terminal) velocity V_T if F_m = mg

$$\frac{B^2V_T\ell^2}{R} = mg \Rightarrow V_T = \frac{mgR}{B^2\ell^2}$$

Ex. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction B(fig.). At the position MNQ, the speed of the ring is V. What is the p.d. developed across the ring at position MNQ.



$d\phi = B.dA = B - 2Rdx = -2RBdx$

Potential difference across the ring $e = -\frac{d\phi}{dt} = -\left(-2BR\frac{dx}{dt}\right)$

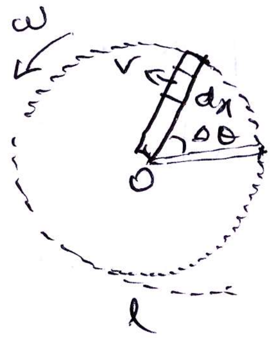
= 2BRV

Induced emf due to Rotation of a conductor rod in a uniform magnetic field

Induced emf in a small element dx is

$de = Bvdx = B\omega xdx$

So net induced emf = $\int_0^{\ell} B\omega xdx$



$\boxed{e = \frac{1}{2}B\omega\ell^2}$

Self Induction

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it also change. As a result of this, in accordance with Faraday's laws of electromagnetic induction, an emf is induced in the coil or circuit. Which opposes the change that causes it according to Lenz Law. This phenomenon is called self induction and the emf induced is called back emf. Current so produced in the coil is called self induced current.

Coefficient of self-induction

Flux linked with the coil is proportional to the current.

i.e. $\phi \alpha I$ or $\phi = LI$. Hence $L = \frac{\phi}{I}$. Coefficient of self-induction or self inductance

The coefficient of self induction of a coil is equal to the flux linked with the coil when the current in it is 1 ampere.

By Faraday's second law, induced emf $e = -\frac{d\phi}{dt}$ ie. $e = -L\frac{dI}{dt}$

Hence coefficient of self induction is numerically equal to the induced emf produced in the coil when the current varying at the rate of 1 ampere / second

Units and dimensional formula of 'L'

If S.I. unit $\Rightarrow \frac{\text{Weber}}{\text{amp}} = \frac{\text{tesla} \times \text{m}^2}{\text{amp}} = \frac{\text{N} \times \text{m}}{(\text{amp})^2}$

= $\frac{\text{Joule}}{(\text{amp})^2} = \frac{\text{coulomb} \times \text{volt}}{(\text{amp})^2} = \frac{\text{volt} \times \text{sec}}{\text{amp}} = \text{ohm} \times \text{sec}$

But practical unit is Henry (H)

Its dimensional formula is $\left[\text{ML}^2\text{T}^{-2}\text{A}^{-2}\right]$

Dependence of self inductance (L)

'L' does not depend on current flowing or change in current flowing but it depends upon number of turns (N), Area of cross section (A) and permeability of medium (μ).

'L' does not play any role till there is a constant current flowing in the circuit. 'L' comes to the picture only when there is a change in current.

Self inductance of a plane coil

Total magnetic flux linked with N turns

$$\phi = \text{NBA} = \text{N}\left(\frac{\mu_0 \text{NI}}{2\text{r}}\right)\text{A} = \frac{\mu_0 \text{N}^2\text{I}}{2\text{r}}\text{A} = \frac{\mu_0 \text{N}^2\text{I}}{2\text{r}} \times \pi \text{r}^2$$

$$\phi = \text{LI} = \frac{\mu_0 \text{N}^2 \text{I} \pi \text{r}}{2} \quad \therefore \text{L} = \frac{\mu_0 \text{N}^2 \pi \text{r}}{2}$$

Self inductance of a solenoid

$$\phi = \text{NBA} = \text{N}\left(\frac{\mu_0 \text{NI}}{\ell}\right)\text{A} = \frac{\mu_0 \text{N}^2 \text{A}}{\ell}\text{I}$$

$\text{L} = \frac{\mu_0 \text{N}^2 \text{A}}{\ell}$ with medium $\text{L}_\text{m} = \frac{\mu_0 \mu_\text{r} \text{N}^2 \text{A}}{\ell}$

Mutual Induction

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is called mutual induction.

Coefficient of mutual induction

Total flux linked with the secondary due to current in the primary is $\text{N}_2\phi_2$ and $\text{N}_2\phi_2 \propto \text{i}_1$ $\quad \text{N}_2\phi_2 = \text{MI}_1$

Where $M = \frac{N_2\phi_2}{I_1}$ called coefficient of mutual induction

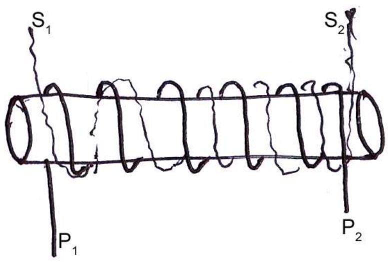
It is numerically equal to the flux linked with the secondary coil when a current of 1A flows through the primary. According to Faraday's second Law emf induced in the secondary $e_2 = -M\frac{dI}{dt}$. Hence coefficient of mutual inductctions is equal to the emf induced in the secondary when current varying at the rate of 1A/s in the primary coil.

M depends on :

- Number of turns (N_1 and N_2)
- Area of cross section (A)
- Distance between two coils (As $d\downarrow \Rightarrow M\uparrow$)
- Coupling factor between two coils
- Coefficient of self inductance (L_1 and L_2)
- Magnetic permeability of medium (μr)
- Orientation between two coils

Diffrent coefficient of mutual inductance

- a) Two-co-axial solenoids (Ms_1s_2)



$$Ms_1s_2 = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left(\frac{\mu_0 N_1 I_1}{\ell}\right)A$$

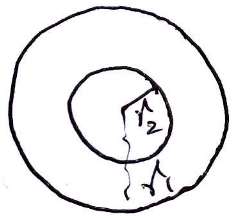
$$Ms_1s_2 = \frac{\mu_0 N_1 N_2 A}{\ell}$$

$A \rightarrow$ Smaller area

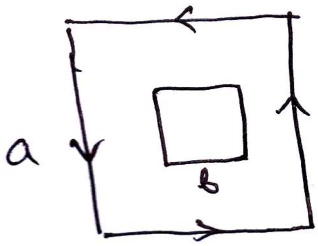
b) Two plane concentric coils (Mc_1c_2)

Two concentric loop

$M \,\alpha\, \frac{r_2^2}{r_1}$

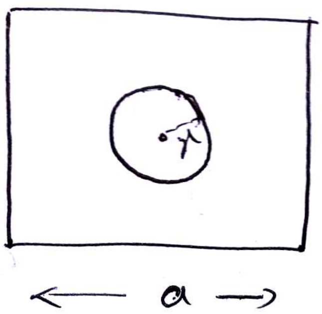


Two concentric square loop



$M \,\alpha\, \frac{b^2}{a}$

A square and a circular loop



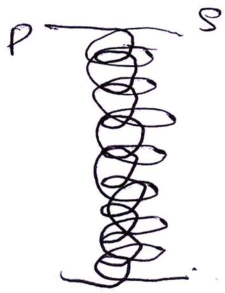
$$M \,\alpha\, \frac{r^2}{a}$$

In terms of L_1 and L_2

$M = K\sqrt{L_1 L_2}$ Here K is coupling factor between two coils.

$0 \geq K \leq 1$

- Different fashion of coupling

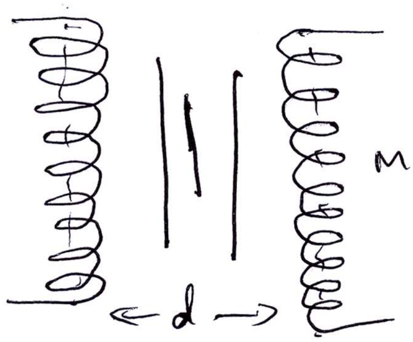


$K = 1$

$M = \sqrt{L_1 L_2}$

ideal coupling

coaxial fashion

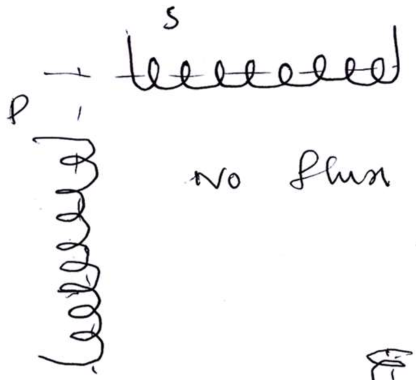


$0 < k < 1$

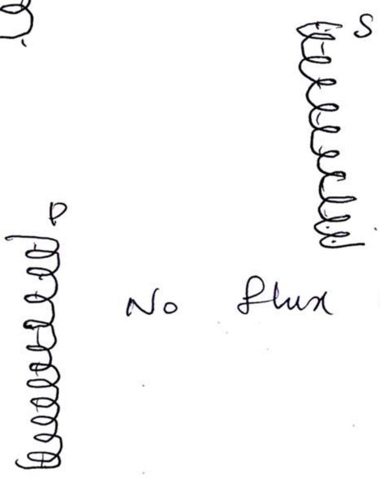
$M = k\sqrt{L_1 L_2}$

normal coupling

If $d \downarrow$ $k \uparrow$ $M \uparrow$.



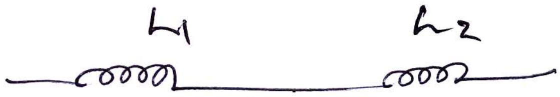
No flux coupling, $K=0$, $M=0$.

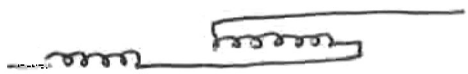


No flux coupling, $K=0$, $M=0$

K is also defined as, $K=\frac{\phi_s}{\phi_p}$

Inductance in series



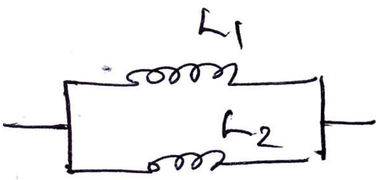


$L = L_1 + L_2 - 2M$

If m = 0, $L = L_1 + L_2$ If $M \neq 0$

$L = L_1 + L_2 + 2M$

Two coils are connected in parallel



If M = 0, $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

If $M \neq 0$, then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

Magnetic potential energy of Inductor

In building a steady current in the circuit, the source of emf has to do work against self inductance of coil and whatever energy consumed for this work stored in magnetic field of coil. This energy called as magnetic potential energy.

$$U = \int_0^i LI\, di = \frac{1}{2} LI^2$$

$$U = \frac{1}{2}\frac{LI \times I}{2} = \frac{\phi I}{2}$$

Magnetic energy density

$$u = \frac{U}{v} = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\frac{(\mu_0 nI)^2}{\mu_0} = \frac{B^2}{2\mu_0}$$

Periodic E.M.I

Suppose a rectangular coil having N turns is placed initially in a magnetic field such that magnetic field is perpendicular to its plane.

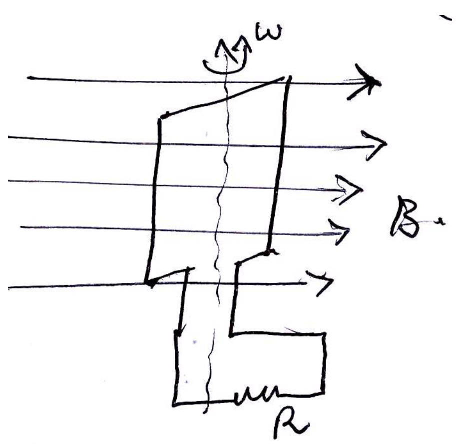
ω – angular speed

ν – frequency of rotation

R – resistance of coil

ϕ – NBA $\cos\theta$

ϕ – NBA $\cos\omega t$



Induced emf in coil : Induced emf also changes in periodic manner that is why this phenomenon called periodic E.M.I

$e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t$ i.e. $e = e_0 \sin \omega t$

$i_0 = \frac{R_0}{R} = \frac{NBA\omega}{R} = \frac{\phi_0 \omega}{R}$

Eddy Currents : are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes. Eddy currents tend to follow the path of least resistance inside the conductor. So they form irregularly shaped loops. However their directions are not random, but guided by Lenz's law.

Applications of eddy currents

1) Induction furnace

2) Electromagnetic damping

3) Electric brakes

4) Speedometers

5) Induction motor

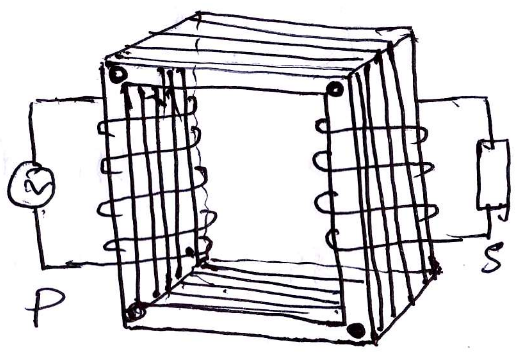
6) Electromagnetic shielding

7) Inductothermy

8) Eergy meters

Transformer

Working principle : Mutual induction transformer has basic two sections



Shell : Consists of primary and secondary coil of copper

Core : Which is between two coil and magnetically coupled two coils. Two coils of transformer are wound on same core.

Work : It regulate A.C. voltage and transfer electrical power without change in frequency of input supply.

Special points :

It can’t work with D.C

It can’t called amplifier as it has no power gain like transistor

It has no moving part, hence there are no mechanical losses in transformer

If no flux leakage $\phi_s = \phi_p = \frac{d\phi_s}{dt} = \frac{d\phi_p}{dt}$

$e_s = e_p = e$ induced emf/turn of each coil is same.

Total induced emf in secondary $E_s = N_s e$

Total induced emf in primary $E_p = N_p e$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

No power loss

$P_{out} = P_{in}$ i.e. $V_s I_s = V_p \times I_p$ $\therefore \frac{V_s}{V_p} = \frac{I_p}{I_s}$

Efficiency of transformer

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_s I_s}{V_p I_p} \times 100$$

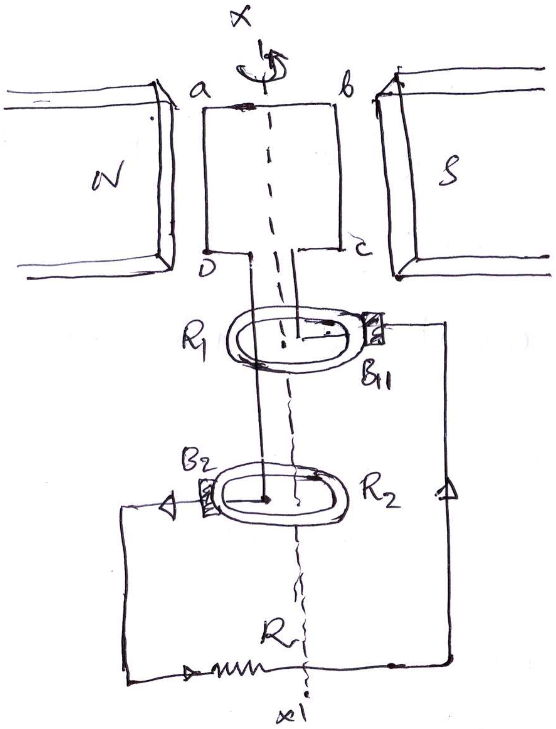
Energy loss in the transformer

- Copper or joule heating losses
- Flux leakage losses
- Iron losses
- Hysteresis losses

Generator or dynamo : It is a device which converts mechanical energy into electrical energy

A.C. Generator

It is based upon the principle of electromagnetic induction.



R_1, R_2 slip rings, B_1 and B_2 Brushes and abcd armature, field magnet.(NS)

Theory : Flux linked with the coil at any instant t.

$$\phi = NBA\cos\omega t$$

$$\frac{d\phi}{dt} = -NBA\omega\sin\omega t$$

emf $E = NBA\omega\sin\omega t$

or $E = E_0\sin\omega t$ where $E_0 = NBA\omega$

ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it changes continuously in magnitude and peridically in direction with time. It can be represented by a sine curve or cosine curve

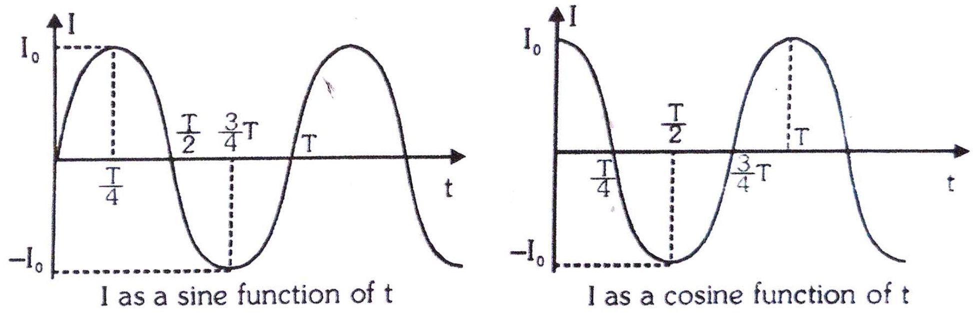
$I = I_0\sin\omega t$ or $I = I_0\cos\omega t$

where I= Instantaneous value of current at time t,

I_0= Amplitude or peak value

ω= Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi f$

T= time period, f= frequency



- **<u>Amplitude of AC</u>** (I_0)

  The maximum value of current in either direction is called peak value or amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

- **<u>Periodic time</u>** (T)

  The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

- **<u>Frequency</u>** $(f \text{ or } \nu)$

  The number of cycle completed by an alternating current in one second is called the frequency of the current.

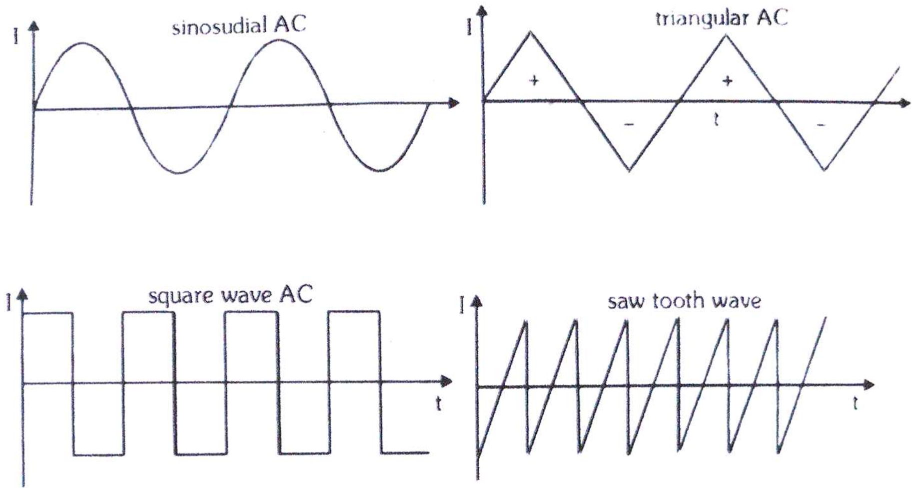
  UNIT: cycle/s; (Hz)
  In India : f= 50Hz, supply voltage= 220 volt (rms)
  In USA : f=60 Hz, supply voltage= 110 volt (rms)

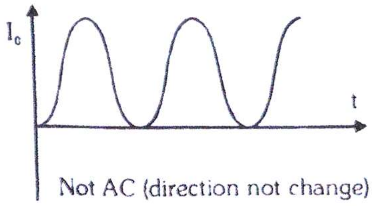
- Condition required for current / voltage to be alternating

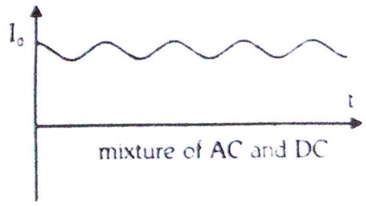
  Amplitude is constant

  Alternate half cycle is positive and half negative

  The alternating current continuously varies in magnitude and periodically reverses its direction.







- The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $\langle I \rangle = \dfrac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin\omega t\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos\omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$\langle \sin\theta \rangle = \langle \sin 2\theta \rangle = 0$
$\langle \cos\theta \rangle = \langle \cos 2\theta \rangle = 0$
$\langle \sin\theta \cos\theta \rangle = 0$
$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$

For symmetric AC, average value over full cycle = 0,
Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

- **Root mean square (rms) value**

  It is the value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin \omega t$:

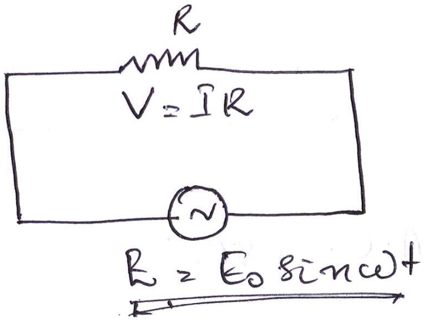
$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin\omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2\omega t\, dt} = I_0\sqrt{\frac{1}{T}\int_0^T\left[\frac{1-\cos 2\omega t}{2}\right]dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2}-\frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular fequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2} I_0$	I_0	ω

For above varieties of current rms $= \frac{\text{Peak value}}{\sqrt{2}}$

- **Different types of A.C circuits**
- **AC circuit containing pure resistance**



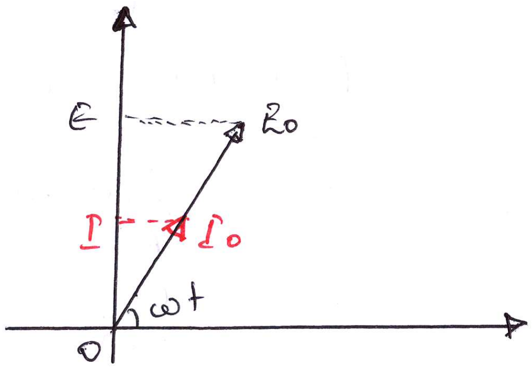
With the help of Kirchoff's circuital law E – IR = 0

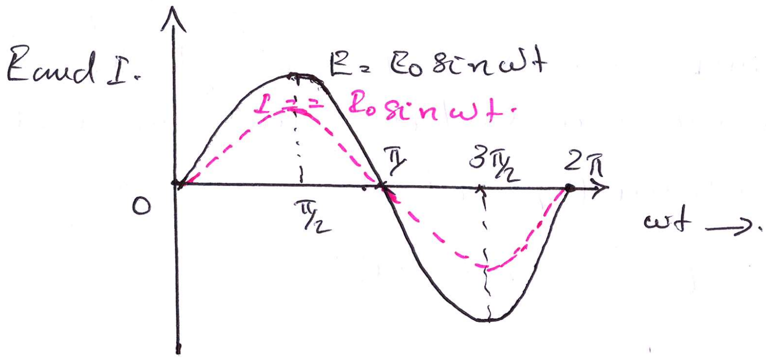
$$E_0 \sin \omega t = IR$$

$$I = \frac{E_0}{R} \sin \omega t$$

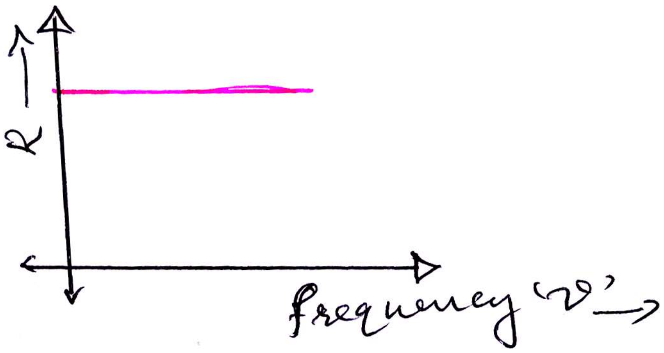
$I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$

So in a resistor, current and emf are in the same phase. The phaser diagram of emf and current are as shown in the figure



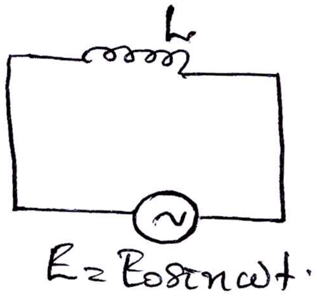


Variation resistance ‘R’ with frequency ν



A.C. circuit containing pure inductance

Let emf E = $E_0 \sin \omega t$



When a.c flows through the circuit, emf induced across inductance

= $-L\frac{dI}{dt}$

$E + -L\frac{dI}{dt} = 0$ or $E = L\frac{dI}{dt}$

$dI = \frac{E_0 \sin \omega t \, dt}{L}$

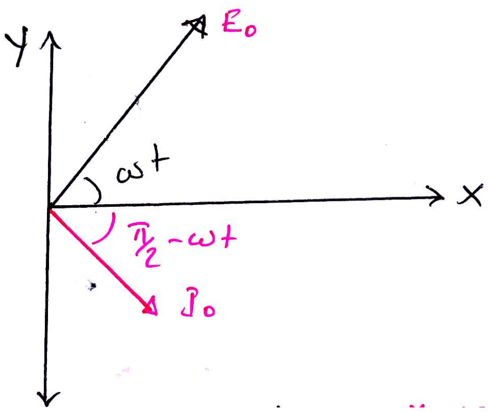
$I = \frac{E_0}{L\omega} \times \sin\left[\omega t - \frac{\pi}{2}\right]$ So current lags due emf with a phase difference of $\frac{\pi}{2}$

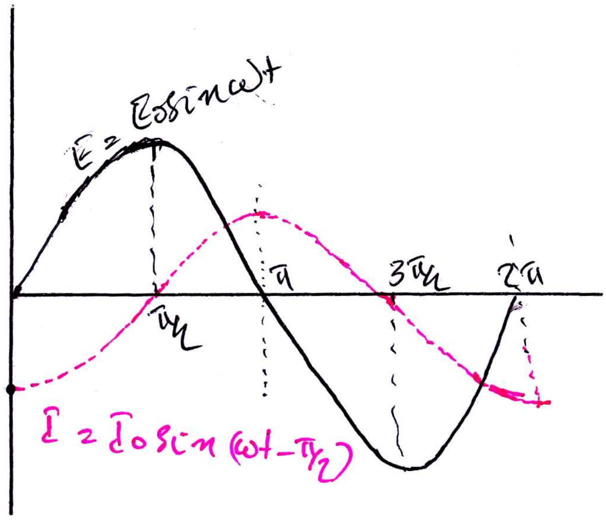
$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ where $I_0 = \frac{E_0}{L\omega}$

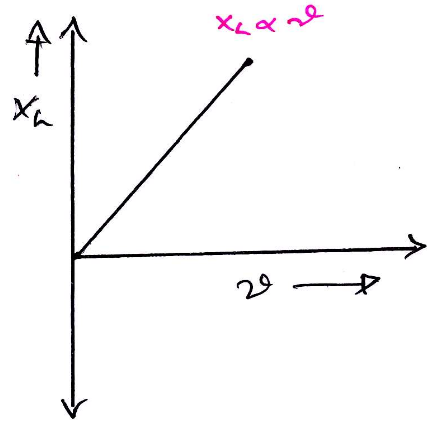
This non resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

$X_L = L\omega = L \times 2\pi\nu$. Its unit is '$\Omega$'

Its phasor diagram is as shown in the figure



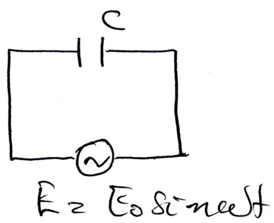




Foer d.c. $\nu = 0$, $X_L = L2\pi\nu = 0$

Hence inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c

A.C. circuit containing pure capacitance

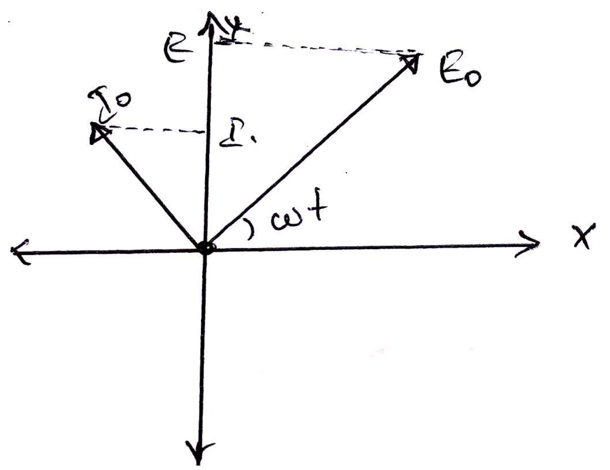


Instantaneous p.d. across the capacitor $E = q/c$

$$q/c = E_0 \sin \omega t$$
$$q = cE_0 \sin \omega t$$
$$\frac{dq}{dt} = cE_0 \omega \cos \omega t$$

$$I = \frac{E_0}{1/c\,\omega} \times \sin\left[\omega t + \pi/2\right]$$ i.e. $I = I_0 \sin\left[\omega t + \pi/2\right]$

$X_C = \frac{1}{c\omega}$ is called capacitive reactance. Its phasor diagram is as shown in the fig.

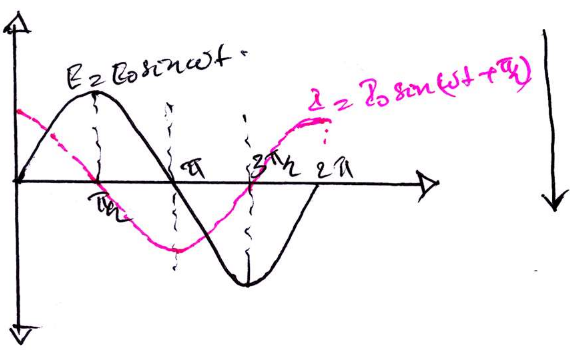


Here current loads the emf with a phase difference of $\pi/2$

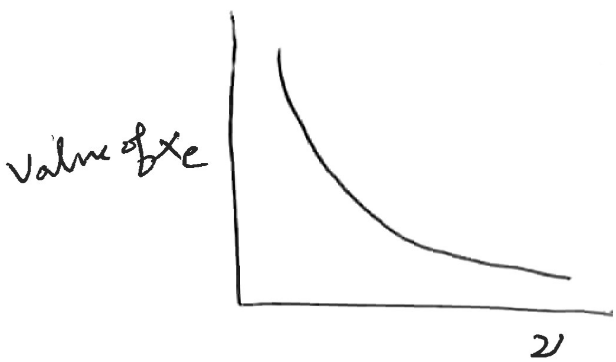
For d.c. $\nu = 0, X_C = \dfrac{1}{c\omega} = \dfrac{1}{c \times 2\pi\nu} = \infty$

So a capacitor applies infinite opposition to the d.c. i.e. It does'nt alow d.c. and an easy path for a.c.

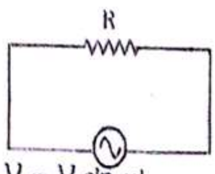
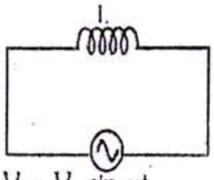
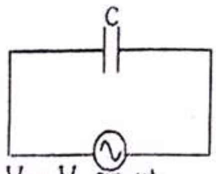
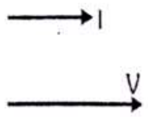
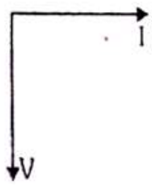
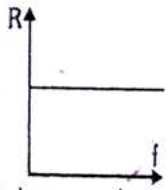
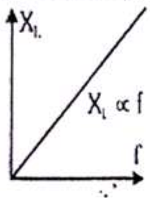
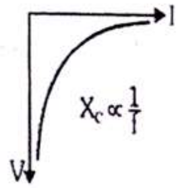
X_C, ν graph is as shown in the figure



unit of X_c is ohm

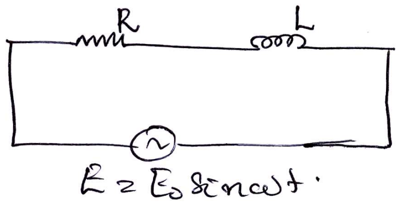


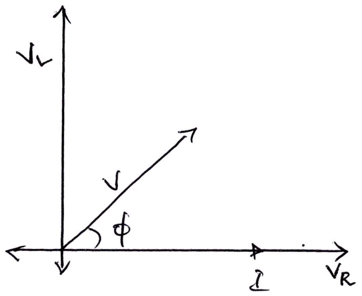
INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f			
G, S_L, S_C (mho, seiman)	R does not depend on f G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

- The phase difference between capacitive and inductive reactance is π
- Inductor called low pass filter because it allow to pass low frequency signal
- Capacitor is called high pass filter because it allow to pass high frequency signal

Resistance and inductance in series (L-R circuit)





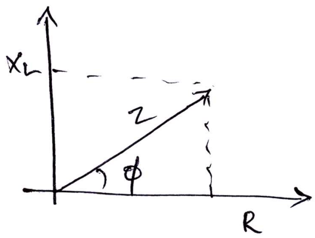
$$V = \sqrt{V_L^2 + V_R^2}$$

$I \times Z = \sqrt{(IR)^2 + (I \times L)^2}\ |Z \Rightarrow$ Impedance of the circuit

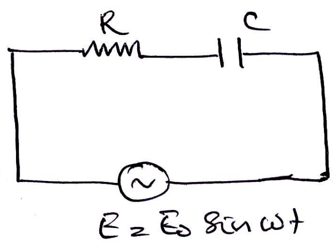
$$Z = \sqrt{R^2 + X_L^2} \qquad I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

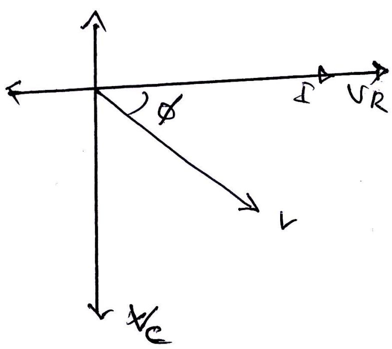
$$\tan\phi = \frac{V_L}{V_R} = \frac{I \times L}{IR} = \frac{X_L}{R} = \frac{L\omega}{R} \qquad I = I_0 \sin(\omega t - \phi)$$

Reciprocal of the impedance is called admittance



Resistance and capacitor are in series (R.C. circuit)





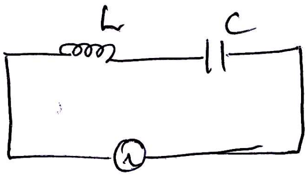
$$V = \sqrt{v_R^2 + V_C^2}$$

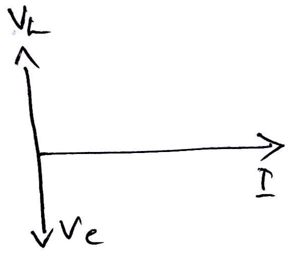
$$Z = \sqrt{R^2 + X_C^2}$$

Here emf lags behind the current with a phase difference of ϕ

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/c\omega}{R} \qquad I = I_0 \sin(\omega t + \phi)$$

L.C. circuit



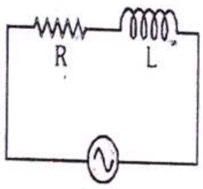
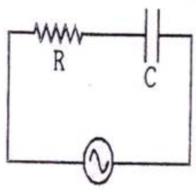
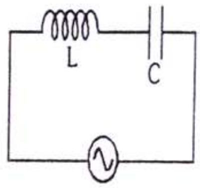
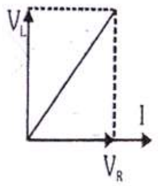
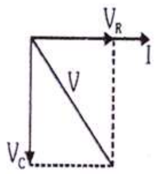
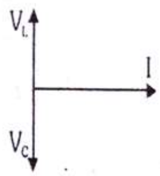
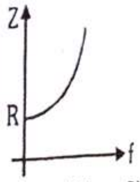
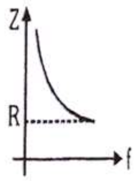
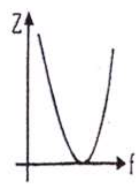


$V = V_L - V_C$

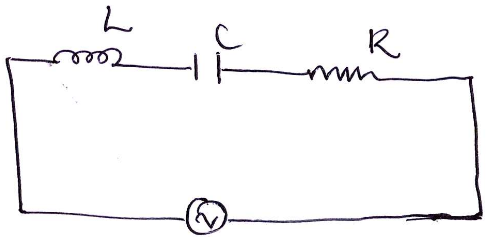
$Z = X_L - X_C$

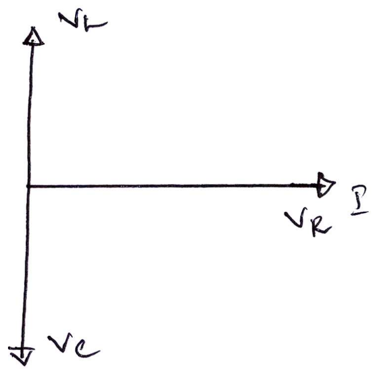
OR $(X_C - X_L)$

COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2 = V_R^2 + V_L^2$	$V^2 = V_R^2 + V_C^2$	$V = V_L - V_C (V_L > V_C)$ $V = V_C - V_L (V_C > V_L)$
Phase difference	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$)
in between V and I			V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_C \rvert$
Variation of Z	as f↑, Z↑	as f↑, Z↓	as f↑, Z first ↓ then↑
with f			
At very low f	$Z \simeq R$ $(X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R$ $(X_C \to 0)$	$Z \simeq X_L$

L.C.R. Series circuit

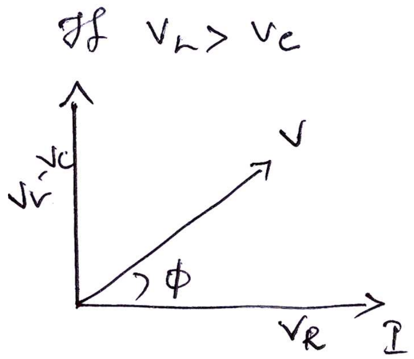




If $V_L > V_C$

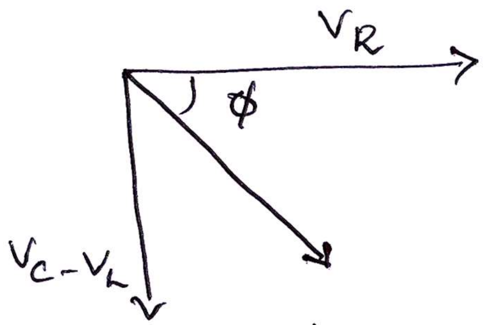
$$V = \sqrt{V_R^2 + \left(V_L - V_C\right)^2}$$

$$Z = \sqrt{R^2 + \left(x_L - X_C\right)^2}$$



Here emf leads the current with a phase difference ϕ and $\tan\phi = \dfrac{V_L - V_C}{VR} = \dfrac{X_L - X_C}{R}$

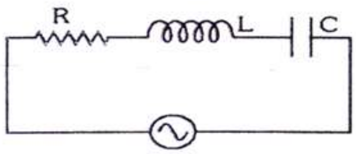
If $V_C > V_L$



Here emf lags behind the current with a phase dfference ϕ

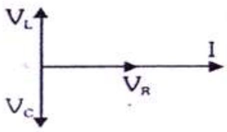
SERIES L-C-R CIRCUIT

1. Circuit diagram

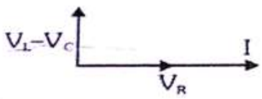


I same for R, L & C

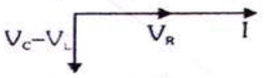
2. Phasor diagram



(i) If $V_L > V_C$ then



(ii) If $V_C > V_L$ then

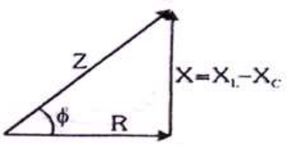


(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$

(iv) Impedance triangle



Resonance : A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

Series Resonance

At resonance $X_L = X_C$, $V_L = V_C$ and $\phi = 0$ V and I in same phase, $Z_{min} = R$

$$I_{max} = \frac{V}{R}$$

Resonance Frequency

$\because X_L = X_C, L\omega_r = \frac{1}{c\omega_r}$ or $\omega_r^2 = \frac{1}{LC}$

$\omega_r = \frac{1}{\sqrt{LC}}$ and $\nu = \frac{1}{2\pi\sqrt{LC}}$

Variation of I with ν

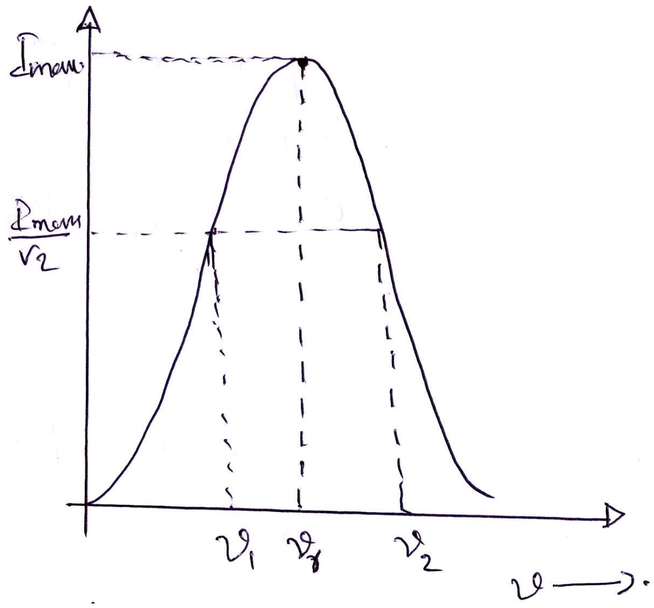
1) If $\nu < \nu_r$ then $X_L < X_C$ circuit nature capacitive, (ϕ negative)

2) At $\nu = \nu_r$ then $X_L = X_C$ circuit nature

Resistive $\phi = 0$

3) If $\nu > \nu_r$ then $X_L > X_C$ circuit nature is inductive, (ϕ positive)

Variation of I with frequency 'ν'



- At resonance impedance of the series resonant circuit is minimum it is called acceptor circuit as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency.

Half Power Frequencies

The frequencies of which, power become half of its maximum value called half power frequencies

Band width $\Delta\nu = \nu_2 - \nu_1$

Quality factor or Q-factor of A.C. circuit basically gives an ideal about stored energy and lost energy.

$$Q = 2\pi \frac{\text{max. energy stored / cycle}}{\text{max. energy loss / cycle}}$$

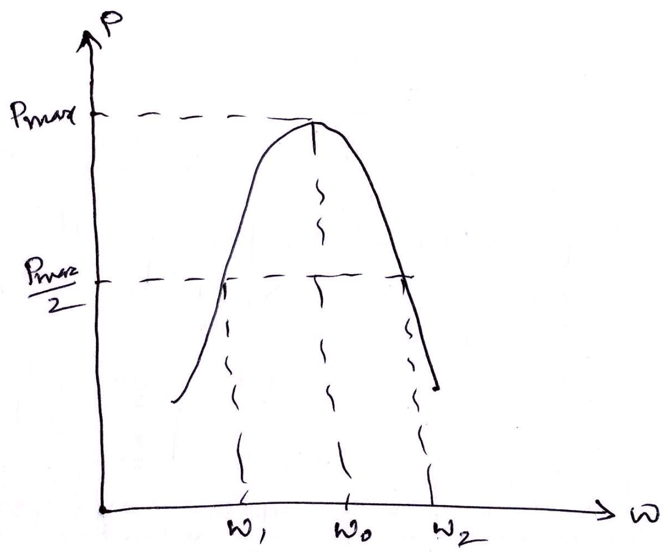
It represents sharpness of resonance. It is a unitless and dimensionless quantity

$$Q = \frac{(V_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\nu_r}{\Delta\nu}$$

Magnification factor = Q $\rightarrow$ factor

Sharpness α Q factor & magnification factor

Power in AC circuit



Power in an AC circuit

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi)) = V_0 I_0 \sin\omega t\,(\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

$$\text{Average power } \langle P\rangle = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2\omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi\, dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi\, dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad \langle P\rangle = \frac{V_0 I_0\cos\phi}{2} = V_{rms} I_{rm,s}\cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = V_{rms} I_{rms} \cos\phi$	$P = V_{rms} I_{rms}$	$P = V_0 I_0$

- $I_{rms} \cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin\phi$ is known as inactive part of current, wattless current, workess current. It is quadrature (90°) with voltage.

Power factor :

Average power $\bar{P} = E_{rms} I_{rms} \cos\phi = \text{r m s power} \times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{r.m.s Power}}$ and $\cos\phi = \dfrac{R}{Z}$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V

ELECTROMAGNETIC WAVES

Electromagnetic Waves

Electromagnetic waves consisit of periodically varying electric and magnetic fields perpendicular to each other propagating through space -time

Displacement Current

Displacement current in a region of space is defined as

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

Where ϕ_E is the flux of electric field in the region

Displacement current can act as source of magnetic field

Hence both conduction current, and displacement current can act as the source of magnetic field.

$\rightarrow$Conduction current $\Rightarrow$ current due to flow of charges in conductors

$\rightarrow$Displacement current $\Rightarrow$ current due to time variation of electric field

Maxwell's modification of Ampere's Law

Maxwell modified Ampere's law by including displacement current

$$\int_L \vec{B} \cdot \vec{dl} = \mu_0 \left(I + I_0 \right)$$

$$\int_L \vec{B} \cdot \vec{dl} = \mu_0 \left(I + \varepsilon_0 \cdot \frac{d\phi_E}{dt} \right)$$

Continuity of Current

The sum of conduction current and displacement current is continuous along a closed path even though individually they may not be continous. In a circuit involving a capacitor, the conduction current in the connecting wires is equal to the displacement current between the plates of capacitor

$$I_C = \frac{dq}{dt}$$

$$I_d = \varepsilon_0 \cdot \frac{d\phi_E}{dt}; = \varepsilon_0 \frac{dE \cdot A}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{A\varepsilon_0} \cdot A\right); = \frac{dq}{dt}; \therefore I_C = I_d$$

Outside the capacitor plates

$$I_C = \frac{dq}{dt}, I_d = 0$$

In between the plates $I_C = 0, I_d = \frac{\varepsilon_0 \cdot d\phi_E}{dt} = \frac{dq}{dt}$

Maxwell's Equations

1) Gauss's law in electrostatics

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$$

2) Gauss's law for magnetism

$$\oint_s \vec{B} \cdot d\vec{s} = 0$$

3) Faraday's law of electromagnetic induction

$$\oint_L \vec{E} \cdot dl = -\frac{d\phi_B}{dt}$$

4) Ampere's Maxwell's law

$$\oint_L \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I_C + Id\right)$$

$$\oint_L \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I_C + \varepsilon_0 \cdot \frac{d\phi_E}{dt}\right)$$

$$\oint_L \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

From Maxwell's equations it can be concluded that time varying electric and magnetic fields give rise to each other. Periodically changing electric and magnetic fields propagating through space-time form electromagnetic waves

$\rightarrow$ In an EM wave electric field and magnetic fields are perpendicular to the dirction of propogation. Hence they are transverse waves

$\rightarrow$ Electric field and magnetic field vectors are perpendicular to each other

$\rightarrow$ The direction of $\vec{E} \times \vec{B}$ gives the direction in which wave is travelling

$\rightarrow$Fields vary sinusoidally. Electric field and magnetic fields vary with same frequency in phase

If the electric field component of an em wave is along y direction and magnetic field is along z direction, then wave propogates along x direction

$\rightarrow$The electric field and magnetic field components can be represented as sinusoidal functions of position and time

$$E = E_0 \sin(kx - \omega t)$$

$$B = B_0 \sin(kx - \omega t)$$

Speed of em waves

Speed of em waves in vacuum is given by

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, ms^{-1}$$

Where ε_0 = permittivity of free space = 8.85×10^{-12}

μ_0 = permeability of free space

In any other medium speed of em wave

$$v = \frac{1}{\sqrt{\mu . \varepsilon}} = \frac{1}{\sqrt{\mu_r \cdot \mu_0 \cdot \varepsilon_r \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_r \cdot \varepsilon_r}} C$$

Refractive index $n = \sqrt{\mu_r \varepsilon_r}$

Mathematical representation of em waves

EM wave, can be represented in the form, sinusoidal functions of position and time. Electric and magnetic field vectors oscillate in phase, perpendicular to each other and are perpendicular to direction of propagation.

$$\vec{E} = E_0 \sin \omega \left(t - \frac{x}{c} \right) \hat{j}$$

$$\vec{B} = B_0 \sin \omega \left(t - \frac{x}{c} \right) \hat{k}$$

$\rightarrow$Peak value of electric and magnetic field can be related as

$$\frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = C = 3 \times 10^8 \, ms^{-1}$$

$$E_0 = C \cdot B_0$$

$\rightarrow$ Propagation constant (wave number)

$$k = \frac{\omega}{C} = \frac{2\pi\upsilon}{C} = \frac{2\pi}{\lambda}$$

where υ is frequency

λ is wavelength

Relation between speed, wavelength and frequency

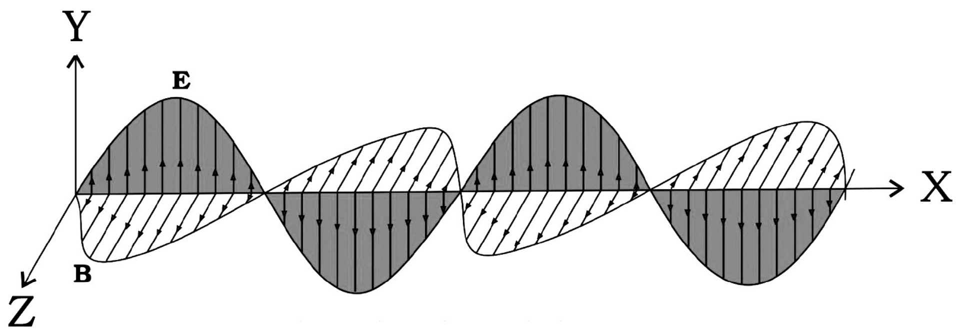
$$C = \upsilon \times \lambda$$

C= speed of light

υ= frequency

λ= wavelength

Diagramatic representation of em wave



Energy density associated with e-m wave

a) Energy density associated with electric field

The energy density associated with an electric field is given by

$$U_E = \frac{1}{2}\varepsilon_0 E^2$$

for an e-m wave

$$U_E(t) = \frac{1}{2}\varepsilon_0 \left[E_0 \sin\left(\omega\left(t - \frac{x}{c}\right)\right)\right]^2$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2\left(\omega\left(t - \frac{x}{c}\right)\right)$$

The average value of $U_E(t)$

$$U_E = \frac{1}{2}\varepsilon_0 \frac{E_0^2}{2};\ U_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 \left(E_{rms}\right)^2$$

b) Energy density associated with magnetic field

Energy density associated with magnetic field is given by

$$U_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

For an e-m wave

$$U_B(t) = \frac{1}{2}\frac{B_0^2 \sin^2\left(\omega\left(t - \frac{x}{c}\right)\right)}{\mu_0}$$

Average value of $U_B(t)$

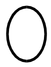
$$U_B = \frac{1}{2}\frac{B_0^2}{\mu_0 \times 2} = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{2}\frac{B_{rms}^2}{\mu_0}$$

Intensity of E-M wave

Intensity of electro magnetic wave at a point is defined as the energy crossing per unit time per unit area normally around that point during the propagation of electromagnetic wave

$$I = \frac{\text{energy}}{\text{area} \times \text{time}} \rightarrow$$

A



Consider a cylindrical volume, with area of cross section A and length $C\Delta t$ along the x-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates with a speed C

Hence, energy contained

$\Delta U =$ average energy density × ΔV

$$\Delta U = U_{av} \times A.c\Delta t$$

Intensity

$$I = \frac{\Delta U}{A\Delta t} = \frac{U_{av} \cdot A \cdot C\Delta t}{A\Delta t}$$

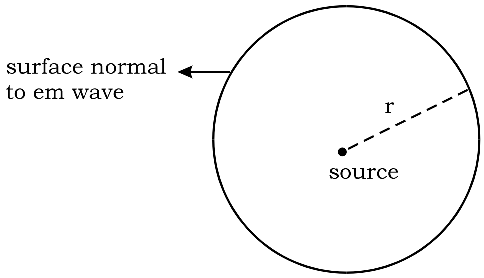
$$I = U_{av} \times c$$

Intensity = Average energy density × speed

$$I = \frac{1}{2}\varepsilon_0 E_0^2 \cdot C = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot C$$

Intensity of E-M wave at a distance r from isotropic point source

P=power emitted source



Intensity $= \dfrac{\text{power crossing the surface}}{\text{area of surface}}$

$$I = \frac{P}{4\pi r^2}$$

$$I \propto \frac{1}{r^2}$$

Poynting vector $\left(\vec{S}\right)$

The rate per unit area at which energy is transported via an electromagnetic wave is given by Poynting vector $\left(\vec{S}\right)$

$$\left(\vec{S}\right) = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

The magnitude is the rate at which energy is transported per unit area at an instant.

The average value of magnitude of poynting vector is numerically equal to the intensity of electromagnetic wave.

$\rightarrow$The direction of poynting vector $\vec{S}$, of an electromagentic wave at a point, gives the direction of energy transport at that point.

Momentum of EM Wave

Electromagnetic waves carry linear momentum with it. The linear momentum carried by a portion of wave having energy U is given by

$$P = \frac{U}{c}$$

If an EM wave is incident on a perfectly absorbing surface, momentum transferred to the surface is

$$\Delta P = \frac{U}{c}$$

If EM wave is incident normally on a perfectly reflecting surface, the momentum transferred to the surface is given by

$$\Delta P = \frac{2U}{c}$$

Radiation Pressure

The force exerted by electromagnetic wave on unit area of the surface as known is radiation pressure.

When em waves are incident normally on a perfectly absorbing surface, radiation pressure

$$P_{rad} = \frac{\Delta P}{A(\Delta t)} = \frac{U}{CA\Delta t} = \frac{I}{C}$$

When em waves are incident normally on a perfectly feflecting surface, radiation pressure is given by

$$P_{rad} = \frac{\Delta P}{\Delta t \cdot A} = \frac{2U}{C \cdot A\Delta t} = \frac{2I}{C}$$

Electromagnetic spectrum

The orderly distribution of electromagnetic waves according to their wavelength or frequency is known as electromagnetic spectrum

In the increasing order of their wavelength electromagnetic waves can be arranged as

Type	Wavelength range
Radio	> 0.1m
Microwave	0.1m to 1mm
Infra-red	1mm to 700 nm
Light	700 nm to 400 nm
Ultraviolet	400 nm to 1nm
X-rays	1nm to 10^{-3} nm
Gamma rays	< 10^{-3} nm

Radio waves

Wavelength range > 0.1m

Frequency range $\rightarrow$500 k Hz $\rightarrow$ 1000 M Hz

$\rightarrow$Produced using oscillating electric circuits

$\rightarrow$Main applications of radio waves are in radio and television communication systems

$\rightarrow$Different frequency ranges are used in different communication systems.

AM transmission : 530 kHz - 1710 k Hz

Short wave bands: upto 54 MHz

FM transmission : 88 MHz - 108 MHz

TW waves : 54 MHz - 890 MHz

Microwave

Wavelength range: 10^{-3} m $\rightarrow$ 10^{-1} m

Frequency range = 3×10^{9} Hz $\rightarrow 3\times10^{11}$Hz

Methods of production: Klystrons, Magnetrons, Gunn diodes

Applications: Radar communication, Microwave oven

Infra-red waves

Wavelength range - 700nm $\rightarrow$ 1mm

Frequency range: $3\times10^{11} \rightarrow 4\times10^{4}$ Hz

Infra-red waves are readly absorbed by different molecules like H_2O, CO_2, NH_3 .

This leads to heating up of objects.

Infra-red waves are emitted by hot bodies and molecules

Infra-red rays are responsible for green-house effect

They are widely used for remote control switches

Visible light

Wavelength range: 4×10^{-7}m $\rightarrow 7\times10^{-7}$ m

Frequency range: 4×10^{14} Hz $\rightarrow$ 8×10^{14} Hz

Human eyes are sensitives to these waves

They are mostly produced during atomic de-excitations

Ultra-violet waves

Wavelength range: 10^{-9} m $\rightarrow 40\times10^{-9}$m

Frequency range: 8×10^{14} Hz $\rightarrow 3\times10^{17}$Hz

Produced when inner shell electrons make a transition from higher energy level to lower energy level

Used as disinfectants in water purifiers, used in LASIK surgery as UV rays can be focused into vary small areas

X-rays

Wavelength range: 10^{-3} mm $\rightarrow$ 1nm

Frequency range: 3×10^{17} Hz $\rightarrow 3\times10^{20}$ Hz

X rays are generally produced when high energy electrons are bombarded with certain metals with large atomic number

X-rays are used as a diagnotic tool in medicine. They are also used to detect cracks and faults in metal products

They are used in radiotherapy to destroy cancer cells.

Gamma Rays

Wavelength range: $10^{-10}m \rightarrow 10^{-14}m$ frequency range: 3×10^{18} Hz $\rightarrow 3\times10^{22}$Hz

They are produced during nuclear reactions and in radioacitve decay. They are used in radiotherapy to destroy cancer cells.

Microwave oven

In microwave oven, microwaves having frequency of the order of 3G Hz are made to pair through food. This is equal to resonant frequency of rotation of water molecules. Hence, water molecules absorb these radiation and the temperature of objects increase.

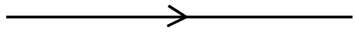
Different types of electromagentic waves

Type	Wavelength range	Production	Deterction
Radio	>0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1mm	Klystron walve or magnetron valve	Point contact diodes
Infra-red	1mm to 700nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1 nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	<10^{-3} nm	Radioactive decay of the nucleus	do

RAY OPTICS
(REFLECTION OF LIGHT)

* **Introduction**

* Light is represented by ray of light. A ray is a light path along which optical energy flows; the direction of energy flow is represented by the arrow sign



Ray of light

* Ray paths are straight line in a homogenous medium i.e. light has rectilinear propagation

Validity of Ray Optics

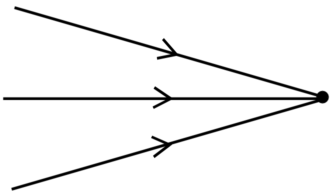
Rectilinear propagation can be applied only when the size of the obstacle is very large compared to the wave length of light. Because diffraction (bending of light) can be neglected at this condition

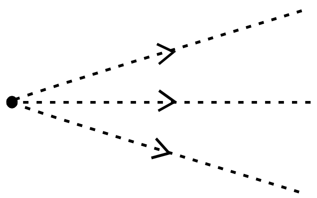
Validity of ray optics can be explained by wave theory of light. Since diffraction is explained by wave theory and rectilinear propagation is possible only in the absence of diffraction.

* **Beam of light** : Collection of rays. Light from a source can be represented by beam of light

Real Images and Virtual Images

Real images are formed when the light rays actually converge to a point after reflection or refraction. They can be obtained on a screen.

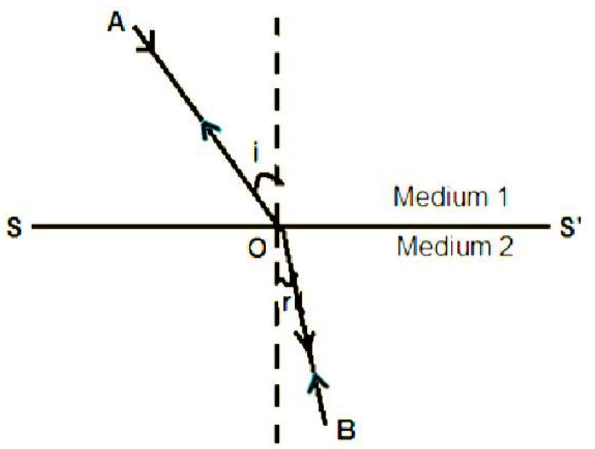


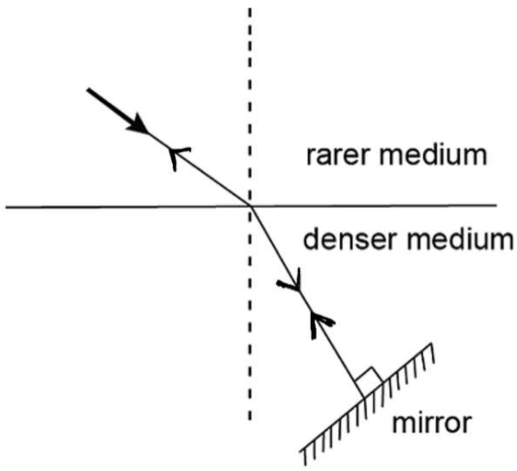


virtual image

Virtual images are formed when the light rays appear to diverge from a point after reflection or refraction. They cannot be obtained on a screen but observable. Virtual rays are represented by doted lines.

Principle of reversibility of ray : When a light, after suffering a number of reflections and refractions, has its final path reversed, it retraces it's own path

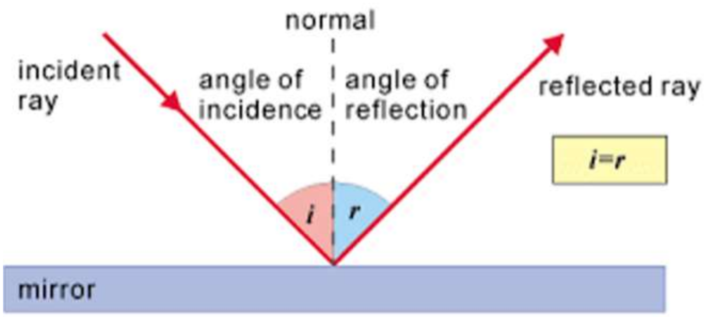




Mutual Independency of ray : Path of the light rays are mutually independent, i.e. they do not disturb each other.

Reflection of light : Bouncing back of light to the same medium. Change in direction of light, without any change in medium

Laws of Reflection

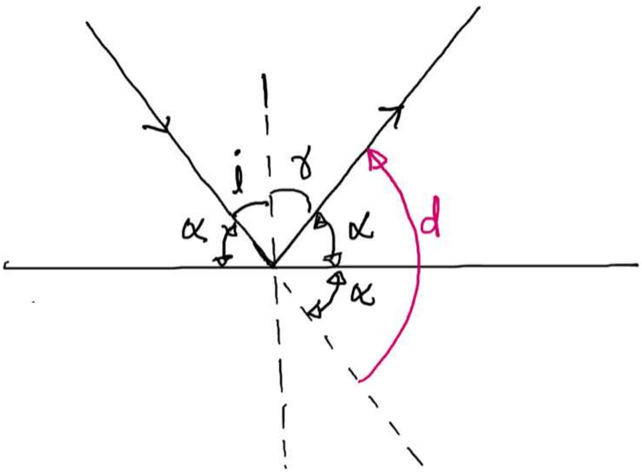


1) angle of incidence is always equal to angle of reflection (i = r)

2) incident ray, reflected ray and the normal at the point of incidence all lie in the same plane

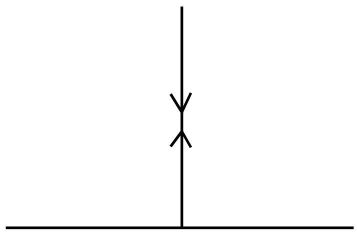
Deviation in Reflection

Angle between directions of incident ray and reflected ray is called angle of deviation (d)

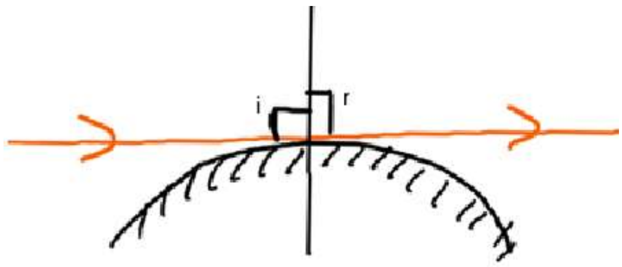


$d = 2\alpha = 2(90 - i)$

$d = 180 - 2i$, direction of deviation is from incident ray to reflected ray



in normal incidence
i = r = zero
∴ light retraces the path after reflection and deviation is 180°



in grazing incident i = r = 90⁰
and deviation = zero

Phase difference due to reflection

When light is reflected from a denser medium phase difference between reflected light and incident light is π rad or 180°

When light is reflected from a rarer medium phase difference between reflected light and incident light is zero.

Reflection by Plane mirror

For a real object image formed by plane mirror

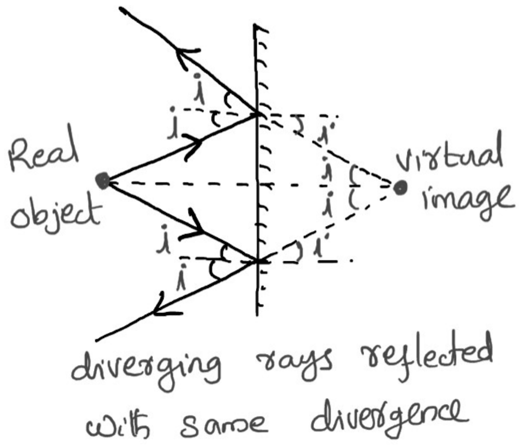
→ Virtual, erect and laterally inverted

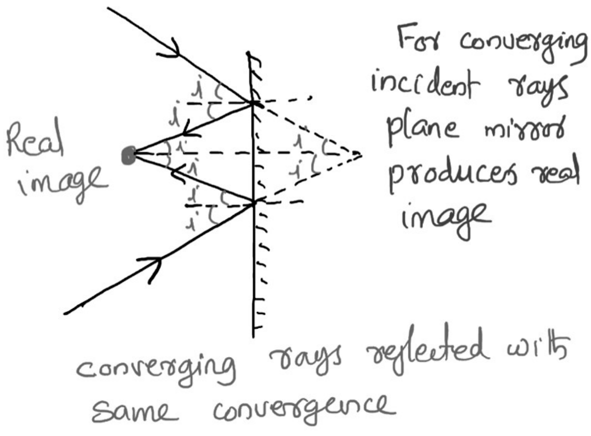
→ image distance = object distance, v = –u

→ image size = object size

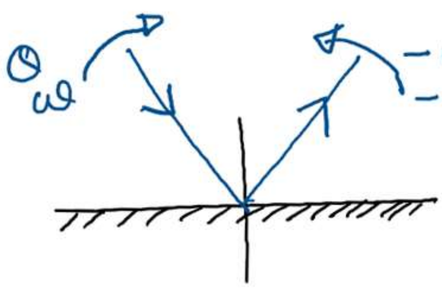
→ image and object are formed in the same plane normal to the mirror

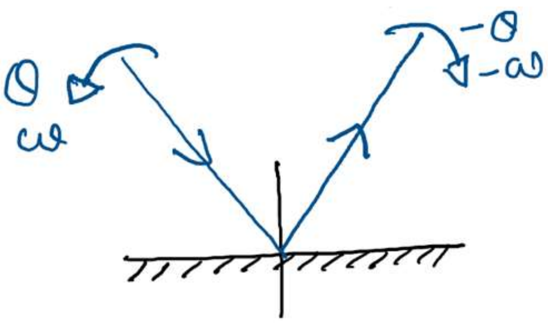
→ When a parallel beam of light is incident on a plane mirror, the reflected rays are also parallel. Thus plane mirror never produces convergence or divergence i.e. focal length of plane mirror is infinity and power is zero.





When the incident ray rotates through an angle θ, with an angular speed ω towards or away from the mirror, the reflected ray rotates through an angle θ with an angular speed ω.
Since angle of incidence = angle of reflection

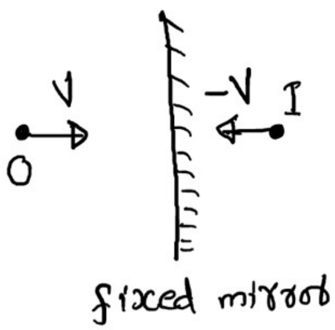


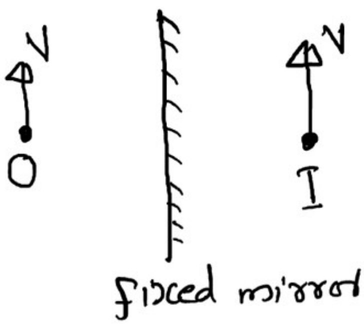


If the mirror rotates through an angle θ, with an angular speed ω, the reflected ray rotates through an angle 2θ, with an angular speed 2ω, with same direction is that of rotation of mirror for a fixed incident ray.

Velocity of image in plane mirror

Image distance = object distance $\therefore$ when the object moves towards or away from the mirror with a velocity $\vec{v}$, then the velocity of the image is $-\vec{v}$





Velocity of object and image are equal, when velocity of object is parallel to the plane of mirror.

→ Thus generally $\left(\vec{V}_0\right)_{\parallel}$ and $\left(\vec{V}_0\right)_{\perp}$ are the velocity of object parallel and normal to the mirror, then object velocity

$\vec{V}_0 = \left(\vec{V}_0\right)_{\parallel} + \left(\vec{V}_0\right)_{\perp}$ and velocity of image

$\vec{V}_I = \left(\vec{V}_0\right)_{\parallel} - \left(\vec{V}_0\right)_{\perp}$

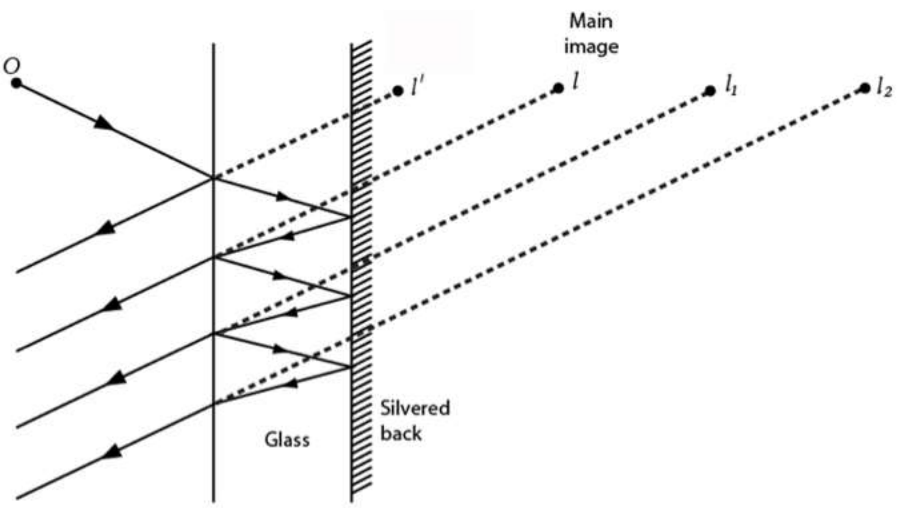
→ If the mirror moves parallel to its mirror, the velocity of image is zero.

→ If $\left(\vec{V}_M\right)_{\perp}$ is the velocity of mirror normal to the mirror, then the velocity of image is $Z\left(\vec{V}_M\right)_{\perp}$

Example :

Thick plane mirror forms a number of images of a point source of light. Which image is brightest ?

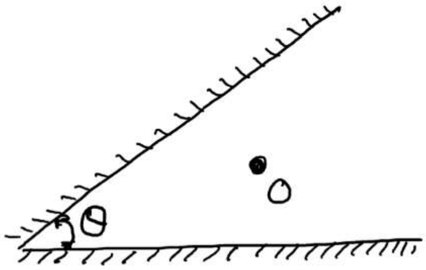
A. First B. Second C. Third D. Fourth



Answer

In the case of a thick plane mirror, reflection takes place at the two surfaces, (top and bottom surfaces) and images are formed due to both. The second surface is silvered and therefore the second image is the brightest. Further images are formed due to multiple reflections and due to absorption of light by medium the images becom fainter.

Images formed by the two plane mirrors : Two plane mirrors subtends an angle θ, and an object is placed as shown. Calculate $\frac{360}{\theta}$



Case 1 : When $\frac{360}{\theta}$ is even, number of images $N = \frac{360}{\theta} - 1$

Case 2 : When $\frac{360}{\theta}$ is odd

a) Object is placed symmetrical between the mirrors $N = \frac{360}{\theta} - 1$

b) Object is placed asymmetrically, $N = \frac{360}{\theta}$

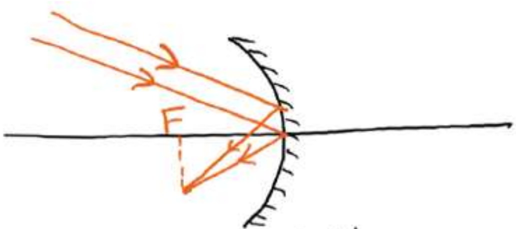
Case 3 : When $\frac{360}{\theta}$ is a fractional value

N = greatest integer value of $\frac{360}{\theta}$

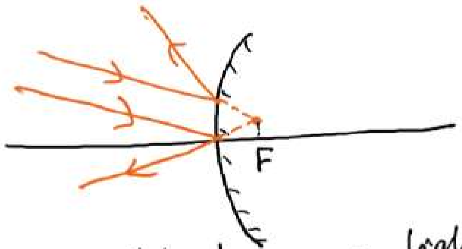
eg. $\left.\begin{aligned} \frac{360}{\theta} &= 3.4 \\ \frac{360}{\theta} &= 3.9 \end{aligned}\right\} N = 3$

Spherical Mirrors

Focal Plane : A plane passing through focus and perpendicular to principal axis is called focal plane



parallel beam of light converges at focal plane, real image formed at focal plane



parallel beam of light appear to diverge from focal plane and a virtual image is formed at focal plane

Sign Conventions in Mirror :

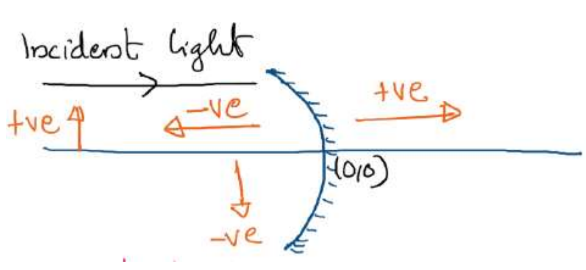
→ Pole of the mirror is taken as the origin, i.e. all the distances are measured from the pole

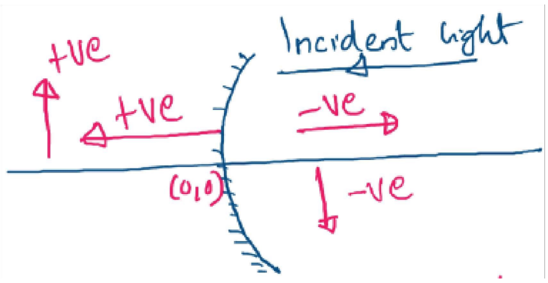
→ All the measurements in the direction of incident light are +ve and measurements opposite to the direction of incident light are –ve

→ Distances upward from the principal axis are positive and vice versa

→ Do not give sign to unknown quantities

→ Anticlockwise angle is taken as +ve and clockwise angle is negative





→ Focal length of a concave or converging mirror is negative

→ Focal length of a convex of diverging mirror is positive

→ Real images are formed in front of the mirror $\therefore$ image distance is negative. Virtual images are behind the mirror and image distance is positive

Relation between f and R

For convex mirror R is positive and for concave mirror R is negative. Focal length of mirror is independent of intervening medium.

Mirror equations and lens equations can be applied only to paraxial rays, but laws of reflection and refraction can be applied to all the rays.

Mirror equation

$$\therefore \boxed{\frac{1}{f}=\frac{1}{v}+\frac{1}{u}} \text{ or } \boxed{f=\frac{uv}{u+v}}$$

For plane mirror R = ∞

$f=\infty$, so u = v

→ The above derived formula can be used for convex mirror also

→ In using these formulas, the signs are given only to known values

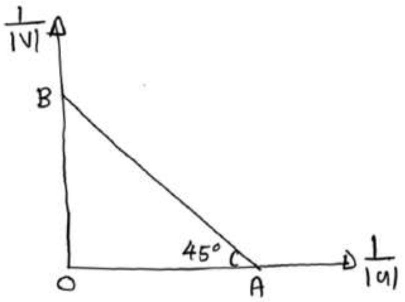
Focal length of concave mirror from graph

a) Graph between $\frac{1}{u}$ vs $\frac{1}{v}$:

We have $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ or $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Comparing this equation with y = mx + c, $y = \frac{1}{v}, c = \frac{1}{f}$,

$x = \frac{1}{u}$ and m = –1



∴ graph between $\frac{1}{u}$ and $\frac{1}{v}$ is a straight line

From graph OA = OB = $\frac{1}{f}$;

$$f = \frac{1}{OB} = \frac{1}{OA}$$

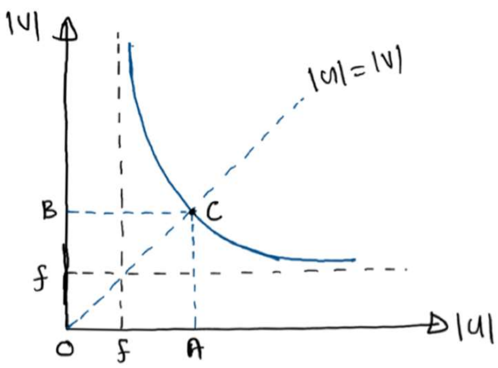
b) Graph between u and v

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ ∴ u vs v graph is hyperbolic.

In the graph coordinates of C are (2f, 2f)

∴ OA = OB = 2f

$f = \frac{OA}{2} = \frac{OB}{2}$



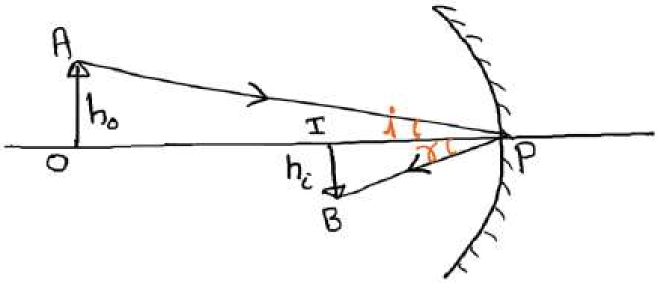
Magnification : The relative size of the image with object can be understood by magnification (m)

i.e. $m = \frac{\text{size of the image}}{\text{size of the object}}$

Depending on the height, length and area of the object, there are three types of magnification

1) Lateral magnification (m) : It is the ratio of the size of the image and object perpendicular to the principal axis.

$$m = \frac{\text{height of the image}}{\text{height of the object}} = \frac{h_i}{h_0}$$



$$\boxed{m = \frac{h_i}{h_0} = -\frac{v}{u}}$$

'm' is negative for inverted images

'm' is positive for erect images

$|v| < |u|$, $|m| < 1$, image is diminished

$|v| > |u|$, $|m| > 1$, image is enlarged

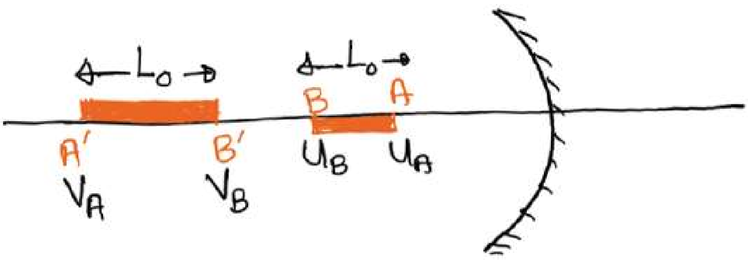
$|v| = |u|$, $|m| = 1$, same size image

'm' in terms of focal length

$$\boxed{m = \frac{f - v}{f}}$$

Longitudinal Magnification (m_L) : It is the ratio of the size of the image and object along the principal axis.

$$m_L = \frac{\text{Length of the image}}{\text{Length of the object}} = \frac{L_i}{L_0}$$



From the diagram $m_L = \frac{v_A - v_B}{u_A - u_B} = \frac{\Delta v}{\Delta u}$

$\frac{1}{v_A} = \frac{1}{f} - \frac{1}{u_A}$ $\frac{1}{v_B} = \frac{1}{f} - \frac{1}{u_B}$

For small objects $m_L = \frac{dv}{du}$

From mirror equation $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

On differentiating w.r.t 'u' $0 = -\frac{1}{v^2}\frac{dv}{du} - \frac{1}{u^2}$

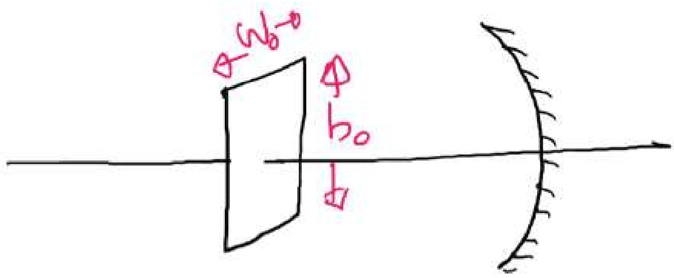
$\frac{dv}{du} = \frac{-v^2}{u^2}$ So $m_L = \frac{dv}{du} = -\frac{v^2}{u^2} = -m^2$

–ve sign shows that imae is inverted

Areal Magnification (m_A) : It is the ratio of the area of the image to the area of the object.

$$m_A = \frac{\text{area of the image}}{\text{area of the object}} = \frac{A_i}{A_0}$$

Area of the object is perpendicular to the principal axis : h_0 and w_0 are the height and width of the object perpendicular to the principal axis.



height of the image $h_i = mh_0$

width of the image $W_i = mW_0$

So area of the image $A_i = h_i W_i = m^2 h_0 W_0$

$A_i = m^2 A_0$ or $m_A = \frac{A_i}{A_0} = m^2$

Here image formed is undistorted i.e. shape of the image = shape of the object.

Newton's equation

If x_1 and x_2 are the object and image distances from the focus of the mirror, then

$$\boxed{f = \sqrt{x_1 x_2}}$$

NOTE :

Sign convention of object distance

For diverging incident rays or for real objects

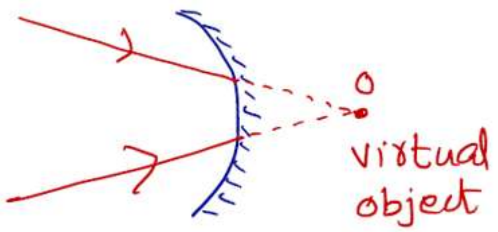
object distance is negative

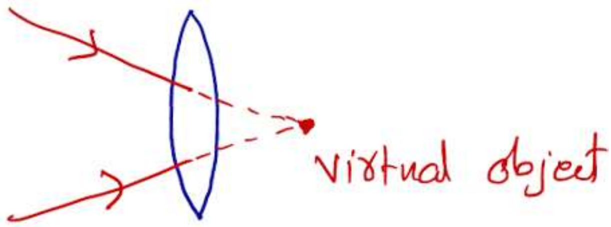
For converging incident rays or for virtual objects

object distance is positive

Virtual Object

When a converging beam of light is incident on a mirror or a lens the position of the object is taken at the point, where the incident rays converge in the absence of the mirror or lens. This assumed object is called virtual object.





Uses of Spherical mirror

Convex mirrors : Used as rear view mirrors, diverging reflectors in street lights. Convex mirrors have large field of view.

Concave mirrors : used as shaving mirrors, used by dentists

Power of mirror : Power measures the degree of convergence or divergence produced by a mirror.

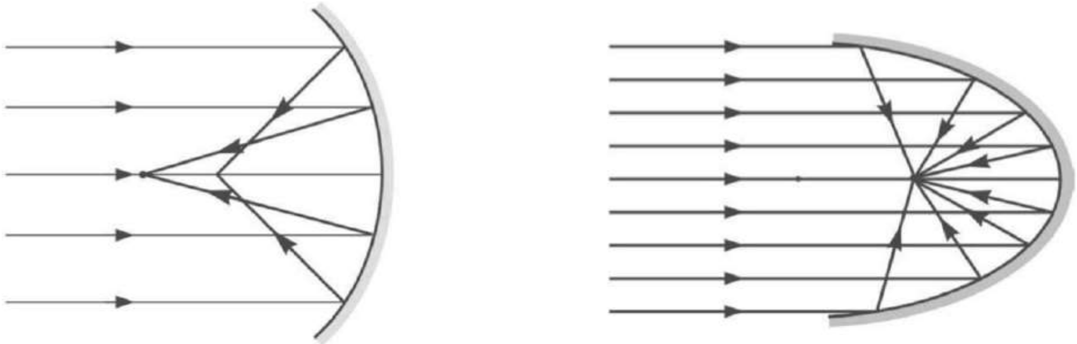
Power, $\boxed{P = -\frac{1}{f}}$ f = focal length. When focal length is in meter, unit of power is dioptre (D).

$$\therefore \boxed{P = \frac{-100}{f(\text{cm})} D}$$

- Power of concave or converging mirror is positive and that of convex or diverging mirror is negative

SPHERICAL ABERRATION IN MIRRORS

In practice spherical mirrors are capapble of forming reasonably sharp images if their apertures are small in comparison to the focal length. In case of large mirror, the rays reflected from the outer edges cross the axis at different distances as shown in figure. This inability to focus all the incident rays at a single point is called spherical aberration. a parabolic mirror, however, brings all rays to a focus at one point. A small source of light located at the focal point of a parabolic reflector becomes a parallel beam after reflection, which is used in automobiles headlights and in search lights



a) Spherical aberration in concave mirror   b) No spherical aberration

REFRACTION OF LIGHT

- The phenomenon of bending away of light from the normal is known as refraction. The bending of the light depends on the two medium that the light is traversing
- **Cause of refraction** : This happens because when a light travels from one medium to another, then the speed of light also changes
- **Refractive index** : This difference in the deflecting property of light in different substances is due to the refractive inded of a material. The refractive index of a medium is defined as the ratio of the speed of the light in vacuum to the speed in the medium. It is denoted by μ or n. Mathematically

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}} = \frac{c}{\nu}$$

- For two mediums $n_1 = \frac{c}{\nu_1}$ and $n_2 = \frac{c}{\nu_1}$

  Relative RI of the mediums $\frac{n_1}{n_2} = \frac{\nu_2}{\nu_1}$

- The medium with higher refractive index is called optically denser medium and medium with lower refractive index is called optically rarer medium
- If f is the frequency of light, λ is the wavelength and 'v' the speed of light in a medium, then the $v = f\lambda$
- Frequency of light is independent of the medium. So for a light in two mediums

$$\nu \,\alpha\, \lambda, \boxed{\frac{\nu_1}{\nu_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}}$$

- Colour of light depends on frequency, thus colour of light is independent of the medium
- Speed of light in vacuum $\boxed{c = \frac{1}{\sqrt{\in_0 \mu_0}}}$, where $\in_0$ = permittivity of free space and

  μ_0 = permeability of free space

Thus in a medium speed of light $v = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \mu_r \in_0 \in_r}} = \frac{c}{\sqrt{\mu_r \in_r}}$

But v = c/n, thus $\boxed{n = \sqrt{\mu_r \in_r}}$. So refractive index or optical density depends on both electric and magnetic properties of the medium.

$\boxed{\lambda_{med} = \frac{\lambda_{vac}}{\lambda_{med}}}$ $\qquad n_{med} > 1 \quad \therefore \lambda_{med} < \lambda_{vac}$

- Optical density is independent of mass density.
- It is possible that mass density of an optically denser medium may be less than or greater than that of an optically rarer medium. For example, mass density of turpentine is less than that of water but its optical density is higher

Cauchy's Dispersion Formula

Refractive index 'n' of a medium depends on wavelength of light 'λ' as

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$$

where A,B,C,.... are constants for a medium usually it is sufficient to use a two term form of the equation, $\boxed{n = A + \frac{B}{\lambda^2}}$

Equation shows that the refractive index of a medium is different for different colours of light.

- If λ_R and λ_V are the wavelengths of red and violet colours of light in a medium, then $\lambda_R > \lambda_V$ and refractive index $n_V > n_R$. So speed of light $V_R > V_V$

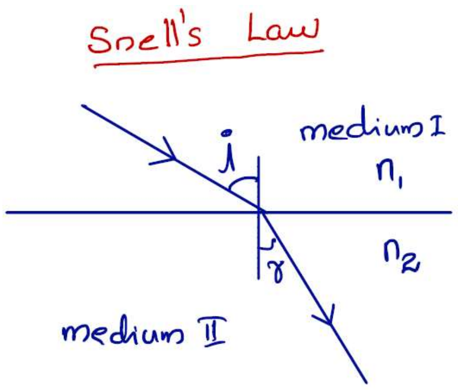
Laws of refraction :

1) The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane

2) The ratio of the sine of the angle of incidence to the sine of the angle of refraction for two mediums is a constant

i.e. $\frac{\sin i}{\sin r} = \text{constant}$

Snell's Law



where,

i = angle of incidence

r = angle of refraction

$$\boxed{\frac{\sin i}{\sin r}=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$$

Where

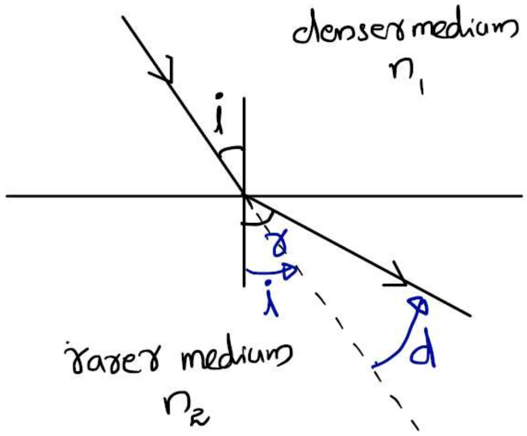
n_1 = refractive index of the first medium

n_2 = refractive index of the second medium

- When light travels from denser to rarer medium, refracted ray bends away from the normal. From Snell's Law $n_1 \sin i = n_2 \sin r$

Here $n_1 > n_2 \quad \therefore \sin r > \sin i$ and r > i

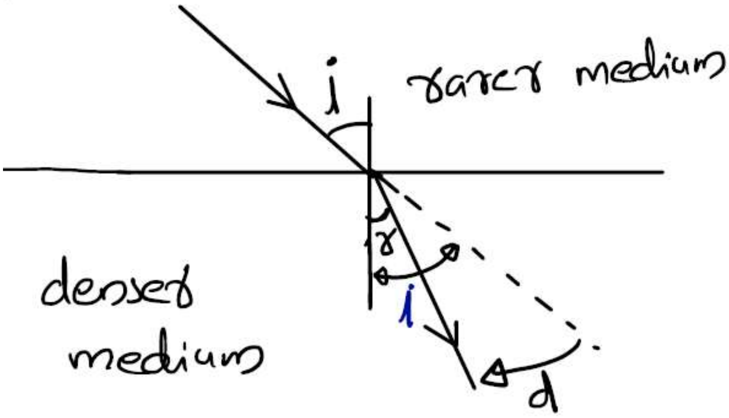
Deviation in Refraction



The angle between the incident ray and refracted ray is called angle of deviation (d).

From the diagram $\boxed{d = r - i}$

- When light travels from rarer to denser medium refracted ray bends towards the normal



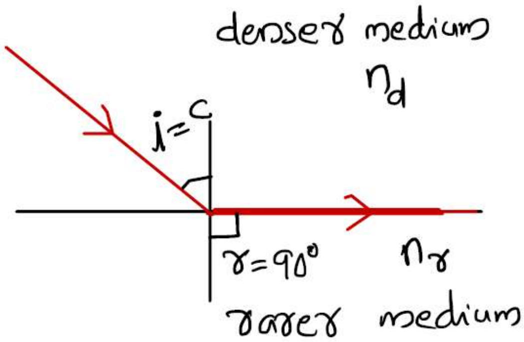
Here deviation

$\boxed{d = i - r}$

- Phase differe between incident light and refracted light is always zero.

Critical Angle (C) :

The angle of incidence in a denser medium at which the angle of refraction in the rarer medium becomes 90° or when the refracted ray passes along the interface is called critical angle (C).



From Snell's Law

$$n_d \sin C = n_r \sin 90$$

$$\boxed{\sin C = \frac{n_r}{n_d} = \frac{V_d}{V_r} = \frac{\lambda_d}{\lambda_r}}$$

Where n_r and n_d are the refractive indices of rarer and denser medium respectively.

- $\sin C \,\alpha\, n_r$ $\quad\therefore$ when $n_r \uparrow$ $\quad C\uparrow$
- $\sin C \,\alpha\, \frac{1}{n_d}$ $\quad\therefore$ when $n_d \uparrow$ $\quad C\downarrow$

If rarer medium is vacuum or air n_r = 1, then $\sin C = \frac{1}{n_d} = \frac{1}{n}, n_V > n_R \quad \therefore C_V < C_R$

C_V and C_R are the critical angles of violet and red colours

- $\frac{\sin i}{\sin r} = \text{constant} \qquad \therefore \sin r \,\alpha\, \sin i$

Thus angle of refraction always increases with increase in angle of incidence

- When light is incident along the interface angle of incidence is maximum i_{max} = 90°. In this case light refracted to the denser medium with maximum angle of refraction r_{max} = critical angle (C)

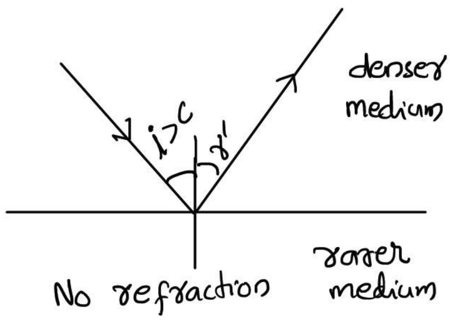
NOTE : Maximum value of angle of refraction in a denser medium is critical angle (C)

$(i_{max})_{rarer} = 90^o, (r_{max})_{denser} = C$

Total Internal Reflection (TIR)

When light travels from denser to rarer medium with an angle of incidence greater than the critical angle, the entire light is reflected back to the same medium without any refraction. This is called total internal reflection. For TIR, $\boxed{i > C}$, sin i > sin C

$\therefore \boxed{\sin i > \frac{n_r}{n_d}}$



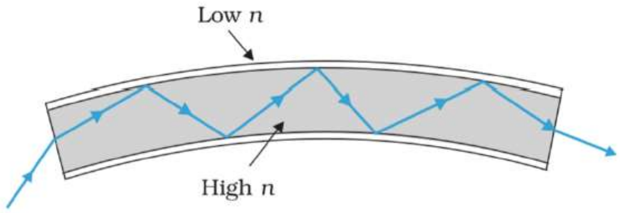
- TIR obeys laws of reflection $(i = r^1)$
- deviation in reflection
  d = 180 – 2i

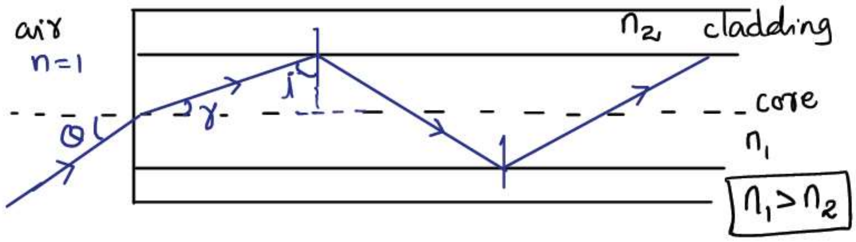
NOTE : Maximum angle of incidence in a denser medium for refraction is critical angle.

APPLICATIONS OF TOTAL INTERNAL REFLECTION

- mirage
- looming
- Brilliance of diamond
- Total reflecting prisms

- **Optical Fibres** : Used for transmitting audio and video signals through long distances, by using total internal reflection. Each fibres consists of a core and a cladding.
  The refractive index of the material of the core is higher than that of the cladding. Light undergoes repeated TIR at the core cladding interface and advances through the optical fibre





For TIR reflection at core-cladding interface angle of incidence i > C

$$\therefore \sin i > \sin C,\ \sin i > \frac{n_2}{n_1}$$

From the diagram $i+r=90^o$ or $i = 90 - r$, $\sin(90-r) = \cos r > \frac{n_2}{n_1}$

$$\sqrt{1-\sin^2 r} > \frac{n_2}{n_1} \text{ or } 1-\sin^2 r > \frac{n_2^2}{n_1^2}$$

$$\sin^2 r < 1 - n_2^2 / n_1^2 \text{ ....(1)}$$

For refraction at air core interface,

$$1\times\sin\theta = n_1\times\sin r,\ \ \sin r = \frac{\sin\theta}{n_1}$$

Put this in eqn. (1)

$$\frac{\sin^2\theta}{n_1^2} < \frac{n_1^2 - n_2^2}{n_1^2}$$

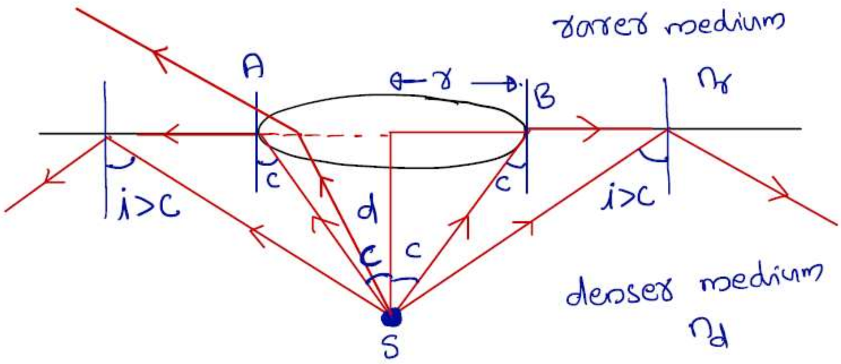
$$\boxed{\sin\theta < \sqrt{n_1^2 - n_2^2}}$$

$\therefore [\sin\theta]_{max} = \sqrt{n_1^2 - n_2^2}$ or $\theta_{max} = \sin^{-1}\sqrt{n_1^2 - n_2^2}$ condition of TIR at core-cladding interface

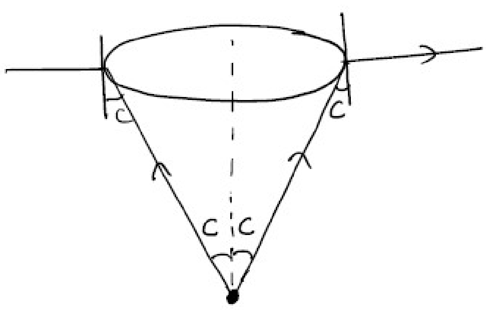
θ_{max} = maximum angle of incidence from air to core for TIR at core cladding interface, which is called acceptance angle.

- **Light from a source in a denser medium enters into a rarer medium only through a circular region due to TIR**

  Consider a source of light at a depth 'd' from the interface as shown



Light incident at critical angle (C) at the point A and B. Thus light from the source undergoes TIR to the left of A and to the right of B. So the light from the source undergoes refraction through a circular region of radius 'r' as shown

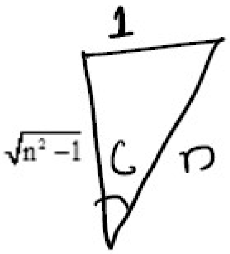


From the diagram, $\tan C = \frac{r}{d}$

$\therefore \boxed{r = d\tan C}$

But $\sin C = \frac{n_r}{n_d} = \frac{1}{n} \Rightarrow \sqrt{n^2 - 1}$

$\therefore \tan C = \frac{1}{\sqrt{n^2 - 1}}$



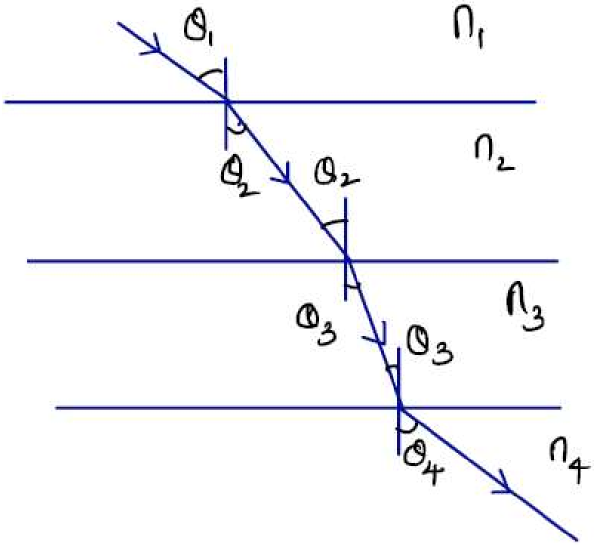
So, $\boxed{r = \frac{d}{\sqrt{n^2 - 1}}}$ Area of the circular region $A = \pi r^2$

→ The angular width of the circular region (2C) is independent of the depth of the source

→ radius 'r' increases with increase in depth 'd'

- Similarly an observer in a denser medium receives light from a rarer medium only through the above mentioned circular region.

Refraction at Parallel Plane Surfaces



From Snell's Law

$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$

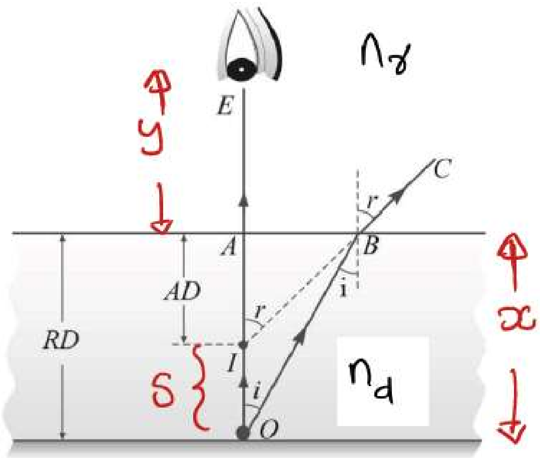
$= n_4 \sin\theta_4$

$\therefore n \sin\theta = \text{constant}$

Image Formation by Refraction

(i) object in denser medium and observer in rarer medium.

Consider an object 'O' placed in an optically denser medium. Ray diagram shows that the refracted rays appear to diverge from the point 'I'. Therefore a virtual image is formed at I.



The distance 'AO' is called real depth or actual depth. 'AI' is called apparent depth. n_r and n_d are the refractive indices of rarer and denser medium respectively.

$\boxed{n = \dfrac{AO}{AI}}$ i.e. $\boxed{n = \dfrac{n_d}{n_r} = \dfrac{\text{real depth (RD)}}{\text{apparent depth (AO)}}}$

$n = \dfrac{x}{AD}$ $\boxed{AD = \dfrac{x}{n}}$ where $n = \dfrac{n_d}{n_r}$

The normal shift produced to the object is, S = real depth – apparent depth

$= RD - \dfrac{RD}{n} = x - \dfrac{x}{n}$, $\boxed{S = x\left[1 - \dfrac{1}{n}\right]}$

Separation between observer and image is,

Z = EA + AI or $\boxed{Z = y + \dfrac{x}{n}}$

On differentiating w.r.t time

$\dfrac{dz}{dt} = \dfrac{dy}{dt} + \dfrac{1}{n}\dfrac{dx}{dt}$ $\boxed{\left[V_{image}\right]_{observer} = V_{oberver} + \dfrac{V_{object}}{n}}$

For example, if a bird observes a fish

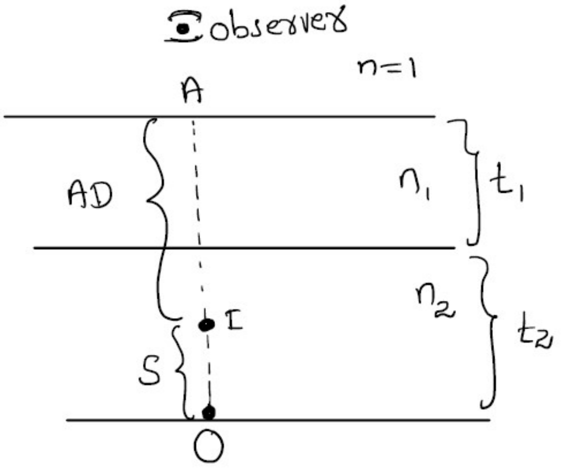
$$\boxed{\left[V_{fish}\right]_{bird} = V_{bird} + \frac{V_{fish}}{n}}$$

NOTE : Shift produced by a denser medium is in the direction of incident light

Sign Conventions of Velocity

Velocities of object and observer are taken as positive, when they move away from the interface. Velocities are taken as negative, when they move towards the interface

Object is situated inside two or more optical mediums



Shift due to number of mediums is the sum of the shifts due to each medium.

In the example two mediums are present,

$\therefore$ Shift, $S = S_1 + S_2$

$$S = t_1\left[1 - \frac{1}{n_1}\right] + t_2\left[1 - \frac{1}{n_2}\right]$$

$\therefore$ apparent depth AD = $t_1 + t_2 - S$ $\qquad AD = \frac{t_1}{n_1} + \frac{t_2}{n_2}$

NOTE : For n mediums of thicknesses; t_1, t_2, ..., t_n of refractive indexes n_1, n_2,....n_n

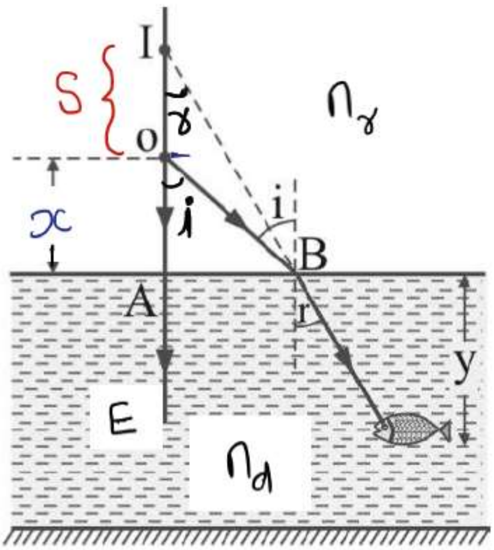
real depth = $t_1 + t_2 + ...t_n$

Apparent depth = $\frac{t_1}{n_1} + \frac{t_2}{n_2} + ... + \frac{t_n}{n_n}$

If n is the effective value of refractive indexes, then $n = \dfrac{\text{real depth}}{\text{apparent depth}}$

$$\boxed{n = \dfrac{t_1 + t_2 + ... + t_n}{\dfrac{t_1}{n_1} + \dfrac{t_2}{n_2} + ... + \dfrac{t_n}{n_n}}}$$

(ii) Object in rarer medium and observer in denser medium



$n = \dfrac{AI}{AO}$; $\boxed{n = \dfrac{\text{Apparent depth (AD)}}{\text{Real depth (RD)}} = \dfrac{n_d}{n_r}}$ AI = n(AO), $\boxed{AI = nx}$

Normal shift due to refraction

S = AD – RD = nx – x

$\boxed{S = x(n-1)}$ (Shift is opposite to the direction of incident light)

Separation between observer and image

Z = EA + AI $\boxed{Z = y + nx}$

On differentiating, $\dfrac{dz}{dt} = \dfrac{dy}{dt} + n\dfrac{dx}{dt}$

$$\boxed{(V_{image})_{observer} = V_{observer} + nV_{object}}$$

For example, when a fish observes a bird

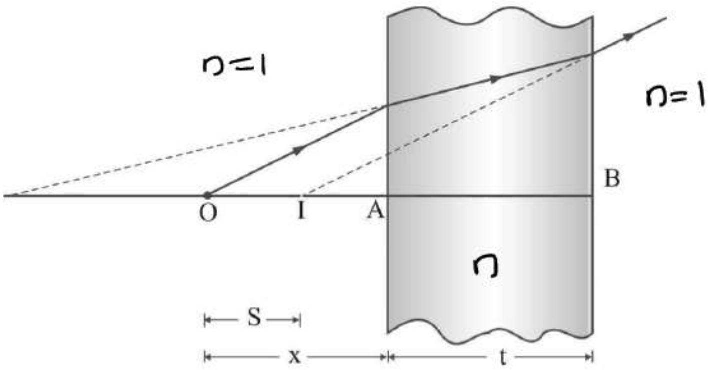
$$\boxed{(V_{bird})_{fish} = V_{fish} + nV_{bird}}$$

Sign conventions mentioned earlier must be applied in this equation.

NOTE : The above equations are used when i and r are very small or in normal observation.

- Shift produced is independent of normal separation of observer from the interface, but shift changes when the observer moves parallel to the interface.

(iii) Shift produced by a glass slab



Conser an object at a distance 'x' from the left face of the glass slab. Thickness of the glass slab is 't' and refractive index 'n'. Actual distance of object from B is (x + t). But for an observer in air apparent thickness of glass slab is $\frac{t}{n}$. Thus the apparent position of the object from B is $BI = x + \frac{t}{n}$

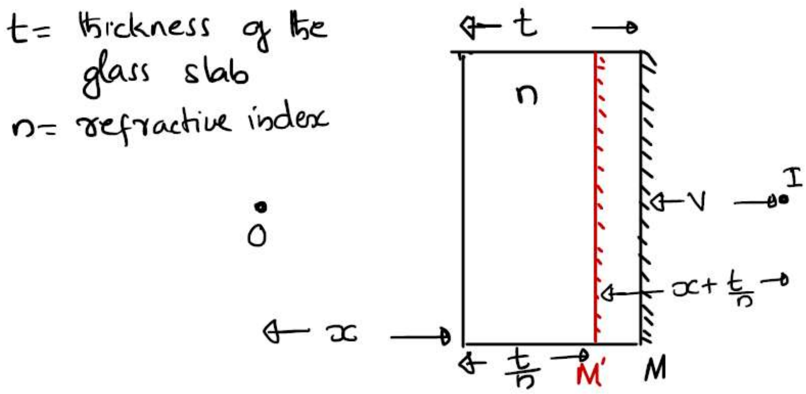
$\therefore$ Normal shift produced by the glass slab is

$$S = BO - BI = (x + t) - \left(x + \frac{t}{n}\right)$$

$$\boxed{S = t\left[1 - \frac{1}{n}\right]}$$

NOTE : Shift produced by the glass slab is in the direction of propagation of light through glass slab and independent of the distance 'x'

Because of multiple reflections and refractions, there will form infinite images; second of them will be brightest, which is formed by the reflection from the silvered face.



Suppose M^1 is the apparent position of the mirror at a distance $\frac{t}{n}$ from the unsilvered face of the mirror.

The object position from $M^1 = x + \frac{t}{n}$

Thus image position from $M^1 = x + \frac{t}{n}$

(behind the mirror)

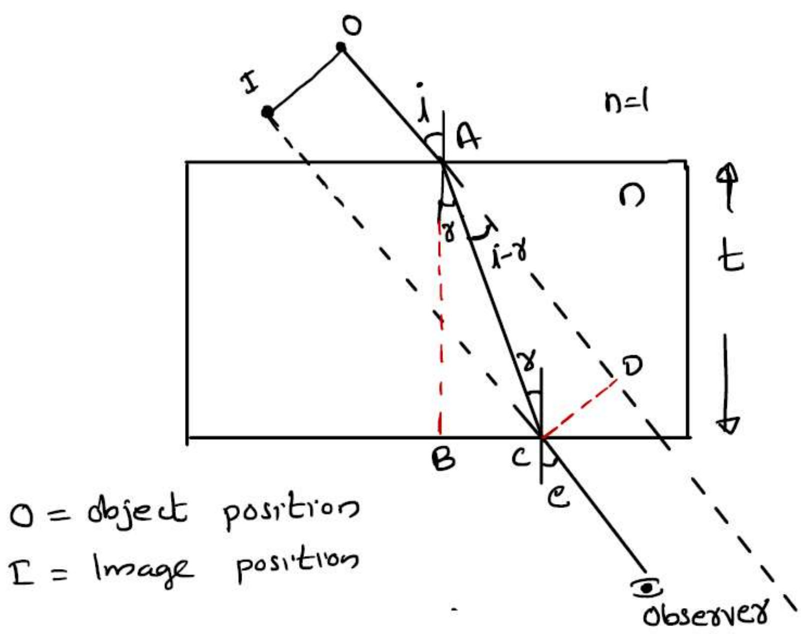
Thus final image distance from the silvered surface M, $V = x + \frac{t}{n} - MM^1$

$$= x + \frac{t}{n} - \left(t - \frac{t}{n}\right)$$

$$\boxed{V = x - t + \frac{2t}{n}}$$

Second image is formed behind the mirror, thus it is virtual.

Lateral shift produced by a glass slab



The shortest (perpendicular) distance between the direction of incident ray and emergent ray is called lateral shift (S_L)

S_L = OI = CD

$$\boxed{S_L = \frac{t\sin(i-r)}{\cos r}}$$

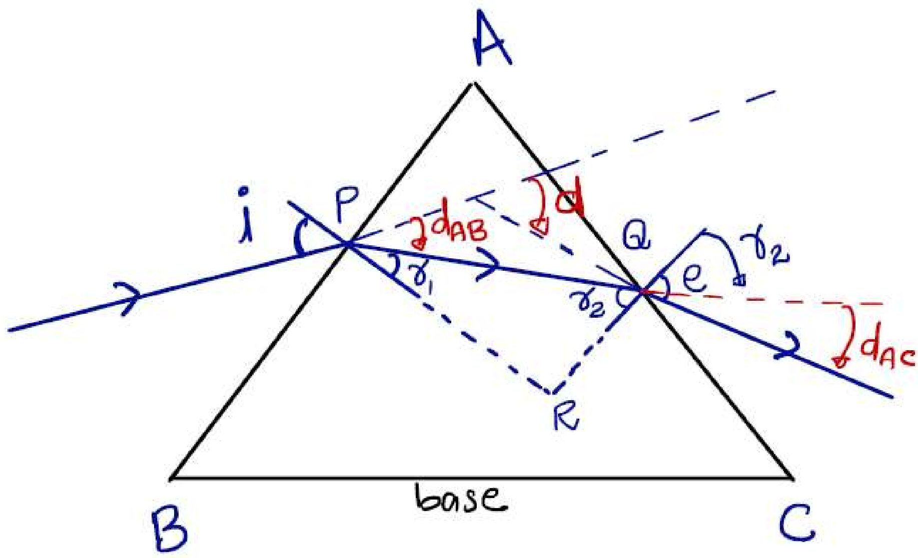
When i and r are very small, then cos r = 1 and sin(i – r) = i – r

$$\therefore \boxed{S_L = t(i-r)}$$

From Snell's law $\frac{\sin i}{\sin r} = \frac{i}{r} = n \quad \text{or} \quad r = \frac{i}{n}$

$$S_L = t\left[i - \frac{i}{n}\right] \quad \boxed{S_L = ti\left[1 - \frac{1}{n}\right]}$$

Refraction through a Triangular Prism



A monochromatic light is incident at an angle 'i' and emerges with an angle 'e' as shown. r_1 is the angle of refraction at the face AB and r_2 is the angle of incidence at the face AC. n_p and n_s are the refractive indices of the prism and the surrounding medium respectively. From Snell's law at the two faces,

$$\boxed{\frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2} = \frac{n_p}{n_s} = n} \quad .... (1)$$

$$\boxed{A = r_1 + r_2}$$

Angle of deviation : The angle between directions of incident ray and emergent ray is called the angle of deviation (d)

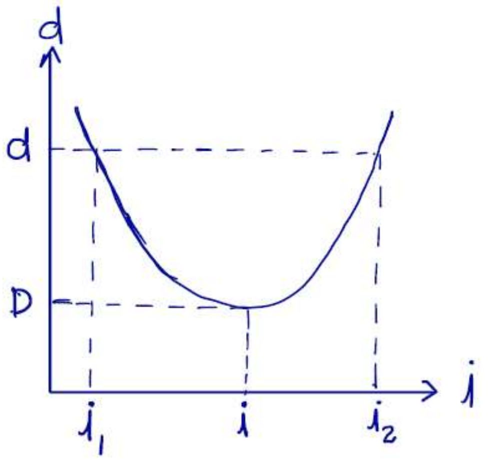
Total deviation d = deviation at face AB + deviation at face AC

$$d = d_{AB} + d_{AC} = (i - r_1) + (e - r_2)$$

$$d = i + e - (r_1 + r_2)$$

$$\boxed{d = i + e - A}$$

Variation of deviation : It first decreases with increase in angle of incidence, reaches a minimum and then increases. Deviation has same value at two angles of incidence, except at angle of minimum deviation (D)



In the graph deviation (d) is same at angles of incidence i_1 and i_2. If 'i' and 'e' are interchanged then we get the same value of deviation because of the reversibility principle of light.

Conditions of minimum deviation

At minimum deviation $r_1 = r_2 = r$ and $i = e$

Thus $A = r_1 + r_2 = 2r$ or $r = \frac{A}{2}$

minimum deviation $\boxed{D = 2i - A}$

or $i = \frac{A + D}{2}$

From Snell's law $\boxed{n = \frac{\sin i}{\sin r} = \frac{\sin\left[\frac{A + D}{2}\right]}{\sin\left[\frac{A}{2}\right]}}$

This equation is known as equation of prism

- When a prism is in minimum deviation position, the refracted ray inside the prism is parallel to the base and passes symmetrically through the prism provided base angles are equal (in the equilateral or isosceles prism)

- At minimum deviation, deviation at both the faces are equal i.e. $d_{AB} = d_{AC}$

$$\therefore D = 2d_{AB} = 2d_{AC} = 2(i - r)$$

- At minimum deviation $r = \frac{A}{2}$ is independent of refractive index.

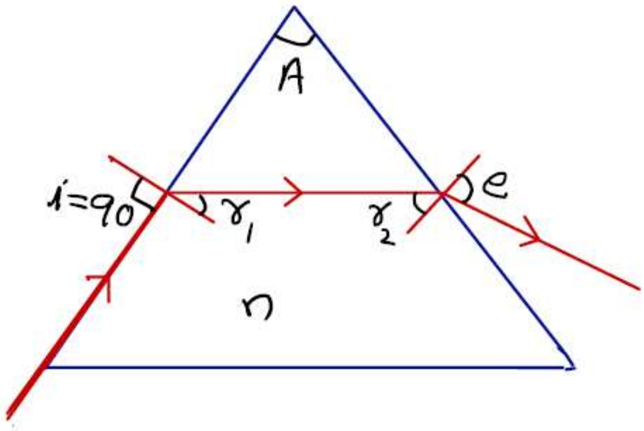
Grazing Incidence, (Maximum deviation)

When i = 90°, r_1 = C, critical angle

$$A = r_1 + r_2$$

$$\therefore r_2 = A - C$$

$r_2 = A - C$



From Snell's law

sin e = n sin r_2

sin e = n sin(A – C)

Here deviation is maximum

$$d_{max} = i_{max} + e - A$$

$$\boxed{d_{max} = 90 + e - A}$$

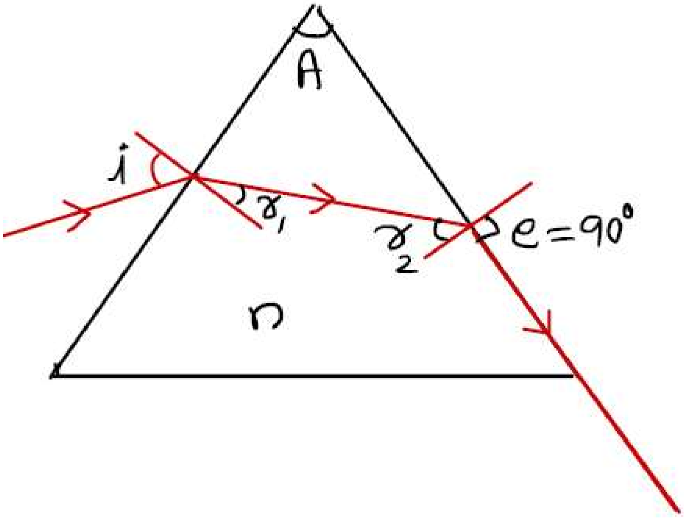
Grazing emergence (maximum deviation)

In grazing emergence

e = 90°, $\therefore r_2 = C$

$$r_1 = A - r_2$$

$r_1 = A - C$



From Snell's law

$\sin i = n \sin r_1$

$\sin i = n \sin(A - C)$

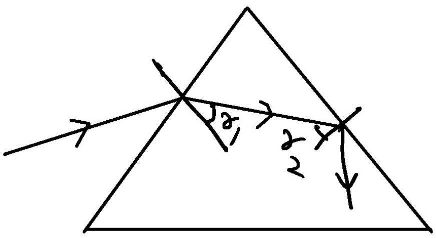
$d_{max} = i + e_{max} - A$

$d_{max} = i + 90 - A$

Condition of no emergence of light

A ray of light is incident on a prism of angle A and refractive index n, will not emerge out of the second face of the prism, if A > 2C, whatever may be the angle of incidence. Where C is the critical angle.

Proof: For no emergence, light must undergo total internal reflection from the second face of the prism. Thus minimum angle of incidence at second face must be greater than critical angle.



$(r_2)_{min} > C,\ r_2 = A - r_1\ \therefore (r_2)_{min} = A - (r_1)_{max}$

r_1 is maximum, when i is maximum.

$i_{max} = 90^o,\ \therefore (r_1)_{max} = C,\ (r_2)_{min} = A - C$

Thus $(r_2)_{min} = (A - C) > C\ \boxed{\therefore A > 2C}$

So, $\boxed{A_{min} = 2C}$

From above condition $\frac{A}{2} > C$ and sin (A/2) > sin C. But $\sin C = \frac{1}{n}$

$$\boxed{\sin(A/2) > \frac{1}{n}}$$

Thin Prism or Small Angled Prism

From equation $n = \frac{\sin\left[\frac{A+D}{2}\right]}{\sin(A/2)}$

When A is very small $\sin(A/2) = \frac{A}{2}$ and $\sin\left(\frac{A+D}{2}\right) = \frac{A+D}{2}$

$$\therefore n = \frac{A+D}{2(A/2)} \quad \boxed{D = A(n-1)}$$

deviation produced by a thin prism is independent of angle of incidence, thus at any angle of incidence deviation,

$\boxed{d = A(n-1)}$, where $\boxed{n = \frac{n_P}{n_S}}$

✶ deviation produced by a thin prism depends on the angle of the prism, material of the prism and surrounding medium.

Dispersion of Light

Splitting up of a light into its composite colours is called dispersion of light.

Cause of Dispersion

Different colours, different frequencies, different wavelengths of light have different speeds and refractive indices in the same medium. When white light is incident on a prism, angle of incidence is same for all the colours.

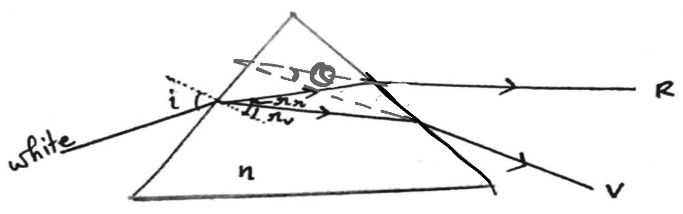
From Snell's Law

$$\sin r = \frac{\sin i}{n} \qquad \therefore \sin r \propto \frac{1}{n}$$

So, different colours refract along diff. paths.

$n_V > n_r$ (From Cauchy's dispersion formula)

$\therefore \boxed{r_V < r_r}$



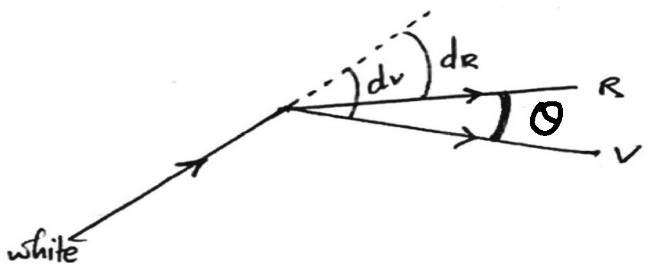
We know; deviation, $d = (n-1)A$

Since $n_v > n_r$

$\boxed{d_v > d_r}$

Angular Dispersion (θ):

It measures the amount of dispersion produced in angle. It is defined as the angle between extreme rays violet and red.



$$\boxed{\theta = d_v - d_r}$$

$\boxed{d_v = (n_v - 1)A}$ $\boxed{d_R = (n_R - 1)A}$

Therefore, $\boxed{\begin{array}{l} \theta = d_v - d_R \\ \theta = (n_v - n_R)A \end{array}}$

Dispersive Power (ω)

If measures the ability of material of prism of produce dispersion.

$$\boxed{\omega = \frac{\theta}{d_y} = \frac{d_v - d_R}{d_y}}$$

d_y = deviation of (mean colour) = yellow

$$d_y = \left(\frac{d_v + d_R}{2}\right), \quad \omega = \frac{A(n_v - n_R)}{A(n_y - 1)}$$

$$\therefore \quad \boxed{\omega = \frac{(n_v - n_R)}{(n_y - 1)}} \quad \Rightarrow n_y = \left(\frac{n_v + n_R}{2}\right)$$

Dispersive power is also defined with blue and red colours.

$$\boxed{\omega = \frac{d_B - d_R}{d} = \frac{n_B - n_R}{n - 1}}$$

d = mean deviation $= \frac{d_B + d_R}{2}$

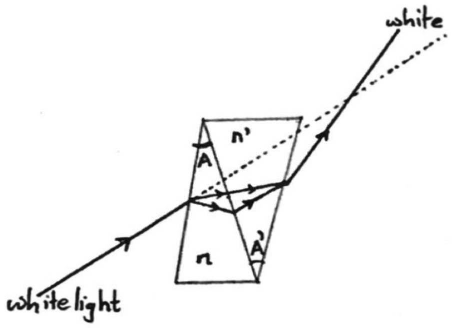
n = mean $R.I. = \frac{n_B + n_R}{2}$

$\rightarrow$ Dispersive power depends on material of the prism

$\rightarrow$ But it is independent of the angle of the prism.

Condition for Deviation without Dispersion:

Two thin prisms made of different materials and different angles are used.



$A \neq A'$

$n \neq n'$

To produce zero dispersion, angular dispersion produced by first prism must be equal and opposite to that of 2[nd] prism.

$$\boxed{\theta = -\theta'}$$

but $n_V - n_R = \omega(n_y - 1)$

$$d_V - d_R = -(d_v' - d_R')$$

$\therefore$ condition becomes

$$\boxed{(n_V - n_R)A = -(n_V' - n_R')A'}$$

$$\boxed{\omega A(n_y - 1) = -\omega' A'(n_y' - 1)}$$

negative sing indicates one prism is inverted to the other.

Condition for Dispersion without Deviation:

Deviation is zero for mean colour yellow.

To produce zero deviation, deviation produced by one prism must be equal and opposite to that of other one.

Two thin prisms made of different materials and different angles are used.

Therefore; $d_y = -d_y'$

$$\left(n_y - 1\right)A = -\left(n_y' - 1\right)A'$$

$$\boxed{A = \frac{-\left(n_y' - 1\right)A'}{\left(n_y - 1\right)}}$$

✶ Two identical thin prisms in the inverted positions produces no dispersion and deviation. Eg: A thin glass slab.

Note: When the refractive index of the prism is doubled deviation produced by a thin prism becomes more than twice the initial value.

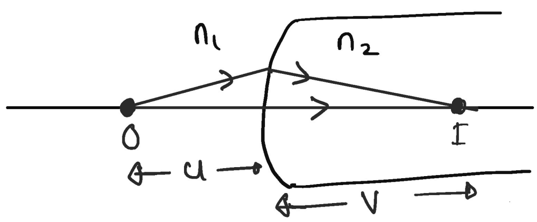
Proof: deviation $d = A\left(n-1\right),\ An = d + A$

when R.I. $n' = 2n$, deviation $d' = A\left(2n-1\right)$

$$d' = 2An - A = 2\left(d + A\right) - A$$

$\underline{d' = 2d + A}$ $\quad\quad\quad\quad \therefore d' > 2d$

Refraction at Spherical Surface



$\boxed{\dfrac{n_2}{v} - \dfrac{n_1}{u} = \dfrac{n_2 - n_1}{R}}$ u = object distance; v = image distance; R = radius of curvature

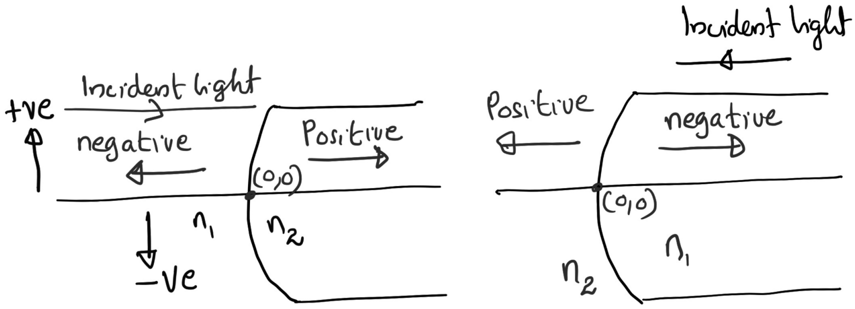
✶ This equation can be used for both convex and concave surfaces by applying sign conventions

✶ When light incident on convex surface R is positive

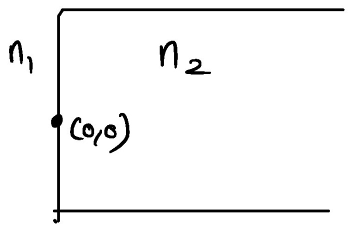
✶ When light incident on concave surface R is negative

✶ Here pole of the surface is the origin

✶ Sign conventions must be applied in the above equations



For a plane surface $R = \infty \therefore \boxed{\frac{n_2}{V} - \frac{n_1}{u} = 0}$



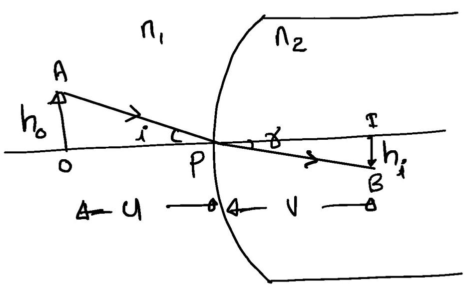
or $\boxed{V = \frac{n_2}{n_1} u}$

Magnification:

✶ Lateral Magnification: It is the ratio of the size of the image and object perpendicular to the principal axis. Thus lateral magnification,

$$m = \frac{\text{height of the image}}{\text{height of the object}} = \frac{h_i}{h_0}$$

For small angle of incidence $\frac{\sin i}{\sin r} = \frac{\tan i}{\tan r} = \frac{OA}{PO} \Big/ \frac{IB}{PI} = \frac{h_0}{-u} \times \frac{V}{-h_i}$



From Snell's law, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$

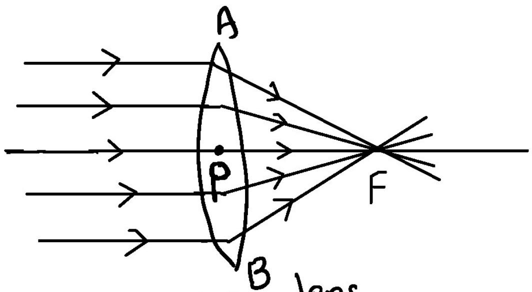
$$\frac{h_0}{h_i}\frac{v}{u} = \frac{n_2}{n_1}, \quad \therefore \quad \boxed{m = \frac{h_i}{h_0} = \frac{v}{u}\frac{n_1}{n_2}}$$

m is positive for erect images

m is negative for inverted images

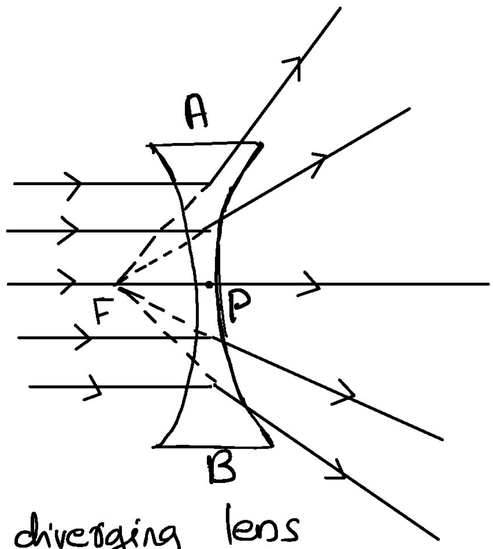
✶ When $|v| = |u|$, $|m| = \frac{n_1}{n_2} \neq 1$, ie, image size is not equal to object size.

Thin Lenses

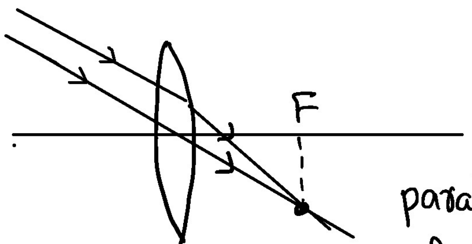


Converging lens

PF = f = focal length



diverging lens



parallel rays converges at focal plane.

<u>Sign Conventions in lens:</u>

✶ Optical centre of the lens is the origin

✶ All the measurements in the direction of incident light are positive.

✶ All the measurements opposite to the direction of incident light are positive.

✶ Upward measurements from principal axis are negative.

✶ Downward measurements from the principal axis are negative

✶ Do not give sign to unknown quantities

✶ Focal length of converging lens is positive

✶ Focal length of diverging lens is negative

✶ Real image distance is positive

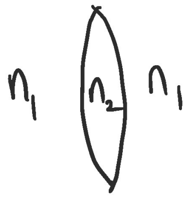
✶ Virtual image distance is negative

Thin lens Formula

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \text{ or } f=\frac{uv}{u-v}$$

Sign conventions and Lens Maker's Formula

$$\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$



→ This equation can be used to all the thin lenses

→ Sign conventions must be applied to this equations.

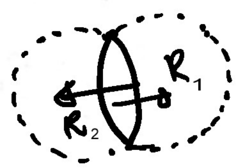
n_1 = refractive index of surrounding medium

n_2 = refractive index of lens

R_1, R_2 = Radius of curvature

For a convex lens; R_1 = +ve, R_2 = -ve

Incident light



i.e., For a convex lens; $\frac{1}{f}=\left({}_1n_2-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

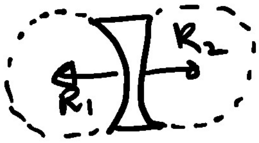
$$\boxed{\frac{1}{f}=\left({}_1n_2-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} \qquad {}_1n_2=\frac{n_2}{n_1}$$

✶ For an equiconvex lens or symmetric lens, $R_1 = R_2 = R$.

Then, $\boxed{\frac{1}{f}=\left({}_1n_2-1\right)\left(\frac{2}{R}\right)}$ $\qquad \therefore \boxed{f=\frac{R}{2\left({}_1n_2-1\right)}}$

✶ ⇒ For a concave lens, R_1 = -ve, R_2 = +ve.

i.e., For a concave lens,

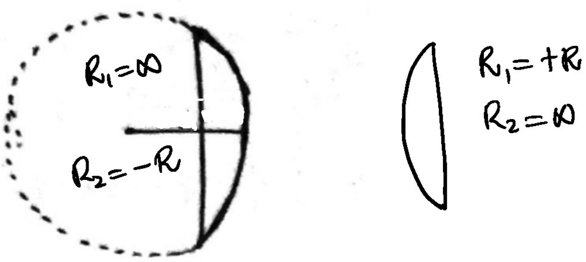


$$\boxed{\frac{1}{f}=\left({}_1n_2-1\right)\left(\frac{-1}{R_1}-\frac{1}{R_2}\right)}$$

✶ For an equiconcave lens; $\boxed{\frac{1}{f}=\left({}_1n_2-1\right)\left(\frac{-2}{R}\right)}$

$\therefore$ $\boxed{f=\frac{-R}{2\left({}_1n_2-1\right)}}$

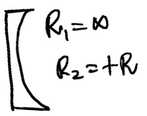
✶ For a plano convex lens;



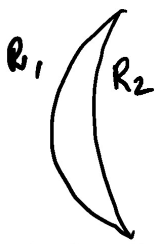
$$\boxed{f=\frac{+R}{{}_1n_2-1}}$$

✶ For a plano concave lens;

$\therefore$ $\boxed{f=\frac{-R}{{}_1n_2-1}}$



✶ Convexo-concave lens

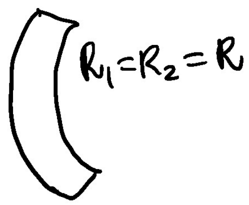


R_1 and R_2 are positive

$$\therefore \frac{1}{f}=\left({}_1n_2-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

when $R_1=R_2=R$

$$\frac{1}{f}=0 \quad \text{or} \quad f=\infty$$



Variation of 'f' with refractive index of the surrounding medium:

In air; $n_1 = 1$

Therefore; $\frac{1}{f_a} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (1)$

In medium; $\frac{1}{f_m} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (2)$

Eq. (1)/(2) $\Rightarrow \frac{f_m}{f_a} = \frac{(n_2 - 1)n_1}{(n_2 - n_1)}$

$\boxed{\frac{f_m}{f_a} = \frac{(n_2 - 1)n_1}{(n_2 - n_1)}}$ f_a = focal length in air; f_m = focal length in a medium of R.I. n_1

When a lens is transferred from air to a medium, with refractive index less than that of the lens, then focal length of the lens, $\uparrow$ ses.

When $n_1 < n_2$

$$n_1 > 1$$
$$(n_2 - 1) > \left(\frac{n_2 - n_1}{n_1}\right)$$
$$\frac{1}{f_m} < \frac{1}{f_a}$$
$$\Rightarrow \underline{\underline{f_m > f_a}}$$

When $n_1 = n_2$, focal length is maximum

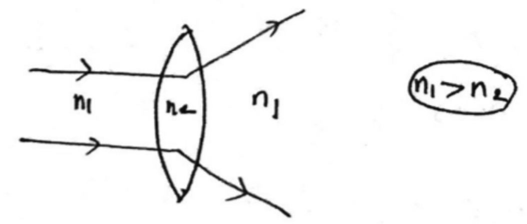
$\frac{1}{f} = 0$ $\underline{\underline{f = \infty}}$ (No refraction).

If the refractive index of the surrounding medium is greater than that of the lens ($n_1 > n_2$), then,

$$(n_2 - n_1) = -ve$$

<u>And *f* reverses it sign</u>

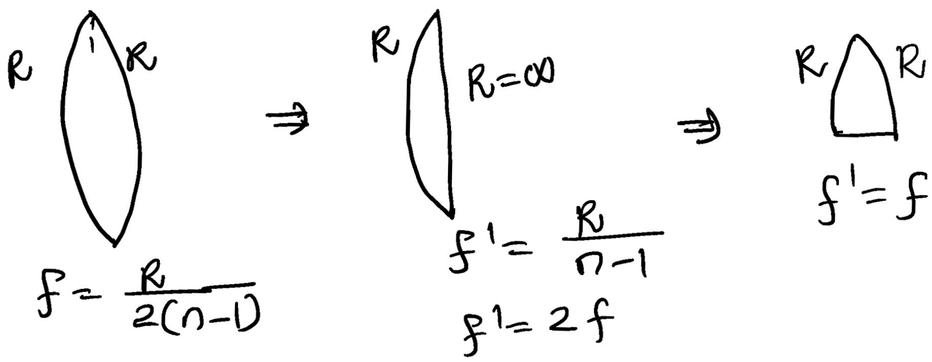
i.e., A converging lens in air becomes a diverging lens in the medium and vice-versa.



Note: Convex and concave lenses can produce both divergence and convergence.

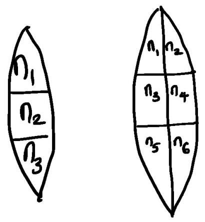
✶ When a lens is cut perpendicular to the principal axis radius of curvature changes, thus focal length changes.

For example, an equiconvex lens is cut as shown, one of the divided parts is a plano convex lens and focal length $f' = 2f$



✶ When a lens is cut parallel to the principal axis, focal length does not change, complete image is formed, but brightness or intensity of the image decreases.

✶ Intensity of the image formed by the lens $I \alpha$ cross sectional area of the lens. If ‘d’ is the diameter of the lens, then area $A = \frac{\pi d^2}{4}$ $\therefore I \alpha d^2$

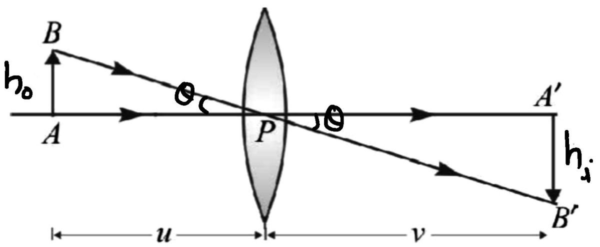
✶ 

Two lenses are made up of different materials as shown in figure. In these two situations lens has three foci and three images are formed.

Lateral/Transverse Magnification (m)

It is the ratio of the size of the image and object normal to the principal axis.

$$m = \frac{\text{height of the image}(h_i)}{\text{height of the object}(h_0)}$$



$$\boxed{m = \frac{h_i}{h_0} = \frac{v}{u}}$$

Sign convention: ‘m’ is positive for erect images and ‘m’ is negative for inverted images.

When $|v| < |u|$, $|m| < 1$ and diminished image

When $|v| > |u|$, $|m| > 1$ and enlarged image

when $|v| = |u|$, $|m| = 1$ and same size image

‘m’ interms of focal length ‘f’

$$\boxed{m = 1 - \frac{v}{f} = \frac{f - v}{f}}$$

$$\therefore \boxed{m = \frac{v}{u} = \frac{f}{f + u}}$$

Longitudinal Magnification (m_L)

When the object is placed parallel to the principal axis, longitudinal magnification,

$$m_L = \frac{\text{Length of the image}}{\text{Length of the object}} = \frac{L_i}{L_0} = \frac{\Delta v}{\Delta u}$$

For small objects, $m_L = \dfrac{dv}{du}$

We have, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

On differentiating with respect ‘u’, $0 = \frac{-1}{v^2}\frac{dv}{du} - \left(\frac{-1}{u^2}\right)$

$\therefore \ \frac{dv}{du} = \frac{v^2}{u^2}$ $\boxed{m_L = \frac{dv}{du} = \frac{v^2}{u^2} = m^2}$

Image formation by concave lens

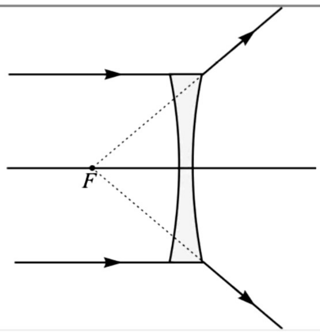
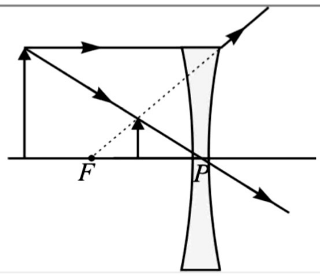
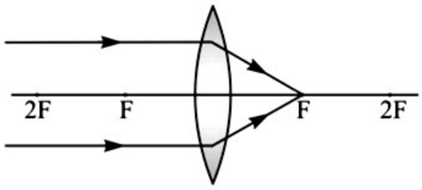
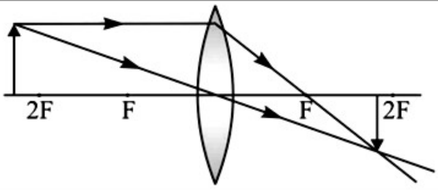
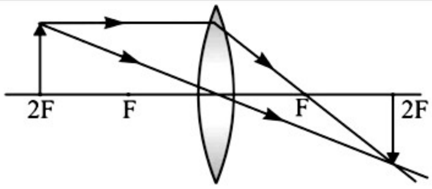
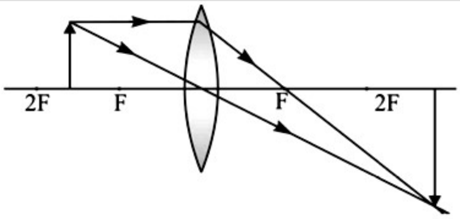
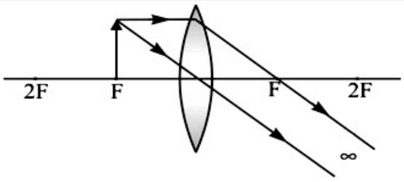
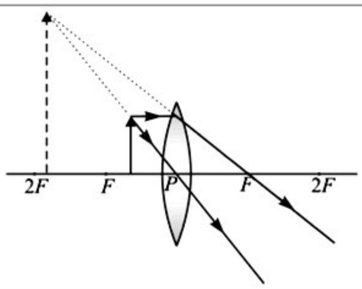
Object position	Ray diagram	Position and nature of image
At ∞		At focus. Virtual erect and diminished.
Anywhere between ∞ and *P*		Between *P* and *F*. Virtual, erect and smaller than object.

Image formation by convex lens

Object position	Ray diagram	Position and nature of image
At ∞		Image at focus. Real, inverted and diminished image.
Between 2F and ∞		Between F and 2F. Real, inverted and diminished.
At 2F		At 2F. Real, inverted and same size of the object.
Between 2F and F		Beyond 2F. Real inverted and larger than object.
At F		At ∞. Real, inverted and very larger than object.
Between F and P		On the side of the object. Virtual, erect and larger than object.

✶ Convex lens cannot product virtual image of a virtual object. But concave lens can produce both real and virtual images of a virtual object.

Velocity of the image in lenses

a) Velocity of the object is parallel to the principal axis

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, on differentiating with respect to 't'

$0=\frac{-1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}$ $\quad\quad \frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$

$\frac{dV}{dt}$ = velocity of image w.r.t. lens = V_{IL}

$\frac{du}{dt}$ = velocity of object w.r.t lens = V_{0L}

$$\boxed{\therefore\ (V_{IL})_{\parallel}=\frac{v^2}{u^2}(V_{0L})_{\parallel}=m^2(V_{0L})_{\parallel}}$$

Velocity of object and image are in the same direction.

The above relation is for the instantaneous velocity

average velocity $V_{aV}=\frac{\text{displacement of the image}}{\text{time}}=\frac{V_f-V_i}{t}$

V_f & V_i are final and initial image positions.

From the above relation $dV=\frac{v^2}{u^2}du=m^2\,du$

Where du = small change in object position and dv = small change in image position.

b) Velocity of object is perpendicular to the principal axis.

When object moves perpendicular to principal axis height of the object and image from the principal axis changes. We have $m=\frac{h_i}{h_0}$ or $h_i = mh_0$

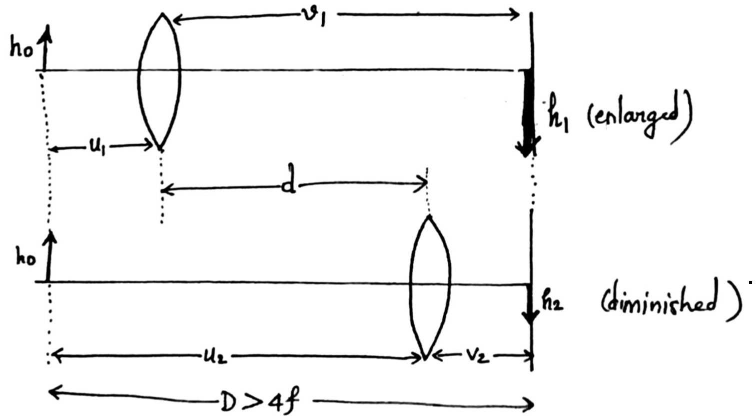
$\therefore \frac{dh_i}{dt}=m\frac{dh_0}{dt}$ $\quad\quad \underline{(V_{IL})_{\perp}=m(V_{0L})_{\perp}}$

$(V_{IL})_{\perp}$ and $(V_{0L})_{\perp}$ are the velocities of image and object perpendicular to the principal axis.

✶ For a convex lens, the minimum separation between object and its real image is 4f. Where f is the focal length of the lens.

Displacement Method:

In displacement method, object and screen are fixed but position of the lens is displaced to produce 2 images, in which one is enlarged and other is diminished.



On solving, $\underline{v_2 = u_1}$ and $\underline{u_2 = v_1}$

From diagram, $u_1 + d + v_2 = D$

$2u_1 + d = D$

$$\boxed{u_1 = \frac{D-d}{2}} \Rightarrow \text{object distance.}$$

$v_1 = D - u_1$

$$\boxed{v_1 = \frac{D+d}{2}} \Rightarrow \text{image distance.}$$

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{\frac{D+d}{2}} - \frac{1}{-\left(\frac{D-d}{2}\right)} = \frac{2(2D)}{D^2 - d^2}$$

$$\boxed{f = \frac{D^2 - d^2}{4D}}$$

$$\boxed{\begin{array}{l} |m_1| = \dfrac{v_1}{u_1} = \dfrac{D+d}{D-d} \\ |m_2| = \dfrac{v_2}{u_2} = \dfrac{D-d}{D+d} \end{array}} \text{ magnifications}$$

$$h_1 h_2 = m_1 h_0 \times m_2 h_0$$

$$h_1 h_2 = h_0^2$$

$\boxed{h_0 = \sqrt{h_1 h_2}}$ (Relation between height of the object and heights of the images). If the area of the object is perpendicular to the principal axis, then the area of the images,

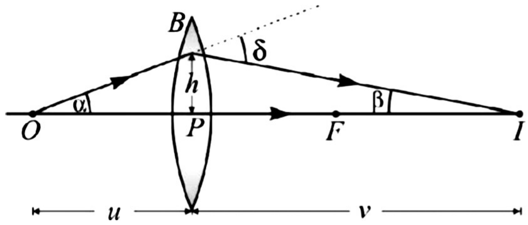
$A_1 = m_1^2 A_0$ and $A_2 = m_2^2 A_0$ $\quad \therefore A_1 A_2 = (m_1 m_2)^2 A_0^2 = A_0^2 \quad \therefore \boxed{A_0 = \sqrt{A_1 A_2}}$

Also focal length $\boxed{f = \dfrac{d}{|m_1| - |m_2|}}$

DEVIATION PRODUCED BY A LENS

Consider a ray OB coming from the object and incident at a height *h* on the lens of focal length *f*. The ray intersect the principal axis at I. So the deviation δ produced by the lens.

$$\delta = \angle BOP + \angle BIP$$
$$= \alpha + \beta$$



For small angles $\alpha \simeq \tan\alpha = \dfrac{h}{-u}$ and $\beta \simeq \tan\beta = \dfrac{h}{v}$

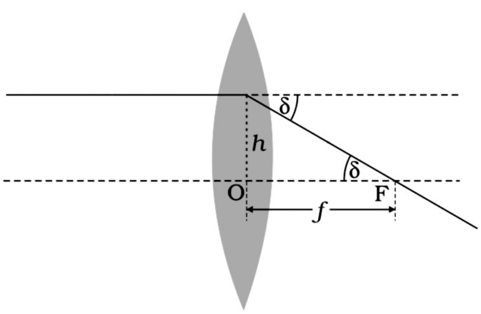
$\therefore \quad \delta = \dfrac{h}{-u} + \dfrac{h}{v}$ or $\delta = h\left[\dfrac{1}{v} - \dfrac{1}{u}\right]$ or $\boxed{\delta = \dfrac{h}{f}}$

The above formula holds for the rays, for which h is small.

Power of a Lens

When light ray is incident on a lens, it bends either towards the principal axis (in convex lens) or away from the principal axis (in concave lens). The ability of a lens to bend the ray towards the principal axis is called power of the lens.

The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre



$\tan\delta = \frac{h}{f}$; if h = 1 $\tan\delta = \frac{1}{f}$ or $\delta = \frac{1}{f}$ for small value of δ. Thus,

$$\boxed{P = \frac{1}{f}}$$

The SI unit for power of a lens is dioptre (D): ID = $1 m^{-1}$. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens. Thus, when an optician prescribes a corrective lens of power +2.5D, the required lens is a convex lens of focal length +40 cm. A lens of power of -4.0 D means a concave lens of focal length -25 cm.

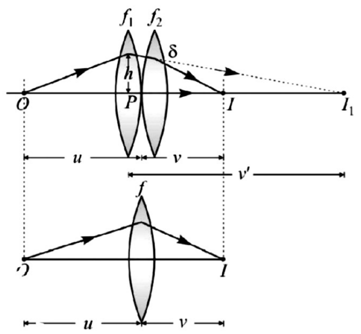
Power of a lens in a medium

$$\boxed{P = \frac{n_{med}}{f_{med}}}$$

where, n_{med} = R.I. of surrounding medium f_{med} = focal length in the medium

Combined Focal Length

i) Two lenses are placed in contact



Also equivalent power; $P = P_1 + P_2$

The above formulas are applicable to any type and any number of thin lenses in contact.

For 'n' thin lenses in contact, $\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + ... + \frac{1}{f_n}$ and equivalent power P = $P_1 + P_2 + ... + P_n$

Note: Proper sign conventions must be applied in this equations.

✶ f_1, f_2, ... are the focal lengths of each lens, when they are separately placed in the medium.

ii) Total Magnification

M= $m_1 m_2$. m_1 and m_2 are the magnifications produced by each lens

For n-thin lenses, we can write $M = m_1 \times m_2 \times ...... \times m_n$

iii) Two lenses separated by a finite distance

Let two lenses of focal lengths f_1 and f_2 are placed on the same optic axis at a separation d.

$$\frac{1}{f_e}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Equivalent power; $P = P_1 + P_2 - d\,P_1P_2$

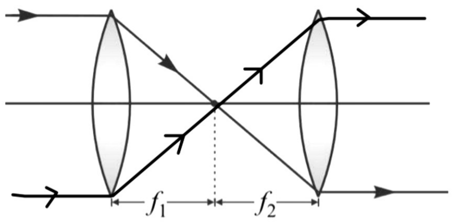
Special case: If parallel incident ray on first lens emerges parallel from the second lens, then $f_e = \infty$.

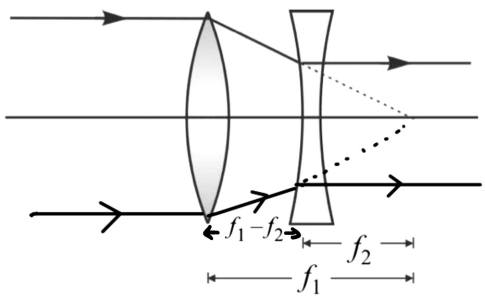
$$\therefore \quad \frac{1}{\infty}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

or $\quad d = f_1 + f_2$

i) If both the lenses are convex, then $d = f_1 + f_2$

ii) If second lens is concave, then $d = f_1 + (-f_2) = f_1 - f_2$

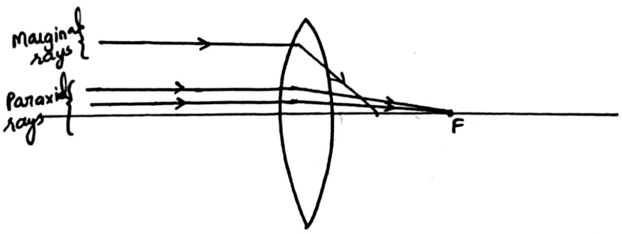




Note: To find out the position and nature of final image formed by combination of lenses, image formed by each lens is considered as the virtual object of each lens in the next stage.

Spherical Abberration:

When light rays incident parallel to the principal axis of a lens or a mirror, the marginal rays do not coverage or diverge to a single point on the principal axis; therefore, the image formed is blurred. This is called spherical abberration.



Corrective Methods:

1. By the use of 'stops' which cut off the marginal rays

2. By the use of two Planoconvex lenses separated by a distance, $\boxed{d = f_1 - f_2}$

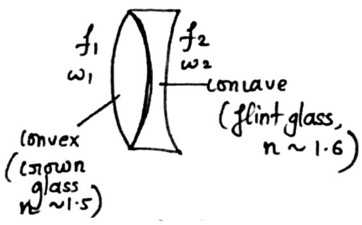
3. For mirrors, parabolic reflectors an be used.

Chromatic Abberation:

When different colours of light incident parallel to the principal axis of a lens, different colours focused at different points on the principal axis due to the dispersion of light. Therefore, the image formed is blurred. This is called chromatic abberation.

Corrective Methods:

1. By the use of Achromatic Doublet (It is the combination of a convex and concave lens placed in contact)



Condition of achromatism:

$$\boxed{\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0} \text{ or } \boxed{\omega_1 P_1 + \omega_2 P_2 = 0}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2} = \frac{-P_2}{P_1}}$$

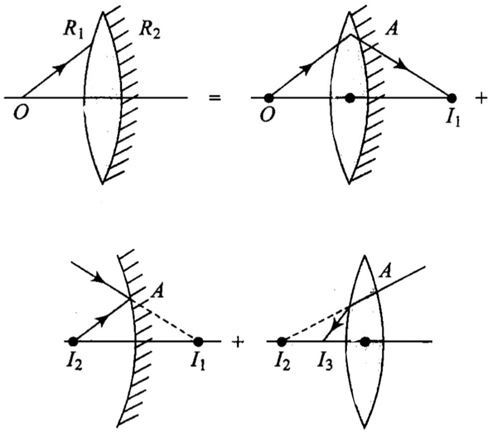
Lens with One Silvered Surface

If the back surface of a lens is silvered and an object is placed in front of it then:

1. First, light will pass through the lens and it will form the image I_1.
2. The image I_1 will act as an object for silvered surface which acts as curved mirror and forms an image I_2 of object I_1.
3. The light after reflection from silvered surface will again pass through the lens and will form final image I_3 of object I_2.

In such a situation, power of the silvered lens will be $P = P_L + P_M + P_L$ $\boxed{P = 2P_L + P_M}$ with

$$P_L = \frac{1}{f_L},\ P_M = -\frac{1}{f_M}$$



Therefore, the silvered lens finally behaves as a mirror with an effective focal length of $\boxed{\dfrac{1}{f_e}=\dfrac{1}{f_m}-\dfrac{2}{f_L}}$

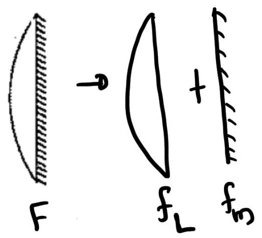
Here, f_m is the focal length of the mirror and f_L is the focal length of the lens.

Thus in solving the problems of silvered lens, first find the focal length by using the above formula and then use mirror formula; $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{1}{f_e}$

If F_e is -ve, then equivalent mirror is converging and if F is +ve, then equivalent mirror is diverging.

<u>Silverd plano-convex lens</u>

Case (I): Plane surface silvered:



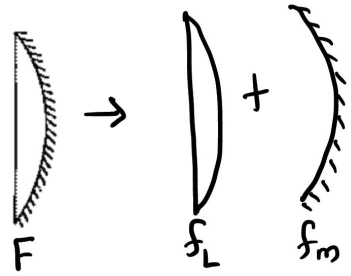
$$\frac{1}{f_L}=(\mu-1)\left(\frac{1}{R}\right)$$

$$f_m=\frac{R}{2}=\infty$$

$$\frac{1}{F}=\frac{1}{f_m}-\frac{2}{f_L}=0-\frac{2}{f_L}$$

$\boxed{F = \frac{-F_L}{2}}$ or $\boxed{F = -\frac{R}{2(\mu-1)}}$

Case (II): Curve surface silvered:



$$\frac{1}{f_L} = (\mu-1)\left(\frac{1}{R}\right)$$

$$f_m = -\frac{R}{2}$$

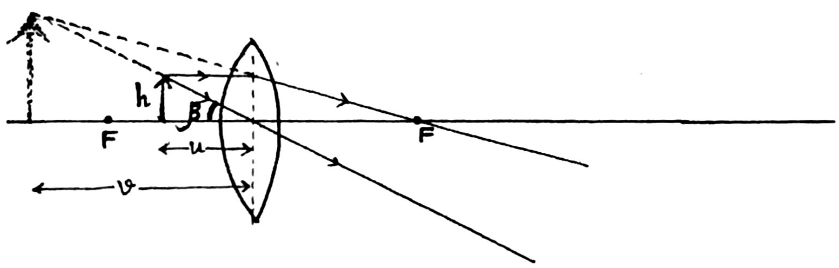
$$\frac{1}{F} = \frac{1}{f_m} - \frac{2}{f_L}$$

$$\frac{1}{F} = \frac{-2}{R} - 2\frac{(n-1)}{R} \qquad \boxed{F = -\frac{R}{2\mu}}$$

<u>Microscope:</u>

<u>Simple Microscope:</u>

A converging lens is used for magnification.



Angular Magnification:

It is the angle made by the image at the eye/lens divided by angle made by object at eye/lens

$$\boxed{M = \frac{\beta}{\alpha}}$$

From the figure; $\tan\beta = \frac{h}{u} \approx \beta$ similarly; $\tan\alpha = \frac{h}{D} \approx \alpha$ $\therefore$ $\boxed{M = \frac{\beta}{\alpha} = \frac{D}{u}}$

1) Near Point Adjustment:

In near point adjustment, image formed at least distance of distinct vision on near point (25 cm).

We have, Linear Magnification, $\boxed{m = \frac{h_i}{h_0} = \frac{v}{u}}$

And Angular Magnification $\boxed{M = \frac{D}{u}}$

Therefore, Angular magnification = Linear magnification only when v = D (ie, in near point adjustment)

$\therefore$ Here $\boxed{m = M}$ (near point adjustment) Hence, $M = \frac{f+v}{f}$ putting v = -D,

$\boxed{M = \frac{f+D}{f} = 1 + \frac{D}{f}}$ (maximum)

Image formed is virtual, erect and enlarged

2) **Far point Adjustment/Normal Adjustment:**

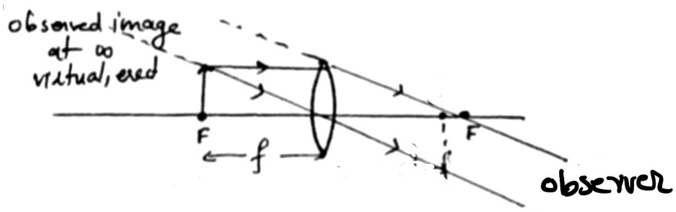
Image is formed at infinity, therefore, object is at focus.

Here, u = F

So, $\boxed{M = \frac{D}{u} = \frac{D}{f}}$ $\boxed{M = \frac{D}{f}}$

Magnification depends on both u and f

✶ Image formed is virtual, erect and enlarged.

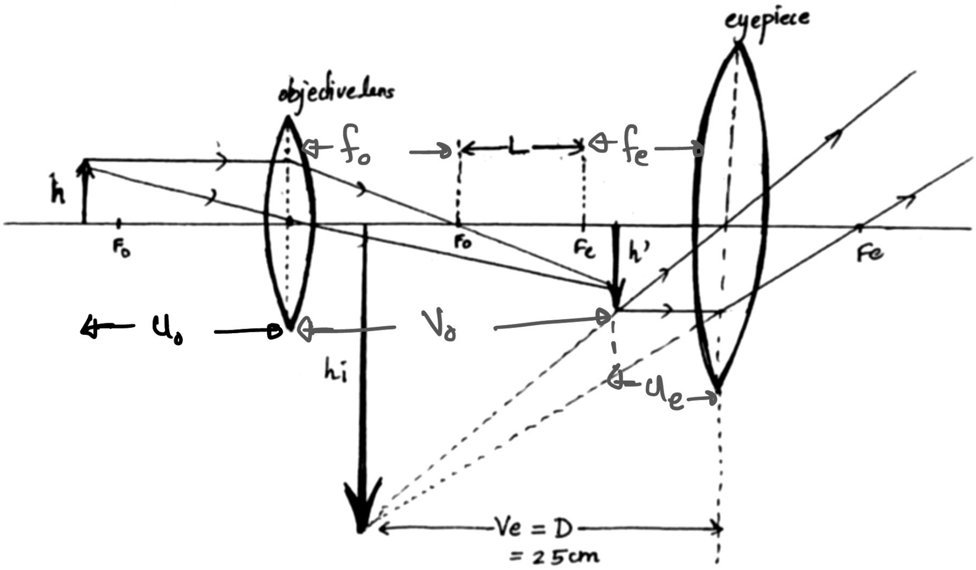


Compound Microscope:

Two converging lenses are used for magnification. Focal length of the eye piece is slightly greater than that object lens.

1) Near Point Adjustment:

Final image formed by the eyepiece is at near point of the normal eye (25 cm).



Separation between 2nd focus of the objective and 1st focus of the eyepiece is called Tube length (L) of the microscope.

Linear Magnification of the objective lens: $\boxed{M_0 = \frac{h'}{h} = \frac{v_0}{u_0}}$

$$\boxed{M_0 = \frac{u_0}{v_0} = \frac{f_0 - v_0}{f_0}}$$

Total magnification; $M = M_0 M_e$

M_0 = linear magnification of objective and

M_e = angular magnification of the eye piece

$$\boxed{M = \frac{u_0}{v_0}\left[1 + \frac{D}{f_e}\right]}$$ (Eye piece is equivalent to simple microscope)

Separation between objective and eye piece:

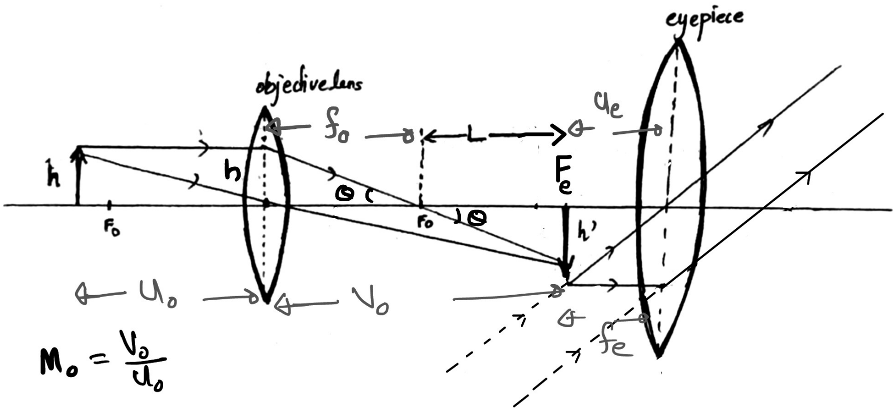
$$\boxed{S = f_0 + L + f_e}$$

or $\boxed{S = v_0 + |u_e|}$

Image formed by the objective is Real, inverted and enlarged. Final image formed (by eye piece) is virtual, inverted and enlarged.

2) <u>Normal setting/far point Adjustment</u>

Final image formed by the eye piece is at infinity.



From the diagram Final image at infinity

$$\frac{h}{f_0} = \frac{h'}{L} \quad \therefore \quad \boxed{M_0 = \frac{h'}{h} = \frac{L}{f_0}}$$

Magnification, $M = M_0M_e$

$$\boxed{M = \frac{L}{f_0}\left(\frac{D}{f_e}\right)} \quad \boxed{M = \frac{v_0}{u_0}\left(\frac{D}{f_e}\right)}$$

Separation between the lenses,

$$\boxed{S = f_0 + L + f_e}$$

$$\boxed{S = v_0 + f_e}$$

Note: Separation between objective and eye piece is also considered as the length of the microscope.

Telescope:

There are two types of Telescopes - Refractive and Reflective

- ✶ In reflective type, objective is a mirror
- ✶ In refractive type, objective is a lens
- ✶ Reflective is better than refractive

Reasons:

Advantages of Reflective Telescope:

1) It is free from chromatic abberration
2) Spherical abberration can be solved easily, (using parabolic reflectors)
3) Mechanical support required is less because mirror is lighter than lens
4) Easy to make a mirror of large aperture

Eg. for Reflective:

1) Newtonian Telescope
2) Cassegrain Telescope

Eg. for Refractive:

1) Astronomical
2) Terrestrial
3) Galilean telescope

Astronomical Telescope

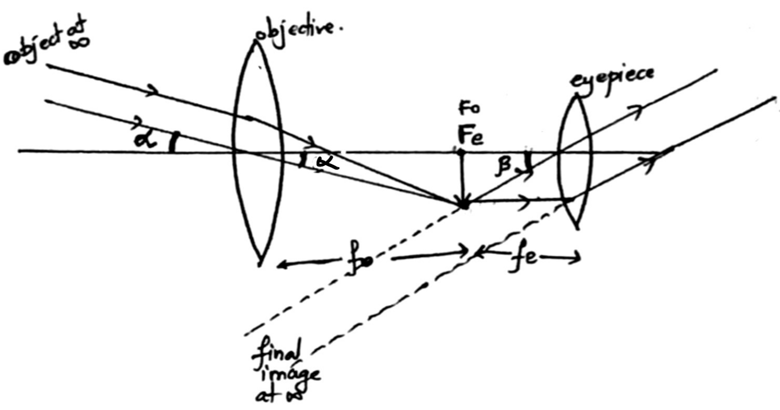
Two converging lenses are used for magnification.

Focal length and aperture of objective lens > that of eye pieces

In telescope; $f_0 >>> f_e$

1) **Normal Adjustment/Far Point Adjustment**

Final image is formed by the eye piece is at infinity, therefore object of the eye piece is at its focus.



Angular Magnification, $M = \frac{\beta}{\alpha} = \frac{\text{Angular size of image}}{\text{Angular size of object}}$

From the diagram; $\tan\alpha = \alpha = \frac{h}{f_0}$, $\tan\beta = \beta = \frac{h}{f_e}$ $\therefore \boxed{M = \frac{f_0}{f_e}}$

✶ Separation between objective and eye piece is called length of the telescope.

$\boxed{L = f_0 + f_e}$

✶ If the object is not at infinity;

Then; $\boxed{L = v_0 + f_e}$

→ Image formed by the objective is Real, inverted and diminished

→ Final image formed is; virtual, inverted and enlarged

When final image is formed at near point

$V_e = D$

$\therefore$ $$M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Length of the telescope is equal to the distance between the lenses and so $L = f_0 + |u_e|$

WAVE OPTICS

Nature of Light

I. Corpuscular theory:

According to Newton light consist of stream of particles called corpuscles which are shot out at high speed by a luminous object and light has greater speed in denser medium than rarer medium. This theory can explain the phenomena like

i) Reflection

ii) Refraction

iii) Rectilinear propagation

II. Huygen's Wave Theory:

According to him light travels in the form of longitudinal waves and propagate through a hypothetical medium called 'ether'.

This theory can explain the phenomena like

i) Reflection

ii) Refraction

iii) Diffraction

iv) Interference

III. Electromagnetic Wave Theory:

According to maxwell light propagate as electric and magnetic field oscillations, these are called electro magnetic wave, transverse in nature and which require no material medium for propagation.

IV. Quantum theory or Photon theory

According to max planck light propagate in the form of small packets of energy called quanta or photons and $E = h\upsilon$

This theory can explain the phenomena like

i) Origin of spectra

ii) Photoelectric effect

iii) Black body radiation

V. Dual Nature:

According to Debroglie light has particle as well as wave characteristics.

Eddington named light as Wavicle (Wave + particle)

Points to remember

- ✶ Light is non mechanical transverse wave
- ✶ When a transverse wave reflected from denser medium (rigid boundary), there is a phase reversal of 180° $(\pi \, \text{radian})$
- ✶ **Optical path**

Let light travels D distance through a medium of refractive index μ with speed v for a time 't'.

$V = \frac{D}{t}$ also $\mu = \frac{C}{V}$

$\frac{C}{V} = \frac{C}{D/t} = \mu$ or ct = light travels through vacuum for same interval 't' = optical path

$$\boxed{\text{optical path} = \mu D}$$

- ✶ **Relation between phase difference $(\Delta\phi)$ and path difference (Δx)**

Let two particle are located at positions x, and x_2 at an instant. Then $\phi_1 = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)$ and $\phi_2 = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)$

$\therefore$ phase difference, $\Delta\phi = \phi_1 - \phi_2 = \frac{2\pi}{\lambda}(x_2 - x_1)$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x = k\Delta x = \frac{\omega}{V}\Delta x}$$

- ✶ **Intensity**

Intensity (I) $= \frac{\text{Energy}}{\text{Area} \times \text{time}}; \text{Wm}^{-2}$

- ✶ **Intensity of a Wave**

$I = \frac{1}{2}\rho\omega^2 A^2 V$

$I \,\alpha\, A^2$

WAVE FRONT

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillates in phase is an example of a wavefront.

Wave front:

It is the continuous locus of all particles of medium which are vibrating in the same phase at any instant.

Characteristics of wavefront

i) All the particles on a wavefront are in same phase.

ii) The phases of consecutive wavefronts are different.

iii) Wavefronts separated by a particular distance, known as the wavelength of wave are also in same phase.

iv) The speed with which the wavefront moves outward from the source is called the phase speed.

v) Wave always propagates in medium in a direction perpendicular to the wavefront.

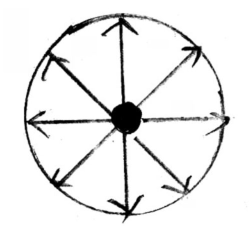
vi) Shape of wavefront varies with the nature of source.

vii) Two wavefronts never cross each other. If they intersect, then there will be two rays or two directions of propagation of energy. It is not possible.

TYPES OF WAVEFRONT

1. Spherical wavefront

In case of waves travelling in all directions from a point source, the wavefronts are spherical shape.



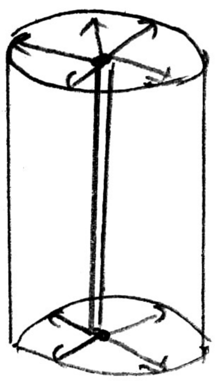
Intensity $\alpha \dfrac{1}{\text{distance}^2}$

$$I \alpha \frac{1}{r^2}$$

$$\therefore A \alpha \frac{1}{r}$$

2. Cylindrical Wavefront

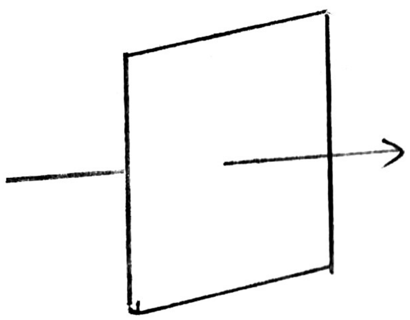
When the source of light is linear such as rectangular slit, the wavefront is cylindrical.



$$I \alpha \frac{1}{r}; \quad A \alpha \frac{1}{\sqrt{r}}$$

3. Plane Wavefront

As spherical or cylindrical wave advances, its curvature decreases progressively. So a small portions of such a wavefront at a large distance from source will be planar.

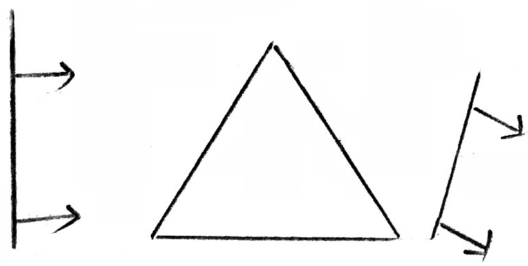


$$I \alpha r^{o} \qquad A \alpha r^{o}$$

BEHAVIOUR OF PLANE WAVEFRONT ON REFLECTION AND REFRACTION

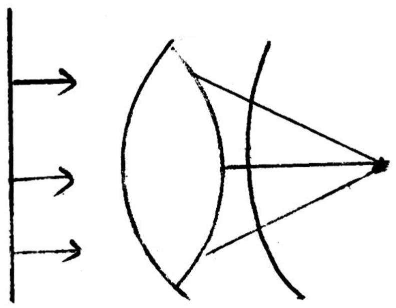
a) Refraction of a plane wavefront by a thin prism

Consider a plane wavefront passing through thin prism. Clearly the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most, since light travels more slowly in glass. So emerging wavefront is planar with a tilt.

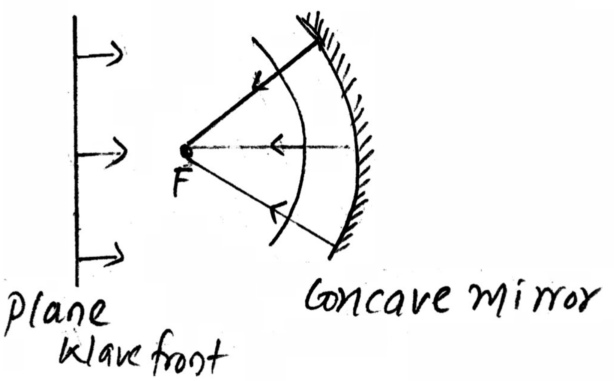


b) **Refraction of plane wavefront by a lens**

Consider a plane wavefront incident on a convex lens. The central part of incident plane wave travels the thickest portion of convex lens and is delayed most. The emerging wavefront has a depression at the centre and is spherical and converges to a focus.



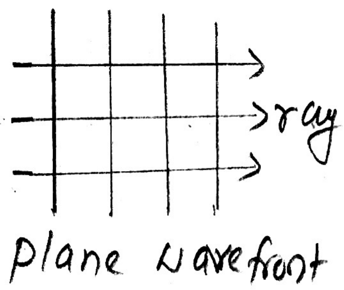
c) A concave mirror, concave lens and convex mirror produces similar effect. The emerging or reflecting wavefronts are converging spherical.

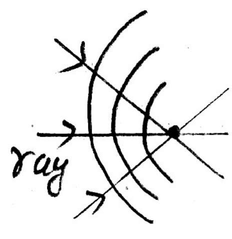


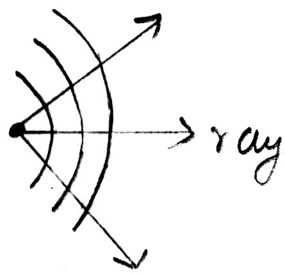
Ray of light

- ✶ It is an arrow drawn perpendicular to wavefront in the direction of propagation.
- ✶ It represent the path along which light travels.
- ✶ The time taken for light to travel from one wavefront to another is the same along any ray.

In case of spherical wavefront, the rays either converge to a point or diverge from a point.



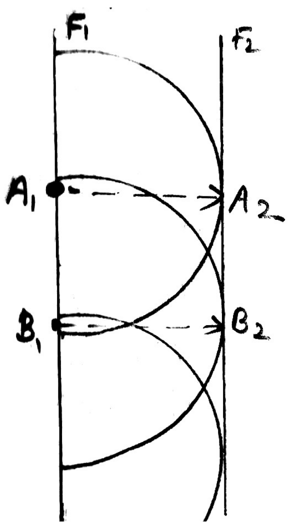




HUYGEN'S PRINCIPLE

1. Each point on a wavefront act as a fresh source of new disturbance called secondary waves or wavelets.
2. The wavelets are spherical and spread out in all directions with the speed of light in the given medium.
3. The new wavefront at any later time is given by the forward envelope (tangential surface in forward direction) of the secondary wavelets at that time.

✶ Huygen's principle illustrated in simple cases of a plane wave.



i) At t = 0, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.

ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time, t = t each of these will have radius r = Vt. These spheres are the secondary wavelets.

iii) After a time 't', the disturbance would now have reached all points with in the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

Principle of superposition of waves

When a number of waves travelling through a medium, superpose each other, the resultant displacement at any point at a given instant is equal to the vector sum of displacements due to individual waves at that point.

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + ...$$

INTERFERENCE

✶ When light waves of same frequency and having zero or constant phase difference travelling in same direction superpose each other, the intensity in the region of superposition get redistributed becoming maximum at some points (constructive interference) and minimum at others (destructive interference)

Constructive interference

When crest of one wave falls over crest of another, or trough of one wave falls over trough of another, resultant intensity is maximum.

Destructive interference

When crest of one wave falls over trough of another, resultant intensity is minimum.

Examples for interference

i) Colour of soap bubbles

ii) Colour of thin films

iii) Irridescence of morpho butterfly

iv) Holography

v) Coloured pattern of wings of peacock

vi) Shift in coloured pattern of security thread in currencies

Sustained interference

✶ In interference pattern, in which the positions of maximum and minimum intensity of light remain fixed all along the screen. Then interference is said to be sustained.

Condition for sustained interference

i) The two sources should emit continuously waves of same frequency or wavelength.

ii) The amplitude of the two waves should be either or nearly equal

iii) The two sources should be narrow

iv) The sources should be close to each other

v) The two sources should be coherent one [This is the necessary condition for interference]

vi) They are in same state of polarisation

vii) They (waves) travel along same direction.

viii) The distance between sources should be small and distance between source and screen is large.

COHERENT SOURCE

✶ Sources are said to be coherent, if they emit light waves continuously of same frequency or wavelength with zero or constant phase difference between them.

✶ LASER is highly monochromatic and highly coherent.

✶ Incoherent sources are those which does not emit light waves continuously with constant phase difference.

✶ Single criterion for coherence is constant phase difference.

Two independent sources can't be coherent

Light is emitted by individual atoms and not by the bulk matter acting as a whole.

Even an atom emits an unbroken wave of about 10^{-8}s due to its transition from a higher energy state to lower energy state. Thus phase difference can remain constant for about 10^{-8}s only ie phase changes (position of bright and dark bands) 10^{8} times in one second. Such rapid change in position of maxima and minima can't be detected by our eye. The interference pattern is lost and almost a uniform illumination is seen on screen.

TWO COHERENT SOURCES CAN BE OBTAINED FROM A SINGLE PARENT SOURCE

1. In Young's double slit experiment

$$\left.\begin{array}{ll} 2. & \text{Fresnel's biprism method (refraction)} \\ 3. & \text{In Lloyd's mirror mehtod} \\ 4. & \text{In Billet's split lens} \\ 5. & \text{In Michleson's interferometer} \end{array}\right\} \text{Beyond our scope}$$

CONDITION FOR OBTAINING TWO COHERENT SOURCES

I) The two sources of light must be obtained from a single source.

ii) The two sources must give monochromatic light.

iii) The path difference between the waves arriving on the screen from the two sources must not be large.

METHOD OF PRODUCING COHERENT SOURCES

1. By division of wavefront (eg: YDSE)
2. By division of amplitude (eg: thin films, soap bubbles)

CONDITION FOR CONSTRUCTIVE & DESTRUCTIVE INTERFERENCE

Consider two waves of amplitudes a_1 and a_2 travelling along same direction with constant phase difference (ϕ), superpose

$$y_1 = a_1 \sin \omega t$$

$$y_2 = a_2 \sin(\omega t + \phi)$$

Resultant displacement, $y = y_1 + y_2$

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t$$

Put $a_1 + a_2 \cos \phi = A \cos \theta ----(1)$

$$a_2 \sin \phi = A \sin \theta ----(2)$$

$$(1)^2 + (2)^2 \Rightarrow A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

Resultant amplitude, $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$ and $I \alpha A^2$

Resultant intensity $I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$; $2\sqrt{I_1I_2} \cos \phi$ is known as interference term.

For constructive interference

Intensity should be maximum, for this $\cos \phi = +1$ ie $\phi = 0, 2\pi, 4\pi, ...$

$$\boxed{\phi = 2n\pi}$$

$$\therefore I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \quad ------(1)$$

$$= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$= \left(a_1 + a_2\right)^2$$

Path difference, $\Delta x = \frac{\lambda}{2\pi}\Delta\phi$

$$= \frac{\lambda}{2\pi} 2n\pi$$

$$\boxed{\Delta x = n\lambda}$$

<u>For destructive interference</u>

Intensity should be minimum, for this $\cos\phi = -1$ ie $\phi = \pi, 3\pi, 5\pi, ...$

$$\boxed{\phi = (2n-1)\pi}$$

$$\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{\lambda}{2\pi}(2n-1)\pi$$

$$\boxed{\Delta x = (2n-1)\frac{\lambda}{2}}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \quad ------(2)$$

$$= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \qquad = \left(a_1 - a_2\right)^2$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2$$

<u>Note:</u>

i) If intensities of two interfering beams are equal ie $I_1 = I_2 = I_0$

$$I_{max} = \left(\sqrt{I_0} + \sqrt{I_0}\right)^2 = 4I_0$$

$$I_{min} = \left(\sqrt{I_0} - \sqrt{I_0}\right)^2 = 0$$

$$I_R = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$

$$= 2I_0\left(1+\cos\phi\right) \qquad = 4I_0\cos^2\frac{\phi}{2}$$

$$I_R = I_{max}\cos^2\frac{\phi}{2}$$

ii) From equations (1) and (2) it is clear that in interference energy is conserved but redistributed.
ie Interference is the modification in energy distribution as a result of superposition of two waves.

iii) For incoherent source interference term $2\sqrt{I_1 I_2}\cos\phi = 0$

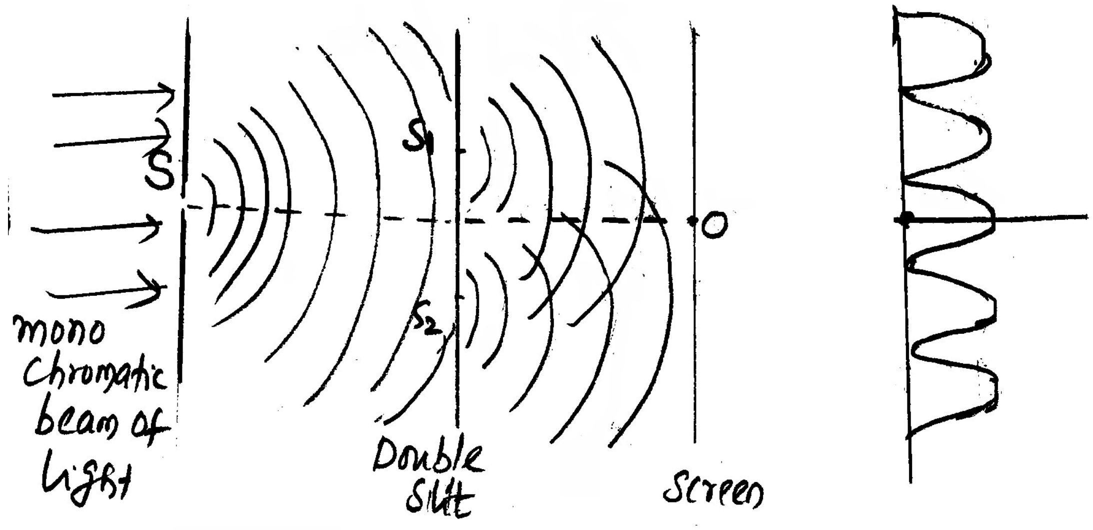
$\therefore\ I_{incoherent} = I_1 + I_2$

if $I_1 = I_2 = I_0$; $\qquad I_{max} = 2I_0$

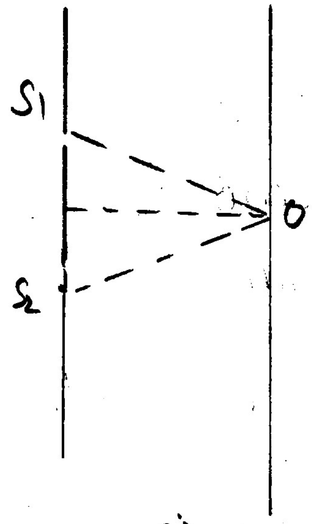
iv) If I_{avg} is the average intensity of two interfering beams of intensities I_1 and I_2

$$I_{avg} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = A_1^2 + A_2^2$$

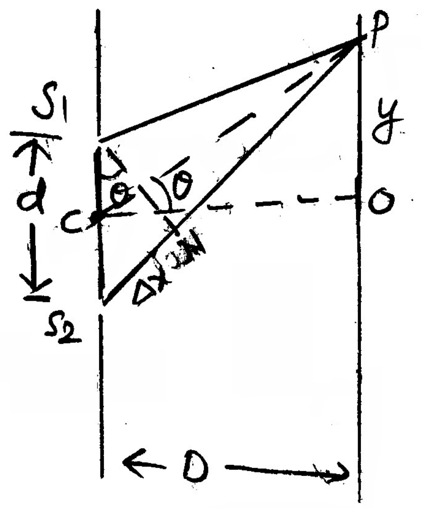
YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)



(i)



(ii)



(iii)

Figure (i) shows arrangement of Young's experiment, in which monochromatic light passes through a single narrow source slit (s) and falls on two closely spaced narrow slits s_1 and s_2. These two slits act as coherent sources of light waves that interfere constructively and destructively at different points on the screen to produce a pattern of alternate bright and dark fringes.

Since point O, the centre of screen is equidistant as shown in figure (ii) from s_1 and s_2, path difference $\Delta x = 0$ for wave fronts coming out of s_1 and s_2. They add constructively thus producing bright fringe at centre.

<u>From fig (iii)</u>

Path difference, $\Delta x = s_2p - s_1p = s_2N$

Form $\Delta s_1 s_2 N; \sin\theta = \frac{\Delta x}{d}$

$\boxed{\Delta x = d\sin\theta}$

and from $\Delta COP, \tan\theta = \frac{y}{D}$

Since θ is very small, $\sin\theta \sim \tan\theta$

$\boxed{\frac{\Delta x}{d} = \frac{y}{D}} \Rightarrow$ for 'd' is very small

Position of nth bright fringe from central bright fringe

From $\frac{\Delta x}{d} = \frac{y}{D}$

$$y_n = \frac{\Delta x\, D}{d}$$

For constructive interference, $\Delta x = n\lambda$

$$\therefore \boxed{y_n = \frac{n\lambda D}{d}}$$

Position of nth dark fringe from central bright fringe

For destructive interference $\Delta x = (2n-1)\frac{\lambda}{2}$

$$\boxed{\therefore\ y_n = (2n-1)\frac{\lambda D}{2d}}$$

Fringe width or Band width (β)

It is the distance between two successive bright fringes or two successive dark fringes.

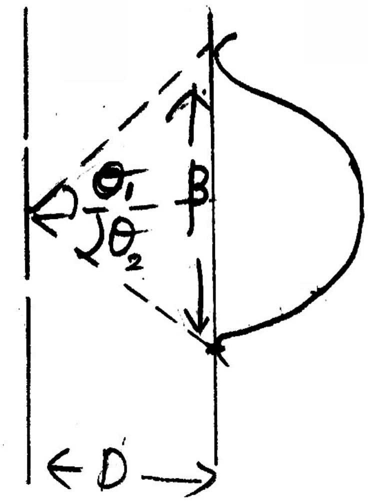
$$\beta = y_{n+1} - y_n = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

✶ In the interference pattern, the fringe width is constant for all the fringes, because fringe width is independent of order of fringes(n)

✶ All bright fringes are of same intensity.

Angular fringe width



$$\theta = \theta_1 + \theta_2 \qquad \theta_1 = \theta_2$$

$$= \frac{\beta}{2D} + \frac{\beta}{2D}$$

$$\theta = \frac{\beta}{D}$$

$$\theta = \frac{\lambda D}{dD}$$

$$\boxed{\theta = \frac{\lambda}{d}}$$

Angular position of n^{th} bright fringe, $\theta_n = \frac{n\lambda}{d}$

Angular position of n^{th} dark fringe, $\theta_n = (2n-1)\frac{\lambda}{2d}$

Note

1. Fringe visibility (V): It quantifies the contrast of interference in any system

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

✶ The two individual wave should have equal amplitude for best visibility

2. If W_1 and W_2 be the width of two slits in YDSE then $\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

3. If entire arrangement of YDSE is immersed in a medium of refractive index μ without any change in experimental setup. Then

   Fringe width decreases by μ times

   $\beta \alpha \lambda$

   $\beta_{med} \, \alpha \, \lambda_{med}$ $\qquad \frac{\beta}{\beta_m} = \frac{\lambda}{\lambda_m} = \frac{\lambda}{\lambda/\mu}$

   $\mu = \frac{\lambda}{\lambda_{med}}$ $\qquad$ ie $\beta_m = \frac{\beta}{\mu}$ also $\theta_m = \frac{\theta}{\mu}$

4. If any one of the slit in YDSE is closed. Then

   i) Interference pattern is replaced by diffraction pattern

   ii) Intensity at central maximum (at any point) become $\frac{I_0}{4}$ because amplitude halved.

5. Intensity interms of fringe width

   $$I_R = I_{max} \cos^2 \phi / 2$$

   $$\frac{\Delta x}{d} = \frac{y}{D}; \ \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\frac{yd}{D}$$

   $$I_R = I_{max} \cos^2 \frac{\pi}{\lambda}\frac{yd}{D} \qquad = I_{max} \cos^2 \left(\frac{\pi y}{\beta}\right)$$

   and also $\Delta x = d \sin\theta; \ \Delta\phi = \frac{2\pi}{\lambda} d \sin\theta$

   $$\therefore I_R = I_{max} \cos^2 \left(\frac{\pi}{\lambda} d \sin\theta\right)$$

6. **<u>Shape of fringes</u>**

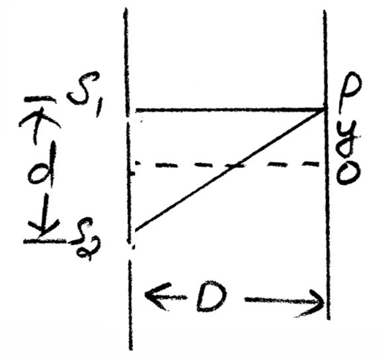
   a) Fringes are usually hyperbolic shape.

   b) When double slit plane and screen are mutually perpendicular, fringes are concentric circles.

   c) When distance between double slit plane (D) is very large compared to separation between slits (d), fringes are straight lines.

7. **Expression for missing wavelength**

At a point on the screen directly infront of one of the slits.



For missing wavelength, intensity will be minimum

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

$$y = \frac{\Delta x D}{d}$$

$$\frac{d}{2} = (2n-1)\frac{\lambda D}{2d}$$

$\therefore \lambda = \dfrac{d^2}{(2n-1)D}$ for n = 1,2,3, ... $\lambda_{missing} = \dfrac{d^2}{D}, \dfrac{d^2}{3D}, ...$

8. **If one of the slit in YDSE is covered by transparent sheet of thickness t and refractive index** μ

i) Fringe width remains same

ii) The path difference at centre will not be zero. It is $\Delta x = (\mu - 1)t$

iii) The entire fringe pattern will shift upward if sheet is placed before the upper slit, and if the sheet is placed before the lower slit, the fringe pattern gets shifted downwards.

Fringe shift $y_0 = \dfrac{\Delta x D}{d} = (\mu - 1)t\dfrac{D}{d}$

$$= \frac{\beta}{\lambda}(\mu - 1)t$$

iv) number of fringes shifted, $n = \dfrac{\text{shift}}{\text{fringe width}}$

$$n = \frac{\beta}{\lambda}\frac{(\mu-1)t}{\beta} = (\mu-1)\frac{t}{\lambda}$$

v) The intensity of light from the covered slit will decrease due to absorption. Hence intensity of bright fringe decreases and dark fringes have some finite intensity. Hence fringe pattern will become less distinct.

9. **If the beam of light has two wavelengths λ_1 and λ_2 (Bichromatic)**

i) their maxima will coincide if $y_1 = y_2$

ie $n_1\lambda_1\frac{D}{d} = n_2\lambda_2\frac{D}{d}$

$$\boxed{n_1\lambda_1 = n_2\lambda_2}$$

DIFFRACTION

✶ It is the phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into the regions of geometrical shadow.

eg: 1) Appearance of a shining circle around the section of sun just before sun rise is due to diffraction.

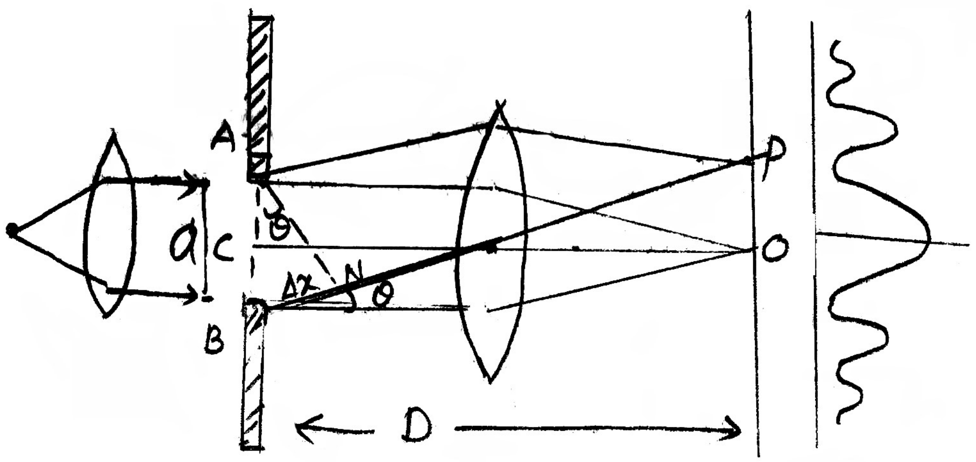
2) Colour of CD

3) Sun light filtering through leaves of trees forms circular patches on ground.

Condition for diffraction

✶ The diffraction effect is more pronounced if the size of the aperture or the obstacle is of the order of wavelength of light used.

DIFFRACTION DUE TO SINGLE SLIT



Let a plane wavefront falling on a single narrow slit AB of width 'a'. According to Huygen's principles, each point on the plane wavefront falling on AB gives rise to secondary wavelets which are initially in the same phase. These wavelets spread out in all directions, thus causing diffraction.

Central maximum

All secondary wavelets going straight across the slit AB are focussed at centre O. The wavelets from any two corresponding points of the two halves of the slit reach the point O in same phase, they add constructively to produce a central bright fringe.

From ΔABN

$$\sin\theta = \frac{\Delta x}{a}; \qquad \therefore \Delta x = a\sin\theta$$

Position of minima

Let the point P be so located on the screen that the path difference $\Delta x = \lambda$. Now divide the slit AB into two halves AC and CB. Then path difference between wavelet from A and C will be $\frac{\lambda}{2}$. Similarly corresponding to every point on the upper half AC, there is a point in lower half CB for which $\Delta x = \frac{\lambda}{2}$.

Hence they reach at point P in opposite phase. They add destructively producing first minimum. Proceeding the same way, it can be shown that the condition for n^{th} minima.

$$\Delta x = n\lambda = a\sin\theta_n$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

If angle is very small, then $\theta_n = \frac{n\lambda}{a}$

Position of maxima

Let point P on the screen be situated such that the path difference $\Delta x = \frac{3\lambda}{2}$

Now consider the slit width to be divided into three equal parts so that the path difference between the wavelets from the first two parts will again be $\frac{\lambda}{2}$. These wavelets will interfere destructively and cancel each other's effect.

The wavelet from the third part will remain uncancelled. Since all of them are in the same phase, they reinforce each other and thus produce the first secondary maximum.

Proceeding in the same way, it can be shown that the condition for n^{th} secondary maximum is

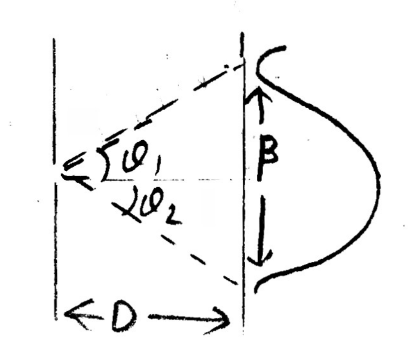
$$\Delta x = (2n+1)\frac{\lambda}{2}$$

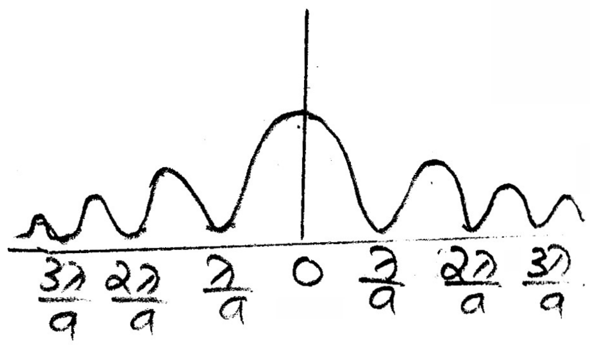
$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

$$\sin\theta_n = (2n+1)\frac{\lambda}{2a}$$

For small angle $\theta_n = (2n+1)\frac{\lambda}{2a}$

Angular width of central maximum and secondary maximum





The angular width of the central maximum is the angular separation between the directions of first minima on the two sides of the central maximum. The directions of first minima on either side of central maximum are given by $\theta = \frac{\lambda}{a}$. This angle is called half angular width of central maximum.

$\therefore$ Angular width of central maximum $= \frac{2\lambda}{a}$

✶ Linear width of central maximum, $\theta_1 + \theta_2 = \frac{\beta}{2D} + \frac{\beta}{2D}$

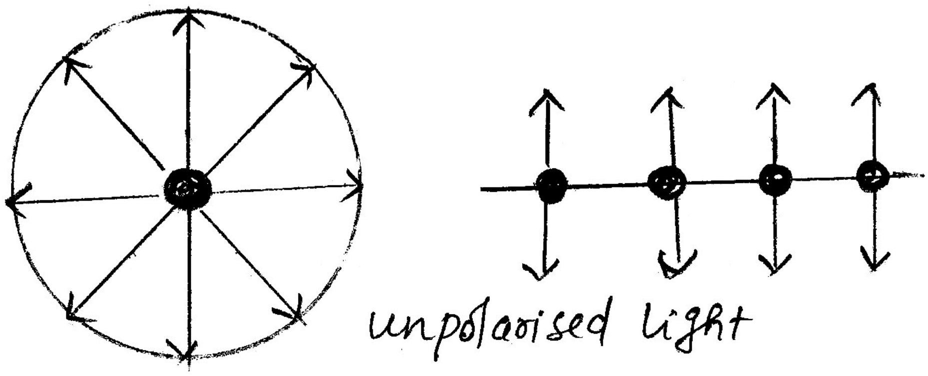
$$\frac{\beta}{D} = \frac{2\lambda}{a}$$

$\therefore \beta = \frac{2\lambda D}{a}$ Since convex lens is placed very close to the slit, then $D \approx f$

$$\therefore \beta = \frac{2\lambda f}{a}$$

POLARISATION

✶ A light which has vibration in all directions in a plane perpendicular to the direction of propagation is called unpolarised light.



- Polarization is the phenomenon of restricting the electric field vibration of light in a particular direction perpendicular to the direction of propagation of wave while passing through certain crystals called polarisers.

  Examples for polariser

  1. Tourmaline crystal
  2. Herapathite crystals
  3. Quartz crystal
  4. Calcite crystal
  5. Nicol prism

- If the EF vector of light wave vibrates just in one direction perpendicular to the direction of propagation is called linearly polarised.

  Since linearly polarised wave, the vibration at all points, at all times, lies in the same plane is called plane polarised.

- In case of linearly polarised light, the magnitude of electric field vector varies periodically with time.
- Polarisation is a characteristics of transverse waves only.

  Examples for polarisation

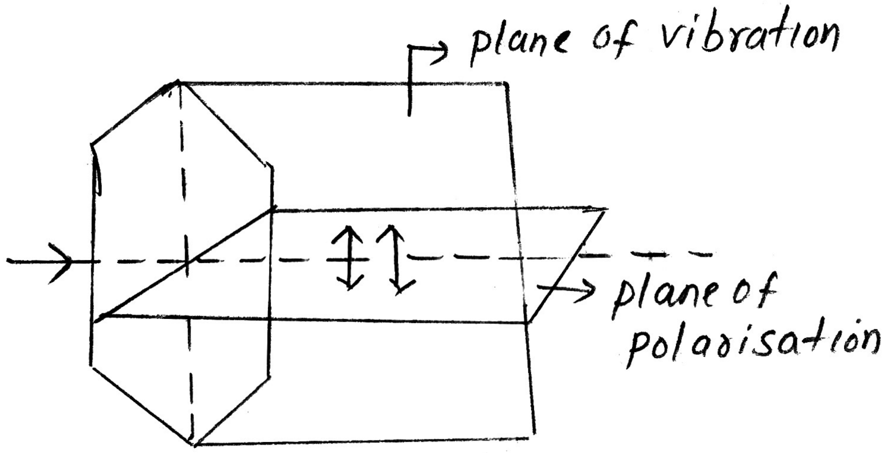
  1. Golden view of sea shell
  2. The wave emitted by radio transmitters are linearly polarised.

  Plane of vibration

- It is the plane with in which the vibrations of polarised light are confined.

Plane of polarisation

✶ It is a plane right angles to the plane of vibration and passing through the direction of propagation of light.

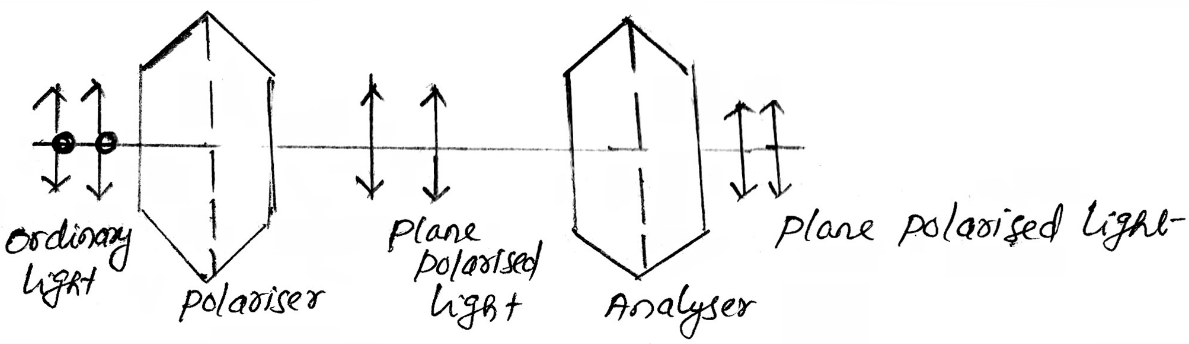


DETECTING PLANE POLARISED LIGHT

✶ We can't make distinction between the unpolarised light and the plane polarised light with the polariser alone. Another such crystal required to analyse the nature of light is called analyser.

A tourmaline crystal or a Nicol prism used to produce plane polarised light is called polariser. If the polariser is rotated in the path of the ordinary light, the intensity of light transmitted from the polariser remains unchanged. It is because, in each orientation of the polariser, the plane polarised light is obtained, which has vibrations in a direction parallel to the axis of the crystal in that orientation.

If we rotate the analyser in the path of the light transmitted from the polariser, so that the axes of the polariser and analyser are parallel to each other, then the intensity of light found to remain unaffected (maximum).



But when the axis of the polariser and analyser are perpendicular to each other, the intensity of light becomes minimum.

MALUS'S LAW

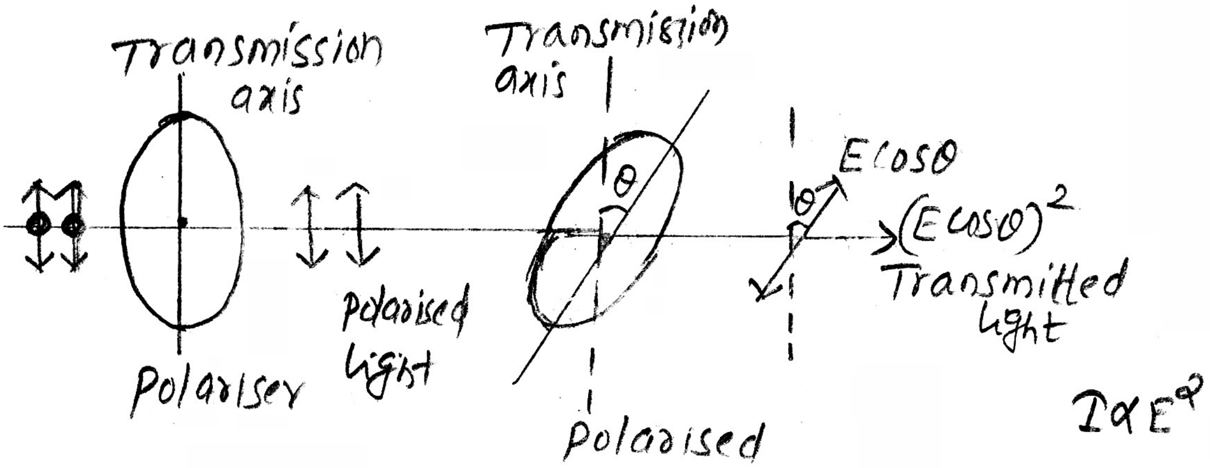
✶ The dependence of intensity of transmitted light on the angle between the analyser and polariser was investigated by E.I. Malus.

✶ It states that when a beam of completely plane polarised light passed through analyser, the intensity I of transmitted light from analyser varies directly as square of cosine of angle between transmission direction of polariser and analyser.

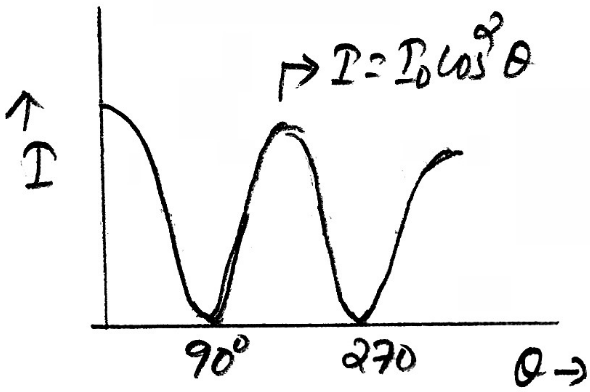
ie $I \alpha \cos^2 \theta$

$I = I_0 \cos^2 \theta$

Here I_0 is the intensity of plane polarised light and is equal to half the intensity of unpolarised light.



Intensity curve:

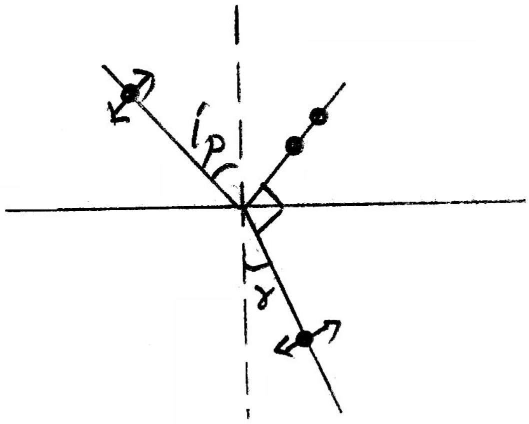


✶ On one complete rotation of analyser, intensity becomes twice maximum and twice minimum.

METHODS OF PRODUCING PLANE POLARISED LIGHT

1. Polarisation by reflection: Brewster's law

✶ When unpolarised light is incident on an interface separating two media, reflected and refracted light are partically polarised. When angle of incidence gradually increases, for a particular value of angle of incidence called polarising angle or Brewster's angle, reflected light is completely polarised. At this stage reflected ray and refracted ray are mutually perpendicular.



When $i = i_p, r = 90 - i_p$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i_p}{\sin(90 - i_p)} = \frac{\sin i_p}{\cos i_p}$$

$\mu = \tan i_p \Rightarrow$ Brewster' law

From Snell's rule, $\mu = \frac{\sin i_p}{\sin r}$

From Brewster's law, $\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$

$$\frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\sin(90 - i_p)}$$

$$\sin r = \sin(90 - i_p)$$

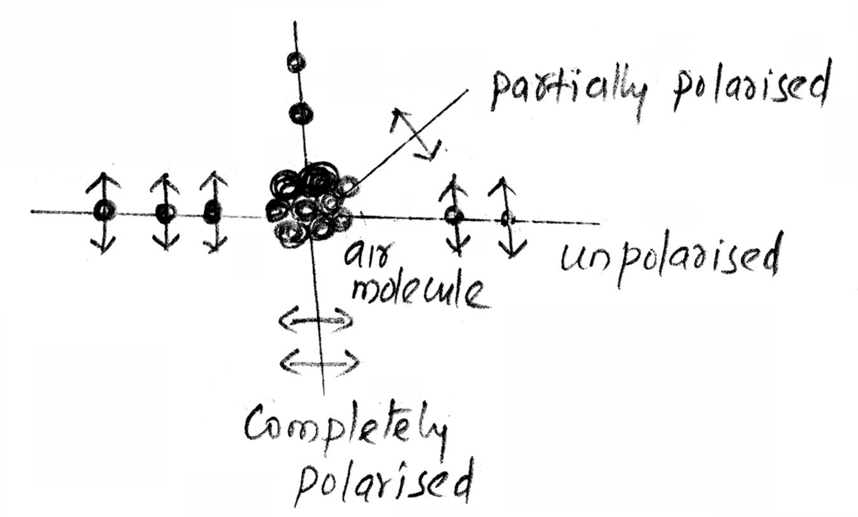
$r = 90 - i_p$

$r + i_p = 90^\circ$

2. Polarisation by scattering

✶ Since light waves are EM in nature they will vibrate the electrons of air molecules perpendicular to the direction in which they are travelling. The electrons then produce radiation that is polarised perpendicular to the direction of the ray.

However there is no polarisation parallel to the original direction of incident light. The light perpendicular to original ray is completely plane polarised and in all other direction, the light scattered by air molecules partially polarised.



DUAL NATURE OF RADIATION AND MATTER

Quantum theory of light

- Electromagnetic radiation is quantized and exist in elementary amounts which are known as photons.

Properties of Photons

- Photons are packets of energy
- A photon does not exist at rest. So rest mass of photon is zero
- A source of radiation emits energy in the form of photons and these photons travels in a straight line with the speed of the light.
- Speed = C
- Photons are electrically neutral and they; can't be deflected by electric and magnetic field.
- Energy of a photon is given by $E = h\upsilon$ or $E = {}^{hc}/_{\lambda}$
- Momentum of a photon is given by $P = {}^{h}/_{\lambda}$
- Equivalent mass of photon is given by $m = {}^{h}/_{C\lambda}$
- If intensity is more number of photons will be more

Number of Photons emitted by source

Consider a light bulb of power P

wave length of light emitted is λ.

$\therefore$ Energy radiated by source in $1\,\text{Sec} = P$

Source emits energy in the form of photons.

$\therefore$ Energy radiated by source in $1\,\text{Sec}$

= no: of photons emitted in $1\,\text{Sec}$ *Energy of one photon

$P = n \times h\upsilon$

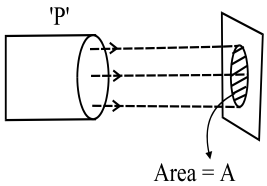
$\boxed{n = \frac{P}{h\upsilon}}$ or $\boxed{n = \frac{P\lambda}{hc}}$ $\left(\upsilon = c/\lambda\right)$

$n \Rightarrow$ no: of photons emitted in 1 Sec

$\therefore$ no: of photons emitted in t sec $= \frac{P\lambda t}{hc}$

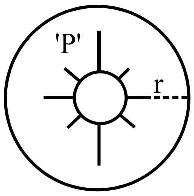
Intensity of light due to light source

The energy crossing per unit area per unit time perpendicular to direction of propagation is called intensity of light wave



In this figure

Intensity $I = P/A$



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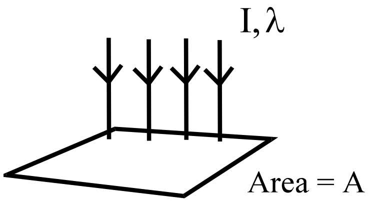
Intensity $I = P/A = \frac{P}{4\pi r^2}$

Number of photons striking in one sec

Normal incidence

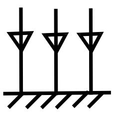


$\boxed{n = \frac{IA\lambda}{hc}}$ $n \rightarrow$ number of photons incidented in 1 sec

Radiation Pressure

The force exerted by radiation on a surface is known as Radiation force and pressure due to this force is known as radiation pressure.

1) Perfectly reflecting Surface (Normal incidence)



Light falling normally on a perfectly reflecting surface of area A.

Total no: of Photons incidented in one sec

$$n = \frac{IA\lambda}{hc}$$

Change in momentum of one photon $= 2\, h/\lambda$

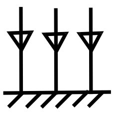
Total change in momentum in one sec $== 2\, h/\lambda \times \frac{IA\lambda}{hc}$

This one is the force.

$\therefore$ force $\boxed{F = \frac{2IA}{c}}$

Pressure is $P = F/A$ $\boxed{P = 2\, I/C}$

2) Perfectly absorbing surface (Normal incidence)



Number of photons striking in one sec

$$\eta = \frac{IA\lambda}{hc}$$

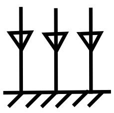
Change in momentum of one photon $= \frac{h}{\lambda}$

Total change in momentum in one sec $= \frac{IA\lambda}{hc} \times \frac{h}{\lambda}$

$\therefore$ Total force $= = I\, A/c$

$\boxed{F = \frac{IA}{C}}$ radiation pressure $P = F/A$ $\boxed{P = I/C}$

3) Partially reflecting surface (Normal incidence)



radiation force $= \frac{IA}{C}(1+r)$ $\boxed{\text{r-ref coeff}}$

radiation pressure $= I/C\,(1+r)$

Photoelectric effect

When electromagnetic radiation of suitable wave length are incidented on metal surface, then electrons are emitted, this phenomenon is called photoelectric effect. The emitted electrons are known as photoelectrons and current so obtained is known as photo electric current.

Work Function (ϕ)

It is the minimum amount of energy required to remove an electron from the surface of a metal.

$\therefore$ for photoelectric effect

Energy of incident photon $\geq$ work function

ie $h\upsilon \geq \phi$

$$\upsilon \geq \phi/h$$

$$\boxed{\upsilon_{min} = \phi/h}$$

Threshold frequency (υ_0)

for photoelectric emission, frequency of incident light should be greater than or equal to a particular value of frequency and this minimum value of frequency is known as threshold frequency.

$$\boxed{\upsilon_0 = \frac{\phi}{h}}$$

Threshold wave length (λ_0)

for photoelectric emission, wave length of incident light should be less than or equal to a particular value and this max value of wave length is known as threshold wave length.

$$h\upsilon \geq \phi \qquad \lambda \leq \frac{hc}{\phi}$$

$$h\, c/\lambda \geq \phi$$

$$\lambda/hc \leq \frac{1}{\phi} \qquad \boxed{\lambda_0 = hc/\phi}$$

Einstein's Photoelectric equation

According ti Einstein, photon energy falling on a metal is utilized for two purposes.

(i) Partly for getting electron free from the atom. This energy is known as work function.

(ii) The balance of photon energy is used up in giving the electron a kinetic energy KE

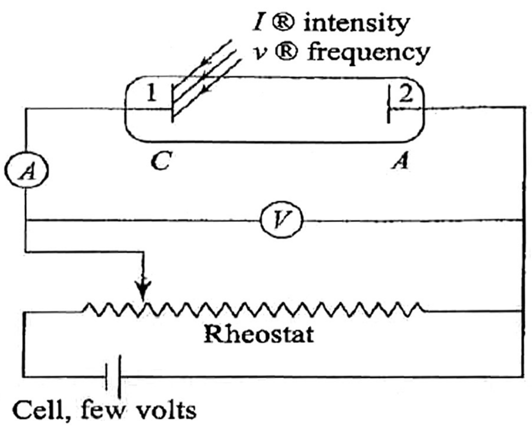
Then we can write

$\boxed{h\upsilon = \phi + KE}$ $\boxed{KE = h\upsilon - \phi}$

If the electron does not lose energy by internal collisions as it escapes from metal, it will still have this much kinetic energy after it emerges.

$\boxed{KE_{max} = h\upsilon - \phi}$ $\boxed{V_{max} = \sqrt{\frac{2(h\upsilon - \phi)}{m}}}$ $m \Rightarrow$ mass of electron

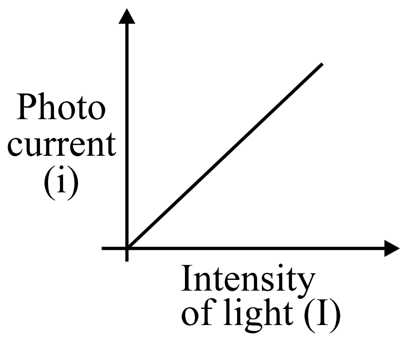
Study of Photoelectric effect



Here plate 1 is called is emitter or cathode and plate 2 is called collector or anode. A potential difference is applied between the plates. On cathode electromagnetic radiation of intensity I and frequency f is allowded to fall on it.

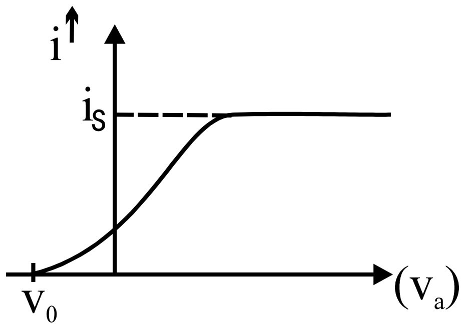
Observations

(i) In this frequency υ and potential diff is kept constant.



ie Photoelectric current is directly proportional to Intensity of incident light

(ii) **Photocurrent versus stopping potential**



If we increase V, then i also increases, but after increasing V up to some values, there is no increase in photocurrent and this value is known as saturation current. At this value all the photo electrons emitted from cathode reaches the anode and there is no possibility of increasing the current by increasing V.

V_0 is known as stopping potential or cut off potential. It is the minimum negative potential given to A w.r.t C at which photocurrent becomes zero.

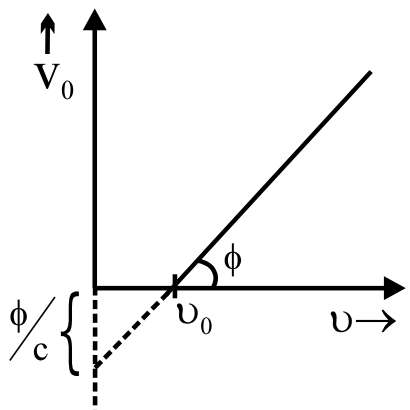
Maximum kinetic energy and stopping potential are related as

$$\boxed{eV_0 = KE_{max}}$$

from Einstein's relation we can write it as $eV_0 = h\upsilon - \phi$

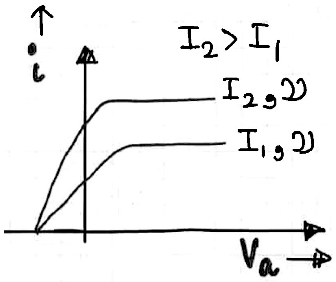
$$\boxed{V_0 = {h}/{e}\,\upsilon - {\phi}/{e}}$$

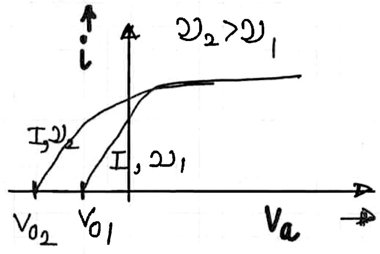
So V_0 versus υ graph is a straight line whose slope is ${h}/{e}$ and y intercept is ${\phi}/{e}$



" There fore slope is same for all the materials." (Slope is constant)

Stopping potential depends only on frequency. It will not depends on intensity of incident light.





Important point

Kinetic energy of photoelectron and stopping potential only depends on the frequency of incident light, it will not depends on the intensity of incident light.

Failure of Wave theory to explain PEE

- If the intensity of radiation is greater, then amplitude of electric and magnetic field is large, so energy absorbed by electrons will be very large, and their kinetic energy will be high.

  But from study of photoelectric effect it is clear that kinetic energy of photoelectrons are independent of intensity.

- From a single wave front, large number of electrons will absorb energy, then the energy absorbed per unit electron per unit time is very small. It can take hours or more for a single electron to pickup sufficient energy to overcome the work function.

  But photoelectric effect is an instantaneous process.

Matter Waves

De broglie suggested that a moving body behaves in certain ways as though it has a wave nature.

The waves associated with moving particles are known as matter waves or de broglie waves.

De broglie wave length is given by

$$\boxed{\lambda = \frac{h}{mv}} \quad \text{or} \quad \boxed{\lambda = \frac{h}{P}}$$

P is the momentum of the particle. Also we know that $K.E = P^2/2m$

$$P = \sqrt{2m\,K.E} \qquad \therefore \boxed{\lambda = \frac{h}{\sqrt{2m\,K.E}}}$$

for a charged particle KE = qV

$$\boxed{\lambda = \frac{h}{\sqrt{2mqv}}}$$

For electron $\lambda = \frac{12.27}{\sqrt{V}}\ A^0 = \frac{1.227}{\sqrt{V}}\ nm$

For protons $\lambda = \frac{0.286}{\sqrt{V}}\ A^0$ $v \Rightarrow$ accelerating potential in volts

For alpha particles $\lambda = \frac{0.101}{\sqrt{V}}\ A^0$

For deuterons $\lambda = \frac{0.202}{\sqrt{V}}\ A^0$

Properties of matter wave

1. Matter waves are different from electromagnetic waves
2. Matter waves are related to moving particles and independent of the fact that whether the particle is neutral or changed.
3. Matter waves travels with speed more than that of light. [Group velocity is same as velocity of body where as individual waves constituting the group may theoretically move with speed greater than that of light]

$$V_{mw} = \upsilon\lambda = \frac{E}{h}.\frac{h}{p} = \frac{mc^2}{mv} = \frac{c^2}{v} > C$$

4. The wave and particle aspect of moving bodies can never be observed at the same time.

ATOMS & NUCLEI

Thomson's Model of Atom

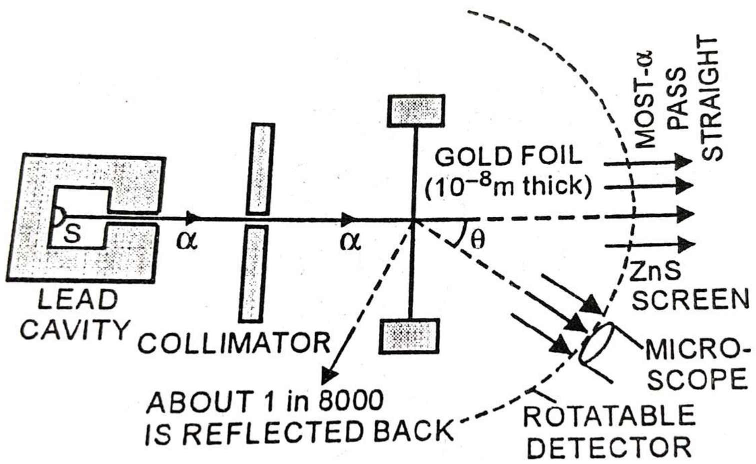
Also known as plum pudding model or water melon model. Atom is a positively charged sphere of radius of the orer of 10^{-10}m $\left(1\text{Å}\right)$. There was no nucleus at the centre of an atom. Instead the positive charge and also the mass was assumed to be spread through out the atom, forming a kind of paste or pudding. The negative electrons were assumed to be suspended like plums. The number of electrons is such that their total negative charge is equal to the total positive charge. Hence atom is electrically neutral.

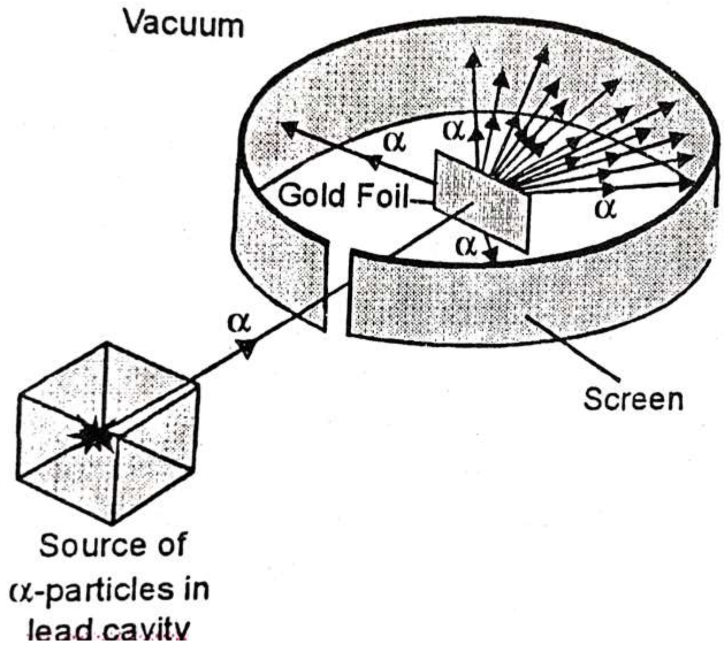
Failures:

✶ Failed to to explain scattering of α-particles by large angle in Rutherford's experiment of α-particle scattering.

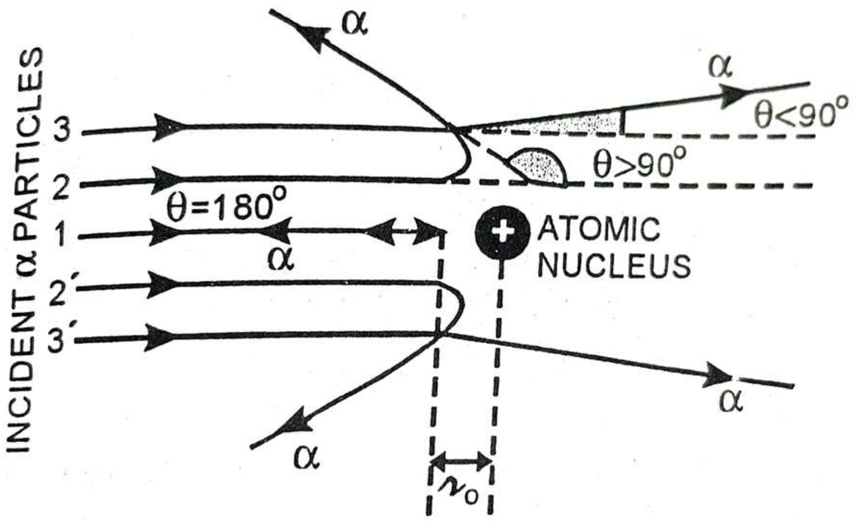
✶ Origine of spectral lines from atom.

Rutherford's Alpha-particle scattering Experiment

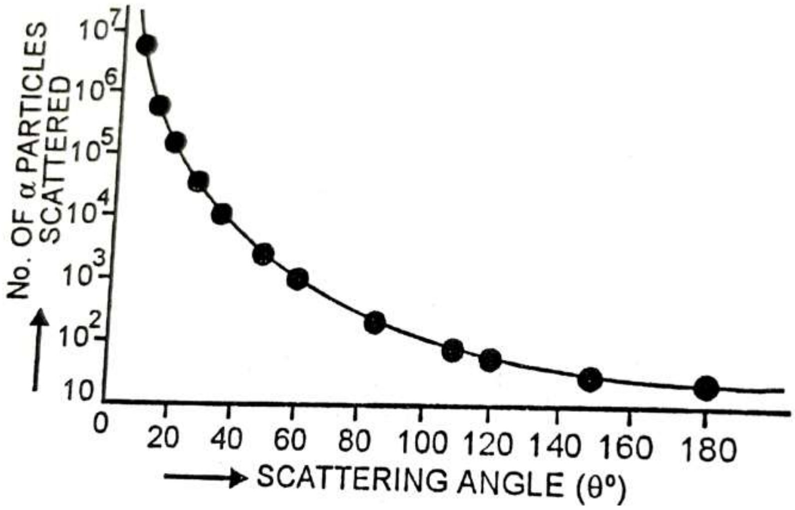




Rutherford passed a parallel beam of α-particles (He nucleus) with the help of lead bricks, into a thin gold foil. The alpha particles get scattered in different directions. The scattered alpha particles on striking the screen produced brief light flashes or scintillations. These are detected by a microscope.



Most of the alpha particles passed through the foil without any deflection. A graph is plotted with total no. of α-particles scattered and the scattering angle.



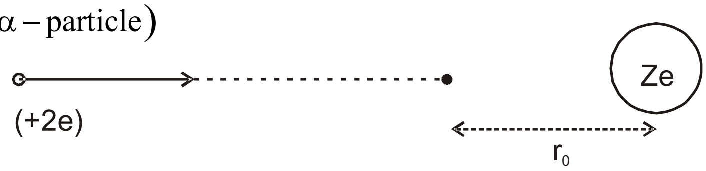
$$N_\theta \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

A few number of α-particles ge scattered by large angle.

The distance of Closest Approach

The distance from the nucleus at which α-particle stops and scattered through 180° ie, reflected back along its path is called distance of closest approach.

$(\alpha - \text{particle})$



P.E. = 0

$$\text{P.E.} = \frac{1}{2}mv^2$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

K.E. = 0

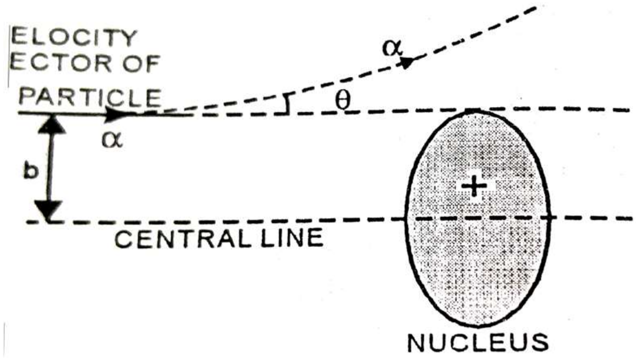
K.E. = P.E.

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{2e\,Ze}{r_0}$$

$$k = \frac{2Ze^2}{4\pi\varepsilon_0 r_0} \text{ or } \boxed{r_0 = \frac{2\,Ze^2}{4\pi\varepsilon_0 k}}$$

Impact Parameter (b)

The perpendicular distance of the initial velocity vector from the centre of nucleus.



$$\boxed{b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{k}\cot\left(\frac{\theta}{2}\right)}$$ $k \rightarrow$ initial K.E.

If b is higher θ will be small. If bis smallar θ will be larger.

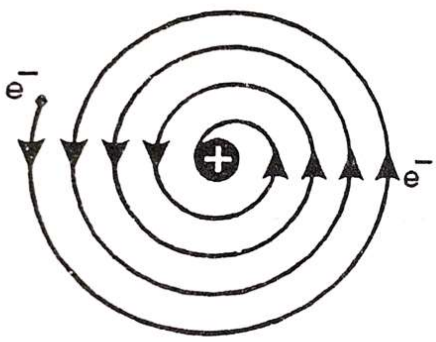
If particle approaches the centre of nucleus, b = $0 \therefore \cot\left(\frac{\theta}{2}\right) = 0 \;\; \frac{\theta}{2} = 90$ or $\theta = 180^{o}$

Rutherford's Nuclear Model of Atom

Most of the atom is empty. Positive charge, instead of being distributed thinly and uniformly throughout the atom, was concentrated in a small region called the nucleus. The electrons are moving around the nucleus in circular orbits, just as planets revolve around the sun. This model is also known as planetary model. The necessary centripetal force is provided by the electrostatic force between nucleus and electron.

Failures

✶ An electron moving in a circular orbit round a nucleus accelerating and according to classical electromagnetic theory, it should therefore emit radiation continuously and there by lose energy. If this happened, the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second.



In Rutherford's model, due to continuously changing radii of circular orbits of electrons, the frequency of revolution of electrons must be changing. As a result electrons will radiate electromagnetic waves of all frequencies, ie., the spectrum of these waves will be continuous in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

BOHR MODEL OF HYDROGEN ATOM

Bohr made some assumptions inorder to combine the new quantum ideas of Planck and Einstein with the traditional description of a particle in unifrom circular motion.

Bohr's Postulates

1. An electron in an atom could revolve in certain stable orbits, without the emission of radiant energy, contrary to the predictions of electromagnetic theory. According to this postulate each atom has definte stable states in which it can exist and each possible state has definite total energy. These are called the stationary states of atom.
2. Electron revolves around only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$ where 'h' is the planck's constant. (h = 6.6 × 10^{-34} Js). The angular momentum (ℓ) of the orbiting electron is quantised. that is $\ell = n\hbar$
3. An electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by $\boxed{h\upsilon = E_i - E_f}$

Radius of Electron Orbit

The electron is revolving around the nucleus of atom. The centripetal force required for revolution of electron is provided by electrostatic force between electron and nucleus.

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

Where $Ze \rightarrow$ charge of nucleus.

From Bohr's second postulate

$$mvr = \frac{nh}{2\pi} \qquad v = \frac{nh}{2\pi mr}$$

$$mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$m\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2Z} \qquad r_n = \frac{h^2\varepsilon_0}{\pi me^2}\frac{n^2}{Z}$$

$r_n = (0.53)\frac{n^2}{Z} A^o$ $\boxed{r_n \, \alpha \, \frac{n^2}{Z}}$ $\Rightarrow$ for Hydrogen like atoms.

Where n is principal quantum number.

$n = 1, 2, 3, ..., \infty$

Hydrogen like atoms $\Rightarrow$ Atoms with one electron.

✶ Stationary orbits are not equally spaced.

Velocity of electron in nth orbit

$v = \frac{nh}{2\pi mr}$ put $r = \frac{n^2 h^2 \varepsilon_0}{\pi Zme^2}$

$$v = \frac{nh \, \pi Zme^2}{2\pi m \, n^2 h^2 \varepsilon_0} = \left(\frac{e^2}{2h\,\varepsilon_0}\right)\frac{Z}{h}$$

Where $\alpha = \frac{e^2}{2h\varepsilon_0 c}$

$\boxed{v\alpha \frac{Z}{n}}$ $v = \left(\frac{e^2}{2h\varepsilon_0 c}\right)\frac{cZ}{n}$

$\alpha \rightarrow$ Sommerfeld's fine structure constant

$\boxed{v = \frac{\alpha c Z}{n}}$

$\alpha = \frac{1}{137}$

c $\rightarrow$ speed of light.

$$v = \frac{1}{137}\left(\frac{cZ}{n}\right)$$

Velocity of electron in Bohr's first orbit of Hydrogen (Z = 1) is $\frac{c}{137}$, in second orbit is $\frac{c}{274}$ and soon.

Orbital frequency of electron

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{e^2}{2\varepsilon_0 h} \times \frac{Z}{n} \; \frac{\pi me^2 Z}{2\pi n^2 h^2 \varepsilon_0}$$

$\boxed{f = \left(\frac{me^4}{4\varepsilon_0^2 h^3}\right)\frac{Z^2}{n^3}}$ $\boxed{f \, \alpha \frac{Z^2}{n^3}}$ $\boxed{f \;\alpha \frac{1}{n^3}}$ for H-atom

$\boxed{T = \frac{4\varepsilon_0^2 h^3}{me^4} \; \frac{n^3}{Z^2}}$ $\boxed{T \;\alpha \frac{n^3}{Z^2}}$

Energy of electron in nth orbit

$F_c = F_e$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze\ e}{r^2}$$

$$mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$$

$$\frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$\boxed{\therefore \text{K.E.} = \frac{Ze^2}{8\pi\varepsilon_0 r}}$$

Potential energy, U

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Ze(-e)}{r}$$

$$\boxed{U = \frac{-Ze^2}{4\pi\varepsilon_0 r}}$$

$$(U = \text{P.E.})$$

Total energy = K.E. + P.E.

$$\text{T.E.} = \frac{Ze^2}{8\pi\varepsilon_0 r} + \frac{-Ze^2}{4\pi\varepsilon_0 r}$$

$$\boxed{\text{T.E.} = \frac{-Ze^2}{8\pi\varepsilon_0 r}}$$

$\Rightarrow$ Total energy = -Kinetic energy

$$\boxed{\begin{array}{l}\text{K.E.}:\text{T.E.} = 1:-1.\\ \text{K.E.}:\text{P.E.} = 1:-2\\ \text{P.E.}:\text{T.E.} = 2:1\end{array}}$$

$\text{T.E.}, E = \dfrac{-Ze^2}{8\pi\varepsilon_0 r}$ but $r = \dfrac{n^2h^2\varepsilon_0}{\pi Zme^2}$

$$\therefore E_n = \frac{-Ze^2}{8\pi\varepsilon_0}\frac{\pi Zme^2}{n^2h^2\varepsilon_0} = \frac{-me^4}{8\varepsilon_0^1 h^2}\frac{Z^2}{n^2}$$

$$\boxed{E_n = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{Z^2}{n^2}} \qquad \boxed{E \,\alpha\, \frac{Z^2}{n^2}}$$

The value of $\dfrac{me^4}{8\varepsilon_0^2h^2}$ is $2.17\times10^{-18}\,J$ = 13.6eV

$\therefore$ Energy of electron in Hydrogen like atoms is $\boxed{E_n = \dfrac{-13.6\,Z^2}{n^2}\text{eV}}$

$$E = -\left[\frac{me^4}{8\varepsilon_0^2 h^3 c}\right]^{hc} \frac{Z^2}{n^2} = \frac{-RhcZ^2}{n^2}$$

Where R = Rhydberg's constant $= \dfrac{me^4}{8\varepsilon_0^2 h^3 c}$

$R = 1.097 \times 10^7\, m^{-1}$

Value of Rhc = 2.17 ×10^{-18}J = 13.6 eV

✶ -Rhc is known as Rhydberg's energy.

It is the energy of electron in the first orbit of hydrogen atom.

$$E_n = \frac{-13.6\, eV}{n^2} [\text{For Hydrogen}]$$

$$E_1 = -13.6\, eV [\text{For Hydrogen}]$$

The -ve sign indicates that electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Total energy of electron in the ground state of Hydrogen atom = -13.6 eV.

its P.E. = -27.2 eV   its K.E. = 13.6 eV

Energy levels of Hydrogen Atom

For Hydrogen Z = 1   T.E., $E = \dfrac{-13.6}{n^2} eV$

For n = 1, E_1 = -13.6 eV (ground state k shell)

For n = 2 $E_2 = \dfrac{-13.6}{4} eV = -3.4\, eV$ (L shell or first excited state)

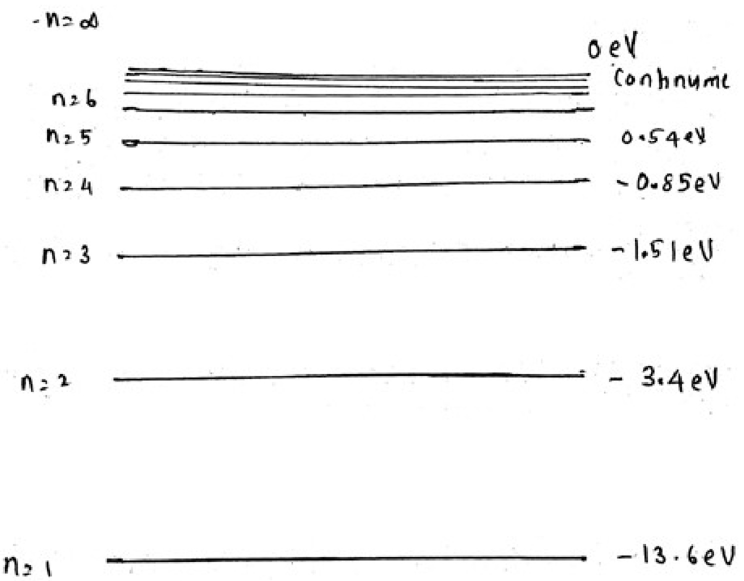
For n = 3 $E_3 = \dfrac{-13.6}{9} eV = -1.51\, eV$ (M shell or second excited state)

For n = 4 $E_4 = \dfrac{-13.6}{16} eV = -0.85\, eV$

For n = 5 $E_5 = \dfrac{-13.6}{25} eV = -0.54\, eV$

For n = 6 $E_6 = \dfrac{-13.6}{36} eV = -0.37\, eV$

For $n = \infty$ $E_\infty = 0$



Total energy of electron in an atom is quantised $E_n = \frac{-13.6\,Z^2}{n^2}\,eV$. Energy levels are not equally spaced.

The separation between the energy levels decreases as the value of n increases.

When $n = \infty$ E=0. In this stage electron is no longer bound to the nucleus. It becomes a free electron.

Atomic Spectra

According to Bohr model electron in an atom will be in a state with certain energy. Normally an electron in an atom will be in the lowest energy state. ie., ground state (n = 1). When we give an external energy to an electron in gorund state it will reach any excited state. The external energy must be equal to the difference between the energy of ground state and any excited state. The electron in excited state will come back to ground state by emitting energy in the form electromagnetic radiation.

Atomic Excitation

An atom can be excited to an energy level above its ground state by

1) Photon absorption 2) Collision

1. Photon Absorption

When an atom absorbs a photon, it becomes excited and returns to its ground state in an average of 10^{-8} second by emitting one or more photons. Atom absorb a photon with energy equal to the energy difference of ground state and any excited state.

Absorption Spectrum

An atom in ground state can absorb light of certain wavelength only. The light having energy equal to energy difference between excited state and ground state will be absorbed. When a white light is passed through an atom it will absorb certain wavelengths. These wavelengths can be represented as dark lines on a bright screen. The wavelength corresponding to dark lines are absorbed by the atom. This is called absorption spectrum.

Emission Spectrum

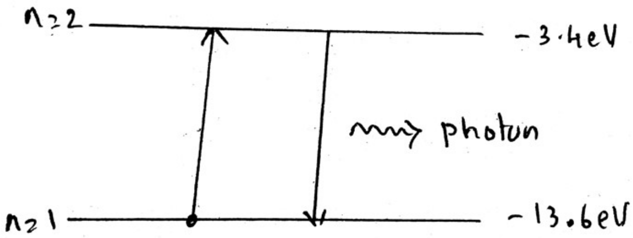
The electron in excited state return to the ground. State by emitting one or more photons. These wavelengths can be represented as bright lines on a dark screen. The wavelengths corresponding to bright lines are emitted by the atom. This is called emission spectrum.

Collision

For atomic excitation, collision has to be inelastic. Here K.E. is not conserved. But linear momentum and total energy will be conserved. K.E. lost in the process is utilised for atomic excitation.

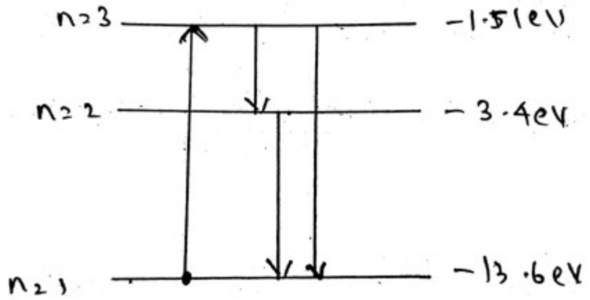
Number of spectral lines emitted

Consider a hydrogen atom in ground state. It absorbs a photon of energy 10.2 eV. It reaches first excited state and returns to ground state by emitting one photon of energy 10.2 eV.



$$\boxed{\text{Number of spectral lines emitted} = \frac{n(n-1)}{2}}$$

If hydrogen atom absorbs a photon of energy 12.1 eV (-1.5 - (-13.6)) it will reach second excited state (n = 3).



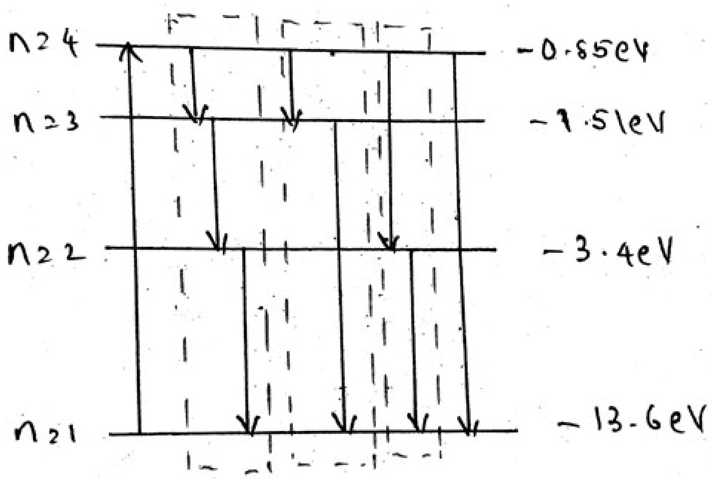
From 3[rd] energy state it will either deexcite as mentioned in process (1) or in process (2). If hydrogen atom is excited using photons of energy 12.1 eV the atom will emit photons energy.

1. 1.9 eV (-1.5 - (-3.4))
2. 10.2 eV (-3.4 - (-13.6))
3. 1.21 eV (-1.5 - (-13.6))

no. of spectral lines emitted $= \frac{n(n-1)}{2}$ where n is the principal quantum number of excited state.

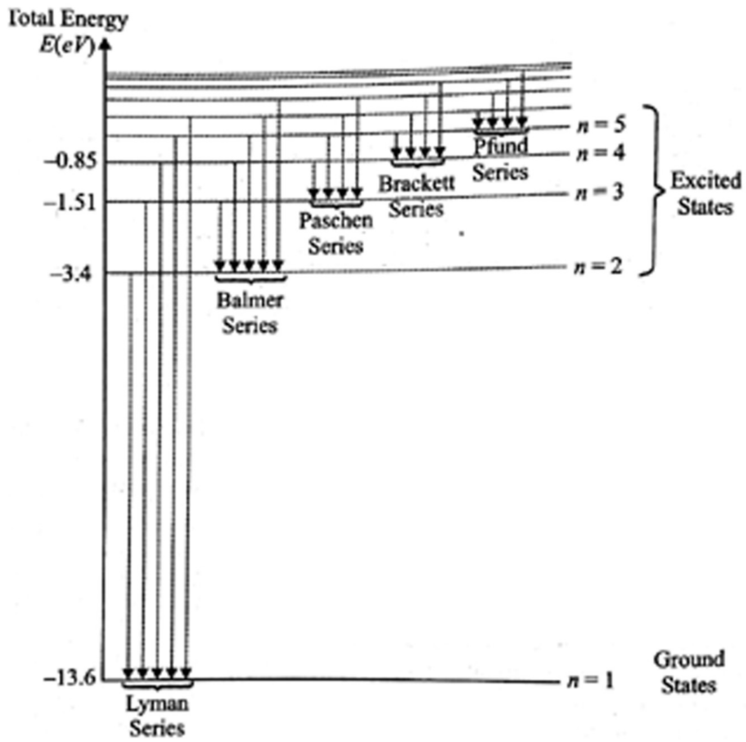
eg: $\frac{3(3-1)}{2} = 3$

another example

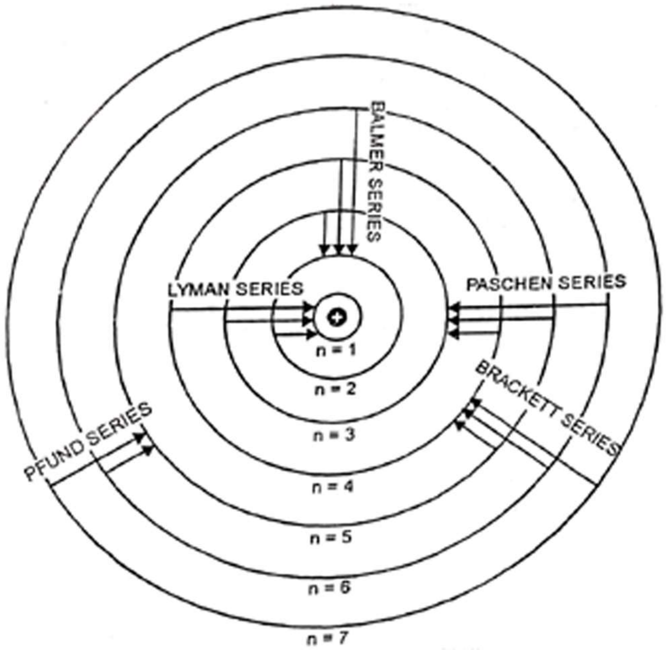


If electron in ground state absorb photons of energy 12.75 eV it will reach with energy level (-0.85 eV - (-13.6) eV = 12.75 eV). From the 4th energy level it can return to ground state in 4 different ways. During different de-excitation we can detect 6 different wavelengths from the emission spectrum. $\frac{4(4-1)}{2}=6$

Spectral Series



Name of series	n_f	n_i	First line (longest λ)	Last line (shortest λ)
Lyman	1	$2,3,...,\infty$	$2 \to 1$	$\infty \to 1$
Balmer	2	$3,4,...,\infty$	$3 \to 2$	$\infty \to 2$
Paschen	3	$4,5,...,\infty$	$4 \to 3$	$\infty \to 3$
Brackett	4	$5,6,...,\infty$	$5 \to 4$	$\infty \to 4$
P fund	5	$6,7,...,\infty$	$6 \to 5$	$\infty \to 5$



If wavelength is large frequency will be small and vice versa.

Frequency of emitted radiation

If electron makes a transition from n_i to n_f during de-excitation the energy of emitted photon is given by

$$E = h\nu = E_i = E_f = -Rhcz^2\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

$$h\nu = Rhcz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \text{ where } n_f < n_i$$

$$\boxed{\nu = Rcz^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$\frac{c}{\lambda} = Rcz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\boxed{\frac{1}{\lambda} = \bar{\nu} = Rz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]}$$ $\bar{\nu}$ or $\frac{1}{\lambda}$ is called wave number.

$$h\nu = Rhcz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{hc}{\lambda} = 13.6\,z^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]eV$$

$$\boxed{\frac{1240\,eV - nm}{\lambda_{(in\,nm)}} = 13.6z^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]eV}$$

Ionization Eenergy

The minimum energy required to remove an electron from ground state to $n = \infty$ is called ionisation energy of atom.

$$E_{ionisation} = E_\infty - E_n = 0 - E_n = 0 - \frac{-13.6\,z^2}{n^2}eV$$

Here n = 1

$$\boxed{E_{ionisation} = 13.6\,z^2 eV}$$

For hydrogen z = 1 ionisation energy = 13.6 eV

For He^+ z = 2 ionisation energy = 54.4 eV

For Li^{2+} z = 3 ionisation energy = 122.4 eV

De Broglie's Explanation of Bohr's 2nd Postulate

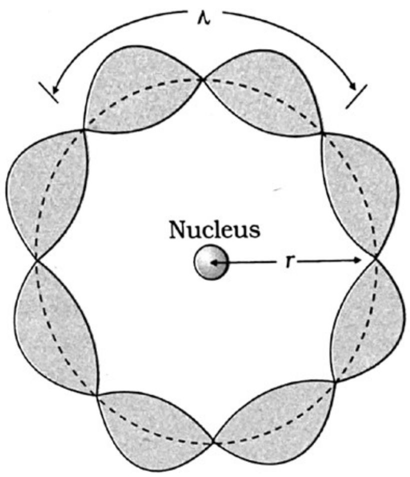
A standing wave is formed on the circular orbit due to the wave nature of electron.

A circular orbit can be taken to be a stationary energy state only if it contain an integral number of de Broglie wavelength. ie., circumference of circular orbit is an integral multiple of de Broglie wavelength.

$$n\lambda = 2\pi r$$

$$\frac{nh}{mv} = 2\pi r$$

$$\therefore \boxed{mvr = \frac{nh}{2\pi}}$$ $\Rightarrow$ Bohr's second postulate.



Standing wave found in n = 4.

Limitations of Bohr Model

1. Bohr Model is limited to hydrogenic atoms. It cannot be even extended to two electron atoms. In two electron atoms electron interact with another electron.
2. Bohr could not explain the relative intensities of frequencies in the spectrum.

NUCLEI

Nucleus consists of neutrons and protons collectively referred to as nucleons

$$\boxed{A = Z + N}$$

$A \rightarrow$ Mass number (Number of nucleons)

$Z \rightarrow$ Atomic number (Number of protons).

$N \rightarrow$ Neutron number (Number of neutrons)

Atomic Mass Unit

1 amu or 1u is defined as $\frac{1}{12}$ th of mass of an atom of carbon 12 isotope.

$$\boxed{1\text{amu} = \frac{1}{12} \times \text{mass of } {}^{12}_{6}\text{C}}$$

Mass of proton = 1836 × mass of electron

$m_p = 1836\ m_e$

mass of neutron = 1837 × mass of electron

$m_e = 1837\ m_e$

Neutron is slightly heavier than proton.

1 amu or 1u = 1.660539 × 10^{-27} kg.

Particle	Electric charge in Coulomb	Mass in amu
Electron	-1.6×10^{-19}C	5.45×10^{-4}
Proton	$+1.6\times10^{-19}$C	1.007
Neutron	0	1.008

Isotopes

Nuclei having same Z but different A are called isotopes.

eg: ${}^{1}_{1}H, {}^{2}_{1}H, {}^{3}_{1}H$ or ${}^{11}_{6}C, {}^{12}_{6}C, {}^{13}_{6}C, {}^{14}_{6}C$

Isobars

Nuclei having same A but different Z are called isobars.

eg: ${}^{3}_{1}H, {}^{3}_{2}He; {}^{14}_{6}C, {}^{14}_{7}N; {}^{16}_{8}O, {}^{16}_{9}F$

Isotones

Nuclei with same number of neutrons but different Z are called isotones.

eg: ${}^{3}_{1}H, {}^{4}_{2}He; {}^{14}_{6}C, {}^{15}_{4}N; {}^{16}_{8}O, {}^{17}_{9}F$

Size of Nuclei

Nucleons combine to form a nucleus as though they were tightly packed sphere. Therefore the shape of nucleus can also be assumed as a sphere.

Volume of a nucleus is found to be proportional to its mass.

$$\frac{4}{3}\pi R^3 \ \alpha\, mA$$

$m \rightarrow$ mass of 1 nucleon $\left(m_p \approx m_n\right)$

$A \rightarrow$ mass number/number of nucleons.

$R^3 \alpha\, A$

$R\, \alpha\, A^{1/3}$

$\boxed{R = R_0\ A^{1/3}}$ $\qquad R_0$ = 1.2 fm

1fm = 1 fermi = 10^{-15}m.

The order of size of atom is $1\mathring{A}$ or 10^{-10}m. The order of size of nucleus is 10^{-15}m or 1 fm.

✶ The ratio of size of atom to size of nucleus will be $= \frac{1\mathring{A}}{1\,fm} = \frac{10^{-10}m}{10^{-15}m} = 10^5$

✶ If a cricket ball is assumed to be nucleus then cricket stadium will be the atom.

Nuclear Density

Density $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{mA}{\frac{4}{3}\pi R^3}$

$m \to$ mass of each nucleon

$A \to$ mass number

$R = R_0 A^{1/3},\ R^3 = R_0^3 A$

Density $= \dfrac{3m\ A}{4\pi R_0^3 A} = \dfrac{3m}{4\pi R_0^3}$

Density of nucleus is independent of mass number. It is same for all nuclei.

$$\boxed{\rho_{\text{nucleus}} = \frac{3m}{4\pi R_0^3}}$$

$$\rho = \frac{3\times1.67\times10^{-27}}{4\times3.14\times\left(1.2\times10^{-15}\right)^3} = 2.3\times10^{17}\,\text{kg}/\text{m}^3$$

$\rho_{\text{nucleus}} = 10^{14}\,\rho_{\text{water}}$

ρ_{nucleus} is comparable to the density of neutron star.

Mass-Energy Equivalence

Mass is another form of energy. We can convert mass energy into other form, say kinetic energy.

Energy and mass are related as $\boxed{E = mc^2}$

Where c is speed of light.

Energy released when 1 kg of mass converted into energy. $E = 1\times\left(3\times10^8\right)^2 = 9\times10^{16}\,\text{J}$

Energy released when 1 amu gets converted into energy.

$E = mc^2 = 1.66\times10^{-27}\times9\times10^{16}\,\text{J}$

$= \dfrac{1.66\times10^{-27}\times9\times10^{+16}}{1.6\times10^{-13}}\,\text{MeV}$ $E = 931.5\,\text{MeV}$.

1 amu c^2 = 931.5 MeV

$$\boxed{1\,\text{amu} = \frac{931.5\,\text{MeV}}{c^2}}$$

Mass defect and Binding energy

The difference in masses of independent nucleons and mass of nucleus is called mass defect.

$m_{nucleus} < m_{independent\ nucleons}$

Mass defect $\boxed{\Delta m = \left[Zm_p + (A-Z)m_n\right] - m_{nucleus}}$

Some amount of mass is converted into energy and released when nucleons bind with each other to form a stable nucleus. Due to this lack of energy, nucleons are bound in nucleus. We can split the nucleus into independent nucleons by supplying this much energy. This energy is called Binding energy.

$\boxed{B.E. = \Delta mc^2}$ $\Delta m \rightarrow$ mass defect

$m_{nucleus} = m_{atom} - m_{electrons}$

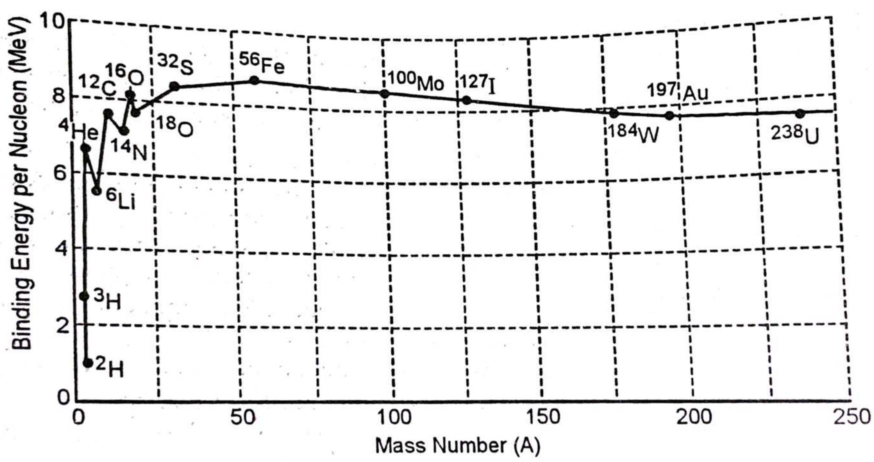
Binding Energy per Nucleon [$(B.E.)_n$]

It is the average energy per nucleon needed to separate the nucleus into its individual nucleons. Average energy to take 1 nucleon from a nucleus.

$\boxed{(B.E.)_n = \frac{B.E.}{A}}$ $A \rightarrow$ number of nucleon, mass number

If a nucleus has more binding energy per nucleon [$(B.E.)_n$], it is more stable.

A graph is plotted comparing $(B.E.)_n$ and A.



- ✶ Almost constant value of $(B.E.)_n$ in the range 30 < A < 170.
- ✶ B.E. of ${}^1_1H$ is zero. Because it contain only one nucleon.
- ✶ For A < 30 and A > 170, $(B.E.)_n$ is less.
- ✶ When a heavy nucleus (A = 240) breaks into 2 almost equal fragments, energy will be released. Some mass of the parent nucleus will be converted into energy. This process is called nuclear fission.
- ✶ When two or more light nuclei fuse together, some mass of nucleon will be lost and energy will be

released. This is called nuclear fusion.

$(B.E.)_{product} > (B.E.)_{reactants}$

✶ Energy released in a nuclear reaction.

$$\boxed{E_{released} = (B.E.)_{products} - (B.E.)_{reactants}}$$

$$\boxed{E_{released} = \left(M_{reactatnts} - M_{products}\right)c^2}$$

B.E. = $(B.E.)_n \times A$

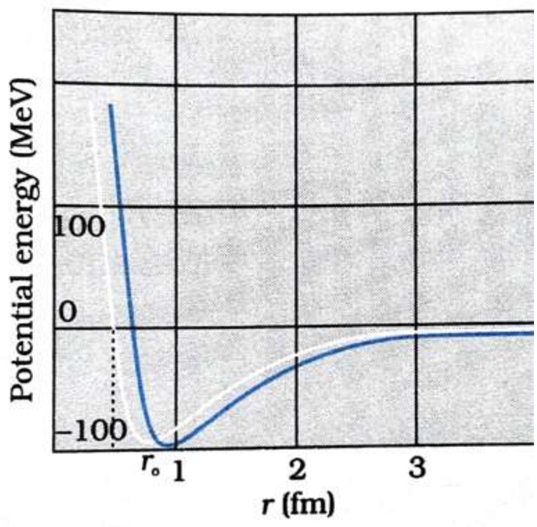
Nuclear Forces

✶ Independent of charge

✶ Proton-proton, neutron-proton, neutron-neutron forces are same.

✶ Strongest force in the nature (100 times of electrostatic force, 10^{38} times of gravitational force)

✶ Short range force act upto 1.5 fm (within nuclear diameter)



✶ The nuclear force is altractive if distance between two nucleons is larger than 0.8 fm and is repulsive if they are separated by distances less than 0.8 fm.

✶ No simple formula like the gravitational force or coulomb force

✶ Non-central force (Not acting along a line)

✶ Doesn't obey inverse square law.

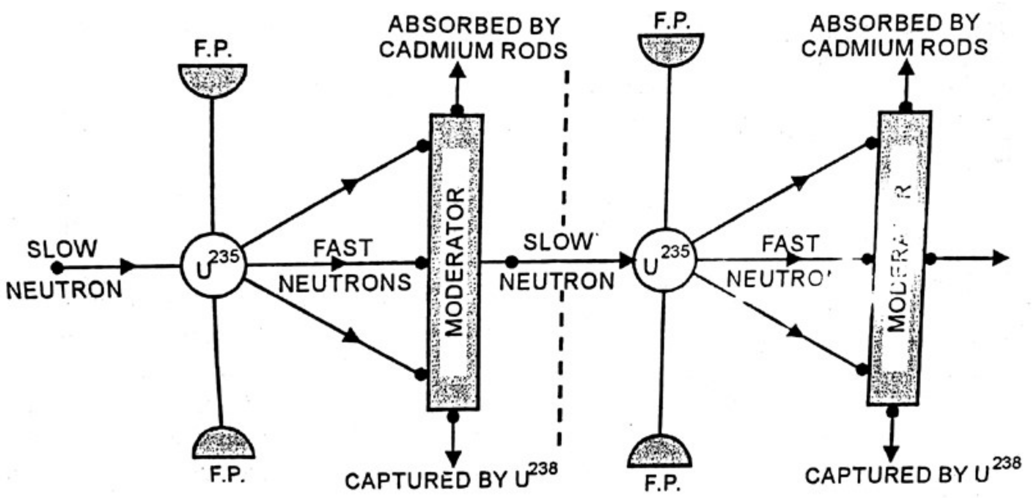
Nuclear Fission

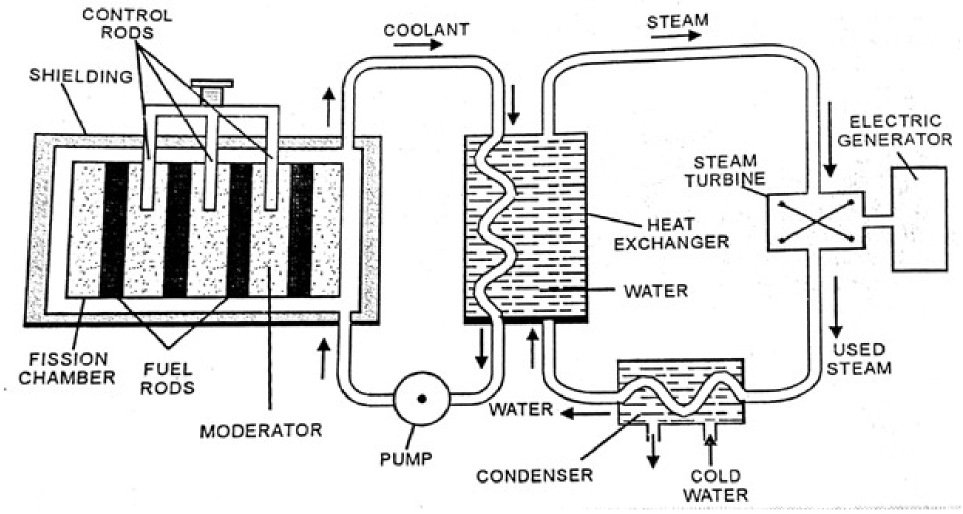
✶ A heavy nucleus is bombarded with proton, neutron or an α – particle. It splits into lighter nuclei with release of energy. The fragment products are radioactive and they emit β -particles in succession to acheive stable and products.

✶ Disintegration energy in fission first appears as the kinetic energy of the fragments and neutrons. Eventually it is transferred to the surrounding matter appearing as heat.

✶ The source of energy in nuclear reactors which produce electricity is nuclear fission.

Nuclear Reactor





Fuel used in nuclear reactor is ^{235}U or ^{239}Pu

Fission reaction of ^{235}U are

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}sr + 2\,{}^{1}_{0}n$$

On an average $2\frac{1}{2}$ neutrons are released per fission of uranium nucleus. There extra neutrons causes another fission reactions, ie., chain reaction. The enormous energy released in an atom bomb comes from uncontrolled chain reaction.

Neutron Multiplication Factor

$$\boxed{K = \frac{\text{number of fission produced by present generation of neutrons}}{\text{number of fission produced by preceding generation of neutrons}}}$$

✶ K is a measure of growth rate of neutron.

1) If $K < 1 \Rightarrow$ subcritical

chain reaction will stop.

2) If $K = 1 \Rightarrow$ critical

controlled chain reaction, which is required for the operation of nuclear reactors.

3) $K > 1 \Rightarrow$ supercritical

uncontrolled chain reaction.

eg: Atom bomb explosion.

Moderator

✶ Chain reaction continue with neutrons having certain energy (thermal neutrons)

✶ Fast neutrons liberated in fission would escape instead of causing another fission reaction.

✶ Moderators are used to slow down the neutrons. eg: water, heavy water (D_2O), graphite.

Control Rods

✶ Reaction rate is controlled through control rods made out of neutron absorbing materials such as B, Cd.

✶ In addition to control rods, safety rods can be inserted into the reactors and K can be reduced rapidly to less than unity.

✶ The core is surrounded by reflectors to reduce leakage.

Coolants

✶ Heat energy released in fission is completely removed by a suitable coolant. eg: water.

Nuclear fusion

When two or more light nuclei $(A \leq 8)$ fuse together, energy will be released. When fusion is acheived by raising the temperature of the system so that particles have enough kinetic energy to overcome the Coulomb repulsive behaviour, it is called thermonuclear fusion. It is the source of energy in the interior or stars. The interior of sun has a temperature about 1.5×10^7K. The fusion reaction in the sun is a multistep process in which the Hydrogen is burnt into Helium.

Proton-Proton Cycle

$${}_{1}^{1}H + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + {}_{+1}^{o}e + \upsilon + 0.42\,MeV \qquad ...(1)$$

$${}_{+1}^{0}e + {}_{-1}^{0}e \rightarrow 2\gamma + 1.02\,MeV \qquad ....(2)$$

$${}_{1}^{2}H + {}_{1}^{1}H \rightarrow {}_{2}^{3}He + \gamma + 5.49\,MeV \qquad ....(3)$$

✶ When first 3 reactions take place 2 times the following reaction will happen.

$${}_{2}^{3}He + {}_{2}^{3}He \rightarrow {}_{2}^{4}He + {}_{1}^{1}H + {}_{1}^{1}H + 12.86\,MeV \qquad ...(4)$$

✶ Four Hydrogen atom combine to form a Helium atom with release of 26.7 MeV.

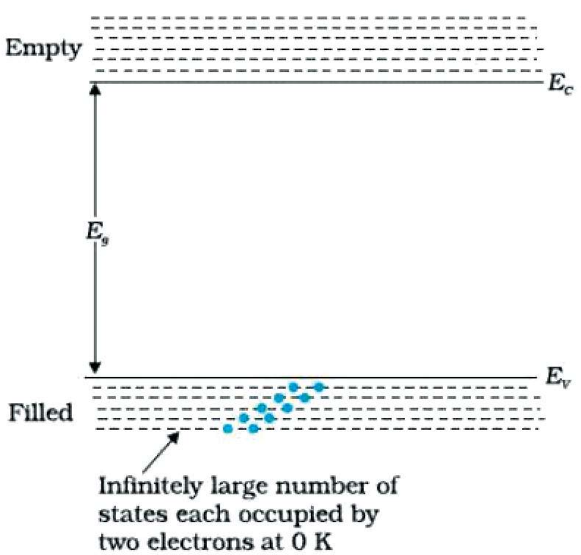
$$4{}_{1}^{1}H + 2{}_{-1}^{0}e \rightarrow {}_{2}^{4}He + 2\nu + 6\gamma + 26.7\,MeV$$

SEMICONDUCTOR ELECTRONICS

Electronics: It is a branch of physics which deals with the study of flow of electrons through a crystal lattice, specifically defined crystal lattices called semiconductors.

Band Theory of solids

- In an isolated Atom, energy level corresponding to its discrete orbital energy can be found.
- When atoms are brought together so closely to form a solid, their interatomic space reduces, their valance orbits may even overlap, hence, to avoid such situations, atoms re-align their energy levels among them self.
- While re-aligning the energy levels, each atom occupies a unique energy level for its valence orbit.
- These closely spaced continuous energy levels may feel as a band of energy.
- The range of energy possessed by valence electrons in the crystal lattice is called valence band.
- The range of energy possessed by free electrons or conduction electrons is called conduction band.
- The energy difference between valence band and conduction band is called forbidden energy band gap



E_c → Lowest energy level of conduction band

E_v → Maximum energy level of valence band

E_g → Energy band gap between valence band and conduction band

- Electrons in valence band are bounded, hence they cannot contribute current flow.
- Electrons in conduction band are free hence support current flow.

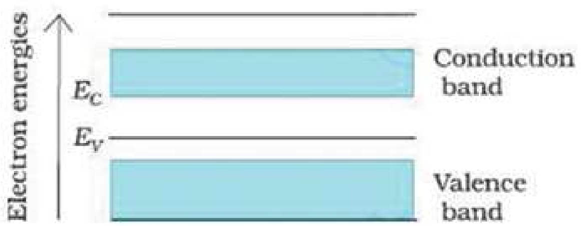
Classification of Materials

- According to Band Theory of Solids, materials are classified into three.
    - Conductor
    - Semiconductor
    - Insulator

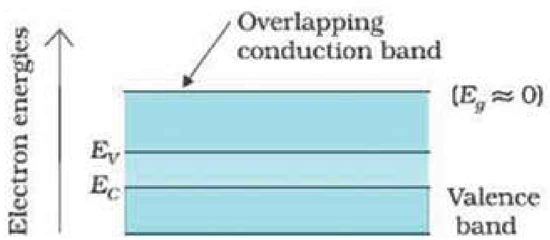
Conductor

According to band theory, a material act as conductor, if it satisfies any of the following two conditions.

- Condition 1 : The valence band and conduction band of the material should overlap each other.
- Condition 2 : If a material has a partially filled conduction band or a partially empty valence band.



Condition 2



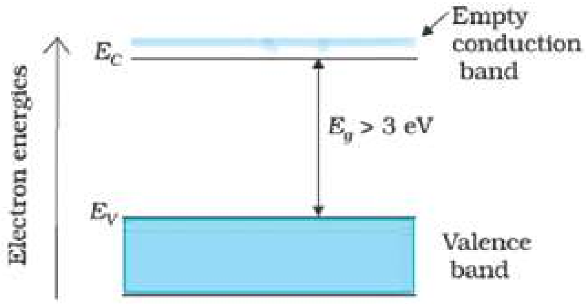
Condition 1

Semiconductor

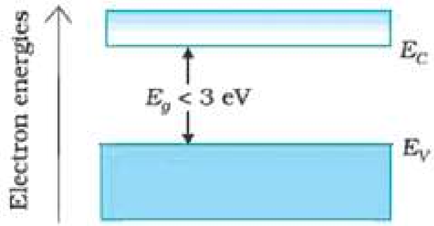
- According to Band Theory, a material act as semiconductor, if the energy difference between the valence band and conduction band of the material is less than 3 eV.

Insulator

- According to Band Theory, a material act as insulator, if the energy difference between valence band and conduction band of the material is greater than 3 eV.



Insulator



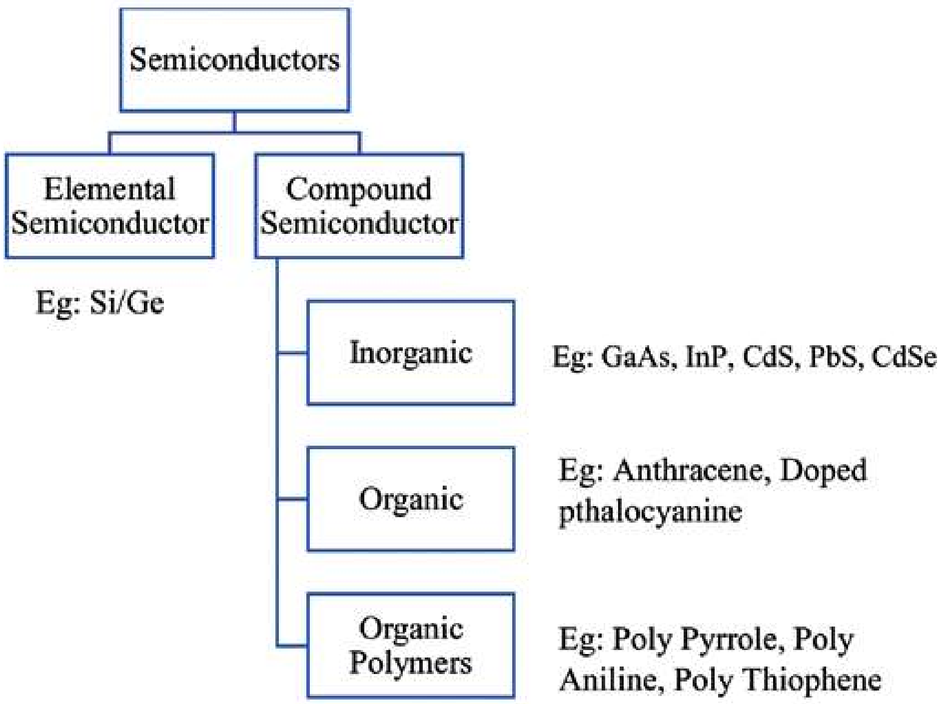
Semiconductor

Semiconductors

Classification of Semiconductors

- Based on chemical composition
- Based on purity

Based on chemical composition

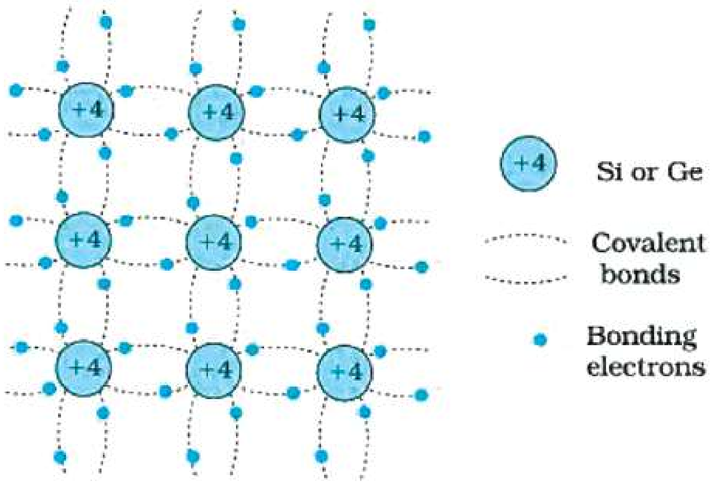


Classification based on purity

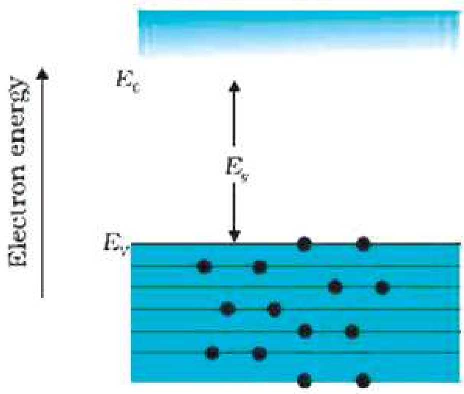
- Based on purity, semiconductors are classified into intrinsic semiconductors (pure) and extrinsic semiconductor (impure).

Intrinsic Semiconductor

- A Semiconductor in its pure form is called Intrinsic Semiconductor.
- In the crystal lattice, Silicon atoms follows, the bonding structure of carbon similar to that in diamond.
- That is, Silicon atoms form covalent bond with adjacent four Silicon atoms and attain chemical and mechanical stability.
- At 0 K (no energy state), all the electrons are in covalent bond, that is, tightly bonded to the parent atoms. Hence, there is no free electron found in the crystal.
- Since there is no free electron found in the crystal, conduction band of the material will be empty.
- If no free electrons are available in the material, such kind of materials will act as insulators.
- That is, intrinsic semiconductor at 0 K act as Insulator.



Crystalline Structure of Intrinsic Semiconductor at 0K



Energy Band Diagram of Intrinsic Semiconductor at 0K

Behaviour of Intrinsic Semiconductor at room temperature

- At 300 K, some of the electrons in covalent bond, absorb thermal energy from room temperature, break the covalent bond and set itself free.
- These free electrons are placed in the inter-atomic spaces and can be moved in these spaces by applying external energy to it (electric field/potential).
- The site from where, electron set freed has a deficiency of electron to complete that covalent bond. It is called as a hole. Hole is considered as '+q' charged particle.
- Electron deficiency in covalent bond alone is called as hole, electron deficiency in any bond other than covalent bond will not be considered as hole.
- Electrons in covalent bond near to the hole site, may have a tendency to break its covalent bond and occupy the hole site. This electron from adjacent site may move to the hole site, that is, we may consider it as, a hole has moved from its own site to the adjacent site.
- Hole moment is actually nothing but, a convenient way of describing bounded electron movement in the crystal lattice.
- Hole movement means, a positively charged particle is moving in the crystal, which can constitute a current in the semiconductor.
- The major difference between a conductor and semiconductor is that, conductor is only having one type of charge carriers that is electrons but the semiconductor is having 2 types of charge carriers in it, electron and other is hole.
- In other words, we can say that, semiconductor support current through 2 types of charge carriers. Through the movement of free electrons and the movement of holes (through the movement of electron which is bounded).
- The mobility (ability to move) of free electron movement will always be greater than the mobility of bounded electron movement, that is, hole mobility will be always less than electron mobility ($\mu_e > \mu_h$).
- The process of generation of electron and hole by breaking covalent bond is called as electron hole pair generation (EHP -Generation/Thermal generation).
- The recombination of electron and hole is called electron hole pair recombination.
- Total current of the semiconductor is the currents due to the free electron movement and the current due to the hole movement.

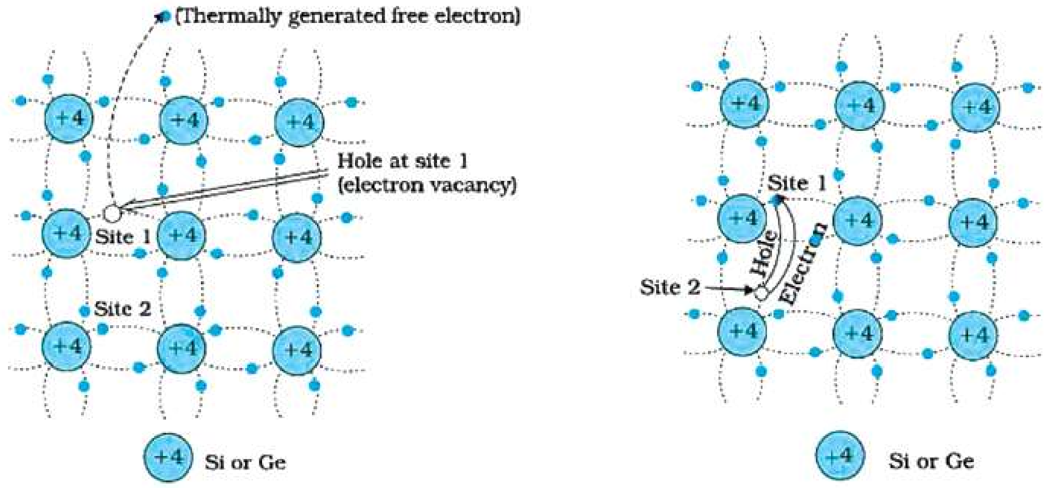
$$I_{sc} = I_e + I_h$$

- The number of free electrons in a semiconductor (n_e) is equal to number of holes (n_h).

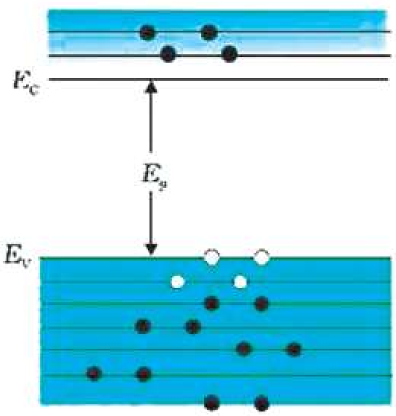
$$n_e = n_h = n_i$$

where n_i is called intrinsic concentration. It is a constant of a material at a particular temperature.

- At equilibrium, rate of electron hole pair generation will be equal to rate of electron hole pair recombination.
- Hence, the net charge carriers available for conduction will be very low (ideally zero).
- Conductivity of an intrinsic semiconductor can be increased by increasing the temperature of the crystal lattice.
- Due to the about 2 reasons intrinsic semiconductors are not used practically.



Crystalline Structure of Intrinsic Semiconductor at 300K



Energy Band Diagram of Intrinsic Semiconductor at 300K

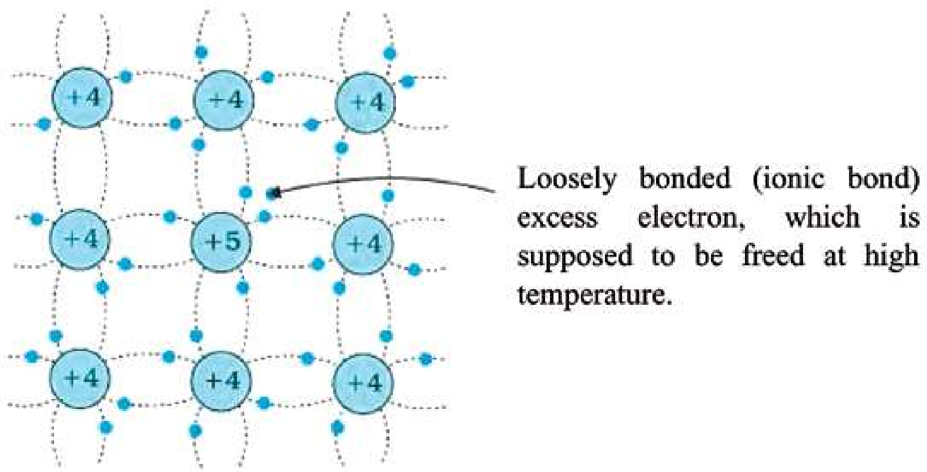
Extrinsic semiconductor

- Semiconductor with added impurity is called extrinsic semiconductor.
- The process of deliberate addition of impurities to an intrinsic semiconductor to improve its conductivity is called doping.

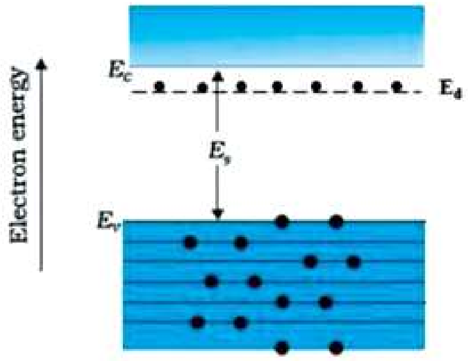
- Externally added impurities are called dopant.
- While choosing the dopant, one should keep in mind that, size of the impurity atom and the size of the semiconductor atom should be same or nearly same. or in other words, we can say like this, doping should not alter stability of the crystalline structure.
- Depending on the type of dopants added extrinsic semiconductor are of 2 types.
    - N - type semiconductor
    - P - type semiconductor

N - type Semiconductor

- N - type semiconductor is formed by doping pentavalent impurities to intrinsic semiconductor (Nitrogen, Phosphorus, Arsenic, Antimony and Bismuth).
- In the crystal lattice, impurity atoms replace some of the semiconductor atoms.
- Among the 5 valence electrons of the pentavalent impurity atom, 4 electrons will form the covalent bond with adjacent 4 Silicon atoms.
- There will be one excess electron available with the pentavalent impurity atom, which can be easily donated to the crystal, and the impurity atom can attain chemical and mechanical stability.
- Since the pentavalent atoms have a tendency to donate an electron to the crystal, they are called as donor impurities.
- Energy possessed by these excess electrons are represented with donor energy level E_D, which is just below the conduction band energy level E_c.
- At 0K, all the electrons are in covalent bond and the excess electron will be loosely bonded to the parent impurity atom.
- Hence, at 0K, there is no free electron found in the crystal, which means N - type semiconductor at 0K act as insulator.



Crystalline Structure of N – Type Semiconductor at 0K



Energy Band Diagram of N – Type Semiconductor at 0K

Behaviour of N – type Semiconductor at Room Temperature

- At 300 K, excess electrons can be ionised easily, along with this process, electron hole pair generation and recombination similar to that an intrinsic semiconductor takes place here also.
- Number of free electrons in N - type semiconductor is contributed through 2 reasons.
  - Electron hole pair generation (thermal generation).
  - Donor given.

  $n_e = n_{e\text{-}ehp} + N_D$

- Holes in N - type semiconductor is contributed only through electron hole pair generation (thermal generation).

  $n_h = n_{h\text{-}ehp}$

- There are only two types of charge carriers in a semiconductor, by comparing their count, number of free electrons is very much greater than number of holes.

  $n_e >> n_h$

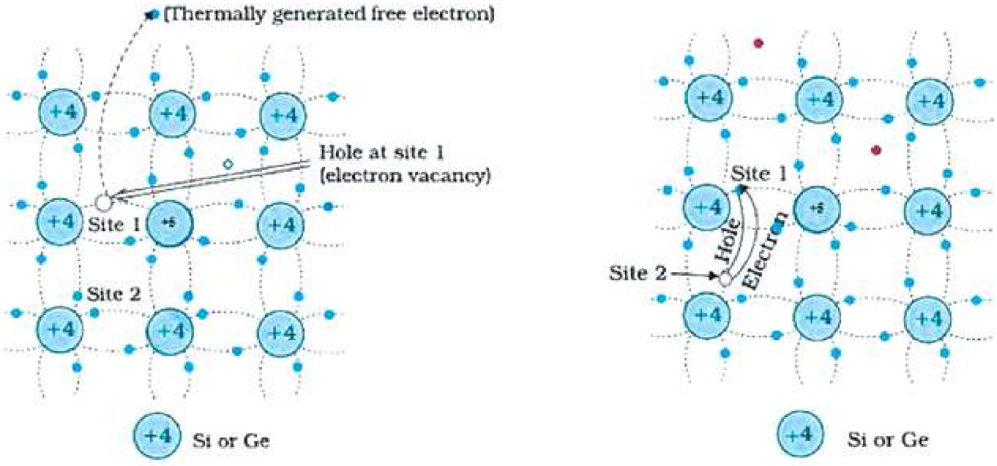
- Hence electrons are called majority charge carriers and holes are called minority charge carriers.
- Total current in N - type semiconductor $I_{SC} = I_e + I_h$.
- Since number of electrons is very much greater than number of holes, electron current will be very much greater than hole current.

  $I_e >> I_h$

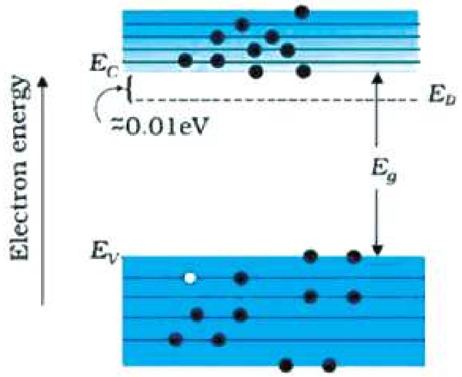
- Electron current is called as majority charge carrier current and hole current is called as minority charge carrier current.
- Total current will be approximately equal to electron current.

  $I_{SC} = I_e + I_h \Rightarrow I_{SC} \approx I_e$

- N - type semiconductors are electrically neutral.
- Conductivity of N - type semiconductor depends on doping concentration.



Crystalline Structure of N – Type Semiconductor at 300K

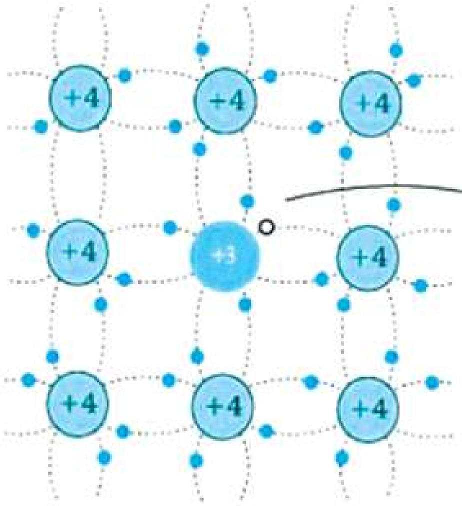


Energy Band Diagram of N – Type Semiconductor at 300K

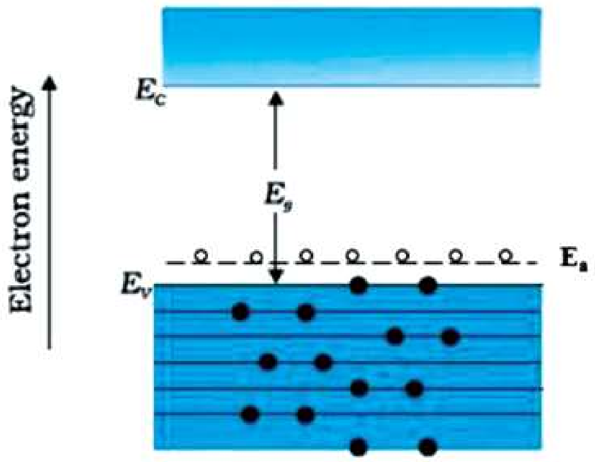
P - type Semiconductor

- P - type semiconductor is formed by dropping trivalent impurities to intrinsic semiconductor (Boron, Aluminium, Gallium, Indium, Thallium).
- All the three valence electrons of trivalent impurity atoms form covalent bond with adjacent three Silicon atoms.
- There is a shortage of an electron to complete its 4th covalent bond, it is considered as hole.
- Since trivalent impurities has a tendency to accept an electron to the crystal, they are called as acceptor impurities.
- Energy possessed by acceptor impurities are represented with acceptor energy level E_A.
- It is represented just about the valence band energy level E_V.

- At 0K, all the electrons are in covalent bond and the excess hole is immobile.
- Hence, P - type semiconductor at zero Kelvin act as insulator.



Crystalline Structure of P – Type Semiconductor at 0K



Energy Band Diagram of P – Type Semiconductor at 0K

Behaviour of P – type Semiconductor at Room Temperature

- At 300 K, excess holes move easily, along with this process, electron hole pair generation and recombination take place similar to that in an intrinsic semiconductor.
- Number of holes n_h in P - type semiconductor is contributed through 2 reasons.
  - Electron hole pair generation (thermal generation).
  - Acceptor given.

$n_h = n_{h\text{-}ehp} + N_A$

- Number of electrons in P-type semiconductor is only due to electron hole pair generation (thermal generation).
- There are only 2 types of charge carriers in semiconductor, by comparing their count, number of holes is very much greater than number of electrons.

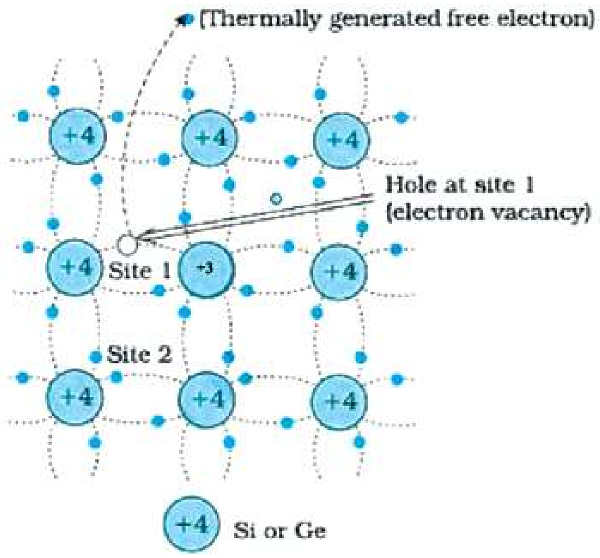
$$n_h >> n_e$$

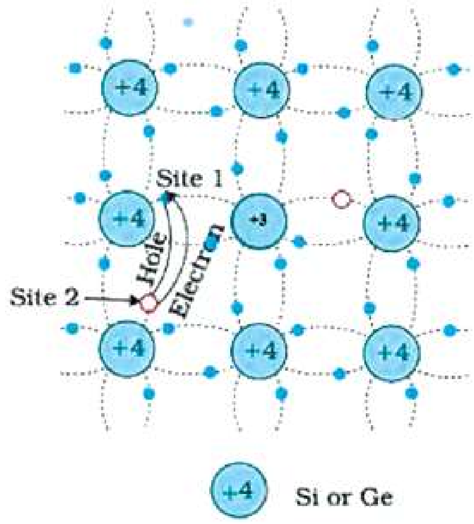
- Hence holes are called majority charge carriers in P-type semiconductor and electrons are called minority charge carriers.
- Total current in the semiconductor is the sum of electron current and hole current.

$$I_{SC} = I_e + I_h$$

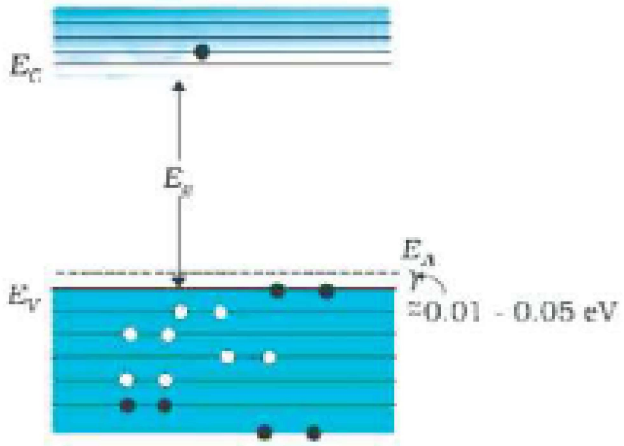
$$I_{SC} \approx I_h$$

- Hole current is called majority charge carrier current. Electron current is called minority charge carrier current.
- Conductivity of P - type semiconductor depends on doping concentration.
- P - type semiconductor electrically neutral.





Crystalline Structure of P – Type Semiconductor at 300K



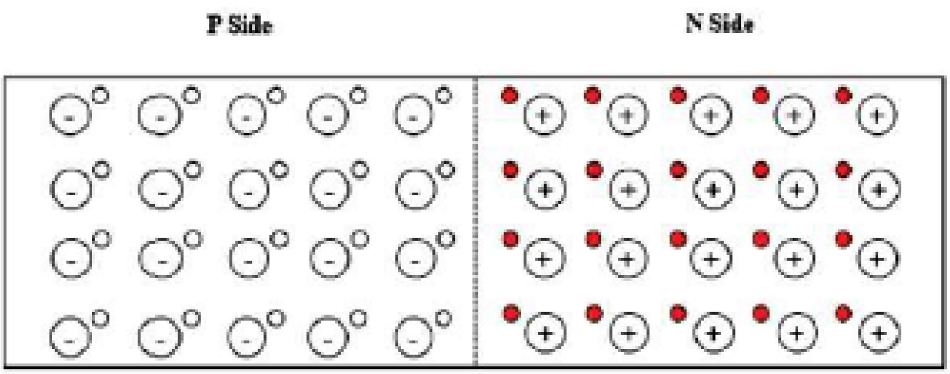
Energy Band Diagram of P – Type Semiconductor at 300K

Law of Mass action $\longrightarrow n_i^2 = n_e \times n_h \rightarrow (7)$

Important Points to Remember

- All semiconductors are insulators at 0 K.
- Extrinsic semiconductors act as intrinsic semiconductors at very high temperature.
- All semiconductors are electrically neutral.
- Law of mass action is valid only at a particular temperature and it will be valid for another set of values in new temperature.

PN Junction



- PN Junction is formed by dropping one side of a P - wafer with excessive pentavalent impurity atom.
- Hence one side of the wafer becomes P - type and other side becomes N – type.
- That is, 2 different regions and a metallurgical region separating 2 regions has been formed on a crystal wafer.
- There are 2 distinct mechanisms involved in the formation of a PN junction.
    - Diffusion
    - Drift
- Diffusion : Movement of charge carriers to neutralise their concentration gradient is called diffusion. Movement due to the difference in concentration. ($\frac{dp}{dx} = 0$) .
- Drift : Movement of charge carrier under the influence of electric field is called drift.

Effect of Majority Charge Carriers on Formation

- Just after the formation of a PN junction, due to the concentration difference on either side of the junction, charge carrier movement will be initiated. That is, diffusion process may take place.
- Means, just after the formation of PN junction, holes from P - side will move to N – side.
- During this moment, holes leave a negative ion near the junction (site from where they moved).
- Since the concentration difference is very large, a large number of holes from P - side will move to N - side and a layer of negative ion will be formed on P - side near the junction.
- Hole movement from P - side to N - side will constitute a hole diffusion current ($I_{h\text{-diffusion}}$) from P - Side to N – side. (in the same direction as flow of hole).
- Similarly, due to the concentration difference, electrons from N - side will move to P – side. during this motion, electrons leave a layer of positive charge on N - side near the junction.
- Electron movement from N- side to P - side will constitute an electron current ($I_{e\text{-diffusion}}$) from P - side to n – side. (electron current will flow in the opposite direction of flow of electrons).
- That is, a net diffusion current ($I_{diffusion}$) will flow from P - Side to N – side.
- Now at the junction, on either side, there is an accumulation of immobile ions, which are acting as a barrier to the flow of charge carriers. Which is called as a barrier or space charge region, or this region is depleted of charge carriers (not available of charge carriers) hence it is called depletion region.

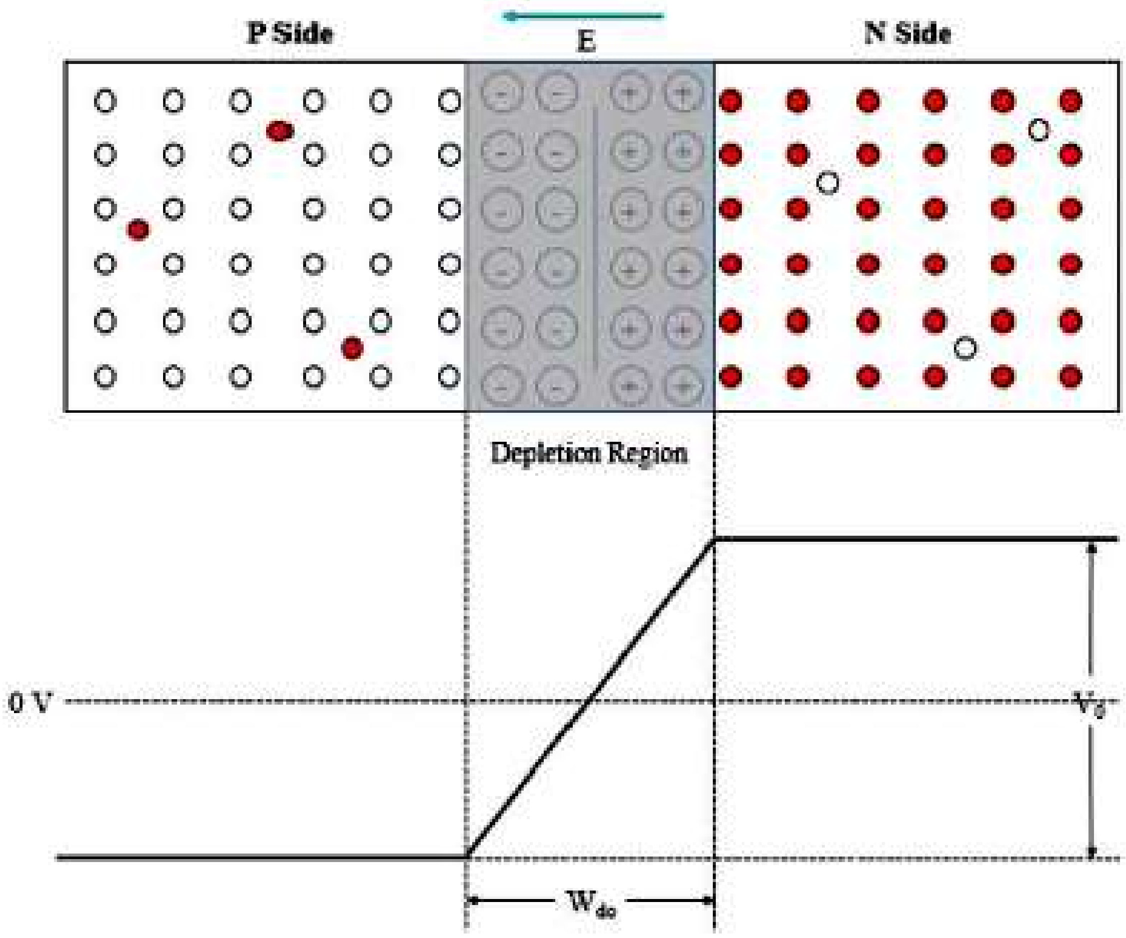
- Due to the formation of depletion region, majority charge carriers on either side require much more energy than the previous time to jump this newly created barrier. That is, the number of charge carriers with sufficient energy to jump the barrier decreases, which means diffusion decreases.
- Though charge carriers jump the barrier, they are lesser in number than the last time. Since this process continuous width of the depletion region increases.
- As the depletion region widens, energy required for the charge carriers to jump the widened depletion region becomes high. That is, number of charge carriers with available energies to jump this required high energy will be lesser than last time, which means diffusion further decreases. In short, as the time progress after formation of depletion region, diffusion decreases slowly.

Effect of Minority Carriers on Formation

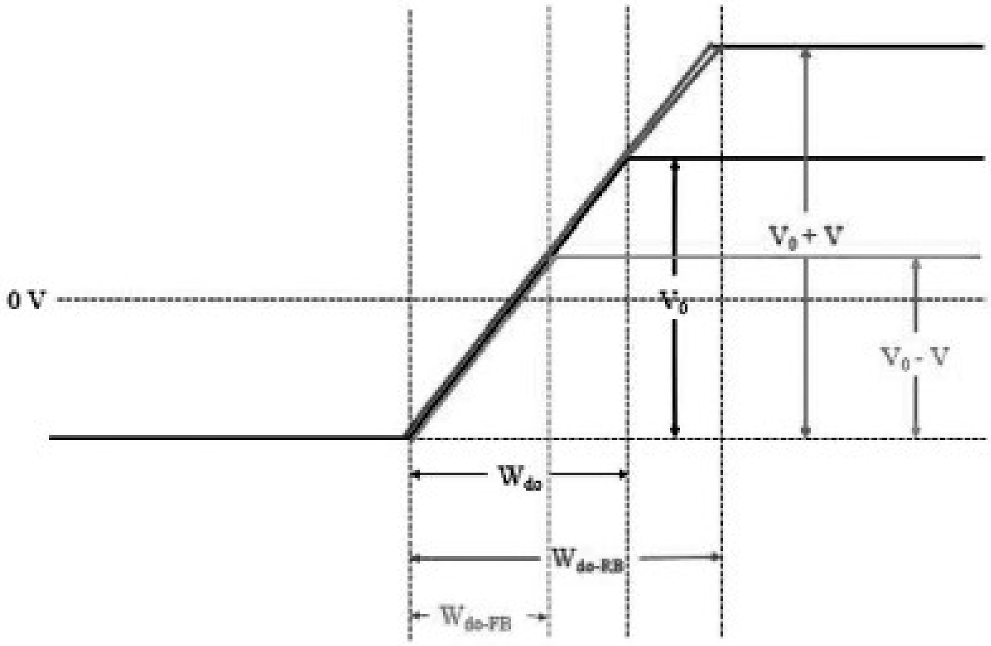
- Due to the formation of depletion region, an electric field is established at the depletion region, which is directed from N - side to P – side.
- Under the influence of this electric field at the junction, minority carriers on either side has a tendency to undergo drift process.
- Electrons from P - side will drift to positive side of the field (that is N – side). Due to this process, an electron drift current ($I_{e\text{-drift}}$) will flow from N – Side to P – Side.
- Similarly holes from N - side drift to negative side of the field (that is P – side). Due to which, a hole drift current ($I_{h\text{-drift}}$) will flow from N – side to P – side.
- Hence an effective drift current (I_{drift}) is constituted from N - side to P – side.
- Initially (just after the formation of PN junction) there was no depletion region, that is, no electric field (that is, no drift current), as the time progresses, diffusion increases, which means depletion region get widens, electric field strength at the junction area increases and drift current increases slowly.
- Effective drift (I_{drift}) current is flowing from N – side to P – side, which means, in the opposite direction of effective diffusion current ($I_{diffusion}$).
- At a particular instant of time after the formation of PN junction, diffusion current and drift current will become equal in magnitude and opposite in direction. This condition is called equilibrium condition.
- Under equilibrium condition, net current flowing through the junction is zero, which means rate of majority carrier diffusion is equal to rate of minority carrier drift.

Potential Curve of a Diode

- During the majority carrier diffusion process, a large number of holes will be accumulated on N-side and a large number of electrons will be accumulated on P - side. Hence, there exists a potential difference between N - side and P - side of the diode. It is called as internal contact potential or barrier potential or built-in potential or barrier height.
- N - side of the diode will be at higher potential than P – side.
- As the barrier height increases, width of the depletion region increases and vice versa
- Barrier height depends on doping concentration on either side, temperature and nature of the material.
- Slope of this curve cannot be altered once it is formed.



Formation of Internal contact potential of a PN Junction Diode



Relation between barrier height and width of depletion region

Biasing of PN Junction

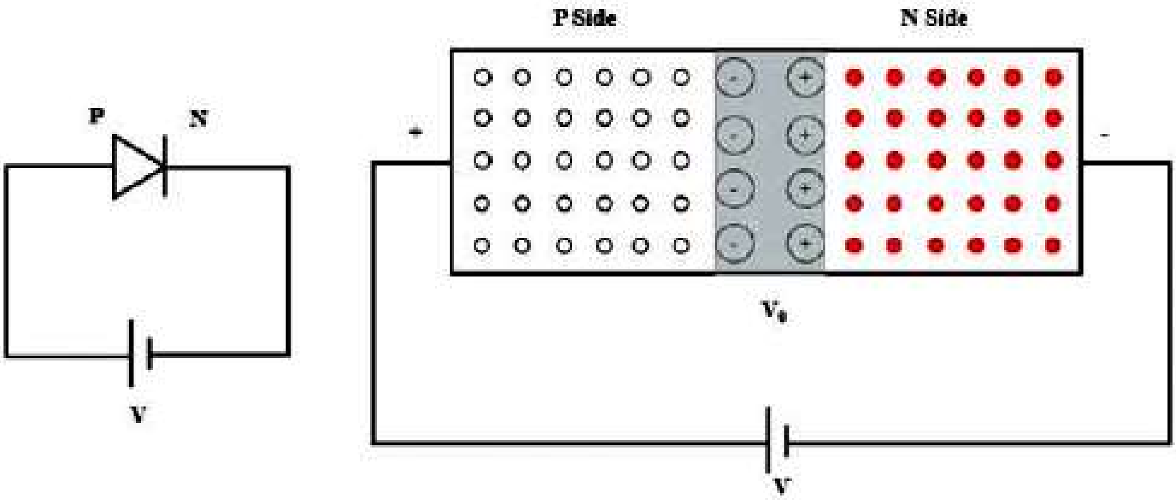
- The process of giving supply to an electronic device is called biassing.
- Depending on the connection given to its terminals, there are 2 possible biasing for a diode.
  - Forward Bias (FB)
  - Reverse Bias (RB)

Diode under Forward Bias Condition

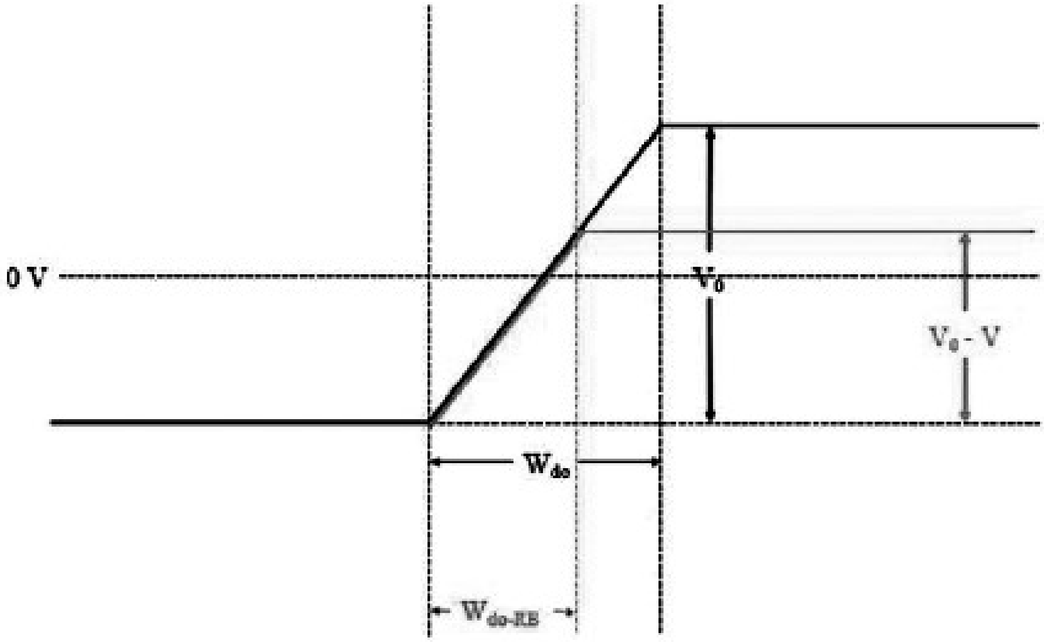
- If positive potential of a battery is connected to P - side and negative potential is connected to N – side, then such biasing is called Forward Bias.
- Or it can be stated in other words, if higher potential is connected to P - side and lower potential is connected to N – side, then such biasing is called Forward Bias.
- Under forward bias condition, externally supplied voltage (V) and internal contact potential(V_0) are in opposite direction.
- Hence, effective barrier height decreases to $| V_0 - V |$, which means width of the depletion region (W_{d0}) decreases.
- Due to the decreased barrier height or decreased width of depletion region, diffusion process initiates easily.

- That is, majority carriers (Holes) from P side diffuses to N - side and becomes minority carrier of that newly injected side (N – side). Similarly majority carriers of N - side (electrons) diffuses from N - side to P - side and becomes minority carrier of newly injected side (P – side), this process is called minority carrier injection (The majority carrier of one side is forced to inject to the opposite side and made as a minority carrier of newly injected side is called minority carrier injection).
- Due to minority carrier injection, concentration of injected minority carriers on either side of the junction increases, hence, they diffuse to far end.
- Hole movement from P side to the far end of N - side will constitute a Hole diffusion current from P side to N – side, similarly electron diffusion from N – side to the far end of P – side constitutes electron diffusion current from P Side to N Side, total diffusion current is constituted, which is of order of milliamperes and flow from P end to N end.
- If, externally supplied voltage(V) is increased further, barrier height keeps on decreasing, and as a consequence, diffusion current increases.
- If, externally supplied voltage is equal to internal contact potential, the effective barrier height will become zero, that is, there is no potential difference or barrier at the junction, it is similar to the condition of a conductor. This external potential is called as knee voltage or threshold voltage or cut-in voltage.

- If, external voltage increases beyond the knee voltage, current increases linearly with the applied forward voltage.
- Forward resistance of a diode (inverse slop of the curve in linear region) is very small, hence, diode under forward bias is considered as a closed switch or on switch.



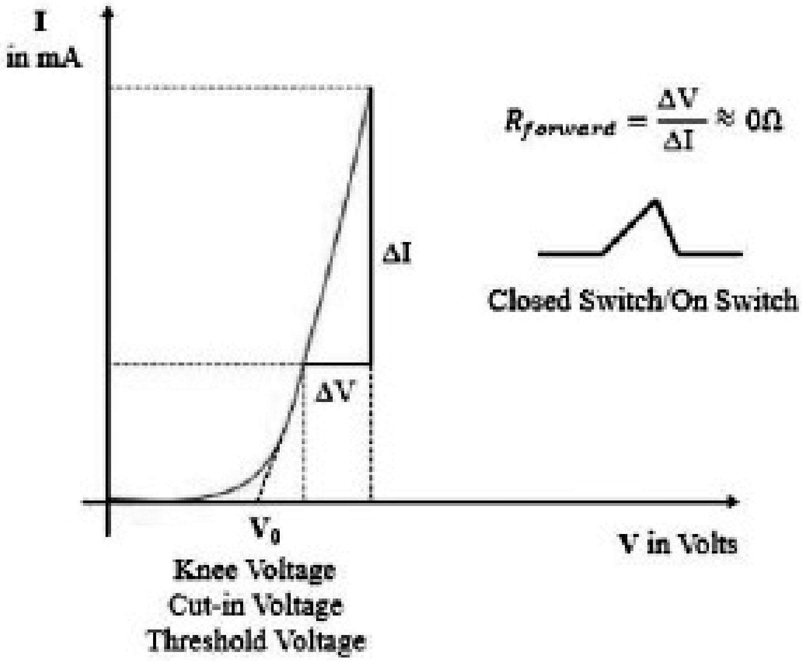
Simplified Forward Biased Diode and its representation with Internal Carriers



Potential Curve for forward Bias.

W_{d0} : Width of depletion region under equilibrium condition

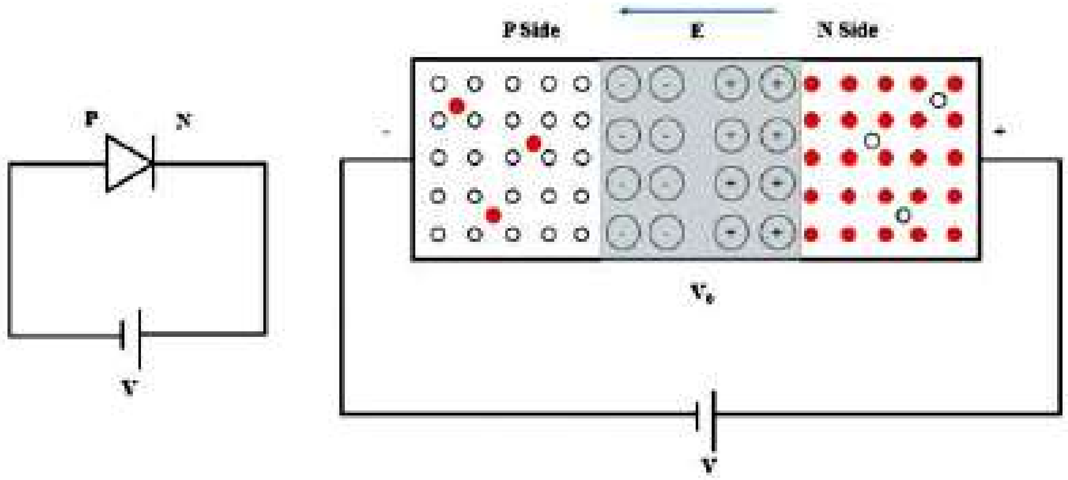
W_{d0-FB} : Width of depletion region under Forward Bias Condition



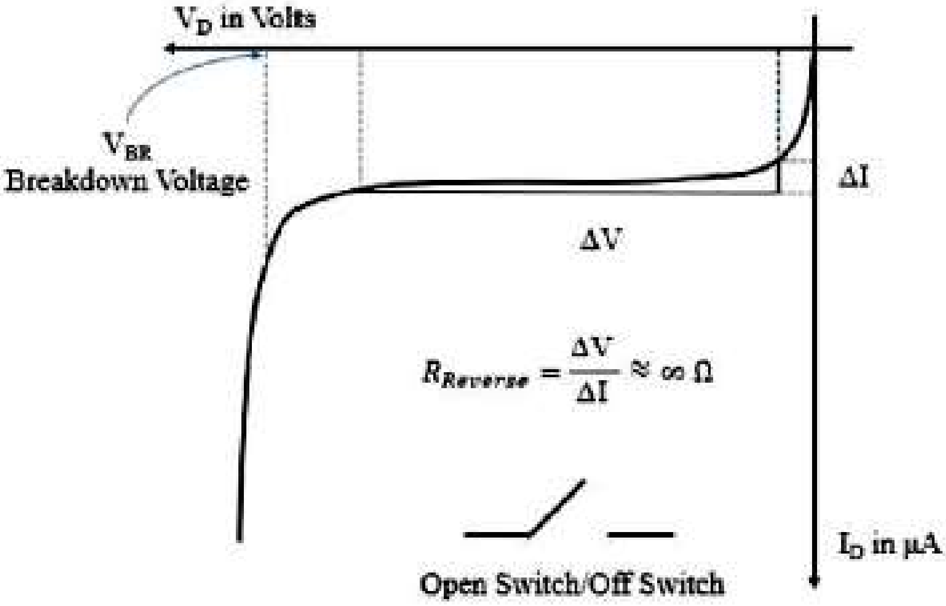
Forward Characteristics of a Diode

Diode under Reverse Bias condition

- If, positive potential of the battery is connected to N - side and negative potential is connected to P – side, then such biasing is called Reverse Bias.
- Or it can be stated as, if higher potential is connected to N - side and lower potential is connected to P - side then such biasing is called reverse bias.
- Under Reverse Bias condition, externally supplied voltage(V) and internal contact potential(V_0) are in same direction. That is, they support each other.
- Due to this, effective barrier height increases to $|V_0 + V|$.
- Since barrier height increases, width of the depletion region (W_{d0}) also increases, which means, the diffusion process comes to a halt.
- Due to the accumulation of immobile ions on either side of the junction (widening of depletion region) electric field strength at the junction increases, due to which minority carrier drift process takes place.
- That is, Holes from N - side drift to negative side of the field (P – side).
- Similarly, electrons from P - side drift to positive side of the field (N – side).
- An effective minority carrier drift current will flow from N - Side to P – side.
- It is of order of micro amperes.
- As applied reverse bias voltage increases, barrier height increases, width of the depletion region increases, electric field strength increases, as a consequence, minority carrier drift process increases, that is, drift current increases slowly and becomes constant at a particular voltage. Even if the reverse voltage increases further drift current remains constant. It is due to the non-availability of minority carriers to take part in further drift process.
- At a large reverse voltage, a wider depletion region causes a high electric field at the junction, due to which, an uncontrollable current flow results in the diode. This process is called Junction breakdown. This voltage is called breakdown voltage.
- There are 2 possible breakdown mechanisms in a diode.
    - Avalanche Breakdown
    - Zener Breakdown
- Reverse resistance of a diode is very high (inverse Slope), hence it is considered as an open circuit or off switch under reverse bias condition.



Simplified Circuit diagram of Diode under Reverse Bias and Diagram showing Internal Carrier



Reverse Characteristics of a diode.

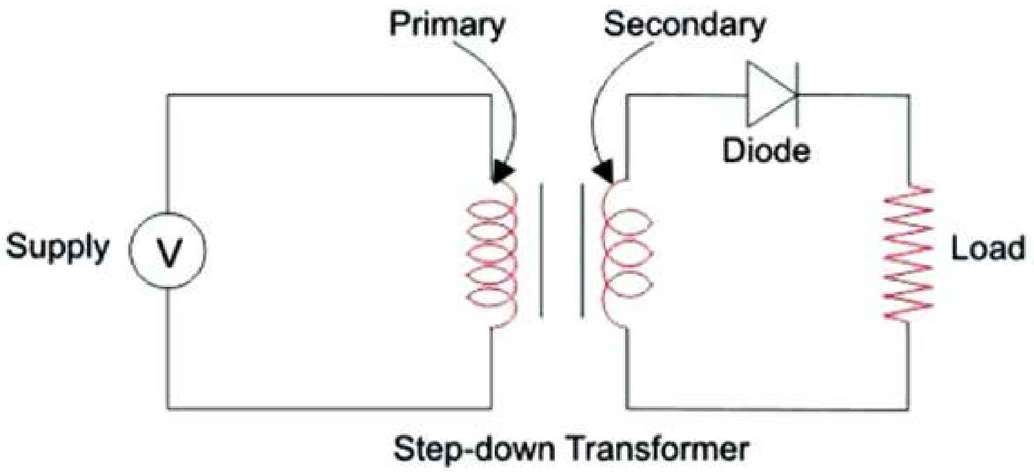
Points to Remember

Particulars	Forward Bias	Reverse Bias
Carriers involved	Majority carriers	Minority carriers
Current flow mechanism	Majority carrier diffusion process	Minority carrier drift process
Direction of current flow	From P - side to N - side	From N - side to P - side
Order of current	Milliampere (mA)	Microampere (μA)
Effective equivalence	ON switch or closed switch	OFF switch or open switch

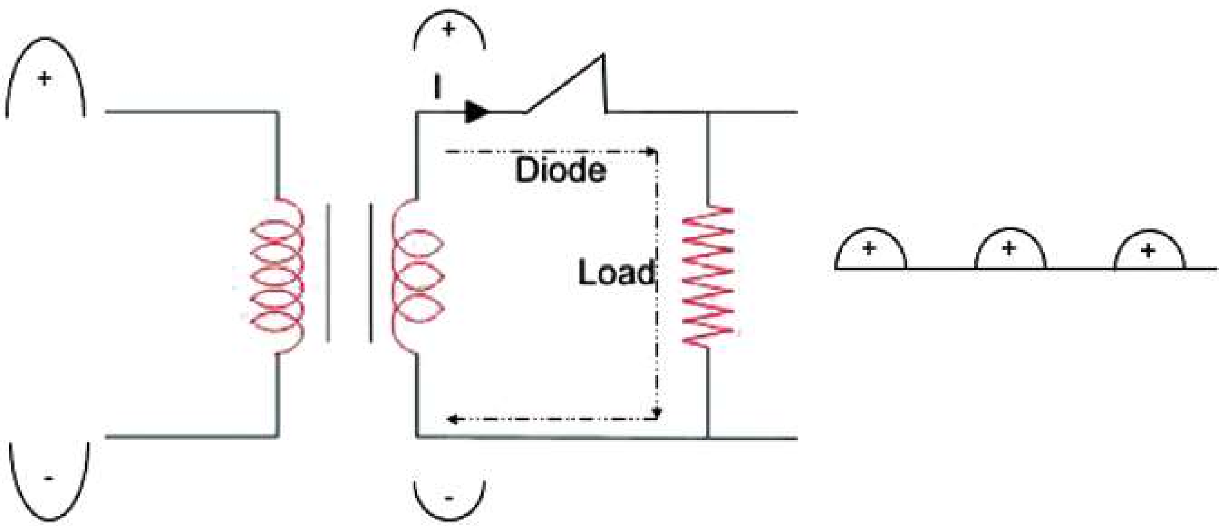
Applications of a Diode

- Diodes are unidirectional devices, that is, they allow current flow only in one direction, diodes are the electrical analogy of mechanical valves.
- Unidirectional conduction nature of diode makes it so useful in the conversion of alternating current to direct current.
- The circuits which are used to convert AC to DC is called rectifiers.
- Depending on the number of half cycles processed by the rectifier circuits, they are classified into half wave rectifier and full wave rectifier.

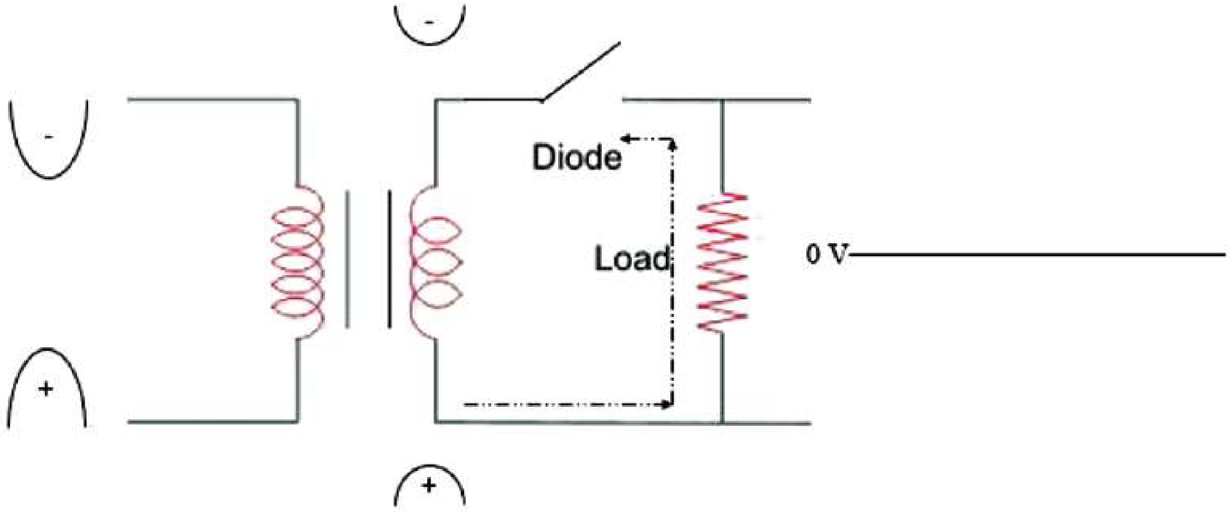
Half Wave Rectifier (HWR)



- Rectifier circuit which converts only one-half cycle of AC to DC is called half wave rectifier
- During the positive half cycle, diode D is forward biased, hence act as a closed switch (ON switch).

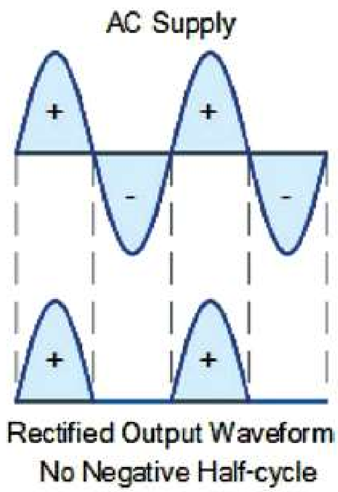


- Input waveform is felt across the load resistor R_L.
- During the negative half cycle, diode D is reverse biased, hence it act as an open switch (OFF switch).



- Under this condition, there is no path for current flow through the load resistor R_L, hence output voltage felt across R_L will be zero.
- The output signal frequency of half wave rectifier will be equal to input signal frequency

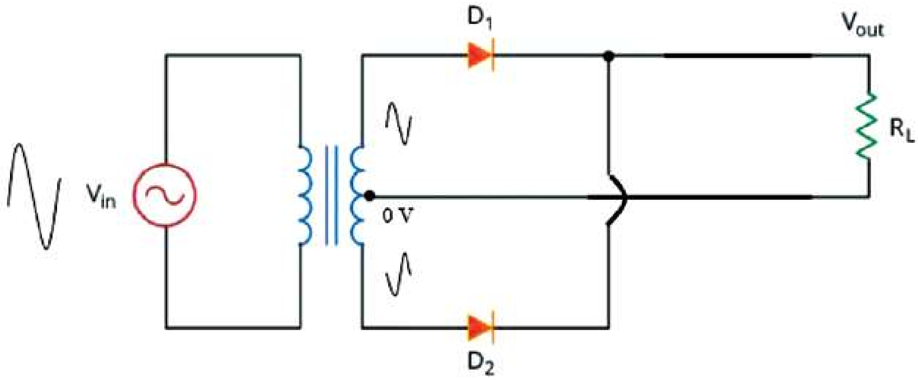
$$f_{out-HWR} = f_{in-ac}$$



- Efficiency of half wave rectifier is only 40.6% .

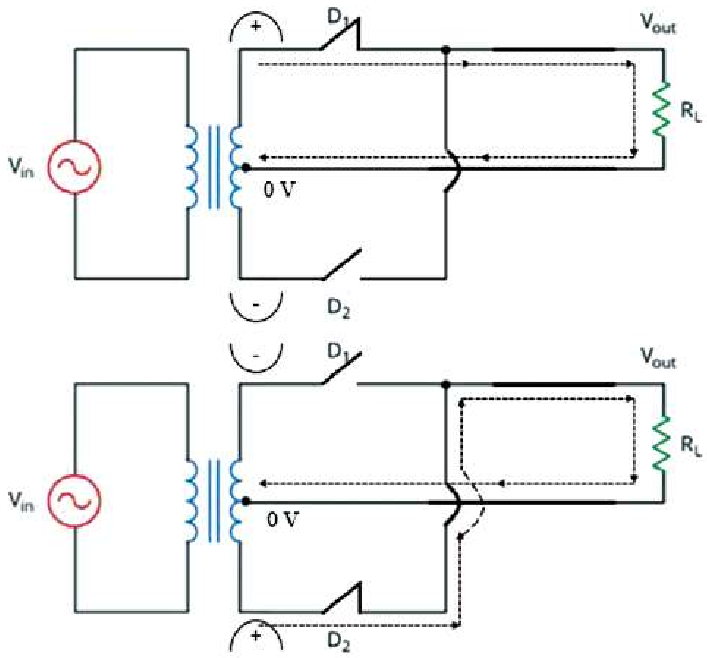
Center-tapped Full Wave Rectifier (CT-FWR)

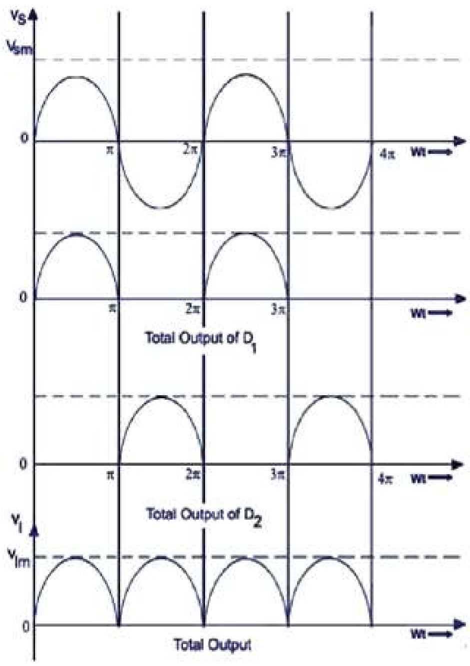
- In center-tap full wave rectifier, the center-tapping terminal of the transformer will always act as reference potential, the circuit is similar to two half wave rectifiers connected in parallel.



- During the positive half cycle diode D_1 is forward biased and diode D_2 is reverse biased.
- Hence, there is a path for current flow through the load resistor R_L (through the upper part of the circuit), hence input waveform is felt across R_L.
- During the negative half cycle, diode D_1 is reverse biased and diode D_2 is forward biased, hence, there is a path for current flow through the lower part of the circuit. In this case also input waveform is felt across R_L.
- Direction of current flow through the load resistor R_L, is same in both the cases. Hence polarity of output waveform is not changing.
- The circuit converts both the half cycles of AC to DC. Hence, it is called full wave rectifier.
- Efficiency of the circuit is 81.2%.
- Output signal frequency is equal to twice the input signal frequency

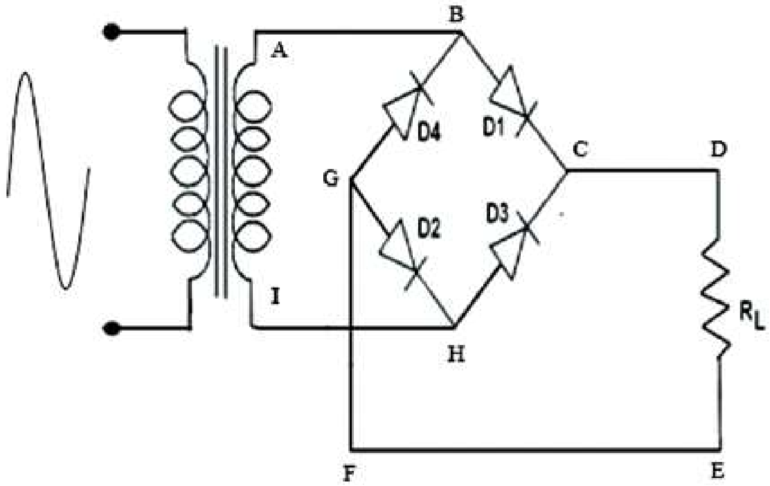
$$f_{out-FWR} = 2 \times f_{in-ac}$$



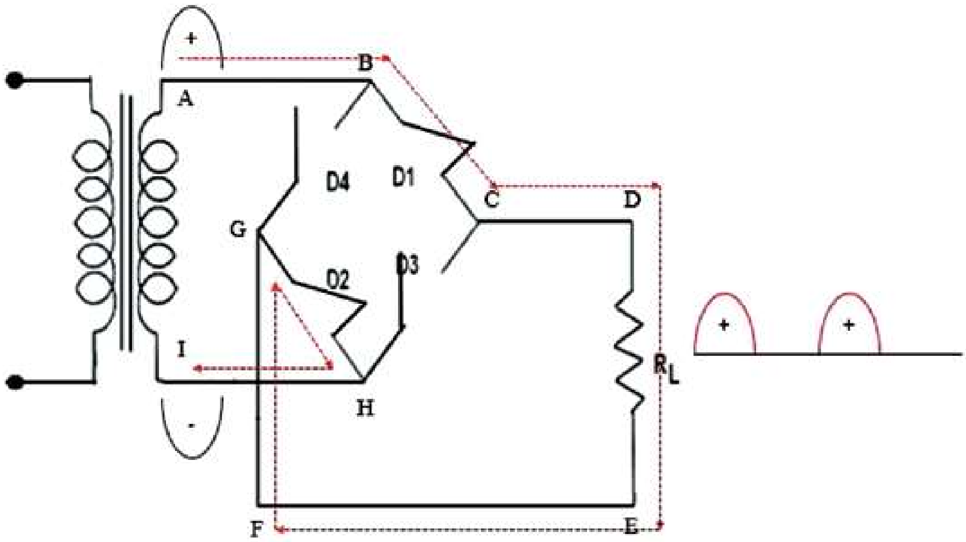


Bridge Rectifier

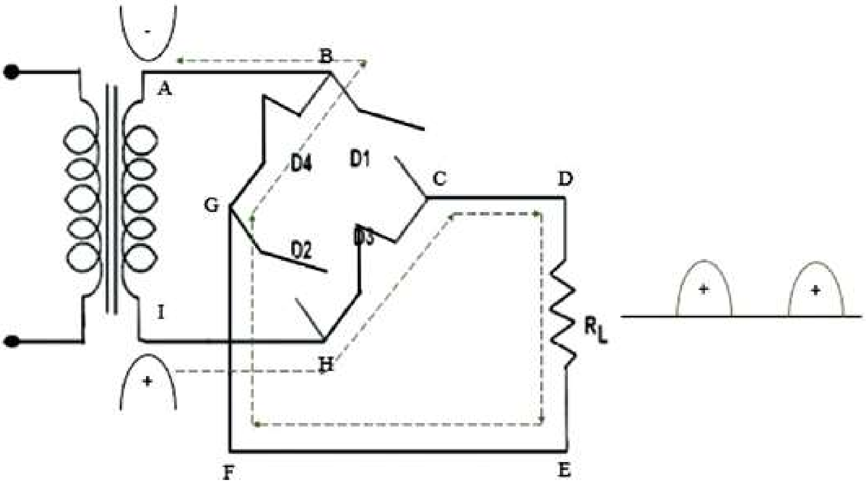
It is a low-cost rectifier compared to center-tapped rectifier.



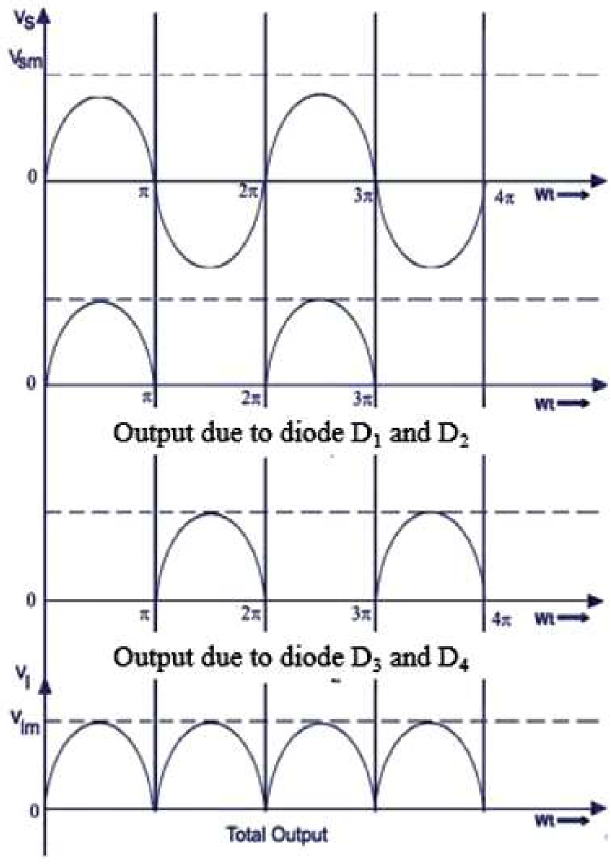
- During the positive half cycle diodes D_1 and D_2 are forward biased and diode D_3 and D_4 are reverse biased. Hence, there is a path for current flow through the load resistor $R_L(A \rightarrow B \rightarrow D_1 \rightarrow C \rightarrow D \rightarrow R_L \rightarrow E \rightarrow F \rightarrow G \rightarrow D_2 \rightarrow H \rightarrow I)$.



- Input waveform will appear at across load resister R_L.
- During the negative half cycle, Diodes D_3 and D_4 are forward biased and D_1 and D_2 are reverse biased. Hence there is a path for current flow through the load resistor $R_L(I \rightarrow H \rightarrow D_3 \rightarrow C \rightarrow D \rightarrow R_L \rightarrow E \rightarrow F \rightarrow G \rightarrow D_4 \rightarrow B \rightarrow A)$.
- Input waveform will appear across load resistor R_L.



- Direction of current flow through the load resister R_L is same in both the cases, hence polarity of output waveform will not change.



- Output signal frequency is equal to twice the input signal frequency.

$$f_{out-FWR} = 2 \times f_{in-ac}$$

- Efficiency of the circuit is 81.2%.

Terms to Remember

- Peak Inverse Voltage (PIV) : It is the maximum reverse bias voltage felt across a diode which is connected under reverse bias condition.
- Ripple Factor : Ripple Factor determines the quality of output DC waveform. It is the ratio RMS value to average value.
- If Ripple Factor is very high, then circuit has poor DC output.
- Ripple Factor of perfect DC signal is zero.

Parameters of Rectifier Circuits

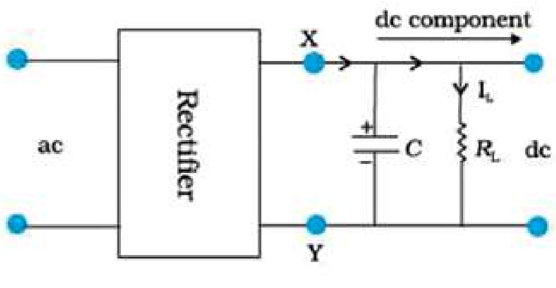
Refer Material Page No. 426 Tabular Column.

Filter

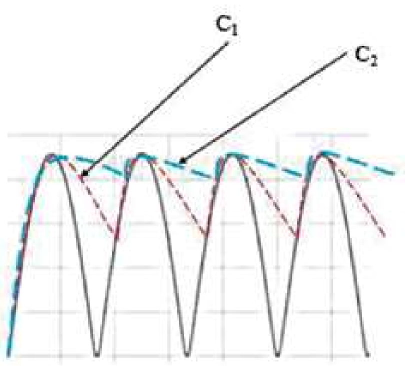
- Filters are the electronic circuits used to remove ripples from a rectified output.
- There are different types of filters namely shunt capacitor filter, series inductor filter, LC filter, π filters etc.
- Prominently used to filter is shunt capacitor filter.

Shunt Capacitor Filter

- Shunt capacitor filter function based on charging and discharging action of a capacitor.
- Capacitor will store energy, if, internally stored potential value is less than externally supplied (available) potential and capacitor will give out energy (discharge), if, externally supplied potential is lower than internally stored potential value.
- Time constant of the filter Indicates, the time required by the capacitor to give out the stored energy completely.
- If a circuit is having large time constant, then it will remove ripples effectively.
- For best DC output, Shunt capacitor filter uses large-valued capacitors.



Time contant of the filter $\tau \propto RC$



$C_1 < C_2$

Special Types of Semiconductor Junction Devices

There are 2 categories of special types of semiconductor diodes

- Breakdown Devices
- Optoelectronic devices

Breakdown Devices

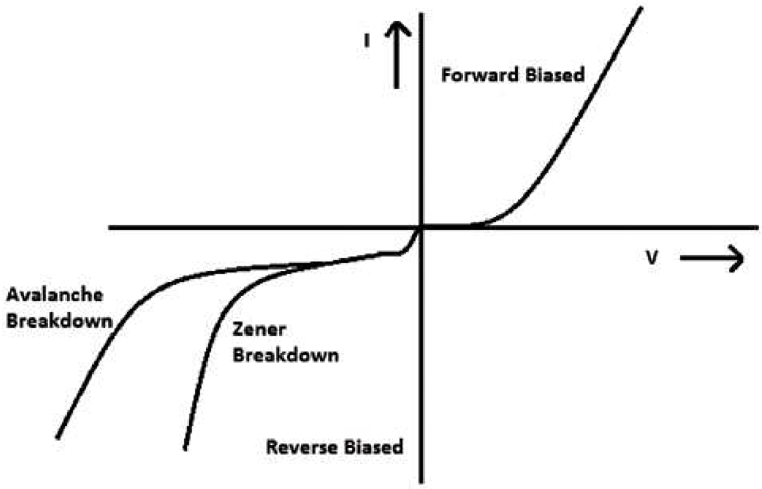
- There are 2 types of breakdown mechanisms involved in a PN junction device.
    - Avalanche breakdown
    - Zener breakdown

Avalanche breakdown

- Avalanche breakdown is the breakdown mechanism involved in lightly doped PN junction devices.
- Under reverse bias condition an electric field is established at the junction. Under the influence of this electric field, an electron from P side drift to N side.
- When these electrons reach, near the depletion region, it gets much more electric field and due to this high electric field, drift velocity of the electron increases. Due to this large velocity electron loses its control and hits the shuttling electrons in covalent bond, these collisions eject out electrons in covalent bond.
- These electrons are again experiencing the same large electric field from the depletion region. Means, the velocity of these 3 electrons increases and they also collide in the adjacent covalent bond positions. Which means carrier multiplication process occurs.
- Due to this large number of electrons generated due to collision, a large current is flowing under breakdown condition. This breakdown mechanism is called avalanche breakdown.

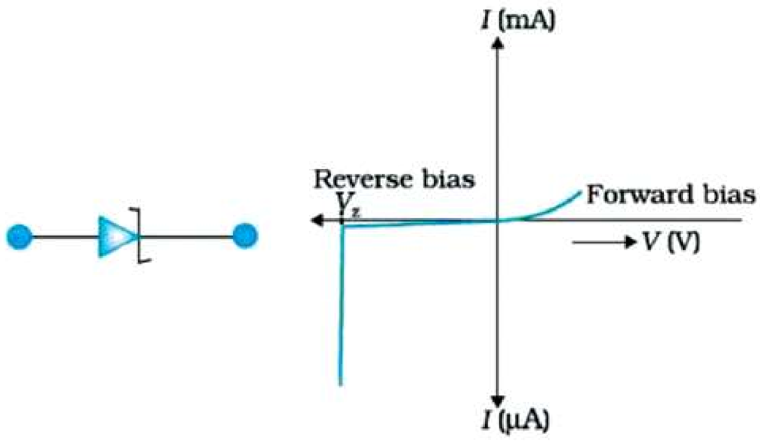
Zener breakdown

- Zener breakdown mechanism is observed by the scientist named C. Zener.
- It is observed in heavily doped PN junction devices.
- When a large electric field of order of 10^6V/m is applied to a semiconductor, its host Silicon atoms will break, its covalent bonds and eject out all the electrons simultaneously. This process called Zener breakdown or field ejection or field ionization or field emission.



Zener Diode

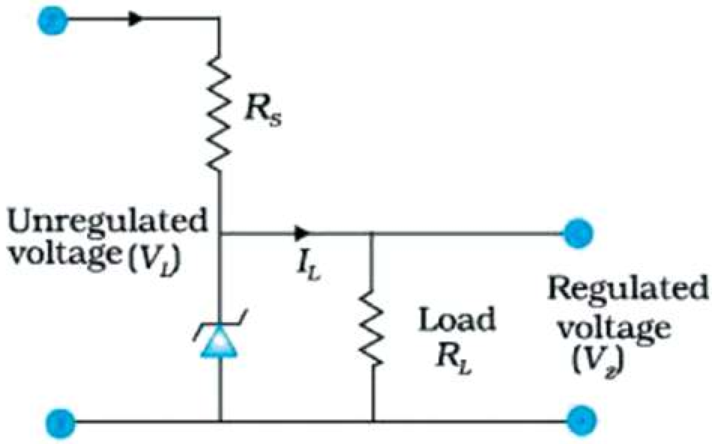
- Zener diode utilises Zener breakdown mechanism.
- Zener diode is essentially a heavily doped PN junction device operated under reverse breakdown conditions.
- While fabricating Zener diode, doping concentration on either side of the junctions are increased such that width of the depletion region of the Zener diode becomes $\frac{1}{10}^{th}$ of the width of the depletion region of the normal diode.
- Under this condition a large electric field of order of 10^6V/m is established at the junction. Due to this large electric field, Zener breakdown occurs at the junction and a large current is flowing through the junction.
- Under reverse breakdown condition, even if the current through the Zener diode is fluctuating by large amount, voltage across the device is maintained constant.
- Hence Zener diode can be used as a voltage regulator under the breakdown condition.



Zener Diode Symbol and Reverse Characteristics

Zener diode as a Voltage Regulator

- For obtaining voltage regulation, Zener diode is connected parallel to the device across which voltage regulation is required.



- By writing Kirchhoff's current law at node A.

  $I_s = I_Z + I_L$

- Voltage fluctuations can be of 2 types.
  - Supply voltage increases.
  - Supply voltage decreases.

- **Case 1:** Supply voltage increases

  When input supply voltage increases, supply current I_s also increases, hence to maintain load current I_L constant, current absorbed by the Zener I_Z decreases. Hence output voltage $V_O(I_L \times R_L)$ is maintained constant.

- **Case 2:** Supply voltage decreases

  When supply voltage decreases, supply current I_s also decreases. hence to maintain load current I_L constant, current absorbed by the Zener I_Z decreases. hence output voltage $V_O(I_L \times R_L)$ is maintained constant.

- Zener diode circuit can give voltage regulation, if and only if, Zener current is varying in between I_{Z-Min} and I_{Z-Max}.

Optoelectronic Devices

These are the devices in which photon-electron interaction takes place. There are two distinct processes involved in optoelectronic devices.

- Photo Generation
- Radiative Recombination

Photo Generation

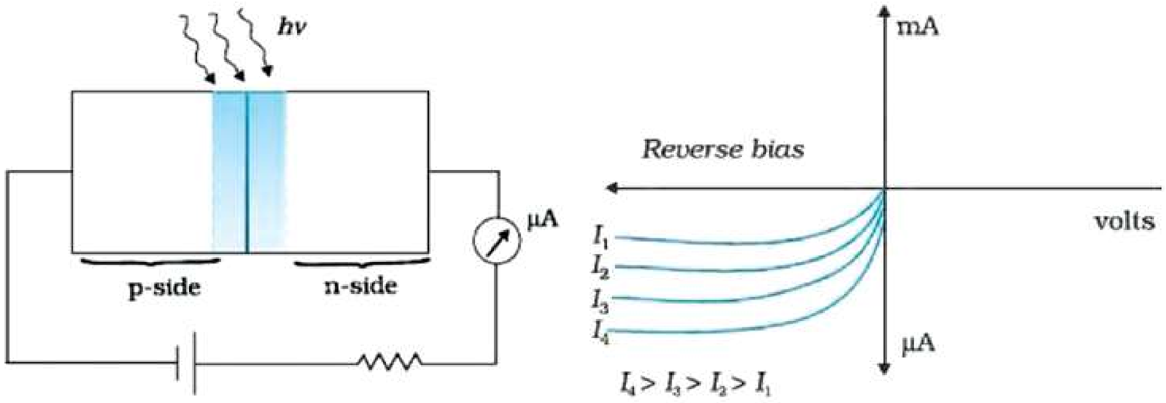
If a photon of energy $h\vartheta$ falls on a semiconductor, which is sufficiently large energy than the energy band gap of the semiconductor(E_g), electron hole pair generation will take place in it. This process is called photo generation.

Radiative Recombination

If the recombination of an electron and a hole give out energy in the form of light, then such combinations are called radiative recombination. for radiative recombination to happen, the energy band gap of the material should be in between 1.8 eV and 3 eV.

Photodiode

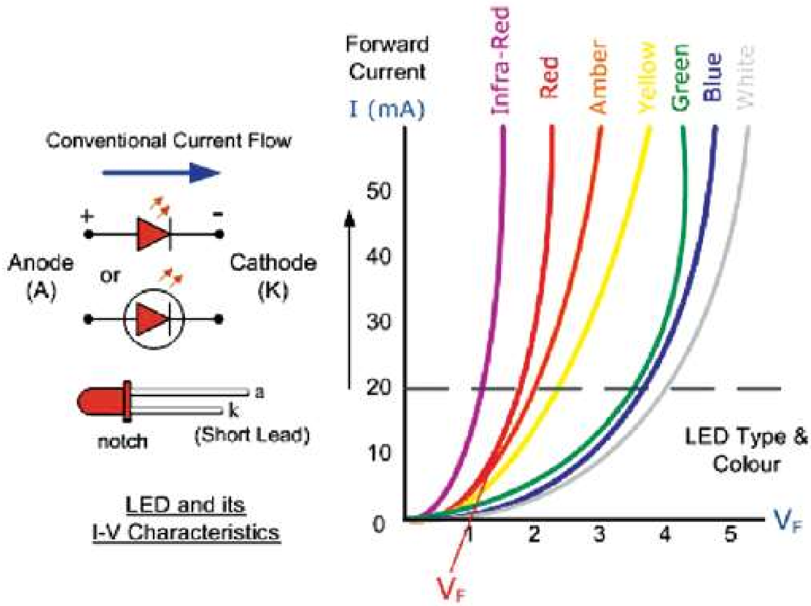
- It is a lightly doped reverse biased junction diode fabricated with slit on the top of its depletion region.
- Working principal of photo diode is photo generation.
- If light falling on the top of the slit is having energy greater than the energy band gap of the semiconductor, photo generation will take place.
- These extra generated (photo carriers) electron hole pairs will be drifted along with the minority carriers under reverse bias. (Ie, reverse current increases).
- As the intensity of light increases, more photo carriers are generated and photo current increases.
- Photodiode is only operated under reverse bias condition. It is because, the fractional change in photo carriers is more easily recognizable in presence of small number of minority carriers in reverse bias, than large number of majority carriers under forward bias condition.



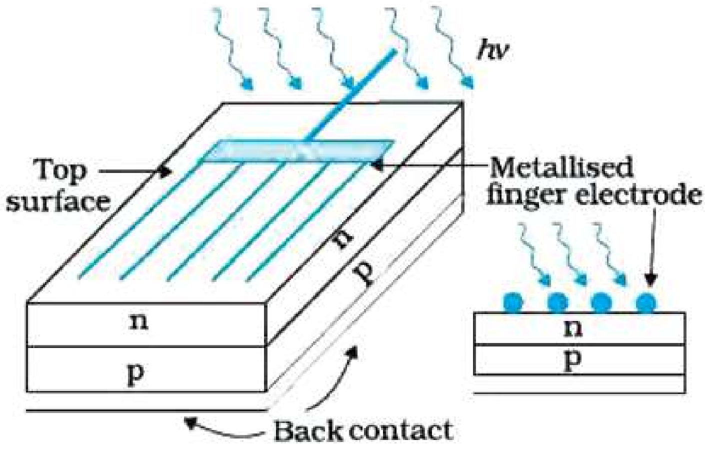
Circuit Connection and Reverse Characteristics of Photo Diode

Light Emitting Diode

- LED is a heavily doped forward biased junction diode with special fabrication feature of a transparent junction area to couple-out the internally produced light effectively.
- The doping concentration is made so high, such that, the probability of recombination increases.
- To make the combinations to happen, opposite carriers should face each other, which happens only in forward bias condition.
- To make recombinations radiative, while fabricating LED, energy band gap of the material is chosen in between 1.8 eV and 3 eV.
- As the applied forward voltage increases intensity of light output also increases, reaches maximum value and then decreases, with further increase in applied voltage.

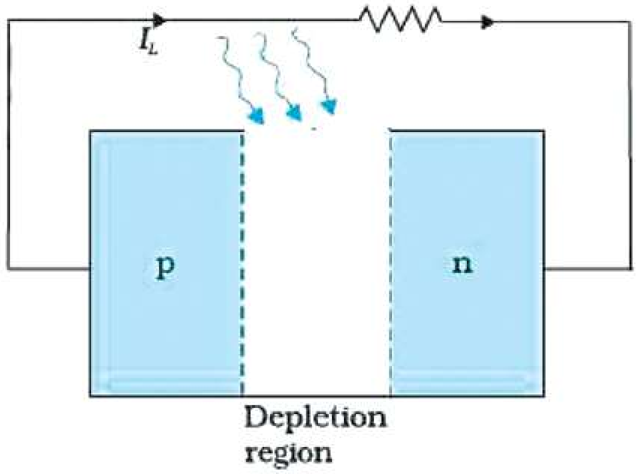


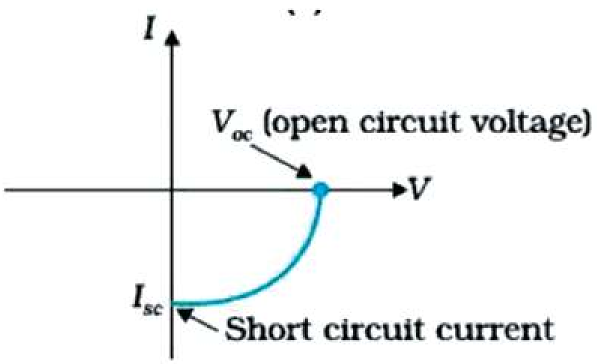
Solar Cell



- Solar cell is a lightly dopped, non-biased, junction diode fabricated with large junction area to handle large power.
- While fabricating the solar cell, N region of size 0.3 micrometer is grown on the top of a thick P - region of size 300 micrometer.
- Internal working principle of solar cell is photo generation/photovoltaic effect.
- When photon of energy $h\vartheta$ which is sufficiently greater than energy band gap E_g falls on the N - region, after penetration, it reaches till the depletion region, where it generates an electron and hole pair (carrier generation process).
- Due to the presence of small electric field at the junction, this thermally generated minority carriers are separated to P and N sides. Thermally generated minority electrons of P - side will drift to positive side of the field (N - side), similarly thermally generated minority carriers on N - side will drift to negative side of the field (P – side) (carrier separation process).
- These carriers are collected by front and back metallic contacts on P and N sides, which gives rise to a potential difference at both the ends of the metallic contacts.
- If an external load is connected to the solar cell, current will flow through the load which means, it acts similar to a battery in presence of suitable radiations.
- It is not compulsory to get a sunlight for its functioning, any radiation with energy greater than energy bandgap of the material is sufficient to ensure operation of the solar cell.
- For the better energy conversion from sunlight, materials with energy bandgap less than or equal to 1.5 eV is used.
- Widely used are Cadmium Telemid, Gallium Arsenide, Silicon, Cupres Indium Selenide etc

- If you choose the materials with the higher energy band gap, example 2.4 eV, it will convert only lower energy spectrum of sunlight into sufficient potential. Higher energy spectrum of sunlight will be wasted.
- Most of the Sun radiation has energy between 1 eV to 1.8 eV, hence the materials having energy band gap in this range is suitable for operation.
- Characteristics of a solar cell is plotted in the 4th quadrant, it is because solar cell does not consume energy, it is a device which gives out energy.
- Solar cell is widely used to power electronic devices in satellite, electric vehicles, used to make power plant at home or industries and other consumer electronic gadgets like calculators.

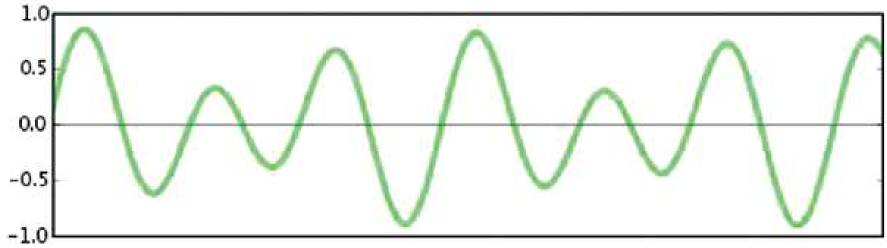




Digital Electronics

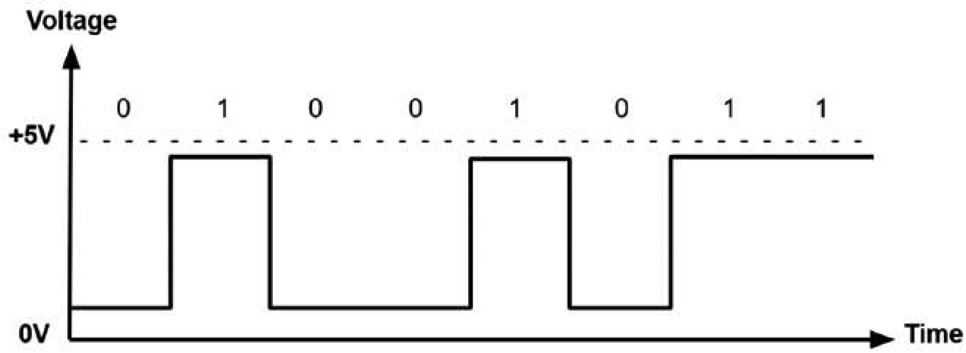
- It is the branch of electronics, which does operations based on a special type signals called digital signals.
- Electronic signals are classified into Analog Signal and Digital Signals
    - Analog signal

      Signal which is continuous both in time axis as well as magnitude axis is called analogue signal. That is, a signal without any break.



    - Digital Signal

      A signal which is continuous in time axis and discretized in magnitude axis is called digital signal.



- Special type of digital signal with only two-level magnitude is called binary signal. These two levels are represented with '0'/ '1' or 'OFF'/ 'ON' or 'LOW' / 'HIGH' or 'FALSE' / 'TRUE' or '+5 V' / '0V'.
- All the operations of digital electronics are based on laws and theorems of mathematical branch called Boolean algebra.

Basic Laws and Theorems of Boolean Algebra

- There are three basic laws in Boolean algebra called as AND Law, OR Law & NOT Law.

- Simplest form of representation in Boolean algebra is tabular form, which express relations between its all-possible inputs and outputs. This table is called Truth Table.

AND Law

A		B	Y
0	.	0	0
0	.	1	0
1	.	0	0
1	.	1	1

OR Law

A		B	Y
0	+	0	0
0	+	1	1
1	+	0	1
1	+	1	1

NOT Law

A	Y
0	1
1	0

- AND Law is similar to the multiplication in normal domain.
- OR Law is similar to the addition in normal domain.
- NOT Law is similar to taking opposite in normal domain.

Commutative Law

Variables are commutable on either side of the operator.

$$A + B = B + A$$
$$A\,.B = B.A$$

Associative Law

$$A + (B + C) = (A + B) + C$$
$$A.(B.C) = (A.B).C$$

Distributive Law

$$A.(B + C) = A.B + A.C$$

$$A + (B.C) = (A + B)\,.(A + C)$$

Idempotent Law

$$A + A = A$$

$$A.A = A$$

Some Basic Identities

$$A + \bar{A} = 1$$

$$A.\bar{A} = 0$$

$$\bar{\bar{A}} = A$$

Absorption Law

$$A + AB = 1$$

$$A.(A + B) = 1$$

De Morgan's Law

$$\overline{A + B} = \bar{A}.\bar{B}$$

$$\overline{A.B} = \bar{A} + \bar{B}$$

Consensus Theorem

$$A.B + \bar{A}.C + B.C = A.B + \bar{A}.C$$

$$(A + B).(\bar{A} + C).(B + C) = (A + B).(\bar{A} + C)$$

Dual Law

If an expression is valid, then its dual will also be valid. Dual is the expression obtained by interchanging AND & OR operators in the expression alone.

Eg: Expression → $A + B = B + A$

Dual → $A\,.B = B.A$

Expression → $A.(B + C) = A.B + A.C$

Dual → $A + (B.C) = (A + B)\,.(A + C)$

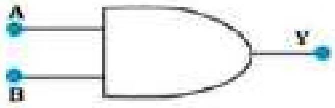
Logic Gates

Logic gates are the electronic circuits, functioning in electronic domain, based on laws and theorems of Boolean Algebra. There are three basic logic gates functioning based on three basic laws of Boolean Algebra and they are called AND Gate, OR Gate & NOT Gate.

AND Gate

- Output of AND gate will be one, if and only if, both the inputs are one. Else, its output will be zero.
- AND Gate is logically equivalent to compulsory conditions in real life.

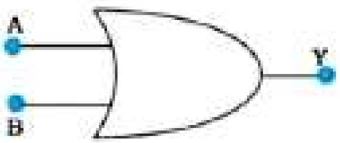
- A sentence having must/should or any other compulsory phrases in language falls under AND Gate implementation.
- Electrical Analogy → Voltage source connected in series with decision making devices, which are connected in series.



Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

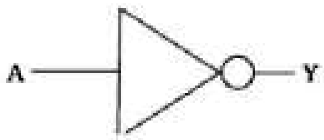
- Output of OR Gate will be one, if any of the input is one, else, output will be zero.
- OR Gate is logically equivalent to minimum requirement condition (at least condition or any of the given condition) in real-life.
- Electrical Analogy → Voltage source connected in series with decision making devices, which are connected in parallel



Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate

- In real-life, it is the complementation or negation of the given expression or value.
- Output will be complement of input.
- A logic gate with single input.
- Electrical Analogy → Voltage source connected in parallel to decision making unit.



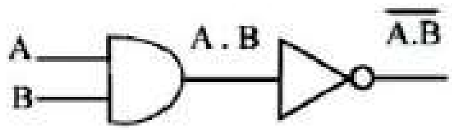
Input	Output
A	Y
0	1
1	0

Combinational Gates

- There are two combinational gates, which are formed by the combination of AND Gate/OR Gate with a NOT Gate. There is only two possibility in such connection.

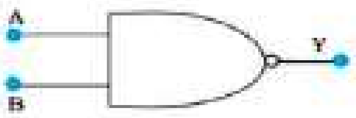
NAND Gate

- NAND gate is logically, a combination of AND Gate & a NOT Gate (connected at the output of AND gate).



Logical Diagram

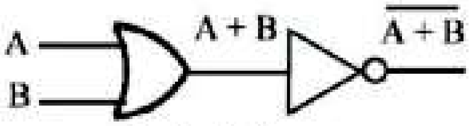
- Output of NAND gate will be zero, if and only if, both the inputs are one, else, output will be 1.
- It is the complement of AND Gate.



Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

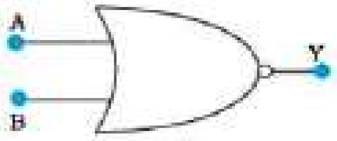
NOR Gate

- NOR gate is logically, a combination of OR Gate & a NOT Gate (connected at the output of OR gate).



Logical Diagram

- Output of NOR Gate will be zero, if any of the input is one. Else output will be 1.
- It is the complement of OR Gate

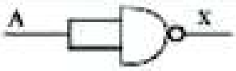
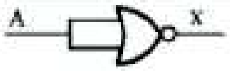
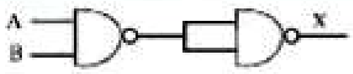
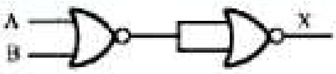
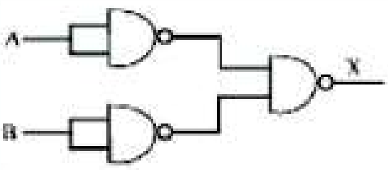
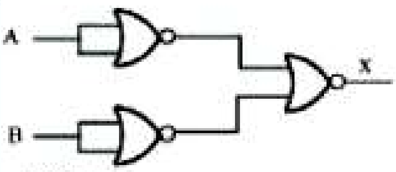
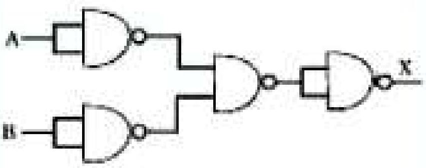
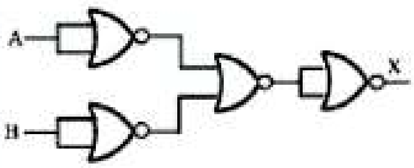


Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

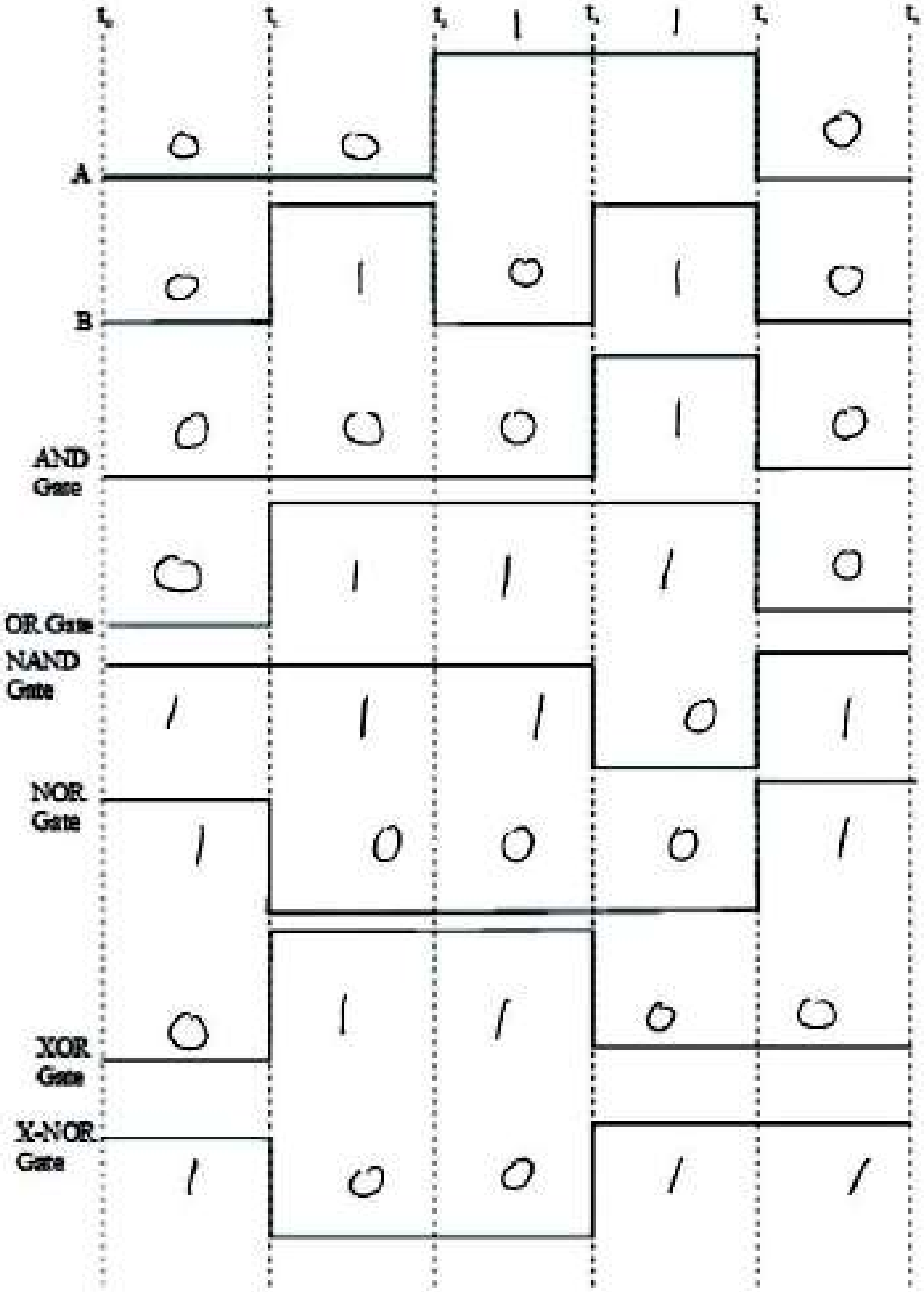
Universal Gates

- NAND Gate and NOR Gate are called universal gates, because all other gates can be implemented with the multiple copies of these two gates.

Realization of other Gates using Universal Gates

No. of gate used	NAND	NOR
1.	 Shorted NAND gate = NOT	 Shorted NOR gate = NOT
2.	 AND gate	 OR gate
3.	 OR gate	 AND gate
4.	 NOR gate	 NAND gate

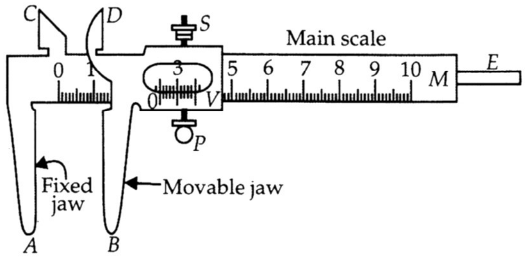
Waveform Drawing



EXPERIMENTAL SKILLS

1. <u>Vernier calipers</u> - It is used to measure internal and external diameter and depth of a vessel.

It is a device used to measure accurately upto $\left(\frac{1}{10}\right)^{th}$ of a millimetre. It was designed by the French Mathematician Pierre Vernier. The vernier calipers is as shown in the figure.



The main parts of vernier calipers are :

Main scale

Vernier scale

Jaws

Metallic strip

Vernier scale : A vernier scale V slides on the metallic strip M. In laboratory vernier calipers, vernier scale has 10 divisions which coincide with 9 mm of the main scale.

Jaws : It has two fixed jaws A and C and two movable jaws B and D.

Vernier Constant

It is the difference between value of one main scale division and one vernier scale division of vernier callipers.

Let n vernier scale divisions (VSD) coincide with (n - 1) main scale divisions (MSD)

$$\therefore\ \text{nVSD} = (n-1)\,\text{MSD}$$

$$1\text{VSD} = \left(\frac{n-1}{n}\right)\text{MSD}$$

Vernier constant, VC = 1 MSD – 1 VSD

Vernier constant, $VC = 1MSD - \left(\frac{n-1}{n}\right)MSD = \frac{1}{n}MSD$

$$VC = \frac{\text{Value of one main scale division}}{\text{Total number of divisions on vernier scale}}$$

Least Count

The smallest value of a physical quantity that can be measured accurately is called the least count of the instrument. For a vernier callipers, its least count is equal to vernier constant.

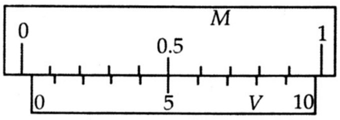
Zero Error

When the jaws A and B touch each other and if the zero of the vernier scale does not coincide with the zero of the main scale, then the instrument has error called zero error.

Zero error is of two types :

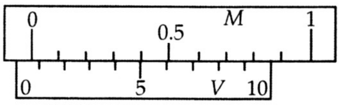
Positive zero error and Negative zero error

Positive zero error: Zero error is said to be positive if the zero of the vernier scale lies on the right of the zero of the main scale as shown in figure (i).



(i) Positive zero error

Zero error is said to be negative if the zero of the vernier scale lies on the left of the main scale as shown in figure (ii).



(ii) Negative zero error

Zero Correction

It has a magnitude equal to zero error but its sign is opposite to that of the zero error.

Zero correction is always algebraically added or substracted to the observed reading depending on the sign of the zero error.

Reading of Vernier Calipers

Place the body between the jaws and the zero of vernier scale lies ahead of N^{th} division of main scale. Then

Main scale reading (MSR) = N

If n^{th} division of vernier scale coincides with any division of main scale, then

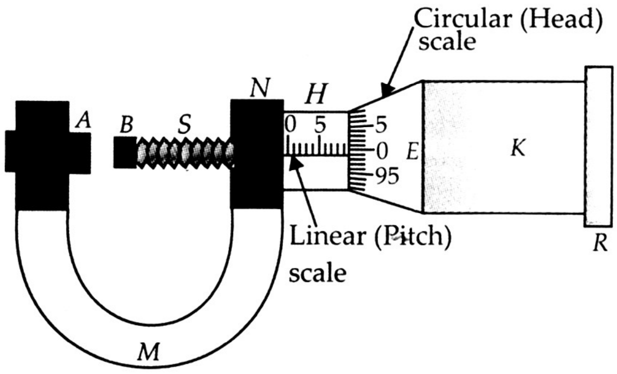
Vernier scale reading (VSR) = n × (VC)

where VC is vernier constant of vernier callipers

Total reading = MSR + VSR = N + n × (VC)

2. Screw gauge : It is used is to determine thickness/diameter of thin sheet/wire

It works on the principle of micrometer screw. A screw gauge is as shown in figure



It consists of a U-shaped metal frame M. At one end of it is fixed with a small metal piece A of gun metal. It is called stud and it has a plane face. The other end N of M carries a cylindrical hub H. On the cylindrical hub along its axis, is drawn a line known as reference line. On the reference line graduations are in millimetre or half millimetre depending upon the pitch of the screw. This scale is called linear scale or pitch scale or main scale. A nut is threaded through the hub and the frame N. Through the nut moves a screw S made of gun metal. The front face B of the screw, facing the plane face A, is also plane. A hollow cylindrical cap K, is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or head scale. A ratchet R is fixed to the right hand side of the cap K. This ratchet avoids undue tightening of the screw.

Pitch

It is defined as the linear distance moved by the screw forward or backward when one complete rotation is given to the circular cap.

$$\text{Pitch of the screw} = \frac{\text{Distance moved on linear scale}}{\text{Number of rotations}}$$

Least Count

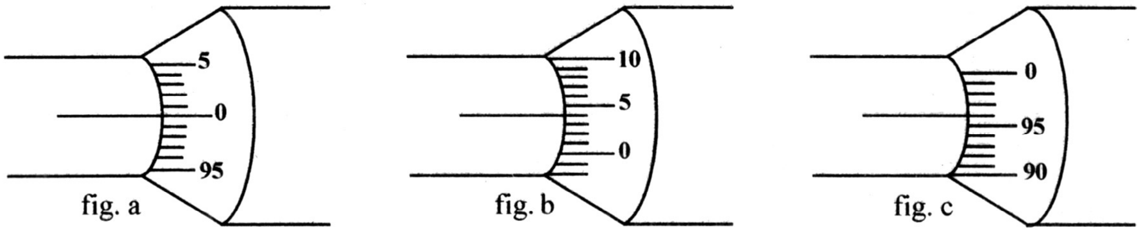
$$\text{Least count of the screw gauge} = \frac{\text{Pitch of the screw}}{\text{Total number of divisions on the circular scale}}$$

Zero Error

When the two studs A and B of the screw gauge are brought in contact and if the zero of the circular scale does not coincide with the reference line then the screw gauge has an error. This error is called zero error.

Positive zero error : Zero error is said to be positive if the zero of the circular scale lies below the reference line as shown in figure (i).

Negative zero error : Zero error is said to negative if the zero of the circular scale lies above the reference line as shown in figure (ii).



Zero correction = nil

Zero coincidence = 4
Zero error = +4
Zero correction = – 4 division

Zero coincidence = 96
Zero error = – 4
Zero correction = + 4 division

Reading of a Screw Gauge

Place a wire between A and B by working the screw forward. The pitch scale reading (PSR) immediately before the head scale is noted. The head scale reading (HSR) coinciding with the line of the pitch scale is also noted. The zero correction is added or subtracted depending on the sign of the zero error to get correct head scale reading.

The pitch scale reading is then added to the corrected head scale reading (multiplied with LC) which gives the diameter of the wire (d).

d = P.S.R. + (corrected H.S.R. × L.C.)

3. <u>Simple pendulum</u> - Dissipation of energy by plotting a graph between square of amplitude and time

When a simple pendulum swings in air, it eventually stops. This happens as the air drag and the friction at the support, oppose the motion of the pendulum and dissipate its energy gradually.

The restoring force acting on the simple pendulum is

$$F_s = -kx$$

where k is force constant $= \dfrac{mg}{L}$

where m is the mass of the bob and L is its effective length.

The damping force acting on the bob is $F_d = -bv$ where b is called damping constant and v is velocity of the bob.

The total force acting on the bob at any time t is

$F = -kx - bx$

Using Newton's second law,

$$m\frac{d^2x}{dt^2} = -kx - \frac{bdx}{dt} \qquad \text{or} \qquad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

The solution of this differential equation is

$$x = A_0 e^{-bt/2m}\cos(\omega' t + \phi) \quad \text{where} \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad \text{---------- (1)}$$

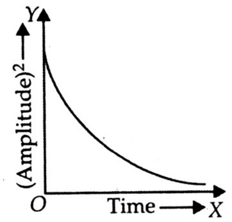
The energy of a simple pendulum at any time t is given by

$$E = E_0 e^{-bt/m} \quad \text{where } E_0 = \frac{1}{2}kA_0^2 \quad \text{---------- (2)}$$

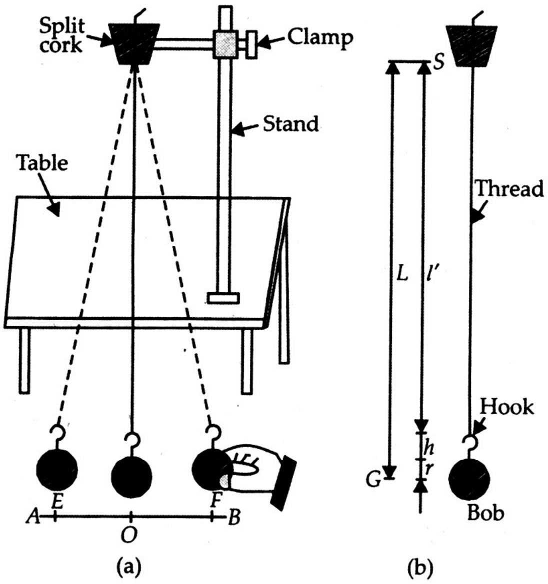
is the energy of undamped (in vacuum) simple pendulum

The energy of simple pendulum is proportional to the square of amplitude, so dissipation of energy of simple pendulum can be studied by measuring the amplitude of oscillations of simple pendulum with time.

The graph between the square of amplitude and time is as shown below.



The experimental setup for the study of dissipation of energy of an oscillating simple pendulum is as shown in figure (a) and effective length of a simple pendulum is as shown in figure (b).



Effective length of the simple pendulum = length of the thread + length of the hook of bob + radius of bob

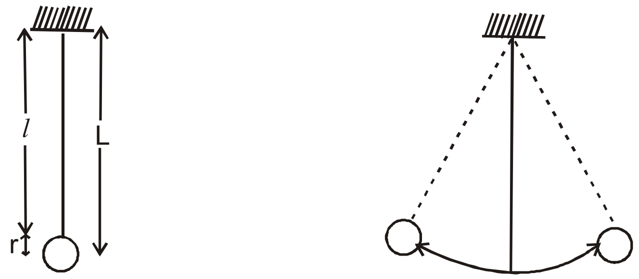
$L = l' + h + r$

Determination of value of 'g' using simple pendulum.

In this experiment, a small spherical bob is hung with a cotton thread. The bob is displaced slightly and allowed to oscillate. To find time period, time taken for 50 oscillations is noted using a stop watch.

Theoretically $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = 4\pi^2 \frac{L}{T^2}$

where L = equivalent length of the pendulum.



$$T = \frac{\text{Time for 50 oscillations}}{50}$$

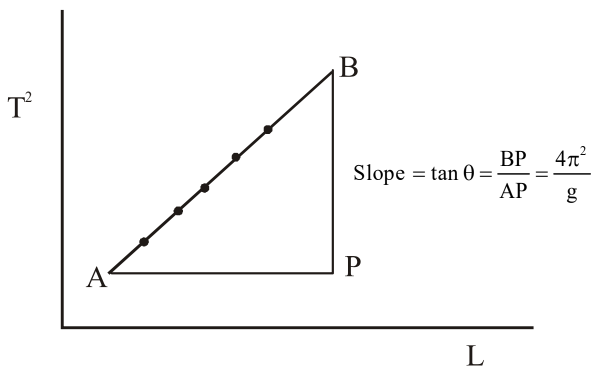
So g can be easily calculated by the equation $g = 4\pi^2 . \frac{L}{T^2}$

Graphical Method

$T^2 = \left(\frac{4\pi^2}{g}\right)L$ So $T^2 \propto L$

Find T for different values of L.

Plot T^2 - L graph. It should be a straight line with slope $= \left(\frac{4\pi^2}{g}\right)$



So g can be calcualted by the equation for slope $= \frac{4\pi^2}{g}$

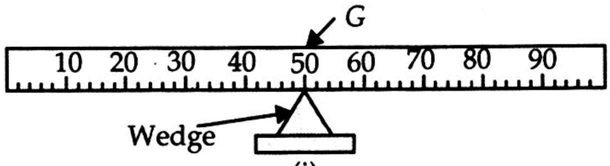
$$\therefore \; g = \frac{4\pi^2}{\text{slope}}$$

4. Metre scale - Mass of a given object by principle of moments

Principle of moments : It states that under the effect of different forces if a beam is balanced, then the algebraic sum of the moments of forces about the balancing point on the beam must be zero. i.e. in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

Like a physical balance, a metre scale can be used as a beam balance making use of principle of moments.

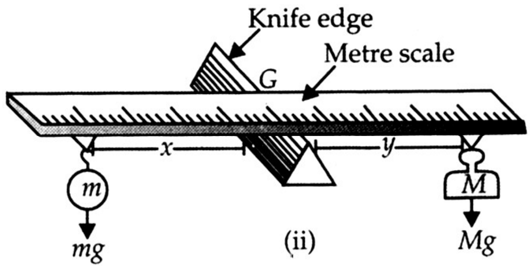
A metre scale is supported at its centre of gravity on a wedge as shown in the figure (i).



(i)

An unknown mass m of weight mg is suspended from the left side at a distance x and a known standard mass M of weight Mg is suspended from the right side at a distance y from the knife edge as shown in figure (ii). Applying the principle of moments when the scale is in equilibrium about G, we get

$$mg \times x = Mg \times y \;\text{ or }\; m = M\left(\frac{y}{x}\right) \text{ ---------- } (3)$$



(ii)

5. Young's modulus of elasticity of the material of a metallic wire

Young's modulus of the material of the given wire is given by

$$Y = \frac{MgL}{\pi r^2 x}\,Nm^{-2} \text{ ---------- } (4)$$

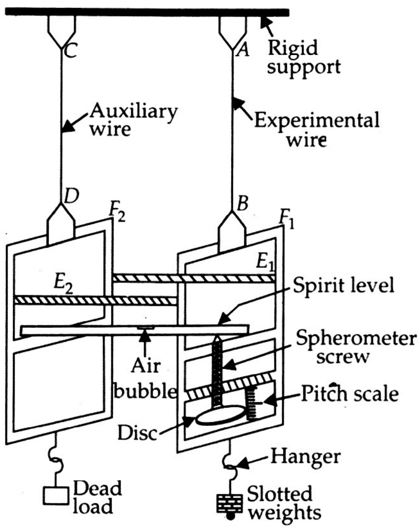
where M = Load applied at the end of the given wire

L = Original length of the wire

r = Radius of the wire

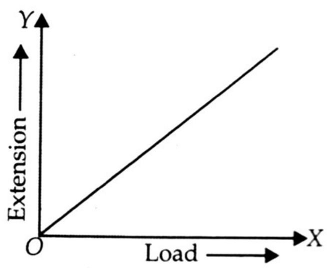
x = Extension produced in the wire.

Young's modulus Y of the material of the given wire is determined by using Searle's apparatus. The labelled diagram of Searle's apparatus is shown in the figure below.



Searle's apparatus : It consists of two metal frames F_1 and F_2. Each frame has a torsion head at the upper side and a hook at the lower side. These frames are suspended from two wires AB and CD of same material, length and cross-section. The upper ends of the wires are screwed tightly in two torsion heads fixed in the same rigid support. The experimental wire AB can be loaded by slipping slotted weights on the hanger. A spirit level rests horizontally with its one end hinged in the frame F_2. The other end of the spirit level rests on the tip of a spherometer screw fitted in the frame F_1 .The spherometer screw can be rotated up and down along a vertical scale marked in millimeter. The two frames are kept together by cross bars E_1 and E_2.

The graph between extension and load is a straight line as shown below.



Maximum permissible error in Y due to error in measurement of M, L, r and x

$$Y = \frac{MgL}{\pi r^2 x}$$

If there is no tolerence in mass, maximum error in Y is

$$\left(\frac{\Delta Y}{Y}\right)_{max} = \frac{\Delta L}{L} + \frac{2\Delta r}{r} + \frac{\Delta x}{x}$$

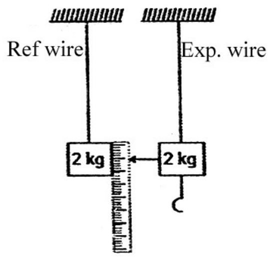
Measurement of Youngs Modulus

To measure extra elongation, compared to initial loaded position, we use a reference wire, also carrying 2 kg load (dead weight). This method of measuring elongation by comparison also cancels the side effect of tamp and yielding of support.

Observations

(i) Initial Reading = x_0 = 0.540 mm (Micrometer Reading without extra load)

(ii) Radius of wire = 0.200 mm. (using screw gauge)

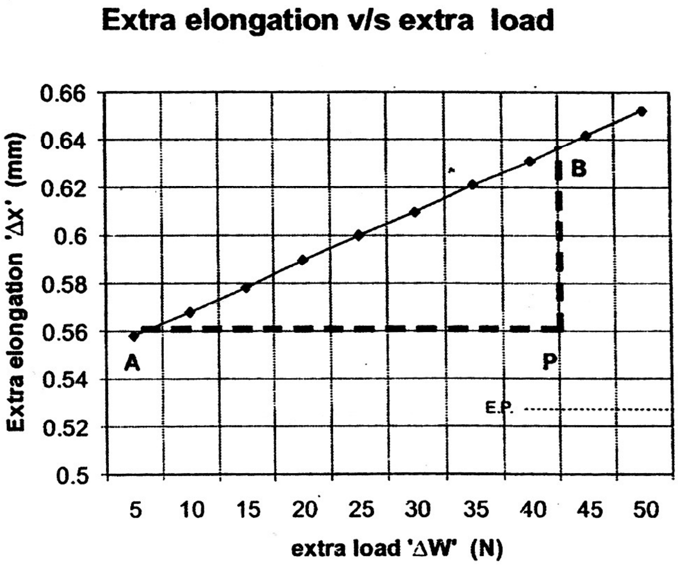


Measurement of Extension due to Extra load

S.No.	Extra load on hanger Δm (kg)	Load increasing (p) (mm)	Load decreasing (q) (mm)	Mean reading (x) (p + q/2) (mm)	Δx extra elongation $(x - x_0)$ (mm)
1	0.5	0.555	0.561	0.558	0.018
2	1.0	0.565	0.571	0.568	0.028
3	1.5	0.576	0.580	0.578	0.038
4	2.0	0.587	0.593	0.590	0.50
5	2.5	0.597	0.603	0.600	0.060
6	3.0	0.608	0.612	0.610	0.70
7	3.5	0.620	0.622	0.621	0.081
8	4.0	0.630	0.632	0.631	0.091
9	4.5	0.641	0.643	0.642	0.102
10	5.0	0.652	0.652	0.652	0.112

Method (1)

Plot Δx v / s $\Delta W(= \Delta m g)$



$$\text{Slope} = \frac{BP}{AP} = \ldots\ldots\ldots \qquad = \frac{L}{Y\left(\pi r^2\right)} \Rightarrow Y = \ldots\ldots\ldots\ldots$$

Method 2

Between observation	(1)	$\rightarrow$	(6)	
and	(2)	$\rightarrow$	(7)	
and	(3)	$\rightarrow$	(8)	2.5 kg extra weight is added
and	(4)	$\rightarrow$	(9)	
and	(5)	$\rightarrow$	(10)	

So elongation from observation (1) $\rightarrow$ (6), (2) $\rightarrow$ (7), (3) $\rightarrow$ (8), (4) $\rightarrow$ (9), and (5) $\rightarrow$ (10) will be due to extra 2.5 kg wt.

So we can find elongation due to 2.5 kg wt from $x_6 - x_1,\ x_7 - x_2,\ x_8 - x_3,$ or $x_{10} - x_5$

and hence we can find average elongation due to 2.5 kg wt.

S.No.	Extra load on hanger Δm (kg)	Micrometer reading		Mean reading (x) (p + q)/2 (mm)	Δx extra elongation due to 2.5 kg extra load (mm)
		Load increasing (p) (mm)	Load decreasing (q) (mm)		
1	0.5	0.555	0.561	0.558	0.052
2	1.0	0.565	0.571	0.568	0.053
3	1.5	0.576	0.580	0.578	0.053
4	2.0	0.587	0.593	0.590	0.052
5	2.5	0.597	0.603	0.600	0.052
6	3.0	0.608	0.612	0.610	
7	3.5	0.620	0.622	0.621	
8	4.0	0.630	0.632	0.631	
9	4.5	0.641	0.643	0.642	
10	5.0	0.652	0.652	0.652	

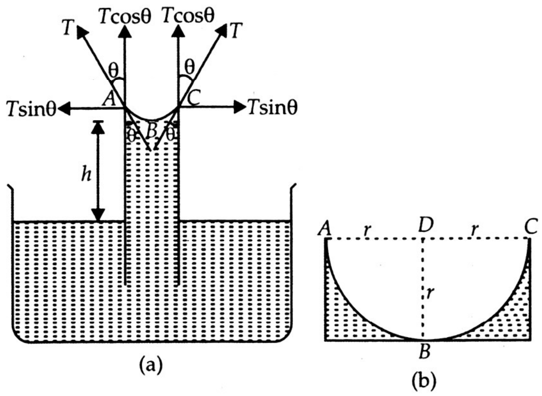
for $\Delta W = 2.5$ kg, average elongation $\Delta x = 0.052$ mm

$$\Delta x = \left(\frac{L}{\pi r^2 Y}\right)(\Delta W) \qquad \text{where } \Delta W = \Delta mg = 25 \text{ N} \qquad \text{and } (\Delta x) \text{ average} = 0.5 \times 10^{-4} \text{ m}$$

Putting the values find Y = ...

6. Surface tension of water by capillary rise and effect of detergents

When a capillary tube of radius r is dipped in a liquid (water) of density ρ, the liquid will rise in the capillary tube upto a height h above the free surface of the liquid and the force of surface tension acts tangentially to the meniscus of the liquid as shown in figure (a).



The component of the surface tension acting vertically upwards is $T\cos\theta$ where θ is the angle of contact. Hence, the total force acting upwards is $2\pi r T\cos\theta$. This force is equal to the weight of the liquid column raised in the capillary tube.

The weight of the liquid column can be calculated as follows :

The volume of the liquid column upto lower meniscus = $\pi r^2 h$

If the capillary tube is of very fine bore, its meniscus will be a hemisphere of radius r as shown in figure (b).

Volume of the liquid in the meniscus above B = Volume of cylinder of radius r and height r – Volume of hemisphere of radius r

$$= \pi r^2 r - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$$

Total volume of the liquid column $= \pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2\left(h + \frac{r}{3}\right)$

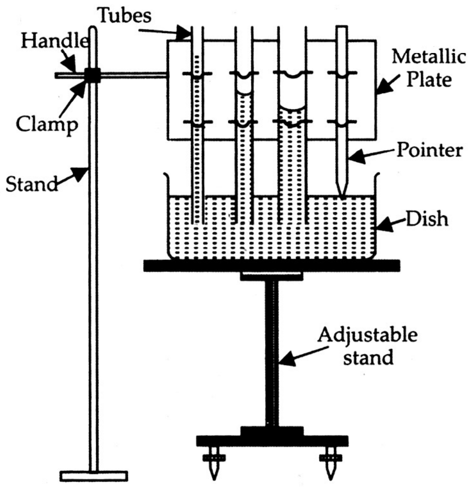
Weight of the liquid column $= \pi r^2\left(h + \frac{r}{3}\right)\rho g$

where ρ is the density of the liquid. For equilibrium, $2\pi r T\cos\theta = \pi r^2\left(h + \frac{r}{3}\right)\rho g$ or $T = \dfrac{r\left(h + \frac{r}{3}\right)\rho g}{2\cos\theta}$

For water θ is very small, so $\cos\theta \approx 1$

$$\therefore\ T = \frac{r\left(h + \frac{r}{3}\right)\rho g}{2} \quad \text{----------} \ (5)$$

The experimental set up for measuring the surface tension of water by capillary rise is as shown in the figure.



Using a travelling microscope, by coinciding the horizontal wire of the cross wire with the tip of the pointer touching the liquid and tangential to the meniscus of the liquid in the tube, the height of liquid column for each capillary tube is calculated. Using the same microscope, the radii of the capillary tubes are calculated. The surface tension of the liquid is calculated by the equation

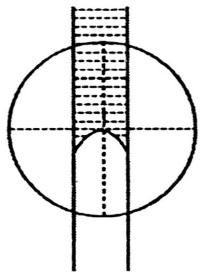
$$T = \frac{r}{2}\left(h + \frac{r}{3}\right)\rho g$$

Observation

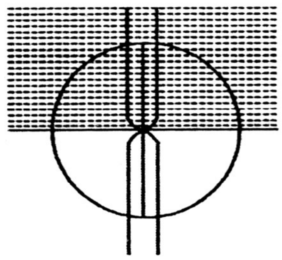
Least count of travelling microscope (L.C.) = ... cm

Table for height of liquid rise

Serial no. of Capillary tube	Reading of Meniscus			Reading of Pointer Tip			Height $h_1 - h_2 = h$(cm)
	M.S.R. N (c.m.)	V.S.R. n×(L.C.) (cm)	Total Reading N+n(L.C.) h_1(cm)	M.S.R. N (cm)	V.S.R. n×(L.C.) (cm)	Total Reading N+n(L.C.) h_2(cm)	
(1)	(2a)	2(b)	(2c)	(3a)	(3b)	(3c)	(4)
1. 2. 3.							



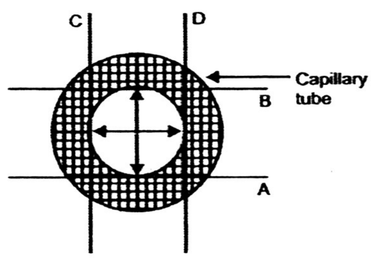
(a)



(b)

Table for internal diameter of the capillary tube

Serial No. of Capillary tube	Microscope Reading for cross wire in position				Internal Diameter			Internal radius
	(A) (cm)	(B) (cm)	(C) (cm)	(D) (cm)	Vertical AB (cm)	Horizontal CD (cm)	Mean $\frac{AB+CD}{2}$ d(cm)	$\frac{d}{2}=r(cm)$
(1)	(2a)	(2b)	(2c)	(2d)	(3a)	(3b)	(3c)	(4)
1. 2. 3.								



(c)

Temperature of water, $(t) = ...{}^{o}C$

Density of water at observed temperature, $\rho = ... \left(g\,cm^{-3}\right)$

Angle of contact of water in glass, $\theta = 8^{o}$

i.e., $\cos\theta = 0.99027$ taken as 1

CALCULATIONS

From formula, $T = \dfrac{r\left(h + r/3\right)\rho g}{2\cos\theta}$

Put values of h (column 4 - first table) and r (column 4 - second table) for each capillary tube separately and find the value of T (in dynes cm^{-1}).

Find mean value, $T = \frac{T_1 + T_2 + T_3}{3} = ...\text{dynes cm}^{-1}$

RESULT

The surface tension of water at t^oC = ... dynes cm^{-1}.

Effect of Detergents

A detergent when added to water reduces the surface tension of water. Therefore, on adding a detergent to water its surface tension decreases. The oil and grease spots on clothes cannot be removed by pure water because pure water which does not wet oily clothes. On the other hand, when detergents (like soap) are added in water, the surface tension of water decreases. As a result of this, wetting power of water increases. Thus, oil, grease and dirt particles get mixed with water easily. Hence clothes are washed easily.

7. <u>Coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body</u>

Viscosity is a measure of resistance of a fluid which is being deformed by either shear stress or tensile stress. Viscosity describes a fluids internal resistances to flow and may be thought of as a measure of fluid friction. The S.I. unit of viscosity is the pascal.second (Pa.s) equivalent to N.s/m^2, or kg/(m.s.).

According to Stoke's law, if a sphere of radius r is allowed to move freely through an infinite homogeneous, incompressible fluid of viscosity η, it acquires a uniform terminal velocity v, which is given by the relation

$$F = 6\pi\eta vr \text{ ---------- } (6)$$

where F is the retarding viscous force acting on the sphere in upward direction.

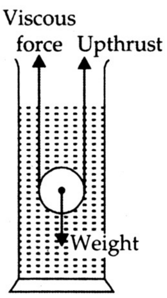
When the sphere acquires terminal velocity, the upward force must be equal to the resultant downward force.

F = Resultant downward force

= Weight of the sphere – upthrust

= Weight of the sphere – weight of the liquid displaced

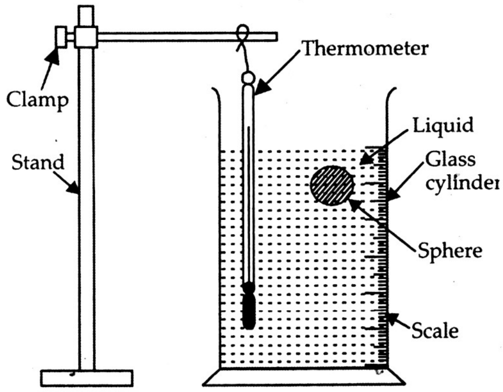
$$\text{or} \quad 6\pi\eta vr = \frac{4}{3}\pi r^3\rho g - \frac{4}{3}\pi r^3\sigma g = \frac{4}{3}\pi r^3(\rho - \sigma)g \quad \text{or} \quad v = \frac{2}{9}(\rho - \sigma)g\frac{r^2}{\eta} \text{ ---------- } (7)$$



where ρ is the density of material of the sphere, σ is the density of the liquid or $\eta = \frac{2}{9}\frac{r^2}{v}(\rho - \sigma)g$ ---------- (8)

Knowing r, ρ and σ and measuring v, η can be calculated.

The experimental set up to determine the coefficient of viscosity of a given liquid by measuring terminal velocity of a given spherical body is as shown in the figure below.



PROCEDURE

1. Determine mean radius of the ball using a screw gauge.
2. Drop the ball gently in the liquid. It falls down in the liquid with accelerated velocity for about one-third of the height. Then it falls with uniform terminal velocity.
3. Start the stop clock/watch when the ball reaches some convenient division (20 cm, 25 cm, ...).
4. Stop the stop clock/watch just when the ball reaches lowest convenient division (45 cm).
5. Find and note the distance fallen and time taken by the ball.
6. Repeat steps two times more.
7. Record the observations as given below.

OBSERVATIONS:

Least count of the vertical scale = ... m

Least count of stop clock/watch = ... s

Zero error of stock clock/watch = ... s

Pitch of the screw (p) = 1 mm

Number of divisions on the circular scale = 100

Least count of screw gauge (L.C.) $= \frac{1}{100} = 0.01$ mm

Zero error of screw gauge (e) = ... mm

Zero correction of screw gauge (C) (–e) = ... mm

Diameter of spherical ball

(i) Along one direction, D_1 = ...mm

(ii) In perpendicular direction, D_2 = ...mm

Terminal velocity of spherical ball

Distance fallen $S = ...mm$

Time taken, $t_1 = ...s$

$t_2 = ...s$

$t_3 = ...s$

CALCULATIONS

Mean diameter $D = \frac{D_1 + D_2}{2}$ mm

Mean radius $r = \frac{D}{2}$ mm $= ....cm$

Mean time $t = \frac{t_1 + t_2 + t_3}{3} = ...s$

Mean terminal velocity, $v = \frac{S}{t} = ...$ cm s^{-1}

From formula, $\eta = \frac{2r^2(\rho - \sigma)g}{9v}$ C.G.S. units

RESULT

The coefficient of viscosity of the liquid at temperature $(\theta°C) = ...$ C.G.S. units

PRECAUTIONS

1. Liquid should be transparent to watch motion of the ball.
2. Balls should be perfectly spherical.
3. Velocity should be noted only when it becomes constant.

8. <u>Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures</u>

Law of Mixtures

The law of mixtures states that when two substances at different temperatures are mixed, i.e., brought in thermal contact with each other, then the heat is exchanged between them, the substance at higher temperature loses heat and that at lower temperature gains heat. Exchange of heat energy continues till both the substances attain a common temperature called equilibrium temperature.

The amount of heat energy lost by the hotter body is equal to the amount of heat energy gained by colder body, provided (i) no heat is lost to the surroundings and (ii) the substances mixed do not react chemically to produce or absorb heat.

In brief, the law of mixtures is also known as principle of calorimetry.

Principle of Calorimetry

On mixing of two substances at different temperatures, if no heat is lost to the surroundings, at the equilibrium temperature, heat lost by hotter body is equal to heat gained by colder body

i.e. Heat gained = Heat lost

Specific Heat Capacity

It is defined as the amount of heat required to raise the temperature of unit mass of the substance through 1°C.

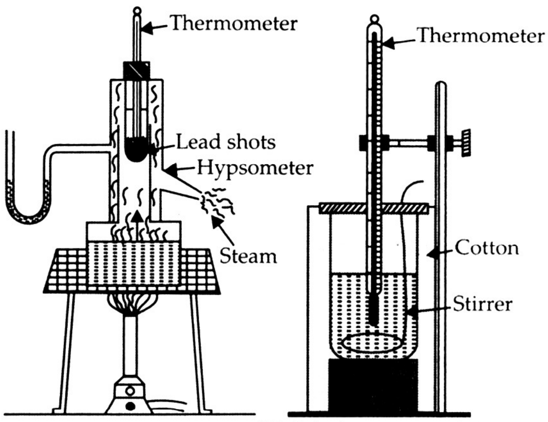
For body of mass m, having a specific heat s, the amount of heat gained or lost ΔQ is given by

$\Delta Q = ms\Delta T$

where ΔT is the rise or fall in the temperature of the body.

The Specific Heat of a Solid (Lead Shots)

The experimental set up to determine the specific heat of a given solid is as shown in the figure.



(i) Hypsometer for generating steam and heating the given solid

(ii) Calorimeter containing known mass of water, stirrer and a thermometer placed inside a jacket.

According to principle of calorimetry

Heat lost by given solid (lead shots) = Heat gained by water and calorimeter

$$(m_3 - m_2)s_s(T_s - t_f) = (m_2 - m_1 + W) \times s_w (T_f - T_w)$$

$$s_s = \frac{(m_2 - m_1 + W) \times s_w \times (T_f - T_w)}{(m_3 - m_2)(T_s - T_f)} \quad \text{---------}(9)$$

where,

m_1 = mass of (empty calorimeter + stirrer)

m_2 = Mass of (calorimeter + water + stirrer)

m_3 = Mass of (calorimeter + water + stirrer + solid (lead shots))

T_w = Initial temperature of water in calorimeter

T_s = Steady temperature upto which solid (lead shots) heated in hypsometer

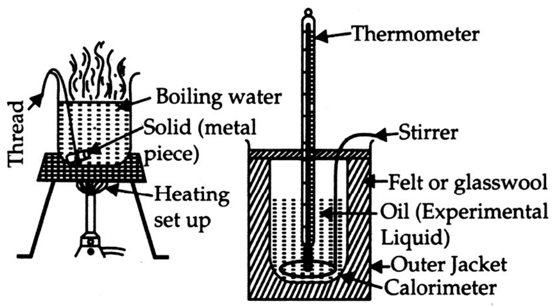
T_f = Final temperature of the mixture

W = water equivalent $\frac{m_1 s_c}{s_w}$

s_c = Specific heat of the material of calorimeter.

The specific heat of a given liquid (kerosene or turpentine oil)

The experimental set up to determine the specific heat of a given liquid (kerosene or turpentine oil) is as shown in the figure below.



According to the principle of calorimetry

Heat lost by solid (metal piece) = Heat gained by liquid and calirmeter

$$(m_3 - m_2)s_s(T_s - T_f) = m_1 s_c (T_f - T_l) + (m_2 - m_1)s_1(T_f - T_l)$$

$$s_l = \frac{(m_3 - m_2)s_s(T_s - T_f) - m_1 s_c(T_f - T_l)}{(m_2 - m_1)(T_f - T_l)} \text{ ---------(10)}$$

m_1 = Mass of (empty calorimeter + stirrer)

m_2 = Mass of (calorimeter + liquid + stirrer)

m_3 = Mass of (calorimeter + stirrer + liquid + solid (metal piece)

T_l = Initial temperature of liquid in calorimeter.

T_s = Initial temperature of the solid (metal piece)

T_f= Final temperature of mixture

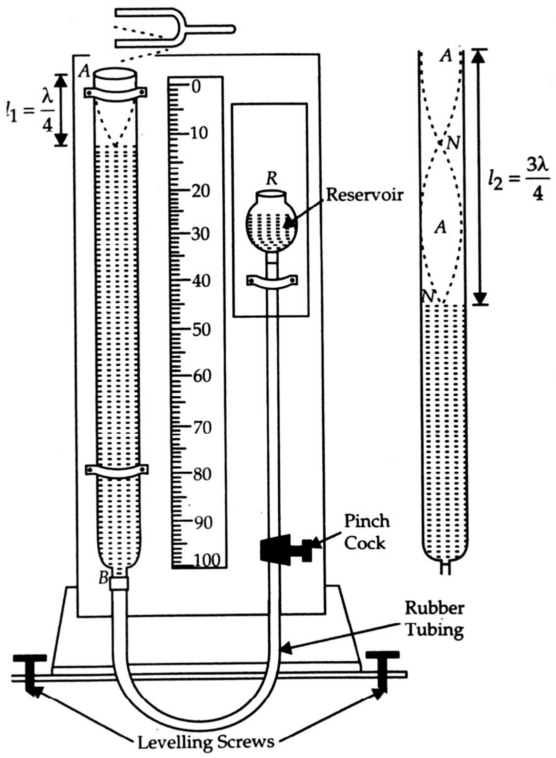
s_c = Specific heat capacity of material of calorimeter

s_l = Specific heat capacity of given liquid

s_s = Specific heat capacity of the solid (metal piece)

9. <u>Speed of sound in air at room temperature using a resonance tube</u>

A resonance tube apparatus consists of a long glass tube AB whose lower end is connected to a rubber tube which in turn is connected to a reservoir R filled with water. The whole apparatus is mounted on a wooden board on which a metre scale is fixed to note the length of air column in resonance (glass) tube as shown in the figure.



A tuning fork is set into vibrations and held over the glass tube. Stationary waves are formed in the air column due to interference of reflected and incident waves. For a particular position of water level, intensity of sound is maximum. At this position, resonance of air column with tuning fork takes place as frequencies of two become equal. If l_1, and l_2 are the lengths of air column for the first and second positions of resonances, then

$$l_1 + e = \frac{\lambda}{4} \text{ ---------(11)}$$

$$l_2 + e = \frac{3\lambda}{4} \text{ ---------(12)}$$

where e is the end correction and λ the wavelength of the sound wave.

Substracting (11) from (12),

$$l_2 - l_1 = \frac{\lambda}{2}$$

or $2(l_2 - l_1) = \lambda$ ---------(13)

$$v = \nu\lambda \quad \text{---------(14)}$$

From equations (13) and (14), we get ,

$$v = 2\nu(l_2 - l_1) \text{---------(15)}$$

Therefore, speed of sound at room temperature is given by $v = 2\nu(l_2 - l_1)$

where ν is frequency of a tuning fork.

Working: Resonance tube is a 100 cm tube. Initially it is filled with water. To increase the length of air column in the tube, water level is lowered. The air column is forced with a tuning fork of frequency f_0. Let at length l_1, we get a first resonance (loud voice) then

$$l_{eq_1} = \frac{V}{4f_0} \Rightarrow l_1 + \varepsilon = \frac{V}{4f_0} \text{---------(16)} \text{ where } \varepsilon \text{ is end correction.}$$

If we further lower the water level, the noise becomes moderate. But at l_2. We, again get a loud noise (second resonance) then

$$l_{eq_2} = \frac{3V}{4f_0} \Rightarrow l_2 + \varepsilon = \frac{3V}{4f_0} \text{---------(17)}$$

For (i) and (ii)

$$V = 2f_0(l_2 - l_1)$$

Observation table:

Room temperature in beginning = 26°C, Room temperature at end = 28°C

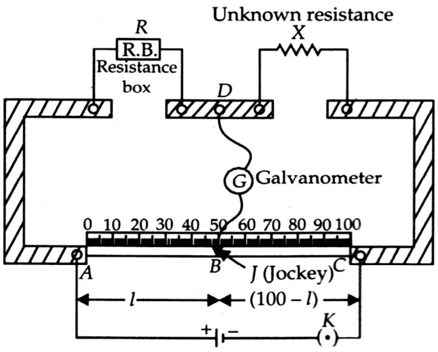
		Position of water level (cm)			
Freq. of tuning fork in (Hz) (f_0)	Resonance	Water level is falling	Water level is rising	Mean resonant length	Speed of sound V = $2f_0$ ($I_2 - I_1$)
340 Hz	1^{st} Resonance	23.9	24.1	I_1 = 24.0	V = ……
	2^{nd} Resonance	73.9	74.1	I_2 = 74.0	

10. Resistivity of the material of a given wire using metre bridge

Metre bridge is the practical form of Wheatstone bridge.

In a metre bridge, P and Q are the ratio arms of fixed resistance, R is an adjustable or variable resistance of known value and S is replaced by unknown resistance X and balance point is obtained at B on the metre bridge wire. Since a jockey is slided over the wire. It is called slide wire bridge.

The circuit diagram to determine resistivity of the material of a given wire using metre bridge is shown in the figure below.



Let AB = l cm then BC = (100 – l) cm

$$\frac{\text{Resistance of AB}}{\text{Resistance of BC}}=\frac{P}{Q}=\frac{lx}{(100-l)x}=\frac{l}{(100-l)}\text{---------}(18)$$

where x is the resistance per unit length (cm) of the bridge wire.

According to Wheatstone's bridge principle

$$\frac{P}{Q}=\frac{R}{X}\text{ or }X=\frac{Q}{P}R\text{ or }X=\frac{(100-l)R}{l}$$

Specific resistance (ρ) of the material of the given wire is given by

$$\rho=\frac{X\pi D^2}{4L}\text{---------}(19)$$

where, L is the length and D is the diameter of the given wire.

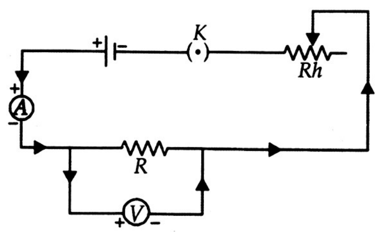
11. <u>Resistance of a given wire using Ohm's law</u>

Ohm's Law : It states that the current flowing through a conductor is directly proportional to the potential difference across its ends provided the physical conditions (like temperature etc.) of the conductor remain same.

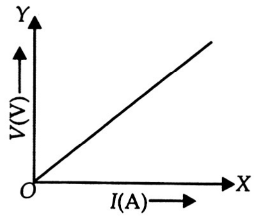
ie, $V \propto I$ or $V = RI$ or $R=\frac{V}{I}$---------(20)

where R is a constant of proportionality called resistance of the conductor. The unit of resistance is volt per unit ampere or ohm (Ω). Its value depends upon the nature of the conductor, its dimensions and the physical conditions.

The circuit diagram to determine the resistance of a given wire using Ohm's law is as shown in the figure below.



The graph between potential difference V and current I is a straight line as shown in the figure.

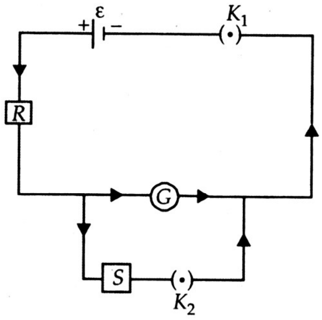


We can find that the specific resistance of a material using Ohm's law experiment,

$$\rho=\frac{RA}{L}=\frac{\pi D^2}{4L}\cdot\frac{V}{I}$$

12. Resistance and figure of merit of a galvanometer by half deflection method

The circuit diagram for finding the resistance of a galvanometer by half deflection method is as shown in the figure below.



To find the resistance of the galvanometer

Put a high resistance from R. (say 2000 Ω). The key k_1 is closed with k_2 opened. Adjust the value of R so that the deflection is maximum. Note the deflection. The deflection in the galvanometer is θ.

The current through the galvanometer

$$I = \frac{E}{R+G} \propto \theta \text{ -------- } (21)$$

Insert the key k_2 also and without changing the value of R' adjust the value of S, such that deflection in the galvanometer reduces to exactly half the value $\frac{\theta}{2}$. Repeat the above steps taking different values of R and adjusting S every time.

The current through the galvanometer

$$I_1 = \frac{E}{\left[R + \frac{GS}{G+S}\right]} \times \frac{S}{G+S} \propto \frac{\theta}{2} \text{ --------- } (22)$$

$\frac{(21)}{(22)}$ gives

$$\frac{R(G+S)+GS}{RS+GS} = 2$$

$$RG + RS + GS = 2RS + 2GS$$

$$RG = RS + GS$$

$$RG - GS = RS$$

$$G(R-S) = RS$$

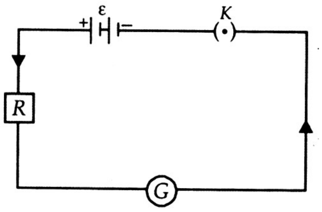
$$G = \frac{RS}{R-S}$$

The resistance of the galvanometer as found by half deflection method is $G = \frac{RS}{R-S}$ ---------(23)

where R is the resistance connected in series with the galvanometer and S is the shunt resistance.

Figure of merit of a galvanometer : It is defined as the current required to produce a deflection of one division in the scale of the galvanometer. It is generally denoted by symbol k.

The circuit diagram for the determination of the figure of merit of a galvanometer is as shown in the figure.



The figure of merit, $k = \dfrac{\varepsilon}{(R+G)\theta}$ ---------(24)

where ε is the emf of the cell and θ is the deflection produced with resistance R.

13. <u>Focal length of</u>

<u>(i) Concave mirror</u>

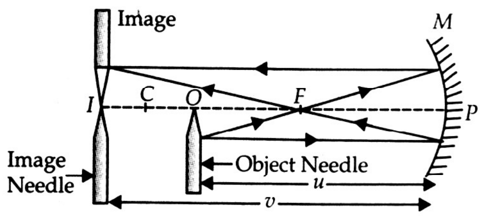
<u>(ii) Convex mirror</u>

<u>(iii) Convex lens</u>

To Determine the Focal Length of a Concave Mirror

To perform the experiment, we require an optical bench with three uprights, concave mirror, a mirror holder, two optical needles, a knitting needle and a half metre scale.

The ray diagram is as shown in the figure.



According to mirror formula $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$ or $f = \dfrac{uv}{u+v}$ ---------(25)

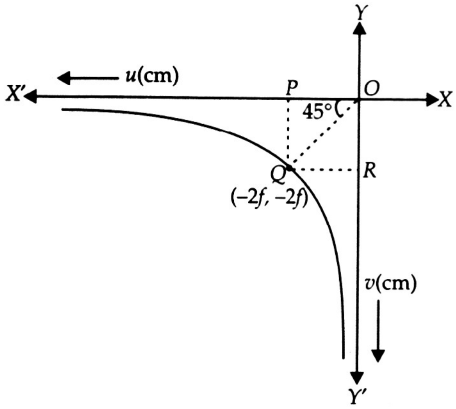
where, f = focal length of concave mirror

u = distance of object needle from pole of the mirror

v = distance of image needle from pole of the mirror

According to new cartesian sign convention both u and v are negative and f is also negative for concave mirror.

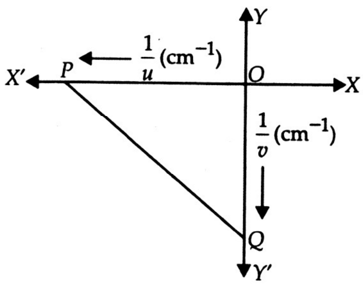
u-v graph : The graph between the u (along negative X axis or X' axis) and v (along negative Y axis or Y' axis) is a rectangular hyperbola as shown in the figure below.



Draw a line OQ bisecting the angle X'OY' and meeting the curve at point Q. The coordinates of this point Q are (–2f, –2f). Draw QP and QR perpendicular on X' and Y' axes respectively. The distances OP = OR = –2f. Half of these distances give the focal length of the concave mirror. Focal length of the concave mirror,

$$f = \frac{OP}{2} = \frac{OR}{2} \qquad \therefore \qquad f = \frac{OP + OR}{4} \text{---------}(26)$$

$\frac{1}{u}$ and $\frac{1}{v}$ graph : The graph between $\frac{1}{u}$ (along negative X axis or X' axis) and $\frac{1}{v}$ (along negative Y axis or Y' axis) is a straight line as shown below.



The straight line cuts the two axes X' and Y' at an angle of 45° at points P and Q, respectively and making equal intercepts of the axes.

Focal length of concave mirror, $f = \frac{1}{OP} = \frac{1}{OQ}$

According to mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

If $\frac{1}{u} = 0$, then $\frac{1}{v} = \frac{1}{f}$

Thus, intercept $OQ = \frac{1}{v} = \frac{1}{f}$

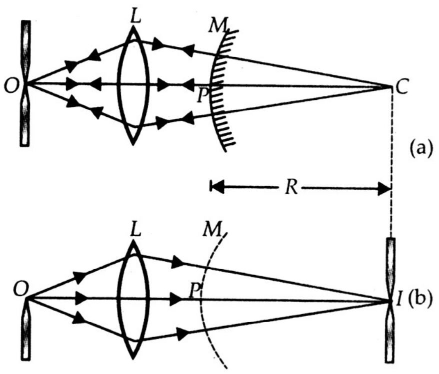
If $\frac{1}{v} = 0$, then $\frac{1}{u} = \frac{1}{f}$

Thus, intercept $OP = \frac{1}{u} = \frac{1}{f}$ $\therefore\ f = \frac{2}{OP + OQ}$ ---------(27)

To Determine the Focal Length of a Convex Mirror using a Convex Lens

To perform the experiment, we require an optical bench with four uprights, convex lens, convex mirror, a lens holder, a mirror holder, two optical needles, a knitting needle, and a half metre scale.

The ray diagram is as shown in the figure below.



As a convex mirror always forms a virtual image, its focal length cannot be found directly as for a concave mirror. For this purpose, indirect method is used, as described below.

A convex lens L is introduced between the convex mirror M and object needle O as shown in figure (a) above. Keeping the object needle at distance about 1.5 times rough focal length of the convex lens, the position of convex mirror behind convex lens is so adjusted that a real and inverted image of object needle O, is formed at O itself. Under such condition, the light rays are incident normally over the convex mirror to retrace their path. In the absence of convex mirror, these rays would have met at centre of curvature C of the convex mirror. The distance PC gives the radius of curvature R of the mirror.

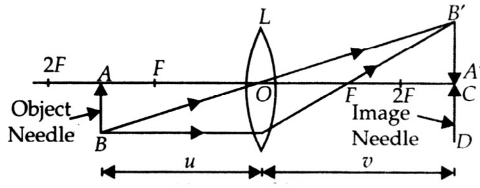
To locate the position of C, convex mirror is removed (without disturbing the object needle O and convex lens L). An image needle I is put behind the convex lens and moved to a position at which there is no parallax between tip of inverted image of O needle and tip of I needle. Position of image needle I gives position of centre of curvature C of mirror M.,

Then, PC = PI = R and $f = \frac{R}{2} = \frac{PI}{2}$

To Determine the Focal Length of a Convex Lens

To perform the experiment, we require an optical bench with three uprights, a convex lens with lens holder, two optical needles, a knitting needle and a half metre scale.

The ray diagram is as shown in the figure below.



$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \text{---------}(28)$$

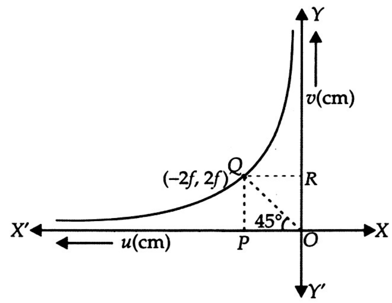
where, u = distance of object needle from optical centre of the lens

v = distance of image needle from optical centre of the lens

f = focal length of convex lens.

According to new cartesian sign convention, u is negative and v is positive and f is positive for convex lens.

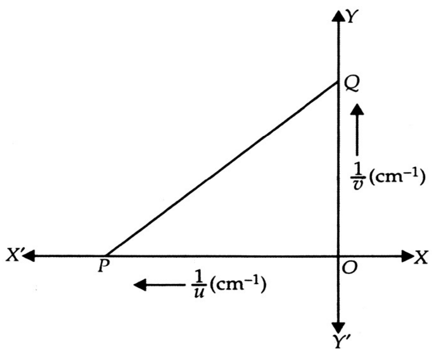
u-v graph : The graph between u (along negative X axis or X' axis) and v (along positive Y axis or Y' axis) is a rectangular hyperbola as shown in the figure below.



Draw a line OQ bisecting the angle X'OY and meeting the curve at point Q. The coordinates of this point Q are (–2f, 2f). Draw QP and QR perpendicular on X' and Y axes respectively.

Focal length of convex lens, $f=\frac{|OP|}{2}=\frac{OR}{2} \quad \therefore\ f=\frac{OP+OR}{4}$ ---------(29)

$\frac{1}{u}$ and $\frac{1}{v}$ graph : The graph between $\frac{1}{u}$ (along negative X axis or X' axis) and $\frac{1}{v}$ (along positive Y axis or Y' axis) is a straight line as shown in the figure below.



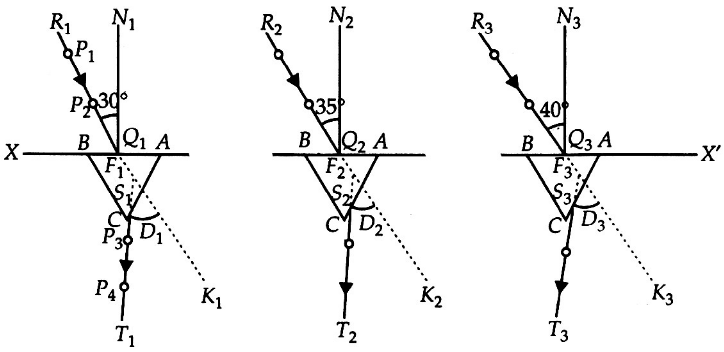
The straight line cuts the two axes X' and Y at an angle of 45° at points P and Q respectively and making equal intercepts on the axes.

Focal length of the convex lens, $f = \frac{1}{|OP|} = \frac{1}{OQ}$ $\quad \therefore f = \frac{2}{OP + OQ}$ ---------(30)

14. Using parallax method, Plot of angle of deviation vs angle of incidence for a triangular prism

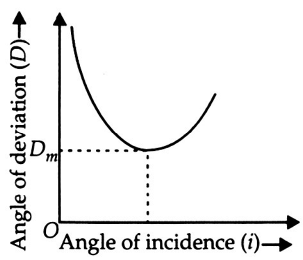
The apparatus required is a drawing board, white sheet of paper, prism, drawing pins, pencil, half metre scale, office pins, graph paper and a protractor.

The white paper is pinned on the drawing board and the prism is fixed at a place. The outline of the prism is marked on the paper. Then an incident ray (say at 30°) is drawn on one of the faces of the prism and two pins are inserted at P_1 and P_2 on the ray as shown.



When viewed from the other face, the pins are along a particular line. We place pins P_3 and P_4 in line with the images of P_1 and P_2 as seen in the prism. Thereafter line P_3P_4 is drawn which represents the emergent ray. Lift the prism and make the dotted lines and discover the angle of deviation D_1 here. This angle can be measured by a protractor. Several such observations for angles of incidence ranging from 30° to 60° can be made and the corresponding angle of deviation can be measured.

The graph between angle of deviation (D) and angle of incidence (i) is as shown in the figure below.

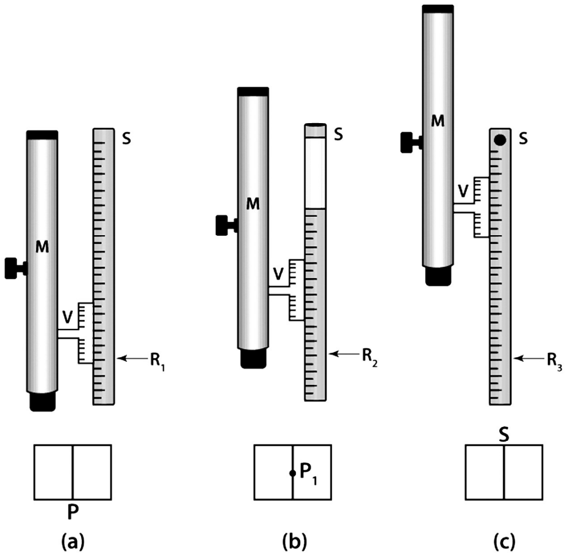


15. Refractive index of a glass slab using a travelling microscope

Travelling microscope: It is a compound microscope fitted vertically on a vertical scale. It can be moved up and down, carrying a vernier scale moving along the main scale. In any position, the reading is taken by combining main scale and vernier scale readings.

The apparatus required is a travelling microscope, glass slab and lycopodium powder

Refractive index of a glass slab, or $\mu = \dfrac{\text{Real thickness of slab}}{\text{Apparent thickness of slab}}$



Real and apparent thickness

Adjustment of a travelling microscope

1. To make the base of the microscope horizontal, adjust the levelling screw.
2. For clear visibility of the cross wire, adjust the position of the eyepiece.
3. Mark point P on the base of the microscope using black ink.
4. To avoid the parallax between the cross-wires and the mark P, make the microscope vertical and focus on P.
5. Let R_1 be the combination of vernier scale and main scale reading on the vertical scale.
6. Place the glass slab with least thickness over the mark P.
7. Let P_1 be the image of the cross mark. Move the microscope upwards and focus on P_1.
8. Let R_2 be the reading of P_1.
9. Sprinkle a few particles of lycopodium powder on the surface of the slab.
10. To focus the particle near S, raise the microscope further upward.
11. Let, R_3 be the reading of S.
12. Repeat the above steps for different thickness glass slabs.
13. Record the observations.

Real thickness = $(R_3 – R_1)$

Apparent thickness = $(R_3 - R_2)$

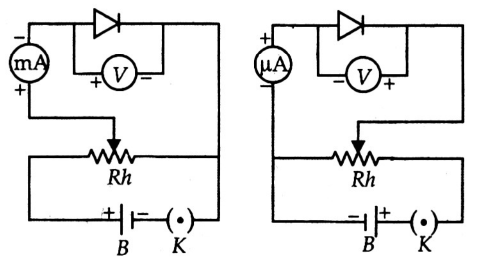
Refractive index, $\mu = \dfrac{(R_3 - R_1)}{(R_3 - R_2)}$ ---------(31)

16. Characteristic curves of a p-n junction diode in forward and reverse bias

Forward biasing : A p-n junction is said to be forward biased if the positive terminal of the external battery is connected to p-side and the negative terminal to the n-side of p-n junction.

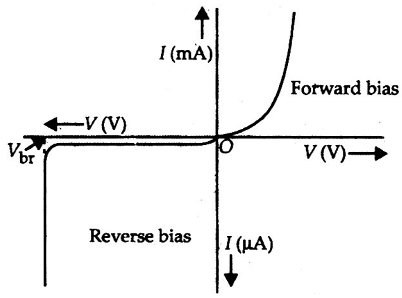
Reverse bias : A p-n junction is said to be reverse biased if the positive terminal of the external battery is connected to n-side and the negative terminal to p-side of the p-n junction,

The circuit diagrams for studying the characteristic curves of a p-n junction diode in forward and reverse biasing is shown in figure (i) and (ii).



(i) Forward Bias (ii) Reverse Bias

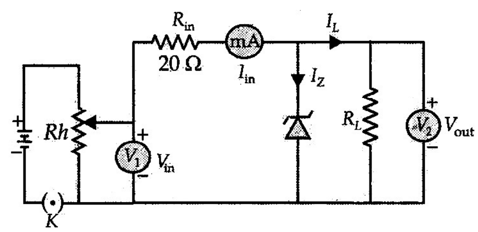
The characteristic curve (I-V curve) of a p-n junction diode is as shown below.



17. Characteristic curves of a Zener diode and finding reverse breakdown voltage

Zener diode : It was invented by C. Zener. It is designed to operate under reverse bias in the breakdown region and is used as a voltage regulator. The symbol for Zener diode is shown in the figure below.

The circuit diagram for plotting the characteristics of a Zener diode and determine its reverse breakdown voltage is as shown in the figure below.



Circuit parameters : In the circuit as shown in the figure above,

V_{in} = Input voltage

V_{out} = Output voltage

R_{in} = Input resistance

R_L = Load resistance

I_{in} = Input current

I_Z = Zener diode current

I_L = Load current

Relations

$I_L = I_{in} - I_Z$ ---------(32)

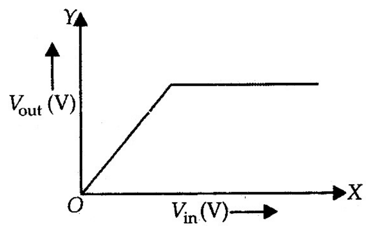
$V_{out} = V_{in} - R_{in}I_{in}$ ---------(33)

$V_{out} = R_L I_L$ ---------(34)

Initially as V_{in} is increased, I_{in} increases a little then V_{out} increases.

At breakdown, increase of V_{in} increases I_{in} by large amount, so that $V_{out} = V_{in} - R_{in} I_{in}$ becomes constant. This constant value of V_{out} which is the reverse breakdown voltage is called Zener voltage.

The graph between input voltage (V_m) and output voltage V_{out} is as shown below.



The constant output voltage of the above graph is the reverse breakdown voltage of Zener diode.

18. <u>Identification of Diode, LED, Resistor, a capacitor from a mixed collection of such items.</u>

The diode and LED (light emitting diode) conduct when forward biased and do not conduct when reverse biased. The LED when forward biased, not only conducts but emits light also, which helps to distinguish between junction diode and LED.

Connect the two probes of the multimeter to the two end terminals of the diode and note whether the resistance is low or high. Then interchange the two probes and again note the resistance whether it is high or low. If the resistance of the diode is high in the first case, it will be low in the second case or vice versa. It shows the unidirectional flow of current in diode.

For LED : Repeat the above process for LED. The LED glows by emitting light, when its resistance is low. This shows unidirectional flow of current through an LED.

Checking Whether Diode, Transistor is in Working Order

A diode will conduct only in one direction i.e., first connect the ends of diode to the two metal ends of the probes and reverse the connecting points. If it conducts in one case, then diode is in working order. If it conducts in both cases or does not conduct in both cases, then it is damaged.

A resistor is a two terminal device. It conduct both with d.c. voltage and a.c. voltage. Further, a resistor conducts equally even when terminals of d.c. battery are reversed.

A capacitor is a two terminal device which does not conduct with d.c. voltage applied either way. But conducts with a.c. voltage.

Procedure

1. The component having two legs may either be a junction diode or capacitor or resistor or a light emitting diode. These items can be distinguished from each other by using a multimeter.
2. Touch the probles to the two ends of each item and observe the deflection on the resistance scale. After this, interchange the two probes and again observe the deflection.
3. a) If the same constant deflection is observed in the two cases (before and after interchanging the probes) the item under observation is a resistor.

   b) If unequal deflections are observed in the two cases, it is a junction diode.

   c) If unequal deflections are observed in the two cases along with emission of light in the case when deflection is large, the item under observation is a light emitting diode (LED).

   d) On touching the probes, if large deflection is observed, which then gradually decreases to zero, the item under observation is a capacitor.

In case of capacity of the capacitor is of the order of picofarad, then the deflection will become zero within no time.