USN 10CS34

Third Semester B.E. Degree Examination, June 2012

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Let $S = \{21, 22, 23, \ldots, 39, 40\}$. Determine the number of subsets A of S such that :
   i) $|A| = 5$
   ii) $|A| = 5$ and the largest element in A is 30
   iii) $|A| = 5$ and the largest element is at least 30
   iv) $|A| = 5$ and the largest element is at most 30
   v) $|A| = 5$ and A consists only of odd integers. **(10 Marks)**

b. Prove or disprove: For non-empty sets A and B, $P(A \cup B) = P(A) \cup P(B)$ where P denotes power set. **(05 Marks)**

c. In a group of 30 people, it was found that 15 people like Rasagulla, 17 like Mysorepak, 15 like Champakali, 8 like Rasagulla and Mysorepak, 11 like Mysorepak and Champakali, 8 like Champakali and Rasagulla and 5 like all three. If a person is chosen from this group, what is the probability that the person will like exactly 2 sweets? **(05 Marks)**

2 a. Verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. **(05 Marks)**

b. Write dual, negation, converse, inverse and contrapositive of the statement given below :
   If Kabir wears brown pant, then he will wear white shirt. **(05 Marks)**

c. Define $(p \uparrow q) \Leftrightarrow \neg(p \wedge q)$. Represent $p \vee q$ and $p \rightarrow q$ using only $\uparrow$. **(05 Marks)**

d. Establish the validity or provide a counter example to show the invalidity of the following arguments : **(05 Marks)**

i)
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg r \\ \hline \therefore q \end{array}$$

ii)
$$\begin{array}{l} p \\ p \rightarrow r \\ p \rightarrow (q \vee \neg r) \\ \neg q \vee \neg s \\ \hline \therefore s \end{array}$$

3 a. For the universe of all polygons with three or four sides, define the following open statements:
   i(x): all the interior angles of x are equal
   h(x): all sides of x are equal
   s(x): x is a square
   t(x): x is a triangle

Translate each of the following statements into an English sentence and determine its truth value:
   i) $\forall x\, [s(x) \leftrightarrow (i(x) \wedge h(x))]$
   ii) $\exists x\, [t(x) \rightarrow (i(x) \leftrightarrow h(x))]$

Write the following statements symbolically and determine their truth values.
   iii) Any polygon with three or four sides is either a triangle or a square
   iv) For any triangle if all the interior angles are not equal, then all its sides are not equal. **(08 Marks)**

3 b. Let p(x, y) denote the open statement x divides where the universe consists of all integers. Determine the truth values of the following statements. Justify your answers.
i) $\forall x\ \forall y\ [p(x, y) \wedge p(y, x) \rightarrow (x = y)]$ ii) $\forall x\ \forall y\ [p(x, y) \vee p(y, x)]$ **(06 Marks)**

c. Prove that for every integer n, n^2 is even if and only if n is even. **(06 Marks)**

4 a. Prove $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \in z^+$. **(06 Marks)**

b. Prove $2^n < n! \quad \forall\ n > 3$ and $n \in z^+$. **(06 Marks)**

c. Define an integer sequence recursively by
$a_0 = a_1 = a_2 = 1$
$a_n = a_{n-1} + a_{n-3}\ \forall\ n \geq 3$.
Prove that $a_{n+2} \geq (\sqrt{2})^n \quad \forall\ n \geq 0$. **(08 Marks)**

PART – B

5 Let $A = \{\alpha, \beta, \gamma\}$, $B = \{\theta, \eta\}$, $C = \{\lambda, \mu, \nu\}$.

a. Find $(A\cup B)\times C$, $A\cup(B\times C)$, $(A\times B)\cup C$ and $A\times(B\cup C)$. **(08 Marks)**

b. Give an example of a relation from A to B × B which is not a function. **(04 Marks)**

c. How many onto functions are there from (i) A to B, (ii) B to A? **(02 Marks)**

d. i) Write a function $f : A\rightarrow C$ and a function $g : C\rightarrow A$. Find $g\circ f : A\rightarrow A$.
ii) Write an invertible function $f : A\rightarrow C$ and find its inverse. **(06 Marks)**

6 a. Let $A = \{1, 2, 3, 4\}$, $B = \{w, x, y, z\}$ and $C = \{p, q, r, s\}$. Consider $R_1 = \{(1, x), (2, w), (3, z)\}$ a relation from A to B, $R_2 = \{(w, p), (z, q), (y, s), (x, p)\}$ a relation from B to C.
i) What is the composite relation $R_1\circ R_2$ form A to C?
ii) Write relation matrices $M(R_1)$, $M(R_2)$ and $M(R1\circ R2)$
iii) Verify $M(R_1)\cdot M(R_2) = M(R_1\circ R_2)$ **(06 Marks)**

b. Let $A = \{1, 2, 3, 6, 9, 12, 18\}$ and define a relation R on A as xRy iff x|y. Draw the Hasse diagram for the poset (A, R). **(06 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ and define R as $(x_1, y_1)R(x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation on A.
ii) Determine the equivalence class [(1, 3)].
iii) Determine the partition induced by R. **(08 Marks)**

7 a. Define a binary operation * on Z as $x * y = x + y - 1$. Verify that (Z, *) is an abelian group. **(07 Marks)**

b. Let $f : G \rightarrow H$ be a group homomorphism onto H. If G is an abelian group, prove that H is also abelian. **(07 Marks)**

c. The encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.
i) Determine all the code words.
ii) Find the associated parity-check matrix H. **(06 Marks)**

8 a. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$, prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a unit of this ring if and only if $ad - bc \neq 0$. **(08 Marks)**

b. Let R be a ring with unity and a, b be units in R. Prove that ab is a unit of R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(06 Marks)**

c. Find multiplicative inverse of each (non-zero) element of Z_7. **(06 Marks)**

* * * * *