Summary of research

***Abstract*—**

I. Introduction

About BICM

For achieving higher spectral efficiencies coded modulation combines higher order modulation with channel coding.

Bit Interleaved Coded Modulation (BICM) [1] creates code-diversity at individual bit level. This is achieved by bit-wise interleaving at encoder output. This breaks the correlation induced by the modulation and adds additional redundancy against bit errors.

About DBICM

Delayed Bit Interleaved Coded Modulation (DBICM) is an extension of BICM. Interleaved codeword is segmented into $log_2(M) = m$ parts and a pre-defined delay sequence of length m (for eg: [0 1 0 ... 1]) will determine which segments will be delayed. Initially decoded segments of a particular codeword can be used to demap symbols containing segments of the same codeword. This improves the performance.

- In other words, extrinsic information generated from decoder can be used again for better demapping of symbols [2], which leads to improved accuracy in information bit estimation.

II. Low complexity Delay Scheme Optimization

In this reseach, we consider AWGN channel, where X denotes the transmit symbol, Y denotes the received symbol and Z denotes the Gaussian noise.

$$Y = X + Z \tag{1}$$

DBICM as a modified version of BICM, can decrease the achievable rate gap between coded modulation (CM) and BICM. Capacity of DBICM depends on the specific delay schemes used in the bit delay module before modulation. Depending on the choice of constellation, a best delay scheme that maximizes the DBICM capacity can be obtained.

Authors of [3] have considered gray labelled QAM constellations and analysed the effect of the delay scheme on the performance. The task of obtaining the best delay scheme for gray labelled QAM constellations has been converted to a simpler problem of obtaining the best delay scheme of the constituent PAM constellation in [3].

Authors of [4] have considered APSK constellations of various sizes in order to analyse the effect of delay schemes on the performance. Bit delay scheme optimization requires calculation of the DBICM capacity using expensive numerical integration (For example when using 16-QAM, BICM mutual

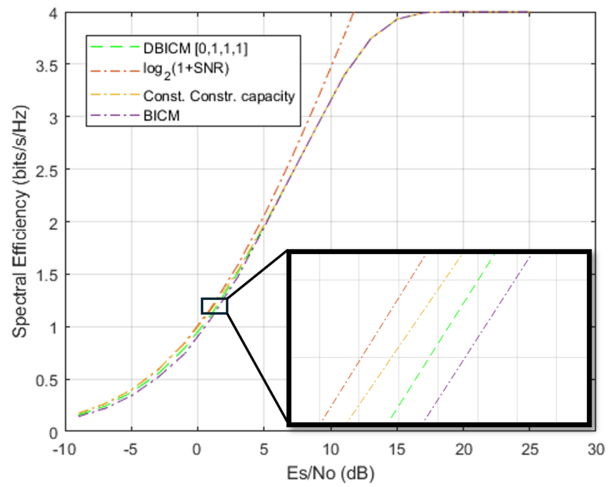

Fig. 1: DBICM mutual information for $[0, 1, 1, 1]$ delay scheme, BICM mutual information, Constellation constrained (CM) capacity and $\log_2(1 + \mathrm{SNR})$ for 16-QAM gray labelled constellation.

information calculation for the accuracy of third decimal requires generating 10^6 random modulated symbols for monte-carlo averaging).

Considering an M-ary constellation ($\log_2 M = m$), capacity of BICM using mutual information notation is given by

$$I_{\mathrm{BICM}} = \sum_{i=0}^{m-1} I\left(B_i; Y\right), \tag{2}$$

where $I\left(B_i; Y\right)$ denotes the i^{th} bit-channel capacity for BICM, which can be expressed as below expectation [1],

$$I\left(B_i; Y\right) = 1 - \mathop{\mathbb{E}}_{b,\mathbf{y}}\left[\log_2 \frac{\sum_{\mathbf{z}\in\mathcal{X}} p(\mathbf{y} \mid \mathbf{z})}{\sum_{\mathbf{z}\in\mathcal{X}_b^i} p(\mathbf{y} \mid \mathbf{z})}\right]. \tag{3}$$

B_i denotes the i^{th} bit of labels corresponding to constellation symbols.

Coded modulation (CM) capacity which is also known as constellation constrained capacity is given by below mutual information expression.

$$I(X; Y) = I\left(B_0, \ldots, B_{m-1}; Y\right) \tag{4}$$

Let's consider an 16-ary constellation and a delay scheme $T = [0, 1, 1, 1]$. DBICM capacity can be expressed using mutual information as follows,

$$I^{T}_{\mathrm{DBICM}} = I\left(B_0;Y|B_1,B_2,B_3\right) + I\left(B_1;Y\right) + I\left(B_2;Y\right) + I\left(B_3;Y\right) \quad (5)$$

All the defined capacity curves are plotted in Figure 1. DBICM capacity curves vary in between BICM and CM capacity based on the selected DBICM delay scheme. In order to obtain the best delay scheme for a constellation at a given SNR, we need to numerically obtain the mutual information for all possible delay schemes and then obtain the delay scheme that results in the largest capacity. This task becomes more complex with increasing constellation size.

Each delay time slot can take values from 0 to $m-1$, the maximum of which is called the maximum delay T_{max}, $0 \leq T_{\text{max}} \leq m-1$. Delay schemes can be found through exhaustive search, but m^m possibilities need to be checked through numerical integration.

Key idea in this paper aligns with a low complexity algorithm used in bit-labeling optimization of BICM-ID systems [5]. In their work, instead of using mutual information as the cost function for the binary switching algorithm for bit labeling optimization, low complexity cost functions in equations 6, 7 are used. D^r and D are attributable to rayleigh and AWGN channels. $\frac{1}{D^r}$ is known as the *harmonic mean* of the squared euclidean distance. D^r and D are associated with BICM Union Bound (UB) for pair-wise error probability (PEP) in [1].

$$D^r = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{s_k\in\chi_b^i}\sum_{\hat{s}_k\in\chi_{\bar{b}}^i}\frac{1}{\left|s_k-\hat{s}_k\right|^2} \quad (6)$$

$$D = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{s_k\in\chi_b^i}\sum_{s_k\in\chi_{\bar{b}}^i}\exp\left(-\frac{E_s}{4N_0}\left|s_k-\hat{s}_k\right|^2\right) \quad (7)$$

In a similar way, we propose an algorithm for delay scheme optimization in AWGN channel based on a low complexity cost function. For this purpose we use equation 8 associated with the BICM expurgated bound (EX) in [1]. The last summation in equation 7 is reduced to a single element, which is $\hat{z}_k$ the nearest neighbour of s_k in $\chi_{\bar{b}}^i$.

$$D_{ex} = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{s_k\in\chi_b^i}\exp\left(-\frac{E_s}{4N_0}\left|s_k-\hat{z}_k\right|^2\right) \quad (8)$$

In order to take into account the effect of apriori information due to the delay scheme in DBICM, we use the modified version of equation 8. An almost similar cost function is introduced for bit labeling optimization for DBICM-ID in [6]. The cost function below is only applicable to maximum delay $T_{\text{max}} = 1$ case.

$$D^T_{dbicm} = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{s_k\in\chi_b^i}\exp\left(-\frac{E_s}{4N_0}\left|s_k-\hat{x}_k\right|^2\right) \quad (9)$$

where T is the delay scheme and,

$$\hat{x}_k = \begin{cases}\hat{z}_k & \text{if } T(k)\neq 0\\ \hat{z}^T_k & \text{if } T(k)=0\end{cases} \quad (10)$$

$\hat{z}^T_k$ is the nearest neighbour of s_k in $\chi_{\bar{b}}^i$, whose bits in the delayed positions are exactly equal to that of the s_k.

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TABLE I: Optimum Delay schemes of Gray labelled M-QAM compared to best delay schemes obtained using our method.

Modulation	Rate	Optimal Delay Scheme	Gap to CM (dB)	Gain over BICM (dB)	Best Delay Scheme (Our)	Gap to Optimal Scheme (dB)
16-QAM	$1/4$	$[0, 1, 0, 1]$	0	0.55		
	$1/3$	$[0, 1, 0, 1]$	0	0.4		
	$2/5$	$[0, 1, 0, 1]$	0	0.3		
	$1/2$	$[0, 1, 0, 1]$	0	0.2	$[0, 1, 0, 1]$	0
64-QAM	$1/4$	$[1, 0, 1, 1, 0, 1]$	0.15	0.7		
	$1/3$	$[0, 1, 0, 0, 1, 0]$	0.15	0.6		
	$2/5$	$[0, 0, 1, 0, 0, 1]$	0.1	0.55		
	$1/2$	$[0, 0, 1, 0, 0, 1]$	0.01	0.45	$[0, 0, 1, 0, 0, 1]$	0
256-QAM	$1/4$	$[0, 0, 1, 1, 0, 0, 1, 1]$	0.3	0.65		
	$1/3$	$[1, 1, 0, 1, 1, 1, 0, 1]$	0.25	0.65		
	$2/5$	$[0, 0, 1, 1, 0, 0, 1, 1]$	0.25	0.65		
	$1/2$	$[0, 0, 0, 1, 0, 0, 0, 1]$	0.15	0.6	$[0, 0, 0, 1, 0, 0, 0, 1]$	0
1024-QAM	$1/4$	$[1, 1, 0, 1, 1, 1, 1, 0, 1, 1]$	0.25	0.85		
	$1/3$	$[1, 0, 0, 1, 1, 1, 0, 0, 1, 1]$	0.25	0.8		
	$2/5$	$[0, 0, 0, 1, 1, 0, 0, 0, 1, 1]$	0.25	0.8		
	$1/2$	$[0, 0, 0, 1, 1, 0, 0, 0, 1, 1]$	0.25	0.65	$[0, 0, 0, 1, 1, 0, 0, 0, 1, 1]$	0