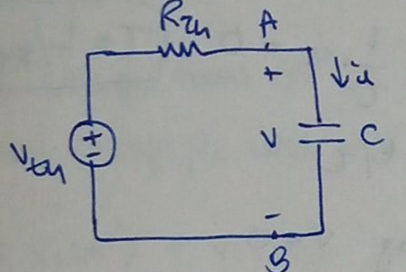
Week 2 : Lecture 7

RC/RL circuits in time domain - 1

* RC circuits with DC sources

If all sources are DC (const.), V_{TH} = constant.



KVL : $V_{TH} = R_{TH} i + v$

$$V_{TH} = R_{TH} C \frac{dv}{dt} + v$$

$$\frac{dv}{dt} + \frac{v}{R_{TH} C} = \frac{V_{TH}}{R_{TH} C}$$

- Homogeneous soln.

$$\frac{dv}{dt} + \frac{1}{\tau} v = 0 \qquad \boxed{\tau = R_{TH} C}$$

Time const.

$$\boxed{v^{(h)} = K e^{-t/\tau}}$$

- Particular soln → specific fn that satisfies differential equation.

Consider sinusoidal steady state;

time derivatives vanish at $t \to \infty$ ⇒ $i = 0$

$$\boxed{v^{(p)} = V_{TH}}$$

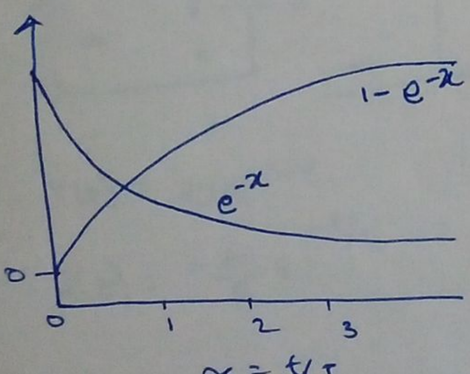
$$v = v^{(h)} + v^{(p)} = K e^{-t/\tau} + V_{TH}.$$

$$\boxed{v(t) = A e^{-t/\tau} + B}$$

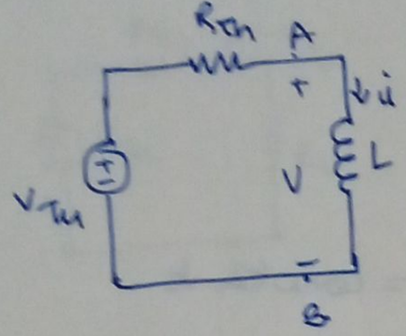
where A & B are known condns of v.

$$i(t) = C \frac{dv}{dt} = C \left(A e^{-t/\tau}\right)\left(\frac{-1}{\tau}\right) = A' e^{-t/\tau}$$

@ $t \to \infty$, $i \to 0$ ⇒ capacitor behaves like an <u>open circuit</u>.



* RL circuits with DC sources.



If all sources are DC (constant),

V_{Th} = constant.

KVL: $V_{Th} = R_{Th} i + L\frac{di}{dt}$.

Homogeneous solution

$$\frac{di}{dt} + \frac{1}{\tau} i = 0 \qquad \boxed{\tau = L/R_{Th}}$$

$$i^{(h)} = ke^{-t/\tau}.$$

Particular soln $\Rightarrow$ $t \to \infty$ making $v = 0$

$$i^{(p)} = V_{Th}/R_{Th}.$$

$$i = i^{(h)} + i^{(p)} = ke^{-t/\tau} + V_{Th}/R_{Th}.$$

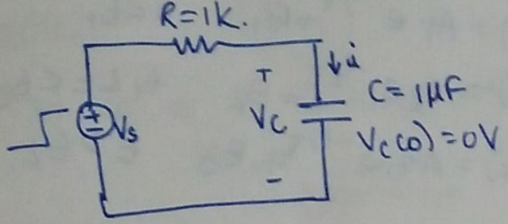
$$i(t) = Ae^{-t/\tau} + B.$$ where A & B are known condns of i.

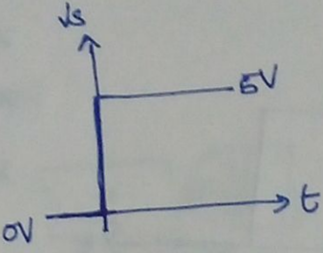
$$v(t) = L\frac{di}{dt} = L \times Ae^{-t/\tau}\left(-\frac{1}{\tau}\right) = A'e^{-t/\tau}$$

As $t \to \infty$, $v \to 0$ i.e the inductor behaves like a short circuit.

Lecture 8 — RC/RL circuits in time domain. 2

* Can V_c change "suddenly"?





V_s changes from 0V (at $t=0^-$) to 5V (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?

$\Rightarrow V_c(0^+) = V_c(0^-)$ $\Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.

Say, V_c changes 1V in 1μs $\Rightarrow \frac{1V}{1\mu s} = 10^6 \, V/s$

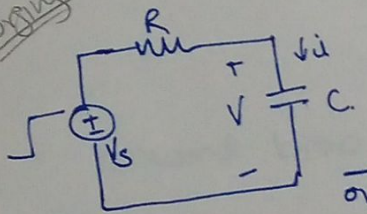
$$i = C\frac{dV_c}{dt} = 1\mu F \times 10^6 \frac{V}{s} = 1A$$

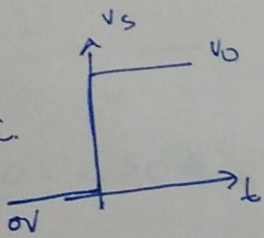
$R = 1000V \rightarrow$ Not possible!

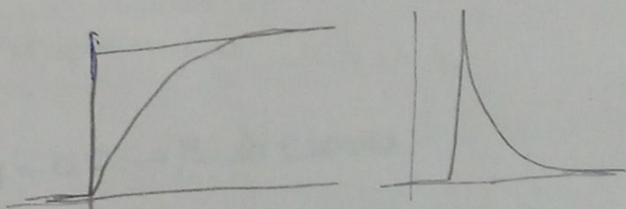
$\Rightarrow$ Similarly an inductor does not allow abrupt changes in i_L.

* RC circuits: Charging and discharging transients

charging



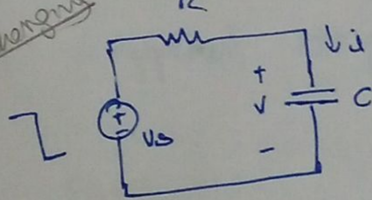


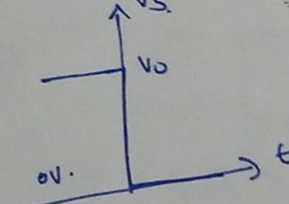


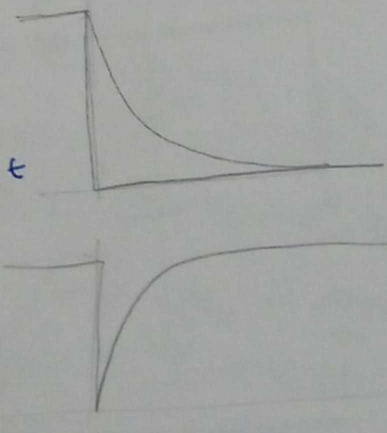
$$i(t) = \frac{V_0}{R} e^{-t/\tau}.$$

$$V(t) = V_0\left[1 - e^{-t/\tau}\right]$$

discharging





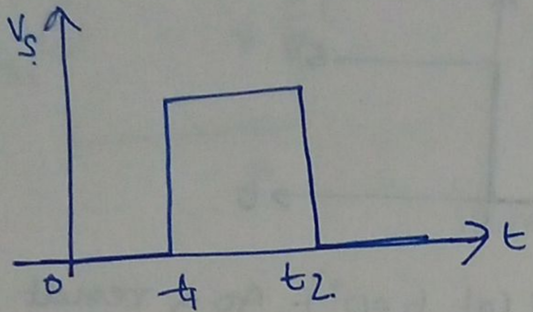


$$V(t) = V_0 e^{-t/\tau}.$$

$$i(t) = -\frac{V_0}{R} e^{-t/\tau}$$

Lecture 9:

Analysis of RC/RL circuits with a piece wise constant source



$x(t) = A_1 e^{-t/\tau} + B_1 \quad t < t_1$

$x(t) = A_2 e^{-t/\tau} + B_2 \quad t_1 < t < t_2$

$x(t) = A_3 e^{-t/\tau} + B_3 \quad t > t_2$

Steps: Identify $A_1 \cdots A_3$, $B_1 \cdots B_3$, τ, & $x(t)$ in each piece.

Calculate $v(t)$

Then use that to Calculate $i(t)$

(or)

Calculate $i(t)$ from scratch.

How? – Consider to time instances $t(0^+)$ & $t(\infty)$

use that to find A & B.

Substitute back in Eqn with $v(t)$ or $i(t)$ value,

to get final Equation.