Program Evaluation and Review Technique (PERT): A Step-by-Step Guide

PERT is a great method to estimate project durations where there is uncertainty around project activity times.

In this example, you are shown three task durations. This is because you do not know exactly how long each task will take.

You will use the Rooftop Garden scenario to prepare a PERT analysis to answer the following questions:

1. What is the mean project completion time?
2. What is the standard deviation of the project completion time?
3. What is the probability that the project will be completed within 85 days?
4. What is the probability that the project will be completed within 70 days?

Step 1

Determine the task duration estimates of optimistic duration (A), most likely duration (M), and pessimistic duration (B). These have been completed for you in the spreadsheet.

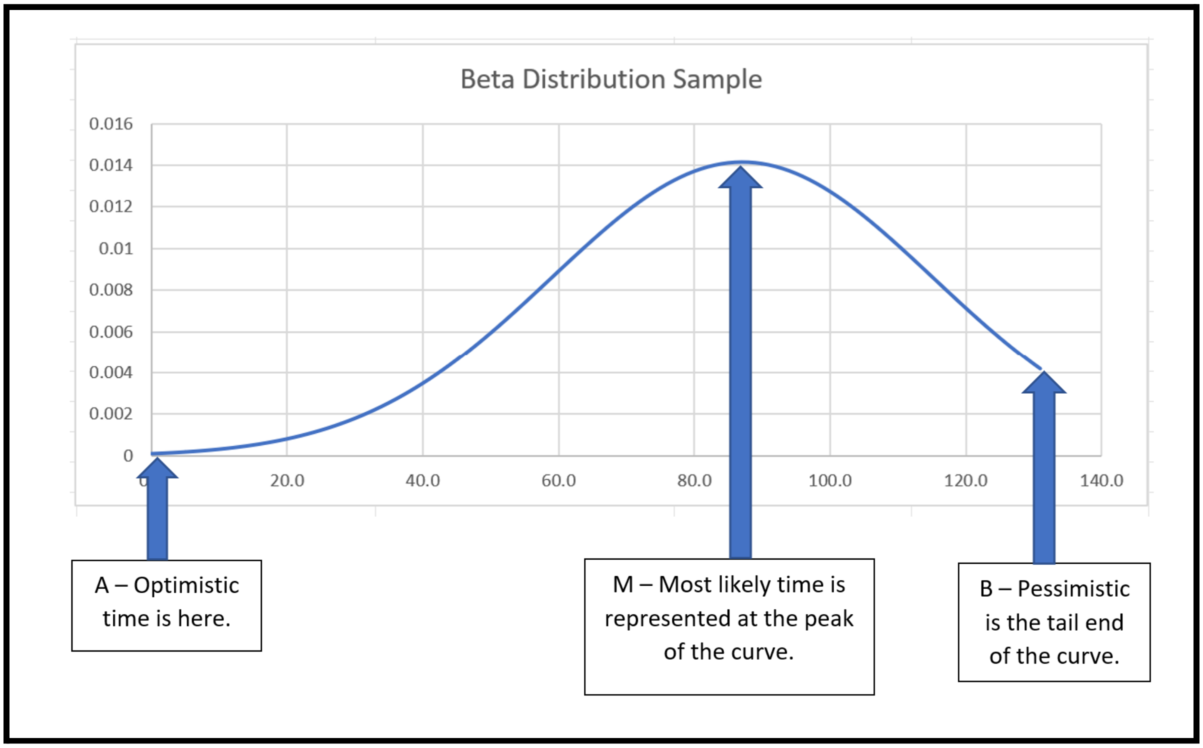
Step 2

Assume that each activity is a random variable that has a beta distribution. There are three variables for the beta distribution, A, M, and B (optimistic, most likely, and pessimistic). Compute the mean expected time TE and the time variance TV for each activity using the following formulas:

$$TE = \frac{A+4M+B}{6}$$

$$TV = \left(\frac{B-A}{6}\right)^2$$

The beta distribution of each task will look like the curve in the Beta Distribution Sample graphic below. Look for the leading tail of the curve (optimistic duration, A), the peak of the curve (most likely duration, M), and the tail end of the curve (pessimistic duration, B).



For demonstration purposes, the TE and TV have been calculated for you. Look at the mean time and variance calculations for tasks A, B, and C.

Task ID	Task name	Predecessor	A — Optimistic Duration (days)	M — Most Likely Duration (days)	B — Pessimistic Duration (days)
A	Allocation of spaces	Start	14	16	21
B	Creating modular spaces	A	10	12	18
C	Building the planting structure (beds)	A	5	6	9

Task A:

$$TE = \frac{14+4(16)+21}{6} = 16.5\ (mean\ expected\ time)$$

$$TV = (\frac{21-14}{6})^2 = 1.36\ (time\ variance)$$

Task B:

$$TE = \frac{10+4(12)+18}{6} = 12.7\ (mean\ expected\ time)$$

$$TV = \left(\frac{18-10}{6}\right)^2 = 1.78\ (time\ variance)$$

Task C:

$$TE = \frac{5+4(6)+9}{6} = 6.3\ (mean\ expected\ time)$$

$$TV = \left(\frac{9-5}{6}\right)^2 = 0.44\ (time\ variance)$$

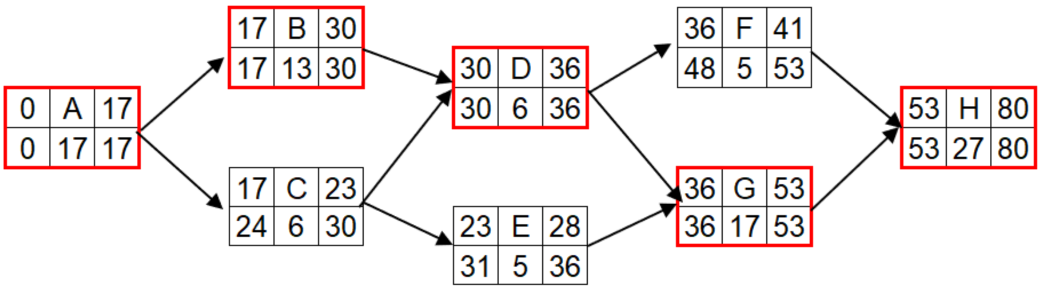
Note: Review the formulas in cells G12 and I12 in the Excel spreadsheet.

Step 3

Find the critical path for the project using the mean expected time as the task duration. Draw the network diagram and use the forward pass and backward pass methods to determine the critical path.

Task ID	Task name	Predecessor	A — Optimistic Duration (days)	M — Most Likely Duration (days)	B — Pessimistic Duration (days)
A	Allocation of spaces	Start	14	16	21
B	Creating modular spaces	A	10	12	18
C	Building the planting structure (beds)	A	5	6	9
D	Installing irrigation and testing	B, C	5	6	9
E	Installing lighting	C	3	5	7
F	Setting up the recreational space	D	3	5	7
G	Reservation process (identify and release)	D, E	14	16	22
H	Gathering and assigning building volunteers	F, G	21	26	38

For the Rooftop Garden project, the critical path follows A B D G H for a total expected duration of 80 days. Remember, you do not know exactly how long the project will take; this is the mean or average of the optimistic, most likely, and pessimistic time estimates.



The project duration is a sum of the critical activity times. The **central limit theorem** states that if you have a population with mean μ and standard deviation σ and take sufficiently large random samples from the population with replacement, then the distribution of the sample means will be approximately normally distributed. Therefore, the results from the sum of the duration of critical activities will provide you with an approximately normal distribution.

Step 4

Determine the normal distribution and likelihood that the project will complete on time. Because the project duration (D) is the sum of the critical path task times, assume that D will follow a normal distribution. For any distribution, you need to know the mean and the standard deviation or variation. With this information, you can determine the probability that a project will be completed by a specified time.

For the Rooftop Garden project, you need the following information to determine the probability that the project will be completed by a specified time:

$$\mu = E(D) = sum\ of\ expected\ times\ for\ the\ critical\ path\ activities$$

$$\sigma^2 = sum\ of\ the\ variance\ of\ critical\ path\ activities$$

$$\sigma = square\ root\ of\ \sigma^2$$

In this example, the sum of expected times for the critical path activities has been calculated:

$$\mu = 80$$

To determine σ^2, the sum of the variance of critical path activities, add the variances of the critical path tasks (highlighted in the table below).

Task ID	Task name	Predecessor	Mean Expected Time (TE)	Time Variance (TV)
A	Allocation of spaces	Start	17	1.36
B	Creating modular spaces	A	13	1.78
C	Building the planting structure (beds)	A	6	0.44
D	Installing irrigation and testing	B, C	6	0.44
E	Installing lighting	C	5	0.44
F	Setting up the recreational space	D	5	0.44
G	Reservation process (identify and release)	D, E	17	1.78
H	Gathering and assigning building volunteers	F, G	27	8.03

$$\sigma^2 = 1.36 + 1.78 + 0.44 + 1.78 + 8.03 = 13.39$$

$$\sigma = \sqrt{13.39} = 3.66$$

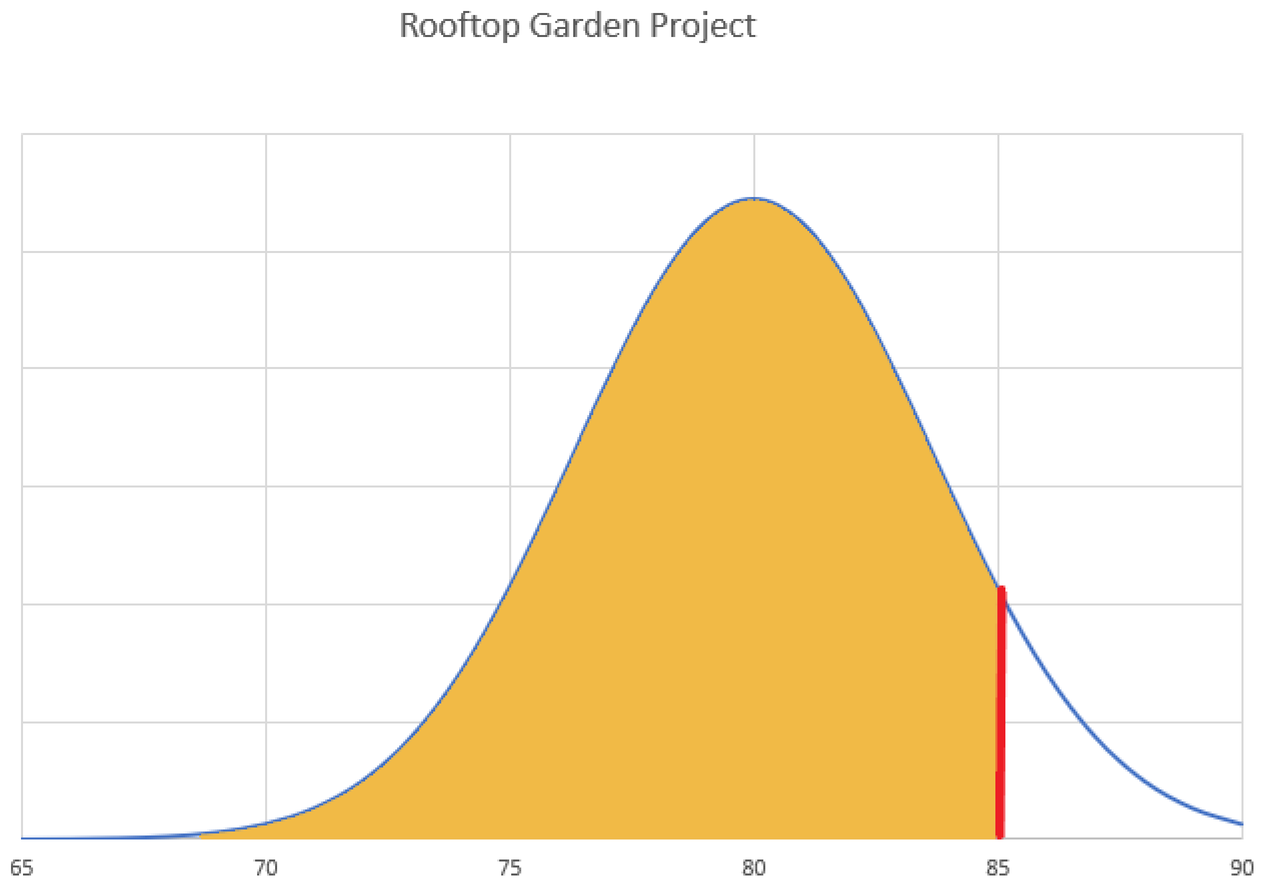
So far, you are able to answer Questions 1 and 2 based on the information above:

1. What is the mean project completion time? **80 days**
2. What is the standard deviation of the project completion time? **3.66**

With the first two questions answered, you can now answer Questions 3 and 4:

3. What is the probability that the project will be completed within 85 days?
4. What is the probability that the project will be completed within 70 days?

For Question 3, you want to know if the project can be completed within 85 days. You want to determine everything that is under the curve up to day 85 (the highlighted area in the graph below):



For this task, you must convert the number 85 to its **z-value**. The z-value is a test statistic for z-tests that measures the difference between an observed statistic and its hypothesized population parameter in units of the standard deviation. The z-value is the number of standard deviations away from the mean.

You can convert the target number to the z-value using the following formula:

$$z = \frac{x - \mu}{\sigma}$$

To find the probability that the project will be completed in less than or equal to 85 days, first calculate the z-value:

$$z = \frac{85 - 80}{3.66} = 1.37$$

Using the z-value 1.37, you can find the z-score in a normal distribution table (ZTable.net) by finding the number that corresponds to the row labeled with “1.3” and the column labeled with “.07”:

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

What is the probability that the project will be completed within 85 days?

With a z-score of .91466, you can say with confidence that the project has about a 92% probability of completing within 85 days.

What is the probability that the project will be completed within 70 days?

Using the z-value formula, you can determine the z-score and the probability:

$$z = \frac{70-80}{3.66} = \; -2.73$$

The z-score for − 2.73 is 0.00317, or 0.3% probability of completing the project within 70 days.

You can conclude the following from the PERT analysis for the Rooftop Garden project:

- The time estimates indicate that regardless of the optimistic and pessimistic times, the project will operate on a very tight time schedule.
- There is little to no float time available.
- Project scheduling and resource availability will be critical.

Note: You can also use Excel to calculate the z-score using the following formula:

=NORM.DIST (x, mean, standard_dev, cumulative)

The target value is *x*. Cumulative should be set to 1 or 0, the equivalent of “true” or “false”. If cumulative is set to true, Excel returns the area under the curve. If cumulative is set to false, Excel finds the height of the curve. Using Excel, the formula for Question 3 should look like this:

=NORM.DIST(85, 80, 3.66, true)

You will notice that the result (0.914049383) is slightly different from the z-score but not too far off.