Formula (Energy & power signals)

Infinite series summation formula

1. $$\sum_{n=0}^{\alpha} a^n = \left| \frac{1}{1-a} \right| \qquad a<1$$

2. $$\sum_{n=-\alpha}^{\alpha} 1 = \alpha$$

Finite series summation formula

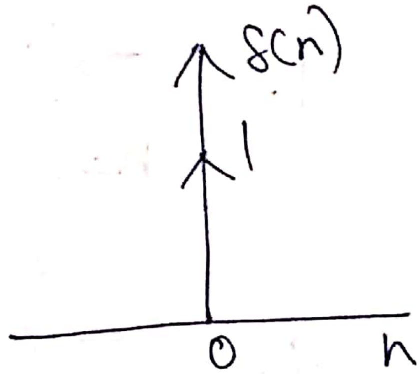
3. $$\sum_{n=0}^{N} (a)^n = \frac{1-a^{N+1}}{1-a} \qquad a<1$$

4. $$\sum_{n=-N_1}^{N_2} 1 = N_2 - (-N_1) + 1$$

Elementary Signal

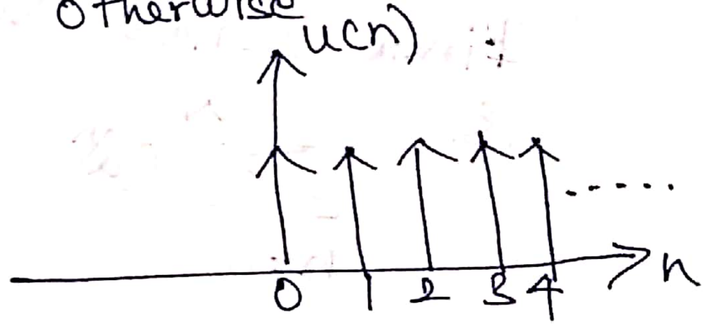
1. Unit impulse signal (or Sample signal)

$$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & n \neq 0 \end{cases}$$



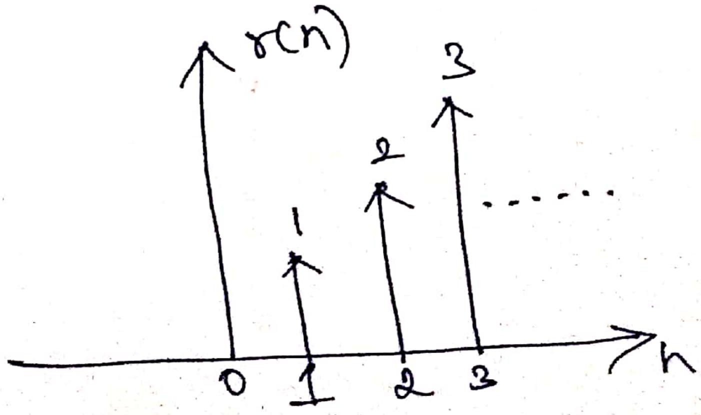
2. Unit step signal

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



3. Unit ramp signal

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



Energy signal:

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2 \text{ Joules}$$

$$0 < E < \infty \qquad P = 0$$

Power signal:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \text{ Watts}$$

$$0 < P < \infty \qquad E = \infty$$

pbm1 Check whether the signal is Energy (or) power signal.

$$x(n) = u(n)$$

soln Energy:

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \sum_{n=0}^{N} 1$$

$$\sum_{n=-N_1}^{N_2} 1 = N_2 - (-N_1) + 1$$

$$= \lim_{N \to \infty} N + 1$$

$$\boxed{E = \infty}$$

Power:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} N+1$$

$$= \lim_{N \to \infty} \frac{1 + 1/N}{2 + 1/N}$$

$$P = \frac{1}{2}$$

$$0 < \frac{1}{2} < \infty$$

$E = \infty$

$P = \frac{1}{2}$ watts $\Rightarrow$ Hence the signal is power signal

pbm2 Check whether the signal is Energy (or) power signal

$$y(n) = \delta(n)$$

Soln Energy:

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} \delta(n)$$

$$E = \lim_{N \to \infty} \sum_{n=0} 1$$

$$\boxed{E = 1 \text{ Joules}}$$

Power!

Power $P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |\delta(n)|^2$$

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=0} 1$$

$$P = 0$$

$0 < 1 < \alpha$

$E = 1$ Joules (finite) value

$P = 0$

Hence the signal is Energy signal.

pbm3 Check whether signal is power (or) Energy signal

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

soln Energy!

$$E = \lim_{N\to\alpha} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \lim_{N\to\alpha} \sum_{n=-N}^{N} \left|\left(\frac{1}{2}\right)^n u(n)\right|^2$$

$$E = \lim_{N\to\alpha} \sum_{n=-N}^{N} \left|\left(\frac{1}{2}\right)^2 u(n)\right|^n$$

(b)

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} \left(\frac{1}{4}\right)^n u(n)$$

$$E = \lim_{N \to \infty} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$\sum_{n=0}^{\infty} (a)^n = \left|\frac{1}{1-a}\right| \quad a < 1$$

$$E = \left|\frac{1}{1-\frac{1}{4}}\right| = \left|\frac{1}{\frac{4}{3}}\right|$$

$$E = \frac{3}{4} \text{ Joules}$$

$$0 < E < \infty \rightarrow 0 < 3/4 < \infty$$

Finite Value

Power:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$P = \lim_{N \to \infty} \left(\frac{1}{2N+1}\right) \frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-\frac{1}{4}}$$

$$P = \frac{1}{\infty} = 0 \qquad P = 0$$

$E = 3/4$

$P = 0$

$\rightarrow$ Energy sgl.

Pbm 4 Check whether the signal is Energy (or) power. 7

soln $x(n) = \sin\left(\frac{\pi}{4} n\right)$

Energy :-

$$E = \lim_{N \to \infty} \cancel{\frac{1}{2N+1}} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} \sin^2 \frac{\pi}{4} n$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} \frac{1 - \cos \cancel{2} \frac{\pi}{\cancel{4}2} n}{2}$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} \frac{1}{2}$$

$$= \frac{1}{2} \lim_{N \to \infty} \sum_{n=-N}^{N} 1$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} 1 = \frac{1}{2} \times \infty$$

$$\boxed{E = \infty}$$

Power!

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2 \frac{\pi}{4} n$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos 2\pi/_{2} n}{2}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \frac{1}{2} \sum_{n=-N}^{N} 1$$

$$= \frac{1}{2N+1} \cdot \frac{1}{2} (N-(-N)+1)$$

$$= \frac{1}{\cancel{2N+1}} \cdot \frac{1}{2} \cdot (\cancel{2N+1})$$

$$P = \frac{1}{2} \text{ watts}$$

$$P = 0 < \frac{1}{2} < \infty \qquad E = \infty$$

Hence the signal is Finite power signal.

pbm5 check whether the signal is Energy (or) power.

$$x(n) = 3e^{j3n}$$

soln

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |3e^{j3n}|^2$$

$$= \lim_{N \to \alpha} 9 \sum_{n=-N}^{N} 1$$

$$= \lim_{N \to \alpha} 9 \left[N - (-N) + 1 \right)$$

$$= \lim_{N \to \alpha} 9 (2N+1)$$

$$\boxed{E = \alpha}$$

Power:

$$P = \lim_{N \to \alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \alpha} \frac{1}{2N+1} 9 \sum_{n=-N}^{N} 1$$

$$P = \lim_{N \to \alpha} 9 \frac{1}{2N+1} (N - (-N) + 1)$$

$$= \lim_{N \to \alpha} 9 \frac{1}{\cancel{(2N+1)}} \cdot \cancel{(2N+1)}$$

$P = 9$ watts.

$P = 9$ watts $\quad E = \alpha$

Hence the signal is power signal.

pbmb

Check whether the ssl is Energy (or) power. 10

$$x(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$$

soln Energy:

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)} \right|^2$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} 1$$

$$E = \lim_{N\to\infty} N-(-N)+1$$

$$E = \lim_{N\to\infty} 2N+1$$

$$E = \infty$$

Power:

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)} \right|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} (N-(-N)+1)$$

$$P = \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cdot \cancel{2N+1}$$

$$P = 1 \text{ watt.}$$

$E = \infty$ Joules

$P = 1$ watt

Hence the signal is power signal.

Short trick:

(i) if $n = \infty$
Amplitude $= 0$ $\rightarrow$ Energy signal

(ii) if $n = \infty$
Amplitude $=$ Constant $\rightarrow$ power signal

(iii) if $n = \infty$
Amplitude $= \infty$ $\rightarrow$ Neither Energy Nor power signal.

(University Question) Find the Energy of the signal

1. $$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 3^n & n < 0 \end{cases}$$

Soln

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^n\right|^2 + \sum_{n=-\infty}^{-1} |2^n|^2$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^2\right|^n + \sum_{n=1}^{\infty} \left|(3^{-1})^n\right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n + \sum_{n=1}^{\infty} \left|(3^{-1})^2\right|^n$$

$$= \left|\frac{1}{1-\frac{1}{4}}\right| + \sum_{n=1}^{\infty} \left(\left(\frac{1}{3}\right)^2\right)^n$$

$$= \frac{4}{3} + \sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{4}{3} + \left(\frac{1}{9}\right)^1 + \left(\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right)^3 + \cdots$$

$$= \frac{4}{3} + \frac{1}{9}\left(1 + \frac{1}{9} + \left(\frac{1}{9}\right)^2 + \left(\frac{1}{9}\right)^3 + \cdots\right)$$

$$= \frac{4}{3} + \frac{1}{9}\left(\frac{1}{1-\frac{1}{9}}\right)$$

$$= \frac{4}{3} + \frac{1}{9} \cdot \frac{1}{\frac{9-1}{8}}$$

$$= \frac{4}{3} + \frac{1}{9} \cdot \frac{1}{\frac{9-1}{9}}$$

$$= \frac{4}{3} + \frac{1}{\cancel{9}} \cdot \frac{\cancel{9}}{8}$$

$$= \frac{4}{3} + \frac{1}{8}$$

$$= \frac{32+3}{24} = \frac{35}{24} \text{ Joules}$$

$$\boxed{E = \frac{35}{24} \text{ Joules}}$$

7. Determine whether the following signals are periodic (or) Not. if periodic determine the fundamental period.

(i) $\sin(0.01\pi n)$

(ii) $\cos(4n)$

(iii) $\cos\frac{\pi}{4}n + \sin\frac{\pi}{3}n$

soln (i) $\sin(0.01\pi n)$

soln $x(n) = \sin(0.01\pi n)$

$x(n) = \sin \omega n$

$\omega = 0.01\pi$

$$2\pi f = 0.01\pi$$

$$f = \frac{0.01\cancel{\pi}}{2\cancel{\pi}}$$

$$f = \frac{1}{200} = \frac{k}{N_0}$$ $k = 1$, $N_0 = 200$

Integer

→ periodic N_0 → Fundamental period $N_0 = 200$

(ii) $x(n) = \cos 4n$

soln $x(n) = \cos \omega n$

$$\omega = 4$$

$$2\pi f = 4$$

$$f = \frac{4}{2\pi} = \frac{2}{\pi} = \frac{k}{N_0}$$ $k \neq 1$, N_0 → Not integer

Hence the signal is

→ Not periodic.

(iii) $x(n) = \cos \frac{\pi}{4} n + \sin \frac{\pi}{3} n$

soln $x(n) = \cos \omega_1 n + \sin \omega_2 n$

$x_1(n) + x_2(n)$

$\omega_1 = \frac{\pi}{4}$	$\omega_2 = \frac{\pi}{3}$
$2\pi f_1 = \frac{\pi}{4}$	$2\pi f_2 = \frac{\pi}{3}$

$$f_1 = \frac{\frac{\pi}{4}}{2\pi} = \frac{\cancel{\pi}}{4 \times 2\cancel{\pi}} = \frac{1}{8} = \frac{k_1}{N_1}$$

$N_1 = 8$

$k_1 = 1$

(integer)

periodic

$$w_2 = \frac{\pi}{3}$$

$$2\pi f_2 = \frac{\pi}{3}$$

$$f_2 = \frac{\frac{\pi}{3}}{2\pi} = \frac{\pi}{3 \times 2\pi}$$

$$f_2 = \frac{1}{6} = \frac{k_2}{N_2}$$

$N_2 = 6$

$k_2 = 1$

(integer)

periodic

Fundamental period

$$\begin{array}{r|ll} 2 & 8 & 6 \\ \hline 2 & 4 & 3 \\ \hline & 2 & 3 \end{array}$$

$2 \times 2 \times 2 \times 3$

$= 24$

$\therefore x(n) \rightarrow$ periodic

period $N = 24$.

II Determine even & odd part of the signal.

$$x(n) = \{3, 2, \underset{\uparrow}{1}, 4, 5\}$$

Even sgl $x_e(n) = \dfrac{x(n)+x(-n)}{2}$

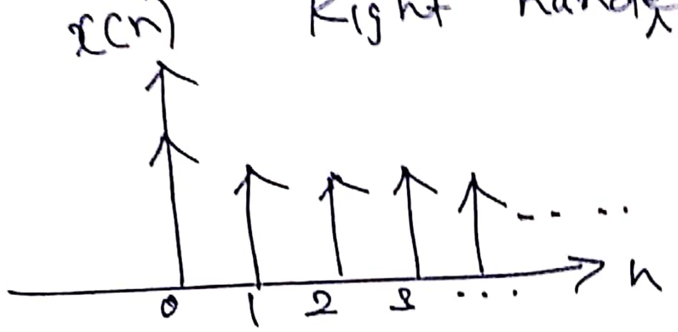
odd signal $x_o(n) = \dfrac{x(n)-x(-n)}{2}$

Causal & Non Causal signal

Causal

$x(n) = 0 \qquad n<0$

Right hand sided sequence



Non causal

$x(n) = 0 \qquad n>0$

Left hand sided sequence