Model No 5.3: **Test of significance for single proportion**

**(i) Null Hypothesis** $(H_0)$: $P = p$ i.e., "there is no significance difference between the sample proportion and population proportion" or "the sample has been drawn from the population"

**(ii) Alternative Hypothesis** $(H_1)$: (i) $P \neq p$ or (ii) $P < p$ or (iii) $P > p$

**(iii) Level of Significance** $(\alpha)$: Set a level of significance

**(iv) Test Statistic:** The test statistic $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}}$

**(v) Conclusion:** (i) If $|z| < z_\alpha$ we accept the Null Hypothesis $H_0$

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis $H_0$ i.e., we accept the Alternative Hypothesis $H_1$

**Problem 15: A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance. Also find the confidence interval.**

**Solution:** Sample size $n = 200$, faulty Items $= 18$, Good Items $= 200 - 18 = 182$

Sample proportion $p = \dfrac{182}{200} = 0.91$

Population proportion $P = 95\% = 0.95$, $Q = 1 - P = 1 - 0.95 = 0.05$

(i) Null Hypothesis $(H_0)$: $P = 95\%$

(ii) Alternative Hypothesis $(H_1)$: $P < 95\%$. Left Tailed Test

(iii) Level of Significance $(\alpha)$: $\alpha = 5\% = 0.05$ $\quad 0.5 - 0.05 = 0.45$ $\quad \boxed{Z_{tab} = 1.645}$

(iv) Test Statistic: The test statistic

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}} = -2.5955$$

(v) Conclusion: Tabulated value of $z_\alpha = 1.645$

Calculated value of $|z_\alpha| = 2.5955$

Calculated value of $|z_\alpha|$ > Tabulated value of $z_\alpha$

Null Hypothesis is Rejected

The confidence interval is

$p \pm Z_{\alpha/2}\sqrt{\dfrac{PQ}{n}}$ $\quad \alpha = 0.05$, $\alpha/2 = 0.025$

$0.5 - 0.025 = 0.475$

$\boxed{Z_{\alpha/2} = 1.96}$

$= 0.91 \pm 1.96\sqrt{\dfrac{(0.95)(0.05)}{200}}$

$= [0.8797, 0.9402]$

**Problem 16: In a random sample of 125 cool drinkers, 68 said they prefer thumsup to pepsi. Test the null hypothesis P=0.5 against the alternative hypothesis P>0.5.**

**Solution:** Sample Size $n=125$, Sample Proportion $p=\frac{68}{125}=0.544$

Population proportion $P=0.5$, $Q=0.5$

(i) Null Hypothesis $(H_0)$: $P=0.5$

(ii) Alternative Hypothesis $(H_1)$: $P>0.5$ [Right tailed Test]

(iii) Level of Significance $(\alpha)$: $\alpha=5\%=0.05$ $0.5-0.05=0.45$ $Z_{tab}=1.645$

(iv) Test Statistic: The test statistic

$$Z_{cal}=\frac{p-P}{\sqrt{\frac{PQ}{n}}}=\frac{0.544-0.5}{\sqrt{\frac{0.5\times0.5}{125}}}=0.9838$$

(v) Conclusion: Tabulated value of $Z_{tab}=1.645$

Calculated value of $Z_{cal}=0.9838$

Calculated value of $<$ Tabulated value of

Null Hypothesis is Accepted.

**Problem 17: In a sample of 500 from a village in Rajasthan, 280 are found to be wheat eaters and the rest rice eaters. Can we assume that the both articles are equally popular.**

**Solution:** Sample Size $n=500$

Sample Proportion $p=\frac{280}{500}=0.56$

Population proportion $P=0.5$

(i) Null Hypothesis $(H_0)$: $P=0.5$

(ii) Alternative Hypothesis $(H_1)$: $P\neq0.5$

(iii) Level of Significance $(\alpha)$: $\alpha=0.05$ $\alpha/2=0.025$ $0.5-0.025=0.475$ $Z_{tab}=1.96$

(iv) Test Statistic: The test statistic

$$Z_{cal}=\frac{p-P}{\sqrt{\frac{PQ}{n}}}=\frac{0.56-0.5}{\sqrt{\frac{0.5\times0.5}{500}}}=2.68$$

(v) Conclusion: Tabulated value of $Z_{tab}=1.96$

Calculated value of $Z_{cal}=2.68$

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected.

**Problem 18: A die was thrown 9000 times and of these 3220 yielded a 3 or 4. Is this consistent with the hypothesis that the die was unbiased?**

**Solution:** Sample Size $n=9000$

Sample proportion $p=\frac{3220}{9000}=0.3577$

Population proportion $P=(3 \text{ or } 4)=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}=0.3333$

$Q=0.6666$

(i) Null Hypothesis $(H_0)$: $P=\frac{1}{3}$

(ii) Alternative Hypothesis $(H_1)$: $P \neq \frac{1}{3}$

(iii) Level of Significance $(\alpha)$: $\alpha=0.05$ $\alpha/2=0.025$ $0.5-0.025=0.475$ $Z_{tab}=1.96$

(iv) Test Statistic: The test statistic

$$Z_{cal}=\frac{p-P}{\sqrt{\frac{PQ}{n}}}=\frac{0.3577-0.3333}{\sqrt{\frac{(0.3333)(0.6666)}{9000}}}=4.9108$$

(v) Conclusion: Tabulated value of $Z_{tab}=1.96$

Calculated value of $Z_{cal}=4.9108$

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected.

**Problem 19:** In a big city 325 men out of 600 men found to be smokers Does this information support the conclusion that the majority of men in this city are smokers ?

**Solution:** Sample Size $n=600$

Sample proportion $p=\frac{325}{600}=0.5416$

Population proportion $P=0.5$

(i) Null Hypothesis $(H_0)$: $P=0.5$

(ii) Alternative Hypothesis $(H_1)$: $P>0.5$ [Right Tailed Test]

(iii) Level of Significance $(\alpha)$: $\alpha=0.05$ $0.5-0.05=0.45$ $Z_{tab}=1.645$

(iv) Test Statistic: The test statistic

$$Z=\frac{p-P}{\sqrt{PQ/n}}=\frac{0.5416-0.5}{\sqrt{\frac{0.5\times0.5}{600}}}=2.0379$$

(v) Conclusion Tabulated value of $Z_{tab}=1.645$ $Z_{cal}=2.0379$

Calculated value of

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected.

**Problem 20:** A social worker believes that fewer than 25% of the couples in a certain area have used any form of birth control. A random sample of 120 couples was contacted. Twenty of them said that they have used. Test the belief of the social worker at 0.05 level.

**Solution:** Sample Size $n = 120$

Sample proportion $p = \frac{20}{120} = 0.1666$

Population proportion $P = 25\% = 0.25$

(i) Null Hypothesis $(H_0)$: $P = 0.25$

(ii) Alternative Hypothesis $(H_1)$: $P < 0.25$

(iii) Level of Significance $(\alpha)$: $\alpha = 0.05$ $0.5 - 0.05 = 0.45$ $Z_{tab} = 1.645$

(iv) Test Statistic: The test statistic

$$Z_{cal} = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.1666 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{120}}} = -2.1098$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.645$

Calculated value of $Z_{cal} = 2.1098$

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected.

**Problem 21:** Among 900 people in a state 90 are found to be chapati eaters. Construct 99% confidence interval for the true proportion.

**Solution:** Sample Size $n = 900$

Sample proportion $p = \frac{90}{900} = 0.1$

Population proportion $P = 0.5$

(i) Null Hypothesis $(H_0)$: $P = 0.5$

(ii) Alternative Hypothesis $(H_1)$: $P \neq 0.5$

(iii) Level of Significance $(\alpha)$: $\alpha = 0.01$ $\alpha/2 = 0.005$ $0.5 - 0.005 = 0.495$ $Z_{tab} = 2.58$

(iv) Test Statistic: The test statistic

$$Z_{cal} = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.1 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{900}}} = 24$$

(v) Conclusion: Tabulated value of $Z_{tab} = 2.58$

Calculated value of $Z_{cal} = 24$

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected.



Confidence Level: $p \pm Z_{\alpha/2} \sqrt{\frac{PQ}{n}}$

$$0.1 \pm (2.58) \sqrt{\frac{0.5 \times 0.5}{900}}$$

$$[0.057, 0.143]$$