Unit 6

Image restoration and reconstruction

Image Degradation Mode, Noise Models, and Restoration in Presence c Noise in spatial Domain. Inverse Filtering, wiener filtering, Introduction to Image reconstruction from projections applications of Image Processing.

IMAGE RESTORATION:

Restoration improves image in some predefined sense. It is an objective process. Restoration attempts to reconstruct an image that has been degraded by using a prior knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and then applying the inverse process in order to recover the original image.

Restoration techniques are based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a “good” enhancement result. Image Restoration refers to a class of methods that aim to remove or reduce the degradations that have occurred while the digital image was being obtained.

All natural images when displayed have gone through some sort of degradation:

- During display mode
- Acquisition mode, or
- Processing mode
    - § Sensor noise
    - § Blur due to camera mis focus
    - § Relative object-camera motion
    - § Random atmospheric turbulence
- Others

Degradation Model:

Degradation process operates on a degradation function that operates on an input image with an additive noise term. Input image is represented by using the notation f(x,y), noise term can be represented as η(x,y).These two terms when combined gives the result as g(x,y). If we are given g(x,y), some knowledge about the degradation function H or J and some knowledge about the additive noise teem η(x,y), the objective of restoration is to obtain an estimate *f'(x,y)* of the original image.

We want the estimate to be as close as possible to the original image. The more we know about h and η , the closer f(x,y) will be to f'(x,y). If it is a linear position invariant process, then degraded image is given in the spatial domain by

g(x,y)=f(x,y)*h(x,y)+η(x,y)

h(x,y) is spatial representation of degradation function and symbol * represents convolution. In frequency domain we may write this equation as

G(u,v)=F(u,v)H(u,v)+N(u,v)

The terms in the capital letters are the Fourier Transform of the corresponding terms in the spatial domain.

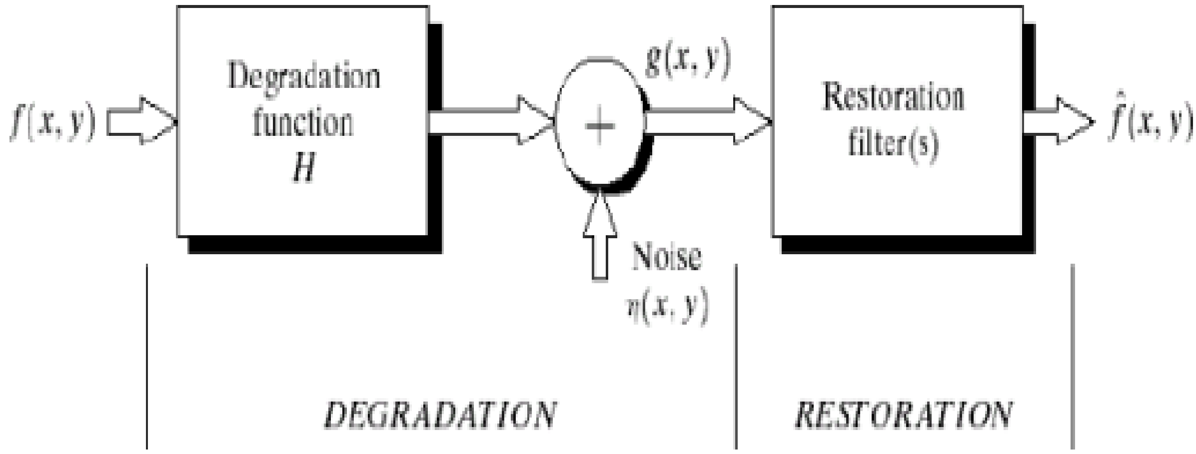


Fig: A model of the image Degradation / Restoration process

Noise Models:

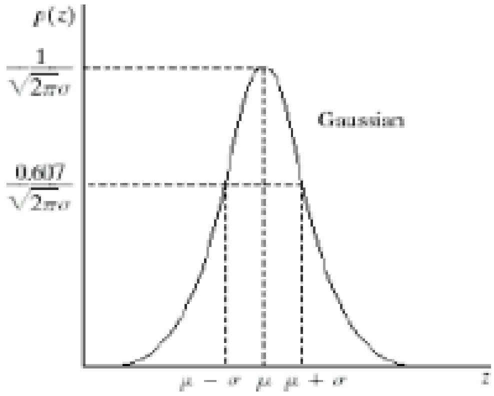
The principal source of noise in digital images arises during image acquisition and /or transmission. The performance of imaging sensors is affected by a variety of factors, such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. Images are corrupted during transmission principally due to interference in the channels used for transmission. Since main sources of noise presented in digital images are resulted from atmospheric disturbance and image sensor circuitry, following assumptions can be made i.e. the noise model is spatial invariant (independent of spatial location). The noise model is uncorrelated with the object function.

Gaussian Noise:

These noise models are used frequently in practices because of its tractability in both spatial and frequency domain. The PDF of Gaussian random variable is

$$p_z(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Where z represents the gray level, μ= mean of average value of z, σ= standard deviation.



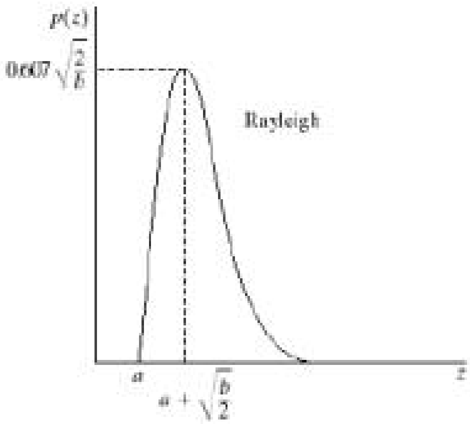
Rayleigh Noise:

Unlike Gaussian distribution, the Rayleigh distribution is no symmetric. It is given by the formula.

$$p_z(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & z \ge a \\ 0 & z < a \end{cases}$$

The mean and variance of this density is

$$m = a + \sqrt{\pi b/4}, \sigma^2 = \frac{b(4-\pi)}{4}$$



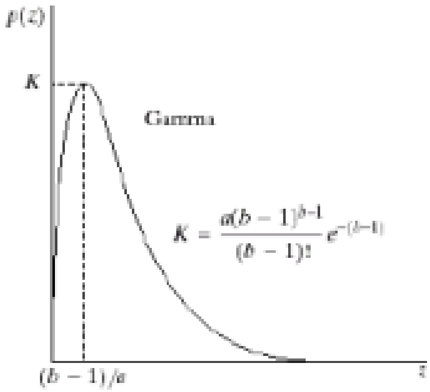
(iii) Gamma Noise:

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az}, & \text{for } z \geq 0 \\ 0, & \text{for } z < 0 \end{cases}$$

The mean and variance of this density are given by

$$\text{mean} : \mu = \frac{b}{a} \quad \text{variance} : \sigma^2 = \frac{b}{a^2}$$



Its shape is similar to Rayleigh disruption. This equation is referred to as gamma density it is correct only when the denominator is the gamma function.

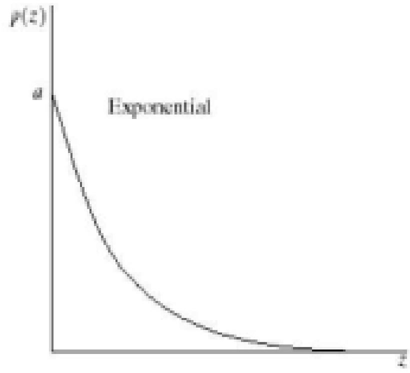
(iv) Exponential Noise:

Exponential distribution has an exponential shape. The PDF of exponential noise is given as

$$p_z(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Where a>0. The mean and variance of this density are given by

$$m = \frac{1}{a}, \quad \sigma^2 = \frac{1}{a^2}$$



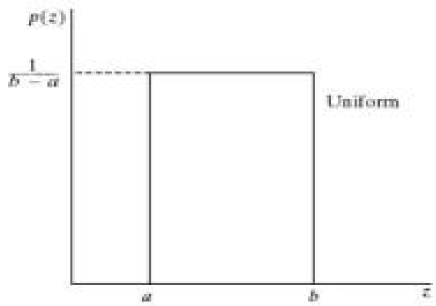
(v) Uniform Noise:

The PDF of uniform noise is given by

$$p_z(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this noise is

$$m = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$



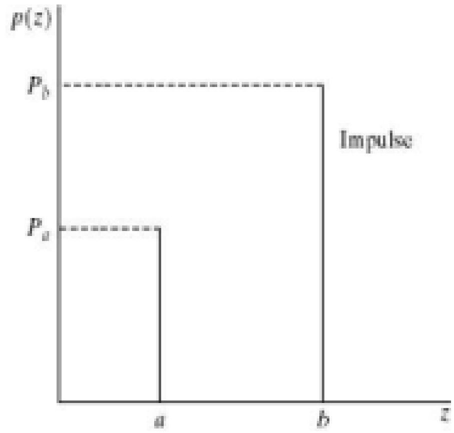
(vi) Impulse (salt & pepper) Noise:

In this case, the noise is signal dependent, and is multiplied to the image.

The PDF of bipolar (impulse) noise is given by

$$p_z(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \qquad b > a$$

If b>a, gray level b will appear as a light dot in image. Level a will appear like a dark dot.



Restoration in the presence of Noise only- Spatial filtering:

When the only degradation present in an image is noise, i.e.

$$\mathbf{g(x,y)=f(x,y)+\eta(x,y)}$$

or

$$\mathbf{G(u,v)= F(u,v)+ N(u,v)}$$

The noise terms are unknown so subtracting them from g(x,y) or G(u,v) is not a realistic approach. In the case of periodic noise it is possible to estimate N(u,v) from the spectrum G(u,v). So N(u,v) can be subtracted from G(u,v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present.

The following techniques can be used to reduce the noise effect:

i) Mean Filter:

(a) Arithmetic Mean filter:

It is the simplest mean filter. Let Sxy represents the set of coordinates in the sub image of size m*n centered at point (x,y). The arithmetic mean filter computes the average value of the corrupted image g(x,y) in the area defined by Sxy. The value of the restored image f at any point (x,y) is the arithmetic mean computed using the pixels in the region defined by Sxy.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn. A mean filter smoothes local variations in image Noise is reduced as a result of blurring. For every pixel in the image, the pixel value is replaced by the mean value of its neighboring pixels with a weight .This will resulted in a smoothing effect in the image.

(b) Geometric Mean filter:

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left(\prod_{(s,t) \in S_{xy}} g(s,t) \right)^{1/mn}$$

Here, each restored pixel is given by the product of the pixel in the sub image window, raised to the power 1/mn. A geometric mean filters but it to loose image details in the process.

(c) Harmonic Mean filter:

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x,y) = \sum_{(s,t) \in S_{xy}} g(s,t)^{Q+1} \Big/ \sum_{(s,t) \in S_{xy}} g(s,t)^{Q}$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well with Gaussian noise also.

(d) Order statistics filter:

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

(e) Median filter:

It is the best order statistic filter; it replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x,y) = \underset{(s,t)\in Sxy}{\text{median}}\{g(s,t)\}$$

The original of the pixel is included in the computation of the median of the filter are quite possible because for certain types of random noise, the provide excellent noise reduction capabilities with considerably less blurring then smoothing filters of similar size. These are effective for bipolar and unipolor impulse noise.

(e) Max and Min filter:

Using the 100th percentile of ranked set of numbers is called the max filter and is given by the equation

$$\hat{f}(x,y) = \max_{(s,t)\in Sxy}\{g(s,t)\}$$

It is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area sky. The 0th percentile filter is min filter.

$$\hat{f}(x,y) = \min_{(s,t)\in Sxy}\{g(s,t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise of the min operation.

(f) Midpoint filter:

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by

$$\hat{f}(x,y) = \left(\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right) \Big/ 2$$

It comeliness the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise.

Explain about Wiener filter used for image restoration.

The inverse filtering approach makes no explicit provision for handling noise. This approach incorporates both the degradation function and statistical characteristics of noise into the restoration process.

The method is founded on considering images and noise as random processes, and the objective is to find an estimate f of the uncorrupted image f such that the mean square error between them is minimized. This error measure is given by

$e^2 = E\ \{(f- f)^2\}$

where E{•} is the expected value of the argument. frequency domain by the expression.

It is assumed that the noise and the image are uncorrelated; that one or the other has zero mean; and that the gray levels in the estimate are a linear function of the levels in the degraded image. Based on these conditions, the minimum of the error function is given in the

$$\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)} \right] G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \\
&= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)
\end{aligned}$$

where we used the fact that the product of a complex quantity with its conjugate is equal to the magnitude of the complex quantity squared. This result is known as the Wiener filter, after N.

Wiener [1942], who first proposed the concept in the year shown. The filter, which consists of the terms inside the brackets, also is commonly referred to as the minimum mean square error filter or the least square error filter. The Wiener filter does not have the same problem as the inverse filter with zeros in the degradation function, unless both H(u, v) and Sη(u, v) are zero for the same value(s) of u and v.

The terms in above equation are as follows:

H (u, v) = degradation function

H*(u, v) = complex conjugate of H (u, v)

$|H(u, v)|^2 = H^*(u, v) * H(u, v)$

$S_\eta(u, v) = |N(u, v)|^2$ = power spectrum of the noise

$S_f(u, v) = |F(u, v)|^2$ = power spectrum of the undegraded image.

As before, H (u, v) is the transform of the degradation function and G (u, v) is the transform of the degraded image. The restored image in the spatial domain is given by the inverse Fourier transform of the frequency-domain estimate F (u, v). Note that if the noise is zero, then the noise power spectrum vanishes and the Wiener filter reduces to the inverse filter.

When we are dealing with spectrally white noise, the spectrum $|N(u, v)|^2$ is a constant, which simplifies things considerably. However, the power spectrum of the undegraded image seldom is known. An approach used frequently when these quantities are not known or cannot be estimated is to approximate the equation as

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K}\right] G(u, v)$$

where K is a specified constant.

Explain the Adaptive Filters.

Adaptive filters are filters whose behavior changes based on statistical characteristics of the image inside the filter region defined by the m X n rectangular window Sxy. **Adaptive, local noise reduction filter:**

The simplest statistical measures of a random variable are its mean and variance. These are reasonable parameters on which to base an adaptive filler because they are quantities closely related to the appearance of an image.

The mean gives a measure of average gray level in the region over which the mean is computed, and the variance gives a measure of average contrast in that region.

This filter is to operate on a local region, Sxy. The response of the filter at any point (x, y) on which the region is centered is to be based on four quantities:

(a) g(x, y), the value of the noisy image at (x, y);
(b) a^2, the variance of the noise corrupting /(x, y) to form g(x, y);
(c) ray, the local mean of the pixels in Sxy; and
(d) σ^2L , the local variance of the pixels in Sxy.

1. If $\sigma^2\eta$ is zero, the filler should return simply the value of g (x, y). This is the trivial, zero-noise case in which g (x, y) is equal to f (x, y).
2. If the local variance is high relative to σ2η the filter should return a value close to g (x, y). A high local variance typically is associated with edges, and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in Sxy. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging. Adaptive local noise filter is given by,

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right].$$

The only quantity that needs to be known or estimated is the variance of the overall noise, a^2. The other parameters are computed from the pixels in Sxy at each location (x, y) on which the filter window is centered.

Adaptive median filter:

The median filter performs well as long as the spatial density of the impulse noise is not large (as a rule of thumb, Pa and Pb less than 0.2). The adaptive median filtering can handle impulse noise with probabilities even larger than these. An additional benefit of the adaptive median filter is that it seeks to preserve detail while smoothing nonimpulse noise, something that the "traditional" median filter does not do.

The adaptive median filter also works in a rectangular window area Sxy. Unlike those filters, however, the adaptive median filter changes (increases) the size of Sxy during filter operation, depending on certain conditions. The output of the filter is a single value used to replace the value of the pixel at (x, y), the particular point on which the window Sxy is centered at a given time.

Consider the following notation:

z_{min} = minimum gray level value in S_{xy}

z_{max} = maximum gray level value in S_{xy}

z_{med} = median of gray levels in S_{xy}

z_{xy} = gray level at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}.

The adaptive median filtering algorithm works in two levels, denoted level A and level B, as follows:

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If A1 > 0 AND A2 < 0, Go to level B

Else increase the window size

If window size $\leq S_{max}$ repeat level A

Else output z_{xy}

Level B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

If B1> 0 AND B2 < 0, output zxy

Else output z_{med}

Inverse filtering.

The simplest approach to restoration is direct inverse filtering, where F (u, v), the transform of the original image is computed simply by dividing the transform of the degraded image, G (u, v), by the degradation function.

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}.$$

The divisions are between individual elements of the functions.

But G (u, v) is given by

$$G(u, v) = F(u, v) + N(u, v)$$

Hence

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}.$$

It tells that even if the degradation function is known the un-degraded image cannot be recovered [the inverse Fourier transform of F(u, v)] exactly because N(u, v) is a random function whose Fourier transform is not known.

If the degradation has zero or very small values, then the ratio N(u, v)/H(u, v) could easily dominate the estimate F(u, v). One approach to get around the zero or small-value problem is to limit the filter frequencies to values near the origin. H (0, 0) is equal to the average value of h(x, y) and that this is usually the highest value of H (u, v) in the frequency domain. Thus, by limiting the analysis to frequencies near the origin, the probability of encountering zero values is reduced.

The differences between the image enhancement and image restoration.

(i) Image enhancement techniques are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human system. Whereas image restoration techniques are basically reconstruction techniques by which a degraded image is reconstructed by using some of the prior knowledge of the degradation phenomenon.

(ii) Image enhancement can be implemented by spatial and frequency domain technique, whereas image restoration can be implement by frequency domain and algebraic techniques.

(iii) The computational complexity for image enhancement is relatively less when compared to the computational complexity for irrrage restoration, since algebraic methods requires manipulation of large number of simultaneous equation. But, under some condition computational complexity can be reduced to the same level as that required by traditional frequency domain technique.

(iv) Image enhancement techniques are problem oriented, whereas image restoration techniques are general and are oriented towards modeling the degradation and applying the reverse process in order to reconstruct the original image.

(v) Masks are used in spatial domain methods for image enhancement, whereas masks are not used for image restoration techniques.

(vi) Contrast stretching is considered as image enhancement technique because it is based on the pleasing aspects of the review, whereas removal of' image blur by applying a deblurring function is considered as a image restoration technique.