Model No 5.6: Test of significance for difference of means (Students's t- test):

(i) **Null Hypothesis** (H_0): $\mu_1 = \mu_2$ i.e., "there is no significance difference between the two means

(ii)Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$

(iii) Level of Significance (α): Set a level of significance

(iv) Test Statistic: The test statistic $t = \dfrac{\bar{x}-\bar{y}}{s\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$ *where* $s^2 = \dfrac{\sum (x_i-\bar{x})^2 + (y_i-\bar{y})^2}{n_1+n_2-2}$

(v) Conclusion: (i) If $|t| < t_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|t| > t_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

With ($n_1 + n_2$ -2) degrees of freedom.

$$S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}}$$

$$S^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

i, SD are Given ⇒

$$S = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2 + \Sigma(x_i-\bar{x}_2)^2}{n_1+n_2-2}}$$

ii, If S.D's aren't Given&

$$S_1 = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1}} \qquad S_2 = \sqrt{\frac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1}}$$

Problem 9: Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results.

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	--

$\bar{x}_1 = 31.2857$

$\bar{x}_2 = 28.1666$

Test whether the two horses have the same running capacity.

Solution: The Given Samples are Small Sample, because $n_1=7$, $n_2=6$. Here S.D's aren't Given.

$$S_1 = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1}} \Rightarrow \Sigma(x_i-\bar{x}_1)^2 = S_1^2(n_1-1) = 6(5.2381) = 31.2857$$

$$S_2 = \sqrt{\frac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1}} \Rightarrow \Sigma(x_i-\bar{x}_2)^2 = S_2^2(n_2-1) = 5(5.3666) = 28.8333$$

$$S = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2+\Sigma(x_i-\bar{x}_2)^2}{n_1+n_2-2}} = \sqrt{\frac{31.4286+28.8332}{11}} = 2.3014$$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii) Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$ (Two Tailed Test)

(iii) Level of Significance (α): $\alpha = 0.05$, $\alpha/2 = 0.025$

(iv) Test Statistic: The test statistic

$$t = \frac{\bar{x}_1-\bar{x}_2}{S\sqrt{1/n_1+1/n_2}} = \frac{31.2857-28.8323}{2.3014\sqrt{1/7+1/6}} = 2.4360$$

(v) Conclusion: Degrees of freedom= $n_1+n_2-2 = 7+6-2 = 11$

Tabulated value of t_α = 2.207

Calculated value of $|t_\alpha|$ = 2.4360

Calculated value of $|t_\alpha|$ Tabulated value of t_α

∴ Null Hypothesis is Rejected.

Problem 10: To examine the hypothesis that the husbands are more intelligent than the wives, an investigator took a sample of 10 couples and admininistrated them a test which measures the I.Q. The results as follows:

$\bar{x}_1 = 103$

Husbands	117	105	97	105	123	109	86	78	103	107
Wives	106	98	87	104	116	95	90	69	108	85

$\bar{x}_2 = 95.8$

Test the hypothesis with a reasonable test at the level of significance of 0.05.

Solution: The Given Samples are Small Samples. $n_1=10$, $n_2=10$, As SD's aren't Given.

$$S_1 = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2}{n}} \Rightarrow \Sigma(x_i-\bar{x}_1)^2 = S_1^2(n_1-1) \Rightarrow 9(178.444) = 1605.996$$

$$S_2 = \sqrt{\frac{\Sigma(x_i-\bar{x}_2)^2}{n}} \Rightarrow \Sigma(x_i-\bar{x}_2)^2 = S_2^2(n_2-1) \Rightarrow 9(186.622) = 1679.598$$

$$S = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2 + \Sigma(x_i-\bar{x}_2)^2}{n_1+n_2-2}} = \sqrt{\frac{1605.996+1679.598}{10+10-2}} = 13.5104$$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii)Alternative Hypothesis (H_1): $\mu_1 > \mu_2$ (Right Tailed)

(iii) Level of Significance (α): 0.05

(iv) Test Statistic: The test statistic

$$t_{cal} = \frac{\bar{x}_1-\bar{x}_2}{S\sqrt{1/n_1+1/n_2}} = \frac{103-95.8}{13.5104\sqrt{1/10+1/10}} = 1.19165$$

(v) Conclusion: Degrees of freedom= $n_1+n_2-2 = 18$

Tabulated value of t_α = 1.734

Calculated value of $|t_\alpha|$ = 1.19165

Calculated value of $|t_\alpha|$ < Tabulated value of t_α

∴ Null Hypothesis is Accepted.

Problem 11: Ten soldiers participated in a shooting competition in the first week. After intensive training they participated in the competition in the second week. Their scores before and after training are given as follows:

Scores before	67	24	57	55	63	54	56	68	33	43
Scores after	70	38	58	58	56	67	68	75	42	38

Solution:

Given $n_1 = 10$, $n_2 = 10$, $\bar{x}_1 = 52$, $\bar{x}_2 = 57$

$$S_1 = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1}} \Rightarrow \Sigma(x_i-\bar{x}_1)^2 = S_1^2(n_1-1) \Rightarrow 9(209.111) = 1881.999$$

$$S_2 = \sqrt{\frac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1}} \Rightarrow \Sigma(x_i-\bar{x}_2)^2 = S_2^2(n_2-1) = 9(184.888) = 1663.992$$

$$S = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2 + \Sigma(x_i-\bar{x}_2)^2}{n_1+n_2-2}} = \sqrt{\frac{1881.999+1663.992}{10+10-2}}$$

$$= 14.0356$$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii)Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$ [Two-Tailed Test]

(iii) Level of Significance (α): $\alpha = 0.05 \quad \alpha/2 = 0.025$

(iv) Test Statistic: The test statistic $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{52-57}{14.0356\sqrt{\frac{1}{10}+\frac{1}{10}}} = 0.7965$

(v) Conclusion: Degrees of freedom= $n_1+n_2-2 = 10+10-2 = 18$

Tabulated value of t_α = $2.101 \rightarrow t_{tab}$

Calculated value of $|t_\alpha|$ = $0.7965 \rightarrow t_{cal}$ Null Hypothesis is Accepted.

Calculated value of $|t_\alpha|$ < Tabulated value of t_α

Problem 12: Samples of two types of electric light bulbs were tested for length of life and following data were obtained

Type I	Type II
Sample number, n1 = 8	$n_2 = 7$
Sample mean, $\bar{x}$ = 1234 hrs	$\bar{y}$ =1036 hrs
Sample S.D., s_1=36 hrs	s_2 = 40 hrs

Is the difference in the means sufficient to warrant that type I is superior to type II regarding length of life. Given that $n_1 = 8$, $n_2 = 7$, $\bar{x}_1 = 1234$ $\bar{x}_2 = 1036$

Solution: SD's are Given: $s_1 = 36$, $s_2 = 40$

$$S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}} = 40.7317$$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii)Alternative Hypothesis (H_1): $\mu_1 > \mu_2$ [Right Tailed Test]

(iii) Level of Significance (α): $\alpha = 0.05$;

(iv) Test Statistic: The test statistic $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{1834-1036}{40.7317\sqrt{\frac{1}{8}+\frac{1}{7}}} = 9.3924$

(v) Conclusion: Degrees of freedom= $n_1+n_2-2 = 8+7-2 = 13$

Tabulated value of t_α = $Z_{tab} = 1.771$

Calculated value of $|t_\alpha|$ = $Z_{cal} = 9.3924$

Calculated value of $|t_\alpha|$ > Tabulated value of t_α

Null Hypothesis is Rejected

Img Problem 13: The means of two random samples of sizes 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviations from the mean are 26.94 and 18.73 respectively. Can the sample be considered to have been drawn from the same normal population. Given $n_1 = 9$, $n_2 = 7$; $\bar{x}_1 = 196.42$ $\bar{x}_2 = 198.82$

Solution: Sum of Squares Of Deviations:

$\Sigma(x_i-\bar{x}_1)^2 = 26.94$ $\Sigma(x_i-\bar{x}_2)^2 = 18.73$

$$S = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2+\Sigma(x_i-\bar{x}_2)^2}{n_1+n_2-2}} = \sqrt{\frac{26.94+18.73}{9+7-2}} = 1.8061$$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii) Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$ Two-Tailed Test

(iii) Level of Significance (α): $\alpha = 0.05$ $\alpha/2 = 0.025$

(iv) Test Statistic: The test statistic $t_{cal} = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{196.42-198.82}{1.8061\sqrt{\frac{1}{9}+\frac{1}{7}}} = -2.6368$

(v) Conclusion: Degrees of freedom= $n_1+n_2-2 = 14$

Tabulated value of t_α = 2.145

Calculated value of $|t_\alpha|$ = 2.6368

Calculated value of $|t_\alpha|$ > Tabulated value of t_α

Null Hypothesis is Rejected.