Analysis of Variance (ANOVA)

①

A test for Homogeneity of Mean.

The technique of "Analysis of variance is referred to as ANOVA. The technique of ANOVA is to split the variation into its various components. They are (i) Variance between samples.
(ii) Variance within samples.

The observations (or data) may be classified according to one factor or two factors. which are called one-way classification and two-way classification.

One-way ANOVA: In This if we consider the influence of any one factor, then it is called one-way classification.

Eg: The yields of several plots of land may be classified according to one or more types of fertilizati

The techniques for ANOVA one-way classification model are:

(i) Direct Method
(ii) Short-Cut Method
(iii) Coding method.

(i) Direct method

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$, where $\mu_1, \mu_2, \cdots, \mu_k$ are the arithmetic means of the k populations from which k samples are drawn at random.

$H_1 : \mu_1 \neq \mu_2 \neq \cdots \neq \mu_k$.

a) Calculation of variance between the Samples

It is the sum of the squares of the deviations of the means of the various samples from the grand mean.

(i) Calculate the sample means $\bar{x}_1, \bar{x}_2, \bar{x}_3, \cdots \bar{x}_k$ of all the k samples.

(ii) Calculate the mean of the sample means:

$$\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k} \quad \text{or} \quad \boxed{\bar{x} = \frac{T}{N}}$$

(iii) Evaluate the deviations of the sample means from the grand mean i,e find

$\bar{x}_1 - \bar{x}, \quad \bar{x}_2 - \bar{x}, \cdots, \quad \bar{x}_k - \bar{x}$.

(IV) SSB (or SSC) = Sum of the squares of the variations between the samples or between the columns $= \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2$.

MSB or MSC = Variance or the mean square between the samples (or between the columns)

$$= \frac{SSB}{\nu_1}, \quad \nu_1 = \text{degrees of freedom}$$

$= \text{no: of samples} - 1$

$= k - 1$

b) Calculation of variance within the samples.

SSW (or SSE) = Sum of the squares of the variations within the samples

(or)

Sum of the squares due to errors.

$$= \Sigma (x_1 - \bar{x})^2 + \Sigma (x_2 - \bar{x})^2 + \cdots + \Sigma (x_k - \bar{x}_k)^2$$

MSW or MSE = Variance or ~~of~~ mean square within the samples.

$$= \frac{SSW}{\nu_2}$$

$\nu_2 = d.f = \text{total no: of observations} - \text{No: of samples}$

$= N - K.$

c) $F = \frac{MSB}{MSW}$ or $\frac{MSC}{MSE} = \frac{\text{Variance between the samples}}{\text{Variance within the samples}}$

D.f $= \nu_1 = K-1, \quad \nu_2 = N - K.$

Annova table (One-way classification)

Source of Variation	Sum of Squares SS	Degrees of freedom	Mean Squares MS	Test Statistic (F-test)
Between Samples or columns.	SSB	$K-1$	$MSB = \frac{SSB}{K-1}$	
Within Samples (Error)	SSW	$N-K$	$MSW = \frac{SSW}{N-K}$	$F = \frac{MSB}{MSW}$.
Total	SST	$N-1$	—	—

Short-Cut Method

1. Caluclate $T = \Sigma X_1 + \Sigma X_2 + \cdots + \Sigma X_K$.
2. Calculate $\frac{T^2}{N}$.
3. Compute SST = Total sum of the squares of deviation
$$= \Sigma X_1^2 + \Sigma X_2^2 + \cdots + \Sigma X_k^2 - \frac{T^2}{N}$$
4. Calculate $SSB = \left[\frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \cdots + \frac{(\Sigma X_k)^2}{n_k}\right] - \frac{T^2}{N}$
5. Calculate $SSW = SST - SSB$.
6. Now proceed as in Direct Method to obtain MSB, MSW and F and arrive at the final decision.

Annova for two-way classification (3)

(Manifold Calssification)

In two-way classification, observations are classified according to two different factors or criteria.

Eg: Fertilizers may be tried on different soil textures.

Working Rule

1. Calculate SSC i,e the sum of squares (or variance) between the columns.

2. SSR = Sum of squares (or Variance) between the rows

3. SSE = the sum of squares for the residuals

4) SST = SSC + SSR + SSE

C — no: of columns, R — no: of rows.

$\therefore$ total no: of degrees of freedom $= cr - 1$

D.f between columns $= c - 1$

" " rows $= r - 1$

D.f between residuals $= (cr-1) - (c-1) - (r-1)$

$= (c-1)(r-1)$

5) Calculate $MSC = \frac{SSC}{c-1}$

$$MSR = \frac{SSR}{r-1}$$

$$MSE = \frac{SSE}{(c-1)(r-1)}$$

6) <u>Calc</u> . $F_C = \frac{MSC}{MSE}$, $MSC > MSE$

$$F_R = \frac{MSR}{MSE} \quad MSR > MSE$$

if $MSE > MSC$, $F_C = \frac{MSE}{MSC}$

Illly if $MSE > MSR$, $F_R = \frac{MSE}{MSR}$.

7. Write conclusions for F_C and F_R :

Analysis Of Variance (ANOVA) UNIT-5 Part-C, 26.05.2022

1 way Classification:

only 1 Parameter is Calculated.

1. Short-cut Method
2. Direct Method
3. Coding Method

Hint: ANOVA is More than 2 Samples

1. Short cut Method:

* K = No of Samples
* N = Total No of Observations
* $T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$
* Correction Factor [CF] $= \frac{T^2}{N}$
* Sum of Squares Between Samples [SSB]:

$$SSB = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$$

* Total Sum of Squares of Samples [SST]:

$$SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - \frac{T^2}{N}$$

* $SSW = SST - SSB$
* $MSB = \frac{SSB}{\nu_1} = \frac{SSB}{K-1}$ * $MSW = \frac{SSW}{\nu_2} = \frac{SSW}{N-K}$
* $F_{cal} = \frac{MSB}{MSW}$ or $\frac{MSW}{MSB}$ * $F_{tab} = F_{0.05}(\underbrace{K-1, N-K}_{\text{degrees of Freedom}})$

$\mu_1 = \mu_2 = \mu_3$

2. Direct Method

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ | & | & | \\ \bar{x}_1 & \bar{x}_2 & \bar{x}_3 \\ \searrow & \downarrow & \swarrow \\ & \bar{x} & \end{array}$$

$$SSB = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2$$

$$SSW = \Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2 + \Sigma(x_3 - \bar{x}_3)^2$$

1) Three different Machines are used for Production. On Basis Of their Output the Machines are Equally effective?

MACHINE-I	MACHINE-II	MACHINE-III
10	9	20
5	5	16
11	7	10
10	6	4

Sol: It belongs to the Category Of one way ANOVA; Since It Contains More than 2 Samples & we Studied only 1 Parameter

* Null Hypothesis (H_0): $\mu_1 = \mu_2 = \mu_3$ (or)
  The three Machines are Equally effective (or) Homogenity Of Means.

* Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2 \neq \mu_3$ (or)
  The Three Machines are Not Equally working (or) Non-Homogenity Of Means.

* Level of Significance (α): 0.05

* Test Statistic:

1, SHORT-CUT METHOD: K = No. of samples = 3
N = Total No. Of observations = 12

MACHINE-I	MACHINE-II	MACHINE-III			
x_1	x_2	x_3	x_1^2	x_2^2	x_3^2
10	9	20	100	81	400
5	5	16	25	25	256
11	7	10	121	49	100
10	6	4	100	36	16
$\Sigma x_1 = 36$	$\Sigma x_2 = 27$	$\Sigma x_3 = 50$	$\Sigma x_1^2 = 346$	$\Sigma x_2^2 = 191$	$\Sigma x_3^2 = 7$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 36 + 27 + 50 = 113$

$CF = \frac{T^2}{N} = \frac{113}{12} = 1064.0833$

$SSB = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N} = \frac{1296}{4} + \frac{729}{4} + \frac{2500}{4} - 1064.0833$

$= \frac{\cancel{5729}\ 4525}{4} - 1064.0833$

$= 1131.25 - 1064.0833$

$= 67.1667$

$SST = \Sigma(x_1)^2 + \Sigma x_2^2 + \Sigma x_3^2 - \frac{T^2}{N}$

$= 346 + 191 + 772 - 1064.0833 =$

$= 1309 - 1064.0833 = 244.9167$

$SSW = SST - SSB = 244.9167 - 67.1667 = 177.75$

$MSB = \frac{SSB}{\nu_1} = \frac{SSB}{K-1} = \frac{67.1667}{2} = 33.5833$

$MSW = \frac{SSW}{\nu_2} = \frac{SSW}{N-K} = \frac{177.75}{12-3} = 19.75$

$F_{cal} = \frac{MSB}{MSW} = \frac{33.5833}{19.75} = \boxed{1.7004}$

$F_{tab} = F_{0.05}(K-1, N-K) = F_{0.05}(2,9) = \boxed{4.26}$

$\therefore$ $F_{cal} < F_{tab}$ $\boxed{\text{Null Hypothesis is Accepted}}$

Hence, the 3 Machines are Equally Effective.

(or) II METHOD

DIRECT METHOD:

x_1	x_2	x_3	$(x_1-\bar{x}_1)^2$	$(x_2-\bar{x}_2)^2$	$(x_3-\bar{x}_3)^2$
10	9	20	1	5.0625	56.25
5	5	16	16	3.0625	12.25
11	7	10	4	0.0625	6.25
10	6	4	1	0.5625	72.25
36	27	50	22	8.75	147

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{36}{4} = 9$

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{27}{4} = 6.75$

$\bar{x}_3 = \frac{\Sigma x_3}{n_3} = \frac{50}{4} = 12.5$

Grand Mean $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3}{3}$

$= \frac{9 + 6.75 + 12.5}{3} = 9.4167$

$SSB = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2$

$= 4(9 - 9.4167)^2 + 4(6.75 - 9.4167)^2 + 4(12.5 - 9.4167)^2$

$= 0.6946 + 28.4452 + 38.0270 = 67.1668$

$SSW = \Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2 + \Sigma(x_3 - \bar{x}_3)^2$

$= [22] + [8.75] + [147] = 177.75$

$MSB = \frac{SSB}{K-1} = \frac{67.1667}{2} = 33.583$

$MSW = \frac{SSW}{N-K} = \frac{177.75}{9} = 19.75$

iii) $F_{cal} = \frac{MSB}{MSW} = \frac{33.583}{19.75} = 1.7004$ $\boxed{F_{cal} < F_{tab}}$

$F_{tab} = F_{0.05}(2,9) = 4.26$

(or)

III CODING METHOD

In this Method: we Have to ADD (or) SUBRACT (or) DIVIDE (or) MULTIPLY with a Constant Value with each of the Observat[ions]

In this Problem, we subract 10 from each Observed value, As '10' is repeated many times in Given Table.

M-I	M-II	M-III
0	-1	10
-5	-5	6
1	-3	0
0	-4	-6

Remaining Procedure can be done using Short-Cut Method Only

Q) Three Samples of five, five, four Motor Car tyres are drawn respectively from 3 Branches 'A', 'B', 'C', Manufact[ured] by 3 Machines. The Lifetime of 3 tyres in 1000 Miles is Given below.

Test whether Average lifetime of 3 Brands tyres are Equal or Not.

A	B	C
35	30	28
40	35	24
33	34	30
36	28	26
31	33	---

Sol $n_1 = 5$ $n_2 = 5$ $n_3 = 4$ $k = 3$ $N = 14$

SC Method

A x_1	B x_2	C x_3	x_1^2	x_2^2	x_3^2
35	30	28	1225	900	784
40	35	24	1600	1225	576
33	34	30	1089	1156	900
36	28	26	1296	784	676
31	33		961	1089	
175	160	108	6171	5154	2936

Null Hypothesis (H_0): $\mu_1 = \mu_2 = \mu_3$

Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2 \neq \mu_3$

Level of Significance $\alpha = 0.05$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 175 + 160 + 108 = 443$, $CF = \frac{T^2}{N} = \frac{(443)^2}{14} = 14017.78$

$$SSB = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$$

$$= \frac{30625}{5} + \frac{25600}{5} + \frac{11664}{4} - 14017.78$$

$$= 6125 + 5120 + 2916 - 14017.78 = 143.22$$

$$SST = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - \left(\frac{T^2}{N}\right) = 6171 + 5154 + 2936 - 14017.78$$
$$= 243.22$$

$$SSW = SST - SSB = 243.22 - 143.22 = 100$$

$$MSB = \frac{SSB}{\gamma_1} = \frac{SSB}{K-1} = \frac{SSB}{2} = \frac{143.22}{2} = 71.61$$

$$MSW = \frac{SSW}{\gamma_2} = \frac{SSW}{N-K} = \frac{100}{14-3} = 9.0909$$

$$F_{cal} = \frac{MSB}{MSW} = \frac{71.61}{9.0909} = \boxed{7.8771}$$

$$F_{tab} = F_{0.05}(K-1, N-K) = F_{0.05}(\overset{\gamma_1}{2}, \overset{\gamma_2}{11}) = \boxed{3.98}$$

$\boxed{\therefore F_{cal} > F_{tab}}$

Null Hypothesis is Rejected

DIRECT-METHOD

x_1	x_2	x_3	$(x_1-\bar{x}_1)^2$	$(x_2-\bar{x}_2)^2$	$(x_3-\bar{x}_3)^2$
35	30	28	0	4	1
40	35	24	25	9	9
33	34	30	4	4	9
36	28	26	1	16	1
31	33	---	16	1	--
175	160	108	46	34	20

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{175}{5} = 35$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{160}{5} = 32$$

$$\bar{x}_3 = \frac{\Sigma x_3}{n_3} = \frac{108}{4} = 27$$

$$\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3}{3} = \frac{35+32+27}{3} = \frac{94}{3} = 31.3333$$

$$SSB = n_1(\bar{x}_1 - \bar{x})^2 + n_2(\bar{x}_2 - \bar{x})^2 + n_3(\bar{x}_3 - \bar{x})^2$$
$$= 5(35 - 31.3333)^2 + 5(32 - 31.3333)^2 + 4(27 - 31.3333)^2$$
$$= 144.5558$$

$$SSW = \Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2 + \Sigma(x_3 - \bar{x}_3)^2$$
$$= 46 + 34 + 20 = 100$$

$$MSB = \frac{SSB}{\gamma_1} = \frac{SSB}{K-1} = \frac{144.5558}{2} = 72.2779$$

$$MSW = \frac{SSW}{\gamma_2} = \frac{SSW}{N-K} = \frac{100}{14-3} = 9.0909$$

$$F_{cal} = \frac{MSB}{MSW} = \boxed{7.9506}$$

$$F_{tab} = F_{0.05}(2, 11) = \boxed{3.98}$$

$\boxed{F_{cal} > F_{tab}}$

Null Hypothesis is Rejected.

ANOVA Two Way Classification:

$N = $ No. of rows × No. of columns $= (r \times c)$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$

$CF = \frac{T^2}{N}$

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$

$SSR = \frac{(\Sigma T_1)^2}{n_1} + \frac{(\Sigma T_2)^2}{n_2} + \frac{(\Sigma T_3)^2}{n_3} - \frac{T^2}{N}$

Error

$\boxed{SSE = SST - (SSC + SSR)}$

$SST = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N}$

$MSC = \frac{SSC}{C-1} =$

$MSR = \frac{SSR}{r-1}$

$MSE = \frac{SSE}{(C-1)(r-1)}$

$F_{cal} = \frac{MSC}{MSE}$ or $\frac{MSE}{MSC}$ $\underbrace{(C-1, (C-1)(r-1))}_{\text{degrees of Freedom}}$

$F_{cal} = \frac{MSR}{MSE}$ or $\frac{MSE}{MSR}$ $\underbrace{((r-1), (C-1)(r-1))}_{\text{degrees of Freedom}}$

1, A Farmer Applies 3 types of Fertilizers on four Separate plots. The Figure on yield per acre are tabulated below:

Find out If the Plots are Materially different in fertility also If 3 fertilizers make any Material difference in yields.

Yields:

Plots & Fertilizers	A	B	C	D
Nitrogen F_1	6	4	8	6
Potassium F_2	7	6	6	9
Phosphorus F_3	8	5	10	9

Sol: Here, we Study 2 Parameters: Plots A, B, C, D and Fertilizers F_1, F_2, F_3. So Here we have to apply 2 way classification of ANOVA.

Null Hypothesis (H_0): $A = B = C = D$, $F_1 = F_2 = F_3$

The effect of 3 Fertilizers are Same.

Alternative Hypothesis (H_1): $A \neq B \neq C \neq D$ & $F_1 \neq F_2 \neq F_3$

Level of Significance $\alpha = 0.05$

Test Statistics:

r = No. of rows = 3

C = No. of Columns = 4

Plots / Fertilizers	A (x_1)	B (x_2)	C (x_3)	D (x_4)	
Nitrogen (F_1) T_1	6	4	8	6	$\Sigma T_1 = 24$
Potassium (F_2) T_2	7	6	6	9	$\Sigma T_2 = 28$
Phosphorus (F_3) T_3	8	5	10	9	$\Sigma T_3 = 32$
	$\Sigma x_1 = 21$	$\Sigma x_2 = 15$	$\Sigma x_3 = 24$	$\Sigma x_4 = 24$	

Yield

$N = rC = 3 \times 4 = 12$

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 21 + 15 + 24 + 24 = 84$

$CF = \frac{T^2}{N} = \frac{(84)^2}{12} = 588$

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - \frac{T^2}{N} = \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - \frac{588}{\cancel{01}} = 18$

$SSR = \frac{(\Sigma T_1)^2}{n_1} + \frac{(\Sigma T_2)^2}{n_2} + \frac{(\Sigma T_3)^2}{n_3} + \cancel{\frac{(\Sigma T_4)}{n_4}} - \frac{T^2}{N}$

$= \frac{(24)^2}{4} + \frac{(28)^2}{4} + \frac{(32)^2}{4} - 588 = 8$

$SST = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N} = 6^2 + 4^2 + 8^2 + 6^2 + 7^2 + 6^2 + 6^2 + 9^2 + 8^2 + 5^2 + 10^2 + 9^2 - 588$

$= 36$

Error $SSE = SST - (SSC + SSR) = 36 - (18 + 8) = 10$

$MSC = \frac{SSC}{C-1} = \frac{18}{3} = 6$

$MSR = \frac{SSR}{r-1} = \frac{8}{2} = 4$

$MSE = \frac{SSE}{(r-1)(C-1)} = \frac{10}{6} = 1.6666$

$F_C\,cal = \frac{MSC}{MSE} = \frac{6}{1.6666} = 3.6001$

$F_R\,cal = \frac{MSR}{MSE} = \frac{4}{1.6666} = 2.40009$

$F_C\,tab = F_C((c-1),(r-1)(c-1)) = F_{0.05}(3,6) = 4.76$

$F_R\,tab = F_R((r-1),(r-1)(c-1)) = F_{0.05}(2,6) = 5.14$

I $\boxed{\therefore F_C cal < F_C tab}$ $A = B = C = D$

All plots are equally effective.

II $\boxed{\therefore F_R cal < F_R tab}$ $F_1 = F_2 = F_3$

All Fertilizers are Equally Effective

2, To Study the Performance of 3 Detergents of 3 different temperatures, the following whiteners were observed. Perform a 2 way Analysis of Variance using 5% Level of Significance.

Water temperature	Detergent A	Detergent B	Detergent C
Cold water	57	55	67
Warm water	49	52	68
Hot water	54	46	58

Sol: Null Hypothesis: $D_A = D_B = D_C$

Cold water = Warm water = hot water

Alternative Hypothesis: $D_A \neq D_B \neq D_C$

Level of Significance: $\alpha = 0.05$

Test Statistic: No of rows$(r) = 3$ $N = rc = 9$

No of columns$(c) = 3$

Water/ Temperature		Detergent-A (x_1)	Detergent (x_2)	Det (x_3)	Total
Cold water	T_1	57	55	67	$\Sigma T_1 = 179$
warm water	T_2	49	52	68	$\Sigma T_2 = 169$
Hot water	T_3	54	46	58	$\Sigma T_3 = 158$
		$\Sigma x_1 = 160$	$\Sigma x_2 = 153$	$\Sigma x_3 = 193$	

By ANOVA 2 way Method:

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 160 + 153 + 193 = 506$

$CF = \frac{T^2}{N} = \frac{(506)^2}{9} = 28448.4$

$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N}$

$= \frac{(160)^2}{3} + \frac{(153)^2}{3} + \frac{(193)^2}{3} - 28448.4 = 304.2222$

$SSR = \frac{(\Sigma T_1)^2}{n_1} + \frac{(\Sigma T_2)^2}{n_2} + \frac{(\Sigma T_3)^2}{n_3} - \frac{T^2}{N}$

$= \frac{(179)^2}{3} + \frac{(169)^2}{3} + \frac{(158)^2}{3} - 28448.4 = 73.6$

$SST = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N} = 57^2 + 49^2 + 54^2 + 55^2 + 52^2 + 46^2 + 67^2 + 68^2 + 58^2 - 28448.4$

$= 439.6$

Error SSE:

$SSE = [SST - (SSC + SSR)] = \overset{439.6}{\cancel{28888}} - (304.2666 + 73.6) = 61.7782$

$MSC = \frac{SSC}{C-1} = \frac{304.2222}{3-1} = 152.1111$

$MSR = \frac{SSR}{R-1} = \frac{73.6}{3-1} = 36.7778$

$MSE = \frac{SSE}{(C-1)(r-1)} = \frac{\overset{61.7782}{\cancel{439.6}}}{(2)(2)} = 15.4445$

$F_C Cal = \frac{MSC}{MSE} = \frac{152.3333}{109.9} = 9.8498$

$F_R Cal = \frac{MSE}{MSR} = \frac{109.9}{36.8} = 2.38141$

$F_C tab = ((C-1), (r-1)(C-1))$

$= F_{0.05}(2, 4)$

$= 6.94$

$F_R tab = ((r-1), (C-1)(r-1))$

$= F_{0.05}(2, 4)$

$= 6.94$

$\boxed{F_C Cal \not< F_C tab}$ → Null Hypothesis is Rejected

The detergents are Not Equally effective. $D_A \neq D_B \neq D_C$

$\boxed{F_R Cal < F_R tab}$ → Null Hypothesis is Accepted

The Temperature of water is Equally Effective.

Coding Method:

In this Coding Method, we multiply or divide or Subraction or Addition of any value for Smaller Values.

Water/ Temperature	Det A	Det B	Det C
Cold water	5	3	15
Warm water	-3	0	16
Hot water	2	-6	6

SHORT-CUT Method

	A x_1	B x_2	C x_3	x_1^2	x_2^2	x_3^2	
T_1	5	3	15	25	9	225	$\Sigma T_1 = 23$
T_2	-3	0	16	9	0	256	$\Sigma T_2 = 13$
T_3	2	-6	6	4	36	36	$\Sigma T_2 = 2$
	$\Sigma x_1 = 4$	$\Sigma x_2 = -3$	$\Sigma x_3 = 37$	38	45	517	

Null Hypothesis: $D_A = D_B = D_C$

Cold water = Warm water = Hot water

All types of water has Equal effect on all Detergents.

Alternative Hypothesis: $D_A \neq D_B \neq D_C$

Level of Significance: $\alpha = 0.05$

Test Statistics: No. of rows $(r) = 3$ $\quad N = r \times c = 9$

No. of columns $(C) = 3$

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 = 4 - 3 + 37 = 38$$

$$CF = \frac{T^2}{N} = \frac{38^2}{9} = 160.4444$$

$$SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - \frac{T^2}{N} = \frac{16}{3} + \frac{9}{3} + \frac{1369}{3} - 160.4444$$
$$= 304.2222$$

$$SSR = \frac{(\Sigma T_1)^2}{n_1} + \frac{(\Sigma (T_2))^2}{n_2} + \frac{(\Sigma T_3)^2}{n_3} - \frac{T^2}{N} = \frac{(23)^2}{3} + \frac{13^2}{3} + \frac{4}{3} - 160.4444$$
$$= 73.5556$$

$$SST = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N} = 25 + 9 + 225 + 9 + 0 + 256 + 4 + 36 + 36$$
$$- 160.4444$$
$$= 439.5556$$

$SSE = SST - (SSC + SSR) = 439.5556 - (304.2222 + 73.5556)$
$= 61.7778$

$MSC = \frac{SSC}{C-1} = \frac{304.2222}{2} = 152.1111$

$MSR = \frac{SSR}{r-1} = \frac{73.5556}{2} = 36.7778$

$MSE = \frac{SSE}{(C-1)(r-1)} = \frac{61.7778}{4} = 15.4444$

$F_C Cal = \frac{MSC}{MSE} = \frac{152.1111}{15.4444} = 9.8489$ | $F_C tab = F_{0.05}(2,4) = 6.94$

$F_R Cal = \frac{MSR}{MSE} = \frac{36.7778}{15.4444} = 2.3813$ | $F_R tab = F_{0.05}(2,4) = 6.94$

$\boxed{\therefore F_C Cal > F_C tab}$ → Null Hypothesis is Rejected
The Detergents are Not Equally Effective. $\boxed{D_A \neq D_B \neq D_C}$

$\boxed{\therefore F_R Cal > F_R tab}$ → Null Hypothesis is Accepted.
The Temperature Of water is Equally Effective