21/1/19

UNIT - 4

DYNAMIC PROGRAMMING

* All pairs shortest path problem (or) Floyd Warshall
* Single source shortest paths general weights (Dijkstra's)
* String Edition.
* Zero by one 0/1 Knapsack problem
* Reliability Design

* Dynamic programming is applicable for only optimization problems.
* For a given problem we may get any no. of solutions from all those solutions we seek for the optimal solutions. and such an optimal solution becomes the solution of to the given problem.

*2M

Principle of optimality:-

* The dynamic programming algorithm obtains the solutions using principle optimality.
* The principle of optimality states that in an optimal sequences of decisions (or) choices.
* Each sub sequences must be optimal.
* When it is not possible to apply the principle of optimality it is almost impossible to obtain the solution using the dynamic programming approach.
* For example finding of shortest path in a given graph uses the principle of optimality.

All pairs shortest path problem:-

* Suppose the given graph is weighted and connected graph
* The objective of all pairs shortest path problem is to find shortest path between each and every pair of nodes present in the given graph
* To find shortest path between every pair of nodes using dynamic programming we need to find $A^{k}[i,j]$

where $k = 0$ to n

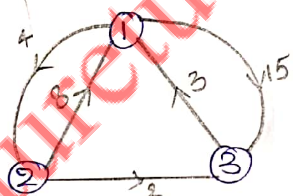
$A^{k}_{[i,j]}$ represents path between i to j via k.

* If $k=0$ we can write A^0 from given graph i.e $A^0 = W[i,j]$

If $k \geq 1$ we can compute the following

$$\boxed{A^{k}[i,j] = \min\{A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\}}$$

Solve Minimum distances (or) All pair shortest path problem using Dynamic programming.



Now convert the given graph into matrix form.

$$\begin{bmatrix} 0 & 4 & 15 \\ 8 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix} = A^0$$

Now find out A^1 Matrix

$A^1_{11} = 0$

$A^1_{12} = \min\{A^{1-1}[1,2],\ A^{1-1}[1,1] + A^{1-1}[1,2]\}$

$= \min\{A^0[1,2], A^0[1,1] + A^0[1,2]\}$

$= \min\{4, 0+4\}$

$= 4$

$A^1_{13} = \min\{A^{1-1}[1,3], A^{1-1}[1,1] + A^{1-1}[1,3]\}$

$= \min\{15, 0+15\}$

$= 15$

$A^1_{21} = \min\{A^{1-1}[2,1], A^{1-1}[2,1] + A^{1-1}[1,1]\}$

$= \min\{8, 8+0\}$

$= 8$

$A^1_{22} = 0$

$A^1_{23} = \min\{A^{1-1}[2,3], A^{1-1}[2,1] + A^{1-1}[1,3]\}$

$= \min\{2, 8+15\}$

$= \min\{2, 23\}$

$= 2$

$A^1_{31} = \min\{A^{1-1}[3,1], A^{1-1}[3,1] + A^{1-1}[1,1]\}$

$= \min\{3, 3+0\}$

$= \cancel{0}\ 3$

$A^1_{32} = \min\{A^{1-1}[3,2], A^{1-1}[3,1] + A^{1-1}[1,2]\}$

$= \min\{\infty, 3+4\}$

$= \min\{\infty, 7\}$

$= 7$

$A^1_{33} = 0$

$\cancel{A^2_{11} = \min\{}$ $A^1 = \begin{bmatrix} 0 & 4 & 15 \\ 8 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$

$A^2_{11} = \min\{A^{2-1}[1,1], A^{2-1}[1,2] + A^{2-1}[2,1]\}$

$= \min\{0, 4+8\}$

$= 0$

$A^2_{12} = \min\{A^{2-1}[1,2], A^{2-1}[1,2] + A^{2-1}[2,2]\}$

$= \min\{4, 4+0\}$

$= 4$

$A^2_{13} = \min\{A^{2-1}[1,3],\ A^{2-1}[1,2] + A^{2-1}[2,3]\}$

$= \min\{15, 4+2\}$

$= 6$

$A^2_{21} = \min\{A^{2-1}[2,1],\ A^{2-1}[2,2] + A^{2-1}[2,1]\}$

$= \min\{8, 0+8\}$

$= 8$

$A^2_{22} = 0$

$A^2_{23} = \min\{A^{2-1}[2,3],\ A^{2-1}[2,2] + A^{2-1}[2,3]\}$

$= \min\{2, 0+2\}$

$= 2$

$A^2_{31} = \min\{A^{2-1}[3,1],\ A^{2-1}[3,2] + A^{2-1}[2,1]\}$

$= \min\{3, 7+8\} = \min\{3, 15\}$

$= 3$

$A^2_{32} = \min\{A^{2-1}[3,2],\ A^{2-1}[3,2] + A^{2-1}[2,2]\}$

$= \min\{7, 7+0\}$

$= 7$

$A^2_{33} = 0$

$$A^2 = \begin{bmatrix} 0 & 4 & 6 \\ 8 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Now findout A^3 matrix, here $k=3$

$A^3_{11} = 0$

$A^3_{12} = \min\{A^2[1,2],\ A^2[1,3] + A^2[3,2]\}$

$= \min\{4, 6+7\}$

$= 4$

$A^3_{13} = \min\{A^2[1,3],\ A^2[1,3] + A^2[3,3]\}$

$= \min\{6, 6+0\}$

$= 6$

$A^3_{21} = \min\{A^2_{[2,1]}, A^2_{[2,3]} + A^2_{[3,1]}\}$

$= \min\{8, 2+3\}$

$= 5$

$A^3_{22} = 0$

$A^3_{23} = \min\{A^2_{[2,3]}, A^2_{[2,3]} + A^2_{[3,3]}\}$

$= \min\{2, 2+0\}$

$= 2$

$A^3_{31} = \min\{A^2_{[3,1]}, A^2_{[3,3]} + A^2_{[3,1]}\}$

$= \min\{3, 0+3\}$

$= 3$

$A^3_{32} = \min\{A^2_{[3,2]}, A^2_{[3,3]} + A^2_{[3,2]}\}$

$= \min\{7, 0+7\}$

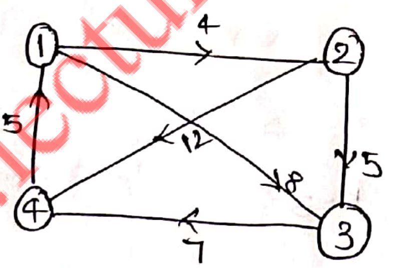
$= 7$

$A^3_{33} = 0$

$$A^3 = \begin{pmatrix} 0 & 4 & 6 \\ 5 & 0 & 2 \\ 3 & 7 & 0 \end{pmatrix}$$

Hence the distances between all pairs obtain

ex:



Now convert the given graph into matrix form

$$\begin{bmatrix} 0 & 4 & 8 & \alpha \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & \alpha & \alpha & 0 \end{bmatrix}$$

Now find out A^1 Matrix

$A^1_{11} = 0$

$A'_{12} = \min\{A^{1-1}_{[1,2]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,2]}\}$

$= \min\{4, 0+4\}$

$= \cancel{\alpha}\ 4$

$A'_{13} = \min\{A^{1-1}_{[1,3]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,3]}\}$

$= \min\{8, 0+8\}$

$= 8$

$A'_{14} = \min\{A^{1-1}_{[1,4]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,4]}\}$

$= \min\{\alpha, 0+\alpha\}$

$= \alpha$

$A'_{21} = \min\{A^{1-1}_{[2,1]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,1]}\}$

$= \min\{\alpha, \alpha+0\}$

$= \alpha$

$A'_{22} = 0$

$A'_{23} = \min\{A^{1-1}_{[2,3]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,3]}\}$

$= \min\{5, \alpha+8\}$

$= 5$

$A'_{24} = \min\{A^{1-1}_{[2,4]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,4]}\}$

$= \min\{12, \alpha+\alpha\}$

$= 12$

$A'_{31} = \min\{A^{0}_{[3,1]}, A^{0}_{[3,1]} + A^{0}_{[1,1]}\}$

$= \min\{\alpha, \alpha+0\}$

$= \alpha$

$A'_{32} = \min\{A^{0}_{[3,2]}, A^{0}_{[3,1]} + A^{0}_{[1,2]}\}$

$= \min\{\alpha, \alpha+4\}$

$= \cancel{4}\ \alpha$

$A'_{33} = 0$

$A'_{34} = \min\{A^{0}_{[3,4]}, A^{0}_{[3,1]} + A^{0}_{[1,4]}\}$

$= \min\{7, \alpha+\alpha\}$

$= 7$

$A^1_{41} = \min\{A^0_{[4,1]}, A^0_{[4,1]} + A^0_{[1,1]}\}$

$= \min\{5, 5+0\}$

$= 5$

$A^1_{42} = \min\{A^0_{[4,2]}, A^0_{[4,1]} + A^0_{[1,2]}\}$

$= \min\{\alpha, 5+4\}$

$= 9$

$A^1_{43} = \min\{A^0_{[4,3]}, A^0_{[4,1]} + A^0_{[1,3]}\}$

$= \min\{\alpha, 5+8\}$

$= 13$

$A^1_{44} = 0$

$$A^1 = \begin{bmatrix} 0 & 4 & 8 & \alpha \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

Now find A^2 matrix, here $k=2$

$A^2_{11} = 0$

$A^2_{12} = \min\{A^1_{[1,2]}, A^1_{[1,2]} + A^1_{[2,2]}\}$

$= \min\{4, 4+0\}$

$= 4$

$\cancel{A^2_{22} = 0}$

$A^2_{13} = \min\{A^1_{[1,3]}, A^1_{[1,2]} + A^1_{[2,3]}\}$

$= \min\{8, 4+5\}$

$= 8$

$A^2_{14} = \min\{A^1_{[1,4]}, A^1_{[1,2]} + A^1_{[2,4]}\}$

$= \min\{\alpha, 4+12\}$

$= 16$

$A^2_{21} = \min\{A^1[2,1],\ A^1[2,2] + A^1[2,1]\}$

$= \min\{\alpha,\ 0 + \alpha\}$

$= \alpha$

$A^2_{22} = 0$

$A^2_{23} = \min\{A^1[2,3],\ A^1[2,2] + A^1[2,3]\}$

$= \min\{\cancel{8}5,\ 0 + 5\}$

$= 5$

$A^2_{24} = \min\{A^1[2,4],\ A^1[2,2] + A^1[2,4]\}$

$= \min\{12,\ 0 + 12\}$

$= 12$

$A^2_{31} = \min\{A^1[3,1],\ A^1[3,2] + A^1[2,1]\}$

$= \min\{\alpha,\ \alpha + \alpha\}$

$= \alpha$

$A^2_{32} = \min\{A^1[3,2],\ A^1[3,2] + A^1[2,2]\}$

$= \min\{\alpha,\ \alpha + 0\}$

$= \alpha$

$A^2_{33} = 0$

$A^2_{34} = \min\{A^1[3,4],\ A^1[3,2] + A^1[2,4]\}$

$= \min\{7,\ \alpha + 12\}$

$= 7$

$A^2_{41} = \min\{A^1[4,1],\ A^1[4,2] + A^1[2,1]\}$

$= \min\{5,\ 9 + \alpha\}$

$= 5$

$A^2_{42} = \min\{A^1[4,2],\ A^1[4,2] + A^1[2,2]\}$

$= \min\{9,\ 9 + 0\}$

$= 9$

$A^2_{43} = \min\{A^1[4,3],\ A^1[4,2] + A^1[2,3]\}$

$= \min\{13,\ 9 + 5\}$

$= 13$

$A^2_{44} = 0$

$$A^2 = \begin{bmatrix} 0 & 4 & 8 & 16 \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

$A^3_{11} = 0$

$A^3_{12} = \min\{A^2[1,2],\ A^2[1,3] + A^2[3,2]\}$

$= \min\{4,\ 8+\alpha\}$

$= 4$

~~$A^3_{21} = \min\{A^2[1,2]$~~

~~$A^3_{24} = \alpha$~~

$A^3_{13} = \min\{A^2[1,3],\ A^2[1,3] + A^2[3,3]\}$

$= \min\{8,\ 8+0\}$

$= 8$

$A^3_{14} = \min\{A^2[1,4],\ A^2[1,3] + A^2[3,4]\}$

$= \min\{16,\ 8+7\}$

$= 15$

$A^3_{21} = \min\{A^2[2,1],\ A^2[2,3] + A^2[3,1]\}$

$= \min\{\alpha,\ 5+\alpha\}$

$= \alpha$

$A^3_{22} = 0$

$A^3_{23} = \min\{A^2[2,3],\ A^2[2,3] + A^2[3,3]\}$

$= \min\{5,\ 5+0\}$

$= 5$

$A^3_{24} = \min\{A^2[2,4],\ A^2[2,3] + A^2[3,4]\}$

$= \min\{12,\ 5+7\}$

$= 12$

$A^3_{31} = \min\{A^2[3,1],\ A^2[3,3] + A^2[3,1]\}$

$= \min\{\alpha,\ 0+\alpha\}$

$= \alpha$

$A^3_{32} = \min\{A^2[3,2],\ A^2[3,3] + A^2[3,2]\}$

$= \min\{\alpha,\ 0+\alpha\}$

$= \alpha$

$A^3_{33} = 0$

$A^3_{34} = \min\{A^2_{[3,4]}, A^2_{[3,3]} + A^2_{[3,4]}\}$

$= \min\{7, 0+7\}$

$= 7$

$A^3_{41} = \min\{A^2_{[4,1]}, A^2_{[4,3]} + A^2_{[3,1]}\}$

$= \min\{5, 13+\alpha\}$

$= 5$

$A^3_{42} = \min\{A^2_{[4,2]}, A^2_{[4,3]} + A^2_{[3,2]}\}$

$= \min\{9, 13+\alpha\}$

$= 9$

$A^3_{43} = \min\{A^2_{[4,3]}, A^2_{[4,3]} + A^2_{[3,3]}\}$

$= \min\{13, 13+0\}$

$= 13$

$A^3_{44} = 0$

$$A^3 = \begin{bmatrix} 0 & 4 & 8 & 15 \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

$A^4_{11} = 0$

$A^4_{12} = \min\{A^3_{[1,2]}, A^3_{[1,4]} + A^3_{[4,2]}\}$

$= \min\{4, 15+9\}$

$= 4$

$A^4_{13} = \min\{A^3_{[1,3]}, A^3_{[1,4]} + A^3_{[4,3]}\}$

$= \min\{8, 15+13\}$

$= 8$

$A^4_{14} = \min\{A^3_{[1,4]}, A^3_{[1,4]} + A^3_{[4,4]}\}$

$= \min\{15, 15+0\}$

$= 15$

$A^4_{21} = \min\{A^3_{[2,1]}, A^3_{[2,4]} + A^3_{[4,1]}\}$

$= \min\{\alpha, 12+5\}$

$= 17$

$A^4_{22} = 0$

$A^4_{23} = \min\{A^3_{[2,3]}, A^3_{[2,4]} + A^3_{[4,3]}\}$

$= \min\{5, 12+13\}$

$= 5$

$A^4_{24} = \min\{A^3_{[2,4]}, A^3_{[2,4]} + A^3_{[4,4]}\}$

$= \min\{12, 12+0\}$

$= 12$

$A^4_{31} = \min\{A^3_{[3,1]}, A^3_{[3,4]} + A^3_{[4,1]}\}$

$= \min\{\alpha, 7+5\}$

$= 12$

$A^4_{32} = \min\{A^3_{[3,2]}, A^3_{[3,4]} + A^3_{[4,2]}\}$

$= \min\{\alpha, 7+9\}$

$= 16$

$A^4_{33} = 0$

$A^4_{34} = \min\{A^3_{[3,4]}, A^3_{[3,4]} + A^3_{[4,4]}\}$

$= \min\{7, 7+0\}$

$= 7$

$A^4_{41} = \min\{A^3_{[4,1]}, A^3_{[4,4]} + A^3_{[4,1]}\}$

$= \min\{5, 0+5\}$

$= 5$

$A^4_{42} = \min\{A^3_{[4,2]}, A^3_{[4,4]} + A^3_{[4,2]}\}$

$= \min\{9, 0+9\}$

$= 9$

$A^4_{43} = \min\{A^3_{[4,3]}, A^3_{[4,4]} + A^3_{[4,3]}\}$

$= \min\{13, 0+13\}$

$= 13$

$A^4_{44} = 0$

$$A^4 = \begin{bmatrix} 0 & 4 & 8 & 15 \\ 17 & 0 & 5 & 12 \\ 12 & 16 & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

4/1/19 All pairs shortest path Algorithm:-

```
Algorithm AllPaths(cost, A, n)
{
  for i:=1 to n do
     for j:=1 to n do
          A[i,j] := cost[i,j];

  for k:=1 to n do
    for i:=1 to n do
        for j:=1 to n do
            A[i,j] := min(A[i,j], A[i,k] + A[k,j]);
}
```

Note:-

Time complexity all pairs shortest path problem is $O(n^3)$

0/1 Knapsack problem:-

* 0/1 Knapsack problem represents 0 means not consider particular object total weight, 1 means consider total weight. M(W) represents total weight of the Knapsack.

* Maximum profit is $\sum_{i=1}^{n} p_i x_i$

where x_i is exact 0 (or) 1

Total weight is $\sum_{i=1}^{n} w_i x_i$ then Maximize

$$\boxed{\text{Maximized } \sum_{i=1}^{n} p_i x_i \text{ subject to } \sum_{i=1}^{n} w_i x_i \leq m}$$

PURGING RULE (or) Dominance Rule:-

* If S^{i+1} contains (P_j, W_j) & (P_k, W_k) is two pairs such that $P_j \leq P_k$ & $W_j \geq W_k$ then (P_j, W_j) can be eliminated

* In Dominance Rule remove the pair with less profit and more weight

eg: Obtain the solutions for the following $n=3$ & $m=6$ profits $(P_1, P_2, P_3) = (1,2,4)$ corresponding weights $(W_1, W_2, W_3) = (2,3,3)$

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$

$= \{S^0 + (1,2)\}$

$= \{(0,0) + (1,2)\}$

$S_1^0 = \{(1,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (1,2)\}$

$S_1^2 = \{$Select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (2,3)\}$

$S_1^2 = \{(2,3), (3,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_1^3 = \{$select next pair $(4,3)$ and add it with $S^2\}$

$= \{S^2 + (4,3)\}$

$= \{(4,3), (5,5), (6,6), (7,8)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

$= \{(0,0), (1,2), (2,3), (3,5), (4,3), (5,5), (6,6), (7,8)\}$

Now apply PURGING RULE on $(3,5)$ and $(4,3)$

$3 \leq 4$ (T) and $5 \geq 3$ (T)

Hence Remove $(3,5)$

apply purging rule $(2,3)$ and $(4,3)$

$2 \leq 4$ (T) and $3 \geq 3$ (T)

Hence Remove $(2,3)$

$S^3 = \{(0,0), (1,2), (4,3), (5,5), (6,6), (7,8)\}$

S^3 contain pair (7,8) but our bag capacity is only 6.
Hence remove (7,8) pair from S^3.

$S^3 = \{(0,0), (1,2), (4,3), (5,5), (6,6)\}$

From S^3 consider (6,6) pair

$(6,6) \in S^3_3$ select third object pair (4,3)

$\{(6-4), (6-3)\} = (2,3)$

now set x_3 $x_3 = 1$

Now $(2,3) \in S^2$ consider second object pair (2,3)

$\{(2-2), (3-3)\} = \{(0,0)\}$

now set $x_2 = 1$

now $\sum_{i=1}^{n} w_i x_i = w_1 x_1 + w_2 x_2 + w_3 x_3$
$= 2 \times 0 + 3 \times 1 + 3 \times 1$
$= 0 + 3 + 3$
$= 6$.

The maximum profit is $\sum_{i=1}^{n} p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3$
$= 1 \times 0 + 2 \times 1 + 4 \times 1$
$= 0 + 2 + 4$
$= 6$.

Eg:- Consider $n=7$, $(w_1, w_2, w_3, w_4, w_5, w_6, w_7) = (2,3,5,7,1,4,1)$
$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10,5,15,7,6,18,3)$ $M=15$ solve 0/1 knapsack problem using dynamic programming

~~consider~~ $S^0 = \{(0,0)\}$.

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$
$= \{S^0 + (10,2)\}$
$= \{(0,0) + (10,2)\}$

$S_1^0 = \{(10,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (10,2)\}$

$S_1^2 = \{$select next ~~pair (P_2, W_2)~~ and add it with $S^1\}$

Q: Consider $n=7$ $(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2,3,5,7,1,4,1)$
$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10,5,15,7,6,18,3)$ $M=15$ Solve
0/1 Knapsack problem using dynamic programming.

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$

$= \{S^0 + (10,2)\}$

$= \{(0,0) + (10,2)\}$

$P_j \le P_k$, $W_j \ge W_k$

$S_1^0 = \{(10,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (10,2)\}$

Now apply purging rule on

$0 \le 10$ T

$0 \ge 2$ F

$S_1^2 = \{$select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (5,3)\}$

$S_1^2 = \{(5,3), (15,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (5,3), (10,2), (15,5)\}$

apply purging rule $(5,3)$ and $(10,2)$

$5 \le 10$ (T) and $3 \ge 2$ (T)

Hence remove $(5,3)$

$S^2 = \{(0,0), (10,2), (15,5)\}$

$S_1^3 = \{$select next pair (P_3, W_3) and add it with $S^2\}$

$= \{S^2 + (15,5)\}$

$S_1^3 = \{(15,5), (25,7), (30,10)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

(15,5)

$S^3 = \{(0,0), (10,2), (15,5), (25,7), (30,10)\}$

purging rule $(15,5)$ and $(15,5)$

$15 \le 15$ (T) and $(5 \ge 5)$ (T)

Hence remove (15,5)

$S^3 = \{(0,0), (10,2), (15,5), (25,7), (30,10)\}$

$S_1^4 = \{$Select next pair (7,7) and add it with $S^3\}$

$= \{S^3 + (7,7)\}$ $X_2 = 1$

$= \{(7,7), (17,9), (22,12), (32,14), (37,17)\}$ 15 5 (10,2)

$S^4 = \{$merge S^3 and $S_1^4\}$

$S^4 = \{(0,0), \cancel{(7,7)}, (10,2), (15,5), \cancel{(17,9)}, \cancel{(22,12)}, (25,7), (30,10), (32,14), \cancel{(37,17)}\}$

apply purging rule on (7,7) and (10,2)

$7 \leq 10$ (T) and $7 \geq 2$ (T)

Now remove (7,7)

apply purging rule on (22,12) and (25,7)

$22 \leq 25$ (T) and $12 \geq 7$ (T)

Hence remove (22,12)

apply purging rule on (17,9) and (25,7)

$17 \leq 25$ (T) and $9 \geq 7$ (T)

Hence remove (17,9)

$S^4 = \{(0,0), (10,2), (15,5), (25,7), (30,10), (32,14)\}$

$S_1^5 = \{$select next pair (6,1) and add it with $S^4\}$

$= \{S^4 + (6,1)\}$

$= \{(6,1), (16,3), (21,6), (31,8), (36,11), (38,15)\}$

$S^5 = \{$merge S^4 and $S_1^5\}$

$S^5 = \{(0,0), (6,1), (10,2), \cancel{(15,5)}, (16,3), (21,6), (25,7), (30,10), (31,8), \cancel{(32,14)}, (36,11), (38,15)\}$

apply purging rule (15,5) and (16,3)

$15 \leq 16$ (T) and $(5 \geq 3)$ (T)

Hence remove (15,5)

$X_1 = 1$ X_1 (10,2) 10,2 0

apply purging rule (30,10) and (31,8) $x_4 = 0$

$30 \le 31$ (T) and $10 \ge 8$ (T) $x_5 = 1$

Hence remove (30,10) $x_3 = 1$

apply purging rule (32,14) and (36,11)   (36,11) 6,1

$32 \le 36$ (T) and $14 \ge 11$ (T)   (30,10)

Hence remove (32,14)

$S^5 = \{(0,0), (6,1), (10,2), (16,3), (21,6), (25,7), (31,8), (36,11), (38,15)\}$

$S_1^6 = \{$select next pair (18,4) and add it with $S^5\}$

$= \{S^5 + (18,4)\}$

$= \{(18,4), (24,5), (28,6), (34,7), (39,10), (43,11), (49,12), (54,15), (56,19)\}$

$S^6 = \{$merge S^5 and $S_1^6\}$

$= \{(0,0), (6,1), (10,2), (16,3), (18,4), (21,6), (24,5), (25,7), (28,6), (31,8), (34,7), (36,11), (38,15), (39,10), (43,11), (49,12), (54,15), (56,19)\}$

apply purging rule (21,6) and (24,5)   (30,10) 15,5

$21 \le 24$ (T) and $6 \ge 5$ (T)

Hence remove (21,6)   (15 5)

apply purging rule (25,7) and (28,6)

$25 \le 28$ (T) and $7 \ge 6$ (T)

Hence remove (25,7)

apply purging rule (31,8) and (34,7)

$31 \le 34$ (T) and $8 \ge 7$ (T)

Hence remove (31,8)

apply purging rule (38,15) and (39,10)

$38 \le 39$ (T) and $15 \ge 10$ (T)

Hence remove (38,15)

$S^6 = \{(0,0), (6,1), (10,2), (16,3), (18,4), (24,5), (28,6), (34,7), (36,11), (39,10), (43,11), (49,12), (54,15)\}$

$S_1^7 = \{$select next pair (3,1) and add it with $S^6\}$

$= \{S^6 + (3,1)\}$

$= \{(3,1), (9,2), (13,3), (19,4), (21,5), (27,6), (31,7), (37,8), (39,12),$

(42,11), (46,12), (52,13), (57,16)}

$S^7 = \{\text{merge } S^6 \text{ and } S_1^7\}$

$= \{(0,0), \cancel{(3,1)}, (6,1), \cancel{(9,2)}, (10,2), \cancel{(13,3)}, (16,3), \cancel{(18,4)}, (19,4), \cancel{(21,5)},$
$(24,5), \cancel{(27,6)}, (28,6), \cancel{(31,7)}, (34,7), \cancel{(36,11)}, (37,8), \cancel{(39,10)}, \cancel{(39,12)},$
$\cancel{(42,11)}, (43,11), \cancel{(46,12)}, (49,12), (52,13), \cancel{(54,15)}, \cancel{(57,16)}\}$

apply purging rule (3,1) and (6,1)

$3 \le 6$ (T) and $(1 \ge 1)$ T

remove (3,1)

apply purging rule (9,2) and (10,2)

$9 \le 10$ (T) and $2 \ge 2$ (T)

remove (9,2)

apply purging rule (13,3) and (16,3)

$13 \le 16$ (T) and $3 \ge 3$ (T)

remove (13,3)

apply purging rule (18,4) and (19,4)

$18 \le 19$ (T) and $4 \ge 4$ (T)

remove (18,4)

apply purging rule (21,5) and (24,5)

$21 \le 24$ (T) and $5 \ge 5$ (T)

remove (21,5)

apply purging rule (27,6) and (28,6)

$27 \le 28$ (T) and $(6 \ge 6)$ T

remove (27,6)

apply purging rule (31,7) and (34,7)

$31 \le 34$ (T) and $7 \ge 7$ (T)

remove (31,7)

apply purging rule (36,11) and (37,8)

$36 \le 37$ (T) and $11 \ge 8$ (T)

remove (36,11)

apply purging rule (39,12) and (42,11)

$39 \le 42$ (T) and $12 \ge 11$ (T)

remove (39,12)

apply purging rule (42,11) and (43,11)

$42 \le 43$ (T) and $11 \ge 11$ (T)

remove (42, 11)

apply purging rule (46,12) and (49,12)

$46 \le 49$ (T) and $12 \ge 12$ (T)

remove (46, 12)

$S^7 = \{(0,0), (6,1), (10,2), (16,3), (19,4), (24,5), (28,6), (34,7), (37,8), (39,10), (43,11), (49,12), (52,13), (54,15)\}$

from S^7 consider (54, 15) pair

$(54,15) \notin S^7$ Select seventh object pair

$x_7 = 0$

we take $x_6 = 1$ $(54-18, 15-4) = (36,11) \in S^5$

$x_5 = 1$ $(36-6, 11-1) = (30,10) \notin S^4$

$x_4 = 0$ $(30,10) \in S^3$

$x_3 = 1$ $(30-15, 10-5) = (15,5) \in S^2$

$x_2 = 1$, $(15-5, 5-3) = (10,2) \in S^1$

$x_1 = 1$, $(10-10, 2-2) = 0$

$x_7 = 0$

$x_6 =$

$S_6 =$

18,4

(54-18)

(15-4)

= (36,11)

eg:- consider $n = 4$, $m = 8$ $(P_1, P_2, P_3, P_4) = (1,2,5,6)$, and weights $(w_1, w_2, w_3, w_4) = (2,3,4,5)$.

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, w_1) and add it with $S^0\}$

$= \{S^0 + (1,2)\}$

$S_1^0 = \{(1,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (1,2)\}$

Now apply purging rule on

$0 \le 1$ (T) and $0 \ge 2$ (F)

$S_1^2 = \{$select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (2,3)\}$

$S_1^2 = \{(2,3), (3,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_1^3 = \{$select next pair (5,4) and add it with $S^2\}$

$= \{S^2 + (5,4)\}$

$= \{(5,4), (6,6), (7,7), (8,9)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

$= \{(0,0), (1,2), (2,3), (3,5), (5,4), (6,6), (7,7), (8,9)\}$

apply purging rule (3,5) and (5,4)

$3 \le 5$ (T) and $(5 \ge 4)$ T

Hence remove (3,5)

$S^3 = \{(0,0), (1,2), (2,3), (5,4), (6,6), (7,7), (8,9)\}$

$S_1^4 = \{$Select next pair (6,5) and add it with $S^3\}$

$= \{S^3 + (6,5)\}$

$= \{(6,5), (7,7), (8,8), (9,10), (11,9), (12,11), (13,11), (13,12), (14,14)\}$

$S^4 = \{$merge S^3 and $S_1^4\}$

$= \{(0,0), (1,2), (2,3), (5,4), (6,6), (6,5), (7,7), (7,7), (8,9), (8,8), (9,10), (11,9), (12,11), (13,11), (13,12), (14,14)\}$

apply purging rule on (6,6) and (6,5)

$6 \le 6$ (T) and $6 \ge 5$ (T)

Hence remove (6,6)

apply purging rule on (7,7) and (7,7)

$7 \le 7$ (T) and $7 \ge 7$ (T)

remove (7,7)

apply purging rule on (8,9) and (8,8)

$8 \le 8$ (T) and $9 \ge 8$ (T)

Hence remove (8,9)

apply purging rule on (12,11) and (13,11)

$12 \le 13$ (T) and $11 \ge 11$ (T)

Hence remove (12,11)

apply purging rule on (9,10) and (11,9)

$9 \leq 11$ (T) and $10 \geq 9$ (T)

Hence remove (9,10)

$S^4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), (7,7), (8,8)\}$

from S^4 consider (8,8) pair

$(8,8) \in S^4$ select fourth object pair

$S^4 = \phi$

$(8-6, 8-5) = (2,3)$

$(2-2, 3-3) = (0,0)$ $S^2 = 1$

(8,8)

$x_4 = 1$

(5,4)

$x_2 = 1$

(6,5)

$x_1 = 0$

(2,3)

$x_3 = 0$

(2,2)

(0,0)

(2,3,4,)

$\sum w_i x_i \leq M$

$8 \leq 8$

$2 \times 0 + 3 \times 1 + 4 \times 0 + 5 \times 1 = 8$

0/1 KNAPSACK ALGORITHM:-

Algorithm DKP(p, w, n, m)
{
$S^0 := \{(0,0)\}$;
for i := 1 to n-1 do
{
$S_1^{i-1} := \{(P, W) \mid (P - p_i, W - w_i) \in S^{i-1}$ and $W \le m\}$;
$S^i :=$ MergePurge (S^{i-1}, S_1^{i-1});
}
$(PX, WY) :=$ last pair in S^{n-1};
$(PY, WY) := (P' + p_n, W' + w_n)$ where W' is the largest W in any pair in S^{n-1} such that $W + w_n \le m$;
if $(PX > PY)$ then $x_n := 0$;
else $x_n := 1$;
TraceBackFor $(x_{n-1}, \ldots, x_1)$;
}

12) SINGLE SOURCE SHORTEST PATH PROBLEM

* Single source shortest path problem in dynamic programming is solved by using BellMan & Ford algorithm

* Greedy single source shortest path problem does not allow negative cost. But where as Bellman ~~forde~~ Ford algorithm allows negative

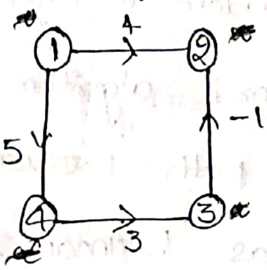
* To solve single source shortest path problem in dynamic programming, BellMan & Ford introduced Relaxiation Rule that is

$\boxed{\text{if } d(u) + cost(u,v) < d(v) \text{ then change as } d(v) = d(u) + cost(u,v)}$

* The differences between single source shortest path in Greedy Method and single source shortest path in Dynamic programming

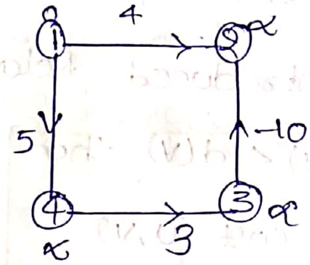
SSSP in GM	SSSP in DP
* Greedy single source shortest path problem is solved by using Dijkstra's algorithm.	* Dynamic programming single source shortest path problem is solved by using BellMan & Ford algorithm
* It is connected and weighted graph	* It is also a connected and weighted graph.
* In greedy sssp we can obtain an optimal solution from possible no. of feasible solutions.	* In Dynamic programming SSSP we can satisfy all choices (or) decisions.
* In greedy sssp we can't solve -ve edge cost in the given graph.	* In Dynamic programming we can solve the problem even the edge cost may be -ve
* In greedy sssp, finally we can written shortest path and minimum distance.	* In dynamic SSSP we can written individual minimum distances.

eg:- Solve SSSP using Dynamic programming.



Sol:- Initially, the distance of source vertex is "0" and remaining vertex distances is α

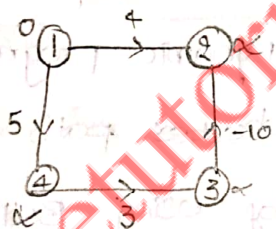
$d(1) = 0$
$d(2) = \alpha$
$d(3) = \alpha$
$d(4) = \alpha$



Now form edge list is (1,2), (1,4), (3,2), (4,3)

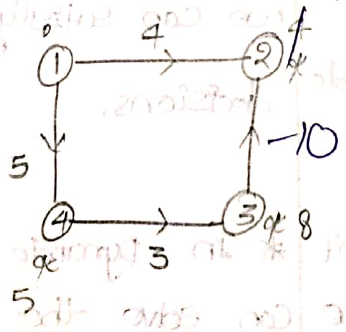
Now apply relaxation Rule

i.e $\boxed{d(u) + cost(u,v) < d(v) \text{ then change as } d(v) = d(u) + cost(u,v)}$



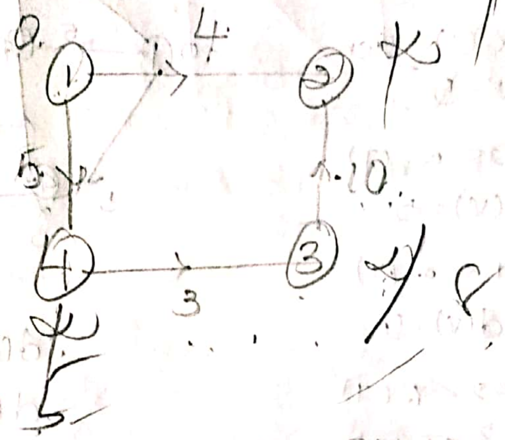
$0+4 < \alpha$ (T) then
$d(v) = 0+4 = 4$
$0+5 < \alpha$ (T)
$d(v) = 0+5 = 5$

$5+3 < \alpha$ (T)
$8 < \alpha$
$d(v) = 5+3 = 8$



1. $0+4 < 4$ (F)

$4 - 1$



(1,2) (1,4), (3,2), (4,3)

$d(1) = 0$

$d(2) = \alpha$

$d(3) = \alpha$

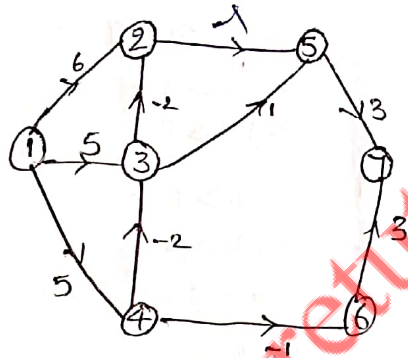
$d(4) = \alpha$

(1) $0 + 4 < \alpha$ True

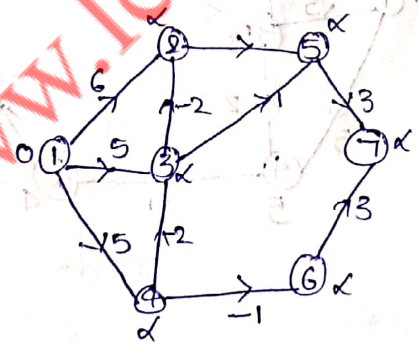
$d(2) = 0 + 4 = 4$

$d(v)$

Solve



Initially the distance of source vertex is "0" and remaining vertex distances is 'α'



$d(1) = 0$
$d(2) = \alpha$
$d(3) = \alpha$
$d(4) = \alpha$
$d(5) = \alpha$
$d(6) = \alpha$
$d(7) = \alpha$

Now form edge list is (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7).

i.e $d(u) + cost(u,v) < d(v)$ then change as.

$d(v) = d(u) + cost(u,v)$.

① $0+6 < \alpha$ (T) then

$d(v)=6$

$0+5 < \alpha$ (T) then

$d(v)=5$

$0+5 < \alpha$ (T)

$d(v)=5$

$6-1 < \alpha$ (T)

$d(v)=5$

$5-2 < 6$ (T)

$3 < 6$ (T)

$d(v)=3$

$5+1 < 5$

$6 < 5$ (F)

$5-2 < 5$

$3 < 5$ (T)

$d(v)=3$

$5-1 < \alpha$

$4 < \alpha$ (T)

$d(v)=4$

$5+3 < \alpha$

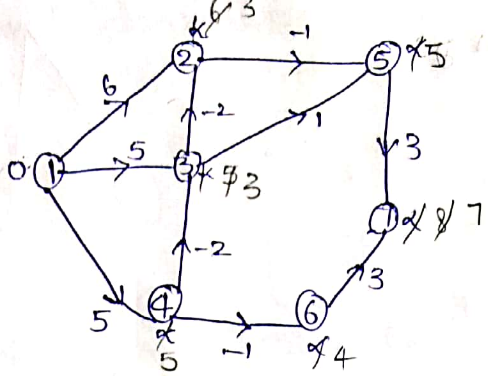
$8 < \alpha$

$d(v)=8$

$4+3 < 8$

$7 < 8$ (T)

$d(v)=7$



$d(1)=0$

$d(2)=3$

$d(3)=3$

$d(4)=5$

$d(5)=5$

$d(6)=4$

$d(7)=7$

② $0+6 < 3$ (F)

$0+5 < 3$ (F)

$0+5 < 5$ (F)

$3-1 < 5$

$2 < 5$ (T)

$d(v)=2$

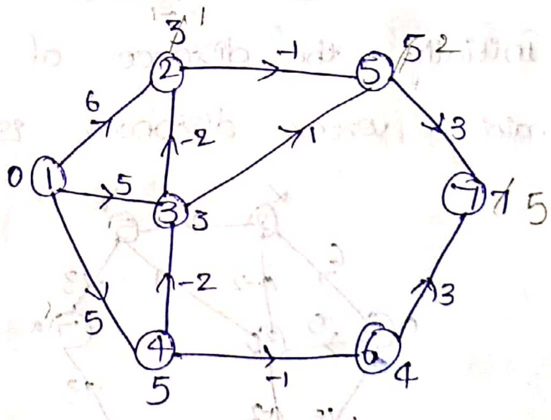
$3-2 < 3$

$1 < 3$ (T)

$d(v)=1$

$3-1 < 2$

$2 < 2$ (F)



$5-2<3$

$3<3$ (F)

$5-1<4$

$4<4$ (F)

$2+3<7$

$5<7$ (T)

$d(v)=5$

$4+3<5$

$7<5$ (F)

$d(1)=0$
$d(2)=1$
$d(3)=3$
$d(4)=5$
$d(5)=2$
$d(6)=4$
$d(7)=5$

(3) $0+6<1$ (F)

$0+5<3$ (F)

$0+5<5$ (F)

$1-1<2$

$0<2$ (T)

$d(v)=0$

$3-2<1$

$1<1$ (F)

$3-1<0$

$2<0$ (F)

$5-2<3$

$3<3$ (F)

$5-1<4$

$4<4$ (F)

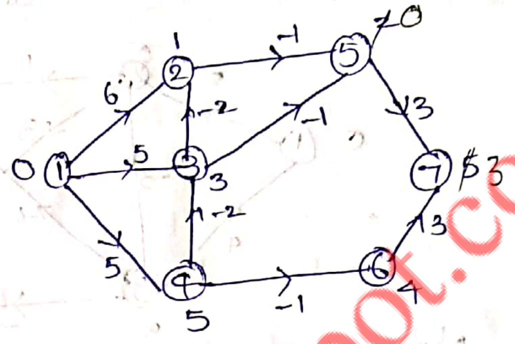
$0+3<5$

$3<5$ (T)

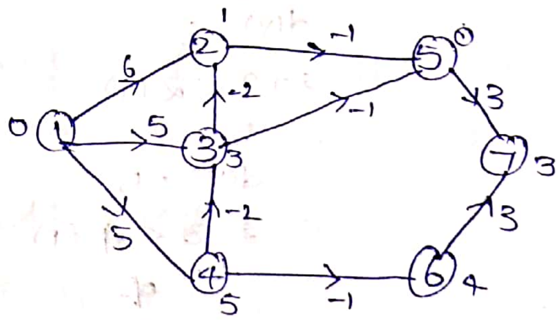
$d(v)=3$

$4+3<3$

$7<3$ (F)



$d(1)=0$
$d(2)=1$
$d(3)=3$
$d(4)=5$
$d(5)=0$
$d(6)=4$
$d(7)=3$



(4) $0+6<1$ (F)

$0+5<3$ (F)

$0+5<5$ (F)

$1-1<0$

$0<0$ (F)

$3-2<1$

$1<1$ (F)

$3-1<0$ (F)

$5-2<3$

$3<3$ (F)

$5-1<4$

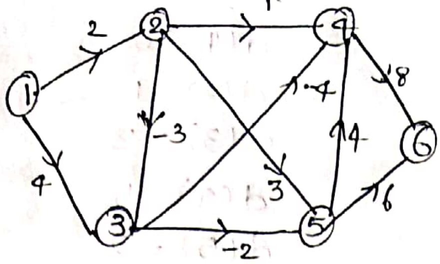
$4<4$ (F)

$0+3<3$ (F)

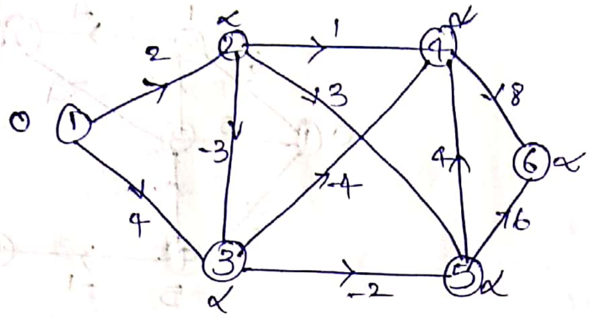
$4+3<3$ (F)

$d(1)=0$
$d(2)=1$
$d(3)=3$
$d(4)=5$
$d(5)=0$
$d(6)=4$
$d(7)=3$

eg:



Initially the vertex distance of source vertex is "0" and remaining vertex distances is "α"



$d(1)=0$
$d(2)=\alpha$
$d(3)=\alpha$
$d(4)=\alpha$
$d(5)=\alpha$
$d(6)=\alpha$

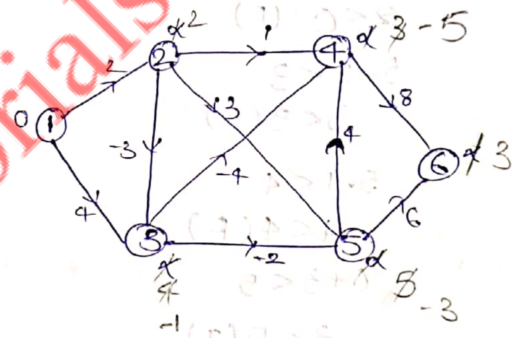
Now form edge list is (1,2), (1,3), (2,4), (2,3), (2,5), (3,5), (3,4), (4,6), (5,4), (5,6).

$d(u)+cost(u,v) < d(v)$ then change as

$d(v) = d(u) + cost(u,v)$

① $0+2<\alpha$ (T) then
$d(v)=2$

$0+4<\alpha$ (T) then
$d(v)=4$



$2+1<\alpha$ (T)
$3<\alpha$
$d(v)=3$

$2+3<\alpha$ (T)
$5<\alpha$
$d(v)=5$

$2-3<\cancel{4}$ (T)
$-1<\cancel{4}$
$d(v)=-1$

$-1-2<5$
$-3<5$ (T)
$d(v)=-3$

$-1-4<3$
$-5<3$ (T)
$d(v)=-5$

$-5+8<\alpha$
$3<\alpha$ (T)
$d(v)=3$

$-3+6<3$
$3<3$ (F)

$-3+4<-5$
$1<-5$ (F)

$d(1)=0$
$d(2)=2$
$d(3)=-1$
$d(4)=-5$
$d(5)=-3$
$d(6)=3$

② $0+2 < 2$ (F)

$0+4 < -1$ (F)

$2+1 < -5$ (F)

$2+3 < -3$ (F)

$2-3 < -1$

$-1 < -1$ (F)

$-1-4 < -5$

$-5 < -5$ (F)

$-1-2 < -3$

$-3 < -3$ (F)

$-5+8 < 3$

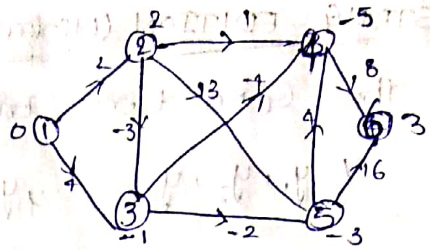
$3 < 3$ (F)

$-3+4 < -5$

$1 < -5$ (F)

$-3+6 < 3$

$3 < 3$ (F)



d(1) = 0

d(2) = 2

d(3) = -1

d(4) = -5

d(5) = -3

d(6) = 3.

Algorithm for SINGLE SOURCE SHORTEST PATH :-

```
Algorithm Bellmanford (v, cost, dist, n)
{
  for i:=1 to n do
    dist [i]:=cost (v,i);
  for k:=2 to n-1 do
   for each u such that u ≠ v and u has
     at least one incoming edge do
    for each (i,u) in the graph do
      if dist [u] > dist [i] + cost [i,u] then
        dist [u] := dist [i] + cost [i,u];
}
```

Time complexity :-

Time complexity for SSSP General weight problem is $O(n^3)$

STRING EDITION (Minimum Edit Distance):-

* The Given two strings $X = x_1, x_2, x_3 \dots x_n$ and $Y = y_1, y_2, y_3, \dots, y_m$ where x_i is $1 \le i \le n$

y_j is $1 \le j \le m$

are members of a finite set of symbols

* We want to transform X Y using a sequence of edit operations on X.

* The permissible edit operations are insert, Delete (or) Remove and Replace (or) change.

* The cost of a sequence of operations is the sum of the costs of individual operations in the sequence.

* The problem of string editing is to identify a minimum cost sequence of editing operations that will transform X into Y.

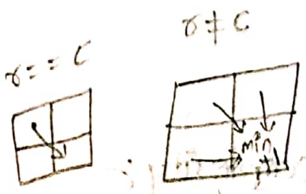
* let $D(x_i)$ be the cost of Deleting the symbol x_i from X.

* let $I(y_j)$ be the cost of inserting the symbol y_j into X

* let $C(x_i, y_j)$ be the cost of changing the symbol x_i of X into y_j of Y

* To solve string edition problem we can use the formula

$$\boxed{cost(i,j) = \min \begin{Bmatrix} cost(i-1, j) + D(x_i), \\ cost(i-1, j-1) + C(x_i, y_j), \\ cost(i, j-1) + I(y_j) \end{Bmatrix}}$$



Consider string Edit problem of $X = aabaababaa$, $Y = babaabab$.

consider string $X = aabaababaa$

string $Y = babaabab$

Now Generate cost Table.

i \ j	0	b	a	b	a	a	b	a	b
0	0	1	2	3	4	5	6	7	8
a	1	1	1	2	3	4	5	6	7
a	2	2	1	2	2	3	4	5	6
b	3	2	2	1	2	3	3	4	5
a	4	3	2	2	1	2	3	3	4
a	5	4	3	3	2	1	2	3	4
b	6	5	4	3	3	2	1	2	3
a	7	6	5	4	3	3	2	1	2
b	8	7	6	5	4	4	3	2	1
a	9	8	7	6	5	4	4	3	2
a	10	9	8	7	6	5	5	4	3

Now $X = a\ a\ b\ a\ a\ b\ a\ b\ a\ a$

$Y = b\ a\ b\ a\ a\ b\ a\ b$

If $a \neq b$ then apply remove operation and add with '1'

Hence the string a is remove from 'X'

If $a \neq b$ then apply remove operation and add with '1'

Hence the string a is removed from 'X'.

If $b = b$ then apply replace operation without adding of "1" hence we can not perform any operation on x & y

likewise perform the replace operations on x & y until we get r = c.

Now if $a \neq b$ then apply replace operation from minimum cost add with 1.

Finally we can perform three operations on X transform to Y.

i.e One replace operation a→b

Two remove operations a, a

Hence final minimum cost is 3.

2) Consider the string Editing problem X = a a b a b, Y = b a bb.

Now x = a a b a b

Y = b a b b

i\j	0	b	a	b	b
0	0	1	2	3	4
a	1	1	1	2	3
a	2	2	1	2	3
b	3	2	2	1	2
a	4	3	2	2	2
b	5	4	3	2	2

→ If b = b then apply replace operation without adding 1, hence we can not perform any operation on x & y

→ If a ≠ b then apply remove operation and add with 1 hence the string a is removed from 'X'.

→ If b = b then apply replace operation without adding 1, hence we can not perform any operation on x & y.

→ If a = a then apply replace operation without adding 1, hence the can not perform any operation on x & y.

→ If a ≠ b then apply replace operation from minimum cost add with 1

Finally we can perform two operations on x transform to y.

i.e one remove operation a

one replace operation a→b

Hence final minimum cost is 2

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RELIABLE DESIGN:-

* let m_i no. of devices in D_i are connected in parallel with reliability r_i
* $(1-r_i)$ is the probability that one copy of the device will malfunction $\rightarrow (1-r_i)$
* The probability that all the devices of this type malfunction at the same time is $(1-r_i)^{m_i}$
* Hence the reliability of device d_i D_i can be expressed as

$$\boxed{\phi_i(m_i) = (1-(1-r_i)^{m_i})}$$

* To find no. of devices in a particular machine we can use the formula as U_i

$$\boxed{U_i = \left\lfloor \left(C + c_i - \sum_{j=1}^{n} c_j \right) \Big/ c_i \right\rfloor}$$

where C = Total cost.

c_i = Individual costs c_1, c_2, c_3

$c_j = c_1 + c_2 + c_3$

Dominance Rule:-

(f_1, x_1) dominates (f_2, x_2) if $f_1 \geq f_2$ for $x_1 \leq x_2$ which means that if we can achieve more reliability by spending less, discard all tuples which spend more for less reliability.

Ex:- construct m_1, m_2, m_3 with the help of $c_1 = 30$, $c_2 = 15$ & $c_3 = 20$ and its corresponding reliabilities are $r_1 = 0.9$, $r_2 = 0.8$ and $r_3 = 0.5$. Total cost $C = 105$

Now $U_i = \left\lfloor \left[C + c_i - \sum_{j=1}^{n} c_j \right] \Big/ c_i \right\rfloor$

$U_1 = C + c_1 - (c_1 + c_2 + c_3) / c_1$

$= \lfloor (105 + 30 - (30 + 15 + 20)) / 30 \rfloor$

$= \lfloor (135-65)/30 \rfloor = \lfloor 2.333 \rfloor = 2$

$U_2 = C + C_2 - (C_1+C_2+C_3)/C_2$

$= \lfloor (105+\cancel{60}\ 15-(30+15+20))/15 \rfloor$

$= \lfloor (120-65)/15 \rfloor = \lfloor 3.66 \rfloor = 3$

$U_3 = \lfloor (105+20-(30+15+20))/20 \rfloor$

$= \lfloor (125-65)/20 \rfloor = \lfloor 3 \rfloor = 3$

Hence $U_1 = 2$ $U_2 = 3$, $U_3 = 3$

The reliability, the cost pair is (r, x) that is $(1,0)$

Hence $S^0 = \{(1,0)\}$

Now

In S^1_1, $i=1$, $j=1$ i.e $m_1=1$

Now $\phi_i m_1 = (1-(1-r_i)^{m_i})$

$= (1-(1-0.9)^1)$

$= (1-(0.1)^1)$

$= 0.9$

cost $C_1 = 1 \times C_1$ i.e $1 \times 30 = 30$

$S^1_1 = \{(0.9, 30)\}$

In S^1_2, $i=1$, $j=2$ i.e $m_1=2$

Now $\phi_i m_1 = (1-(1-0.9)^2)$

$= 1-(0.01)$

$= 0.99$

cost $C_1 = 2 \times C_1$ i.e $2 \times 30 = 60$.

S^1 can be obtained by merging the sets S^1_1, S^1_2

$S^1 = \{(0.9, 30), (0.99, 60)\}$

calculate S^2_1

here $i=2$, $j=1$ i.e $m_2=1$

Now $\phi_2(m_2) = 1-(1-r_2)^{m_2}$

$= 1-(1-0.8)^1 = 1-(0.02) = 0.8$

Since $j=1$, $1 \times c_2 = 1 \times 15 = 15$ is added to s' pair

$s_1^2 = \{(0.9 \times 0.8, 30+15), (0.99 \times 0.8, 60+15)\}$

$= \{(0.72, 45), (0.792, 75)\}$

Now s_2^2 $j=2$, $i=2$, $m_2=2$

$\phi_2(m_2) = 1-(1-r_2)^{m_2} = 1-(1-0.8)^2 = 1-0.02$
$= 0.96$

Since $j=2$, $2 \times c_2 = 2 \times 15 = 30$ is added to s' pairs

$s_2^2 = \{(0.9 \times 0.96, 30+30), (0.99 \times 0.96, 60+30)\}$

$= \{(0.864, 60), (0.9504, 90)\}$

Now find s_3^2

Here $i=2$, $j=3$, $m_2=3$

$\phi_2(m_2) = 1-(1-r_2)^{m_2} = 1-(0.02)^3$
$= 0.992$

Since $j=3$, $3 \times c_2 = 3 \times 15 = 45$

$s_3^2 = \{(0.9 \times 0.992, 30+45), (0.99 \times 0.992, 60+45)\}$

$= \{(0.8928, 75), (0.98208, 105)\}$

s^2 can be obtained by merging set s_1^2, s_2^2, s_3^2

$s^2 = \{(0.72, 45), \cancel{(0.792, 75)}, (0.864, 60), (0.9504, 90),$
$(0.8928, 75), (0.98208, 105)\}$

$0.792 \leq 0.864$ (T) $75 \geq 60$ (T)

Remove $(0.792, 75)$

$s^2 = \{(0.72, 45), (0.864, 60), (0.8928, 75), (0.98208, 105)\}$

Calculate s_1^3

Here $i=3$, $j=1$, $m_3=1$

$\phi_3(m_3) = 1-(1-r_3)^{m_3} = 1-(1-0.5)^1 = 0.5$

Since $j=1$, $c_3=20$

$s_1^3 = \{(0.72 \times 0.5, 45+20), (0.864 \times 0.5, 60+20), (0.8928 \times 0.5,$
$75+20), (0.98208 \times 0.5, 105+20)\}$

last tuple is removed from S_1^3, since cost 125 exceeding the given cost 105.

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$

In S_2^3

Here $i=3$; $j=2$, i.e $m_3=2$

since $j=2$, $2 \times C_3 = 2 \times 20 = 40$

$\phi_3(m_3) = 1-(1-r_3)^{m_3} = 1-(1-0.5)^2 = 1-0.25 = 0.75$.

$S_2^3 = \{(0.72 \times 0.75, 45+40), (0.864 \times 0.75, 60+40)\}$

$= \{(0.54, 85), (0.648, 100)\}$

In S_3^3:

Here $i=3$, $j=3$, $m_3=3$

since $j=3$, $3C_3 = 3 \times 20 = 60$

$\phi_3(m_3) = 1-(1-r_3)^{m_3}$

$= 1-(1-0.5)$

$= 1-0.125 = 0.875$

$S_3^3 = \{(0.72 \times 0.875, 45+60)\}$

S^3 can be obtained by merging the set S_1^3, S_2^3, S_3^3

$S^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95), (0.54, 85), (0.648, 100), (0.63, 105)\}$

Apply purging rule

$S^3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.648, 100)\}$

⇒ The best design has reliability of 0.648 and cost 100

(0.648, 100) pair present in S_2^3, $j=2$ so $m_3=2$

⇒ The (0.648, 100) pair obtained from (0.864, 60), which is present in S_2^2, $j=2$ $m_2=2$

⇒ The (0.864, 60) pair obtained from (0.9, 30) S_1^1, $j=1$ so $m_1=1$

$m_1=1$ $m_2=2$ $m_3=2$

1 device of D_1 | 2 device of D_2 | 2 device of D_3

construct m_1, m_2 & m_3 $c_1 = 25$ $c_2 = 18$, $c_3 = 25$ Total cost is 120, $r_1 = 0.5$, $r_2 = 0.8$, $r_3 = 0.7$.

Now $u_i = \left\lfloor \left[c + c_i - \sum_{j=1}^{n} c_j\right] / c_i \right\rfloor$

$U1 = c + c_1 - (c_1 + c_2 + c_3) / c_1$

$= \left[120 + 25 - (25 + 18 + 25) / 25\right]$ 145 68 77

$= (145 - 68) / 25$ 25) 77 (3 75 20

$= 3$

$U_2 = c + c_2 - (c_1 + c_2 + c_3) / c_2$

$= 120 + 18 - (25 + 18 + 25) / 18$

$= (138 - 68) / 18$ 138 68 70 18) 70 (3 54 16

$= 3$

$U_3 = c + c_3 - (c_1 + c_2 + c_3) / c_3$

$= 120 + 25 - (25 + 18 + 25) / 25$

$= (145 - 68) / 25$

$= 3$

Hence $U1 = 3$, $U2 = 3$ and $U3 = 3$

the reliability, the cost pair is (r, x) that is $(1, 0)$

Hence, $S^0 = \{(1, 0)\}$

now S_1^1 $i = 1$, $j = 1$ $m_1 = 1$

$\phi_i m_1 = (1 - (1 - r_i)^{m_i})$

$= (1 - (1 - 0.5)^1)$

$= (1 - 0.5)$

$= 0.5$

$C_1 = 1 \times C_1$ i.e $1 \times 25 = 25$ is added to S^0 pair

$S_1^1 = \{(0.5, 25)\}$

In S_2^1 $i = 1$, $j = 2$ $m_1 = 2$

$\phi_i m_1 = (1 - (1 - 0.5)^2)$

$= (1 - (0.5)^2)$

$= (1 - 0.25) = 0.75$

$C_1 = 2 \times C_1$ i.e $2 \times 25 = 50$ is added to S^0 pair.

$S_2^1 = \{(0.75, 50)\}$

now S_3^1 $i=1$, $j=3$, $m1=3$

$\phi_1 m_1 = (1-(1-0.5)^3)$

$= (1-(0.125))$

$= 0.875$

$0.75 \times 0.5 = 375$, $000 \times$, 0.375

$C_1 = 3 \times C_1$ i.e $3 \times 25 = 75$ is added to S^0 pair

$S_3^1 = \{(0.875, 75)\}$

S^1 can be merged S_1^1, S_2^1, S_3^1

$S^1 = \{(0.5, 25), (0.75, 50), (0.875, 75)\}$

Calculate S_1^2

here $i=2$, $j=1$ $m2=1$

$\phi_2 m_2 = (1-(1-\cancel{0.2})^{m_2})$

$= (1-(1-0.8)^1)$

$= (1-0.2)$

$= 0.8$

$1 \times C_2 = 1 \times 18 = 18$ is added to S^1 pair

$S_1^2 = \{(0.5 \times 0.8, 25+18), (0.75 \times 0.8, 50+18), (0.875 \times 0.8, 75+18)\}$

1.00, 0.04, 0.96

$= \{(0.4, 43), (0.6, 68), (0.7, 93)\}$

1, 0.2×0.2, 04, $00 \times$, 0.04

Now S_2^2 $i=2$, $j=2$ $m2=2$

$\phi_2 m_2 = (1-(1-0.8)^2) = 1-(0.2)^2 = 1-0.04 = 0.96$

~~$= (1-(1-0.64)) = 1-0.36 = 0.64$~~

$2 \times C2 = 2 \times 18 = 36$ is added to S^1 pair

$S_2^2 = \{(0.5 \times \overset{0.96}{\cancel{0.64}}, 25+36), (0.75 \times \overset{0.96}{\cancel{0.64}}, 50+36), (0.875 \times \overset{0.96}{\cancel{0.64}}, 75+36)\}$

$= \{(0.32, 61), (0.48, 86), (0.56, 111)\}$

Now find S_3^2 $i=2$, $j=3$, $m2=3$

$\phi_2 m_2 = (1-(1-0.8)^3)$

$= (1-(1-0.512))$

$S_2^2 = \{(0.5\times0.96, 25+36), (0.75\times0.96, 50+36), (0.875\times0.96, 75+36)\}$

$\{(0.480, 61), (0.7200, 86), (0.84000, 111)\}$

Now S_3^2 $i=2$, $j=3$, $m2=3$

$\phi_2 m_2 = (1-(1-0.8)^3)$

$= (1-(0.2)^3) = 1-0.008$

$= 0.992$

$3\times C2 = 3\times18 = 54$ is added to s^1 pair

$S_3^2 \{(0.5\times0.992, 25+54), (0.75\times0.992, 50+54), (0.875\times0.992, 75+54)\}$

$= \{(0.4960, 79), (0.744, 104), (0.868, 129)\}$

S^2 can be merged S_1^2, S_2^2, S_3^2

$S^2 = \{(0.4, 43), \cancel{(0.6, 68)}, \cancel{(0.7, 93)}, (0.480, 61), \cancel{(0.7200, 86)}, \cancel{(0.84000, 111)},$

$(0.4960, 79), (0.744, 104), \cancel{(0.868, 129)}\}$

$S^2 = \{(0.4, 43), (0.480, 61), (0.4960, 79), (0.744, 104)\}$

calculate S_1^3 $i=3$, $j=1$, $m3=1$

$\phi_3 m_3 = 1-(1-0.7)^1 = 1-0.3 = 0.7$

$1\times C3 = 1\times25 = 25$ is added to S^2 pair

$S_1^3 = \{(0.4\times0.7, 43+25), (0.48\times0.7, 61+25), (0.496\times0.7, 79+25),$

$(0.744\times0.7, 104+25)\}$

$= \{(0.28, 68), (0.336, 86), (0.3472, 104), \cancel{(0.5208, 129)}\}$

Last tuple is removed from S_1^3, since cost 129 exceed the given cost 120.

Now S_2^3 $i=3$, $j=2$, $m3=2$

$\phi_3 m_3 = 1-(1-0.7)^2 = 1-(0.3)^2 = 1-0.09 = 0.91$

$2\times C3 = 2\times25 = 50$ is added to S^2 pair

$S_2^3 = \{(0.4\times0.91, 43+50), (0.48\times0.91, 61+50), (0.496\times0.91, 79+50),$

$(0.744\times0.91, 104+50)\}$

$= \{(0.364, 93), (0.4368, 111), \cancel{(0.45136, 129)}, \cancel{(0.67704, 154)}\}$

last two tuples are removed from S_2^3 since cost 129, 154 exceeding the given cost 120

$S_2^3 = \{(0.364, 93), (0.4368, 111)\}$

Now S_3^3 $i=3$, $j=3$, $m_3=3$

$$\phi_3 m_3 = 1-(1-0.7)^3 = 1-(0.3)^3 = 1-0.027$$
$$= 0.973$$

$3 \times C_3 = 3 \times 25 = 75$ is added to S^2 pair

$S_3^3 = \{(0.4 \times 0.973, 43+75), (0.48 \times 0.973, 61+75), (0.496 \times 0.973, 79+75),$
$(0.744 \times 0.973, 104+75)\}$

$= \{(0.3892, 118), (0.46704, 136), (0.482608, 154), (0.723912, 179)\}$

Last three tuples are removed from S_3^3 since cost is 136, 154, 179 exceeding the given cost is 120.

$S_3^3 = \{(0.3892, 118)\}$

S^3 can be merged S_1^3, S_2^3, S_3^3

$S^3 = \{(0.28, 68), (0.336, 86), \cancel{(0.3472, 104)}, (0.364, 93),$
$(0.4368, 111), \cancel{(0.3892, 118)}\}$