Part-B

Formulas for small samples

Small Samples: $n \le 30$		Test Statistic	Degrees of freedom
1	**Student's 't' test for single mean (Single sample)**		
a.	S.D. is not given	$t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$	$\nu = n-1$ $s = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n-1}}$
b.	S.D. is given directly	$t = \dfrac{\bar{x}-\mu}{s/\sqrt{n-1}}$	$\nu = n-1$
2	**Student's "t" test for difference of means (Two Sample Means)**		
a.	Direct	$t = \dfrac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ or $t = \dfrac{(\bar{x}-\bar{y})-(\mu_1-\mu_2)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ $(\mu_1-\mu_2)=0$ $(\mu_1-\mu_2)\ne 0$	$\nu = n_1+n_2-2$ $S = \sqrt{\dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$
3	F-Test- Variances **(Two Sample Variances)**		
a	Direct	$F = \dfrac{Greater\ \ Variance}{Smaller\ \ Variance}$	$\nu = (n_1-1,\ n_2-1)$ $S_1 = \sqrt{\dfrac{n_1 s_1^2}{n_1-1}} = \sqrt{\dfrac{\sum(x-\bar{x})^2}{n_1-1}}$ $S_2 = \sqrt{\dfrac{n_2 s_2^2}{n_2-1}} = \sqrt{\dfrac{\sum(y-\bar{y})^2}{n_2-1}}$
3	**CHI-SQUARE (χ^2) Test FOR GOODNESS OF FIT (Attributes)**		
a	Direct	$\chi^2 = \sum\dfrac{(O_i-E_i)^2}{E_i}$	$\nu = n-1$ $E_i = \dfrac{Sum\ of\ all\ observations}{number\ of\ observations}$
b	Expected frequencies by Binomial distribution	$\chi^2 = \sum\dfrac{(O_i-E_i)^2}{E_i}$	$\nu = n-1$
c	Expected frequencies by Poisson distribution	$\chi^2 = \sum\dfrac{(O_i-E_i)^2}{E_i}$	$\nu = n-2$

$$S^2 = \frac{\left[\sum\left[(x_i-\bar{x})^2+\sum(y_i-\bar{y})^2\right]\right]}{n_1+n_2-2}$$

F-Test:

$$F = \frac{\text{Greater Variance}}{\text{Smaller Variance}}$$

i) $\dfrac{S_1^2}{S_2^2}$ If $[S_1^2 > S_2^2]$

Degrees of Freedom (n_1-1, n_2-1)

ii) $\dfrac{S_2^2}{S_1^2}$ If $(S_2^2 > S_1^2)$

DOF : (n_2-1, n_1-1)

4. CHI-SQUARE $\left(\chi^2\right)$ Test for Independence Attributes

(Matrix Type or Habitual activities)

Test Statistic: $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Let us consider two attributes A and B, and they are divided into two classes. The various frequencies can be expressed as follows:

A	a	b
B	c	d

a	b	$a+b$
c	d	$c+d$
$a+c$	$b+d$	$a+b+c+d=N$

The expected frequencies are given given by:

$E(a)=\frac{(a+c)(a+b)}{N}$	$E(b)=\frac{(b+d)(a+b)}{N}$	$a+b$
$E(a)=\frac{(a+c)(c+d)}{N}$	$E(b)=\frac{(b+d)(c+d)}{N}$	$c+d$
$a+c$	$b+d$	$a+b+c+d=N$

Degrees of freedom $=(n-1)(m-1)$

Let us consider three attributes A, B and C they are divided into three classes. The various frequencies can be expressed as follows: Degrees of freedom $=(n-1)(m-1)$

A	a	b	c
B	d	e	f
C	g	h	i

$E(a)=\frac{(a+d+g)(a+b+c)}{N}=$	$E(b)=\frac{(b+e+h)(a+b+c)}{N}=$	$E(b)=\frac{(c+f+i)(a+b+c)}{N}=$
$E(c)=\frac{(a+d+g)(d+e+f)}{N}=$	$E(b)=\frac{(b+e+h)(d+e+f)}{N}=$	$E(b)=\frac{(c+f+i)(d+e+f)}{N}=$
$E(c)=\frac{(a+d+g)(g+h+i)}{N}=$	$E(b)=\frac{(b+e+h)(g+h+i)}{N}=$	$E(b)=\frac{(c+f+i)(g+h+i)}{N}=$

Model No 5.5: Test of significance for single mean (Students's t- test)

Model No 5.6: Student's "t" test for difference of means (Two Sample Means)

Model No 5.7: F-Test- Variances **(Two Sample Variances)**

Model No 5.8: CHI-SQUARE (χ^2) Test FOR GOODNESS OF FIT (Attributes)

Model No 5.9: CHI-SQUARE (χ^2) Test for Independence Attributes

TEST OF SIGNIFICANCE FOR SMALL SAMPLES:

Model No 5.5: Test of significance for single mean (Students's t- **test**):

(i) Null Hypothesis (H_0): $\bar{x} = \mu$ i.e., "there is no significance difference between the sample mean and population mean" or "the sample has been drawn from the population"

(ii)Alternative Hypothesis (H_1): (i) $\bar{x} \neq \mu$ or (ii) $\bar{x} < \mu$ or (iii) $\bar{x} > \mu$

(iii) Level of Significance (α): Set a level of significance

(iv) Test Statistic: The test statistic $t = \dfrac{\bar{x} - \mu}{s / \sqrt{n-1}}$ *or* $\dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

(v) Conclusion: (i) If $|t| < t_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|t| > t_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

Problem 1: A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the test statistic you would use to test whether the work is meeting the specifications. Also state how you would proceed further.

Solution: Here we are given,

μ=0.700 inch, $\bar{x}$=0.742 inches, s=0.040 inch and n=10

Null Hypothesis: H_0: μ=0.700 inch, i.e., the product is confirming to specifications

Alternative hypothesis: H_1: $\mu \neq$0.700inches

Level of significance: $\alpha = 0.05$

Test statistic: Under H_0, the test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{0.742 - 0.700}{0.040/\sqrt{9}} = 3.15$$

How to proceed further: Here the test statistic 't' follows student's t-distribution with 10-1=9 degrees of freedom. We will now compare this calculated value with the tabulated value for t for 9 degrees of freedom and at a certain level of significance, say 5%.

i) If calculated 't' =3.15> t-table value, we say that the value of t is significant. This implies that $\bar{x}$ differs significantly from μ and H_0 is rejected at this level of significance and we conclude that the product is not meeting the specifications.

ii) If calculated t<t-table value, we say that the value of t is not significant. There is no significant difference between $\bar{x}$ and μ. We may take the product conforming to specifications.

$t_{0.05}$=2.26 , $\boxed{t_{cal} > t_{tab}}$

Therefore H_0 is rejected. Hence the product is not meeting the specification.

****** **Problem 2:** A random sample of 10 boys had the following I. Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

a) Do these data support the assumption of a population mean I. Q. of 100?
b) Also, Find a reasonable range in which most of the mean I. Q. values of samples of 10 boys lie.

Solution: Null Hypothesis H_0: The data are consistent with the assumption of a mean I. Q. of 100 in the population, i.e., H_0: μ=100.

Alternative Hypothesis: H_1: $\mu \neq 100$.

Level of Significance (α): 5%

Test statistic: Under H_0, the test statistic is: $t = \frac{\bar{x}-\mu}{S/\sqrt{n}}$

Where $\bar{x}$ and s^2 are to be computed from the sample values of I. Q.'s.

Calculations for Sample Mean and Standard deviation:

Here n=10, $\bar{x}$=972/10=97.2 and $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = 1833.60/9 = 203.73$

$$|t| = \frac{2.8}{14.27/\sqrt{10}} = 0.62$$

T-table value at 5% LOS for 9 degrees of freedom for two-tailed test is 2.262.

Conclusion:

Since calculated t is less than tabulated t ($t_{cal} < t_{tab}$). Null Hypothes is H_0, may be accepted at 5% level of significance. Hence we conclude that the data are consistent with the assumption of mean I.Q. of 100 in the population.

The 95% confidence limits within which the mean I. Q. values of samples of 10 boys will be are given by:

$\boxed{\bar{x} \pm t_{0.05} S/\sqrt{n}}$=97.2±2.262*4.514=107.41 and 86.99

Hence the required 95% confidence interval is [86.99, 107.41].

Problem 3: **Producer of gutkha, claims that the nicotine content in his "gutkha" on the average is 1.83 mg. Can this claim accepted if a random sample of 8 gutkha of this type have the nicotine contents of 2.0, 1.7, 2.1, 1.9, 2.2, 2.0, 1.6 mg? Use a 0.05 L.O.S.**

Solution: Given $n=8$, $\mu=1.83$, $\bar{x}=1.95$ (From Calci)

$$S=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=0.2070$$

(i) Null Hypothesis (H_0): $\bar{x}=\mu$ (or) $\mu=1.83$

(ii)Alternative Hypothesis (H_1): $\mu \neq 1.83$ (Two tailed Test)

(iii) Level of Significance (α): $\alpha=0.05$, $\alpha/2=0.025$

(iv) Test Statistic: The test statistic

$$t=\frac{\bar{x}-\mu}{\left(\frac{S}{\sqrt{n}}\right)}=\frac{1.95-1.83}{\left(\frac{0.2070}{\sqrt{8}}\right)}=1.6396$$

(v) Conclusion: Degrees of freedom=

$n-1$
$=8-1$
$=7$

Tabulated value of t_{tab} = 2.365

Calculated value of $|t_{cal}|$ = 1.6396

Calculated value of $|t_{cal}|$ < Tabulated value of t_{tab}

∴ Null Hypothesis is Accepted

Problem 4: The life time of electric bulbs for a random sample of 10 from a large consignment gave the following data.

Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Life in	1.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6
1000hrs	1200	4600	3900	4100	5200	3800	3900	4300	4400	5600

Can we accept the hypothesis that the average life time of bulbs is 4000hrs?

Use a 0.05 L.O.S.

Solution Given $n = 10, \mu = 4000; \bar{x} = 4100$

$$S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = 1174.73$$

(i) Null Hypothesis (H_0): $\bar{x} = \mu$ (or) $\mu = 4000$

(ii)Alternative Hypothesis (H_1): $\mu \neq 4000$ (Two Tailed Test)

(iii) Level of Significance (α): $\alpha = 0.05$, $\alpha/2 = 0.025$

(iv) Test Statistic: The test statistic $t = \frac{\bar{x} - \mu}{(S/\sqrt{n})} = \frac{4100 - 4000}{\frac{1174.73}{\sqrt{10}}} = 0.2691$

(v) Conclusion: Degrees of freedom= $10 - 1 = 9$

Tabulated value of $t_{tab} = 2.262$

Calculated value of $|t_{cal}| = 0.2691$

Calculated value of $|t_{cal}| <$ Tabulated value of t_{tab}

$\therefore$ Null Hypothesis is Accepted

Problem 5: A sample of 26 bulbs gives a mean life of 990 hours with a S.D of 20 hours. The manufacturer claims that the mean life of bulbs is 1000 hours. Is the sample not up to the standard?

$n = 26, \bar{x} = 990$, $S = 20, \mu = 1000$

Solution

(i) Null Hypothesis (H_0): $\bar{x}=\mu=990$

(ii) Alternative Hypothesis (H_1): $\mu<1000$ (Left Tailed Test)

(iii) Level of Significance (α): $\alpha=0.05$

(iv) Test Statistic: The test statistic $t=\dfrac{\bar{x}-\mu}{\frac{S}{\sqrt{n-1}}}=\dfrac{990-1000}{\frac{20}{\sqrt{26-1}}}=-2.5$

(v) Conclusion: Degrees of freedom= $n-1=26-1=25$

Tabulated value of t_{tab} = 1.708

Calculated value of $|t_{cal}|$ = 2.5

Calculated value of $|t_{cal}|$ > Tabulated value of t_{tab}

∴ Null Hypothesis is Rejected

Problem 6: The average breaking strength of the steel rods is specified to be 18.5 thousands pounds. To test this sample of 14 rods were tested. The mean and standard deviations obtained were 17.85 and 1.955 respectively. Is the result of experiment significant.

Solution: Given $\mu=18.5$, $n=14$, $\bar{x}=17.85$, $S=1.955$

(i) Null Hypothesis (H_0): $\bar{x}=\mu$, $\mu=18.5$

(ii) Alternative Hypothesis (H_1): $\mu\neq18.5$ (2 Tailed Test)

(iii) Level of Significance (α): $\alpha=0.05$, $\alpha/2=0.025$

(iv) Test Statistic: The test statistic $t_{cal}=\dfrac{\bar{x}-\mu}{\frac{S}{\sqrt{n-1}}}=\dfrac{17.85-18.5}{\left(\frac{1.955}{\sqrt{14-1}}\right)}=-1.198$

(v) Conclusion: Degrees of freedom= $14-1=13$

Tabulated value of t_{tab} = 2.160

Calculated value of $|t_{cal}|$ = 1.198

Calculated value of $|t_{cal}|$ < Tabulated value of t_{tab}

∴ Null Hypothesis is Accepted

Problem 7: The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches ? Test at 5% significance level assuming that for 9 degree of freedom (t=1.833 at α =0.05) .

Item	1	2	3	4	5	6	7	8	9	10
Life in 1000hrs	1.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

} No need

Solution: Given $n=10$, $\bar{x}=66$, $S=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=3.1622$; $\mu=64$

(i) Null Hypothesis (H_0): $\mu=64$; $\bar{x}=\mu$

(ii)Alternative Hypothesis (H_1): $\mu>64$ (Right Tailed Test)

(iii) Level of Significance (α): $\alpha=0.05$

(iv) Test Statistic: The test statistic

$$t_{cal}=\frac{\bar{x}-\mu}{S/\sqrt{n}}=\frac{66-64}{\frac{3.1622}{\sqrt{10}}}=2.00004$$

(v) Conclusion: Degrees of freedom= $10-1=9$

Tabulated value of t_{tab} = 1.833

Calculated value of $|t_{cal}|$ = 2.00004

Calculated value of $|t_{cal}|$ > Tabulated value of t_{tab}

∴ Null Hypothesis is Rejected.

Problem 8: A new process of producing synthetic diamonds can be operated at a profitable level only if the average weight of the diamonds is greater than 0.5 carat. To test the probability of the process, 6 diamonds are produced with weights 0.46, 0.60, 0.52, 0.49, 0.58 and 0.54 carat respectively. Do the 6 measurements present sufficient evidence to indicate that the average weight of the diamonds produced by the process is in excess of 0.5 carat?

Solution: $n=6$, $\bar{x}=0.5316$, $\mu=0.5$

$$S=\sqrt{\frac{(\Sigma x_i-\bar{x})^2}{n-1}}=0.053$$

(i) Null Hypothesis (H_0): $\mu = 0.5$, $\bar{x} = \mu$

(ii) Alternative Hypothesis (H_1): $\mu > 0.5$ (Right-Tailed Test)

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic

$$t_{cal} = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{0.5316 - 0.5}{0.053/\sqrt{6}} = 1.4604$$

(v) Conclusion: Degrees of freedom= $6-1=5$

Tabulated value of t_{tab} = 2.015

Calculated value of $|t_{cal}| = 1.4604$

Calculated value of $|t_{cal}| <$ Tabulated value of t_{tab}

$\therefore$ Null Hypothesis is Accepted.