25.05.2022 Wednesday

Model No 5.8: CHI- SQUARE TEST (χ^2) FOR GOODNESS OF FIT

(i) **Null Hypothesis (H_0):** There is no significant difference between expected frequency and observed frequency

(ii) **Alternative Hypothesis (H_1):** There is a significant difference between expected frequency and observed frequency

(iii) **Level of Significance (α):** set a lelvel of significance

(iv) **Test Statistic:** The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

(v) **Conclusion:** Degrees of freedom = $n-1$

(i) If Calculated value of $\chi^2 <$ Tabulated value of χ^2, we accept H_0

(ii) If Calculated value of $\chi^2 >$ Tabulated value of χ^2, we reject H_0

No. appeared on Die	1	2	3	4	5	6
Observed (O_i)	40	32	28	58	54	52
Probability $p(x_i)$	1/6	1/6	1/6	1/6	1/6	1/6
Expected (E_i) $264 \times p(x_i)$	44	44	44	44	44	44

Problem 21: A die is thrown 264 times with the following results. Show that the die is biased.

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

Solution: Given n= 6

(i) Null Hypothesis (H_0): There is no significant difference b/w Observed & Expected frequencies

(ii) Alternative Hypothesis (H_1): There is significant difference.

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
40	44	16	0.3636
32	44	144	3.2727
28	44	256	5.8182
58	44	196	4.4545
54	44	100	2.2727
52	44	64	1.4545
	44		
			$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 17.6362$

(v) Conclusion: Degrees of freedom $= n-1 = 6-1 = 5$

Calculated value of $\chi^2_{cal} = 17.6362$

Tabulated value of $\chi^2_{tab} = \chi^2_{0.05}(n-1) = \chi^2_{0.05}(5) = 11.070$

Calculated value of χ^2 Tabulated value of χ^2

$\chi^2_{cal} > \chi^2_{tab}$ Null Hypothesis is Rejected

Problem 22: The following figures show the distribution of digits in numbers chosen at random from a telephone directory. There is No significant difference b/w Expected frequencies & Observed frequencies (O_i)

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

Test whether the digits may be taken to occur equally frequently in the directory.

Solution: *Given n=* 10

(i) Null Hypothesis (H_0): The digits may be taken to occur equally frequently in directory.

(ii) Alternative Hypothesis (H_1): There is a significant difference b/w Observed & Expected frequencies

(iii) Level of Significance (α):

$\alpha = 0.05$

Expected Frequencies of each one is average.

$$avg = \frac{1026 + 1107 + \cdots + 853}{10} = 1000$$

$\boxed{E_i = 1000}$ → for Each.

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
1026	1000	676	0.676
1107	1000	11449	11.449
997	1000	9	0.009
966	1000	1156	1.156
1075	1000	5625	5.625
933	1000	4489	4.489
1107	1000	11449	11.449
972	1000	784	0.784
964	1000	1296	1.296
853	1000	21609	21.609
			$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 58.542$

(v) Conclusion: Degrees of freedom $= n - 1 = 10 - 1 = 9$

Calculated value of $\chi^2 = \chi^2_{cal} = 58.542$

Tabulated value of $\chi^2_{tab} = \chi^2_{0.05}(n-1) = \chi^2_{0.05}(9) = 16.919$

Calculated value of χ^2 Tabulated value of χ^2

$\chi^2_{cal} > \chi^2_{tab}$ Null Hypothesis is Rejected.

Problem 23: A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had scored a third class, 90 were placed in second class and 20 got a first class. Do these figures commensurate with the general examination result which is in the ratio 4:3:2:1 for the various categories respectively.

Solution: Given n= ~~500~~ 4

(i) Null Hypothesis (H_0): There is no Significant difference b/w Observed & Expected frequency

(ii) Alternative Hypothesis (H_1): There is Significant difference b/w Observed & Expected frequencies.

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Problem-23

Here, the Expected frequencies are in the ratio 4:3:2:1

In the form Of Percentages : 40:30:20:10

1, $40\%(500) = \frac{40}{100} \times 500 = 200$

2, $30\%(500) = 30 \times 5 = 150$

3, $20\%(500) = 20 \times 5 = 100$

4, $10\%(500) = 10 \times 5 = 50$

	Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
Fail	220	200	400	2
Third	170	150	400	2.6666
Second	90	100	100	1
First	20	50	900	18
				$\chi^2=\sum\frac{(O_i-E_i)^2}{E_i}=23.6666$

(v) Conclusion: Degrees of freedom $=n-1=4-1=3$

Calculated value of $\chi^2=\chi^2_{cal}=23.6666$

Tabulated value of $\chi^2=\chi^2(0.05)=\chi^2_{0.05}(n-1)=\chi^2_{0.05}(3)=7.815$

Calculated value of χ^2 Tabulated value of χ^2

$\chi^2_{cal} > \chi^2_{tab}$ Null Hypothesis is Rejected.

Problem 24: A pair of dies are thrown 360 times and the frequency of each sum is indicated below:

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

Would you say that the dice are fair on the basis of the chi-square test at 0.05 level of significance?

Solution: Given n=

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$p(x_i)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
Expected Frequencies $=360p(x_i)$	10	20	30	40	50	60	50	40	30	20	10

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $\chi^2=\sum\frac{(O_i-E_i)^2}{E_i}$

The Possible Outcomes are (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6),

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
8	10	4	0.4
24	20	16	0.8
35	30	25	0.8333
37	40	9	0.2250
44	50	36	0.72
65	60	25	0.4167
51	50	1	0.02
42	40	4	0.100
26	30	16	0.5333
14	20	36	1.8
14	10	16	1.6
			$\chi^2 = \sum\frac{(O-E)^2}{E} = 7.4483$

(v) Conclusion: Degrees of freedom $= n-1 = 11-1 = 10$

Calculated value of $\chi^2_{cal} = 7.4483$

Tabulated value of $\chi^2_{tab} = \chi^2_{(0.05)(n-1)} = \chi^2_{0.05}(10) = 18.307$

Calculated value of χ^2 < Tabulated value of χ^2 Null Hypothesis is Accepted.

Problem 25: 4 coins were tossed 160 times and the following results were obtained.

No. of Heads	0	1	2	3	4
Observed Frequency	17	52	54	31	6

$n=5$

Under the assumption that coins are balanced, find the expected frequencies of 0,1,2,3 or 4 heads, and test the goodness of fit at $\alpha = 0.05$ Fit a Binomial Distribution.

Solution: No. of coins = $n=4$

Probability to get a head p = , q = 1 − p =

$X = x_i$	0	1	2	3	4
$p(x_i)$	0.0625	0.25	0.375	0.25	0.0625
Expected Frequencies $E_i = 160\,p(x_i)$	10	40	60	40	10

Binomial Distribution Formula: $p(X=x) = n_{C_x} p^x q^{n-x}$

Given n= 5 [From Table]

(i) Null Hypothesis (H_0): There is No Significant Difference.

(ii)Alternative Hypothesis (H_1): There is Significant difference

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
17	10	49	4.9
52	40	144	3.6
54	60	36	0.6
31	40	81	2.025
6	10	16	1.6
			$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 12.725$

(v) Conclusion: Degrees of freedom $= n-1 = 5-1=4$

Calculated value of $\chi^2_{cal} = 12.725$

Tabulated value of χ^2_{tab} $\chi^2(0.05)(4) = 9.488$

Calculated value of χ^2 > Tabulated value of χ^2

Null Hypothesis is Rejected.

Problem 26: A survey of 240 families with 4 children each revealed the following distribution. To fit a Binomial Distribution

Male Births	4	3	2	1	0
Observed Frequencies	10	55	105	58	12

Can we accept that the male and female births are equally distributed?

Solution: No. of families $= 240$ No. of children $= n = 4$

Probability to have a male birth p = $1/2$, q = 1 − p = $1/2$

$X = x_i$	4	3	2	1	0
$p(x_i)$	0.0625	0.25	0.375	0.25	0.0625
Expected Frequencies $= 240\,p(x_i)$	15	60	90	60	15

The Binomial Distribution $P(X=x) = nC_x p^x q^{n-x}$

Given n= 5

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
10	15	25	1.6666
55	60	25	0.4167
105	90	225	2.5
58	60	4	0.0667
12	15	9	0.6
			$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 5.2498$

(v) Conclusion: Degrees of freedom $= n-1 =$ $5-1=4$

Calculated value of $\chi^2_{cal} =$ 5.2498

Tabulated value of $\chi^2_{tab} =$ $\chi^2_{0.05}(4) = 9.488$ Null Hypothesis is Accepted.

Calculated value of χ^2 < Tabulated value of χ^2

Problem 27: Fit a poisson distribution to the following data and for its goodness of fit at level of significance 0.05? To fit a Possion Distribution

x	0	1	2	3	4
f	419	352	154	56	19

$X = x_i$	0	1	2	3	4
$p(x_i)$	0.4049	0.366	0.1654	0.0498	0.0112
Expected Frequencies $= 1000(Px_i)$	404.9	366	165.4	49.8	11.2

Poission Distribution: $P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$

$$\mu = \frac{\sum fx}{\sum f} = \frac{(0\times419)+(1\times352)+(2\times154)+(3\times56)+(4\times19)}{1000}$$

$$\mu = \frac{904}{1000} = 0.904$$

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Given n= 5

(i) Null Hypothesis (H_0):

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

Observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
~~10~~ 419 ~~55~~ 352 ~~105~~ 154 ~~58~~ 56 ~~12~~ 19	404.9 366 165.4 49.8 11.2	155946.01 96721 3648.16 67.24 0.64	385.1469 264.2650 22.0565 1.3502 0.0571
			$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} =$ 672.8757

(v) Conclusion: Degrees of freedom $= n - 2 = 5 - 2 = 3$

Calculated value of $\chi^2_{cal} = 672.8757$

Tabulated value of $\chi^2_{tab} = \chi^2_{0.05}(n-2) = \chi^2_{0.05}(3) = 7.815$

Calculated value of χ^2 Tabulated value of χ^2

$\chi^2_{cal} > \chi^2_{tab}$ Null Hypothesis is Rejected