

prism type:

Volume = base area \times height

Lateral surface area \times curved surface area:

= base perimeter \times height

Total surface area = LSA + 2 \times base area P

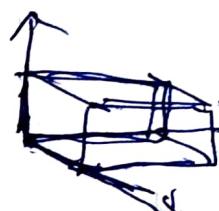
cuboid

Volume = lwh cubic units

Surfaces = 6

Edges = 12

Corners = 8



$$\text{LSA} = 2(l+b) \times h$$

$$\text{TSA} = 2(l+b) \times h + 2lb$$

$$= 2(lb + bh + lh)$$

*Imp:

$$F + V = E + 2$$

F = no. of surfaces

V = corners/vertices

E = edges.

cube: $\sqrt{l^2 + b^2 + h^2}$

Volume = $s \times s \times s = s^3$ cubic units

$$\text{LSA} = 4s^2$$

$$\text{TSA} = 6s^2$$

$$\text{Diag} = \sqrt{3s^2}$$



$1 \rightarrow 2, 3, 5, 4$ $[1 \rightarrow 6]$

$$\begin{array}{r} 36 \\ 25 \\ 20 \\ 18 \\ 10 \\ \hline 100 \end{array}$$

8 cuts \rightarrow same direction

9

two direction

three direction

\leftarrow

$$(1, 7) \text{ CHIX7H}$$

$$\frac{2+1+6+1}{21} = (2, 6) = 16,$$

$$(3, 5) = \frac{3+5+1}{24} = 24$$

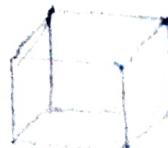
$$\boxed{5+5} = 25$$

$$= (4, 4)$$

Note: If you count maximum number of identical pieces

we should cut the cube in all directions.

If you count minimum number of identical pieces we should cut the cube in same directions.



No. of faces with 2 faces visible = $12 \times (n-2)$

No. of faces with 1 face visible = $6 \times (n-2)^2$

No. of faces with no face visible = $(n-2)^3$

1. If statement one alone is sufficient to answer the question.

2. If statement II alone is sufficient to answer the question.

3. Both the statements I & II are required to answer the question. But either statement is alone not sufficient.

4. Both the statements I & II are not sufficient to answer the question & additional data is required to answer it.

If no. of equations = no. of variables then such type of linear equations have unique solution otherwise no solution or infinite solution.

Analytical Reasoning

Arrangements are 3 types

1. Linear Arrangement

2. Circular Arrangement

a) facing towards center

b) Facing towards outside the center

c) facing towards center and outside the center.

3. Puzzle based arrangement.

Study the following information to answer the given questions:

Eight friends: K, L, M, N, V, W, X & Y are seated in straight line, facing North but not necessarily in the same order.

* N sits fourth to left of X. X sits at one of the extreme ends of the line.

* V sits third to right of Y.

* Only one person sits b/w Y & W. Neither V nor K is an immediate neighbour of V.

* M is an immediate neighbour of N.

W K Y N ~~V~~ M X

W K V N L V M X,

* what is the position of M with respect to N.

* How many ~~people~~ sit b/w M & Y.
people.

* Exactly between V & Y (L, N).

* Extreme left end.

Q Nine persons C, D, E, F, I, J, K, L & M are seated in a straight line facing North, with equal distance b/w each other but not necessarily in the same order.

• As many people sit to the left of D as to the right of D.

• Only two persons sit between F & D.

• J sits third to the right of E.

• E is not an immediate neighbour of D.

• E does not sit at any of the extreme of people sitting between J & I is double as that between E & C.

• K is one of the immediate neighbours of M.

• L is not a immediate neighbour of J.

Time Distance & Speed

- The relation between distance speed & time is $D = S \times T$
- $\frac{D_1}{S_1 T_1} = \frac{D_2}{S_2 T_2}$ (or) $\frac{S_1 T_1}{D_1} = \frac{S_2 T_2}{D_2}$
- Distance = $S \times T$
- Speed = $\frac{D}{T}$
- Time = $\frac{D}{S}$
- Frennal units for speed are km/h or m/s.
- To convert speed from km/h to m/s we have to multiply with $5/18$.
- To convert speed from m/s to km/h we have to multiply with $18/5$.
- If the ratio of speeds of two persons is $a:b$ then the ratio of time taken by the two to travel the same distance is $\frac{1}{a}:\frac{1}{b}$ (or) $b:a$.
- If both speed and time increases by $x\%$ and $y\%$ respectively then the distance travelled increases by $\frac{xy+xy}{100}\%$.

- If both speed & time decreases by $x\%$ & $y\%$ respectively then distance travelled is decreased by $\frac{xy - xy}{100}\%$.
- If speed increases by $x\%$ & time decreases by $y\%$ then the distance travelled is changed by $x - y - \frac{xy}{100}\%$
 - if it is +ve then it increases.
 - if it is -ve then it decreases.
 - if it is 0 then there is no change.
- If speed decreased by $x\%$ and time increased by $y\%$ then distance travelled is changed by $y - x - \frac{xy}{100}\%$
 - if $y - x - \frac{xy}{100}$ is +ve then distance is increased.
 - if $y - x - \frac{xy}{100}$ is -ve then distance is decreased.
 - if $y - x - \frac{xy}{100}$ is 0 then there is no change.
- If speed increases by $x\%$ due to that time decreases by $\left(\frac{x}{100+x}\right) \times 100\%$ so that there is no change in distance travelled.

If speed decreases by $\alpha\%$ due to that time increases by $(\frac{\alpha}{100-\alpha}) \times 100\%$ such that there is no change in distance travelled.

Time increases by $\alpha\%$ due to that speed decreased by $(\frac{\alpha}{100+\alpha}) \times 100\%$ such that there is no change in distance travelled.

Time decreased by $\alpha\%$ due to that speed increased by $(\frac{\alpha}{100-\alpha}) \times 100\%$ such that there is no change in distance travelled.

Average speed:

Average speed is ratio of total distance travelled to that of time taken.

$$\text{Avg speed} = \frac{\text{Total distance}}{\text{Total time}}$$

If two equal distances travelled different speeds $x \text{ km/h}$ & $y \text{ km/h}$ then the avg speed during the whole journey is $\frac{2xy}{x+y}$ (a)

$$\frac{2xy}{x+y}$$

$$\frac{1/x + 1/y}{2}$$

- If a person travelled ~~regular~~ intervals with different speeds x_1, x_2, \dots, x_n km/h then his average speed during the whole journey

$$v_s = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

Relative speed:

Relative speed is the comparison of speed of 1st person with respect to another person.

Two persons travelled with different speeds x km/h & y km/h in the opposite direction.

- The relative speed of 1st person with respect to second person is $x+y$ km/h
- The relative speed of 2nd person with respect to 1st is $y+x$ km/h.

Two persons travelled in ~~different~~ speeds x km/h & y km/h in same direction.

- The relative speed of 1st person with respect to second person is $x-y$ km/h
- The relative speed of 2nd person with respect to 1st person is $y-x$ km/h.

Points for problems based on trains

- If a train crosses a stationary object of negligible length (man, tree..) then it travels a distance of length of the tree.
 - If a train crosses a stationary object having some length (bridge, platform, tunnel..) then it travels a distance of length of train + length of the object which it crosses.
 - If a train crosses the moving objects the distances are same as above but speeds are considered as the relative speed.
- If two trains of lengths ' x ' meters & ' y ' meters are moving in opposite direction with speeds u m/sec & v m/sec then time taken by the trains to cross each other is $\frac{x+y}{u+v}$ sec.
- If two trains of lengths ' x ' meters & ' y ' meters are moving in same direction with speeds u m/sec & v m/sec respectively then the time taken by the faster train to cross slower train is $\frac{x+y}{v-u}$ sec.

If two persons start at the same time, Pn opposite directions from two stations a & b & if passing each other they complete the journey Pn a & b hours respectively then the ratio of their speeds,

$$V_b : V_a =$$

Circular tracks:

If two persons running on a circular track of length cmeters are moving on opp direction with speeds x mls & y mls then the first meeting time of two persons at any point is $\frac{C}{x+y}$ sec.

If two persons running on a circular track of length cmeters in the same direction with speeds x mls & y mls then the first meeting time of two persons at any point is $\frac{C}{x-y}$ sec.

If two persons running on a circular track of length cmeters are moving on any direction with speeds x mls & y mls then the first meeting time of two persons at starting point is $\frac{C}{x+y}$ sec.

$$\frac{C}{x+y} \text{ sec.}$$

RACES & GAMES

- Race: A Race is a contest of speed in running, riding, driving, sailing, rowing etc, over a particular distance.
 - Race Course: Race course is the ground or path on which contests are conducted.
 - Starting point: the point where the race starts.
 - Winning point: The point where the race ends or finishes is winning point or goal.
 - Dead-heat Race: A race is said to be a Dead-heat Race if all the persons contesting the race reach the winning point exactly at the same time.
 - Winner: Winner is the person who first reaches the goal is winner.
- M. I of S. I in R & G:
- Let A & B be two participants in a race. Let us examine some of the general statements & their mathematical interpretations.
 - A beats B by 't' seconds. A finishes race t seconds before B.

If A takes x sec to finish the race B takes $x+t$ sec to finish the race.

* It gives B a start of t sec means that i.e. A starts the race t sec after B starts from the same starting point. i.e.

If B takes x sec to reach the goal then to take $x-t$ sec to reach & win the race.

* It gives B a start of x m means that A starts from starting point & B starts x m ahead of the starting point at the same time.

(or)

To cover a race of y m A will cover y m while B will cover only $y-x$ m.

* A beats B by x m or a race of y m means that while A reaches the goal in the same time.

B is x m behind the goal.
i.e.

In the same time A travels y m & B travels only $y-x$ m.

A game of 100 points means that a game in which a participant who scores 100 points first is winner.

In a game of 100, A can give B $\frac{1}{x}$ points means that while A scores 100 points

Note

If A is n times as fast as B & A gives B a start of x meters, then the length of the race course, so that A & B reaches the coining post at the same time is $x \left(\frac{n}{n-1} \right)$ meters.

If A can run x meters race in t_1 seconds & B in t_2 seconds where $t_1 < t_2$ the A beats B by distance of $\frac{x}{t_2} (t_2 - t_1)$ meters.

A runs $1\frac{2}{3}$ times as fast B if A gives B a start of 80m how far must the coining race post be so that A & B reaches it at the same time.

$$\Rightarrow 80 \left(\frac{1\frac{2}{3}}{1\frac{2}{3} - 1} \right)$$

$$\begin{array}{r} 5 \\ \times 80 \\ \hline 400 \end{array}$$

$$\begin{array}{r} 5 \\ \times 1\frac{2}{3} \\ \hline 8 \end{array}$$

$$\Rightarrow 200m,$$

- * A is $\frac{7}{8}$ times as fast as B. If A gives B a start of 80m how long should the race run be so that both of them reach at the same time.

$$\frac{7}{8} \times 80 = 140 \text{ m},$$

$$\frac{4}{3}$$

- * A runs $1\frac{3}{8}$ times as fast B. If A gives B a start of 90m & they reach the goal at same time.

$$\frac{11}{8} \times 90 = 330 \text{ m},$$

$$\frac{3}{8}$$

A can run 224m in 28 sec & B in 32 sec.

By what distance A beat B.

$$\frac{224}{32} (32 - 28)$$

$$\Rightarrow \frac{224}{32} (4)$$

$$\frac{224}{32}$$

$$\Rightarrow 28 \text{ m},$$

In a 100m race A can give B 10m & C 28m. In the same race B can give C.

$$D_A = 100 \text{ m}$$

$$D_B = 90 \text{ m}$$

$$D_C = 72 \text{ m}$$

$$\frac{90}{72} = \frac{100}{x}$$
$$\Rightarrow 90x = 100 \times 72$$
$$x = \frac{100 \times 72}{90} = 80 \text{ m}$$

B gives $100 - 80 = 20 \text{ m}$ to C.

At a game of billiard A can give B 15 pts in 60 & A can give C 20 pts in 60. How many points can B give to C in a game of 90.

$$P_A = 60$$

$$P_B = 45$$

$$P_C = 40$$

$$\Rightarrow P_B = 90$$

$$\frac{45}{40} = \frac{90}{x}$$

$$\Rightarrow x = \frac{90 \times 40}{45}$$

$$x = 80 \text{ } \therefore 10 \text{ pts}$$

In a 500m race, the ratio of the speeds of two contestants A & B is 3:4. A has a start of 40m. Then A wins by:

$$11 \text{ m}$$

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Sequences:

- A sequence is a series of numbers or letters which maintains a common characteristic property through out the series.
- Generally sequences are also called as progressions.

- A.P
- G.P
- H.P

A.P: The difference between two consecutive terms throughout the sequence is same then that sequence is called A. sequence or A. pro. Here the equal difference is called common difference.

Note:

The terms of A.P are

- $a, at+d, at+2d, \dots, at+(n-1)d$
- n^{th} term of A.P whose first term is a & common difference d is

$$T_n = a + (n-1)d$$

Sum of n terms

$$T_n - T_{n-1} =$$

$$T_n = S_n -$$

AM of

a, b, c are

$$b = \frac{a+c}{2}$$

$$S_n = n*$$

To insert

$$1, 18, 2, \dots$$

$$d = \frac{b-a}{n+1}$$

The recip

To con

them as

To cons

them as

Sum of n terms in A.P.,

$$S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} (T_1 + T_n)$$

$$T_n - T_{n-1} = d, \quad T_n - T_m = (n-m)d$$

$$T_n = S_n - S_{n-1}$$

- AM of a & b is $\frac{a+b}{2}$
- a, b, c are in AP if & only if $b-a = c-b$ or $b = \frac{a+c}{2}$
- $S_n = n * \text{AM}$
- To insert n A.M. b/w a & b they are considered as x_1, x_2, \dots, x_n where $x_i = a + (i-1)d$ where $i=1, 2, 3, \dots, n$.
- $d = \frac{b-a}{n+1}$
- The reciprocals of terms of A.P. are in H.P.
- To consider 3 terms in AP we have to take them as $a-d, a, a+d$
- To consider 4 terms in AP we have to take them as $a-3d, a-d, a+d, a+3d$

for 3 terms form a group
and for 4 terms form a group

GP

= the ratio of any two successive terms throughout the sequence is same is called GP. The constant ratio is called common ratio.

e.g. the several terms of GP are

a → first term.

r → common ratio.

$a, ar, ar^2, \dots, ar^{n-1}$

* nth term of GP whose first term is a then

$$T_n = ar^{n-1}$$

$$\frac{T_n}{T_{n-1}} = r, \frac{T_n}{T_m} = r^{n-m}$$

* Sum of n terms of GP is

$$S_n = \frac{a(r^n - 1)}{r-1} \quad r > 1$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad r < 0$$

Sum of ~~first~~ terms of GP is ∞ if $r > 1$

G.M of a, b is \sqrt{ab}

G.M of numbers which are in G.P. = $\sqrt[n]{T_1 \cdot T_n}$

b = G.M of abc if and only if

b = \sqrt{ac} , b is G.M of a, c.

The reciprocals of terms of GP are given in G.P.

To consider three terms in GP we have to take them as $\frac{a}{q}, a, aq$.

To consider four terms in GP we have to take them as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

HP If the reciprocals of terms are in AP then the series is called H.P.

General terms of H.P. are $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots$

n^{th} term of H.P. $T_n = \frac{1}{a+(n-1)d}$

$$3) \frac{1}{T_n} - \frac{1}{T_{n-1}} = d \quad \frac{1}{T_n} - \frac{1}{T_m} = (n-m)d$$

$$4) \text{H.M. of } a \text{ and } b \text{ is } \frac{ab}{a+b}$$

$$5) \text{G.M.} = \sqrt{A.M. \times R.M.}$$

Sum of interior angles of polygonal to n sides is
 $(n-2)180$

$$\text{and } a+d, a_1+d_1, a_2+d_2 = 180 \quad (n-2)180$$

$$5a + 5d = 3(n-2)180$$

$$a + d = \frac{5(n-2)180}{5} = 108$$

The sum of all even natural numbers less than 100 is

$$2 + 4 + 6 + \dots + 98$$

$$T_n = a + (n-1)d = 98$$

$$= 2 + (n-1)2 = 98$$

$$\Rightarrow 1 + (n-1) = 49$$

$$\therefore n = 49,$$

$$S_n = \frac{n}{2}(T_1 + T_n)$$

$$= \frac{49}{2}(2 + 98)$$

$$= 49 \times \frac{100}{2}$$

$$= 2450,$$

$$S_{50} = \frac{25}{2}[102 + 199]$$

$$101 + 199$$

②

$$a = 101$$

$$d = 2$$

$$101 + (n-1)2 = 199$$

$$\begin{array}{r}
 & 1 \\
 + & - \\
 1 & 3 \\
 \hline
 1 & 5
 \end{array}
 \quad
 \begin{array}{r}
 434182 \\
 \times 3 \\
 \hline
 6 \times 3 = 18
 \end{array}
 \quad
 \begin{array}{r}
 546 \\
 - 728 \\
 \hline
 7826
 \end{array}$$

Odd digit in unit place i.e. unit place can have 3 ways
 tens place have 6 ways.

Hence the no. of two digit odd numbers that can form is $18 \times (6 \times 3)$

$$a = 35$$

$$d = 7$$

$$35 + (n-1)7 = 329$$

$$35 + (n-1)7 = 329$$

$$7n + 28 = 329$$

$$7n = 299 - 28$$

$$n = 43$$

$$S_{43} = \frac{43}{2} [35 + 329]$$

\Rightarrow

$$\Rightarrow 546$$

$$\begin{array}{r}
 7) 329 (47 \\
 28 \\
 \hline
 69
 \end{array}$$

$$322$$

$$\begin{array}{r}
 322 \\
 - 28 \\
 \hline
 294
 \end{array}$$

$$7) 321$$

$$182$$

$$364$$

$$\begin{array}{r}
 7) 82843 \\
 56 \\
 \hline
 264
 \end{array}$$

$$546$$

- reminders divisible by 7: two digit numbers.

$$a = 10$$

$$d = 7$$

$$S_{13} = \frac{13}{2} (10 + 94)$$

$$10 + (n-1)7 = 94$$

$$\frac{13}{2} (10 + 94)$$

$$= 13 \times 52$$

$$7n = 91$$

$$\frac{26}{65}$$

$$n = 13$$

$$\frac{65}{676}$$

- Sum of all 3 digit numbers that give a remainder of 4 when it is divisible by 7.

$$\Rightarrow a = 102$$

$$d = 9\cancel{8}\cancel{7}$$

$$102 + (n-1)7 = 998$$

$$\text{top } n = 889$$

$$\text{bottom } n = 129$$

$$S_n =$$

$$\frac{1}{2} (102 + 998) \times 7$$

$$\frac{1}{2} (102 + 998) \times 7 = 7000$$

$$\frac{998}{7} \quad \frac{102}{7} \quad \frac{896}{7} \quad \frac{25}{7}$$

$$\frac{998}{7} \quad \frac{102}{7} \quad \frac{896}{7} \quad \frac{25}{7}$$

$$\frac{998}{7} \quad \frac{102}{7} \quad \frac{896}{7} \quad \frac{25}{7}$$

• sum of 4th & 12th terms of an A.P. is 120 then

sum(⁴ terms)

$$\Rightarrow a + 3d + a + 11d = 120$$

$$\Rightarrow 2a + 14d = 120$$

$$\Rightarrow a + 7d = 60$$

$$\frac{15}{2} [2a + 14d],$$

$$\frac{15}{2} (120) \Rightarrow 900,$$

$$a + 5d + a + 14d = a + 6d + a + 9d + a + 11d$$

$$\Rightarrow 2a + 19d = 3a + 26d$$

$$a + 7d = 0,$$

$$a + (-1)d = 0$$

$\Rightarrow 8^{\text{th}}$ term,

$$a + 2d + a + 14d = a + 5d + a + 16d + a + 12d$$

$$\Rightarrow 2a + 16d = 3a + 27d$$

$$a + 11d = 0$$

$\Rightarrow 12^{\text{th}}$ term,

$$3, 9, 15, 21, \dots$$

$\{ -3, -1/2, 2, \dots \}$

$a = 3$

$d = 6$

$a_{15} = 3 + (14)6$

$= 3 + 84$

$= 87$

$T_n = -3 + (n-1)\frac{5}{2}$

$\Rightarrow T_n = \frac{-3 + 5n - 5}{2}$

• 1, 11, 21, 31

$a = 1$

$d = 10$

$S_{10} = \frac{10}{2} [2 + 9(10)]$

$= 5[92]$

$= 460$

• Find the sum of all the numbers / by 6 in between 100 to 400

12,500

12,450

11,450

11,500