Model No: 3.7 : NORMAL DISTRIBUTION

Definition: A Continuous Random Variable 'x' is Said to follow the Normal Distribution With Mean 'μ' and Standard Deviation 'σ'. Then:

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}, \quad \begin{array}{l} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{array}$$

Conditions Of Normal Distribution:

* Normal Distribution is a Limiting form Of Binomial distribution under the following Conditions:

  a) 'n' be the No. of are Independently that is $n \to \infty$

  b) Neither p nor q is Very Small

* Normal Distribution can also be Obtained as a Limiting Form Of Poisson Distribution will the Parameter $\mu \to \infty$

* In Normal Distribution $\boxed{\text{Mean} = \text{Median} = \text{Mode}}$

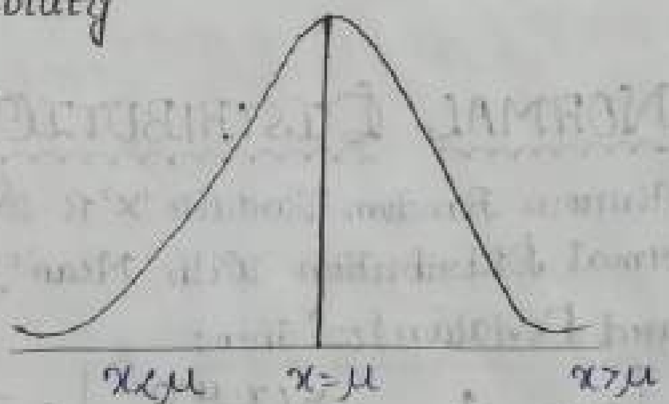
Properties of Normal Distribution:

* The Normal Curve is a Bell Shaped Curve and It is Symmetric about $x = \mu$
* In ND, $\boxed{\text{Mean} = \text{Median} = \text{Mode}}$
* Since, the Curve is Symmetrical, Skewness $\beta_1 = 0$ and Kurtosis $\beta_2 = 3$
* The Standard Normal Variant is $\boxed{Z = \frac{x-\mu}{\sigma}}$

  The Points of Inflections are:

$$Z = \mu \pm \sigma$$
$$Z = \mu \pm 2\sigma$$
$$Z = \mu \pm 3\sigma$$

'Area' In this Case is equal to Probability

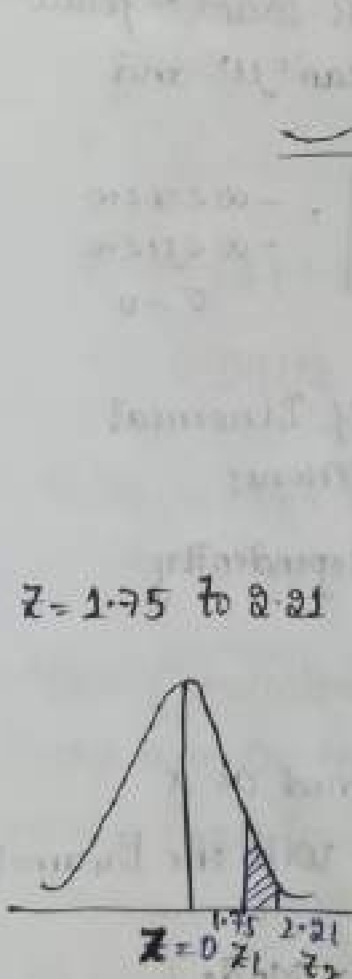


$$Z = \frac{x-\mu}{\sigma}$$

Z -ve   Z = 0   Z +ve

Cases

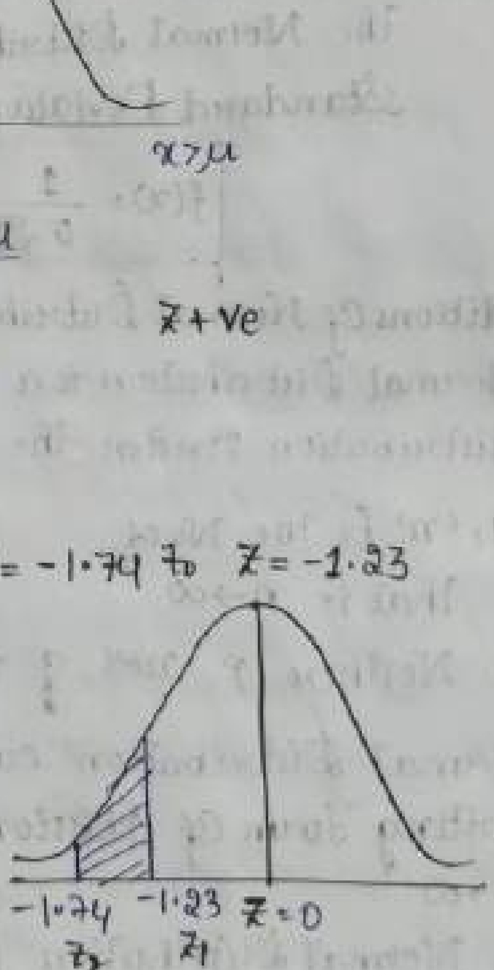
i) $Z = 1.75$ to 2.21



$A(0 \text{ to } 2.21) - A(0 \text{ to } 1.75)$

$= 0.4864 - 0.4599$

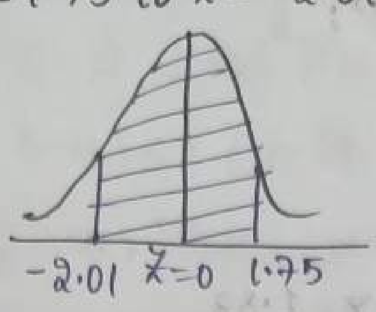
ii) $Z = -1.74$ to $Z = -1.23$



$A(0 \text{ to } 1.74) - A(0 \text{ to } 1.23)$

$= 0.4591 - 0.3907$

iii) $z = 1.75$ to $z = -2.01$.

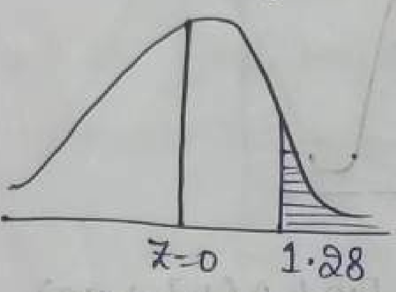


$A(0 \text{ to } 1.75) + A(0 \text{ to } 2.01)$

2) i, To the right of $z = 1.28$

Over, $\geq$

$\therefore z \geq 1.28$

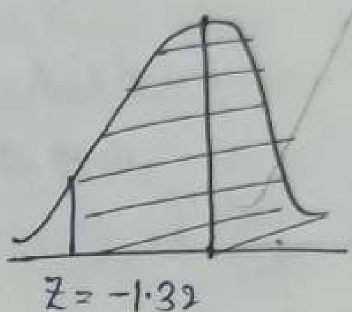


$= 0.5 - A(0 \text{ to } 1.28)$

$= 0.5 - 0.3997$

ii, To the right of $z = -1.32$

$\therefore z \geq -1.32$



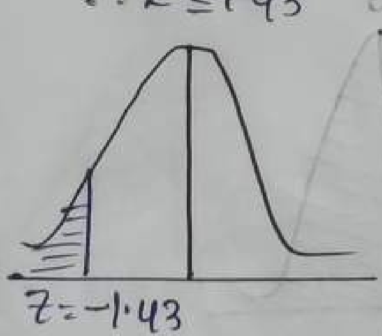
$0.5 + A(0 \text{ to } 1.32)$

$0.5 + 0.4066$

iii, To the left of $z = 1.43$

under, $\leq$

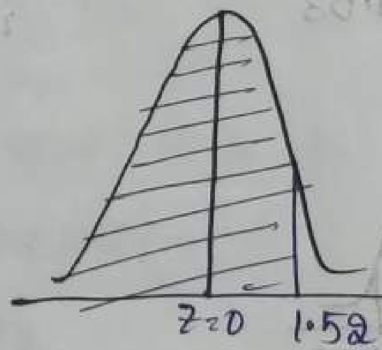
$\therefore z \leq 1.43$



$0.5 - A(0 \text{ to } 1.43)$

$0.5 - 0.4236$

iv, To the left of $z = 1.52$



$= 0.5 + A(0 \text{ to } 1.52)$

$= 0.5 + 0.4357$

1, For a Normally Distributed Variate 'x':

i, To the left of $z = -1.78$

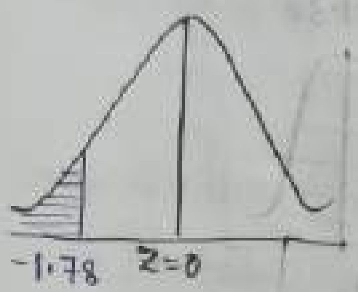
ii, To the right of $z = -1.45$

iii, Corresponding to $-0.8 \le z \le 1.53$

iv, To the left of $z = -2.52$ & right of $\underline{z = -1.78}$ $z = 1.83$

Sol.

i, $z \le -1.78$

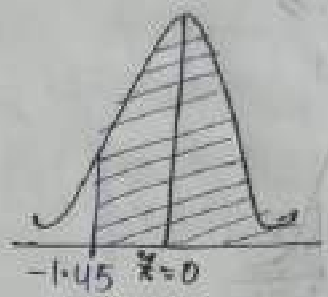


$= 0.5 - A(0 \text{ to } 1.78)$

$= 0.5 - 0.4625$

RA = 0.0375

ii, $z \ge -1.45$

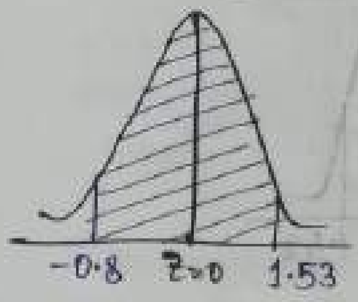


Required Area $= 0.5 + A(0 \text{ to } 1.45)$

$= 0.5 + 0.4265$

$= 0.9265$

iii, $-0.8 \le z \le 1.53$

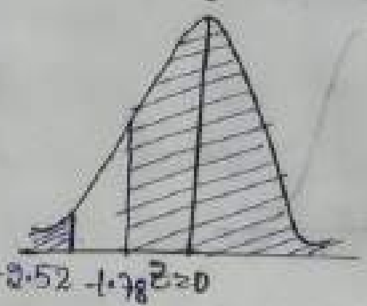


$RA = A(0 \text{ to } 1.53) + A(0 \text{ to } 0.8)$

$= 0.4370 + 0.2881$

0.7251

iv, $z \le -2.52$ & $z \ge -1.78$



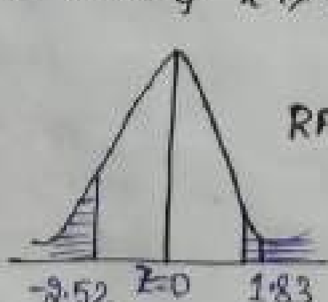
$RA = 0.5 - A(0 \text{ to } 2.52) + [0.5 + A(0 \text{ to } 1.78)]$

$= 0.5 - 0.4941 + [0.5 + 0.4625]$

$= 0.0059 + 0.9625$

$= 0.9684$

v, $z \overset{\text{left}}{\le} -2.52$ & $z \overset{\text{right}}{\ge} 1.83$



$RA = [0.5 - A(0 \text{ to } 2.52)] + [0.5 - A(0 \text{ to } 1.83)]$

$= (0.5 - 0.4941) + (0.5 - 0.4664)$

$= 0.0059 + 0.0336$

$= 0.0395$

2) For a Normally Distributed Variate with Mean ($\mu = 1$) & S.D ($\sigma = 3$). Find the Probability of:

i) $3.43 \le x \le 6.19$ ii) $-1.43 \le x \le 6.19$

Sol: Given; Mean $\mu = 1$ & Standard Deviation $\sigma = 3$

At $x = 3.43$:

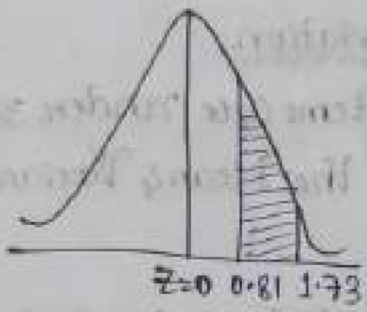
$$z = \frac{x-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81$$

At $x = 6.19$

$$z = \frac{x-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$\therefore$ $3.43 \le x \le 6.19$

$\boxed{0.81 \le z \le 1.73}$



RA = A(0 to 1.73) − A(0 to 0.81)
= 0.4582 − 0.2910
= 0.1672

ii) At $x = -1.43$

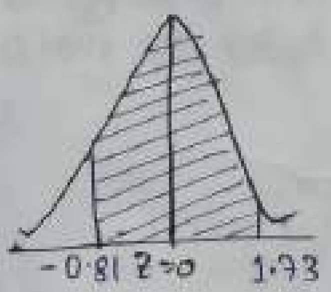
$$z = \frac{x-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81$$

At $x = 6.19$

$$z = \frac{x-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73$$

$-1.43 \le x \le 6.19$

$-0.81 \le z \le 1.73$



RA = A(0 to 1.73) + A(0 to 0.81)
= 0.4582 + 0.2910
= 0.7492

1.36) 3) If 'x' is a Normal Variant with $\mu = 30$ & $\sigma = 5$. Find the Probability that: i) $26 \le x \le 40$ ii) $x \ge 45$

Sol. Given; $\mu = 30$, $\sigma = 5$

i) At $x = 26$

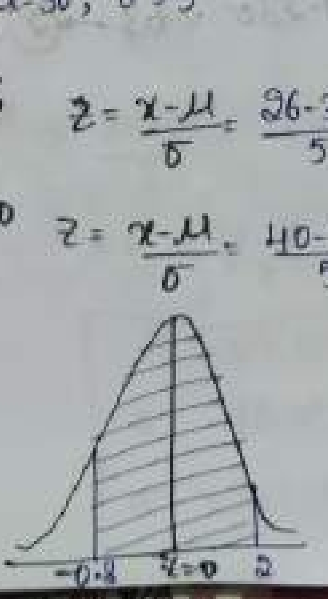
$$z = \frac{x-\mu}{\sigma} = \frac{26-30}{5} = -0.8$$

At $x = 40$

$$z = \frac{x-\mu}{\sigma} = \frac{40-30}{5} = 2$$

$26 \le x \le 40$

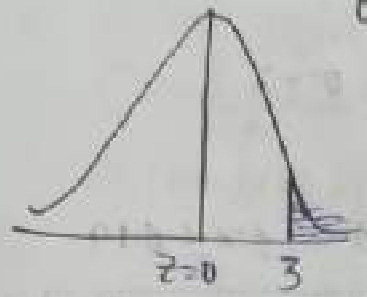
$\boxed{-0.8 \le z \le 2}$



RA = A(0 to 2) + A(0 to 0.8)
= 0.4772 + 0.2881
= 0.7653

iii) $x \geq 45$

At $x=45$ $\quad z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = 3$ $\}$ $z \geq 3$



$RA = 0.5 - A(0 \text{ to } 3)$

$= 0.5 - 0.4987$

$= 0.0013$

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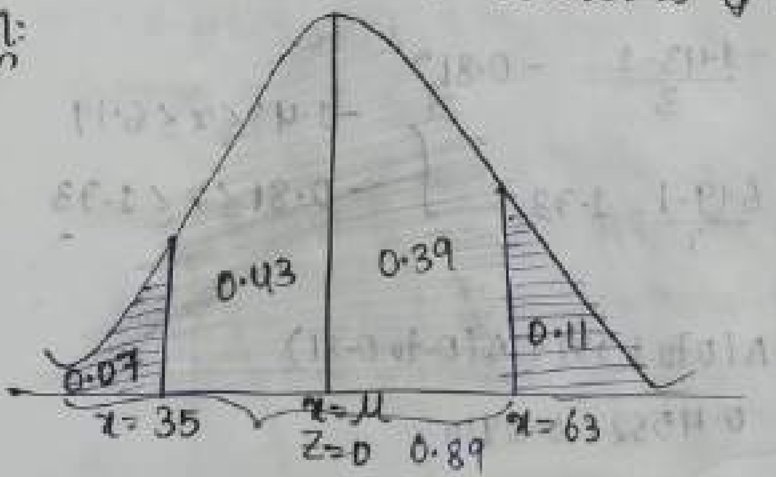
Problems To Find Mean & Standard Deviation:

4) In a Normal Distribution 7% of the Items are under 35 and 89% are under 63. Determine the Mean & Variance Of the Normal Distribution.

(or)

Find the Mean & SD of a Normal Distribution Of which 7% of the Items are under 35 & 89% are under 63

Sol:



$z = -1.48$ (search for 0.43) $\qquad z = 1.23$ (search for 0.39)

$z = \frac{x-\mu}{\sigma} \Rightarrow -1.48 = \frac{35-\mu}{\sigma} \Rightarrow \mu - 1.48\sigma = 35 \rightarrow (1)$

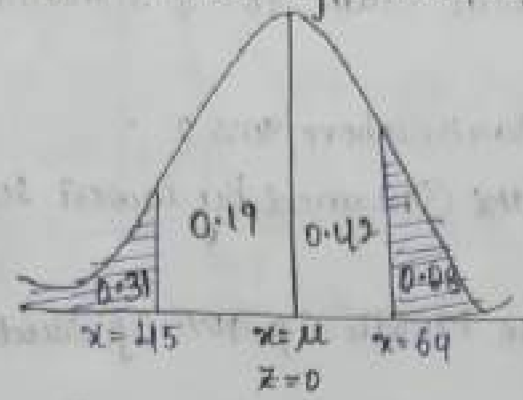
$z = \frac{x-\mu}{\sigma} \Rightarrow 1.23 = \frac{63-\mu}{\sigma} \Rightarrow \mu + 1.23\sigma = 63 \rightarrow (2)$

Solve

$$\boxed{\begin{gathered}\mu = 50.29 \\ \sigma = 10.33 \\ \sigma^2 = 106.70\end{gathered}}$$

5. In ND 31% of items are under 45 & 8% are over 64. Find Mean & Variance of distribution.

Sol:



$Z = -0.5$ (search for 0.19) $Z = 1.41$ (search for 0.42)

$z = \frac{x-\mu}{\sigma} \Rightarrow -0.5 = \frac{45-\mu}{\sigma} \Rightarrow \mu - 0.5\sigma = 45 \rightarrow (1)$

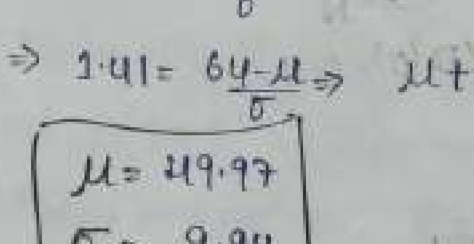
$\Rightarrow 1.41 = \frac{64-\mu}{\sigma} \Rightarrow \mu + 1.41\sigma = 64 \rightarrow (2)$

} Solve

$$\boxed{\begin{aligned} \mu &= 49.97 \\ \sigma &= 9.94 \\ \sigma^2 &= 98.80 \end{aligned}}$$

6. Suppose 10% of probability of ND is below 35 & 5% above 90. What are the values of μ & σ?

Sol:



$Z = -1.29$ $Z = 1.65$

$Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{35-\mu}{\sigma} = -1.29 \Rightarrow \mu - 1.29\sigma = 35 \rightarrow (1)$

$Z = \frac{x-\mu}{\sigma} \Rightarrow 1.65 = \frac{90-\mu}{\sigma} \Rightarrow \mu + 1.65\sigma = 90 \rightarrow (2)$

} Solve

$$\boxed{\begin{aligned} \mu &= 59.13 \\ \sigma &= 18.70 \\ \sigma^2 &= 349.69 \end{aligned}}$$

4) The Marks Obtained in Mathematics by 1000 Students is Normally distributed with Mean 78% & standard deviation 11%. Determine:

i) How many Students Got Marks above 90%?

ii) What was the Highest Marks Obtained by lowest 10% of the Students.

iii) Within what limits did the Middle Of 90% Of Students lie.

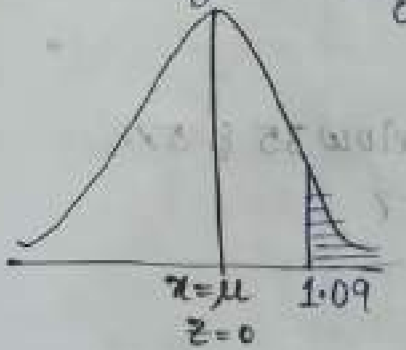
Sol: In Our Given Question, they donot Mention Probability at any where.

Here x = No.of Students = 1000 N

Given Mean $(\mu) = 0.78$ & SD $(\sigma) = 0.11$

i) $p(x \geq 90\%) = p(x \geq 0.9)$

$$z = \frac{x-\mu}{\sigma} = \frac{0.9-0.78}{0.11} = 1.09 \quad z \geq 1.09$$

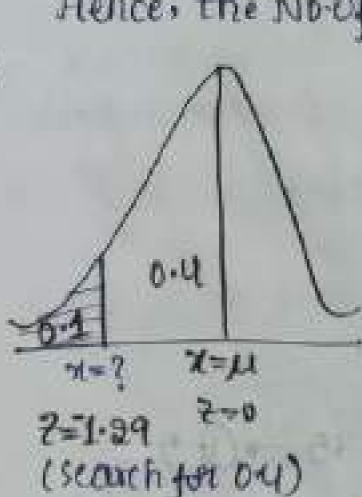


$RA = 0.5 - A(0 \text{ to } 1.09)$
$= 0.5 - 0.3621$
$= 0.1379$

$\boxed{\text{Area} = p(x \geq 0.9) = 0.1379}$

Hence, the No.of Students with Marks more than 90% $= 0.1379 \times 1000 = 137$

ii)



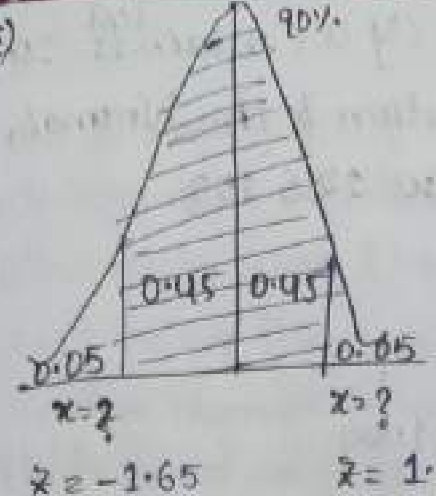
$$z = \frac{x-\mu}{\sigma} \Rightarrow -1.29 = \frac{x-0.78}{0.11}$$

$x = 0.78 + ((0.11)(-1.29))$

$\boxed{x = 0.6381} \times N^{1000} = 638.1$

Hence, the Highest Mark Obtained by the lowest 10% is 63.81%

(ii)



$z = \frac{x-\mu}{\sigma} \Rightarrow -1.65 = \frac{x - 0.78}{0.11} \Rightarrow x = 0.5985$

$z = \frac{x-\mu}{\sigma} \Rightarrow 1.65 = \frac{x - 0.78}{0.11} \Rightarrow x = 0.9615$

$0059.85 \le x \le 96.15$

$59.85 \le x \le 96.15$

$\boxed{60 \le x \le 96}$

Note:

* $|x| \le a \Rightarrow \boxed{-a \le x \le a}$

8. If 'x' is Normally distributed with Mean '2' & $\sigma^2 = 0.1$ then find $P(|x-2| \ge 0.01)$

Sol. Given Mean $\mu = 2$, $\sigma^2 = 0.1$, $\sigma = 0.31$

Given: $P(|x-2| \ge 0.01)$

$= 1 - P(|x-2| < 0.01)$

$= 1 - P(-0.01 < x - 2 < 0.01)$

$= 1 - P(1.99 < x < 2.01)$

At $x = 1.99$

$\Rightarrow z = \frac{x-\mu}{\sigma} =$

$\Rightarrow z = \frac{1.99 - 2}{0.31} = -0.03$

At $x = 2.01$

$z = \frac{x-\mu}{\sigma}$

$z = \frac{2.01 - 2}{0.31} = 0.03$

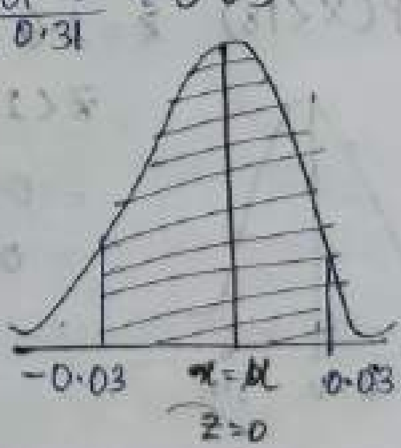
$= 1 - P(-0.03 < z < 0.03)$

$= 1 - [A(0 \text{ to } 0.03) + A(0 \text{ to } 0.03)]$

$= 1 - [0.0120 + 0.0120]$

$= 1 - 0.024$

$= 0.976$



(9) In a Sample Of 1000 Cases, the Mean Of a Certain test is 14, & S.D $\sigma = 2.5$. Assuming the distribution to the Normal, Find: i, How many Students Score b/w 12 & 15?

ii, How many Score above 18?

iii, How many Score below 18?

Sol:-

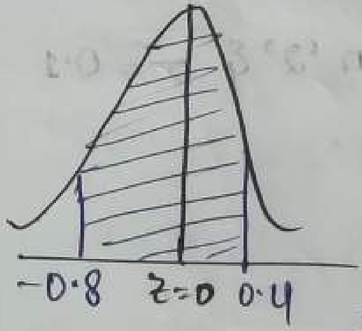
Given Mean $\mu = 14$, SD $\sigma = 2.5$, $N = 1000$

i, $P(12 < X < 15)$

$X = 12$, $Z = \frac{x-\mu}{\sigma} \Rightarrow Z = \frac{12-14}{2.5} = -0.8$

$X = 15$, $Z = \frac{x-\mu}{\sigma} = \frac{15-14}{2.5} = 0.4$

$-0.8 < Z < 0.4$



RA = A(0 to 0.4) + A(0 to 0.8)

= 0.1554 + 0.2881

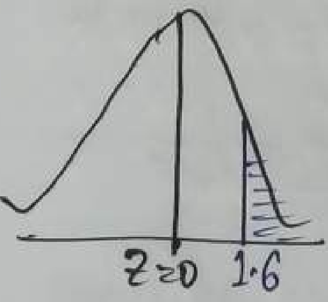
$\boxed{A = P(12 < X < 15) = 0.4435}$

0.4435 × 1000

$= \boxed{443.5 \text{ Students}}$

ii, $P(X > 18)$

$X = 18$ $Z = \frac{x-\mu}{\sigma} = \frac{18-14}{2.5} = 1.6$



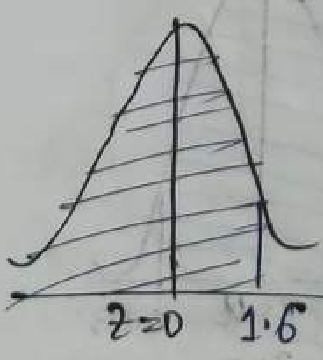
$Z > 1.6$

= 0.5 − A(0 to 1.6)

= 0.5 − 0.4452

= 0.0548 × 1000 = 54.8 = $\boxed{55 \text{ Students}}$

iii, $P(X < 18)$ $Z = 1.6$



$Z < 1.6$

= 0.5 + A(0 to 1.6)

= 0.5 + 0.4452

= 0.9452 × 1000 = 945.2 = $\boxed{945 \text{ Students}}$

05.04.2022 Thursday

1. In a Sample, If the Masses of 300 Students are Normally distributed with Mean 68 kgs & SD of 3 kgs. How many Students have Masses :

i, Greater than 72 kg

ii, Less than or Equal to 64 kg

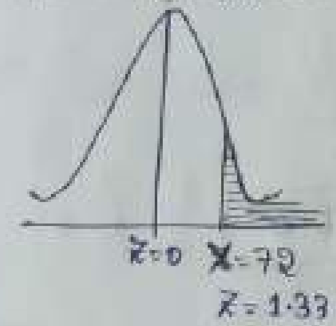
iii, between 65 & 71 kgs

Sol: Given $\mu = 68$, $\sigma = 3$, $N = 300$

i, $P(X > 72)$

we Know; $Z = \frac{x-\mu}{\sigma} = \frac{72-68}{3} = 1.33$

$P(X>72) ; P(Z>1.33)$



$RA = 0.5 - A(0 \text{ to } \overset{x}{72})$

$= 0.5 - A(0 \text{ to } 1.33)$

$= 0.5 - 0.4082$

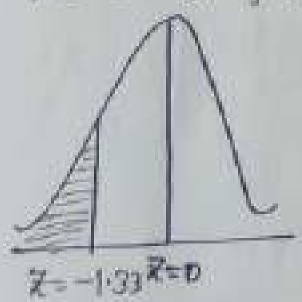
$= 0.0918 \times N$

$\boxed{RA = 0.0918 \times 300 = 27.54}$

ii, $P(X \leq 64)$

$Z = \frac{x-\mu}{\sigma} = \frac{64-68}{3} = \frac{-4}{3} = -1.33$

$P(X \leq 64) = P(Z \leq 1.33)$



$RA = 0.5 - A(0 \text{ to } 1.33)$

$= 0.5 - 0.4082$

$= 0.0918 \times 300$

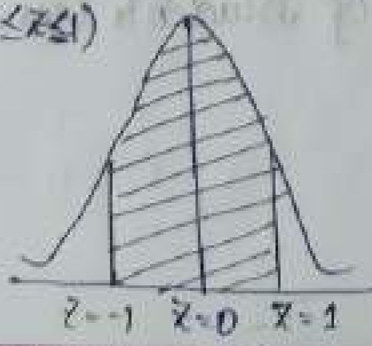
$\boxed{RA = 27.54}$

iii, $P(65 \leq X \leq 71)$

$X = 65 ; Z = \frac{x-\mu}{\sigma} = \frac{65-68}{3} = -1$

$X = 71 ; Z = \frac{x-\mu}{\sigma} = \frac{71-68}{3} = 1$

$P(-1 \leq Z \leq 1)$



$RA = A(0 \text{ to } 1) + A(0 \text{ to } 1)$

$= 0.3413 + 0.3413$

$= 0.6826 \times 300$

$= 204.78 \approx \boxed{RA = 205}$

2, A Sales Tax officer has reported that average sales Of the 500 business that he has deal during a year is 36000/- with a S.D of 10,000/-. Assume that the sales in business are Normally distributed

i, The No. of business as the Sales of which are 40,000/-

ii, The Percentage of business the Sales of which are likely to range b/w 30,000/- & 40,000/-

Sol: Given $\mu = 36000$, $\sigma = 10{,}000$, $N = 500$

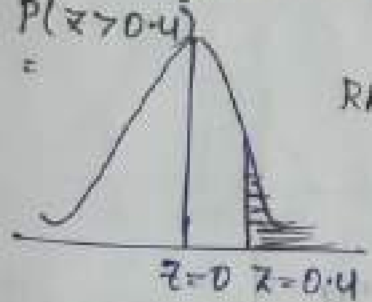
i, $P(X > 40{,}000)$

$X = 40{,}000$

$$Z = \frac{x-\mu}{\sigma} = \frac{40{,}000 - 36000}{10{,}000} = 0.4$$

$P(Z > 0.4)$

=



$RA = 0.5 - A(0 \text{ to } 0.4)$

$= 0.5 - 0.1554$

$$\boxed{RA = 0.3446 \times 500 = 172.3}$$

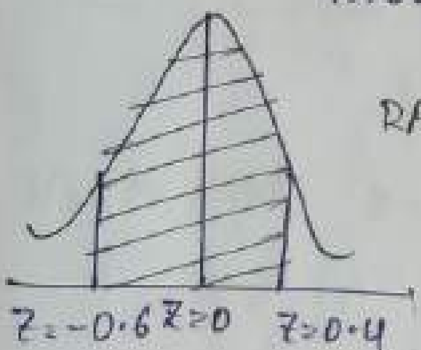
ii, $P(30{,}000 < x < 40{,}000)$

$X = 30{,}000$

$$Z = \frac{30{,}000 - 36000}{10{,}000} = -0.6$$

$X = 40{,}000$

$$Z = \frac{40{,}000 - 36000}{10{,}000} = 0.4$$



$RA = A(0 \text{ to } 0.4) + A(0 \text{ to } 0.6)$

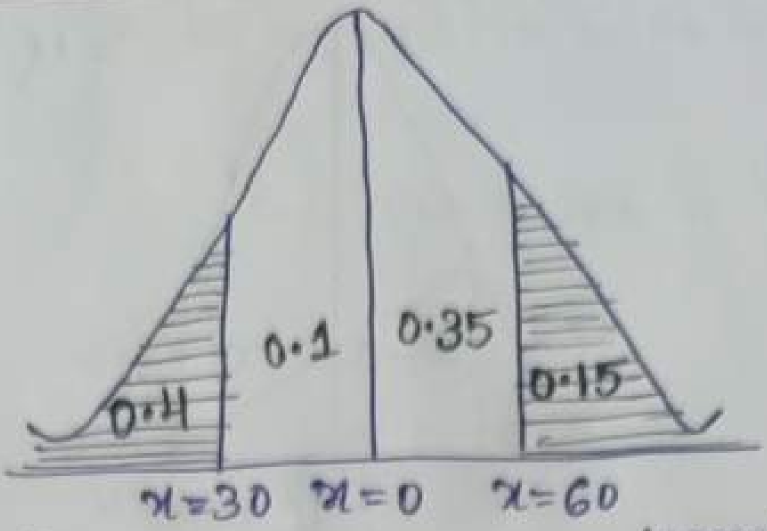
$= 0.1554 + 0.2258$

$= 0.3812 \times 100$

$$\boxed{RA = 38.12}$$

3, The Marks Obtained in Statistics in a Certain Examination found to be Normally distributed, If 15% Of Students Greater or Equal to 60 Marks, 40% of Students less than 30. Find the Mean & SD

3. Sol:



search $\mu z = 0.1$	search $\mu z = 0.35$
$z = 0.26$	$z = 1.04$
$z = -0.26$	$z = 1.04$
$\Rightarrow z = \dfrac{x-\mu}{\sigma}$	$\therefore z = \dfrac{x-\mu}{\sigma}$
$\Rightarrow -0.26 = \dfrac{30-\mu}{\sigma}$	$1.04 = \dfrac{60-\mu}{\sigma}$
$\Rightarrow \mu - 0.26\sigma = 30 \rightarrow (1)$	$\mu + 1.04\,\sigma = 60 \rightarrow (2)$

$$\left.\begin{array}{l}\mu - 0.26\sigma = 30 \rightarrow (1)\\ \mu + 1.04\sigma = 60 \rightarrow (2)\end{array}\right\}$$

By Solving Equations (1) & (2):

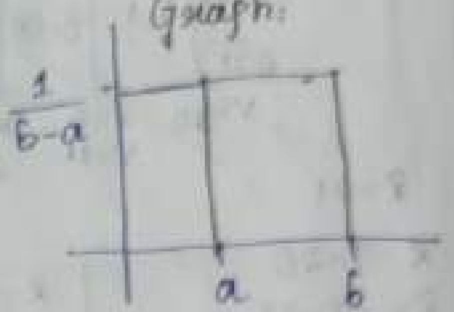
We Get:

$$\boxed{\begin{array}{l}\mu = 36\\ \sigma = 23.07\end{array}}$$

Uniform Distribution:

It is a Continuous Distribution Function:

$$f(x) = K\begin{cases} K & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$



Mean $\mu = \frac{b+a}{2}$

Variance $\sigma^2 = \frac{(b-a)^2}{12}$

$$\boxed{K = \frac{1}{b-a}}$$

We know that: Sum of all Probabilities = 1

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{a} \underbrace{f(x)\,dx}_{0} + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} \underbrace{f(x)\,dx}_{0} = 1$$

$$\Rightarrow \int_{a}^{b} K\,dx = 1 \Rightarrow K[x]_a^b = 1 \qquad \boxed{\therefore\ K = \frac{1}{b-a}}$$

Mean:

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{a} x \underbrace{f(x)\,dx}_{0} + \int_{a}^{b} x f(x)\,dx + \int_{b}^{\infty} x \underbrace{f(x)\,dx}_{0}$$

$$= \int_{a}^{b} Kx\,dx = K\int_{a}^{b} x\,dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \boxed{\mu = \frac{a+b}{2}}$$

Variance:

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{-\infty}^{a} x^2 \underbrace{f(x)\,dx}_{0} + \int_{a}^{b} x^2 f(x)\,dx + \int_{b}^{\infty} x^2 \underbrace{f(x)\,dx}_{0} - \mu^2$$

$$= (K)\int_{a}^{b} (x^2\,dx) - \left(\frac{a+b}{2}\right)^2$$

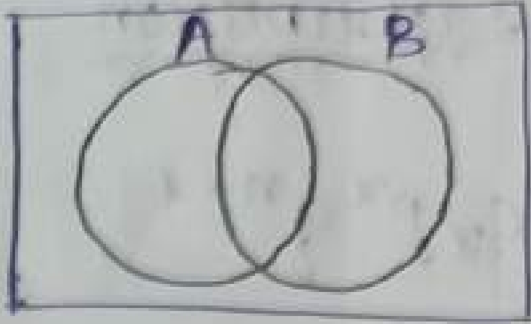
$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \left(\frac{a+b}{2}\right)^2 = \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right) - \left(\frac{a+b}{2}\right)^2$$

$$\sigma^2 = \frac{1}{b-a}\,\frac{(b-a)(b^2+ab+a^2)}{3} - \left(\frac{a+b}{2}\right)^2 = \frac{b^2+a^2-2ab}{12}$$

$$\boxed{\sigma^2 = \frac{(b-a)^2}{12}}$$

Addition Theorem On Probability:

Let A & B be the 2 (Unions) Events



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Clearly from Diagram:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Let 'S' be the Sample Space dividing the above Equation throughout with $n(S)$

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\boxed{\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Fitting of a Binomial Distribution

$$P(X=x) = n_{C_x} p^x q^{n-x}$$

Here

n = No. of Trails

p = Probability of Success

q = Probability of failure

x = No. of Successors
0, 1, 2, ..., n

Mean:
$\mu = np$

Variance:
$\sigma^2 = npq$

Fitting of a Poisson Distribution

$$P(X=x) = \frac{e^{-\mu} \mu^x}{x!}$$

x = No. of Successors
0, 1, 2, ... n

Mean: $\mu = np$

Variance:
$\sigma^2 = np$

1, Fit a Binomial Distribution to the following Frequency

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

$P(X=x) = nC_x p^x q^{n-x}$

Mean $\mu = np$ $\quad N = \Sigma f = 200$

$$\frac{\Sigma xf}{\Sigma f} = np$$

$$\frac{535}{200} = 6(p)$$

$$\boxed{p = 0.4458}$$

$q = 1 - p$
$= 1 - 0.4458$

$$\boxed{q = 0.5542}$$

$x = 0$. ${}^6C_0 (0.4558)^0 (0.5542)^6 \times N$
$= 0.0289 \times 200 = 5.782$ 6

$x = 1 = {}^6C_1 (0.4458)^1 (0.5542)^5$
$= 0.1398 \times 200 = 27.96$ 28

* Fit a Poisson distribution to the following Frequency

x	0	1	2	3	4	5
f	142	156	69	27	5	1

$$P(X=x) = \frac{e^{-\mu} \mu^x}{x!}$$

Mean: $\mu = np$ $\quad N = \Sigma f = 400$

$$\frac{\Sigma fx}{\Sigma f} = np = \mu \quad n = 5$$

$$\mu = \frac{400}{400} = 1$$

$x = 0$ $\quad \frac{e^{-1} 1^0}{0!} = 0.3678 \times \overset{400}{N}$
$= 147.12$
≈ 148

$x = 1$ $\quad \frac{e^{-1} 1^1}{1!} = 147.12$
≈ 148

$x = 2 = \frac{e^{-1} 1^2}{2!} = 0.1839 \times 400$
$= 73.56$
≈ 74

$x = 3 = \frac{e^{-1} 1^3}{3!} = 24.52 \approx 24$

$x=2$ $\quad {}^6C_2 (0.4558)^2 (0.5542)^4$

$= 56.18 = 56$

$x=3$ $\quad {}^6C_3 (0.4558)^3 (0.5542)^3$

$= 60.32 = 60$

$x=4$ $\quad {}^6C_4 (0.4558)^4 (0.5542)^2$

$= 36.42 = 36$

$x=5$ $\quad {}^6C_5 (0.4558)^5 (0.5542)^1$

$= 11.728 = 12$

$x=6$ $\quad {}^6C_6 (0.4558)^6 (0.5542)^0$

$= 1.5732 = 2$

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4
Ef	6	28	56	60	36	12	2

$x=4$ $\quad \frac{e^{-1} 1^4}{4!} = 6.131 \approx 6$

$x=5$ $\quad \frac{e^{-1} 1^5}{5!} = 1.226 \approx 1$

x	0	1	2	3	4	5
f	142	156	69	27	5	1
Ef	148	148	74	24	6	1