Contrast stretching

aims to increase (expand) the dynamic range of an image. It transforms the gray levels in the range {0,1,…, L-1} by a piecewise linear function. The figure below shows a typical transformation used for contrast stretching.

The locations of points (r1, s1) and (r2, s2) control the shape of the transformation function.

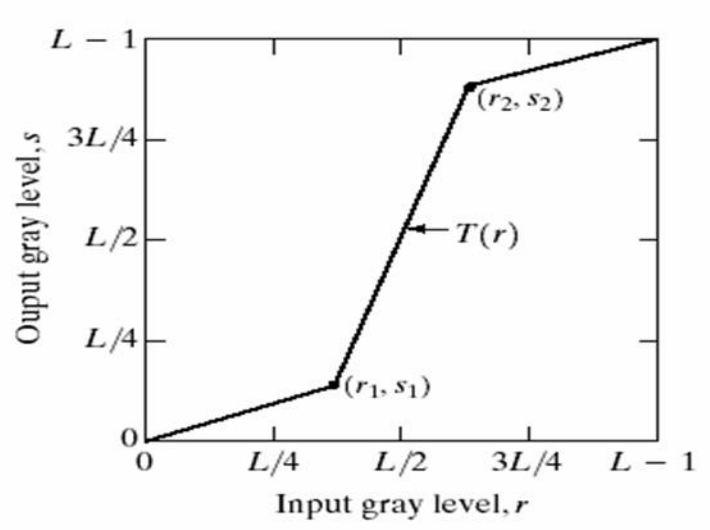


Figure 5.1 Form of transformation function

For example the following piecewise linear function

$$s = \begin{cases} (227 * r - 5040)/47 & if \;\; 28 \le r \le 75 \\ r & otherwise \end{cases}$$

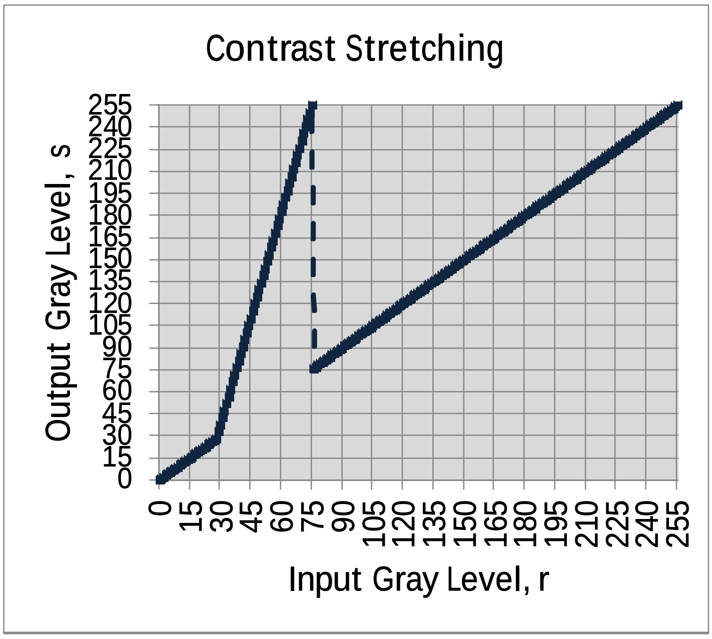
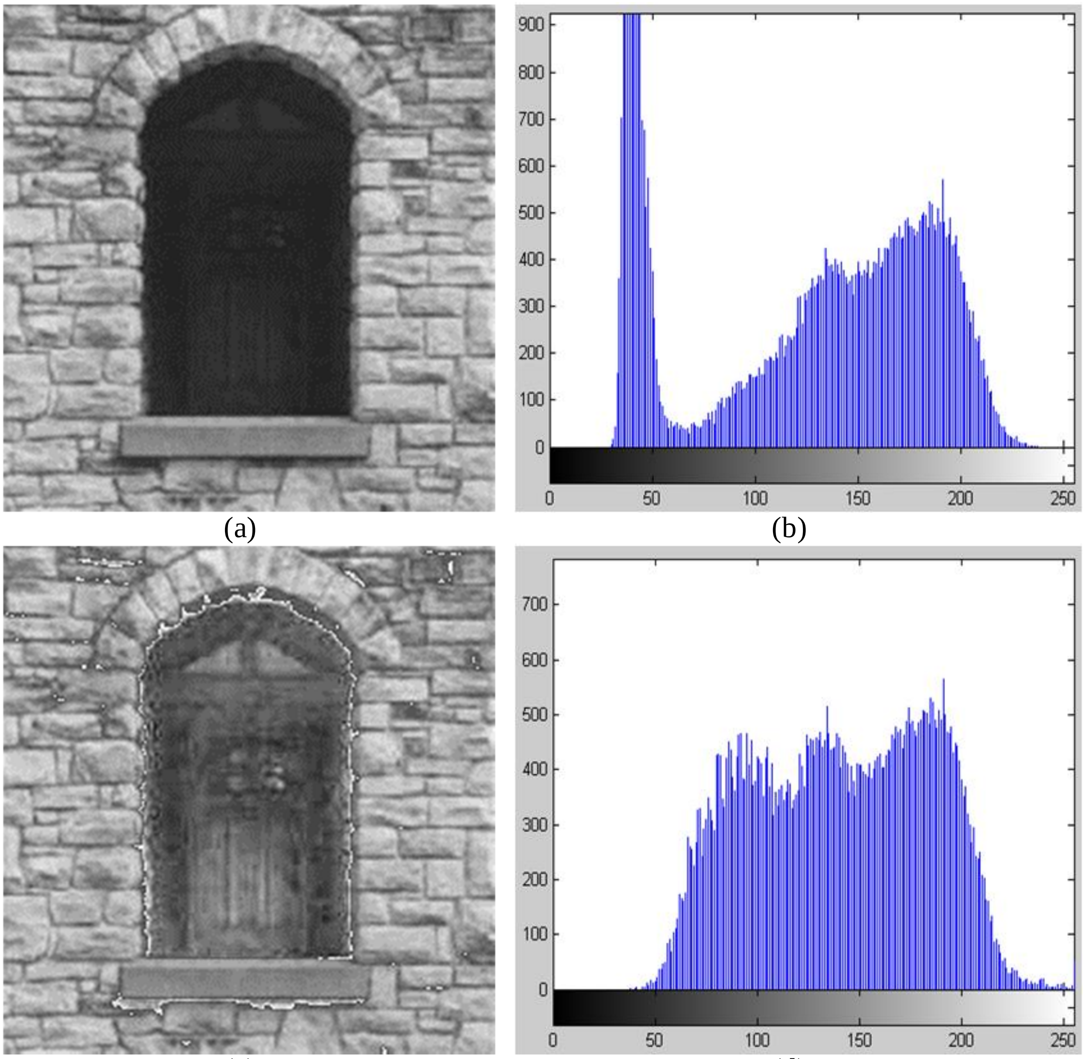


Figure 5.2 Plot of above piecewise linear function

will be used to increase the contrast of the image shown in the figure below:



(a) (b)

(c) (d)

Figure 5.3 Contrast stretching. (a) Original image. (b) Histogram of (a). (c) Result of contrast stretching. (d) Histogram of (c).

For a given plot, we use the equation of a straight line to compute the piecewise linear function for each line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

For example the plot in Figure 5.2, for the input gray values in the interval [28 to 75] we get:

$$y - 28 = \frac{255 - 28}{75 - 28}(x - 28)$$

$$y = (227 * x - 5040)/47 \qquad if \;\; 28 \le x \le 75$$

Similarly, we compute the equations of the other lines.

Another form of contrast stretching is called automatic (full) contrast stretching as shown in the example below:

$$s = \begin{cases} 0 & if \; r < 90 \\ (255 * r - 22950)/48 & if \; 90 \le r \le 138 \\ 255 & if \; r > 138 \end{cases}$$

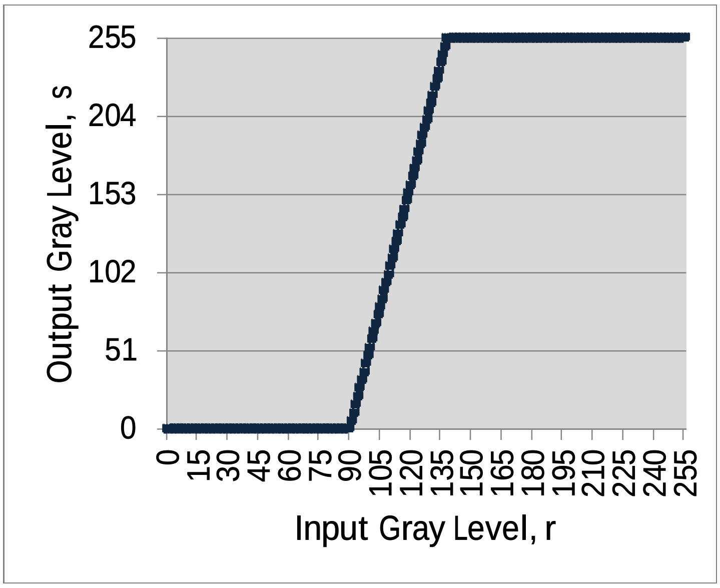


Figure 5.4 Full contrast-stretching

This transform produces a high-contrast image from the low-contrast image as shown in the next figure.

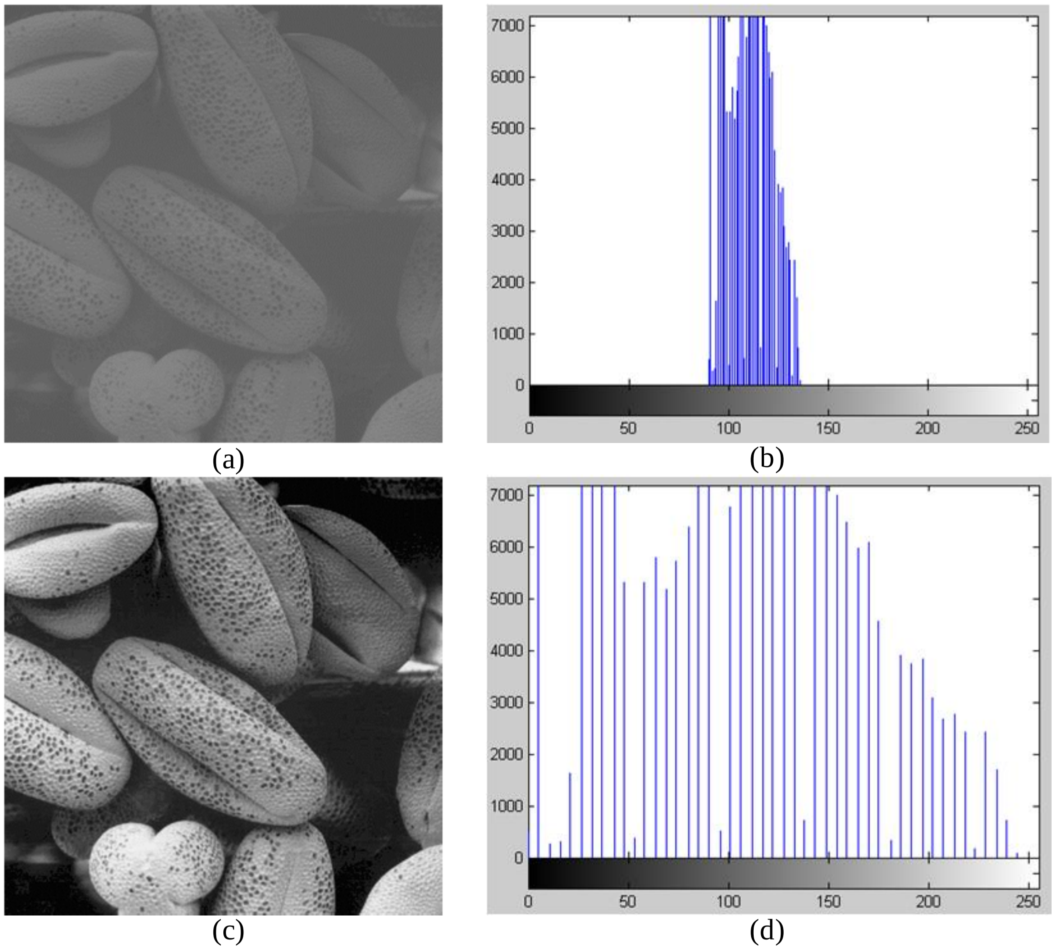
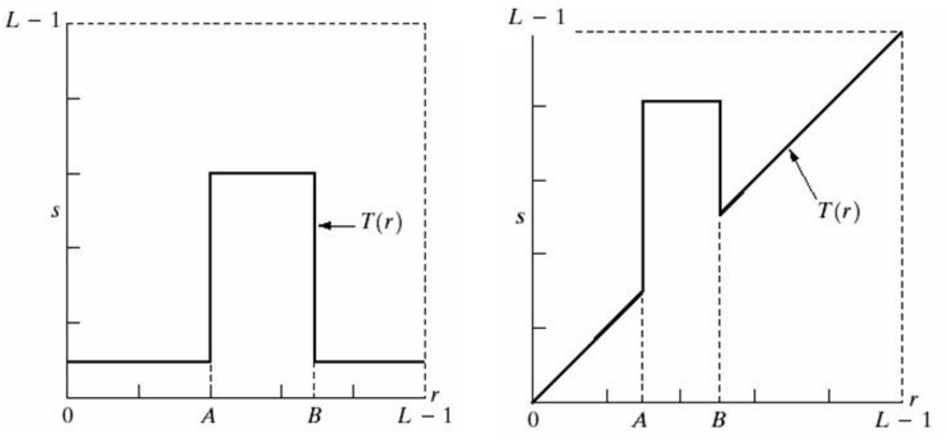


Figure 5.5 (a) Low-contrast image. (b) Histogram of (a). (c) High-contrast image resulted from applying full contrast-stretching in Figure 5.4 on (a). (d) Histogram of (c)

Gray-level slicing

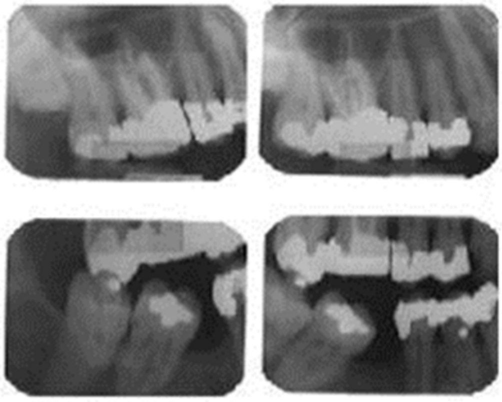
Gray-level slicing aims to highlight a specific range [A…B] of gray levels. It simply maps all gray levels in the chosen range to a high value. Other gray levels are either mapped to a low value (Figure 5.6(a)) or left unchanged (Figure 5.6(b)). Gray-level slicing is used for enhancing features such as masses of water in satellite imagery. Thus it is useful for feature extraction.



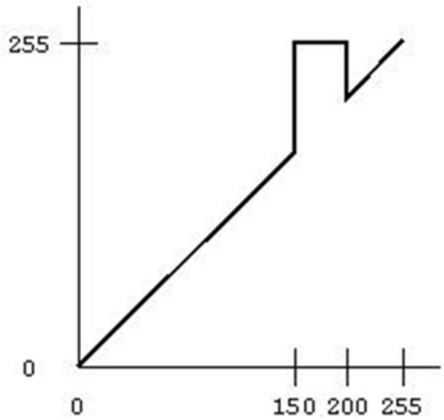
(a) (b)

Figure 5.6 Gray-level slicing

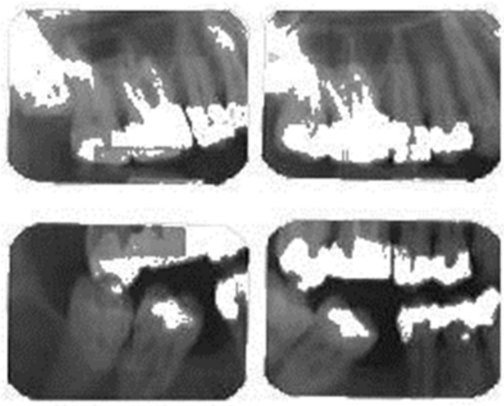
The next figure shows an example of gray level slicing:



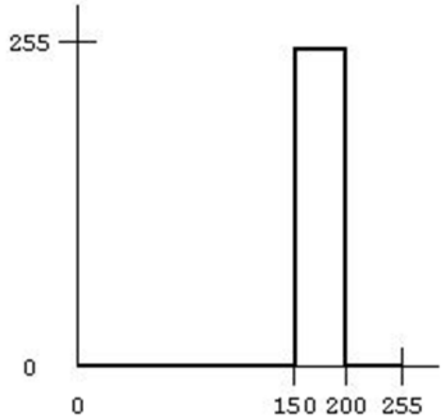
(a) Original image



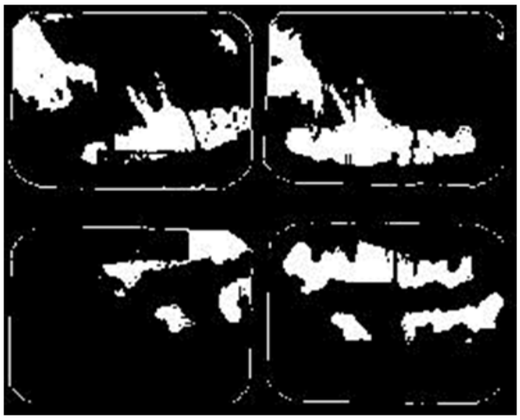
(b) Operation intensifies desired gray level range, while preserving other values



(c) Result of applying (b) on (a) (background unchanged)



(d) Operation intensifies desired gray level range, while changing other values to black



(e) Result of applying (d) on (a) (background changed to black)

Figure 5.7 Example of gray level slicing

Enhancement through Histogram Manipulation

Histogram manipulation aims to determine a gray-level transform that produces an enhanced image that has a histogram with desired properties. We study two histogram manipulation techniques namely Histogram Equalization (HE) and Histogram Matching (HM).

Histogram Equalization

is an automatic enhancement technique which produces an output (enhanced) image that has a near uniformly distributed histogram.

For continuous functions, the intensity (gray level) in an image may be viewed as a random variable with its probability density function (PDF). The PDF at a gray level *r* represents the expected proportion (likelihood) of occurrence of gray level *r* in the image. A transformation function has the form

$$s = T(r) = (L-1)\int_{0}^{r} p_r(w)\, dw$$

where w is a variable of integration. The right side of this equation is called the cumulative distribution function (CDF) of random variable r. For discrete gray level values, we deal with probabilities (histogram values) and summations instead of probability density functions and integrals. Thus, the transform will be:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{M \times N}$$

$$= \frac{(L-1)}{M \times N}\sum_{j=0}^{k} n_j \qquad k = 0,1,2,\dots ,L-1$$

The right side of this equation is known as the cumulative histogram for the input image. This transformation is called *histogram equalization* or *histogram linearization*.

Because a histogram is an approximation to a continuous PDF, perfectly flat histograms are rare in applications of histogram equalization. Thus, the histogram equalization results in a near uniform histogram. It spreads the histogram of the input image so that the gray levels of the equalized (enhanced) image span a wider range of the gray scale. The net result is contrast enhancement.

Example:

Suppose that a 3-bit image (L = 8) of size 64 × 64 pixels has the gray level (intensity) distribution shown in the table below.

r_k	n_k
$r_0 = 0$	790
$r_1 = 1$	1023
$r_2 = 2$	850
$r_3 = 3$	656
$r_4 = 4$	329
$r_5 = 5$	245
$r_6 = 6$	122
$r_7 = 7$	81

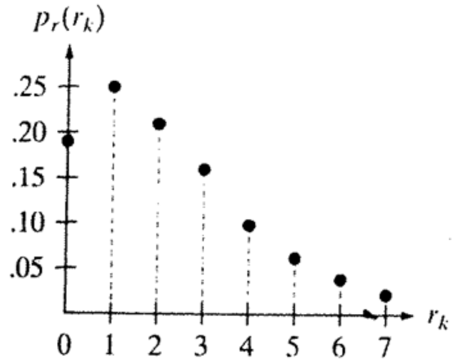
Perform histogram equalization on this image, and draw its normalized histogram, transformation function, and the histogram of the equalized image.

Solution:

$M \times N = 4096$

We compute the normalized histogram:

r_k	n_k	$p_r(r_k) = n_k / MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Normalized histogram

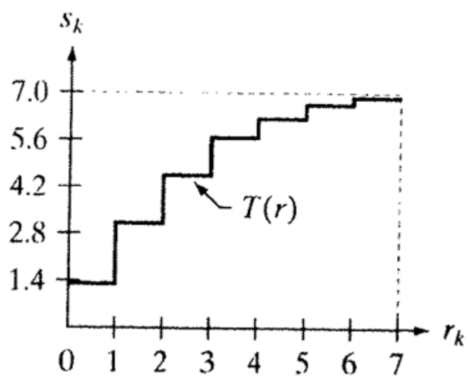
Then we find the transformation function:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and $s_2 = 4.55, s_3 = 5.67, s_4 = 6.23, s_5 = 6.65, s_6 = 6.86, s_7 = 7.00$



Transformation function

We round the values of s to the nearest integer:

$s_0 = 1.33 \rightarrow 1$ $s_1 = 3.08 \rightarrow 3$ $s_2 = 4.55 \rightarrow 5$

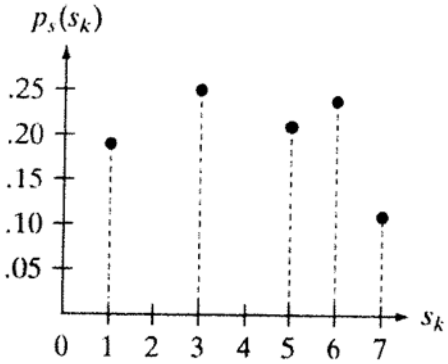
$s_3 = 5.67 \rightarrow 6$ $s_4 = 6.23 \rightarrow 6$ $s_5 = 6.65 \rightarrow 7$

$s_6 = 6.86 \rightarrow 7$ $s_7 = 7.00 \rightarrow 7$

These are the values of the equalized histogram. Note that there are only five gray levels.

r_k	n_k	s_k	$New\ n_k$	$p_s(s_k) = New\ n_k/MN$
$r_0 = 0$	790	$s_0 = 1$	790	0.19
$r_1 = 1$	1023	$s_1 = 3$	1023	0.25
$r_2 = 2$	850	$s_2 = 5$	850	0.21
$r_3 = 3$	656	$s_3 = 6$	985	0.24
$r_4 = 4$	329	$s_4 = 6$		
$r_5 = 5$	245	$s_5 = 7$	448	0.11
$r_6 = 6$	122	$s_6 = 7$		
$r_7 = 7$	81	$s_7 = 7$		

Thus, the histogram of the equalized image can be drawn as follows:



Histogram of equalized image

The next figure shows the results of performing histogram equalization on dark, light, low-contrast, and high-contrast gray images.

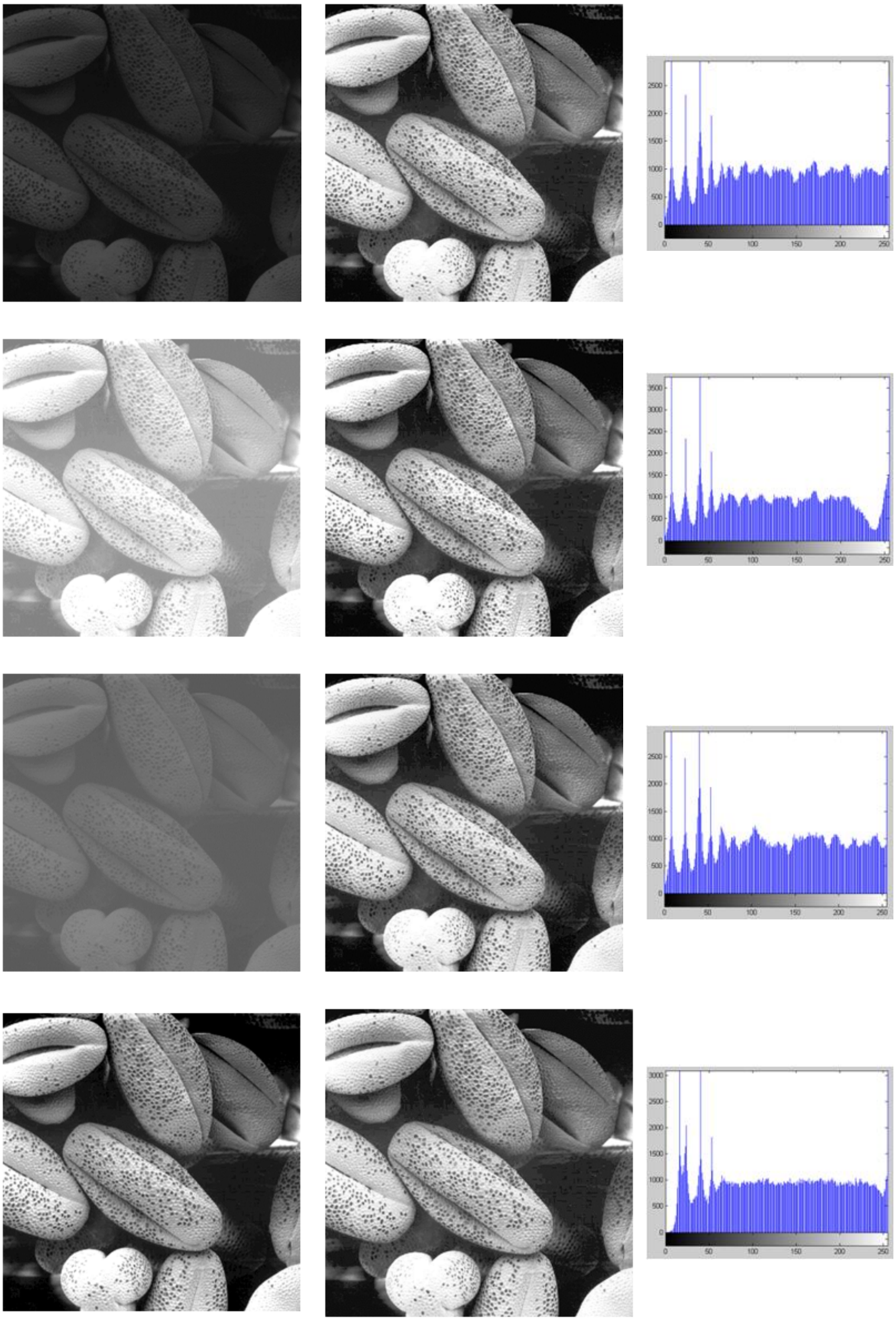
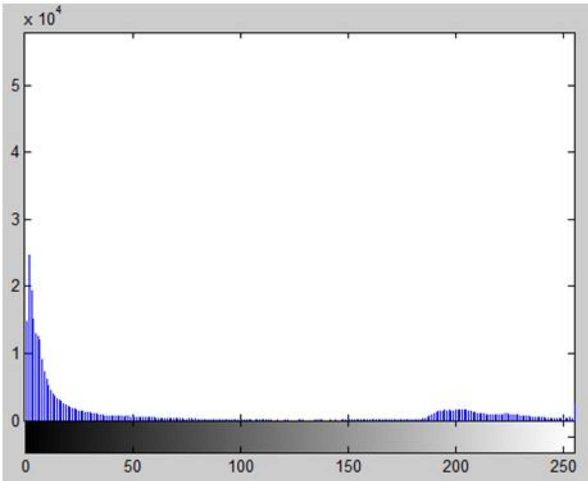


Figure 5.8 Left column original images. Center column corresponding histogram equalized images. Right column histograms of the images in the center column.

Although all the histograms of the equalized images are different, these images themselves are visually very similar. This is because the difference between the original images is simply one of contrast, not of content.

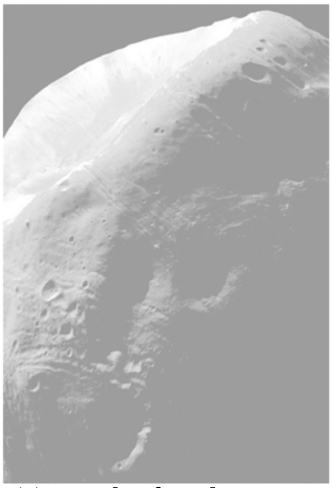
However, in some cases histogram equalization does not lead to a successful result as shown below.

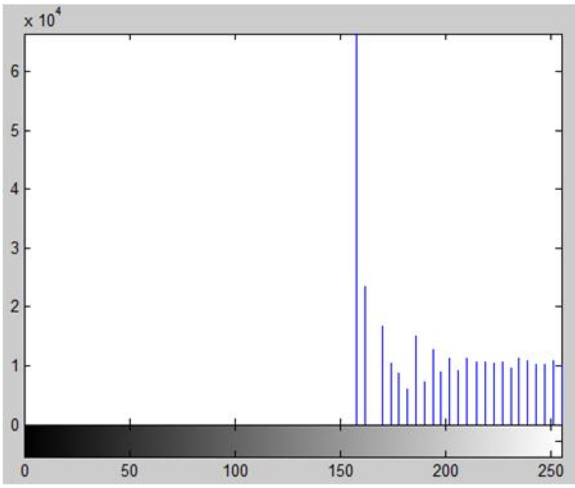


(a) Original image (b) Histogram of (a)

Figure 5.9 Image of Mars moon and its histogram

The result of performing histogram equalization on the above image is shown in the figure below.





(a) Result of applying HE (b) Histogram of (a)

Figure 5.10 Result of applying HE on Figure 5.9 (a)

We clearly see that histogram equalization did not produce a good result in this case. We see that the intensity levels have been shifted to the upper one-half of the gray scale, thus giving the image a washed-out appearance. The cause of the shift is the large concentration of dark components at or near 0 in the original histogram. In turn, the cumulative transformation function obtained from this histogram is steep, as shown in the figure below, thus mapping the large concentration of pixels in the low end of the gray scale to the high end of the scale.

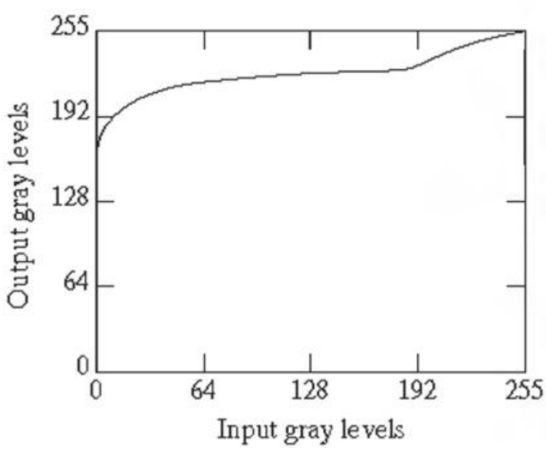


Figure 5.11 HE transformation function of Figure 5.10(a)

In other cases, HE may introduce noise and other undesired effect to the output images as shown in the figure below.

(a) Original image

(b) Result of applying HE on (a)

(c) Original image (d) Result of applying HE on (c)

Figure 5.12 Undesired effects caused by HE

These undesired effects are a consequence of digitization. When digitize the continuous operations, rounding leads to approximations.
From the previous examples, we conclude that the effect of HE differs from one image to another depending on global and local variation in the brightness and in the dynamic range.

Histogram Matching (Specification)

is another histogram manipulation process which is used to generate a processed image that has a specified histogram. In other words, it enables us to specify the shape of the histogram that we wish the processed image to have. It aims to transform an image so that its histogram nearly matches that of another given image. It involves the sequential application of a HE transform of the input image followed by the inverse of a HE transform of the given image.

The procedure of Histogram Specification is as follows:

1. Compute the histogram $p_r(r)$ of the input image, and use it to find the histogram equalization transformation using

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{M \times N}$$

$$= \frac{(L-1)}{M \times N}\sum_{j=0}^{k} n_j \qquad k = 0,1,2,\dots,L-1$$

Then round the resulting values, s_k , to the integer range [0, L-1].

2. Compute the specified histogram $p_z(z)$ of the given image, and use it find the transformation function G using

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) \qquad q = 0,1,2,\dots,L-1$$

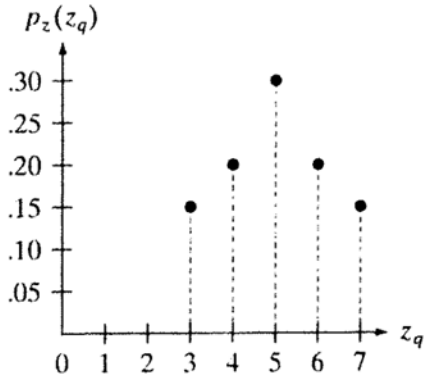
Then round the values of G to integers in the range [0, L-1]. Store the values of G in a table.

3. Perform inverse mapping. For every value of s_k , use the stored values of G from step 2 to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k and store these mappings from s to z. When more than one value of z_q satisfies the given s_k (i.e. the mapping is not unique), choose the smallest value.
4. Form the output image by first histogram-equalizing the input image and then mapping every equalized pixel value, s_k , of this image to the corresponding value z_q in the histogram-specified image using the inverse mappings in step 3.

Example:

Suppose the 3-bit image of size 64 × 64 pixels with the gray level distribution shown in the table, and the specified histogram below.

r_k	n_k
$r_0 = 0$	790
$r_1 = 1$	1023
$r_2 = 2$	850
$r_3 = 3$	656
$r_4 = 4$	329
$r_5 = 5$	245
$r_6 = 6$	122
$r_7 = 7$	81



Perform histogram specification on the image, and draw its normalized histogram, specified transformation function, and the histogram of the output image.

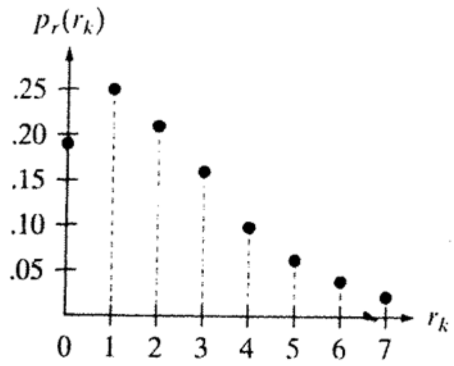
Solution:

Step 1:

$M \times N = 4096$

We compute the normalized histogram:

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02



Normalized histogram

Then we find the histogram-equalized values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and $s_2 = 4.55, s_3 = 5.67, s_4 = 6.23, s_5 = 6.65, s_6 = 6.86, s_7 = 7.00$

We round the values of s to the nearest integer:

$s_0 = 1.33 \rightarrow 1$ $s_1 = 3.08 \rightarrow 3$ $s_2 = 4.55 \rightarrow 5$

$s_3 = 5.67 \rightarrow 6$ $s_4 = 6.23 \rightarrow 6$ $s_5 = 6.65 \rightarrow 7$

$s_6 = 6.86 \rightarrow 7$ $s_7 = 7.00 \rightarrow 7$

Step 2:

We compute the values of the transformation G

$$G(z_0) = 7\sum_{i=0}^{0} p_z(z_i) = 0$$

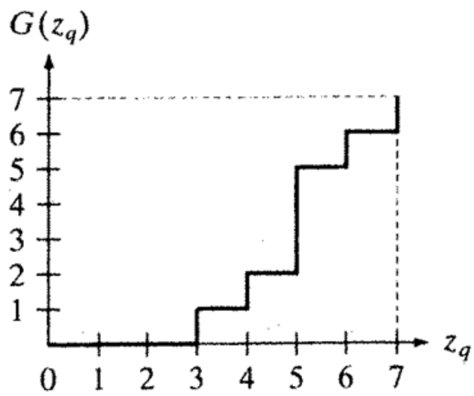
$$G(z_1) = 7\sum_{i=0}^{1} p_z(z_i) = 7[p(z_0) + p(z_1)] = 0$$

and $G(z_2) = 0$, $G(z_3) = 1.05$, $G(z_4) = 2.45$

$G(z_5) = 4.55$, $G(z_6) = 5.95$, $G(z_7) = 7.00$

We round the values of G to the nearest integer:

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7



Transformation function obtained from the specified histogram

Step 3:

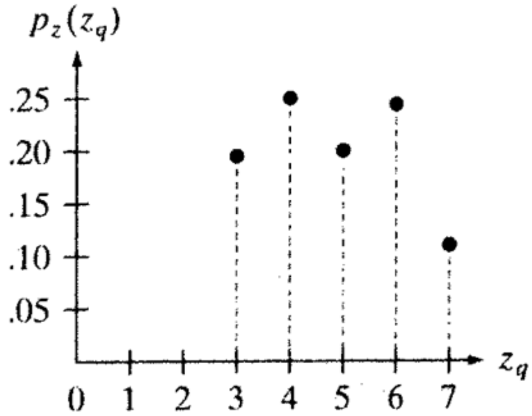
We find the corresponding value of z_q so that the value $G(z_q)$ is the closest to s_k .

s_k	$\longrightarrow$	z_q
1	$\longrightarrow$	3
3	$\longrightarrow$	4
5	$\longrightarrow$	5
6	$\longrightarrow$	6
7	$\longrightarrow$	7

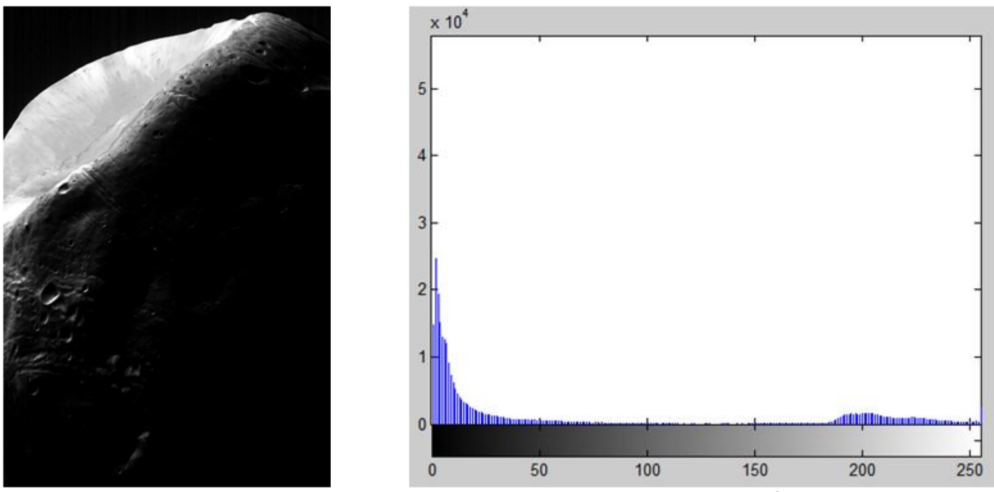
Step 4:

r_k	n_k	s_k	$New\ n_k$	$p_s(s_k) = New\ n_k/MN$
$r_0 = 0$	790	$s_0 = 3$	790	0.19
$r_1 = 1$	1023	$s_1 = 4$	1023	0.25
$r_2 = 2$	850	$s_2 = 5$	850	0.21
$r_3 = 3$	656	$s_3 = 6$	985	0.24
$r_4 = 4$	329	$s_4 = 6$		
$r_5 = 5$	245	$s_5 = 7$	448	0.11
$r_6 = 6$	122	$s_6 = 7$		
$r_7 = 7$	81	$s_7 = 7$		

Thus, the histogram of the output image can be drawn as follows:



To see an example of histogram specification we consider again the image below.



(a) Original image (b) Histogram of (a)

Figure 5.13 Image of Mars moon and its histogram

We use the following specified histogram shown in the figure below to perform histogram specification on the image in the previous figure.

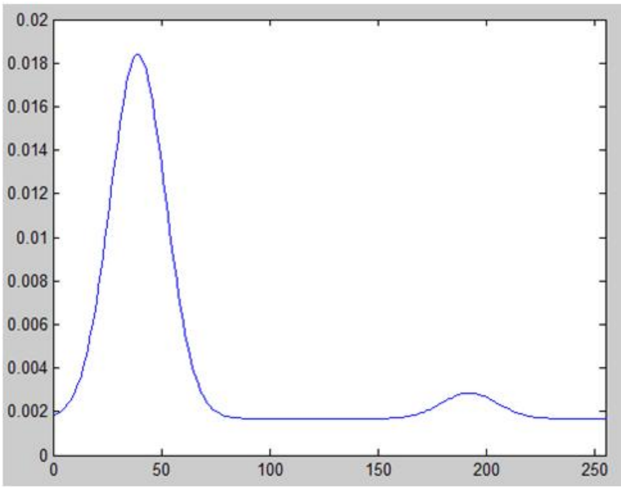
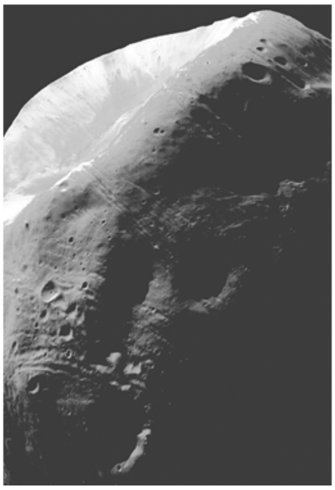
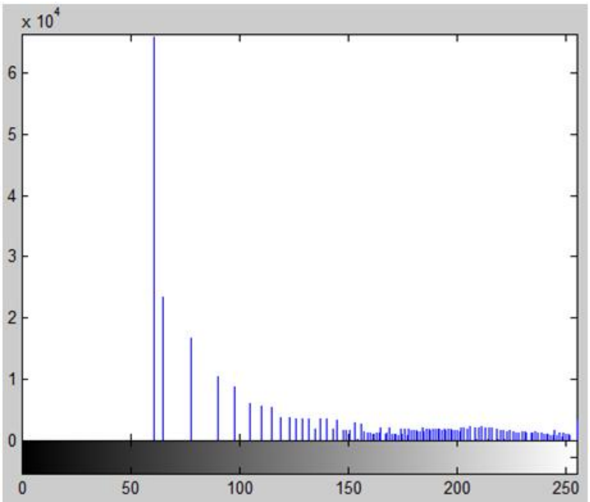


Figure 5.14 Specified histogram for image in Figure 5.13(a)

The output image of histogram specification is shown below.

(a) Result of applying HS (b) Histogram of (a)
Figure 5.15 Result of applying HS on Figure 5.13 (a)