F-Test: If $\underset{s_1, s_2}{\text{S.D's}}$ are Given, then :- $S_1 = \sqrt{\frac{n_1 s_1^2}{n_1 - 1}}$, $S_2 = \sqrt{\frac{n_2 s_2^2}{n_2 - 1}}$

If S.Ds are not Given then :- $S_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}}$, $S_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2 - 1}}$

Model No 5.7: F-Test: Variances

This test is also called as variance ratio test. The objective of this test is to determine whether two independent estimates of the population variance differ significantly or whether the two samples may be regarded as drawn from the normal populations having the same variance, i.e., $\sigma_X^2 = \sigma_Y^2 = \sigma^2$. To carry out this test, we find the ratio F given by

$F = \frac{S_X^2}{S_Y^2}$ where $S_X^2 = \frac{\sum_{i=1}^{n_1}(x_i - \bar{x})^2}{n_1 - 1}$ and $S_Y^2 = \frac{\sum_{i=1}^{n_2}(y_j - \bar{y})^2}{n_2 - 1}$ and the test follows F-distribution with $\gamma_1 = n_1 - 1$ and $\gamma_2 = n_2 - 1$ degrees of freedom. It is to be noted that the numerator is greater than variance.

SNEDECOR'S **F-TEST OF SIGNIFICANCE**

(i) **Null Hypothesis** (H_0): $\sigma_1^2 = \sigma_2^2$ or $s_1^2 = s_2^2$ i.e., the variances of the two populations are same.

(ii)**Alternative Hypothesis** (H_1): $\sigma_1^2 \neq \sigma_2^2$

(iii) **Level of Significance** (α): set a lelvel of significance

(iv)**Test Statistic:** The test statistic

$$\boxed{F = \frac{\text{larger variance}}{\text{smaller variance}}}, \text{where } s_1^2 = \frac{\sum(x - \bar{x})^2}{n_1 - 1}, s_2^2 = \frac{\sum(y - \bar{y})^2}{n_2 - 1}$$

(v) **Conclusion:** Degrees of freedom = $(n, m) = (n_1 - 1, n_2 - 1)$

(i) If Calculated value of F < Tabulated value of F, we accept H_0

(ii) If Calculated value of F > Tabulated value of F, we reject H_0

Problem 14: The time taken by the workers in performing a job by method I and method II is given below:

Method I	**20**	**16**	**26**	**27**	**23**	**22**	**--**
Method II	**27**	**33**	**42**	**35**	**32**	**34**	**38**

Do the data show that the variances of time distribution from population from which these samples are drawn do not differ significantly?

Solution: The Problem Belongs to the case Of Test On Variances. So It belongs to $\boxed{\text{F-Test.}}$

Here S.D's are Not Given, So $S_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}}$ $S_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2 - 1}}$

$S_1 = 4.0331$ $S_2 = 4.7207$

$S_1^2 = 16.2650$ $S_2^2 = 22.2857$

* F-Test : Imp

In this Model, we are always Consider 'α' only.

For Single Tailed Test or two Tailed Test

*** Formulae:

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}}$$

Case-1: $\frac{S_1^2}{S_2^2}$ If $[S_1^2 > S_2^2]$

Degrees of Freedom : $(n_1 - 1, n_2 - 1)$

Case-2: $\frac{S_2^2}{S_1^2}$ If $[S_2^2 > S_1^2]$

Degrees of Freedom : $(n_2 - 1, n_1 - 1)$

(i) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$

(ii)Alternative Hypothesis (H_1): $\sigma_1^2 \neq \sigma_2^2$ [Two Tailed Test]

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $F = \dfrac{\text{Greater Variance}}{\text{Smaller Variance}} = \dfrac{S_2^2}{S_1^2} = \dfrac{22.2857}{16.2650} = 1.3701$

(v) Conclusion: Degrees of freedom = $\nu(n_2-1, n_1-1) = \nu(6,5) = 4.95$ [Table-5]

Tabulated value of F = $F_{tab} = F_{0.05}(5,6) = 4.95$

Calculated value of F = $F_{cal} = 1.3701$

Calculated value of $F <$ Tabulated value of F

Null Hypothesis is Accepted.

Problem 15: The measurements of the output of two units have given the following results. Assuming that both samples have been obtained from the normal population at 10% significant level, test whether the two populations have the same variance.

Unit-A	14.1	10.1	14.7	13.7	14.0
Unit-B	14.0	14.5	13.7	12.7	14.1

Solution:

Given $n_1 = 5$, $n_2 = 5$

$S_1^2 = \dfrac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1} = 3.372$, $S_2^2 = \dfrac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1} = 0.46$

(i) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$

(ii)Alternative Hypothesis (H_1): $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance (α): $\alpha = 0.01$

(iv) Test Statistic: The test statistic $F = \dfrac{\text{Greater Variance}}{\text{Smaller Variance}} = \dfrac{S_1^2}{S_2^2} = \dfrac{3.372}{0.46} = 7.3304$

(v) Conclusion: Degrees of freedom = $\nu(n_1-1, n_2-1) = \nu(4,4)$

Tabulated value of F = $F_{tab} = F_{0.01}(4,4) = 15.98$

Calculated value of F = $F_{cal} = 7.3304$

Calculated value of $F <$ Tabulated value of F

Null Hypothesis is Accepted

$\Sigma(x_i-\bar{x}_1)^2$ & $\Sigma(x_i-\bar{x}_2)^2$

Problem 16: In two independent samples of sizes 8 and 10 the sum of squares of deviations of the sample values from the respective means were 84.4 and 102.6. Test whether the difference of variances of the population is significant or not. Use a 0.05 level of significance.

$n_1=8,\ n_2=10,\ \Sigma(x_i-\bar{x}_1)^2=84.4,\ \Sigma(x_i-\bar{x}_2)^2=102.6$

Solution:

$$S_1^2=\frac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1}=\frac{84.4}{8-1}=12.0571 \qquad S_2^2=\frac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1}=\frac{102.6}{10-1}=11.4$$

(i) Null Hypothesis (H_0): $\sigma_1^2=\sigma_2^2$

(ii)Alternative Hypothesis (H_1): $\sigma_1^2\neq\sigma_2^2$

(iii) Level of Significance (α): $\alpha=0.05$

(iv) Test Statistic: The test statistic $F=\dfrac{\text{Greater Variance}}{\text{Smaller variance}}=\dfrac{12.0571}{11.4}=\dfrac{S_1^2}{S_2^2}=1.0576$

(v) Conclusion: Degrees of freedom = $\gamma(n_1-1, n_2-1)=\gamma(7,9)$

Tabulated value of F = 3.29

Calculated value of F = 1.0576

Calculated value of F < Tabulated value of F

Null Hypothesis is Accepted.

Problem 18: In one sample of 10 observations from a normal population, the sum of the squares of the deviations of the sample values from the sample mean is 102.4 and in another sample of 12 observations from another normal population, the sum of the squares of the deviations of the sample values from the sample mean is 120.5. Examine whether the two normal populations have the same variance.

Solution: Given $\Sigma(x_i-\bar{x}_1)^2 = 102.4$, $\Sigma(x_i-\bar{x}_2)^2 = 120.5$, $n_1 = 10$, $n_2 = 12$

$$S_1 = \sqrt{\frac{\Sigma(x_i-\bar{x}_1)^2}{n_1-1}} = \sqrt{\frac{102.4}{10-1}} = 3.373 \qquad S_2 = \sqrt{\frac{\Sigma(x_i-\bar{x}_2)^2}{n_2-1}} = \sqrt{\frac{120.5}{12-1}} = 3.309$$

$$S_1^2 = 11.3771 \qquad S_2^2 = 10.9494$$

(i) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$

(ii) Alternative Hypothesis (H_1): $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $F = \dfrac{\text{Greater Variance}}{\text{Smaller Variance}} = \dfrac{S_1^2}{S_2^2} = \dfrac{11.3771}{10.9494} = 1.0390$

(v) Conclusion: Degrees of freedom = $\gamma(n_1-1, n_2-1) = \gamma(9, 11)$

Tabulated value of F = $F_{tab} = F_{0.05}(9,11) = 2.90$

Calculated value of F = $F_{cal} = 1.0390$

Calculated value of F < Tabulated value of F

Null Hypothesis is Accepted.

Problem 19: Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins, show the sample standard deviations of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal, test hypothesis that the true variances are equal.

$n_1 = 11,\ n_2 = 9,\ s_1 = 0.8,\ s_2 = 0.5$

$$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.8)^2}{11-1} = 0.704$$

$$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9-1} = 0.2812$$

Solution:

(i) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$

(ii) Alternative Hypothesis (H_1): $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic $F = \dfrac{\text{Greater Variance}}{\text{Smaller variance}} = \dfrac{S_1^2}{S_2^2} = \dfrac{0.704}{0.2812} = 2.5032$

(v) Conclusion: Degrees of freedom = $\gamma(n_1-1, n_2-1) = \gamma(10, 8)$

Tabulated value of F = $F_{tab} = F_{0.05}(10,8) = 3.35$

Calculated value of F = $F_{cal} = 2.5032$

Calculated value of F $<$ Tabulated value of F

Null Hypothesis is Accepted

Problem 20: The nicotine contents in milligrams of two samples of tobacco were found to be as follows. Test whether there is a significant difference between the two samples. or The samples are drawn from same Population

Sample A	24	27	26	21	25	--
Sample B	27	30	28	31	22	36

In this case we can do both T-Test & F-test for small samples only.

Solution: t-Test: $n_1 = 5,\ n_2 = 6,\ \bar{x}_1 = 24.6,\ \bar{x}_2 = 29$

$$S_1^2 = \frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1} \Rightarrow \Sigma(x_i - \bar{x}_1)^2 = S_1^2(n_1 - 1) = (5.3)(5-1) = 21.2$$

$S_1^2 = 5.3$

$S_2^2 = 21.6$

$S_1 =$

$$S_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}} = 2.3021$$ ~~$S_1^2 = 5.2997$~~

$$S_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2 - 1}} = 4.6475$$ ~~$S_2^2 = 21.5943$~~

$$S = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2 + \Sigma(x_i - \bar{x}_2)^2}{n_1 + n_2 - 2}} = \qquad = 4.1645$$

(i) Null Hypothesis (H_0): $\mu_1 =$

(ii) Alternative Hypothesis (H_1):

(iii) Level of Significance (α):

(iv) Test Statistic: The test statistic

(v) **Conclusion:** Degrees of freedom=

Tabulated value of t_α =

Calculated value of $|t_\alpha|$ =

Calculated value of $|t_\alpha|$ Tabulated value of t_α

F-Test: $S_1^2 = 5.2997$ $S_2^2 = 21.5943$

Hence, In Both the cases Null Hypothesis is Accepted, So the Samples are drawn from Same Population. So there is No significant differente.

(i) Null Hypothesis (H_0): $\sigma_1^2 = \sigma_2^2$

(ii)Alternative Hypothesis (H_1): $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of Significance (α): $\alpha = 0.05$

(iv) Test Statistic: The test statistic

$$F = \frac{\text{Greater Variance}}{\text{Smaller Variance}} = \frac{S_2^2}{S_1^2} = \frac{21.5943}{5.2997} = 4.0756$$

(v) Conclusion: Degrees of freedom = $\gamma(n_2-1, n_1-1) = \gamma(5,4) = \gamma(5,4)$.

Tabulated value of F = $F_{tab} = F_{0.05}(4,5) = \cancel{5.19}\ 6.26$

Calculated value of F = $F_{cal} = 4.0756$

Calculated value of $F \angle$ Tabulated value of F

Null Hypothesis is Accepted.

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