Model No 5.2: Test of significance for difference of means:

(i) **Null Hypothesis** (H_0): $\bar{x}_1 = \bar{x}_2$ or $\mu_1 = \mu_2$ i.e., "there is no significance difference between means of the populations" or "the two samples have been drawn from the same population"

(ii) **Alternative Hypothesis** (H_1): $\bar{x}_1 \neq \bar{x}_2$ or $\mu_1 \neq \mu_2$

(iii) **Level of Significance** (α): Set a level of significance

(iv) **Test Statistic:**

Case(i): (a) When the S.D. of the populations σ_1 , σ_2 are given then the test statistic

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

b) When the samples are taken from the same population then the test statistic $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}}$

Case(ii): When the S.D. of the population is not known then the test statistic $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$

(v) **Conclusion:** (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0 i.e., we accept the Alternative Hypothesis H_1

Problem 8: The average marks scored by 32 boys is 72 with a S.D. of 8. While that for 36 girls is 70 with a S.D. of 6. Does this indicate that the boys perform better than girls at level of significance 0.05?

Solution: Given that $n_1 = 32, \bar{x}_1 = 72, \sigma_1 = 8 \rightarrow$ Boys

$n_2 = 36, \bar{x}_2 \; 70, \sigma_2 = 6 \rightarrow$ Girls

The Two Samples can be drawn from the Same Population.

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2 \Rightarrow$

(ii) Alternative Hypothesis (H_1): $\mu_1 > \mu_2$ [Better than Girls] Right tailed test

(iii) Level of Significance (α): $\alpha = 0.05 \Rightarrow 0.5 - \alpha = 0.5 - 0.05 = 0.45$ $\boxed{Z_{tab} = 1.645}$

(iv) Test Statistic: The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{72 - 70}{\sqrt{\frac{64}{32} + \frac{36}{36}}} = \boxed{Z_{cal} = 1.1547}$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.645$
Calculated value of $Z_{cal} = 1.1547$
Calculated value of < Tabulated value of

$\boxed{Z_{cal} < Z_{tab}}$ The Null Hypothesis is Accepted

Problem 9: Two types of new cars produced in U.S.A. are tested for petrol mileage, one sample is consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variance as $\sigma_1^2 = 2.0$ *and* $\sigma_2^2 = 1.5$ respectively. Test whether there is any significance difference in the petrol consumption of these two types of cars. (use level of significance 0.01)

Solution: $n_1 = 42, \; \bar{x}_1 = 15,$ | $n_2 = 80, \; \bar{x}_2 = 11.5$

$\sigma_1^2 = 2.0 \quad \sigma_1 = \sqrt{2},$ | $\sigma_2^2 = 1.5 \quad \sigma_2 = \sqrt{1.5}$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii) Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$ Two Tailed test

(iii) Level of Significance (α): $\alpha = 0.01 \Rightarrow 0.5 - \frac{\alpha}{2} = 0.5 - \frac{0.01}{2} = 0.495$ $\boxed{Z_{tab} = 2.58}$

(iv) Test Statistic: The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{15 - 11.5}{\sqrt{\frac{2.0}{42} + \frac{1.5}{80}}} = \boxed{Z_{tab} = 13.585}$$

(v) Conclusion: Tabulated value of $Z_{tab} = 0.2580 = 2.58$
Calculated value of $Z_{cal} = 13.585$
Calculated value of Tabulated value of

$\boxed{Z_{cal} > Z_{tab}}$ Here Null Hypothesis is Rejected.

Problem 10: The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D. 2.5 inches. Use 5% L.O.S

Solution: Given $n_1 = 1000$ $\bar{x}_1 = 67.5$ $n_2 = 2000$ $\bar{x}_2 = 68.0$ } Common S.D $\sigma = 2.5$

(i) Null Hypothesis (H_0): $\mu_1 = \mu_2$

(ii) Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$ Two Tailed Test $\boxed{Z_{tab} = 1.96}$

(iii) Level of Significance (α): $\alpha = 5\% = 0.05 \Rightarrow 0.5 - \alpha/2 = 0.5 - 0.05 = 0.475 =$

(iv) Test Statistic: The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} = \frac{67.5 - 68.0}{\sqrt{\frac{2.5^2}{1000} + \frac{2.5^2}{2000}}} = \boxed{-5.1639 = Z_{cal}}$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$
Calculated value of $Z_{cal} = -5.1639$
Calculated value of Tabulated value of

$\boxed{|Z_{cal}| > Z_{tab}}$ Null Hypothesis is Rejected

Problem 11: Samples of students were drawn from two universities and from their weights in kilograms, mean and S.D. are calculated and shown below. Make a large sample test to test the significance of the difference between the means.

	Mean	S.D.	Size of the sample
University A	55	10	400
University B	57	15	100

Solution: $n_1 = 400$ $S_1 = 10$ $\bar{x}_1 = 55$

$n_2 = 100$ $S_2 = 15$ $\bar{x}_2 = 57$

(i) **Null Hypothesis** (H_0): $\mu_1 = \mu_2$

(ii) **Alternative Hypothesis** (H_1): $\mu_1 \neq \mu_2$ Two Tailed Test $\boxed{Z_{tab} = 1.96}$

(iii) **Level of Significance** (α): $\alpha = 5\% = 0.05 \Rightarrow 0.5 - \alpha/2 = 0.475$

(iv) **Test Statistic:** The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{55-57}{\sqrt{\frac{10^2}{400} + \frac{15^2}{100}}} = -1.2649$$

(v) **Conclusion:** Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = -1.2649$

Calculated value of Tabulated value of

$\boxed{|Z_{cal}| < Z_{tab}}$ Null Hypothesis is Accepted.

Problem 12: The mean yield of wheat from a district A was 210 pounds with S.D 10 pounds per acre from a sample of 100 pounds. In another district the mean yield was 200 pounds with S.D 12 pounds from a sample of 150 plots. Assuming that the S.D of yield in the entire was 11 pounds, test whether there is any significant difference between the mean yield of crops in the two districts.

Solution: Given $\bar{x}_1 = 210$ $n_1 = 100$ $\bar{x}_2 = 200$ $n_2 = 150$ } common SD $\sigma = 11$

(i) **Null Hypothesis** (H_0): $\mu_1 = \mu_2$

(ii) **Alternative Hypothesis** (H_1): $\mu_1 \neq \mu_2$ Two Tailed Test

(iii) **Level of Significance** (α): $\alpha = 0.05 \Rightarrow 0.5 - \alpha/2 = 0.475$ $\boxed{Z_{tab} = 1.96}$

(iv) **Test Statistic:** The test statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}} = \frac{210-200}{\sqrt{\frac{11^2}{100} + \frac{11^2}{150}}} = 7.0417$$

(v) **Conclusion:** Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 7.0417$

Calculated value of Tabulated value of

$\boxed{Z_{cal} > Z_{tab}}$ Null Hypothesis is Rejected

Problem 13: In a survey of buying habits, 400 women shoppers are chosen at random in super market 'A' located in a certain section of the city. Their average weekly food expenditure is Rs250 with a S.D of Rs40. For 400 women shoppers chosen at random in super market 'B' in another section of the city, the average weekly food expenditure is Rs220 with a S.D of Rs55.

Test at 1% level of significance whether the average weekly food expenditure of the two populations of shoppers are equal.

Solution: $n_1 = 400$ $\bar{x}_1 = 250$ $s_1 = 40$

$n_2 = 400$ $\bar{x}_2 = 220$ $s_2 = 55$

(i) **Null Hypothesis** (H_0): $\mu_1 = \mu_2$

(ii) **Alternative Hypothesis** (H_1): $\mu_1 \neq \mu_2$ Two Tailed Test

(iii) **Level of Significance** (α): $\alpha = 1\% = 0.01 \Rightarrow 0.5 - \alpha/2 = 0.5 - \frac{0.01}{2} = 0.495$

$\boxed{Z_{tab} = 2.58}$

(iv) **Test Statistic:** The test statistic

$$Z_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{250-220}{\sqrt{\frac{40^2}{400} + \frac{55^2}{400}}} = 8.8225$$

(v) **Conclusion:** Tabulated value of $Z_{tab} = 2.58$, $Z_{cal} = 8.8225$

Calculated value of

Calculated value of Tabulated value of

$\boxed{Z_{cal} > Z_{tab}}$ Null Hypothesis is Rejected.