Image Enhancement in the Frequency Domain

- The frequency content of an image refers to the rate at which the gray levels change in the image.
- Rapidly changing brightness values correspond to high frequency terms, slowly changing brightness values correspond to low frequency terms.
- The Fourier transform is a mathematical tool that analyses a signal (e.g. images) into its spectral components depending on its wavelength (i.e. frequency content).

2D Discrete Fourier Transform

The DFT of a digitized function *f(x,y)* (i.e. an image) is defined as:

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\left[\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right) - j\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)\right]$$

The domain of ***u*** and ***v*** values ***u*** = 0, 1, ..., *M*-1, ***v*** = 0,1,…, *N*-1 is called the frequency domain of *f(x,y)*.

$$R(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)$$ is called real part

$$I(u,v) = \frac{-1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)$$ is called imaginary part

The magnitude of *F(u,v)*, $|F(u,v)|=[R^2(u,v)+I^2(u,v)]^{1/2}$, is called the **Fourier spectrum** of the transform.

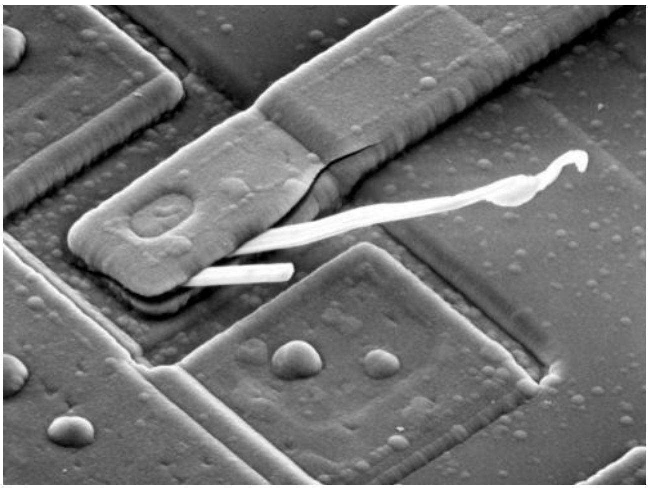
The **phase angle (phase spectrum)** of the transform is:

$$\emptyset(u,v) = tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

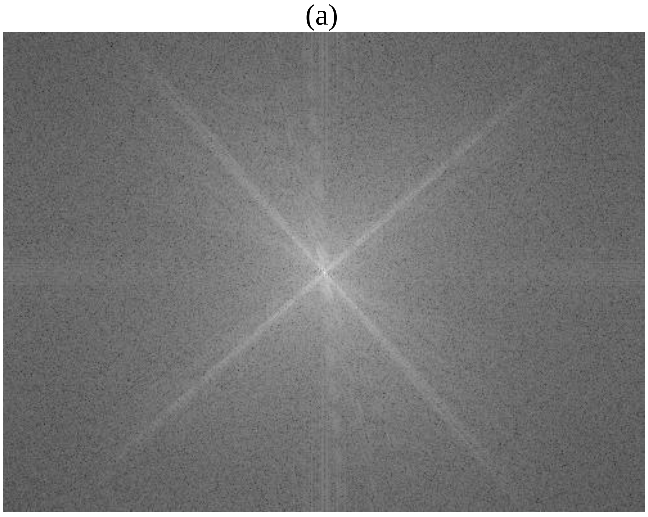
Note that, *F(0,0)* = the average value of *f(x,y)* and is referred to as the *dc component* of the spectrum.

It is a common practice to multiply the image f(x,y) by $(-1)^{x+y}$. In this case, the DFT of $(f(x,y)(-1)^{x+y})$ has its origin located at the centre of the image, i.e. at (u,v) = (*M*/2,*N*/2).

The figure below shows a gray image and its centered Fourier spectrum.



(a)



(b)

Figure 7.1 (a) Gray image. (b) Centered Fourier spectrum of (a)

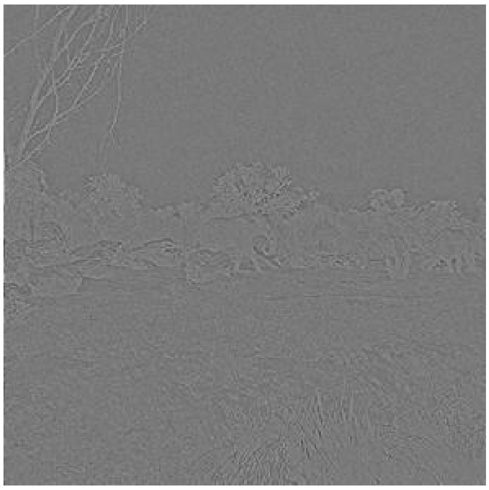
The original image contains two principal features: edges run approximately at ±45°.

The Fourier spectrum shows prominent components in the same directions.

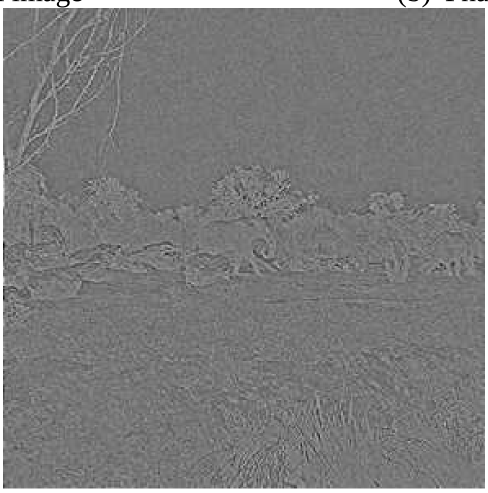
Phase spectrum

Phase data contains information about where objects are in the image, i.e. it holds spatial information as shown in the Figure below.

(a) Original image



(b) Phase only image



(c) Contrast enhanced version of image (b) to show detail

Figure 7.2 Phase spectrum

Fourier transform does not provide simultaneously frequency as well as spatial information.

Inverse 2D-DFT

After performing the Fourier transform, if we want to convert the image from the frequency domain back to the original spatial domain, we apply the *inverse transform*. The inverse *2D*-DFT is defined as:

$$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\left[\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right) + j\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)\right]$$

where $x = 0,1,\ldots, M\text{-}1$ and $y = 0,1,\ldots, N\text{-}1$.

Frequency domain vs. Spatial domain

	Frequency domain	Spatial domain
1.	is resulted from Fourier transform	is resulted from sampling and quantization
2.	refers to the space defined by values of the Fourier transform and its frequency variables (*u*,*v*).	refers to the image plane itself, i.e. the total number of pixels composing an image, each has spatial coordinates (*x*,*y*)
3.	has complex quantities	has integer quantities

Filtering in the Frequency Domain

Filtering in the frequency domain aims to enhance an image through modifying its DFT. Thus, there is a need for an appropriate filter function H(u,v).

The filtering of an image *f(x,y)* works in 4 steps:

1. Compute the centered DFT, $\mathrm{F(u,v)} = \Im((-1)^{\mathrm{x+y}}\mathrm{f(x,y)})$
2. Compute $\mathrm{G(u,v)} = \mathrm{F(u,v)H(u,v)}$.
3. Compute the inverse DFT of G(u,v), $\Im^{-1}(\mathrm{G(u,v)})$.
4. Obtain the real part of $\Im^{-1}(\mathrm{G(u,v)})$.
5. Compute the filtered image $\mathrm{g(x,y)} = (-1)^{\mathrm{x+y}}\mathrm{R}(\Im^{-1}(\mathrm{G(u,v)}))$.

Generally, the inverse DFT is a complex-valued function. However, when *f(x,y)* is real then the imaginary part of the inverse DFT vanishes. Thus, for images step 4, above, does not apply.

The figure below illustrates the filtering in the frequency domain.

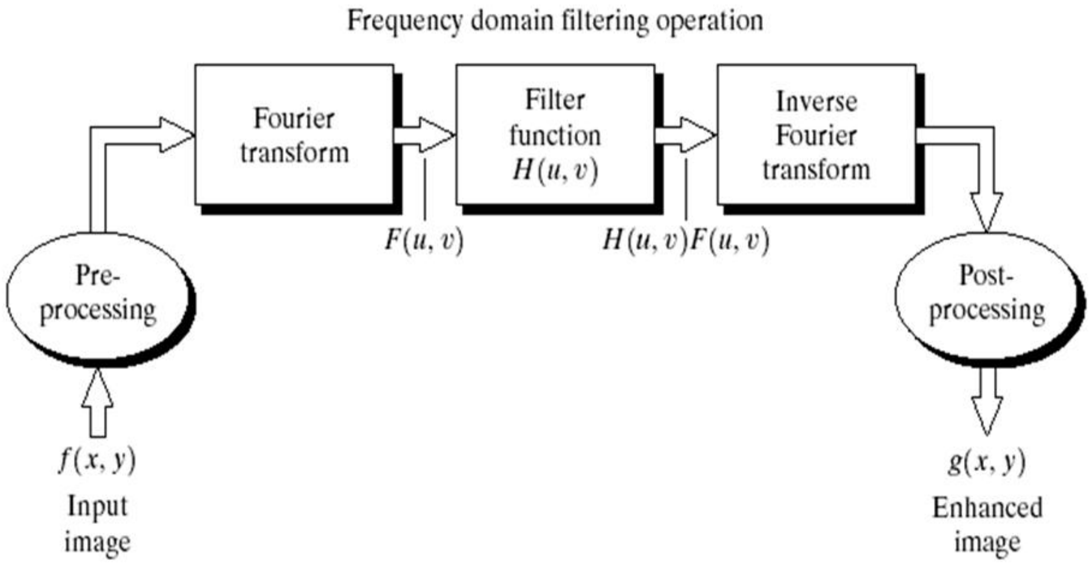


Figure 7.3 Basic steps for filtering in the frequency domain

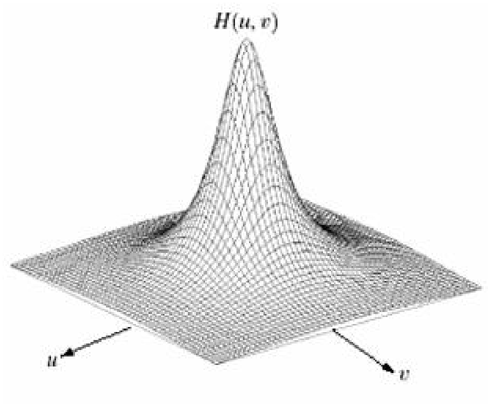
Low-pass and High-pass filtering

- Low frequencies in the DFT spectrum correspond to image values over smooth areas, while high frequencies correspond to detailed features such as edges & noise.
- A filter that suppresses high frequencies but allows low ones is called **Low-pass filter**, while a filter that reduces low frequencies and allows high ones is called **High-pass filter**.
- Examples of such filters are obtained from circular Gaussian functions of 2 variables:

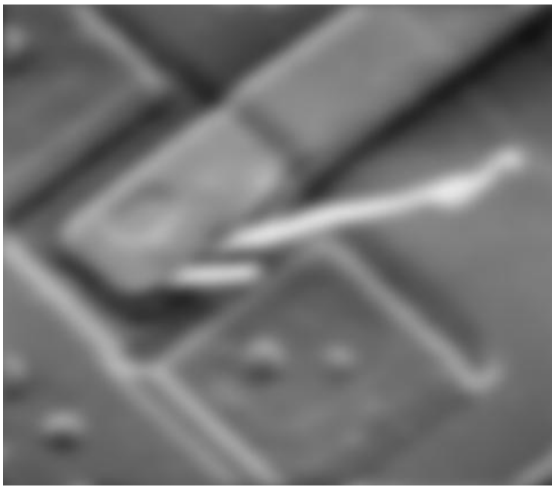
$$H(u,v) = \frac{1}{2\pi\sigma^2} e^{-(u^2+v^2)/2\sigma^2} \quad \text{Low-pass filter}$$

$$H(u,v) = \frac{1}{2\pi\sigma^2} \left(1 - e^{-(u^2+v^2)/2\sigma^2}\right) \quad \text{High-pass filter}$$

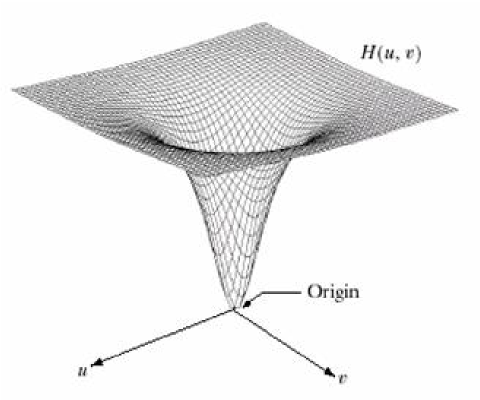
The results of applying these two filters on the image in Figure 6.1(a) are shown in the figure below.



(a) Low-pass filter function



(b) Result of lowpass filtering



(c) Highpass filter function

(d) Result of highpass filtering

Figure 7.4 Low-pass and High-pass Filtering

Low-pass filtering results in blurring effects, while High-pass filtering results in sharper edges.

In the last example, the highpass filtered image has little smooth gray-level detail as a result of setting F(0,0) to 0. This can be improved by adding a constant to the filter, for example we add 0.75 to the previous highpass filter to obtain the following sharp image.

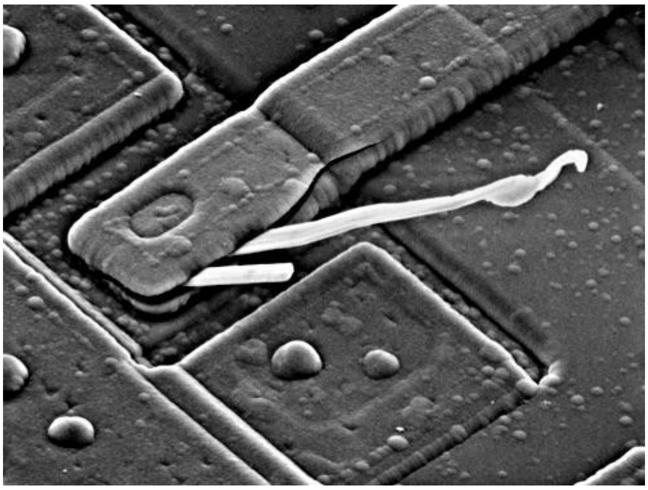


Figure 7.5 Result of highpass filter modified by adding 0.75 to the filter