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Time - distance - speed.

- When Time is constant then distance is directly proporti onal to speed. i.e.) $d \propto S$.

- When speed is constant then distance is directly proportional to Time i.e, $d \propto T$.

- Combining these two relations we have

$$d \propto S \times T \Rightarrow \frac{d_1}{d_2} = \frac{S_1 T_1}{S_2 T_2}.$$

- Distance = Speed × Time ($d = S \times T$).

- If The speed and Time both increases $x\%$ and $y\%$ respectively then distance travelled is increased by $\left(x + y + \frac{xy}{100}\right)\%$.

$$D \propto S \times T \rightarrow y\% \uparrow$$
$$S \downarrow x\% \uparrow$$
$$D \downarrow (x+y+xy/100)\% \uparrow$$

- If speed and time both decreases by $x\%$ and $y\%$ respectively then distance traveled is decreased by $-\left(x + y - \frac{xy}{100}\right)\%$.

$$D = S \times T \rightarrow y\% \downarrow$$
$$S \downarrow x\% \downarrow$$
$$D \rightarrow (x+y-xy/100)\% \downarrow$$

- If speed increases by $x\%$ and time decreases by $y\%$ then distance travelled is changed by $\left(x - y - \frac{xy}{100}\right)\%$

$$D = S \times T \rightarrow y\% \downarrow$$
$$S \downarrow x\% \uparrow$$
$$D \downarrow (x-y-xy/100)\%$$

(i) If $x - y - \frac{xy}{100}$ is positive then the distance travelled is increased.

(ii) If $x-y-xy/100$ is negative then distance travelled decreases and $x-y-xy/100$ is zero then there is no change in distance travelled.

- If speed decreases x% and Time increases y% then the distance travelled is changed by $\left(-x+y-\frac{xy}{100}\right)$%.

(i) If $-x+y-xy/100$ is positive, the distance travelled increases.

(ii) If it is negative the distance travelled decreases

(iii) If it is zero then there is no change in distance.

- If Speed and time both increases 30% and 40% respectively. What is the % increased by distance.

A:- $D = S \times T \rightarrow 40\%$

$\downarrow$

$30\% \uparrow$

$\Rightarrow \left(x+y+\frac{xy}{100}\right) \Rightarrow 30+40+\frac{30 \times 40}{100}$

$\Rightarrow 70+12, = 82\%$.

Distance is increased by 82%.

If speed and time both decreases by 30% and 40% respectively. Then what is the % decreased in distance travelled.

A:- ~~xx~~ $D = S \times T \rightarrow 40\%$

$S \downarrow 30 \downarrow$

$$\Rightarrow \left(x + y - \frac{xy}{100}\right) = 30 + 40 - \frac{30 \times 40}{100}$$

$$= 70 - 12$$

$$= 58$$

The distance is decreased by 58%.

3 If speed increases by 30% and time decreases by 20% then what is the % changed in distance.

$D = S \times T \rightarrow 20\% \downarrow$

$S \downarrow 30\% \uparrow$

$$\left(x - y - \frac{xy}{100}\right)\% = \left(30 - 20 - \frac{60}{100}\right)$$

$$= 10 - 6 = 4\%$$

Distance is increased by 4%.

4. If speed increases by 30% and time decreases by 40% then what is the % change in distance.

A:- $D = S \times T \rightarrow 40\% \downarrow$

$S \downarrow 30\% \uparrow$

$$\left(x - y - \frac{xy}{100}\right)\% = \left(30 - 40 - \frac{30 \times 40}{100}\right)$$

$$= (-10 - 12)$$

$$= -22\%$$

Distance is decreased by 22%.

If speed increases by 25% time decreases by 20%. then what is the % change in distance?

A:- $D = S \times T \longrightarrow 20\% \downarrow$.

S: $25\% \uparrow$

$$= \left(x - y - \frac{xy}{100}\right)\% = 25 - 20 - \frac{25 \times 20}{100}$$

$$= 5 - 5$$

$$= 0.$$

There is no change in distance.

If speed decreases by 25% and time increases by y%. So that the distance travelled is decreased by 6.25%.

$D = S \times T \longrightarrow$

D: $6.25 \downarrow$, S: $25 \downarrow$

$$-6.25 = \left(-25 + y + \frac{-25y}{100}\right)$$

$$18.75 = y + \frac{-25y}{100}.$$

$$18.75 \times 100 = 100y - 25y.$$

$$75y = 1875.$$

$$\boxed{y = 25}$$

Time increases by 25%.

$$\begin{array}{r} 25.00 \\ -\ 6.25 \\ \hline 18.75 \end{array}$$

$$\begin{array}{r} 75 \\ \times 5 \\ \hline 375 \end{array} \quad \begin{array}{r} 75 \\ \times 25 \\ \hline 375 \\ 165 \times \\ \hline 2015 \end{array} \quad \begin{array}{r} 25 \\ \times 15 \\ \hline 375 \\ 75 \times \\ \hline 1125 \end{array}$$

Speed = Distance / Time (units - km/h, m/sec).

$1 \text{ kmph} = \frac{5}{18} \text{ m/sec}$.

$1 \text{ m/sec} = \frac{18}{5} \text{ kmph}$.

- To convert p from kmph to m/sec, we have to multiply with $\frac{5}{18}$.

- To convert p from m/sec to kmph, we have to multiply with $\frac{18}{5}$.

Time = Distance / speed

- If 2 persons travelled with speed x kmph and y kmph then the ratio of times taken by them to cover the same distance is.

$$\frac{T_1}{T_2} = \frac{D_1}{S_1} \times \frac{S_2}{D_2}.$$

$$\frac{T_1}{T_2} = \frac{\frac{D}{x}}{\frac{D}{y}} \Rightarrow \frac{T_1}{T_2} = \frac{\cancel{D}}{x} \times \frac{y}{\cancel{D}}$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = \frac{y}{x}}$$

Average Speed :-

Average speed is the ratio of total distance traveled to that of total time taken.

Note :-

If a person traveled two equal distances with different speeds x kmph and y kmph respectively.

Then his average speed during total journey is:

A: T D' = 2a.

$T T = \frac{a}{x} + \frac{a}{y}$

$\frac{a}{x} \Big| \frac{a}{y}$

$T = \frac{a}{x}$ $T = \frac{a}{y}$

$$\text{Avg speed} = \frac{2\cancel{a}}{\cancel{a}\left[\frac{1}{x}+\frac{1}{y}\right]} = \frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2xy}{x+y}$$

If a person travels three equal distances with 3 different speeds x kmph, y kmph, z kmph. Then his average speed during the journey is.

$$\text{Avg speed} = \left(\frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}\right) \text{ kmph.}$$

If a person travels n equal distances with different speeds. Then his average speed during the journey is

~~Av~~ $$\text{Avg speed} = \left(\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}\right) \text{ kmph}$$

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Relative Speed:-

If Comparison of speed of one person with respect to another is called Relative speed.

Note:-

- Suppose, 2 persons travelled with different speeds x kmph and y kmph respectively then the relative speed of 1st person w.r.t 2nd person if they travel in opposite direction is $(x+y)$ kmph.

Similarly, the relative speed of 2nd person w.r.t 1st person is also $(x+y)$ kmph.

- Suppose 2 persons are traveled with different speeds x kmph and y kmph respectively in the same direction. Then the relative speed of 1st person w.r.t. 2nd is $(x-y)$ kmph.

  Similarly relative speed of 2nd person w.r.t 1st person is $(y-x)$ kmph.

Points to be remember to solve the problems based on trains.

- If a train (passes) crosses a stationary object (pole, tree, signal poles or standing man) then it travels a distance of length of the train.

- If a train (passes) crosses a stationary object having some length (a bridge, a tunnel, platform, standing train) then the train travels a distance of sum of length of the train and length of the object which it crosses.

- In case of moving objects also, we take distances are same as above but speeds are consider as relative speeds.

- If 2 trains of length x meters and y meters respectively are moving in opposite direction ~~to~~ with speeds u m/s and v m/s then the time taken by them to cross each other is $(x+y/u+v)$ sec.

- If 2 trains of lengths x m & y m respectively are moving in same direction with speeds u m/s and v m/s respectively then the time taken by them so that the fast train crosses slower train is $\frac{(x+y)}{|u-v|}$ sec.

and slower train never crosses the faster train.

problems:-

1. A man on tour travels first 160 km at 90 kmph. and the next 160 km he travelled at 54 kmph. Then average speed of man on a total of 320 km tour is.

A:- Here the man travels two equal distances (each 160 kms) travelled with different speeds.

$x = 90$ kmph $\quad y = 54$ kmph. then

Average speed during total journey is

$$\frac{2xy}{x+y} = \frac{2(90)(54)}{90+54} = \frac{2(90)(54)}{144} = \frac{135}{2}$$

$= 67.5$ kmph.

2. A man covers half of his journey by train at 60 kmph, half of the remaining by cycle at 30 kmph and remaining by walk at 10 kmph. then his average speed during a total journey is.

A:- Total distance = x (say)

$\frac{1}{2}x \rightarrow$ train. $\quad \frac{1}{4} \rightarrow$ cycle $\quad \frac{1}{4} \rightarrow$ walk.

Average speed for same distance is.

$$\frac{2xy}{x+y} = \frac{2(30)(10)}{30+10} = \frac{2(3\cancel{0})(\cancel{10}^{5})}{\cancel{4}\cancel{0}_{2}} = 15 \text{ kmph.}$$

Average speed for a total journey is.

$$\frac{2xy}{x+y} = \frac{2(15)(60)}{15+60} = \frac{2(\cancel{15})(\cancel{60}^{12})}{\cancel{75}_{5}} = 24 \text{ kmph.}$$

3. A man walking at the rate of 5 kmph crosses a bridge in 15min. The length of the bridge in mts.

A:- Speed = 5 kmph = $\frac{5}{18} \times 5 = \frac{25}{18}$ m/s

$T = 15$ min. $= 15 \times 60 = 900$ sec.

Length of the bridge = d.

$d = S \times T = \cancel{5} \times \frac{5}{\cancel{18}} \times \cancel{60} \times \cancel{15}$.

$\boxed{d = 1250 \text{ m.}}$

4. The distance of the college and home of Rajesh is 80 km. One day he was late by 1 hr. than normal time, so he increased his speed by 4 kmph. and thus reached to clg at normal time. what is the increased speed of Rajesh.

A:- Let the normal speed of Rajesh is x kmph & increase speed is equals to $x+4$ kmph.

Distance travelled in both cases = 80 km.

$T_1 = \frac{d}{Ns} = \frac{80}{x}$ · Ns $\rightarrow$ Normal speed

Is $\rightarrow$ Increased speed.

$T_2 = \frac{d}{Is} = \frac{80}{x+4}$.

difference of time = 1hr.

$T_1 - T_2 = 1$.

$\frac{80}{x} - \frac{80}{x+4} = 1$.

$80x + 320 - 80x = x(x+4)$.

$x(x+4) = 320.$

$x(x+4) = 16(16+4).$

N.S $\Rightarrow x = 16.$

T.S $\Rightarrow x+4 = 20$ kmph.

5. This distance b/w 2 cities A & B is 330 km. A train starts from A at 8 Am and travels towards B at 160 kmph. another train starts from B at 9 AM. and travels towards A at 75 kmph. At what time do they meet each other.

a) 10 AM b) 10:30 AM c) 11:00 AM D) 11:30 AM.

The two trains meet each other x hrs after 8 AM

$d_1 = S_1T_1$ where $S_1 = 16$ kmp $T_1 = x$ hrs.

$d_1 = 160x.$

$d_2 = S_2T_2$ where $S_2 = 75$ kmp $t_2 = x-1$ hrs.

$d_2 = 75(x-1).$

Given that

$d_1 + d_2 = 320.$

$160x + 75x - 75 = 320.$

$\cancel{91x = 320+75}$

$135x = 405.$

$x = 3$ hrs.

Req time = 8:00 + 3:00 = 11:00 AM.

$$\begin{array}{r} 330 \\ +75 \\ \hline 405 \end{array}$$

$$\begin{array}{r} 75 \\ 16 \\ \hline 91 \end{array}$$

$$\begin{array}{r} 320 \\ 75 \\ \hline 395 \end{array}$$

135
×11
135
135×
1485

575 m long

6. A train crosses a tunnel of long 325 m in 90 sec. what is the speed of the train in kmph.

A: Here the distance travelled $(d) = 575 + 325$

$= 900$ m.

$T = 90$ sec.

~~Average~~ speed $= \frac{d}{T} = \frac{900}{90} = 10$ m/sec.

$= 10 \times \frac{18}{5} = 36$ kmph.

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A person covers 11m in 3 sec. Then How many kms he can travel in 1hr 40 min if he travelled at the same rate.

Sol: Ther relation b/w distance and time when speed is constant is

$$\frac{d_1}{T_1} = \frac{d_2}{T_2}.$$

Given

$d_1 = 11m = \frac{11}{1000}$ km. $T_1 = 3sec = \frac{3}{3600}$ hrs.

$d_2 = x$ km. $T_2 = 1$ hr $+ \frac{40}{60}$ hr.

$= 1 + \frac{2}{3} = \frac{5}{3}$ hr.

$$\frac{\frac{11}{1000}}{\frac{3}{3600}} = \frac{x}{\frac{5}{3}}$$

$$\frac{11 \times \cancel{36}}{\cancel{3} \times \cancel{10}} = \frac{\cancel{3}x}{\cancel{5}}$$

$\boxed{x = 22 \text{ km}}$

The ratio of speeds of A and B is 3:7. If B takes 20 min less time than A to cover a certain distance then what is the time taken by A to cover that distance?

Sol: Here the relation b/w speed and time when distance is constant is $S_1 T_1 = S_2 T_2$.

Given that $S_1 : S_2 = 3:7$ $\Rightarrow S_1 = 3k$ $S_2 = 7k$.

let $T_1 = T$ mins $\Rightarrow T_2 = T - 20$ min.

$S_1T_1 = S_2T_2$.

$\Rightarrow 3\cancel{k}(T) = 7\cancel{k}(T-20)$

$3T = 7T - 140$.

$4T = 140$.

$\boxed{T = 35 \text{ min.}}$

By travelling at $\frac{3}{5}$th of his usual speed, A man is late by 20 min to cover a certain distance. What is the usual time to cover the same distance by travelling at his usual speed.

sol: The relation b/w speed and time when distance is constant is $S_1T_1 = S_2T_2$.

Given that.

Let $S_1 = S$ $S_2 = \frac{3}{5}S$. $T_1 = T$ min $T_2 = (T+20)$ min.

$S_1T_1 = S_2T_2$.

$\cancel{S}(T) = \frac{3}{5}(\cancel{S})(T+20)$

$T = \frac{3}{5}T + \frac{3 \times \cancel{20}^{4}}{\cancel{5}}$

$T - \frac{3}{5}T = 12$.

$\frac{5T - 3T}{5} = 12$.

$\frac{\cancel{2}T}{5} = \cancel{12}^{6}$.

$\boxed{T = 30 \text{ min}}$

If a person takes 20 min more time than his usual time by decreasing a speed by 20%. what is the usual time taken by the person to cover the same distance?

Sol: The relation b/w speed and Time when distance is constant is

$$S_1T_1 = S_2T_2.$$

Let Usual speed $S_1 = S$. $S_2 = S - \frac{20}{100_5}S = \frac{4S}{5}$.

$T_1 = T$ min $T_2 = T+20$ min.

$$\cancel{S}T = \frac{4\cancel{S}}{5}(T+20)$$

$$5T = 4T + 80.$$

$\boxed{T = 80 \text{ min.}}$ $\Rightarrow T = \frac{80}{60}$ hr $= \frac{4}{3}$ hr.

$$\boxed{T = 1\frac{1}{3} \text{ hr}}$$

Travelling from his house at 30 kmph a person is late to his office by 5 min. If he increased his speed by 30 kmph, he would be early by 15 min to his office. what should be his speed so that he reaches his office on time?

Sol: The relation b/w speed and Time when distance is constant is.

$$S_1T_1 = S_2T_2 = S_3T_3.$$

Let S_1 = Usual speed = S. $S_2 = 30$ kmph.

T_1 = Usual Time = T $T_2 = T+5$ min.

$S_3 = 40$ kmph. $T_3 = T-15$ min.

$$ST = 30(T+5) = 40(T-15)$$

$$ST = 30T + 150 = 40T - 600.$$

$\cancel{5}\cancel{0}\cdot 10T = \cdot 750\cdot$

$$\boxed{T = 75\cdot}$$

$$sT = 30(T+5) = 40(T-15).$$

$$s(75) = 30(75+5)$$

$$s(75) = 30(80).$$

$$s = \frac{2400}{75}\cdot$$

$$\boxed{s = 32\text{ kmph.}}$$

A train covered x km at 45 kmph and another x km at 36 kmph. what is the average speed of the train in covering the entire journey.?

sol: A train covers two equal distances (each x km) at speed $a = 45$ kmph $b = 36$ kmph.

$$\text{Average speed} = \frac{2ab}{a+b} = \frac{2(45)(36)}{45+36.} = \frac{2(\cancel{45}^{5})(\cancel{36}^{4})}{\cancel{81}\cdot\cancel{9}}$$

$= 40$ kmph.

A person covered a certain distance in 11 hrs. He covered ~~fo~~ first half of the distance at a speed of ~~1~~30kmph and remaining half at 25 kmph. Find the distance travelled by the man in 11 hrs.

sol: Let the distance $= d$

Given the time $= 11$ hr.

Average speed $= \frac{d}{11}$ kmph.

But he travelled this distance by taking two equal halfs, first half at ~~130~~ 30 kmph and second half at 25 kmph.

$$\text{Average speed} = \frac{2ab}{a+b} = \frac{2(30)(25)}{30+25} = \frac{2(30)(\cancel{25}^{5})}{\cancel{55}\,11}$$

$$= \frac{300}{11}\cdot$$

$\frac{d}{11} = \frac{300}{11}$ $\Rightarrow$ $\boxed{d = 300\,km}$

Varma covers 22.5 kms in 3hr 20mins. He covered 10 km at 8 kmph at what uniform speed did he cover the remaining distance?

Sol: Given that total distance = 22.5 kms.

Total time = 3hr 20mins. $= 3 + \frac{\cancel{2}0}{\cancel{3}60} = 3+\frac{1}{3} = \frac{10}{3}$ hr

Distance traveled $d_1 = 10$ km.

$S_1 = 8$ kmph.

$T_1 = \frac{10}{8}$ hr $= \frac{5}{4}$ hr.

Remaining distance = $22.5 - 10 = 12.5$ km.

Remaining Time = $\frac{10}{3} - \frac{5}{4} = \frac{40-15}{12} = \frac{25}{12}$.

Average speed for remaining distance.

$= \frac{\text{Remaining distance}}{\text{Remaining Time}}$

$= \frac{12.5}{\frac{25}{12}} = \frac{\frac{125}{10}}{\frac{25}{12}} = \frac{\cancel{125}}{\cancel{10}} \times \frac{\cancel{12}}{\cancel{25}}$ 6.

= 6 kmph.

Murali travelled from city A to city B at a speed of 40 kmph from city B to city C ~~at~~ 60 kmph. What is the average speed of murali from A to C. Given that the ratio of distances b/w A to B & B to C is 2:3.

Sol: Given that the ratio of distances = 2:3.

i.e., $d_1 : d_2 = 2:3$.

$d_1 = 2x$, $d_2 = 3x$.

Total distance $= 5x$.

Given that $S_1 = 40$ kmph $\quad S_2 = 60$ kmph.

$\Rightarrow T_1 = \frac{2x}{40}$ hr $\quad T_2 = \frac{3x}{60}$ hr.

$T_1 = \frac{x}{20}$ hr $\quad T_2 = \frac{x}{20}$ hr.

Total time $= T_1 + T_2 = \frac{x}{20} + \frac{x}{20} = \frac{2x}{20} = \frac{x}{10}$ hr.

Average speed $= \frac{\text{Total distance}}{\text{Total Time}} = \frac{5\cancel{x}}{\frac{\cancel{x}}{10}} = 50$ kmph.

Travelling at a speed of 8 kmph A student reaches school 10 min early. If he travels at 6 kmph. he is late by 20 min. Find the distance b/w school & house.

sol: Let.

$S_1 = S \quad T_1 = T$.

$S_2 = 8$ kmph $\quad T_2 = T - 10$.

$S_3 = 6$ kmph $\quad T_3 = T + 20$.

Given that:

Difference of Time intervals is 30 min $= \frac{1}{2}$ hr.

Let the distance b/w school and house is d km.

Difference of time $= \frac{d}{6} - \frac{d}{8} = \frac{1}{2}$ hr.

$\Rightarrow \frac{4d - 3d}{24} = \frac{1}{2}$.

$\Rightarrow \frac{d}{24} = \frac{1}{2}$.

$\Rightarrow d = \frac{24}{2}$.

$\Rightarrow \boxed{d = 12 \text{ km}}$

29/4/23.

Circular tracks:-

i) Two persons are running on a circular track of length c meters from the same point at the same time in opposite directions. Then the first meeting time of the two persons at any point on the circular track is

$\frac{c}{x+y}$ sec.

ii) Two persons are running on a circular track of length c meters from the same point at the same time in same direction then the first meeting time of the two persons at any point on the circular track is $\frac{c}{|x-y|}$ sec.

iii) If two persons are running on a circular track of length c meters from the same point at the same time then the first meeting time of the two person at starting point if they travel in any direction (opp/same) is $LCM\left(\frac{c}{x}, \frac{c}{y}\right)$

Boats and streams.

i) If a boat travels in the same direction of water flow then speed of boat increases in this case we say that boat travels in downstream (or) with tide

ii) If the boat travels in the opposite direction of water flow then speed of boat decreases in this case we say that boat travels in upstream (or) againt tide.

iii) If the speed of boat in stationary water is x km/h, speed of stream is y km/h then speed of boat in downstream is $x+y$ km/h and speed of boat in upstream is $x-y$ km/h.

iv) If the speed of boat in downstream is 'a' km/h and in upstream is 'b' km/h then speed of boat in stationary water is $\frac{a+b}{2}$ km/h and speed of stream is $\frac{a-b}{2}$ km/h.

1. If two persons are running on a circular track of length 600 mts with speeds 15 m/s, 10 m/s from the same point at the same time in opposite direction. Then what is the first meeting time of the two persons at any point on the circular track.

Sol: Given

$C = 600$ mts $\quad x = 15$ m/s $\quad y = 10$ m/s.

$$\text{Time} = \frac{C}{x+y} \text{ sec} = \frac{600}{15+10} = \frac{\cancel{600}^{\cancel{120}\ 24}}{\cancel{25}_{\cancel{5}}} = 24 \text{ sec.}$$

2. In the above problem what is the first meeting time at any point on the circular track if they travel in the same direction.

$$\text{Time} = \frac{C}{|x-y|} = \frac{600}{15-10} = \frac{600}{5} = 120 \text{ sec}$$

3. If 2 persons are running on the circular track of length 720 mts with speeds 12 m/s, 18 m/s ~~first at~~ ~~meeting~~ the same point and at the same time Then what is the first meeting time of 2 persons at starting time if they travel in any direction.

A: $C = 720$. $x = 12 m/s$ $y = 18 m/s$.

LCM of $\left\{ \frac{720}{12}, \frac{720}{18} \right\}$.

$$
\begin{array}{r|l}
2 & 60, 40 \\
2 & 30, 20 \\
2 & 15, 10 \\
5 & 15, 5 \\
3 & 3, 1 \\
 & 1, 1.
\end{array}
$$

LCM $(60, 40) = 2^3 \times 5 \times 3$.

$= 8 \times 15 = 120$ sec.

$$
\begin{array}{r}
15 \\
\times 8 \\
\hline
120
\end{array}
$$

4. A boat sails 1.5 km of a river towards upstream in 5hrs. How long will it take to cover the same distance downstream with the speed of current is 1/4th speed of boat in stationary water.

sol: Let the speed of boat in stationary water is 'x' km/h.

then speed of stream is $x/4$ km/h.

then S_1 = speed of boat in upstream $= x - \frac{x}{4} = \frac{3x}{4}$.

t_1 = time in upstream = 5hrs.

S_2 = speed of boat in downstream $= x + \frac{x}{4} = \frac{5x}{4}$.

$t_2 = ?$

$S_1 t_1 = S_2 t_2$.

$\frac{3\cancel{x}}{\cancel{4}}(\cancel{5}) = \frac{\cancel{5}\cancel{x}}{\cancel{4}}(t_2)$

$\boxed{t_2 = 3 hrs}$

5. A boat moves with a speed of 4.5 km/hr in stationary water to a certain upstream point and comes back to the starting point in a river which flows at 1.5 km/hr? what is the avg speed of the boat for the entire journey?

A: Here the boat travels two equal distances with different speeds then avg speed $= \frac{2xy}{x+y}$.

where $x =$ Speed in upstream $= 4.5 - 1.5 = 3$ km/hr.

$y =$ speed in downstream $= 4.5 + 1.5 = 6$ km/hr.

$$\text{Avg speed} = \frac{2(3)(6)}{3+6} = \frac{2(18)}{9} = 4 \text{ km/h.}$$

6. A boy can swim in stationary water at 4.5 km/h. but takes twice as long to go upstream than downstream then the speed of stream is.

sol: Let the speed of stream is x km/hr.

then $s_1 =$ Speed in upstream $= 4.5 - x$.

$s_2 =$ Speed in downstream $= 4.5 + x$.

given that.

time in upstream $= 2 \times$ time in downstream.

$T =$ time in downstream.

time in upstream $= 2T$.

$$(4.5 - x)\,2\cancel{T} = (4.5 + x)\,\cancel{T}$$

$$9 - 2x = 4.5 + x.$$

$$9 - 4.5 = 3x.$$

$$3x = 4.5.$$

$$x = 1.5 \text{ km/hr.}$$

7. A boat running upstream takes 8hr 48min to cover a certain distance while it takes 4 hr to cover the same distance running downstream. What is the ratio b/w the speed of boat in stationary water and speed of stream respectively.

Let the speed of boat in stationary water is x km/h

Speed of stream is y km/hr

Let $S_1 = S_{US} = x - y$.

$S_2 = S_{DS} = x + y$.

$T_1 = T_{US} = 8 + \frac{48}{60} = \frac{44}{5}$ hrs.

$T_2 = T_{DW} = 4$ hrs.

$S_1 T_1 = S_2 T_2$.

$(x-y)\left(\frac{\cancel{44}^{11}}{5}\right) = (x+y)\cancel{4}$

$11x - 11y = 5x + 5y$.

$11x - 5\cancel{y} = 11y + 5y$.

~~6y =~~ $6x = 16y$.

$\frac{x}{y} = \frac{16}{6} = \frac{8}{3}$.

$\therefore x : y = 8 : 3$.

8. A man can go upstream at 16 km/hr and in downstream at 24 km/hr then what is the ratio of speed of stream to that speed of man in stationary water.

Sol: Given that

Speed of man in upstream b = 16 km/hr

Speed of man in downstream $a = 24$ km/h.

Speed of water (or) stream $= \frac{a-b}{2} = \frac{24-16}{2}$

$= \frac{8}{2} = 4$ km/hr.

Speed of man in stationary water $= \frac{a+b}{2} = \frac{24+16}{2}$

$= \frac{40}{2} = 20$ km/hr.

$\begin{array}{r} 24 \\ 18 \\ \hline \end{array}$

Ratio $= \frac{S_2}{S_1} = \frac{4}{20} = \frac{1}{5}$.

$S_2 : S_1 = 1 : 5$.

9. A man can row 8 km/h in stationary water if he takes thrice as long as to row in upstream as to row in downstream of the river then what is the speed of the water.

Sol. Let

speed of stream $= x$ km/hr.

speed of man $= 8$ km/hr.

$S_1 =$ Speed in upstream $= 8-x$ km/hr.

$S_2 =$ speed in downstream $= 8+x$ km/hr.

$T_1 =$ Time in upstream $= 3T$.

$T_2 = T$.

$S_1T_1 = S_2T_2$.

$(8-x)(3\cancel{T}) = (8+x)\cancel{T}$

$24 - 3x = 8 + x$.

$4x = 24 - 8$.

$4x = 16$.

$\boxed{x = 4 \text{ km/h.}}$

5/8/23.

Races and Games

Race :- The contest b/w 2 or more participants in speed of running, raiding, driving, sailing etc. is called Race.

Race course :- The place (or) ground where the races are conducted is called Race course.

Starting point :- The point where the race begins is called starting point.

Winning point (or) Goal :- The point where the race finishes or ends is called Winning point (or) Goal.

Winner :- The participant who reaches the goal first is said to be winner.

Dead Heat Race :- (or) Cancel Race :- In a race if all the participants reaches the goal at the same time then the race is called Dead Heat Race.

General Stmts involved in Races and Games and their mathematical Interpretation.

Stmt 1 :- In a race of x mts A beats B by y mts means that A reaches the goal first at that time B is y mts behind the goal. Its mathematical interpretation is while A travels 'x' mts in the same time, B travels only '$x-y$' mts.

Stmt 2 :- In a race of x mts A gave a start of y mts to B. means that both of them start the race at the same time but A start the race from the starting point while B start the race y mts ahead of starting point. The mathematical interpretation of this stmt is to win the race A has to travel x mts while B has to travel $x-y$ mts.

Stmt-3:- In a race A beats B by 't' sec means that A reaches the goal first then B reaches t sec after A reached the goal. The mathematical interpretation of this stmt is A takes t_1 sec to reach the goal and B takes 't_1+t' sec to reach the goal.

Stmt-4:- In a race A gave a start of t sec to B, means that both of them start the race from the same starting point but A start race t sec after B start the race. The mathematical interpretation of this stmt is if B takes t_1 sec to reach the goal then A takes 't_1-t' sec to reach the goal.

Stmt-5:- A game of ~~100~~ x pts means that the person who scores 'x' points first is said to be winner.

* In a game of x points A beats B by y points means that while A scores x points in the same time B scores $x-y$ points.

* In a game of x points A gave y points to B means that to win the game A has to score 'x' points while B has to score '$x-y$' points in the same time.

Note:-

* A is n times as fast B and A gave a start of x mts to B then the length of race if the race is

dead heat race is $\frac{nx}{n-1}$ mts.

* A can run x mts race in t_1 sec and B in t_2 sec where $t_1 < t_2$ then A beats B. by a distance of $\frac{x}{t_2}(t_2 - t_1)$ mts.

Problems.

1. If A is $1\frac{2}{3}$ times as fast as B and A gave a start of 80 mts to B. then how far must the winning post so that A & B reaches the goal at the same time?

Sol: Here the length of the race $= \frac{nx}{n-1}$ mts.

where $n = 1\frac{2}{3} = \frac{5}{3}$ $x = 80$ mts.

$$= \frac{\frac{5}{3}(80)}{\frac{5}{3} - 1} = \frac{\frac{5}{\cancel{3}}(80)}{\frac{2}{\cancel{3}}}$$

$$= \frac{5(\cancel{80}^{40})}{\cancel{2}} = 200 \text{ mts.}$$

2. A is $2\frac{1}{3}$ times as fast as B if A gave a start of 80 mts then how long should the race course so that the race is dead heat race?

Sol: The length of the race $= \frac{nx}{n-1}$ = mts

where $n = \frac{7}{3}$ $x = 80$ mts.

$$= \frac{\frac{7}{3}(80)}{\frac{7}{3} - 1} = \frac{\frac{7}{\cancel{3}}(\cancel{80}^{20})}{\frac{\cancel{4}}{\cancel{3}}} = 140 \text{ mts.}$$

3. A runs $1\frac{3}{8}$ times as fast as B and A gave a start of 90 mts so that both of them reaches the goal at the same time then the goal is at a distance of?

Sol: Length of the race $= \frac{nx}{n-1}$ mts

$n = \frac{24}{8}$ $x = 90.$ $= \frac{\frac{24}{8}(90)}{\frac{24}{8} - 1} = \frac{\frac{24}{\cancel{8}}(90)}{\frac{16}{\cancel{8}}} = 330$ mts.

4. In a 100 mts race A covers the distance in 35 sec and B covers in 45 sec. In this race A beats B by how many meters.

Sol: Given $x = 100$. $t_1 = 35$ $t_2 = 45$

$$\text{Length} = \frac{100}{45}(45-35) = \frac{20}{9}(10) = 20 \text{ mts}.$$

5. A can run 224mt in 28 sec and B in 32 sec. By what distance A beat B?

Sol: Given $x = 224$ $t_1 = 28$ $t_2 = 32$.

$$\frac{x}{t_2}(t_2 - t_1) = \frac{224}{32}(32-28) = \frac{\overset{28}{\cancel{224}}}{\underset{\cancel{8}}{\cancel{32}}}(\cancel{4})$$

$= 28$ mts.

6. In a race of 100mts A can give 10 mts to B and 28 mts to C. In the same race B can give how many points to C?

Sol: Given In a 100mt race A gave 10 mts to B. and 28 mts to C.

$\Rightarrow A_1 = 100$ mts $B_1 = 90$ mts $C_1 = 100 - 28 = 72$ mts.

Given in the same race B can give x points to C.

$\Rightarrow$ Then $B_2 = 100$mts $C_2 = 100 - x$ mts.

$$\therefore \frac{B_1}{C_1} = \frac{B_2}{C_2}.$$

$$\Rightarrow \frac{9\cancel{0}}{72} = \frac{10\cancel{0}}{100-x}.$$

$$900 - 9x = 720.$$

$$9x = 900 - 720.$$

$$9x = 180 \quad \boxed{x = 20 \text{mts}}$$

$$\begin{array}{r} 900 \\ 720 \\ \hline 180 \end{array}$$

7. In a game of 60 points A beat B by 15 points and C by 20 points then how many points can B beats C in a game of 90 points.

Sol: Given: In a game of 60 points A beat B by 50 pts and C by 20 pts. $\Rightarrow A_1 = 60$ $B_2 = 60-15 = 45$ $C_1 = 60-20$, $C_1 = 40$.

In a game of 90 points B can beat C by x points.

$\Rightarrow B_2 = 90$ $C_2 = 90 - x$.

$$\Rightarrow \frac{B_1}{C_1} = \frac{B_2}{C_2} = \frac{45}{40} = \frac{90}{90-x}$$

$90 - x = 80$

$\boxed{x = 10}$

8. In a race of 200 mt A can beat B by 31 mts and C by 18 mts. In a race of 350 mt C will beat B by how many mt.

Sol: Given

In a race of 200 mt A can bead B by 31 mts and C by 18 mts.

$\Rightarrow A_1 = 200$ $B_1 = 200-31 = 169$ $\boxed{C_1 = 200-18 = 182}$

In a race of 350 mt C will beat B by.

$\Rightarrow C_2 = 350$, $B_2 = 350 - x$

$$\Rightarrow \frac{C_1}{B_1} = \frac{C_2}{B_2} \Rightarrow \frac{350}{31}$$

$$\frac{182}{169} = \frac{350}{350-x}$$ (14 / 13)

$\Rightarrow 13(350) = 14(350 - x)$

$\Rightarrow 4550 = 4900 - 14x$

$\Rightarrow 14x = 4900 - 4550$

$\Rightarrow 14x = 350 \Rightarrow \boxed{x = 25}$

01/06/23.

1. A can run 22.5 mts while B run 25 mts. In one km race B beats A by how many meters?

A: Given that

A runs 22.5 mts $A_1 = 22.5$ mts

B runs 25 mts $B_1 = 25$ mts.

Given in a km race B beats A by

$A_2 = 1000 - x$ mts. $B_2 = x$ mts

$\Rightarrow \frac{A_1}{B_1} = \frac{A_2}{B_2} = \frac{22.5}{25} = \frac{1000-x}{x}$

$\begin{array}{r} 22.5 \\ 25 \\ \hline 47.5 \end{array}$

$\Rightarrow 22.5x = 25(1000) - 25x$

$\Rightarrow \frac{\cancel{47.5}x}{\cancel{9.5}\ 1.9} = \frac{\cancel{25}(1000)}{\cancel{5}}$

$\Rightarrow \boxed{x = 100.54}$

2. In a 100 mts race A can beat B by 25m and B can beat C by 4 mts. In the same race A can beat C by how many meters.

A: Given in a 100 mt race

A can beat B by 25 m $\Rightarrow A_1 = 100.$

$B_1 = 100 - 25 = 75m$

B can beat C by 4m $\Rightarrow B_2 = 100 - 4$

$C_2 = 96.$

A can beat C by x m $\Rightarrow C_1 = 100 - x$

Let $B_2 = 100.$

$\frac{B_1}{C_1} = \frac{B_2}{C_2} \Rightarrow \frac{\cancel{75}\ 3}{100-x} = \frac{\cancel{100}\ 4}{96}.$

$400 - 4x = 288.$

$96 \times 3 = 288$

$400 - 288 = 4x.$

$4x = 112.$

$400 - 288 = 112$

$\boxed{x = 28}$

3. In a 300 mts race A can beats B by 22.5 m of 6 sec. Then B time over the course is

sol: Here A beats B by 22.5 mts. of 6 sec.

$\Rightarrow d_1 = 22.5 \quad t_1 = 6 sec.$

Now $d_2 = 300 \quad t_2 = x$ sec.

We know that.

$d \propto t. \quad \frac{d_1}{t_1} = \frac{d_2}{t_2} \Rightarrow \frac{22.5}{.6} = \frac{300}{x}.$

$\Rightarrow x = \frac{\cancel{300} \times \cancel{60}}{\cancel{225}} = 80$

$\boxed{x = 80 \text{ sec}}$

B's Time over time course = 80 sec.

4. A beats B by 35m or 7 seconds in a 200 m race. Then the A's time over the course is.

sol: Here A beats B by 35 m or 7 sec.

$\Rightarrow d_1 = 35 \quad t_1 = 7.$

$\Rightarrow d_2 = 200 \quad t_2 = x.$

$d \propto t \Rightarrow \frac{d_1}{t_1} = \frac{d_2}{t_2} \Rightarrow \frac{\cancel{35}}{\cancel{7}} = \frac{\cancel{200}}{x}. \quad 40$

$\Rightarrow \boxed{x = 40 \text{ sec}}$

A's time is $40 - 7 = 33$ sec.

5- In a game of 100 points A can give 20 points to B and 28 points to c then B can give c how many points in a game of 100 points?

A:- Given

In a game of 100 points.

A can give 20 points to B $\Rightarrow A_1 = 100.$

$\Rightarrow B_1 = 100 - 20 = 80.$

A can give 28 points to c $\Rightarrow C_1 = 100 - 28 = 72.$

In a game of 100 points.

$B_2 = 100 \quad C_2 = 100 - x.$

$$\Rightarrow \frac{B_1}{C_1} = \frac{B_2}{C_2} \Rightarrow \frac{\cancel{80}}{\cancel{72}} = \frac{\cancel{100}}{100 - x}.$$

$100 - x = 90.$

$\boxed{x = 10.}$

6- In a km race A can give a start of 40m to B and 64 m to c then in a same ways how many meters start can B give c?

A:- In 1000 m A can give a start of 40m to B and 64m to c.

$A_1 = 1000 \quad B_1 = 1000 - 40 = 960 \quad C_1 = 1000 - 64. = 936.$

$B_2 = 1000 \quad C_2 = 1000 - x.$

$$\frac{\cancel{960}}{\cancel{936}} = \frac{100\cancel{0}}{1000 - x}.$$

$4(1000 - x) = 3900.$

$4000 - 4x = 3900$

$4000 - 3900 = 4x.$

$4x = 100$

$\boxed{x = 25\text{ m}}$

$1000 - 64 = 936$

$12 \times 8 = 96$

7. In a game of 90 pts.. A can give 15 points to B and C 13 points. How many pts can B give to C in a game of 100 pts?

$\begin{array}{r} \cancel{9}0 \\ 13 \\ \hline 77 \end{array}$

sol: Given.

$A_1 = 90$. $B_1 = 90 - 15$. $C_1 = 90 - 130$

$B_1 = 75$. $C_1 = 60$.

$B_2 = 100$ $C_2 = 100 - x$.

$\begin{array}{r} 77 \\ \times 4 \\ \hline 308 \end{array}$

$\frac{\overset{3}{\cancel{75}}}{\cancel{60}} = \frac{\overset{4}{\cancel{100}}}{100 - x}$.

$4(60) = 300 - 3x$.

$\cancel{308 = 300 - 3x}$.

$240 = 300 - 3x$.

$\Rightarrow 3x = 60$

$\boxed{x = 20}$

8. In a race of 600 mts. A can beat B by 60 mts and in a race of 500 mts. B can beat C by 50 mts then how many mts will A beat C in a race of 400 mts?

sol: In a race of 600 mts A beat B by 60 mts. and C by x mts.

$\Rightarrow A_1 = 600$ $B_1 = 600 - 60$. $C_1 = 600 - x$.

$= 540$.

In a race of 500 mts B can beat C by 50 mts.

$B_2 = 500$ $C_2 = 500 - 50$.

$= 450$.

$\Rightarrow \frac{B_1}{C_1} = \frac{B_2}{C_2}$. $\Rightarrow \frac{\cancel{540}}{600 - x} = \frac{\cancel{500}}{\cancel{450}}$.

$600 - x = 54 \times 9$.

$\boxed{x = 486}$

$\begin{array}{r} 1200 \\ 972 \\ 3\overline{)228}(7 \\ 21 \end{array}$

In a race of 400 mts A can beat C by.

$A_2 = 400$ $C_2 = 400 - x$.

$\begin{array}{r} 486 \\ 486 \\ \hline 972 \end{array}$

$\frac{\overset{3}{\cancel{600}}}{486} = \frac{\overset{2}{\cancel{400}}}{400 - x}$.

$3(400 - x) = 2(486)$

$1200 - 3x = 972$.

$3x = 228$. $\boxed{x = 76 m}$