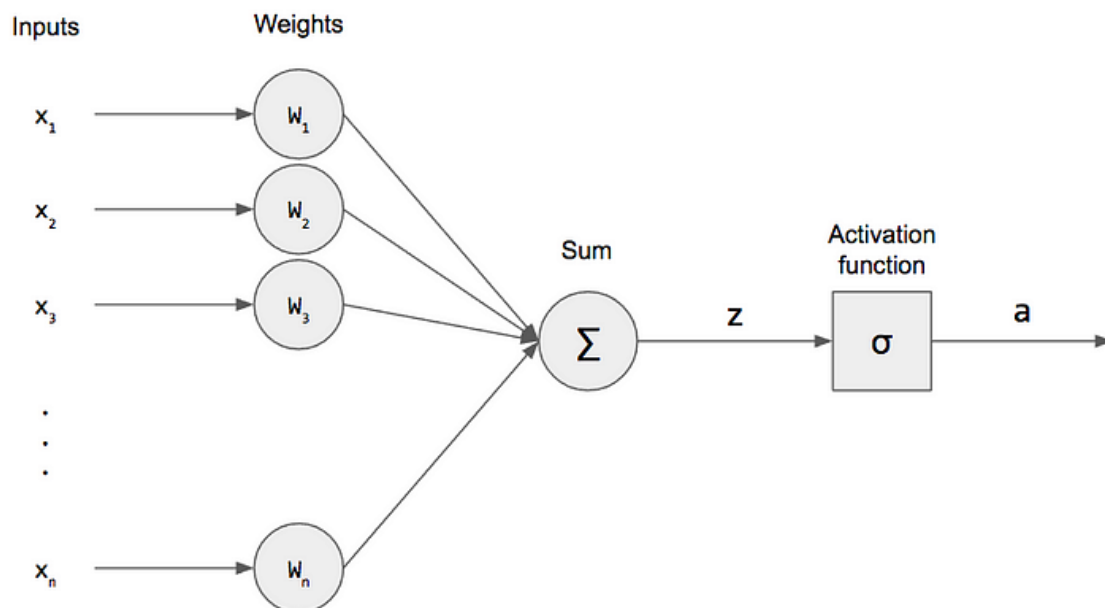


# Neural Representation of AND, OR, NOT, XOR and XNOR Logic Gates (Perceptron Algorithm)



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While taking the Udacity Pytorch Course by Facebook, I found it difficult understanding how the Perceptron works with Logic gates (AND, OR, NOT, and so on). I decided to check online resources, but as of the time of writing this, there was really no explanation on how to go about it. So after personal readings, I finally understood how to go about it, which is the reason for this medium post.

Note: The purpose of this article is NOT to mathematically explain how the neural network updates the weights, but to explain the logic behind how the values are being changed in simple terms.

First, we need to know that the Perceptron algorithm states that:

---

*Prediction ( $y'$ ) = 1 if  $Wx+b > 0$  and 0 if  $Wx+b \leq 0$*

---

Also, the steps in this method are very similar to how Neural Networks learn, which is as follows;

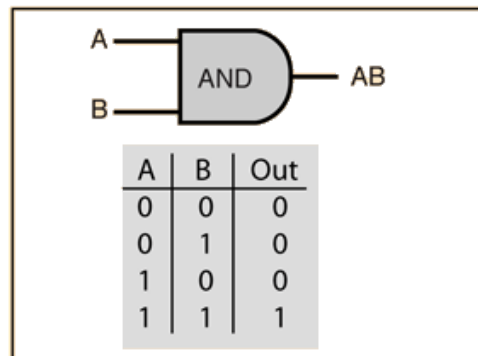
- Initialize weight values and bias
- Forward Propagate
- Check the error
- Backpropagate and Adjust weights and bias

- Repeat for all training examples

Now that we know the steps, let's get up and running:

### AND Gate

From our knowledge of logic gates, we know that an AND logic table is given by the diagram below



AND Gate

The question is, what are the weights and bias for the AND perceptron?

First, we need to understand that the output of an AND gate is 1 only if both inputs (in this case,  $x_1$  and  $x_2$ ) are 1. So, following the steps listed above;

#### Row 1

- From  $w_1 \cdot x_1 + w_2 \cdot x_2 + b$ , initializing  $w_1$ ,  $w_2$ , as 1 and  $b$  as  $-1$ , we get;

---


$$x_1(1) + x_2(1) - 1$$


---

- Passing the first row of the AND logic table ( $x_1=0$ ,  $x_2=0$ ), we get;

---


$$0 + 0 - 1 = -1$$


---

- From the Perceptron rule, if  $Wx + b \leq 0$ , then  $y' = 0$ . Therefore, this row is correct, and no need for Backpropagation.

#### Row 2

- Passing ( $x_1=0$  and  $x_2=1$ ), we get;

---


$$0 + 1 - 1 = 0$$


---

- From the Perceptron rule, if  $Wx + b \leq 0$ , then  $y' = 0$ . This row is correct, as the output is 0 for the AND gate.

- From the Perceptron rule, this works (for both row 1, row 2 and 3).

#### Row 4

- Passing ( $x_1=1$  and  $x_2=1$ ), we get;

---


$$1+1-1 = 1$$


---

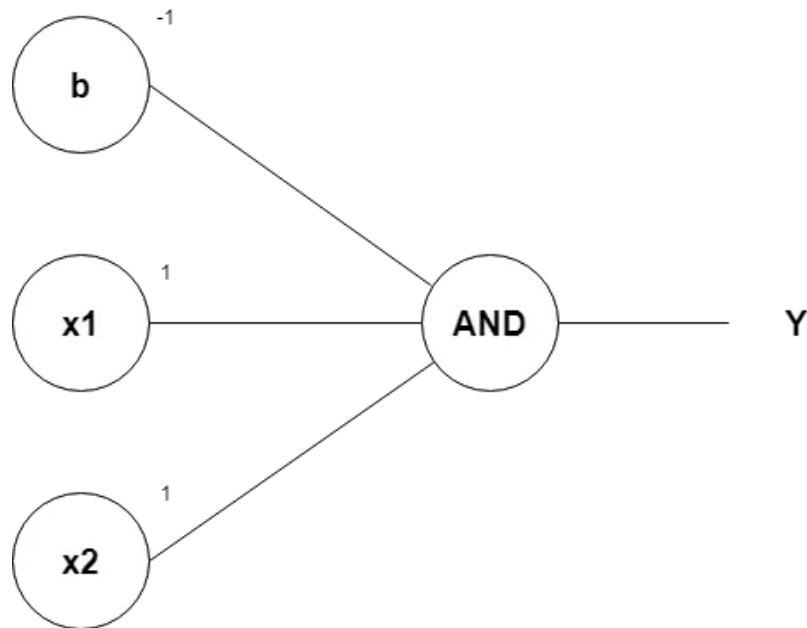
- Again, from the perceptron rule, this is still valid.

Therefore, we can conclude that the model to achieve an AND gate, using the Perceptron algorithm is;

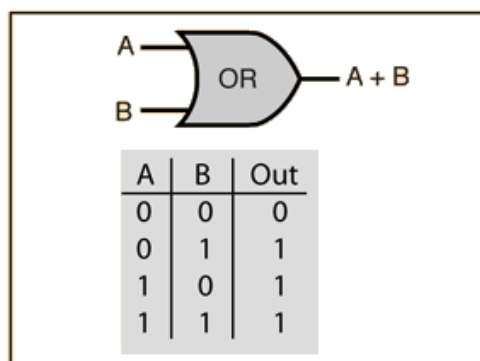
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$$x_1 + x_2 - 1$$


---



### OR Gate



OR Gate

From the diagram, the OR gate is 0 only if both inputs are 0.

### Row 1

- From  $w_1x_1 + w_2x_2 + b$ , initializing  $w_1$ ,  $w_2$ , as 1 and  $b$  as  $-1$ , we get;

---


$$x_1(1) + x_2(1) - 1$$


---

- Passing the first row of the OR logic table ( $x_1=0$ ,  $x_2=0$ ), we get;

---

$$0+0-1 = -1$$

---

- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y'=0$ . Therefore, this row is correct.

### Row 2

- Passing ( $x_1=0$  and  $x_2=1$ ), we get;

---

$$0+1-1 = 0$$

---

- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y'=0$ . Therefore, this row is incorrect.
- So we want values that will make inputs  $x_1=0$  and  $x_2=1$  give  $y'$  a value of 1. If we change  $w_2$  to 2, we have;

---

$$0+2-1 = 1$$

---

- From the Perceptron rule, this is correct for both the row 1 and 2.

### Row 3

- Passing ( $x_1=1$  and  $x_2=0$ ), we get;

---

$$1+0-1 = 0$$

---

- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y'=0$ . Therefore, this row is incorrect.
- Since it is similar to that of row 2, we can just change  $w_1$  to 2, we have;

---

$$2+0-1 = 1$$

---

- From the Perceptron rule, this is correct for both the row 1, 2 and 3.

### Row 4

- Passing ( $x_1=1$  and  $x_2=1$ ), we get;

---

$$2+2-1 = 3$$

---

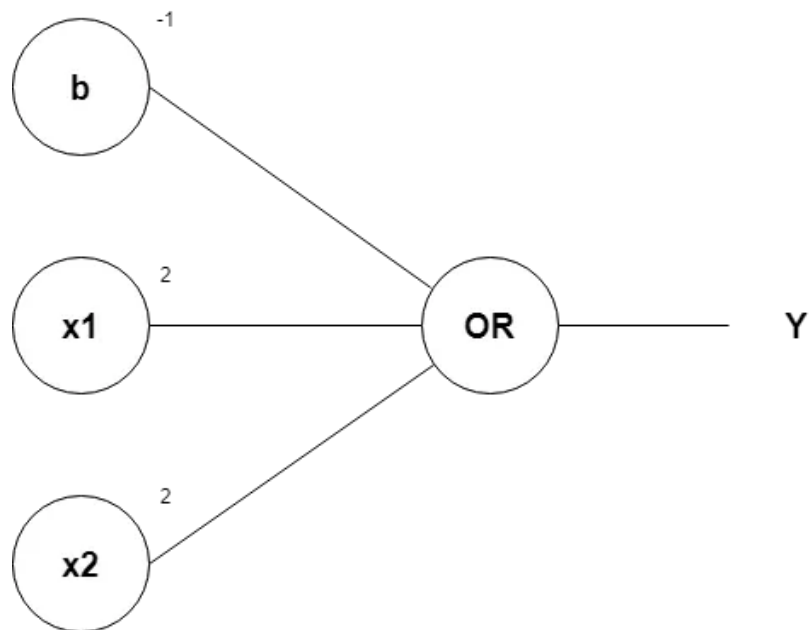
- Again, from the perceptron rule, this is still valid. Quite Easy!

Therefore, we can conclude that the model to achieve an OR gate, using the Perceptron algorithm is;

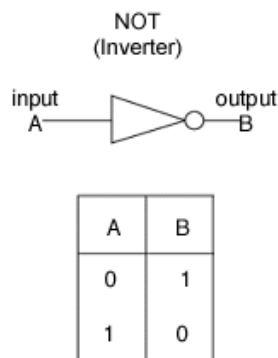
---

$$2x_1+2x_2-1$$

---



## NOT Gate



NOT Gate

From the diagram, the output of a NOT gate is the inverse of a single input. So, following the steps listed above;

## Row 1

- From  $w_1x_1+b$ , initializing  $w_1$  as 1 (since single input), and  $b$  as  $-1$ , we get;

---


$$x_1(1)-1$$


---

- Passing the first row of the NOT logic table ( $x_1=0$ ), we get;

---


$$0-1 = -1$$


---

- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y' = 0$ . This row is incorrect, as the output is 1 for the NOT gate.
- So we want values that will make input  $x_1=0$  to give  $y'$  a value of 1. If we change  $b$  to 1, we have;

---


$$0+1 = 1$$


---

- From the Perceptron rule, this works.

## Row 2

- Passing ( $x_1=1$ ), we get;

---


$$1+1 = 2$$


---

- From the Perceptron rule, if  $Wx+b > 0$ , then  $y'=1$ . This row is so incorrect, as the output is 0 for the NOT gate.
- So we want values that will make input  $x_1=1$  to give  $y'$  a value of 0. If we change  $w_1$  to  $-1$ , we have;

---


$$-1+1 = 0$$


---

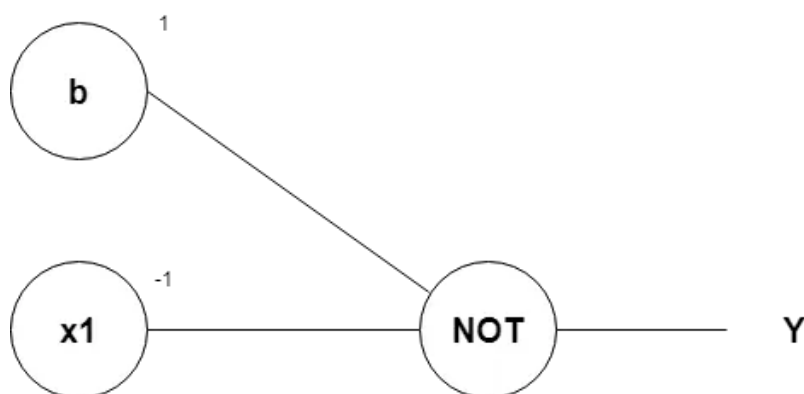
- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y'=0$ . Therefore, this works (for both row 1 and row 2).

Therefore, we can conclude that the model to achieve a NOT gate, using the Perceptron algorithm is;

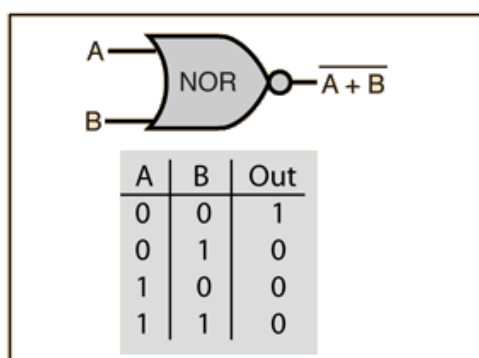
---


$$-x_1+1$$


---



## NOR Gate



NOR Gate

From the diagram, the NOR gate is 1 only if both inputs are 0.

## Row 1

- From  $w_1x_1+w_2x_2+b$ , initializing  $w_1$  and  $w_2$  as 1, and  $b$  as  $-1$ , we get;

---

$$x_1(1)+x_2(1)-1$$

---

- Passing the first row of the NOR logic table ( $x_1=0, x_2=0$ ), we get;

---

$$0+0-1 = -1$$

---

- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y'=0$ . This row is incorrect, as the output is 1 for the NOR gate.
- So we want values that will make input  $x_1=0$  and  $x_2 = 0$  to give  $y'$  a value of 1. If we change  $b$  to 1, we have;

---

$$0+0+1 = 1$$

---

- From the Perceptron rule, this works.

### Row 2

- Passing ( $x_1=0, x_2=1$ ), we get;

---

$$0+1+1 = 2$$

---

- From the Perceptron rule, if  $Wx+b > 0$ , then  $y'=1$ . This row is incorrect, as the output is 0 for the NOR gate.
- So we want values that will make input  $x_1=0$  and  $x_2 = 1$  to give  $y'$  a value of 0. If we change  $w_2$  to  $-1$ , we have;

---

$$0-1+1 = 0$$

---

- From the Perceptron rule, this is valid for both row 1 and row 2.

### Row 3

- Passing ( $x_1=1, x_2=0$ ), we get;

---

$$1+0+1 = 2$$

---

- From the Perceptron rule, if  $Wx+b > 0$ , then  $y'=1$ . This row is incorrect, as the output is 0 for the NOR gate.
- So we want values that will make input  $x_1=0$  and  $x_2 = 1$  to give  $y'$  a value of 0. If we change  $w_1$  to  $-1$ , we have;

---

$$-1+0+1 = 0$$

---

- From the Perceptron rule, this is valid for both row 1, 2 and 3.

### Row 4

- Passing  $(x_1=1, x_2=1)$ , we get;

---


$$-1-1+1 = -1$$


---

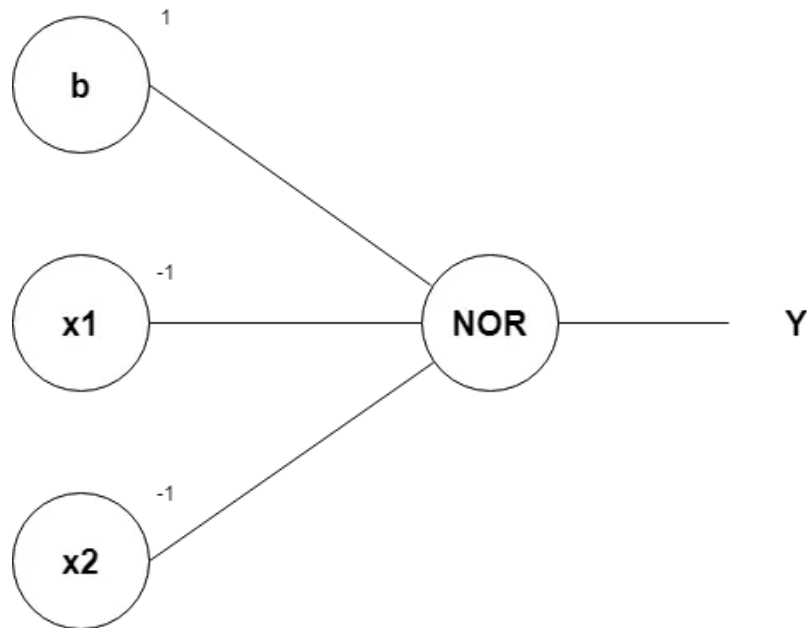
- From the Perceptron rule, this still works.

Therefore, we can conclude that the model to achieve a NOR gate, using the Perceptron algorithm is;

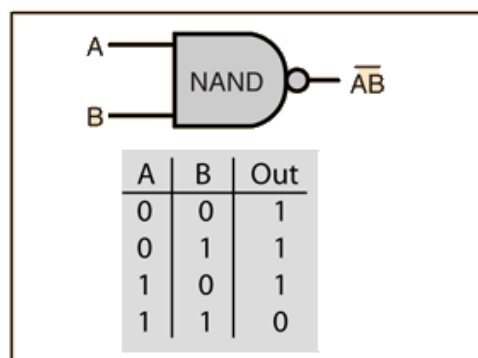
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$$-x_1-x_2+1$$


---



### NAND Gate



From the diagram, the NAND gate is 0 only if both inputs are 1.

### Row 1

- From  $w_1x_1+w_2x_2+b$ , initializing  $w_1$  and  $w_2$  as 1, and  $b$  as -1, we get;

---


$$x_1(1)+x_2(1)-1$$


---

- Passing the first row of the NAND logic table  $(x_1=0, x_2=0)$ , we get;

---


$$0+0-1 = -1$$


---



- From the Perceptron rule, if  $Wx+b \leq 0$ , then  $y' = 0$ . This row is incorrect, as the output is 1 for the NAND gate.
- So we want values that will make input  $x_1=0$  and  $x_2 = 0$  to give  $y'$  a value of 1. If we change  $b$  to 1, we have;

---


$$0+0+1 = 1$$


---

- From the Perceptron rule, this works.

## Row 2

- Passing  $(x_1=0, x_2=1)$ , we get;

---


$$0+1+1 = 2$$


---

- From the Perceptron rule, if  $Wx+b > 0$ , then  $y' = 1$ . This row is also correct (for both row 2 and row 3).

## Row 4

- Passing  $(x_1=1, x_2=1)$ , we get;

---


$$1+1+1 = 3$$


---

- This is not the expected output, as the output is 0 for a NAND combination of  $x_1=1$  and  $x_2=1$ .
- Changing values of  $w_1$  and  $w_2$  to -1, and value of  $b$  to 2, we get;

---


$$-1-1+2 = 0$$


---

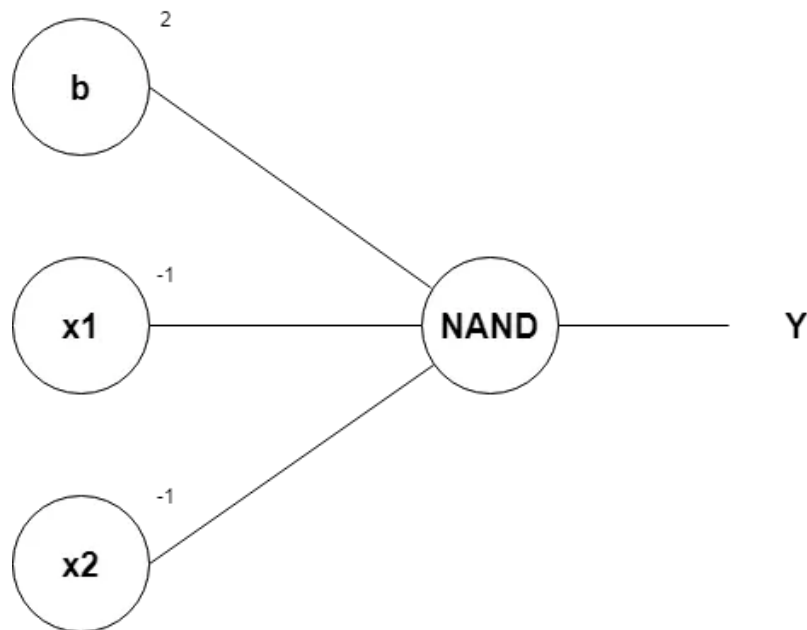
- It works for all rows.

Therefore, we can conclude that the model to achieve a NAND gate, using the Perceptron algorithm is;

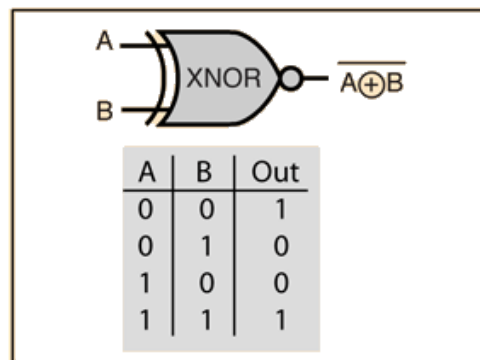
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$$-x_1-x_2+2$$


---



### XNOR Gate



XNOR Gate

Now that we are done with the necessary basic logic gates, we can combine them to give an XNOR gate.

The boolean representation of an XNOR gate is;

---


$$x_1x_2 + x_1'x_2'$$

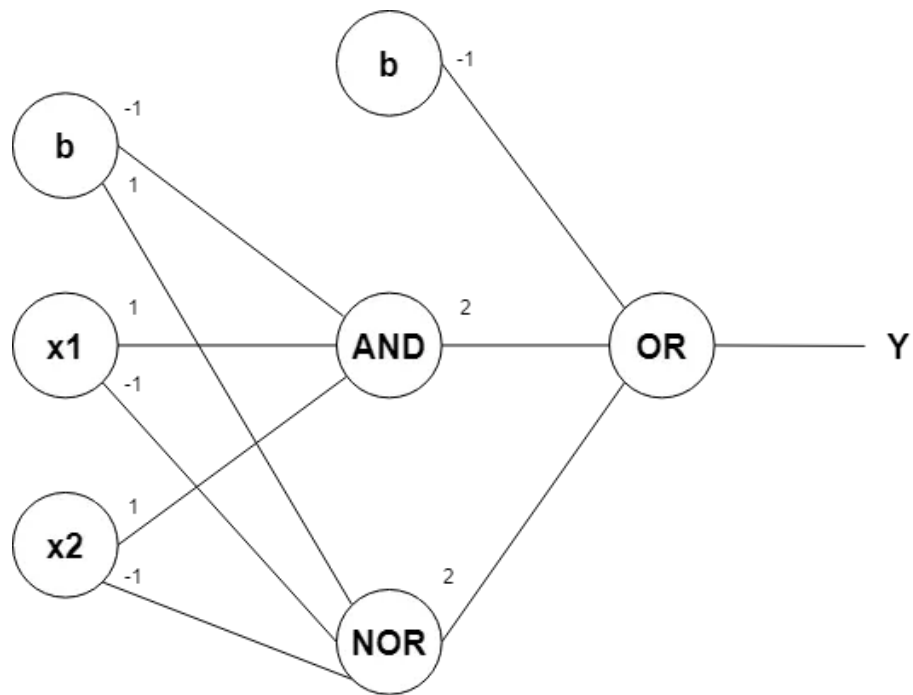

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Where '' means inverse.

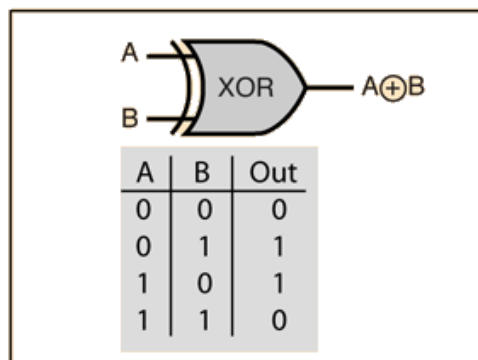
From the expression, we can say that the XNOR gate consists of an AND gate ( $x_1x_2$ ), a NOR gate ( $x_1'x_2'$ ), and an OR gate.

This means we will have to combine 3 perceptrons:

- AND ( $x_1+x_2-1$ )
- NOR ( $-x_1-x_2+1$ )
- OR ( $2x_1+2x_2-1$ )



### XOR Gate



XOR Gate

The boolean representation of an XOR gate is;

$$x_1x_2' + x_1'x_2$$

We first simplify the boolean expression

$$x_1'x_2 + x_1x_2' + x_1'x_1 + x_1x_2$$

$$x_1(x_1' + x_2) + x_2(x_1' + x_2)$$

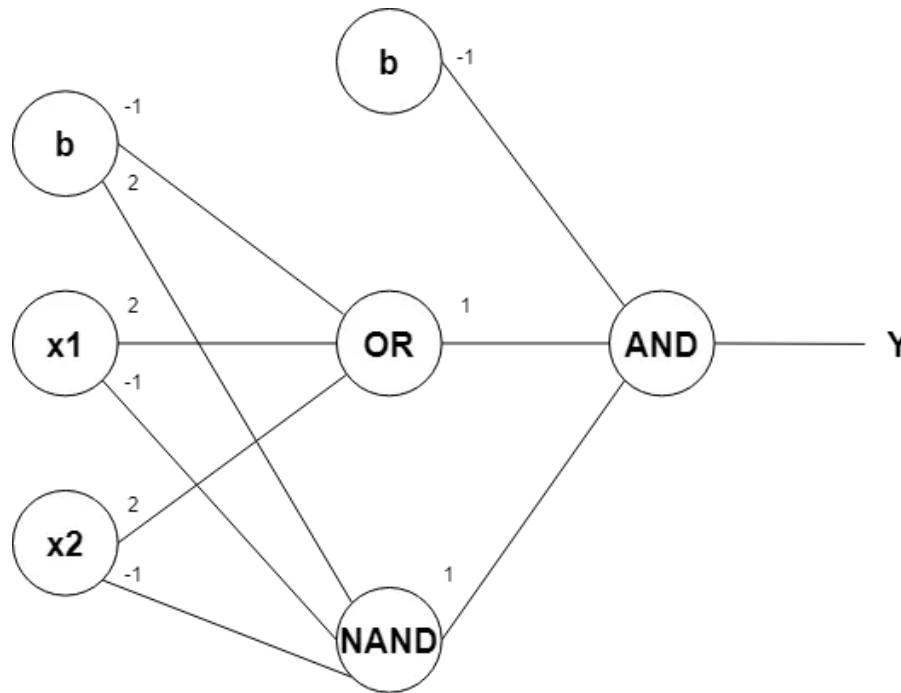
$$(x_1 + x_2)(x_1' + x_2)$$

$$(x_1 + x_2)(x_1x_2)'$$

From the simplified expression, we can say that the XOR gate consists of an OR gate ( $x_1 + x_2$ ), a NAND gate ( $(x_1x_2)'$ ) and an AND gate ( $(x_1 + x_2)(x_1x_2)'$ ).

This means we will have to combine 2 perceptrons:

- OR ( $2x_1+2x_2-1$ )
- NAND ( $-x_1-x_2+2$ )
- AND ( $x_1+x_2-1$ )



## CONCLUSION

In conclusion, this is just a custom method of achieving this, there are many other ways and values you could use in order to achieve Logic gates using perceptrons. For example;

---

*AND* ( $20x_1+20x_2-30$ )

*OR* ( $20x_1+20x_2-10$ )

*NOT* ( $-20x_1+10$ )

---

This will still work.

In my next post, I will show how you can write a simple python program that uses the Perceptron Algorithm to automatically update the weights of these Logic gates.

Thank you....

Udacity, Facebook Research