Contraharmonic Mean Filter

**Description**

Applies a contraharmonic mean filter to an image.

With a contraharmonic mean filter, the color value of each pixel is replaced with the contraharmonic mean of color values of the pixels in a surrounding region.

The contraharmonic mean with order Q is defined as:

$$C_Q = \frac{x_1^{Q+1} + x_2^{Q+1} + \cdots + x_n^{Q+1}}{x_1^Q + x_2^Q + \cdots + x_n^Q}$$

A contraharmonic mean filter reduces or virtually eliminates the effects of salt-and-pepper noise. For positive values of Q, the filter eliminates pepper noise. For negative values of Q it eliminates salt noise. It cannot do both simultaneously.

Note that the contraharmonic filter is simply the arithmetic mean filter if Q = 0, and the harmonic mean filter if Q = −1.

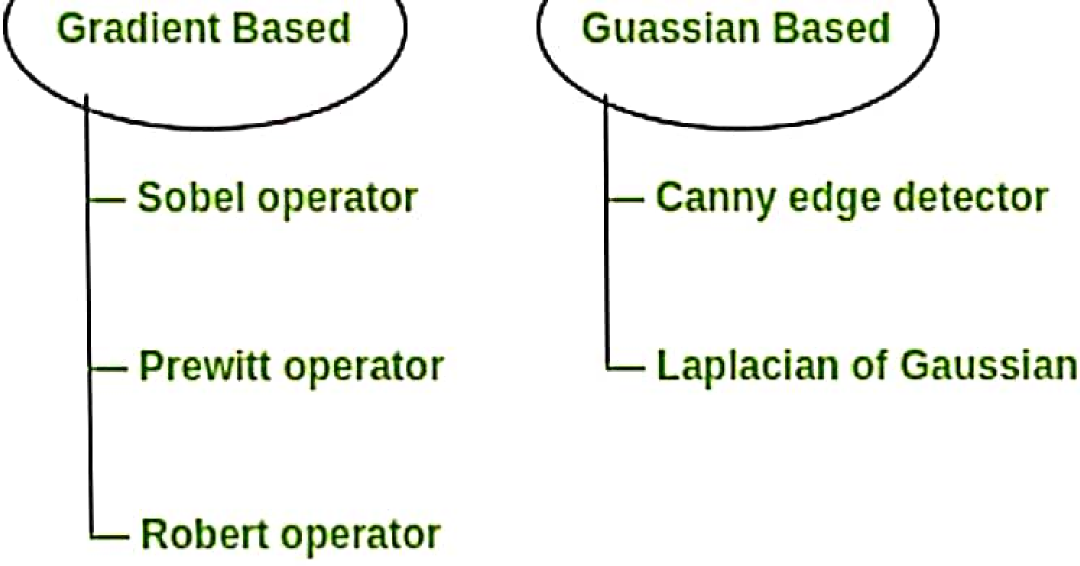
A larger region (filter size) yields a stronger filter effect with the drawback of some blurring.

An **image gradient** is a directional change in the intensity or color in an image. The gradient of the image is one of the fundamental building blocks in image processing. For example, the Canny edge detector uses image gradient for edge detection. In graphics

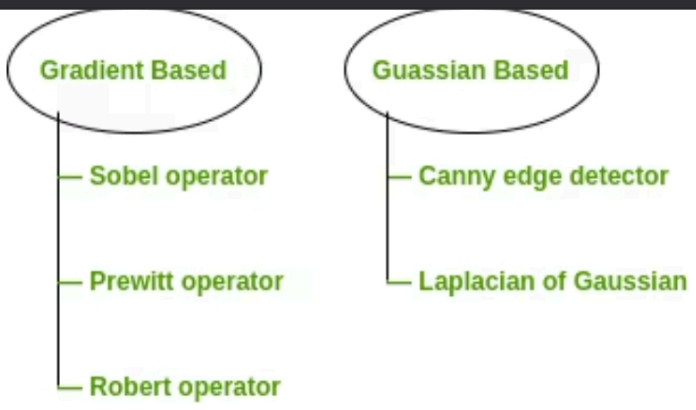
**Edge Detection** is a method of segmenting an image into regions of discontinuity. It is a widely used technique in digital image processing like

- pattern recognition
- image morphology
- feature extraction

Edge detection allows users to observe the features of an image for a significant change in the gray level. This texture indicating the end of one region in the image and the beginning of another. It reduces the amount of data in an image and preserves the structural properties of an image.




**Sobel Operator:** It is a discrete differentiation operator. It computes the gradient approximation of image intensity function for image edge detection. At the pixels of an image, the Sobel operator produces either the normal to a vector or the corresponding gradient vector. It uses two 3 x 3 kernels or masks which are convolved with the input image to calculate the vertical and horizontal derivative approximations respectively –




**Sobel Operator:** It is a discrete differentiation operator. It computes the gradient approximation of image intensity function for image edge detection. At the pixels of an image, the Sobel operator produces either the normal to a vector or the corresponding gradient vector. It uses two 3 x 3 kernels or masks which are convolved with the input image to calculate the vertical and horizontal derivative approximations respectively –

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

not give appropriate results

**Prewitt Operator:** This operator is almost similar to the sobel operator. It also detects vertical and horizontal edges of an image. It is one of the best ways to detect the orientation and magnitude of an image. It uses the kernels or masks –

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

*Advantages:*

1. Good performance on detecting vertical and horizontal edges
2. Best operator to detect the orientation of an image

*Limitations:*

**Robert Operator:** This gradient-based operator computes the sum of squares of the differences between diagonally adjacent pixels in an image through discrete differentiation. Then the gradient approximation is made. It uses the following 2 x 2 kernels or masks –

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad M_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- The basic edge detection method is based on simple filtering without taking note of image characteristics and other information
- **More advanced techniques make attempt to improve the simple detection by taking into account factors such as noise, scaling etc.**

## Laplacian of Gaussian (Marr-Hildrith) Operator

**Discussion of Marr and Hildreth was about:**

- Intensity of changes is not independent of image scale
- Sudden intensity change will cause a zero crossing of the second derivative

**Therefore, an edge detection operator should:**

- Be capable of being tuned to any scale
- Be capable of computing the first and second derivatives

**To minimize the noise susceptibility of the Laplacian Operator, Laplacian of Gaussian (LoG) Operator is often preferred**

## Laplacian of Gaussian (Marr-Hildrith) Operator

**The LoG Algorithm can be written as:**

- Generate the mask and apply LoG to the image
- Detect the Zero Crossing

* For 1D,

$$\nabla^2(f * g) = f * \nabla^2 g = f * LoG$$

Let the 2D, Gaussian $f^n$ is given as →

$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{x^2+y^2}{2\sigma^2}\right] \quad — ①$$

To suppress the noise, the image is convolved with the Gaussian smoothing function before using the Laplacian for edge detection

$$\nabla\left[G_\sigma(x,y) * f(x,y)\right] = \left[\nabla G_\sigma(x,y)\right] * f(x,y) = LoG * f(x,y)$$

The LoG function can be derived as:

$$\frac{\partial}{\partial x} G_\sigma(x,y) = \frac{\partial}{\partial x} e^{\left[-\frac{x^2+y^2}{2\sigma^2}\right]} = -\frac{x}{\sigma^2} e^{\left[-\frac{x^2+y^2}{2\sigma^2}\right]} \quad — ②$$

## Segmentation based on grey lever discontinuities – Edge detection using

## Laplacian of Gaussian (Marr-Hildrith) Operator

Similarly,

$$\frac{\partial^2}{\partial x^2} G_\sigma(x,y) = \frac{x^2}{\sigma^4} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)} - \frac{1}{\sigma^2} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)} = \frac{x^2-\sigma^2}{\sigma^4} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)}$$

Similarly, by ignoring the normalization constant $1/\sqrt{2\pi\sigma^2}$, we get

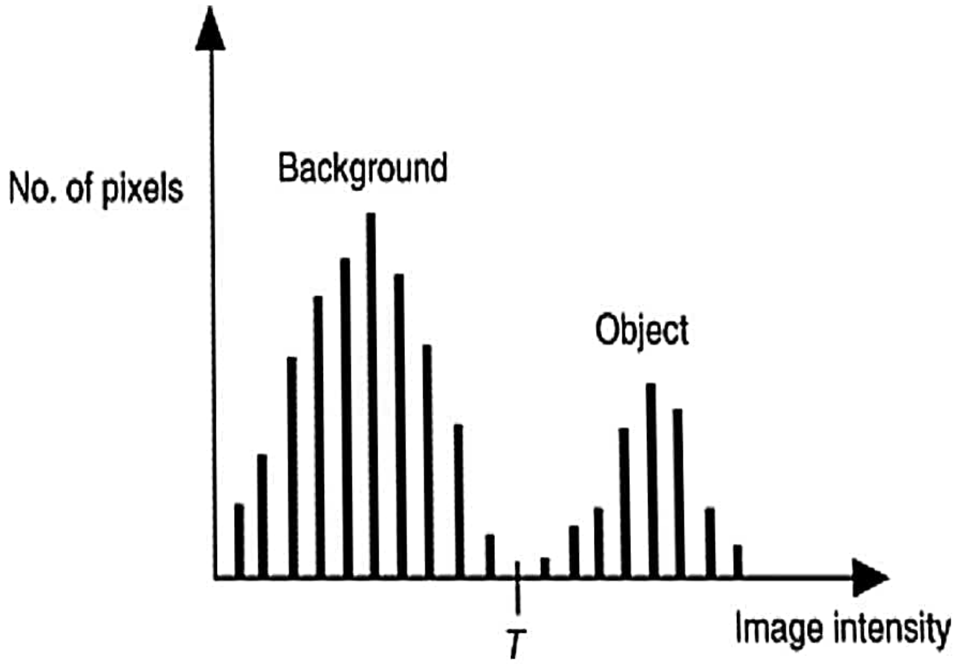
$$\frac{\partial^2}{\partial y^2} G_\sigma(x,y) = \frac{y^2}{\sigma^4} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)} - \frac{1}{\sigma^2} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)} = \frac{y^2-\sigma^2}{\sigma^4} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)}$$

The LoG kernel can be described as:

$$LoG \triangleq \frac{\partial^2}{\partial x^2} G_\sigma(x,y) + \frac{\partial^2}{\partial y^2} G_\sigma(x,y) = \frac{x^2+y^2-2\sigma^2}{\sigma^4} e^{\left(-\frac{x^2+y^2}{2\sigma^2}\right)}$$

### 5.2.1 Global Thresholding

Global thresholding is based on the assumption that the image has a bimodal histogram and, therefore, the object can be extracted from the background by a simple operation that compares image values with a threshold value *T* [32, 132]. Suppose that we have an image *f(x,y)* with the histogram shown on Figure 5.1.




Sign in to download full-size image

FIGURE 5.1. An example of a bimodal histogram with selected threshold *T*.

The object and background pixels have gray levels grouped into two dominant modes. One obvious way to extract the object from the background is to select

The object and background pixels have gray levels grouped into two dominant modes. One obvious way to extract the object from the background is to select a threshold $T$ that separates these modes.

The thresholded image $g(x,y)$ is defined as $g(x, y)$

$$g(x, y) = \begin{cases} 1 & \text{if } (x, y) > T \\ 0 & \text{if } (x, y) \leq T \end{cases}. \tag{5.1}$$

The result of thresholding is a binary image, where pixels with intensity value of 1 correspond to objects, whereas pixels with value 0 correspond to the background.

There are many other ways to select a global threshold. One of them is based on a classification model that minimizes the probability of error [93]. For example, if we have an image with a bimodal histogram (e.g., object and background), we can calculate the error as the total number of background pixels misclassified as object and object pixels misclassified as background. A semiautomated version of this technique was applied by Johnson *et al.* [69] to measure ventricular volumes from 3D magnetic resonance (MR) images. In their method an operator selects two pixels —one inside an object and one in the background. When the distribution of pixel intensities in the circular regions around selected pixels is compared, the threshold is calculated automatically, and it corresponds to the least number of misclassified pixels between two distributions. The result of the thresholding operation is displayed as a contour map and superimposed on the original image. If needed, the operator can manually modify any part of the border. The