IMAGE PROCESSING USING PYTHON

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SYLLABUS

UNIT – I

Fundamentals of Image Processing: Introduction, Fundamental steps in image processing, Image sampling, Quantization, Resolution, Elements of image processing system, Applications of Digital image processing. Color fundamentals, Color image formats and conversion.

UNIT – II

Image Enhancement:

Spatial domain methods: Point & Histogram processing, Fundamentals of Spatial filtering, Smoothing spatial filters, Sharpening spatial filters.

Frequency domain methods: Basics of filtering in frequency domain, image smoothing, image sharpening, Selective filtering.

UNIT – III

Image Restoration and Reconstruction

A model of the image degradation and Restoration process, Noise models, restoration in the presence of noise only-Spatial Filtering, Periodic Noise Reduction by frequency domain filtering, Linear, Position –Invariant Degradations, Estimating the degradation function, Inverse filtering, Minimum mean square error (Wiener) filtering, constrained least squares filtering.

UNIT – IV

Image Segmentation: Fundamentals, point, line, edge detection, thresholding, and region –based segmentation.

Morphological Image Processing: Preliminaries, Erosion and dilation, opening and closing, basic morphological algorithms for boundary extraction, thinning.

UNIT – V

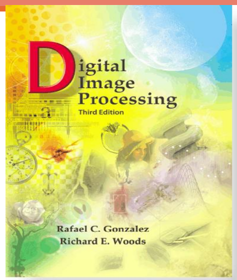
Image Compression: Introduction, Need for image compression, Redundancy in images, Classification of redundancy in images, image compression scheme, Classification of image compression schemes, Fundamentals of information theory, Run length coding, Shannon – Fano coding, Huffman coding, Arithmetic coding, Predictive coding.

Text Books:

- Digital Image Processing – Gonzaleze and Woods, 2nd Ed., Pearson.
- S. Jayaraman, S. Esakkirajan and T. VeeraKumar, "Digital Image processing, Tata McGraw Hill publishers, 2009

One picture is worth more than ten thousand words

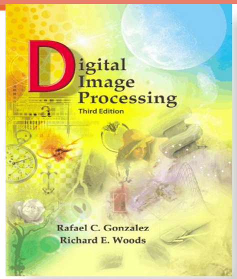
- Anonymous



Syllabus

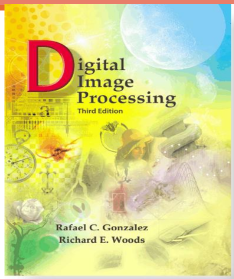
- Fundamentals of Image Processing:
  - Introduction
  - Fundamental steps in image processing
  - Image sampling, Quantization, Resolution
  - Elements of image processing system
  - Applications of Digital image processing

- Color fundamentals, Color image formats and conversion.



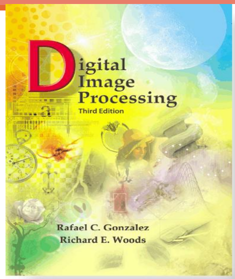
Fundamentals of Image Processing

- Introduction
- Image Sensing and Acquisition
- Image Sampling & Quantization
- Image File Formats
- Some basic Relationships between pixels



Introduction

- What is an Image & Digital Image Processing?
- Types of Digital Images?
- Why we need DIP?
- Applications of DIP?
- Fundamental Steps in Digital Image Processing
- Components of an Image Processing System



What is an Image & DIP?

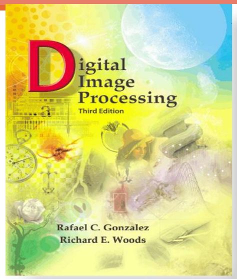
An **Image** is a 2D function, f(x, y)

- Where x and y are spatial coordinates
- Amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image.

When spatial coordinates and amplitude values are all finite, discrete quantities, the image is called as Digital image.

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}.$$

DIP refers Processing digital images by means of a digital computer.



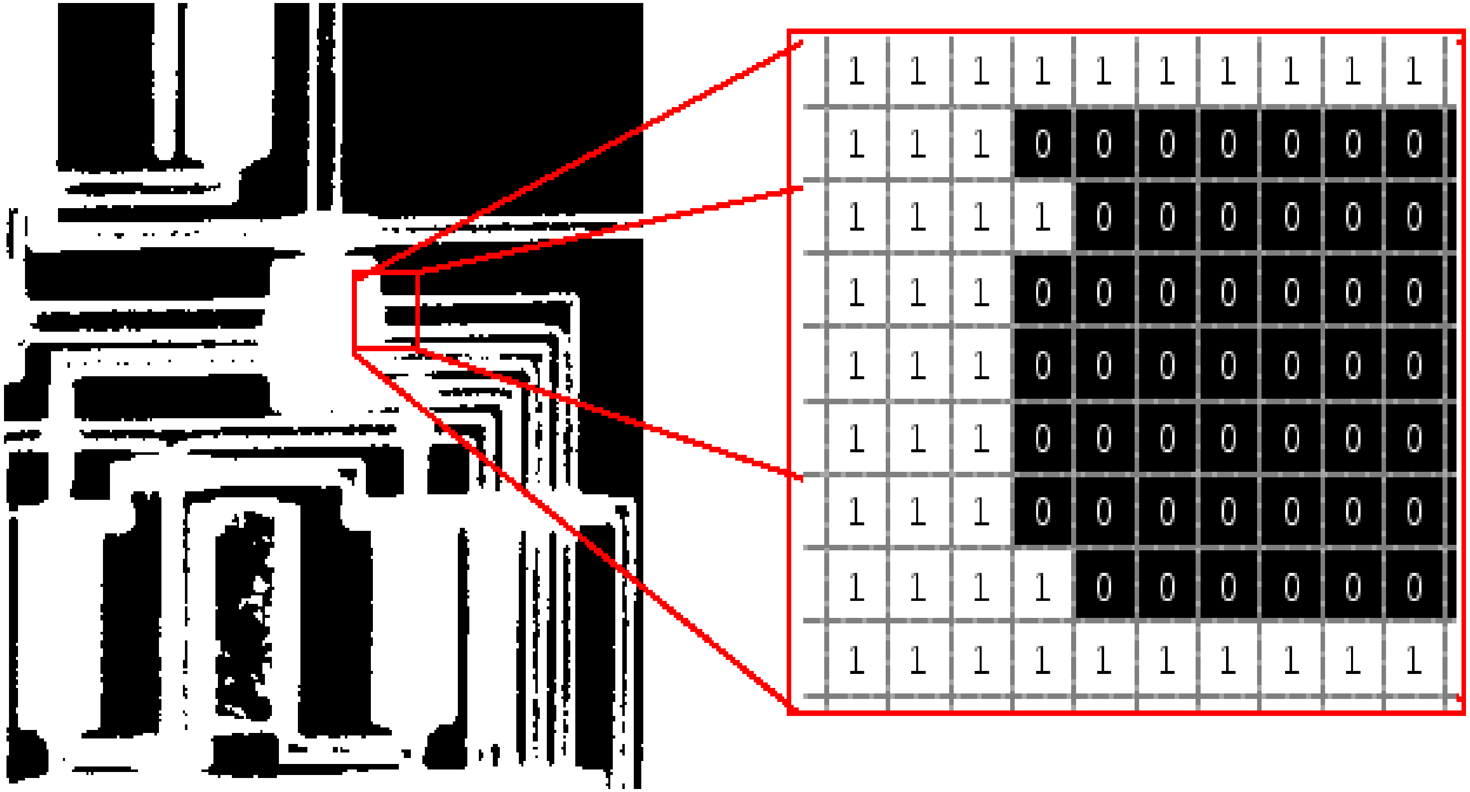
Types of Digital Image

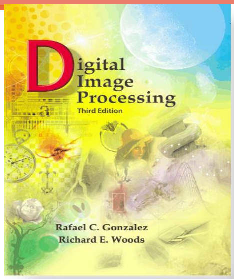
- Black-n-White Image
  - Binary Image (two-tone)
  - Gray level Image (gray-tone)
- Color Image
- Indexed Image

Another classification

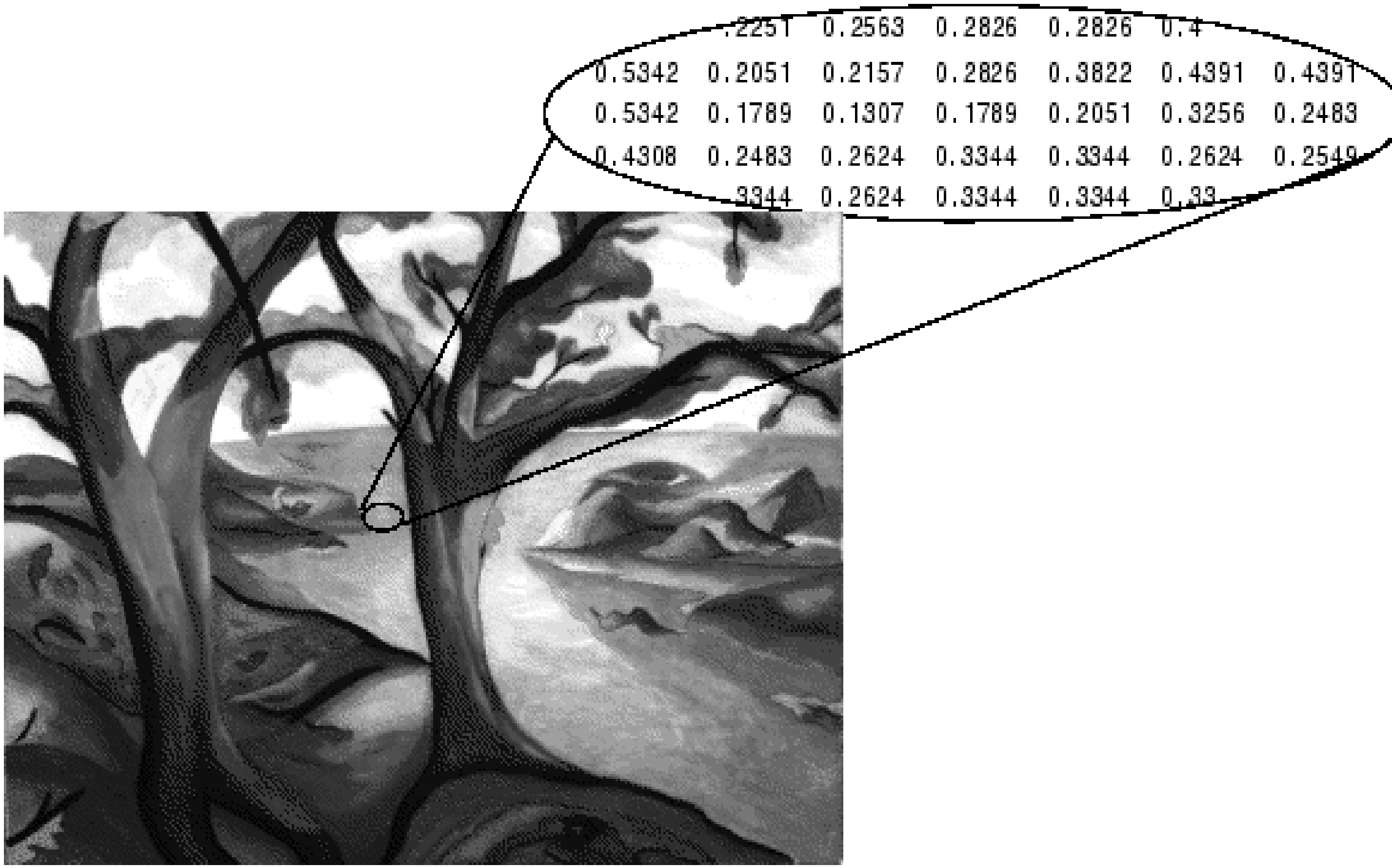
- Still Image
- Movie Image (Video Sequences)

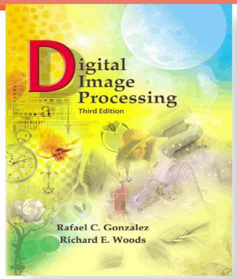
Binary Image



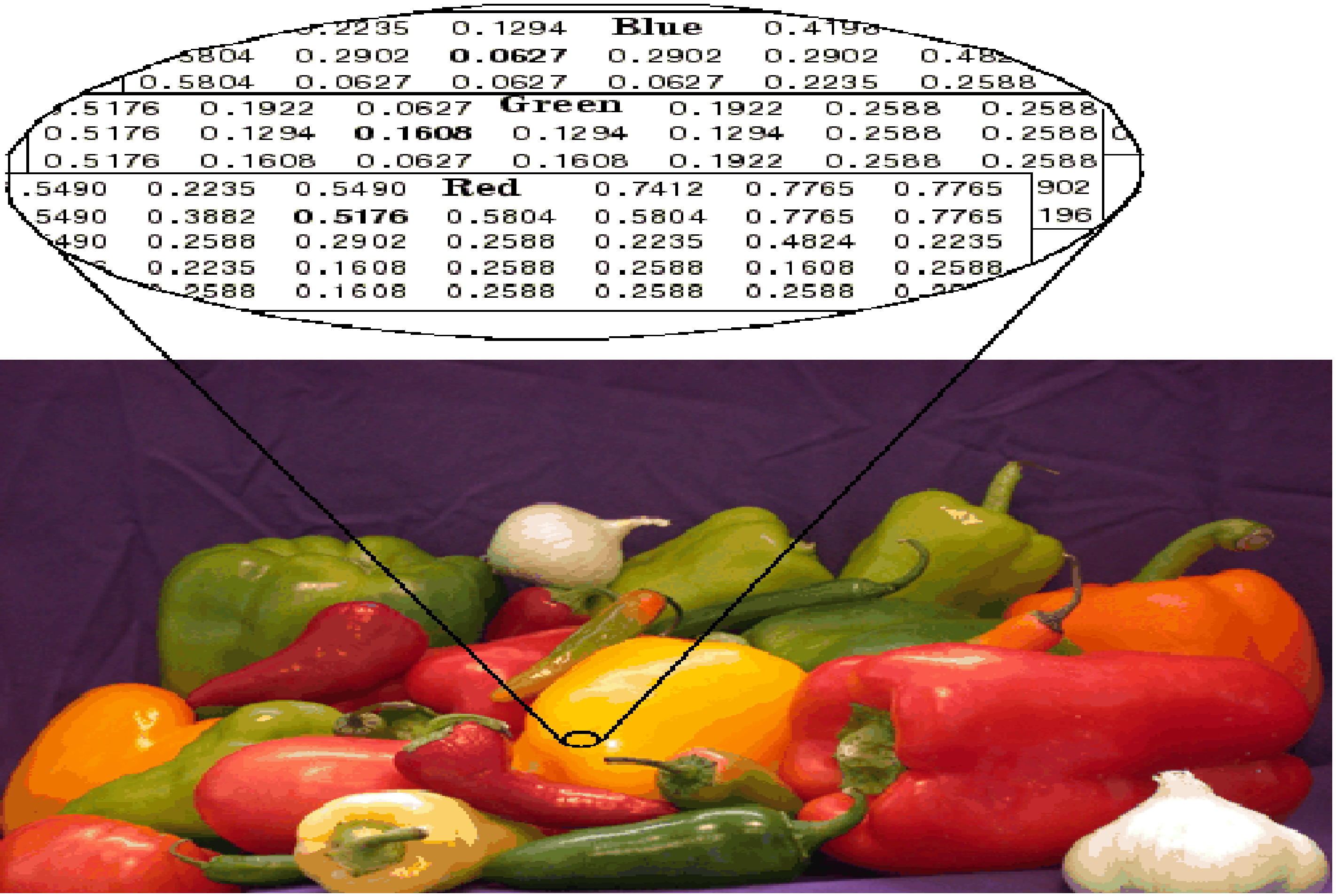


Gray level Image




Digital Image Processing
Third Edition
Rafael C. Gonzalez
Richard E. Woods

Color Image


0.1294 Blue
0.2902 0.0627 0.2902 0.2902
0.5804 0.0627 0.0627 0.0627 0.2235 0.2588
0.1922 0.0627 Green 0.1922 0.2588 0.2588
0.5176 0.1294 0.1608 0.1294 0.1294 0.2588 0.2588
0.5176 0.1608 0.0627 0.1608 0.1922 0.2588 0.2588
0.2235 0.5490 Red 0.7412 0.7765 0.7765
0.3882 0.5176 0.5804 0.5804 0.7765 0.7765
0.2588 0.2902 0.2588 0.2235 0.4824 0.2235
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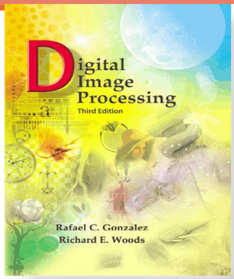
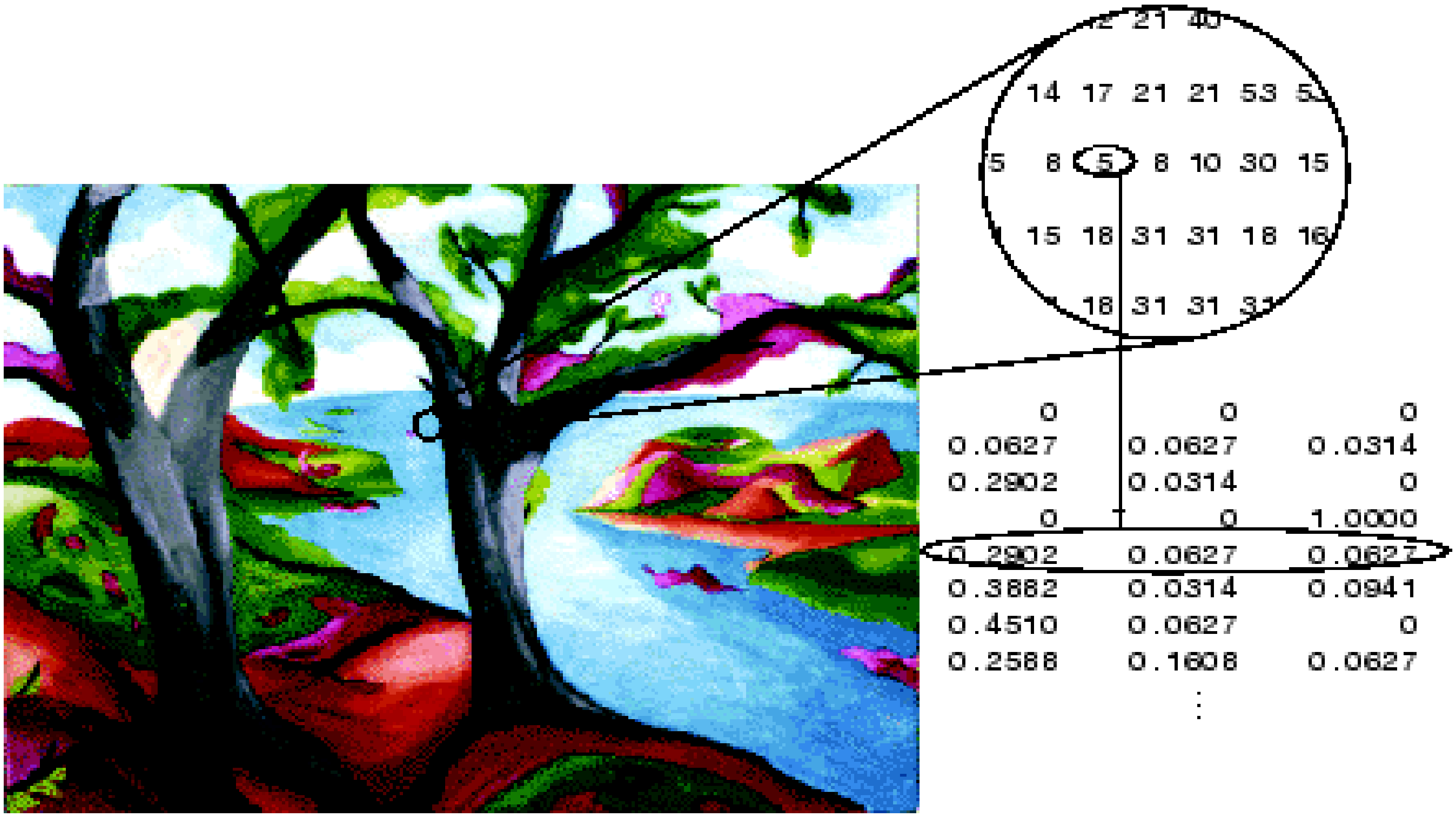
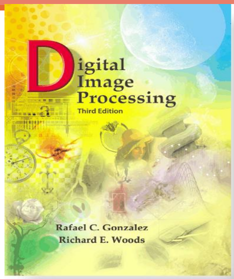


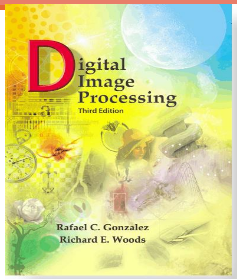
Indexed Image





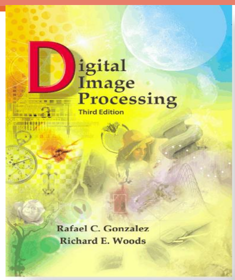
Why we need DIP?

- ❑ Human perception
- ❑ Machine Vision
- ❑ Storage & Transmission



IP Applications: Human Perception

- Noise Filtering
- Contrast Enhancement
- Image Deblurring
- Image Correction
- Image Inpainting
- Image Fusion
- Image Stitching
- Transformations
- Astronomy
- Weather Forecasting
- Medical Imaging



Noise Filtering

The procedure of reducing the noise components of an image so as to enhance its information is known as **Noise filtering**.

Original Image

Noised Image

Mean filtered Image

Contrast Enhancement

Contrast enhancement increases the total contrast of an image by making light colors lighter and dark colors darker at the same time.

Original Image

Enhanced Image

Image Deblurring: Motion Blur

Used to restore deblurred images when the camera or object is moved during exposure

Original Image

Blurred Image

Restored Image

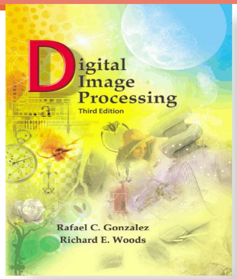


Image Deblurring: Out of Focus Blur

- Used to restore deblurred images when the camera was not focused properly

Original Image

Blurred Image

Restored Image

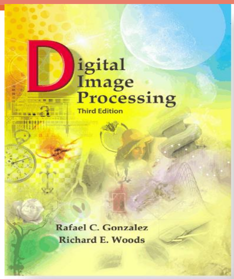
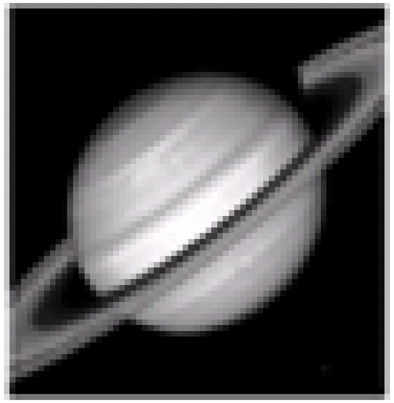


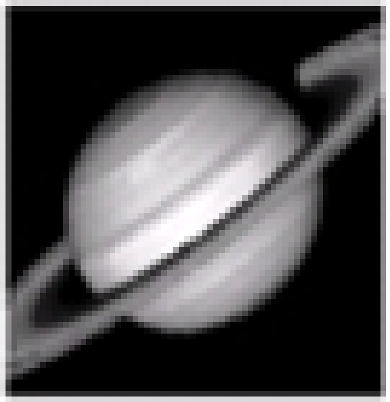
Image Correction

- Needed when image data is erroneous:
  - Bad transmission
  - Bits are missing: Salt & Pepper Noise



Original Image

Noised Image



Corrected Image

Image Inpainting

- Image Inpainting is the process of reconstructing lost or deteriorated parts of images. It also improves brightness, color etc.

Original Image

Processed Image

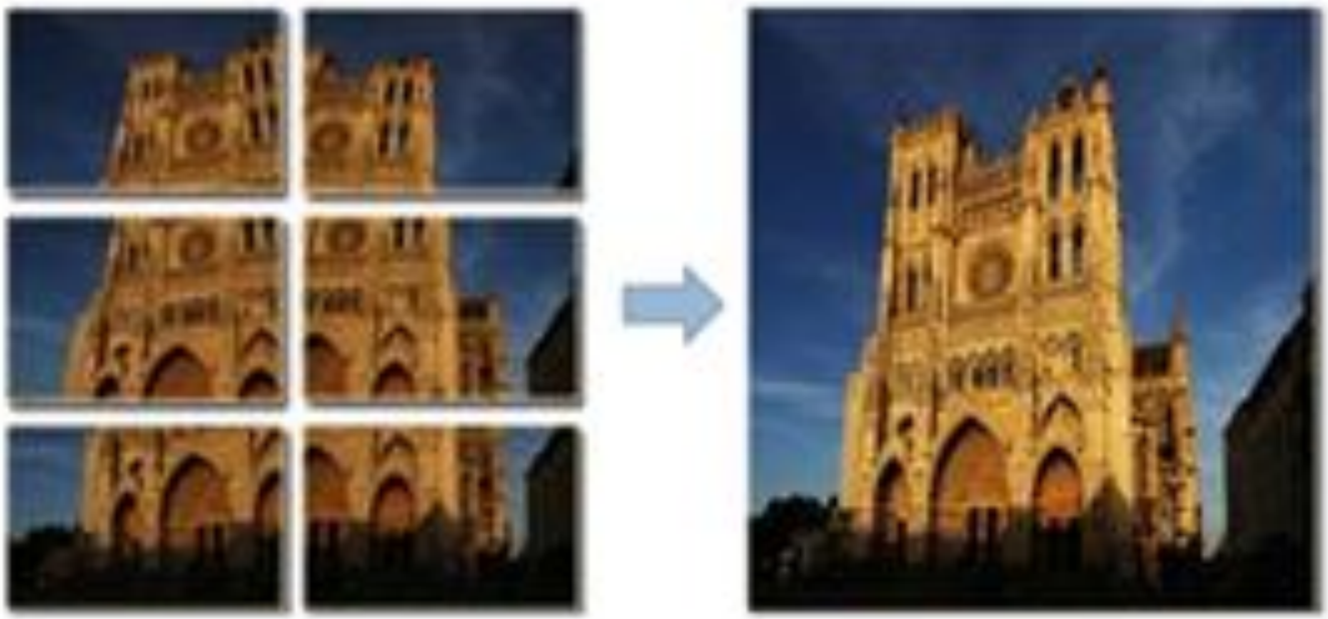
Image Fusion

- **Image fusion** is the process of combining relevant information from two or more images into a single image. The resulting image will be more informative than any of the input images.

Image1 & Image 2 fused to got Image 3

Image Stitching

Image stitching is the process of combining multiple photographic images with overlapping fields of view to produce a segmented panorama or high-resolution image.



Geometric Transformations

Rotate + Scale

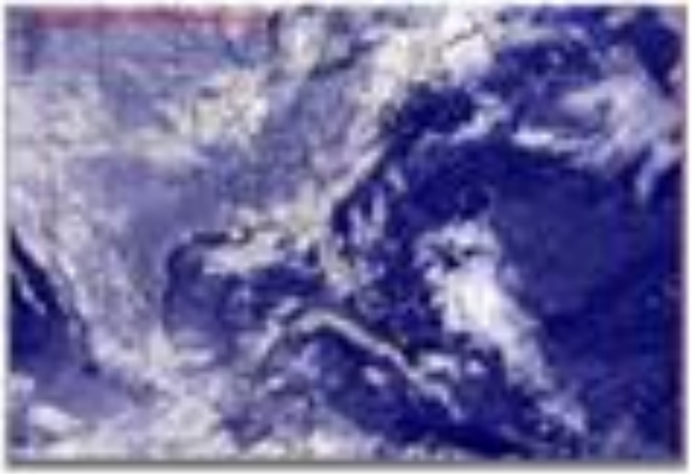
Astronomy

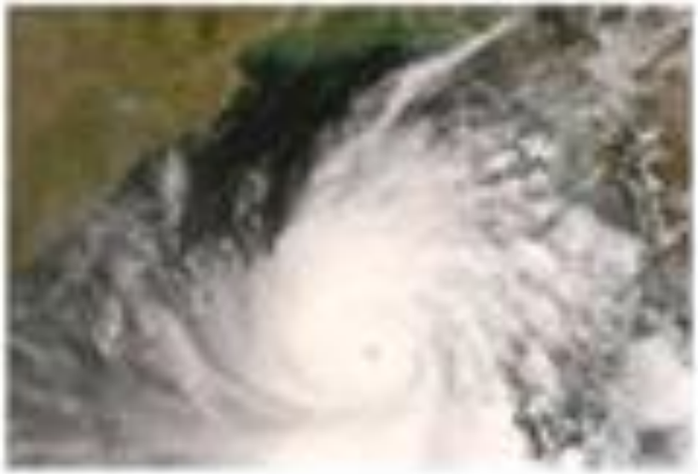
Raw image

Processed image

Weather Forecasting

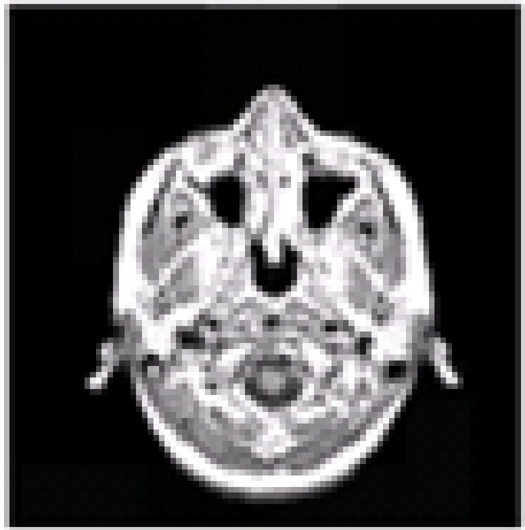
- Image processing combined with the skills and experience of meteorologist, offer solutions to a large range of weather forecasting problems such as defining & modeling weather patterns.



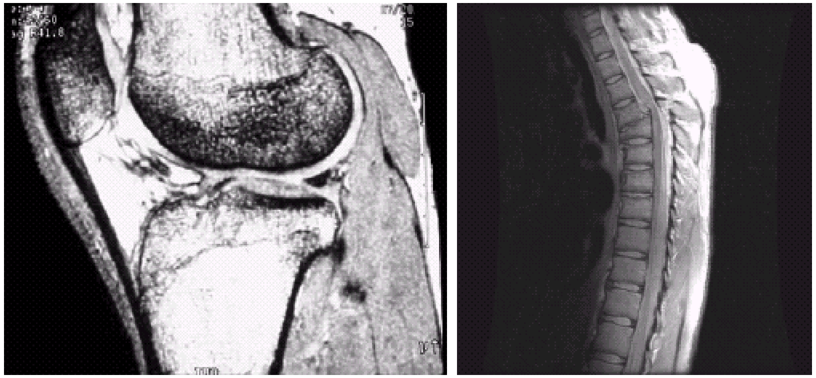


Medical Image Processing

- Image Processing is widely used, e.g. Analysis of microscopic images

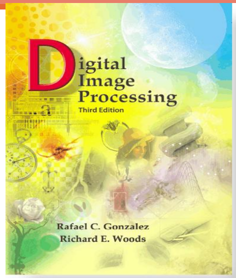


MR/CT Imaging of a human body
- Use for Brain Surgery



a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



IP Applications: Machine Vision

- Object Detection and Tracking
- Foreground Extraction
- Industrial Inspection
    - PCB Inspection
    - License Plate Recognition
    - Biometrics

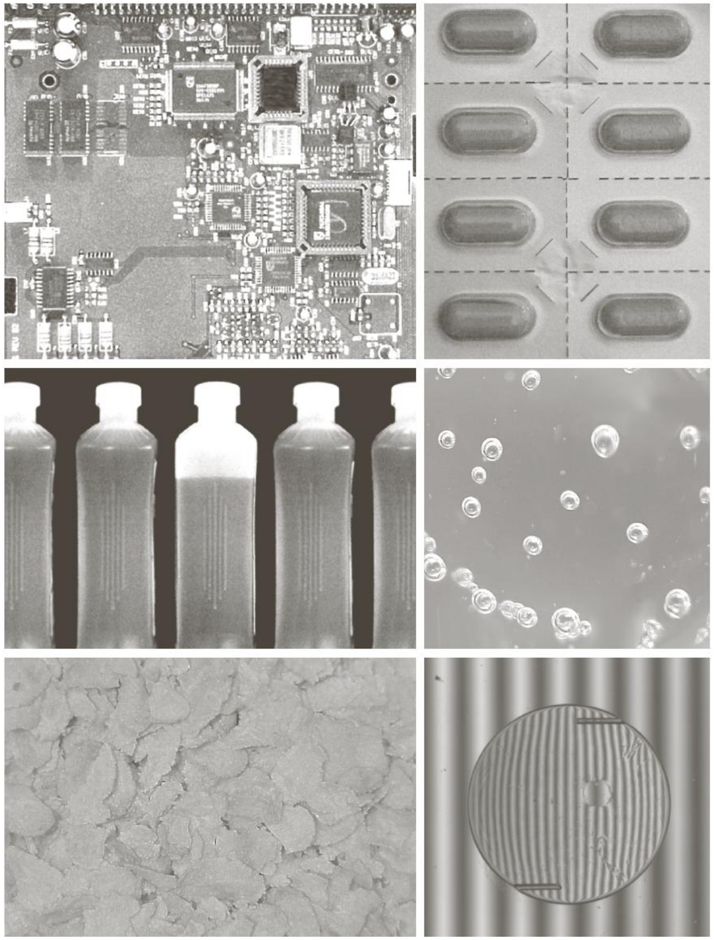
Object Detection and Tracking

Object tracking is important because it enables several important applications such as: Security and surveillance- to recognize people, to provide better sense of security using visual information.

Foreground Extraction

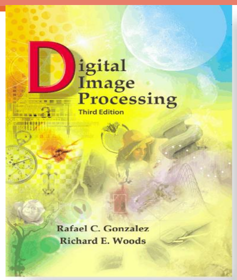
- **Foreground Extraction** is the process of segmenting the foreground objects from the background.

Industrial Inspection



a b
c d
e f

FIGURE 1.14 Some examples of manufactured goods often checked using digital image processing. (a) A circuit board controller. (b) Packaged pills. (c) Bottles. (d) Air bubbles in a clear-plastic product. (e) Cereal. (f) Image of intraocular implant. (Fig. (f) courtesy of Mr. Pete Sites, Perceptics Corporation.)

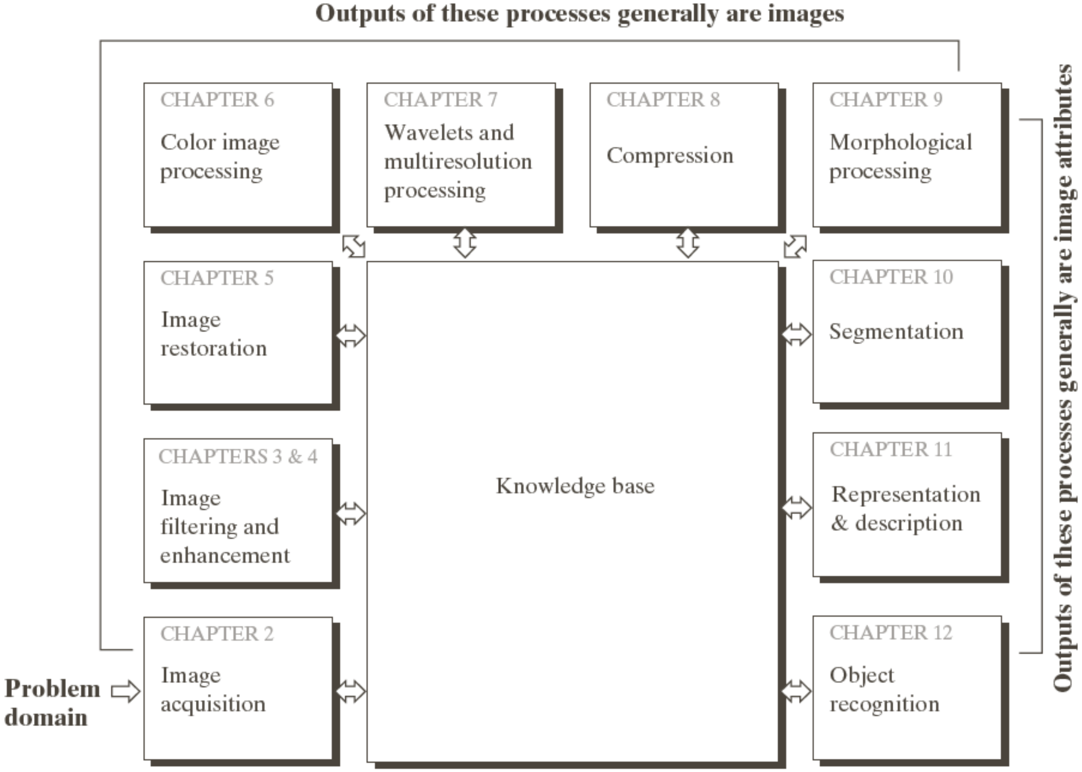


Contd…

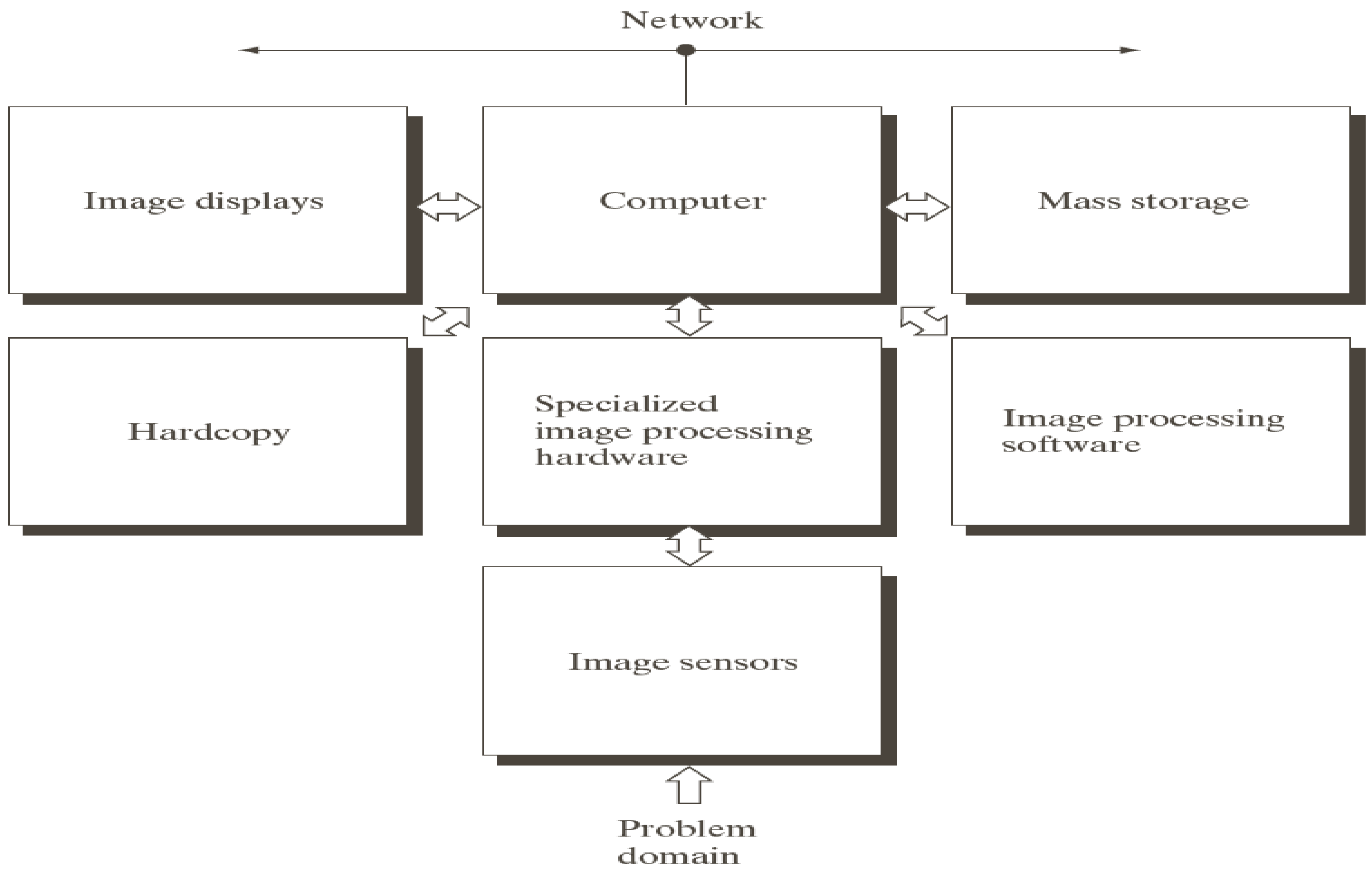
a b
c
d

FIGURE 1.15 Some additional examples of imaging in the visual spectrum. (a) Thumb print. (b) Paper currency. (c) and (d) Automated license plate reading. (Figure (a) courtesy of the National Institute of Standards and Technology. Figures (c) and (d) courtesy of Dr. Juan Herrera, Perceptics Corporation.)

Fundamental Steps in DIP



Components of an IP System



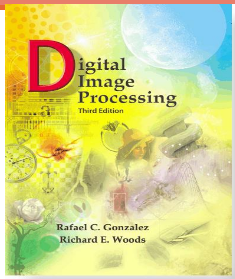
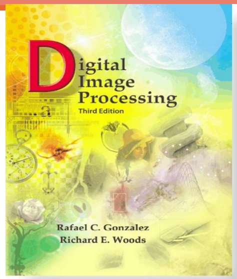


Image Sensing and Acquisition

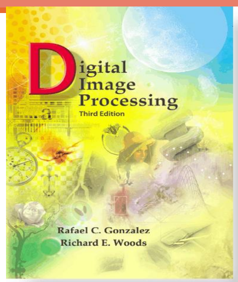
- Light & EM Spectrum
- Image Formation Model
- Image Acquisition using a Single Sensor
- Image Acquisition using Sensor Strips
- Image Acquisition using Sensor Arrays



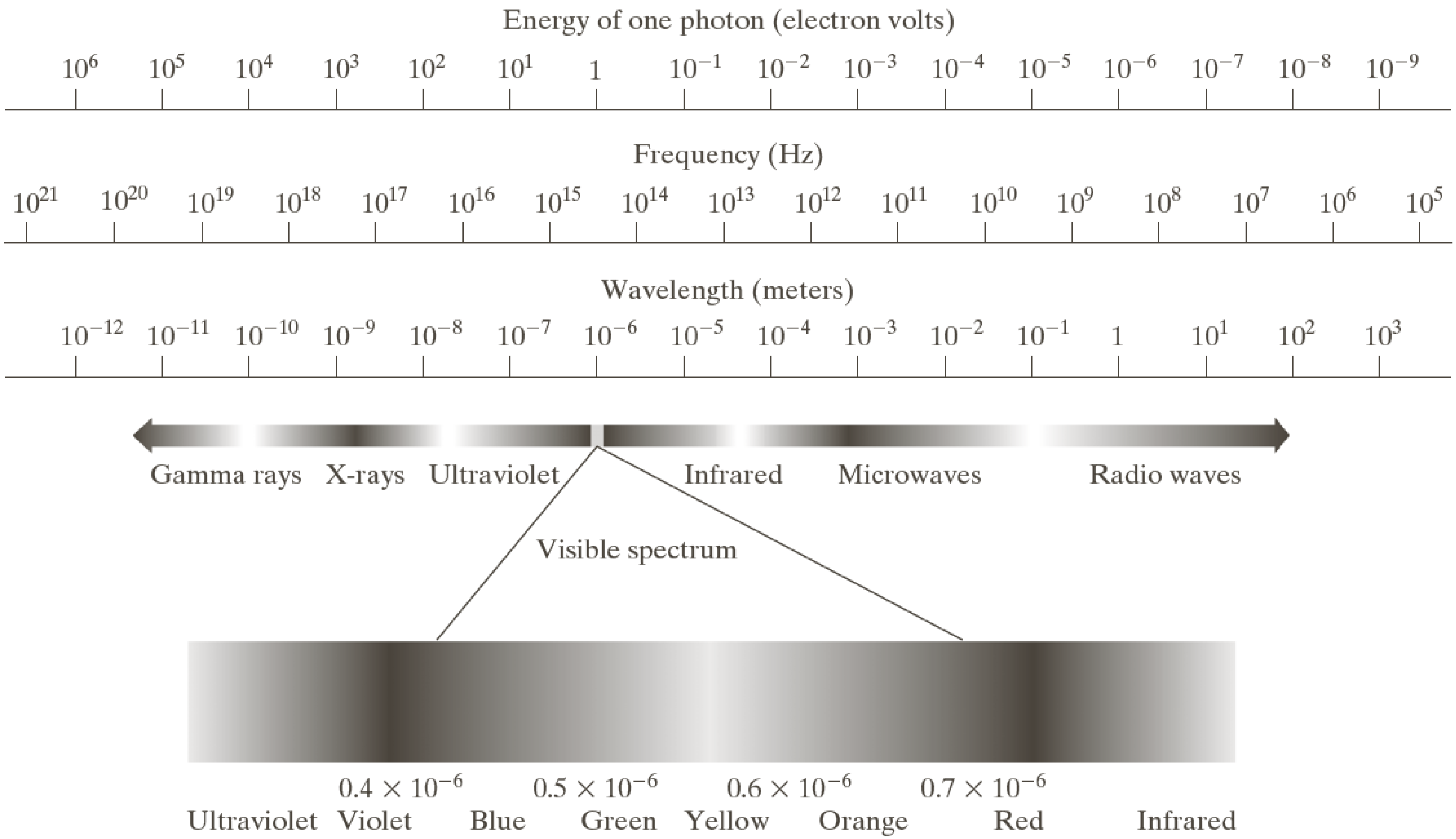
Light

The colors that humans perceive in an object are determined by the nature of the light reflected from the object.

- Monochromatic light
  1. Intensity
- Chromatic light bands
  1. Radiance
  2. Luminance
  3. Brightness



Light & EM Spectrum



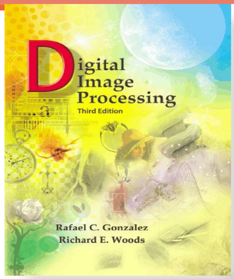


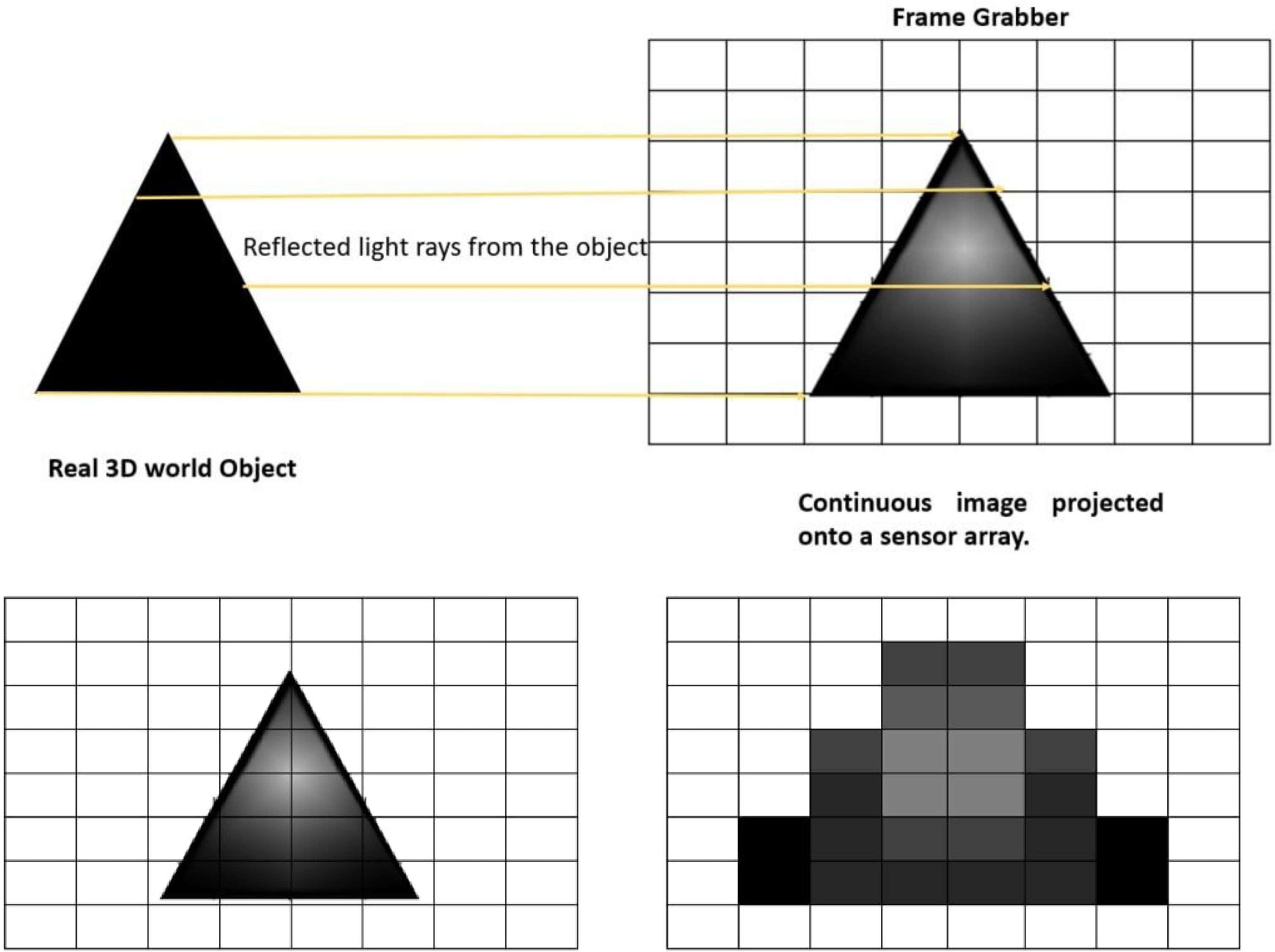
Image Formation Model

- Image formation is an analog to digital conversion of an image with the help of 2D Sampling and Quantization techniques that is done by the capturing devices like cameras.

- In general, we see a 2D view of the 3D world. In the same way, the formation of the analog image took place.

- It is basically a conversion of the 3D world that is our analog image to a 2D world that is our Digital image.

- Generally, a frame grabber or a digitizer is used for sampling and quantizing the analog signals.

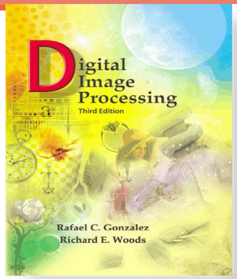


Real 3D world Object

Continuous image projected onto a sensor array.

Continuous image projected onto a sensor array.

Result of image sampling and quantization.



A Simple Image Formation Model

$$f(x,y) = i(x,y) r(x,y)$$

$f(x,y)$: intensity at the point (x,y)

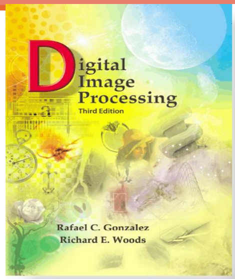
$i(x,y)$: illumination at the point (x,y)

(the amount of source illumination incident on the scene)

$r(x,y)$: reflectance/transmissivity at the point (x,y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x,y) < \infty$ and $0 < r(x,y) < 1$



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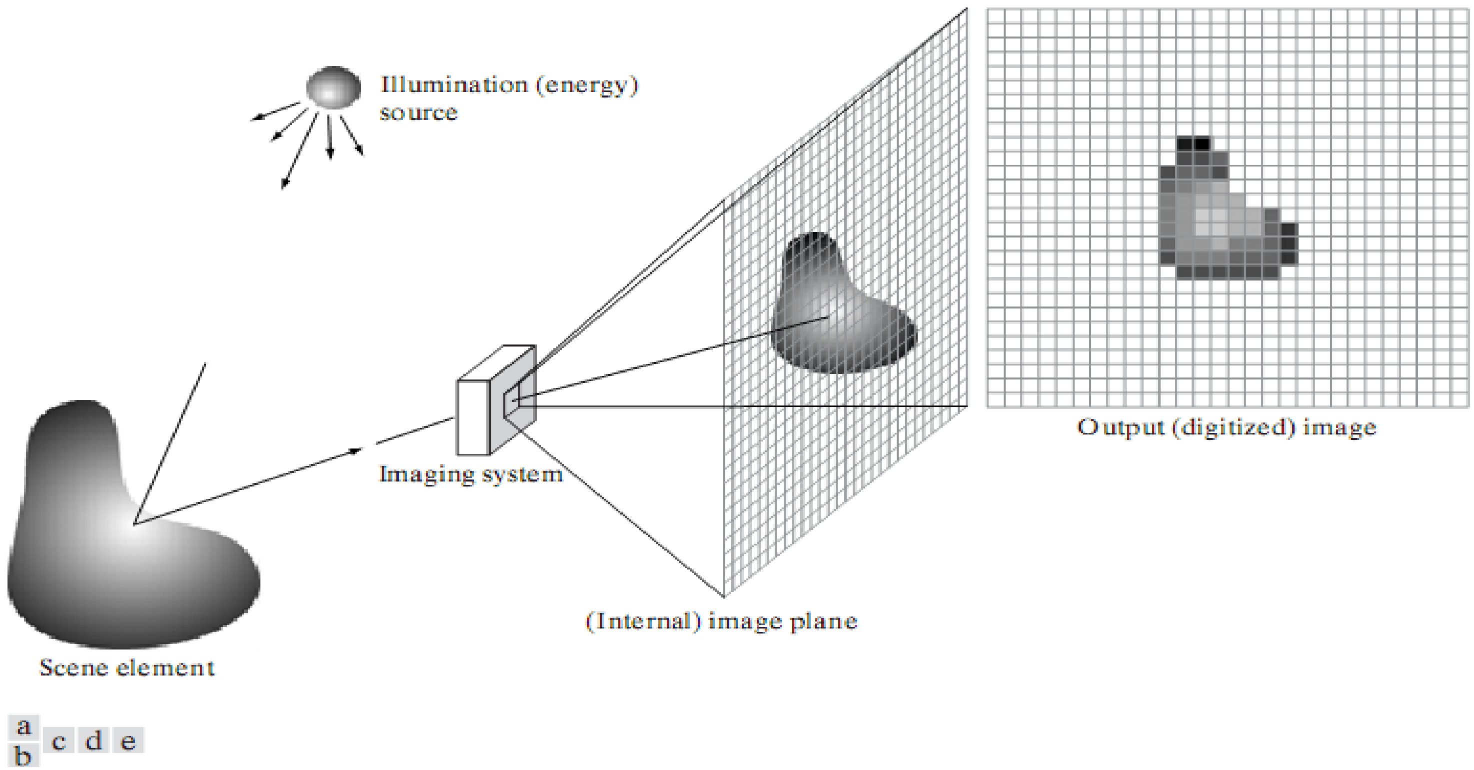
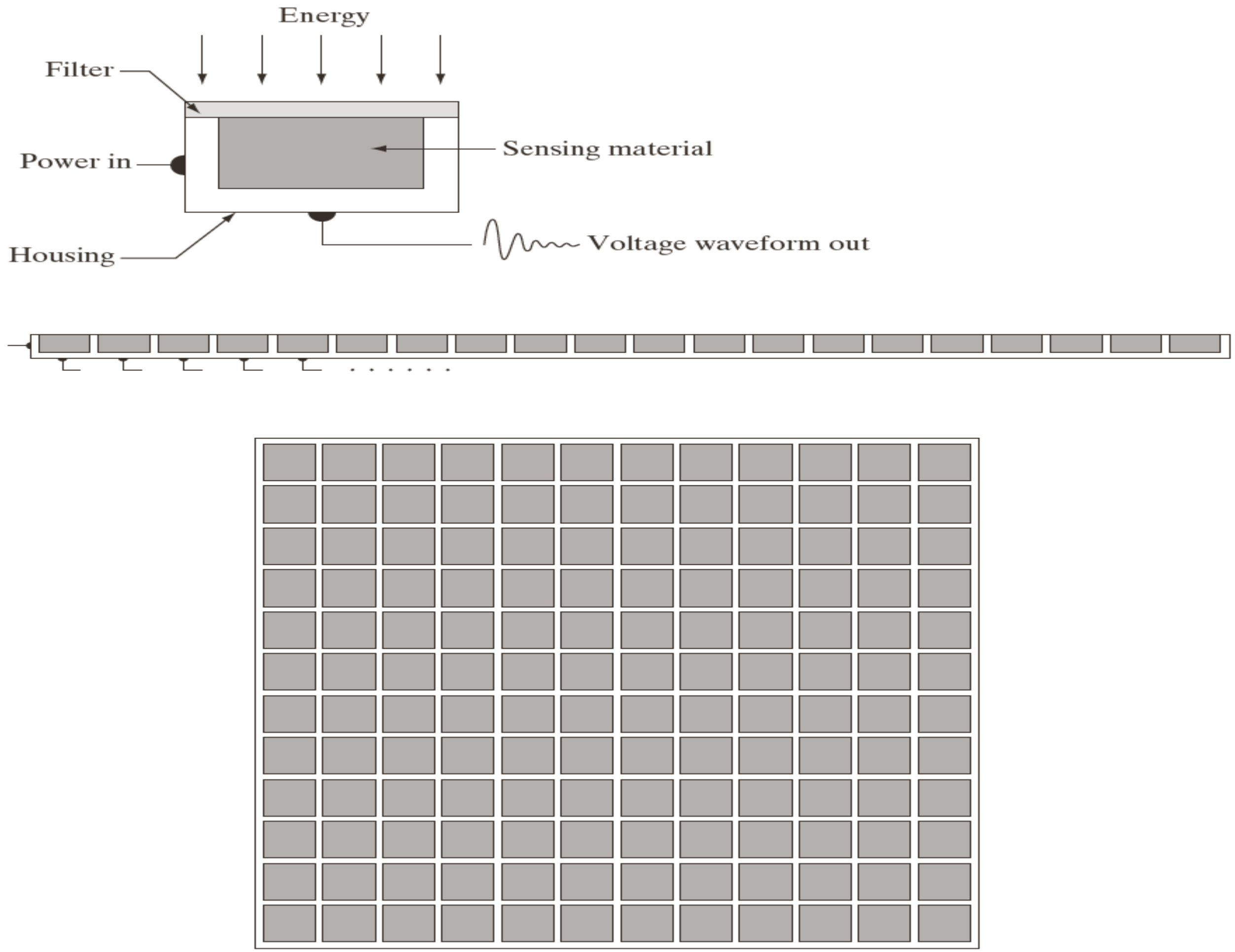


(a) Energy ("illumination") source (b) An element of a scene (c) Imaging system (d) Projection of the scene onto the image plane (e) Digitized image

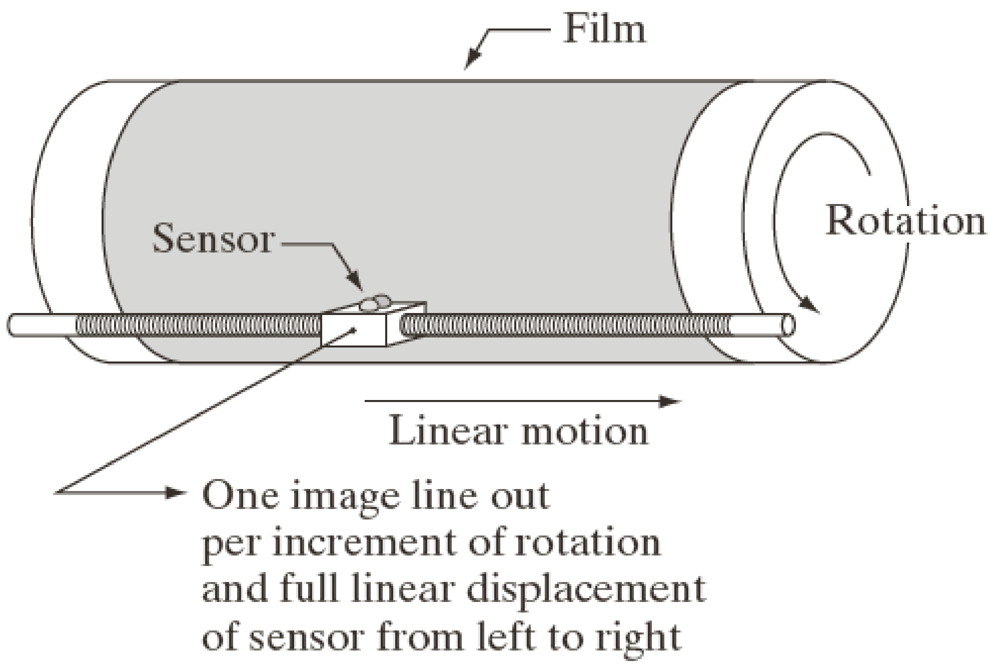
Image Acquisition



a
b
c

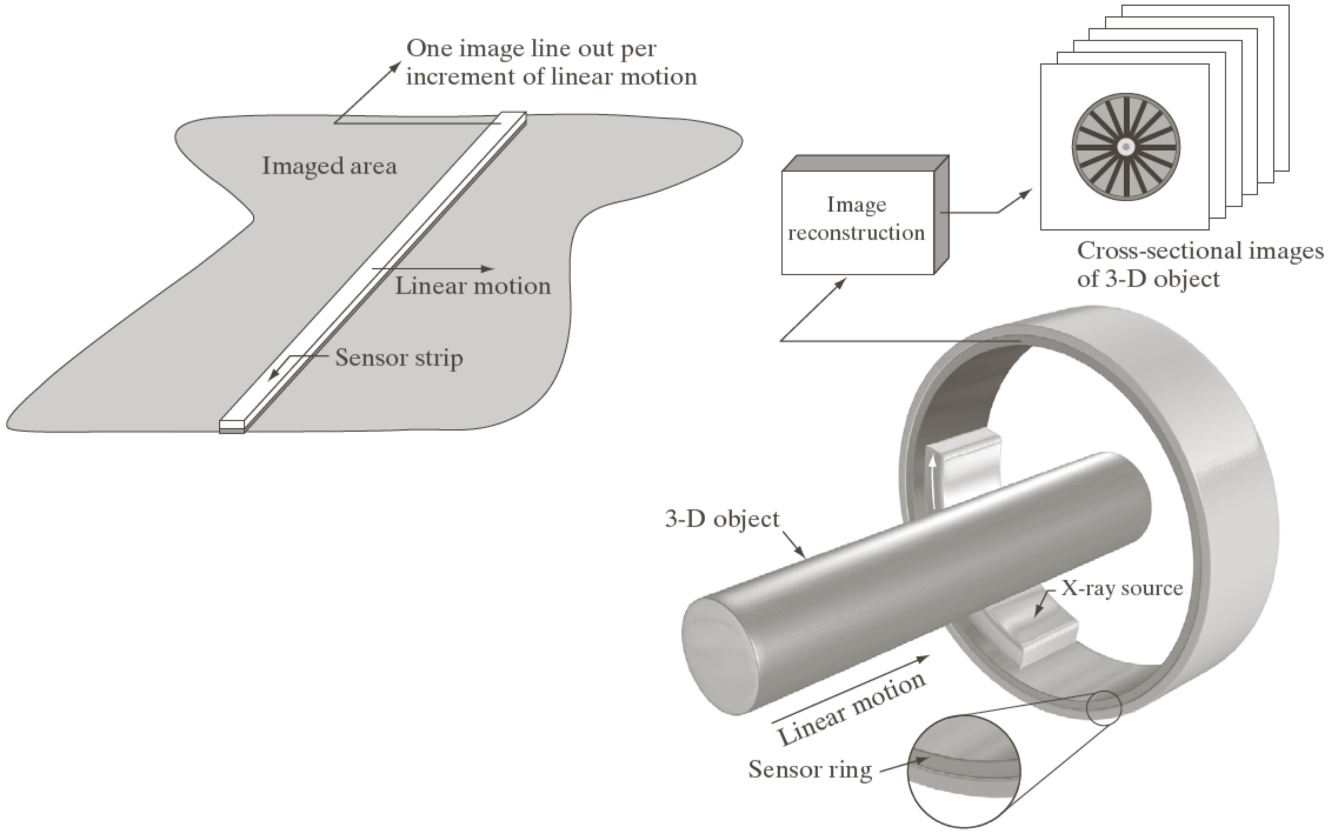
(a) Single imaging Sensor
(b) Line sensor
(c) Array sensor

Image Acquisition using a Single Sensor



Combining a single sensor with motion to generate a 2-D image

Image Acquisition using Sensor Strips



a b

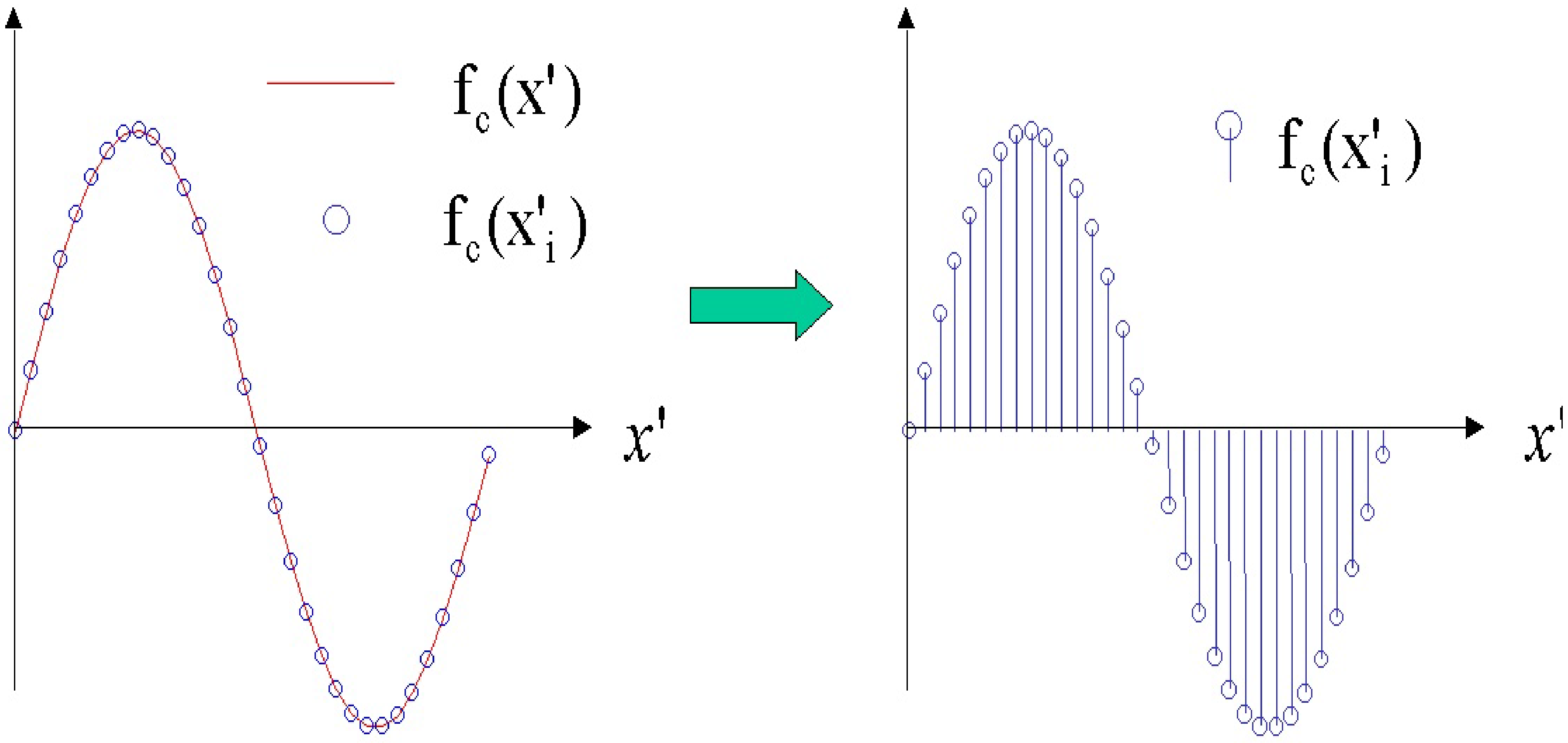
(a) Image acquisition using a linear sensor strip

(b) Image acquisition using a circular sensor strip

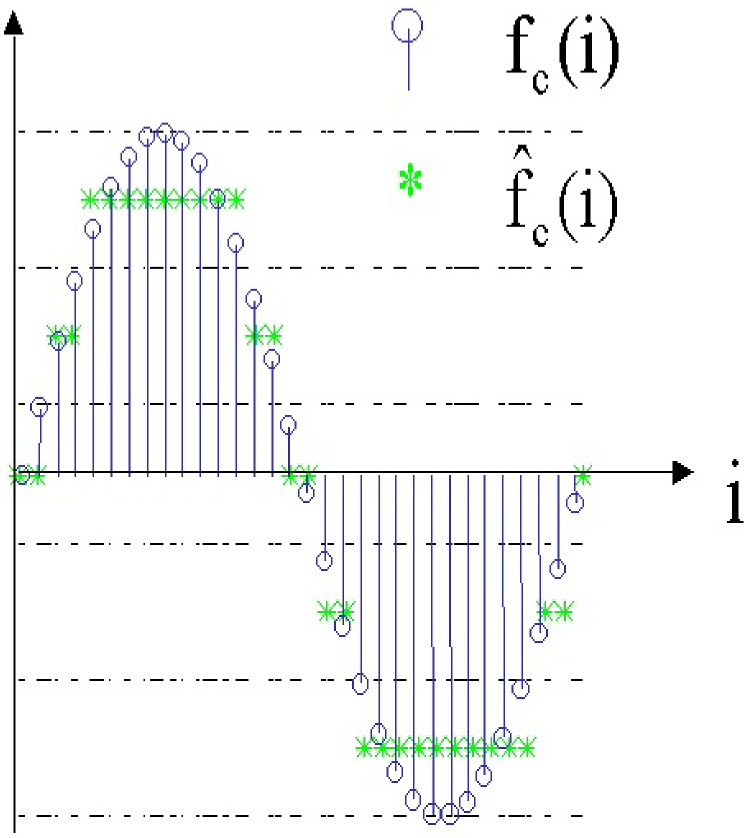
Image Sampling and Quantization

- Basic Concepts in Sampling & Quantization
- Representing Digital Images
- Spatial & Intensity Resolution

Sampling



Sampling and Quantization



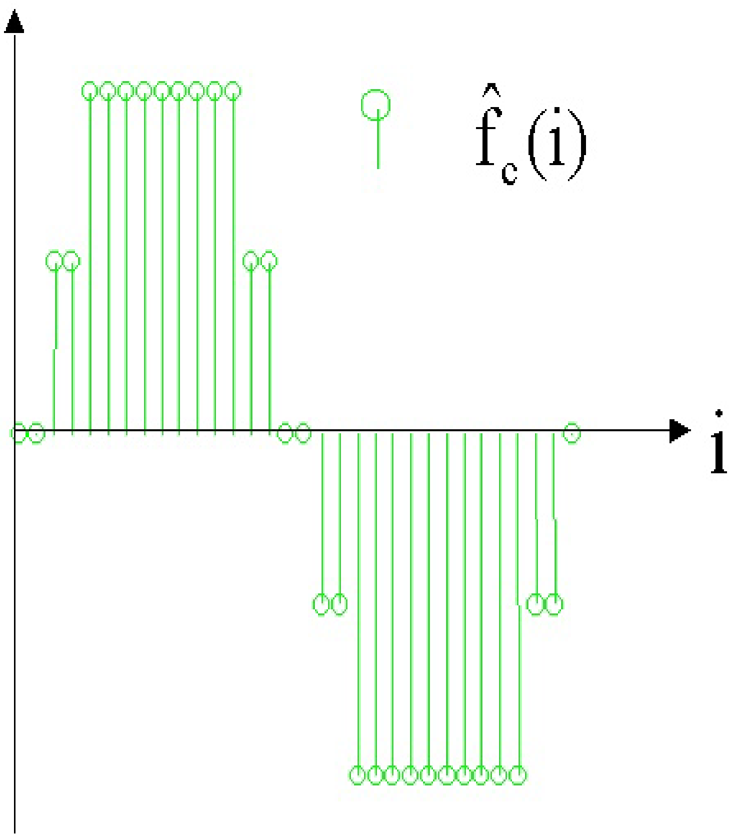
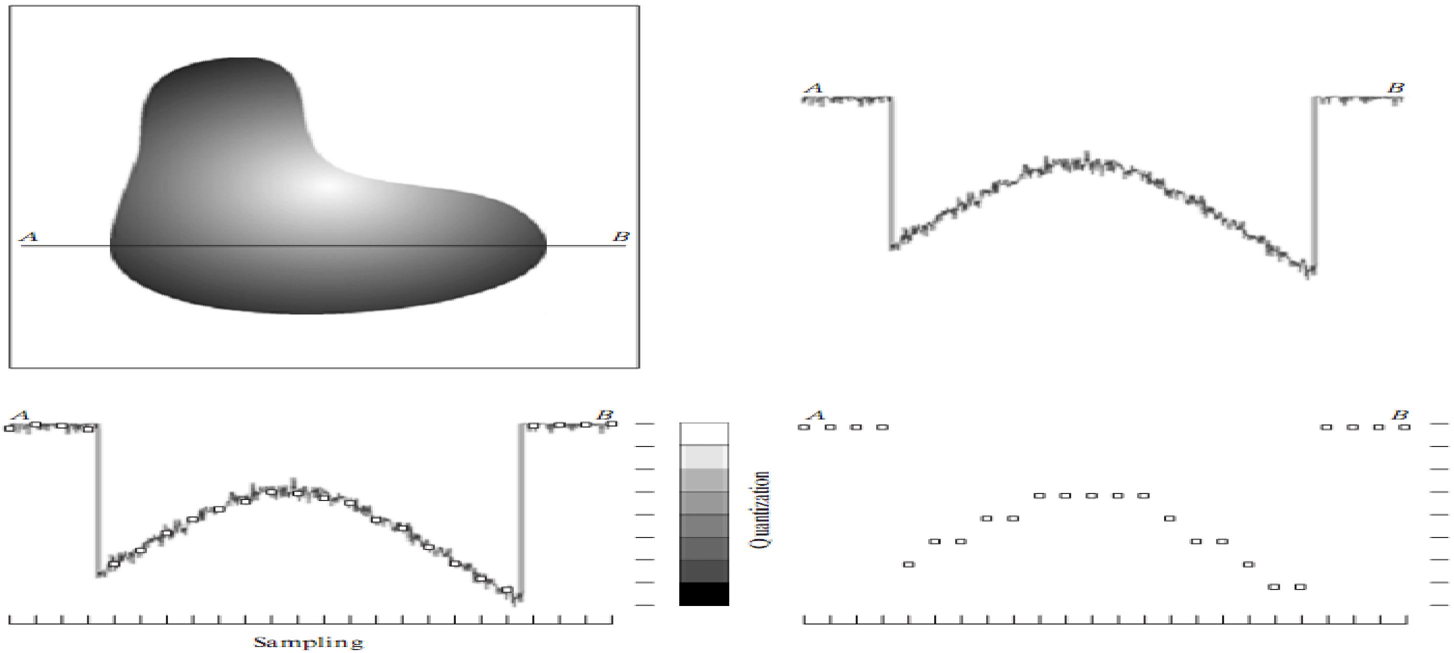
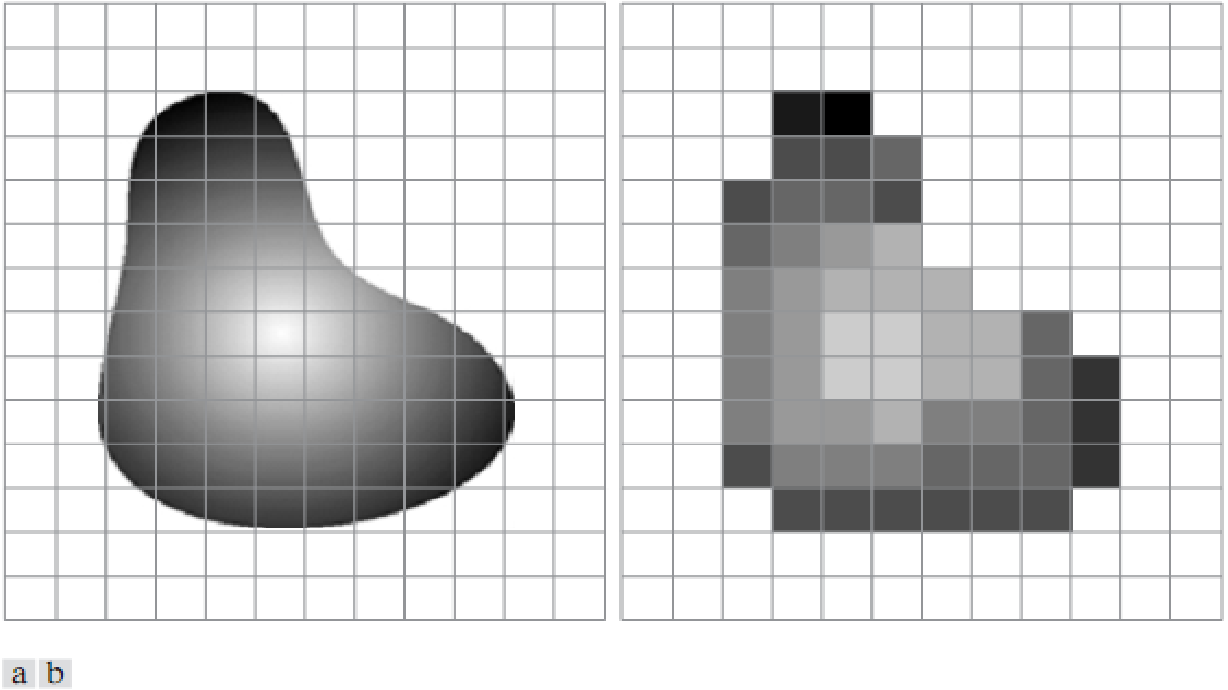


Image Sampling and Quantization

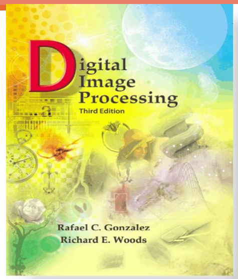


a b
c d
(a) Continuous image (b) A scan line from A to Bin the continuous image, used to illustrate the concepts of sampling and quantization (c) Sampling and quantization. (d) Digital scan line

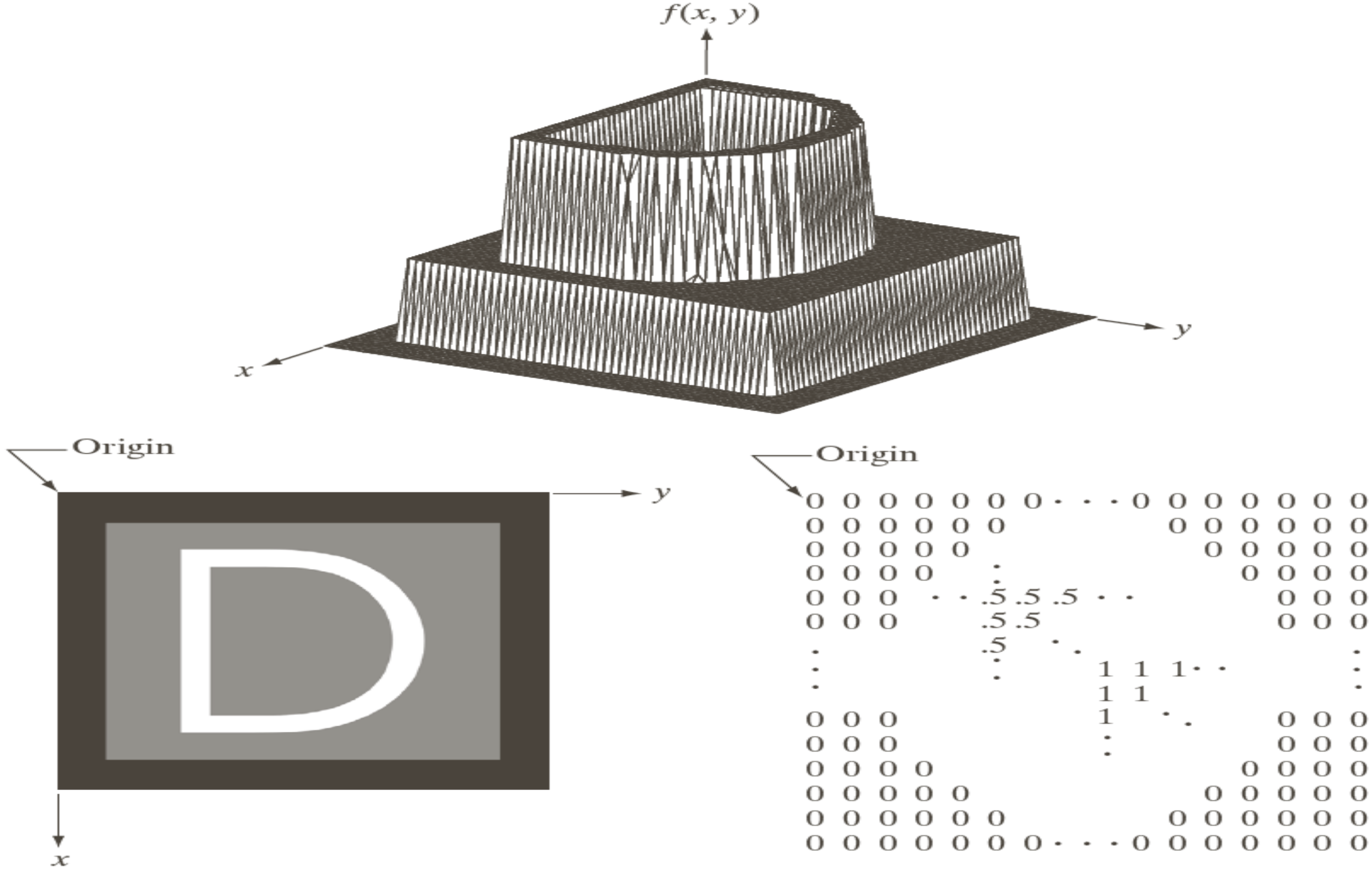
Contd…

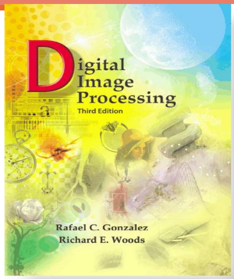


(a) Continuous image projected onto a sensor array (b) Result of image sampling and quantization



Representing Digital Images





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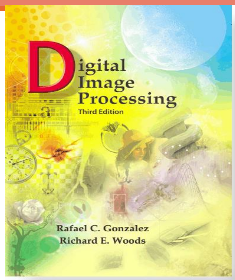
M×N numerical array

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

M×N numerical array in MATLAB

$$f(x,y)=\begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$



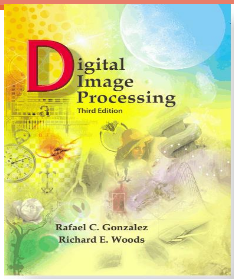
Contd…

Discrete intensity interval [0, L−1], $L=2^k$

The number b of bits required to store a M × N digitized image **$b = M \times N \times k$**

Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912



Resolution

- **Spatial resolution**

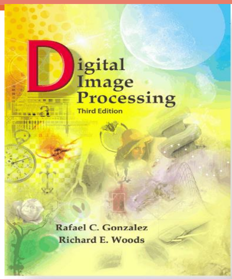
  — A measure of the smallest discernible detail in an image

  — stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

- **Intensity** (or) **Gray-Level resolution**

  — The smallest discernible change in intensity level

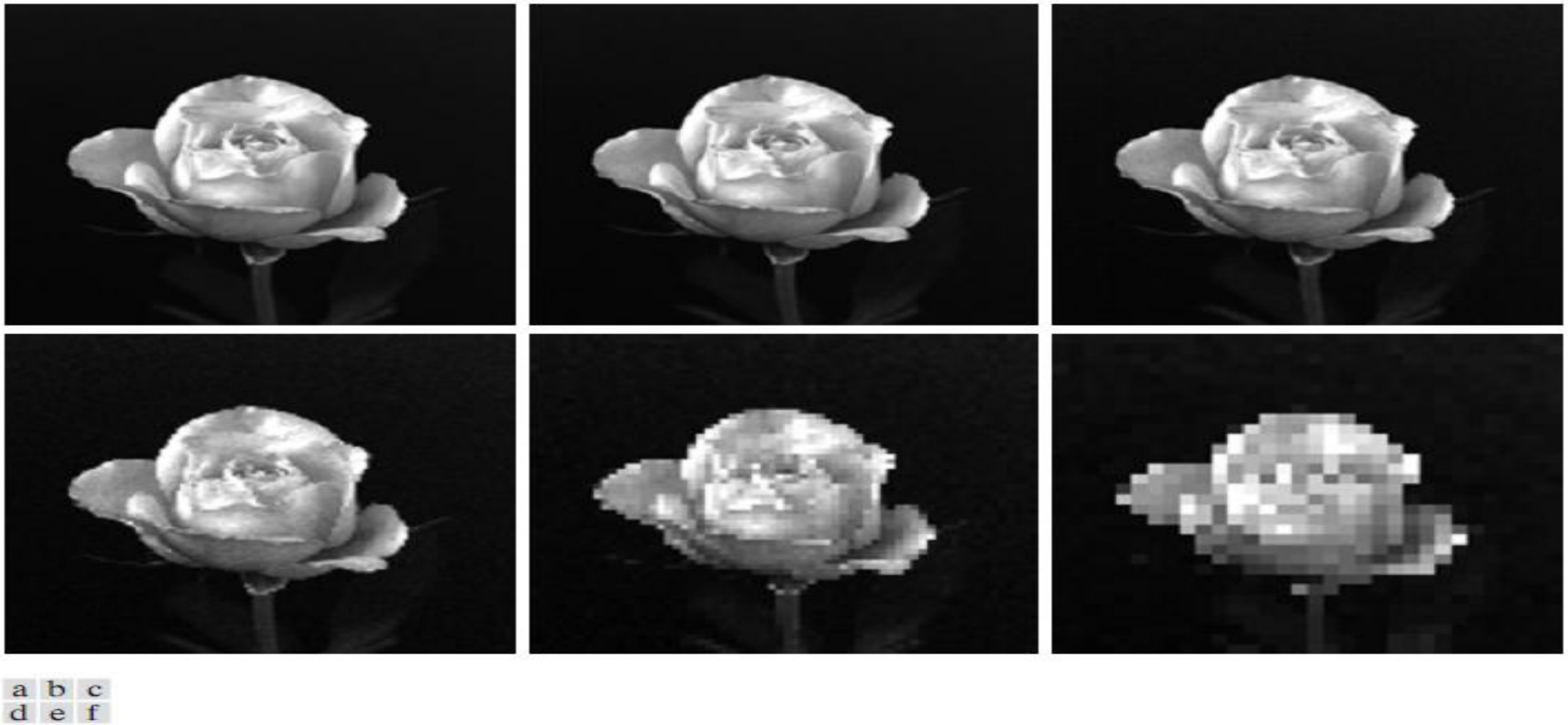
  — stated with *8 bits, 12 bits, 16 bits, etc.*



Spatial Resolution

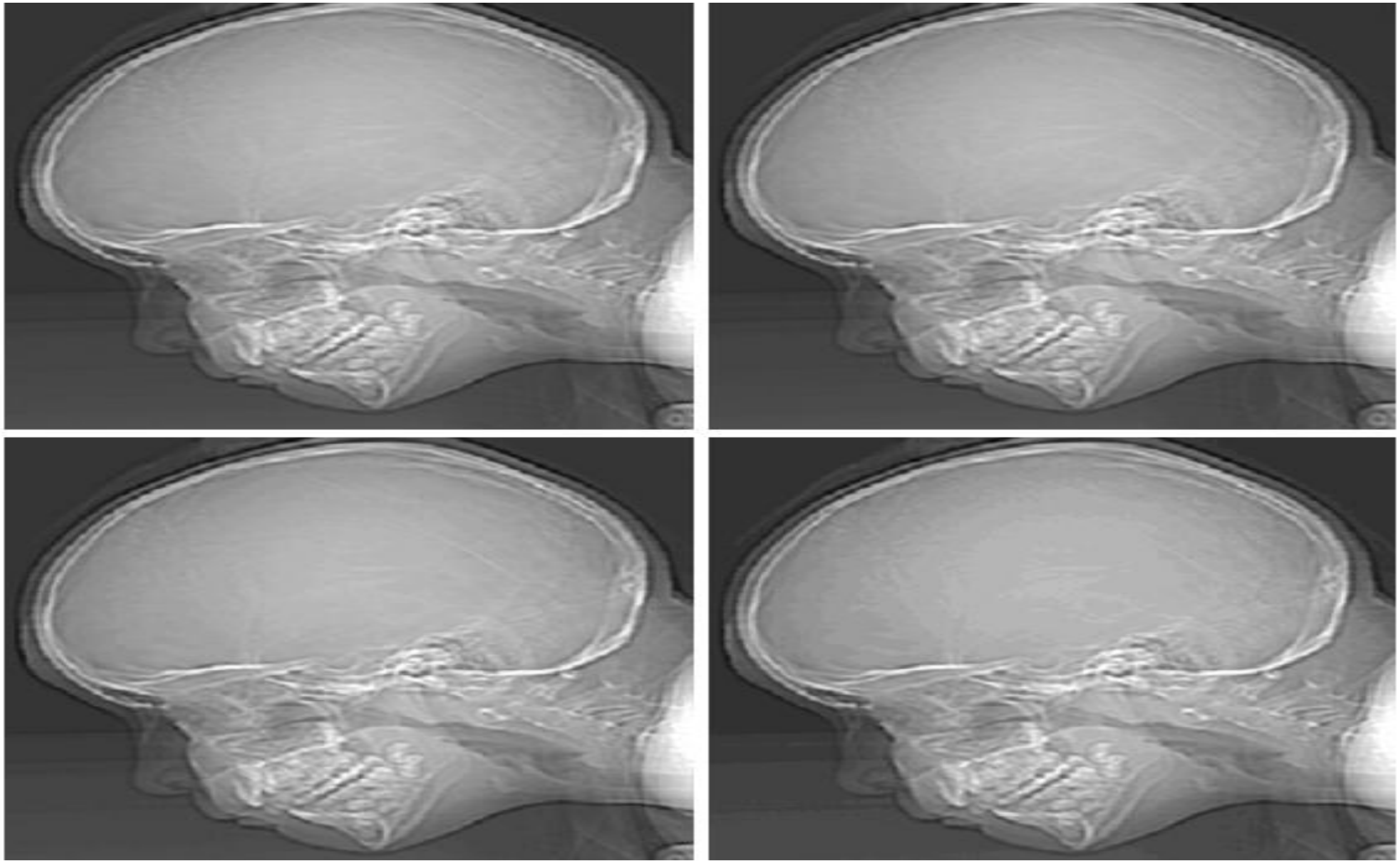
A 1024*1024, 8-bit image sub sampled down to size 32*32 pixels. The number of allowable gray levels was kept at 256.

Contd…

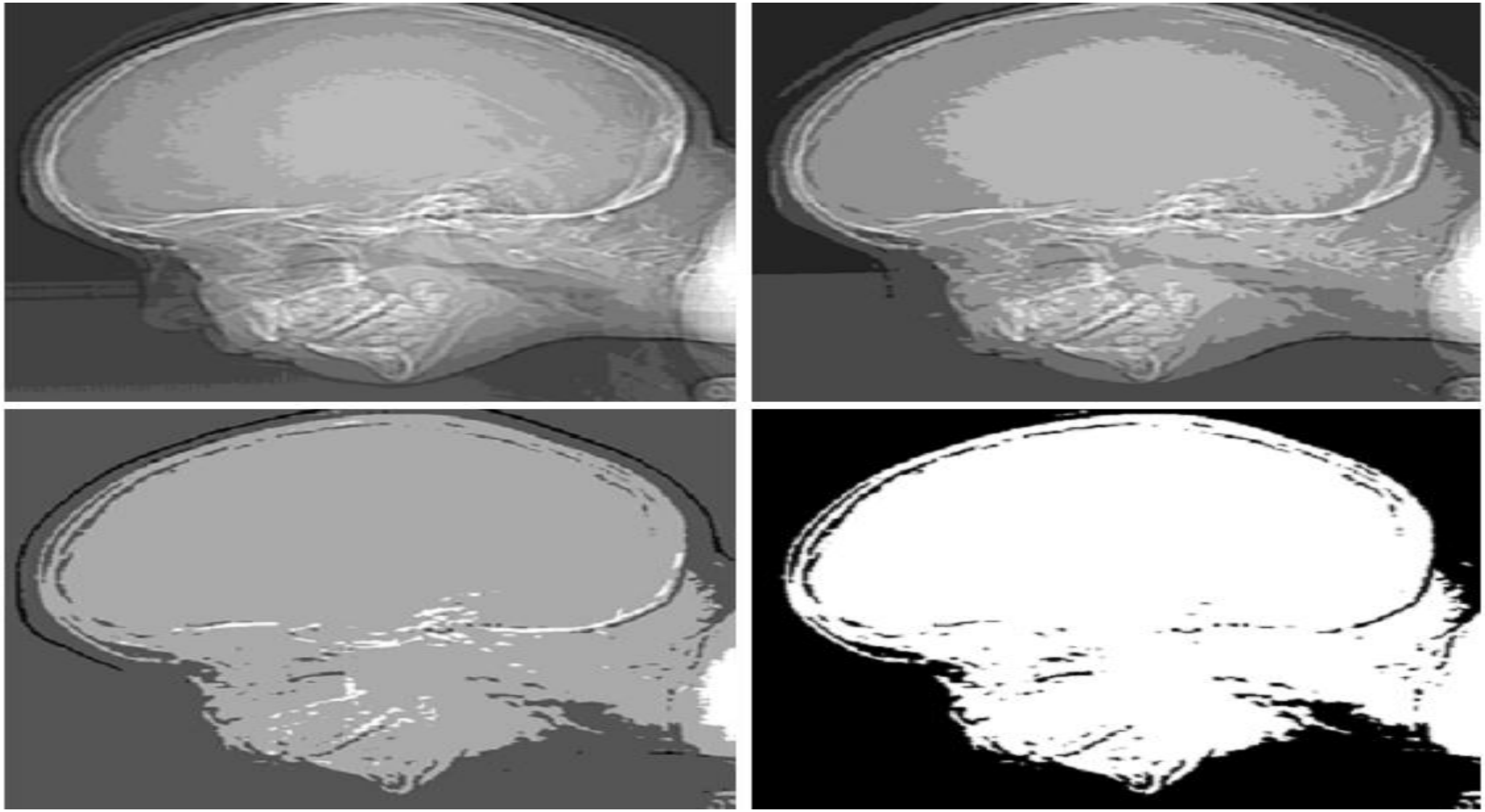


(a) 1024*1024, 8-bit image (b) 512*512 image resampled into 1024*1024 pixels by row and column duplication (c) through (f) 256*256, 128*128, 64*64, and 32*32 images resampled into 1024*1024 pixels

Gray level Resolution



Contd…



(a) 452*374, 256-level image (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant (e)–(g) Image displayed in 16, 8, 4, and 2 gray levels.

Contd….

a b c

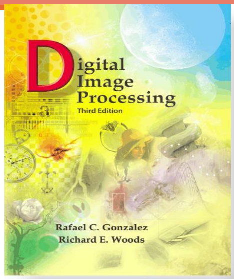
(a) Image with a low level of detail

(b) Image with a medium level of detail

(c) Image with a relatively large amount of detail

Image File Formats

- File Formats
- Image File Formats



File Formats

What are file formats/ extensions?

- A file format/ extension relates to the last three/ four letters at the end of a file name e.g. image_transfer.**doc**

- It is possible that you cannot see them as your computer may be set to hide them

What it the purpose of file formats/ extensions?

- File formats/ extensions are there so that your computer knows what programme to use to open the file
- As you may already know it is not possible to open a music file (**.wma**) in Microsoft Word (**.doc**) and so on

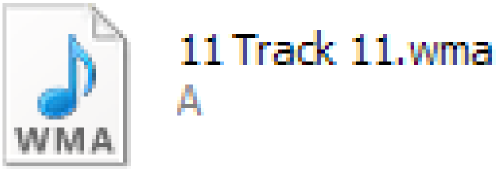


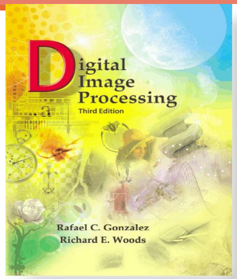
Image File Formats

Why would you use different file formats?

- The reason for using a different file format when saving a document is that each format will have different properties that may be required for your document
- This is particularly important when saving graphics e.g. The **GIF** format is used for still images and simple animations and a **JPEG** will only store static photographic images

Below are some of the file formats/ extensions that are related with image files on your computer:

- **.BMP** – Bit Map
- **.GIF** – Graphics Interchange Format
- **.PNG** – Portable Network Graphics
- **.PGM** – Portable Grey Map
- **.TIFF** – Tagged Image File Format
- **.JPEG** – Joint Photographic Experts Group



Contd…

.BMP

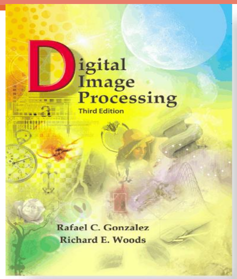
- Bitmap format Windows only?
- NO Compression means LARGE files
- Standard Screen Snapshot is BMP
- The number of bits per pixel (1, 4, 8, 15, 24, 32, or 64) for a given BMP file is specified in a file header. BMP files with 24 bits per pixel are common

.GIF

- **Compression:** Lossless – compression without loss of quality
- **Best For:** Web Images because they are limited to only 256 colors
- **Special Attributes:** Can be Animated, Can Save Transparency

.PNG

- **Compression:** Lossless – compression without loss of quality
- **Best For:** Web Images able to handle up to 16 million colors, unlike the 256 colors supported by GIF
- **Special Attributes:** Can Save Transparency



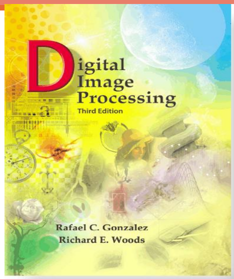
Contd…

.PGM

- Store grayscale 2D images.
- Each pixel within the image contains only one or two bytes of information (8 / 16 bits).
- While that might not sound like a lot of information, PGM files can hold tens of thousands of shades — ranging from pure black to white and every shade of grey in between.

.TIFF

- **Compression:** Lossless - no compression results larger file sizes
- Very high-quality images
- **Best For**: High quality prints, professional publications, archival copies
- **Special Attributes:** Can save transparencies



Contd…

.JPEG (or JPG)

- **Compression:** Lossy – some file information is compressed or lost
- **Best For:** Web Images, Non-Professional Printing, E-Mail, PowerPoint
- **Special Attributes:** Can choose amount of compression when saving in image editing programs like Adobe Photoshop or GIMP.

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Path
- Connectivity
- Regions and boundaries

Neighbors

Neighbors of a pixel p at coordinates (x, y)

- **4-neighbors of p**, denoted by **$N_4(p)$**:

  (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

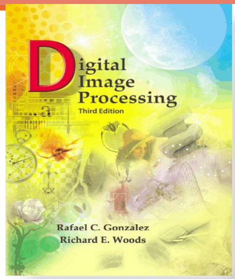
- **4 diagonal neighbors of p**, denoted by **$N_D(p)$**:

  (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

- **8 neighbors of p**, denoted **$N_8(p)$**

  $N_8(p) = N_4(p) \cup N_D(p)$

x-1, y-1	x-1, y	x-1, y+1
x, y-1	x, y	x, y+1
x+1, y-1	x+1, y	x+1, y+1



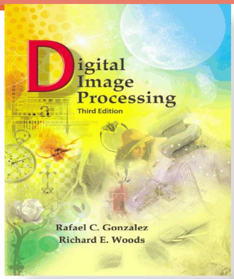
Adjacency

Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set **$N_4(p)$**.
- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set **$N_8(p)$**.
- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

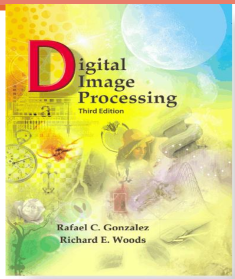
  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.



Example: Adjacency

Let V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$



Example: Adjacency

Let V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

8-adjacency

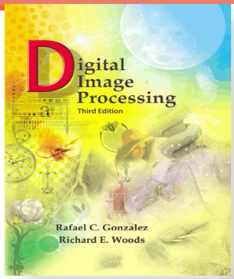
Example: Adjacency

Let V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacency

m-adjacency



Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

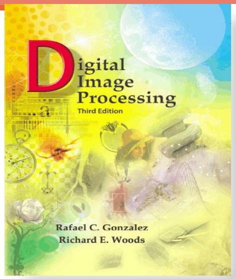
  $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$

  Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \leq i \leq n$.

- Here *n* is the *length* of the path.

- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.

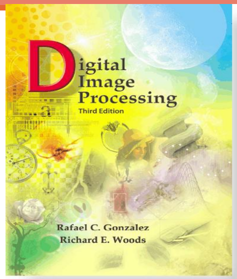
- We can define 4-, 8-, and m-paths based on the type of adjacency used.



Example: Adjacency & Path

Let V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

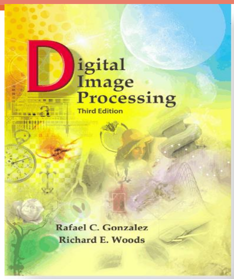


Example: Adjacency & Path

Let V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent



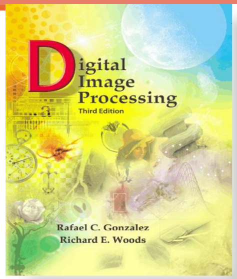
Example: Adjacency & Path

Let V = {1, 2}

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacency

m-adjacency



Example: Adjacency & Path

Let V = {1, 2}

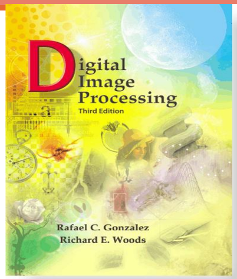
$$\begin{array}{ccc} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacency m-adjacency

The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)



Example: Adjacency & Path

Let V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix}$$

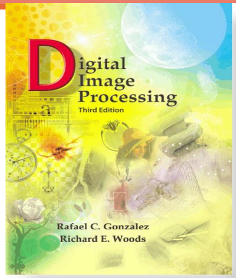
$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacency

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

m-adjacency

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)



Example: Adjacency & Path

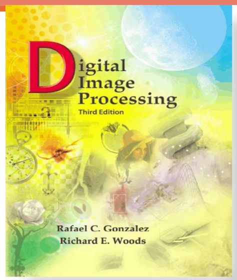
Let V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacency   m-adjacency

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)



Example: Adjacency & Path

Let V = {1, 2}

$$\begin{array}{ccc} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{array}$$

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacency

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

m-adjacency

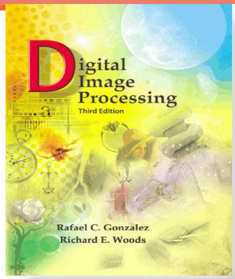
The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

(1,3), (1,2), (2,2), (3,3)



Connectivity

Connected in S

Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where $$\forall i, 0 \le i \le n, (x_i, y_i) \in S$$

Region

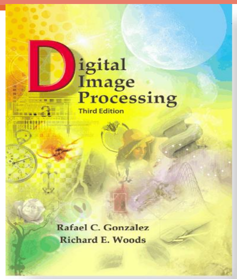
Let S represent a subset of pixels in an image

For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.

If S has only one connected component, then S is called ***Connected Set***.

We call R a **region** of the image if R is a connected set

Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
Regions that are not to be adjacent are said to be ***disjoint***.



Boundary

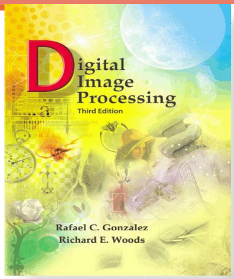
- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Foreground and background

- An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

All the points in R_u is called **foreground;**

All the points in $(R_u)^c$ is called **background**.



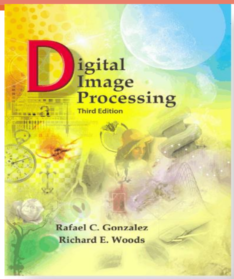
Distance Measures

Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0 \quad [D(p, q) = 0, \text{ if } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$



Contd…

The following are the different Distance measures:

a. **Euclidean Distance** :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. **City Block Distance**:

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. **Chess Board Distance**:

$D_8(p, q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2