Smoothing frequency domain filters

Ideal Lowpass Filter (ILPF)

ILPF is the simplest lowpass filter that "cuts off" all high frequency components of the DFT that are at a distance greater than a specified distance D_0 from the origin of the (centered) transform.

The transfer function of this filter is:

$$H(u,v) = \begin{cases} 1 & if \ \ D(u,v) \leq D_0 \\ 0 & if \ \ D(u,v) > D_0 \end{cases}$$

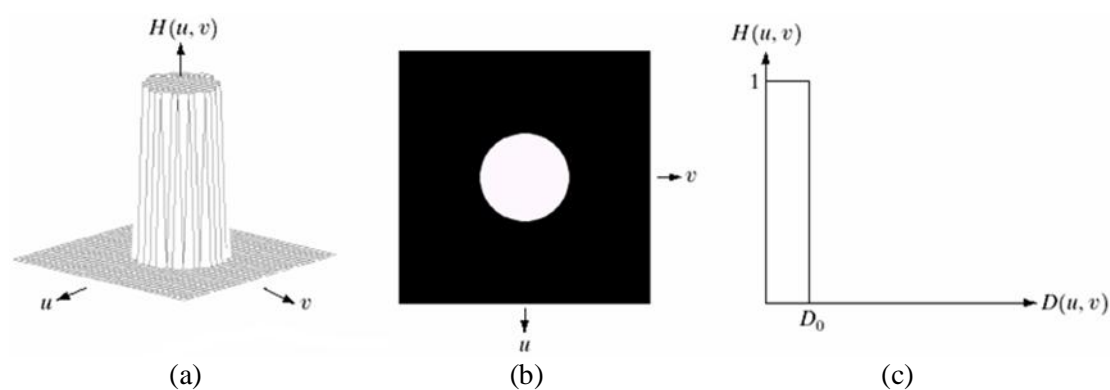where D_0 is the cutoff frequency, and $D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$

Figure 8.1 (a) ILPF transfer function. (b) ILPF as an image. (c) ILPF radial cross section

The ILPF indicates that all frequencies inside a circle of radius D_0 are passed with no attenuation, whereas all frequencies outside this circle are completely attenuated.

The next figure shows a gray image with its Fourier spectrum. The circles superimposed on the spectrum represent cutoff frequencies 5, 15, 30, 80 and 230.
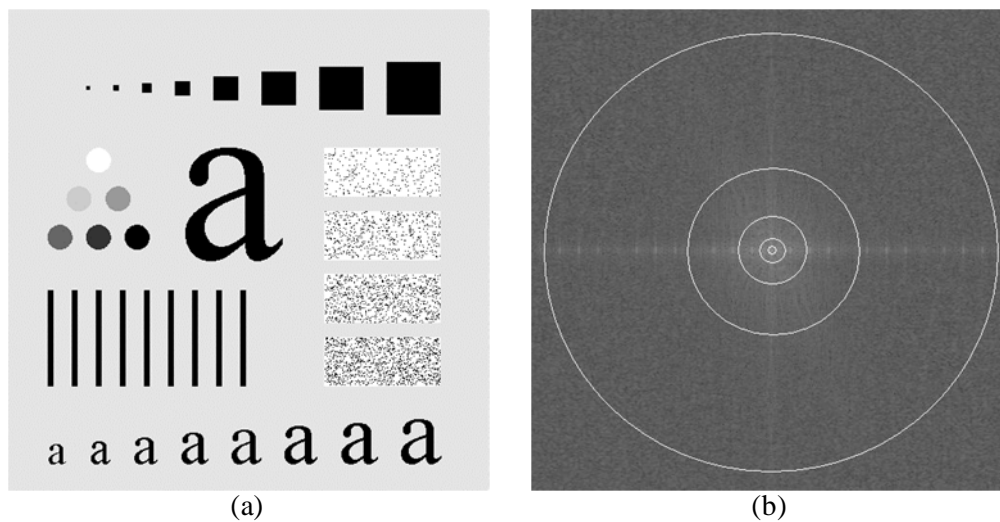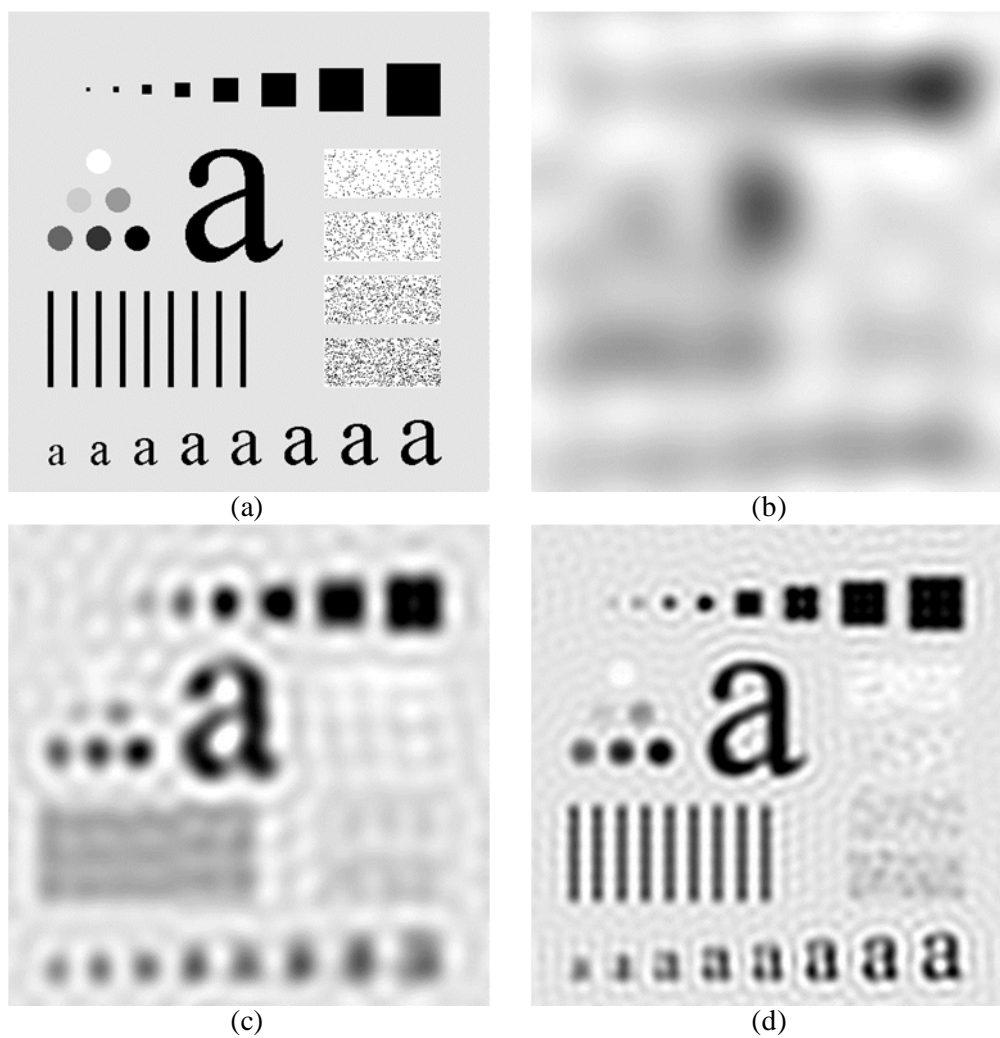
(a)   (b)
Figure 8.2 (a) Original image. (b) its Fourier spectrum

The figure below shows the results of applying ILPF with the previous cutoff frequencies.

(a)   (b)

(c)   (d)

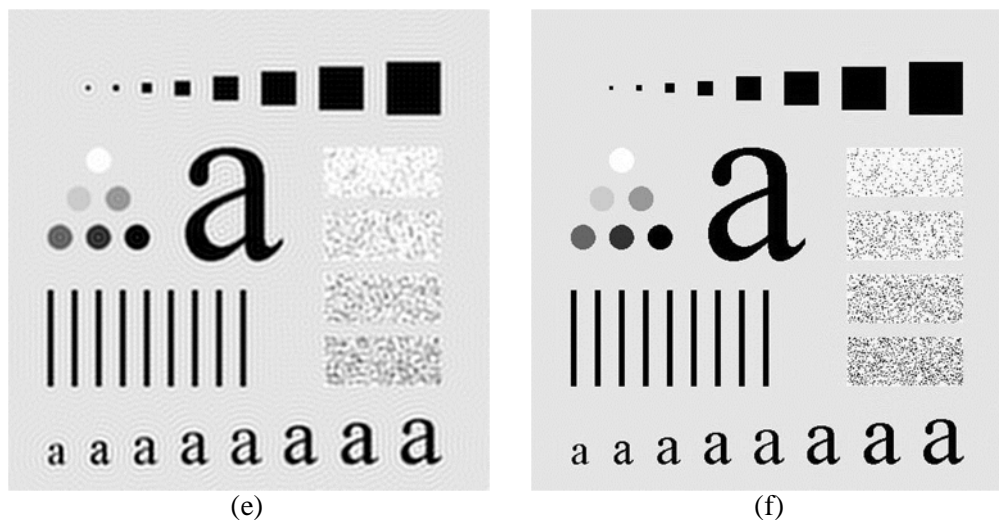(e)   (f)

Figure 8.3 (a) Original image. (b) - (f) Results of ILPF with cutoff frequencies 5, 15, 30, 80, and 230 respectively.

We can see the following effects of ILPF:

1. Blurring effect which decreases as the cutoff frequency increases.

2. Ringing effect which becomes finer (i.e. decreases) as the cutoff frequency increases.

Butterworth Lowpass Filter (BLPF)

The BLPF of order n and with cutoff frequency at a distance D_0 from the origin is defined as:

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

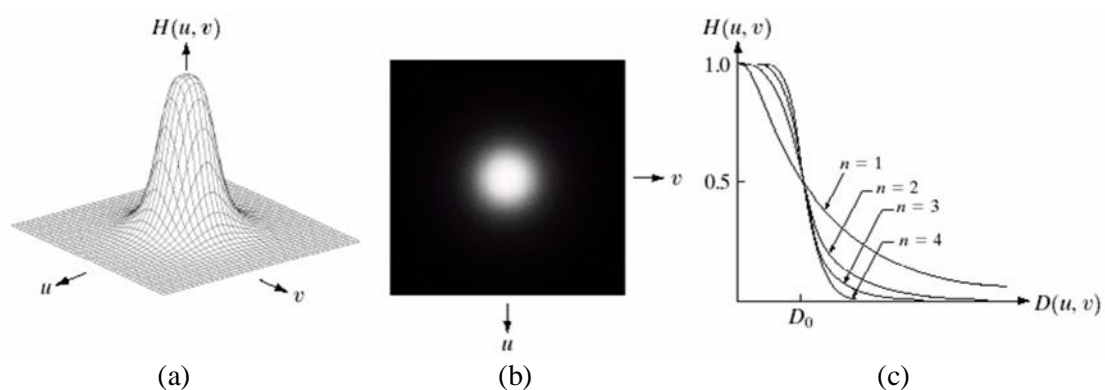(a)   (b)   (c)

Figure 8.4 (a) BLPF transfer function. (b) BLPF as an image. (c) BLPF radial cross section
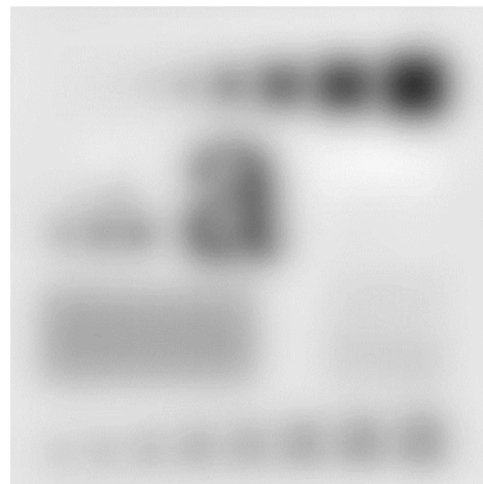
Unlike ILPF, the BLPF transfer function does not have a sharp transition that establishes a clear cutoff between passed and filtered frequencies. Instead, BLPF has a smooth transition between low and high frequencies.
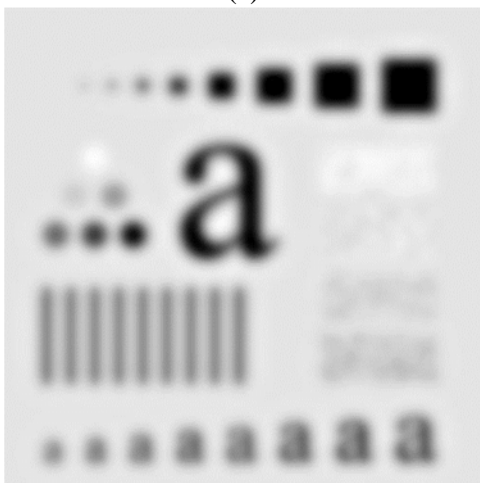
The figure below shows the results of applying BLPF of order 2 with the same previous cutoff frequencies.
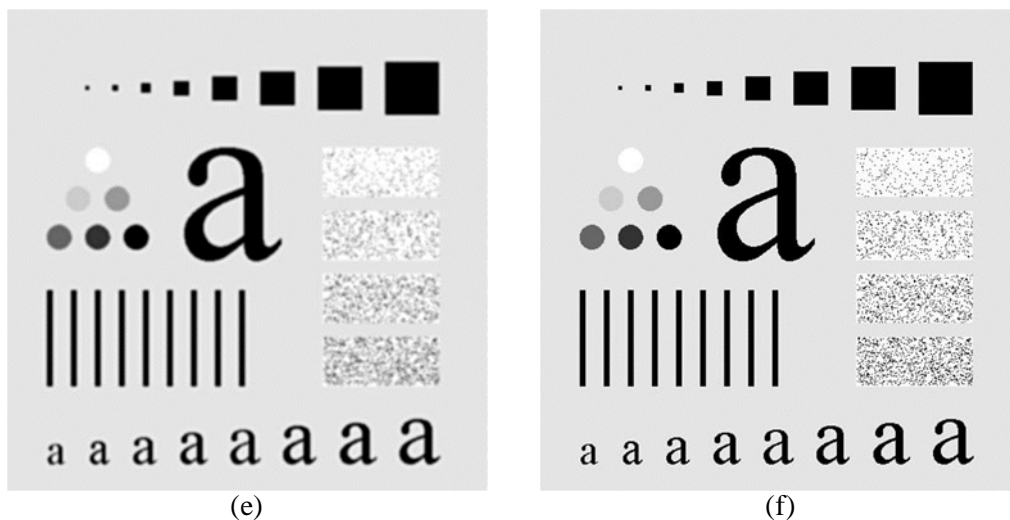

(a)

(b)

(c)

(d)

(e)   (f)
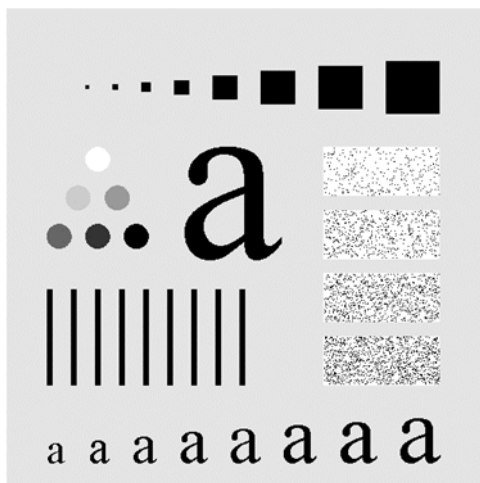
Figure 8.5 (a) Original image. (b) - (f) Results of BLPF of order n = 2 with cutoff frequencies 5, 15, 30, 80, and 230 respectively.

We can see the following effects of BLPF compared to ILPF:

1- Smooth transition in blurring as a function of increasing cutoff frequency.

2- No ringing is visible because of the smooth transition between low and high frequencies.

3- BLPF of order 1 has no ringing. Ringing is imperceptible in BLPF of order 2, but can become significant in BLPF of higher order as shown in the figure below.

(a)   (b)

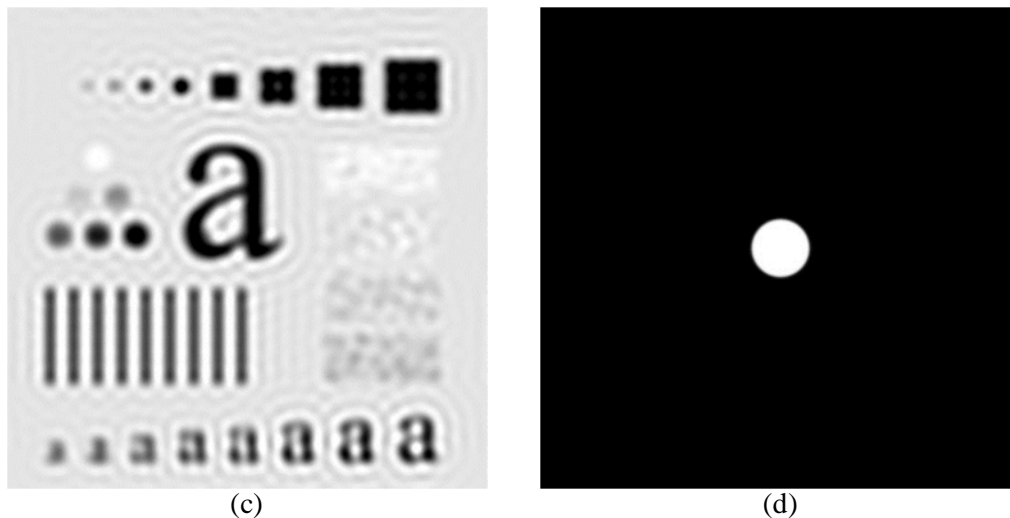(c)   (d)
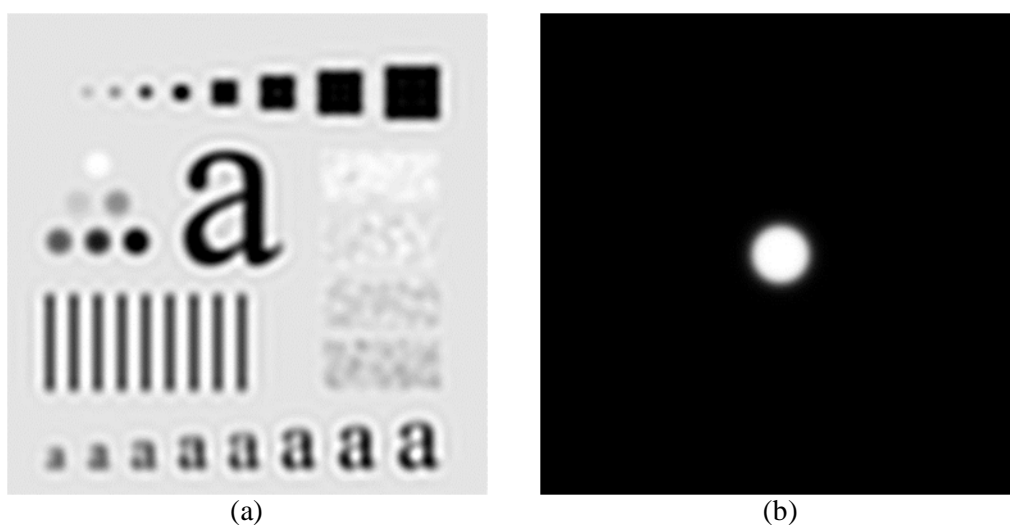
Figure 8.6 (a) Result of BLPF with order 5. (b) BLPF of order 5. (c) Result of BLPF with order 20. (d) BLPF of order 20. (cutoff frequency 30 in both cases).

BLPF is the preferred choice in cases where the tight control of the transition between low and high frequencies are needed. However, the side effect of this control is the possibility of ringing.

Gaussian Lowpass Filter (GLPF)

The GLPF with cutoff frequency D_0 is defined as:

$$H(u, v) = e^{-D^2(u,v)/2D_0^2}$$

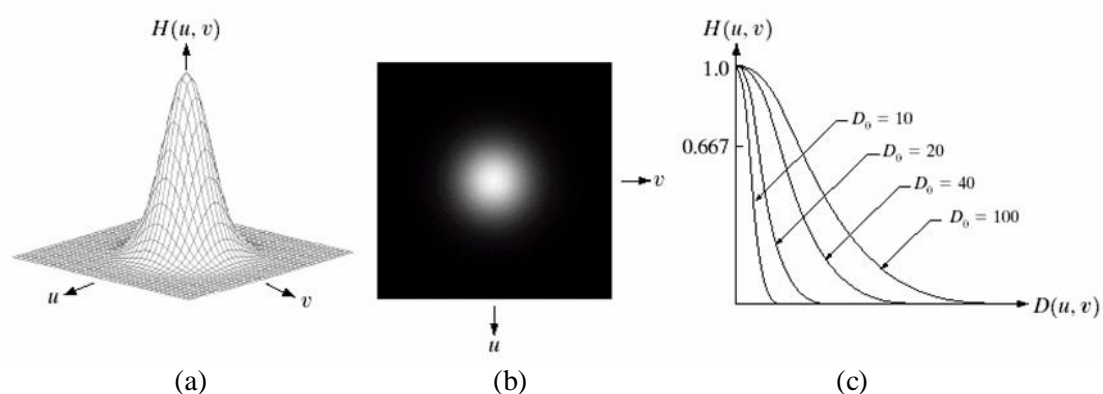(a)   (b)   (c)

Figure 8.7 (a) GLPF transfer function. (b) GLPF as an image. (c) GLPF radial cross section

Unlike ILPF, the GLPF transfer function does not have a sharp transition that establishes a clear cutoff between passed and filtered frequencies.

Instead, GLPF has a smooth transition between low and high frequencies. The figure below shows the results of applying GLPF.
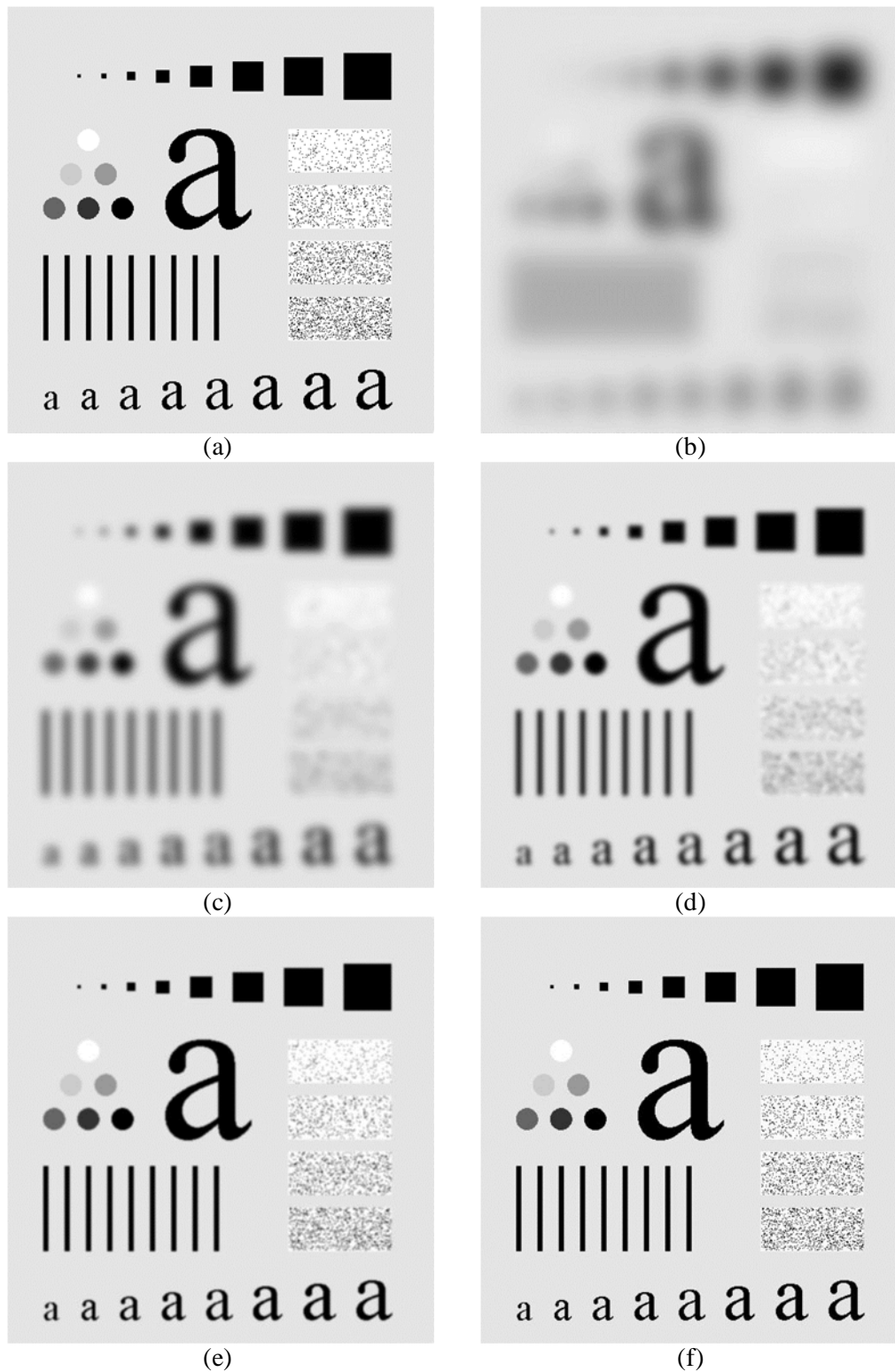

Figure 8.8 (a) Original image. (b) - (f) Results of GLPF with cutoff frequencies 5, 15, 30, 80, and 230 respectively.

We can see the following effects of GLPF:

1. Smooth transition in blurring as a function of increasing cutoff frequency.

2. GLPF did not achieve as much smoothing as BLPF of order 2 for the same cutoff frequency.

3. No ringing effect. This is important in situations (e.g. medical imaging) where any type of artifact is not acceptable.

Smoothing (lowpass) filtering is useful in many applications. For example, GLPF can be used to bridge small gaps in broken characters by blurring it as shown in the figure below. This is useful for automatic character recognition system.

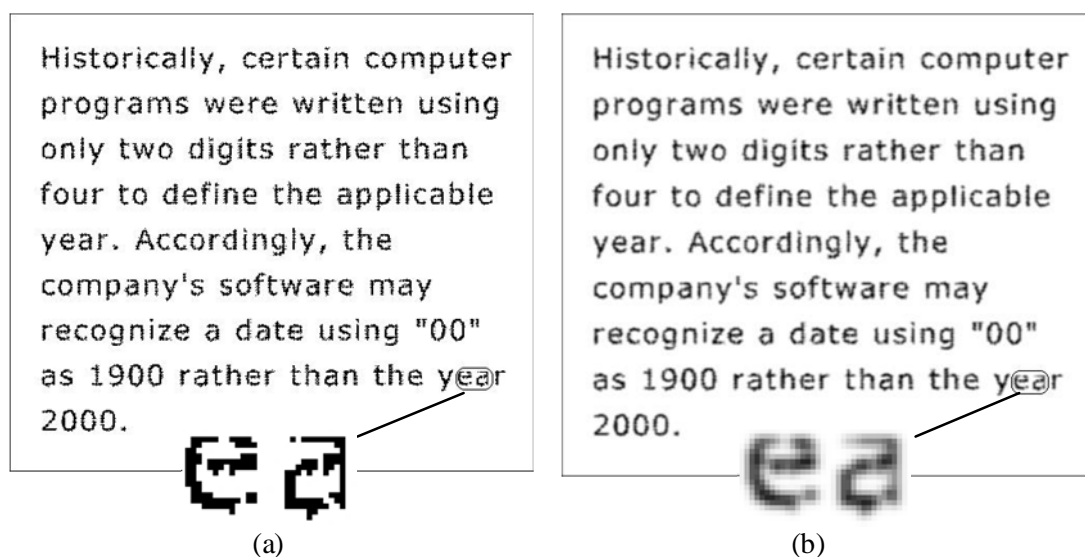(a)   (b)

Figure 8.9 (a) Text of poor resolution. (b) Result of applying GLPF with cutoff=80 on (a)

GLPF can also be used for cosmetic processing prior to printing and publishing as shown in the next figure.
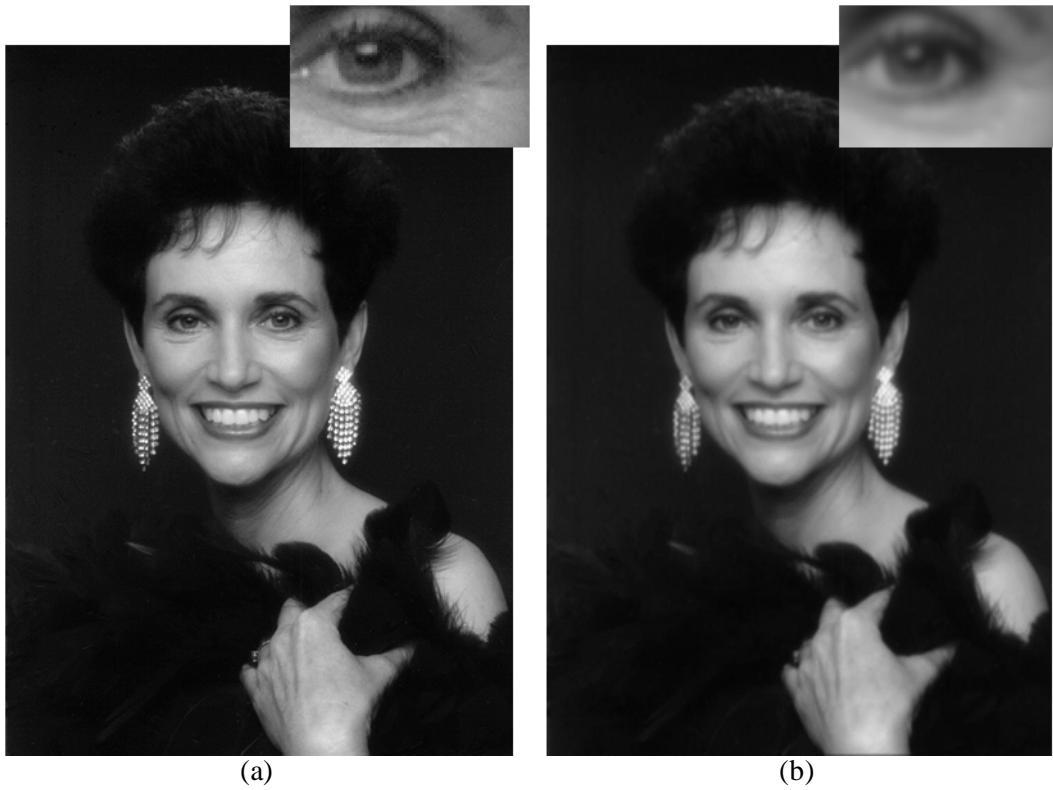
(a)   (b)

Figure 8.10 (a) Original image. (b) Result of filtering using GLPF with cutoff=80

Sharpening frequency domain filters

Edges and sudden changes in gray levels are associated with high frequencies. Thus to enhance and sharpen significant details we need to use highpass filters in the frequency domain

For any lowpass filter there is a highpass filter:

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

Ideal Highpass Filter (IHPF)

The IHPF cuts off all low frequencies of the DFT but maintain the high ones that are within a certain distance from the center of the DFT.

$$H(u, v) = \begin{cases} 1 & if \ D(u, v) > D_0 \\ 0 & if \ D(u, v) \leq D_0 \end{cases}$$

where D_0 is the cutoff frequency, and $D(u, v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$

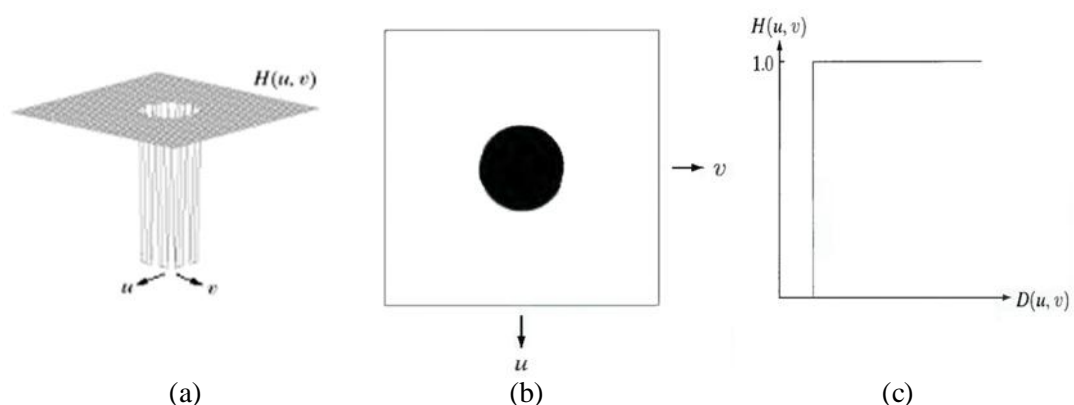(a)   (b)   (c)

Figure 8.11 (a) IHPF transfer function. (b) IHPF as an image. (c) IHPF radial cross section

The IHPF sets to zero all frequencies inside a circle of radius D_0 while passing, without attenuation, all frequencies outside the circle.

The next figure shows the results of applying IHPF with cutoff frequencies 15, 30, and 80.
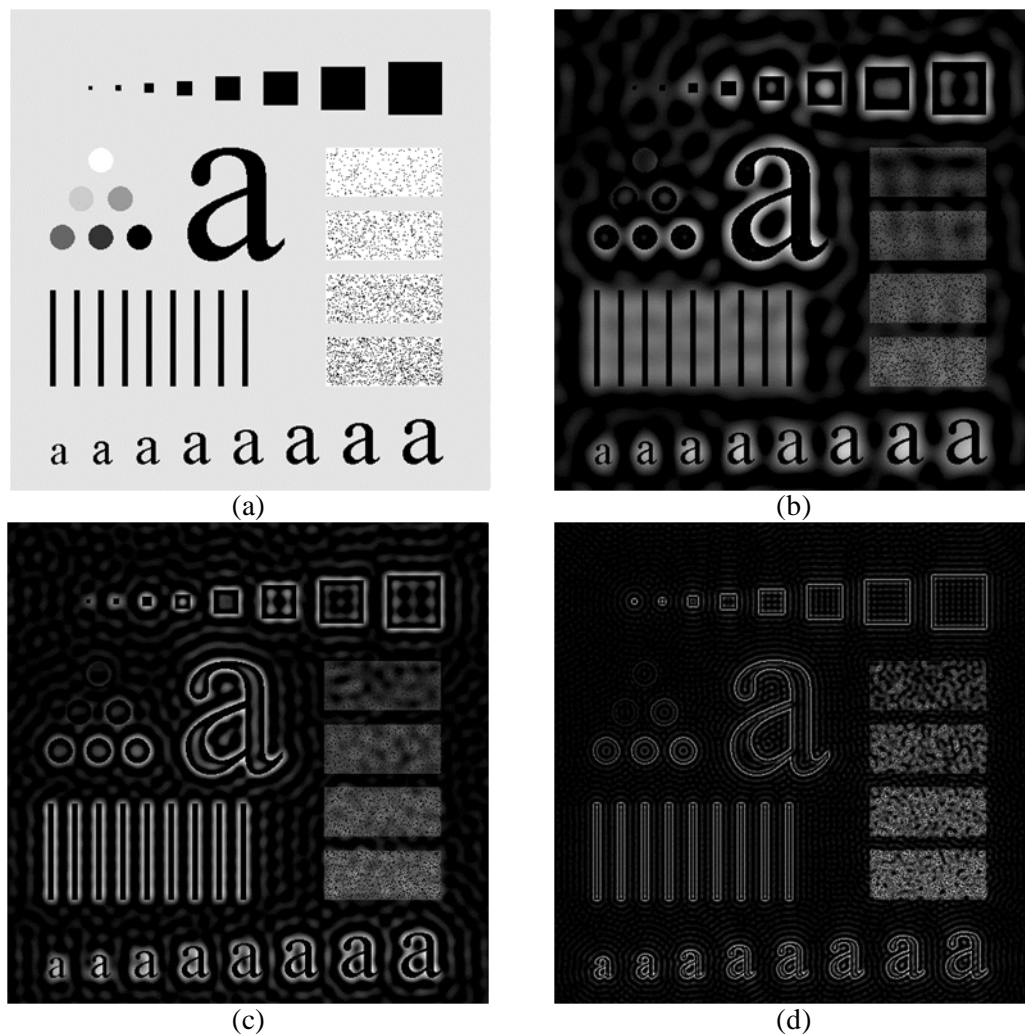
Figure 8.12 (a) Original image. (b) - (d) Results of IHPF with cutoff frequencies 15, 30, and 80 respectively.

We can see the following effects of IHPF:

1. Ringing effect.

2. Edge distortion (i.e. distorted, thickened object boundaries).

Both effects are decreased as the cutoff frequency increases.

Butterworth Highpass Filter (BHPF)

The transfer function of BHPF of order n and with cutoff frequency at distance D_0 is defined as:

$$H(u,v) = \frac{1}{1 + [D_0/D(u,v)]^{2n}}$$

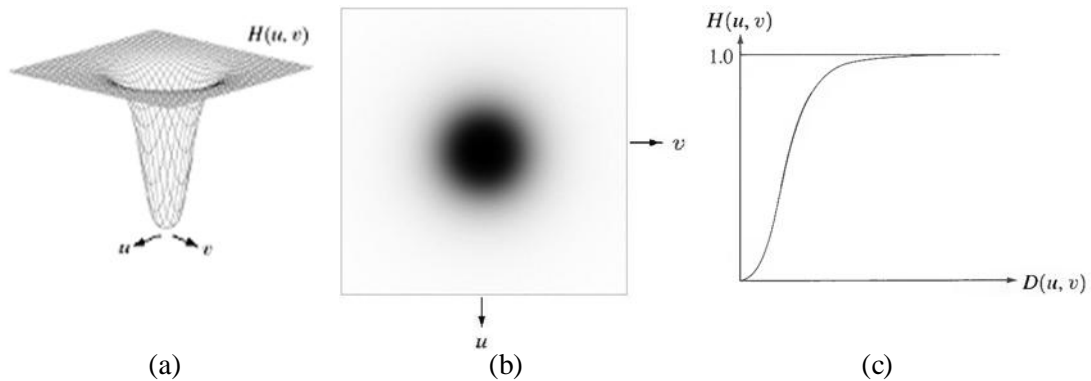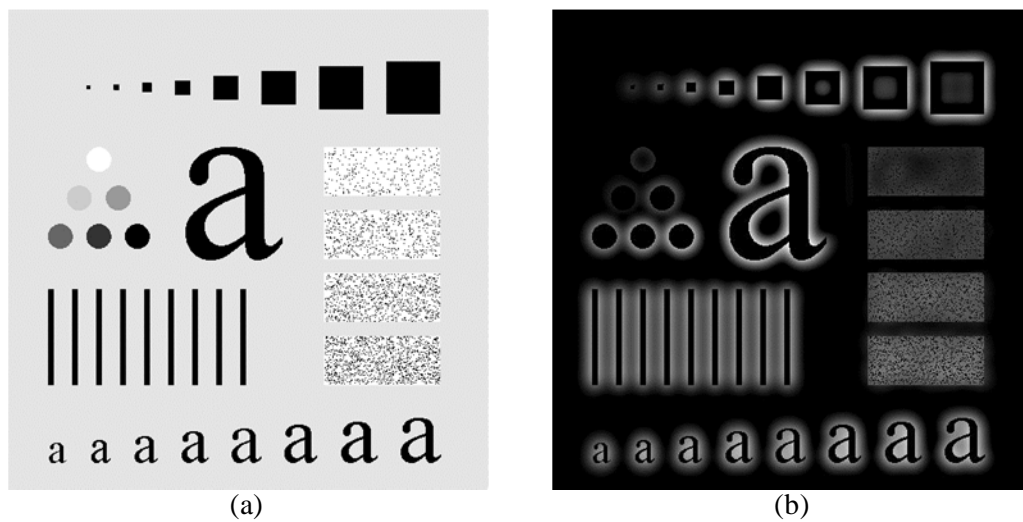|		
(a)	(b)	(c)

Figure 8.13 (a) BHPF transfer function. (b) BHPF as an image. (c) BHPF radial cross section

The figure below shows the results of applying BHPF with cutoff frequencies 15, 30 and 80.

(a)	(b)

<div align="center">(c)   (d)</div>
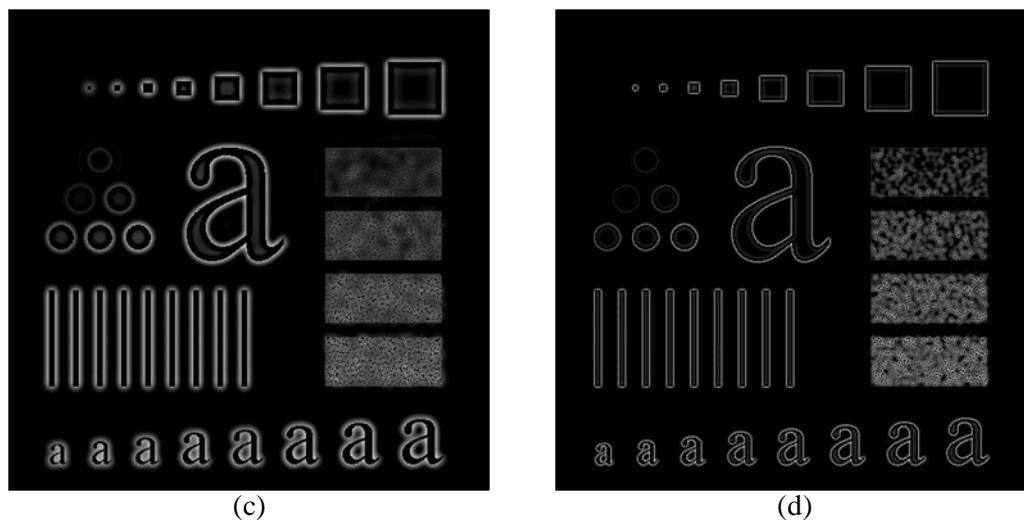
Figure 8.14 (a) Original image. (b) - (d) Results of BHPF with cutoff frequencies 15, 30, and 80 respectively.

We can clearly see the following effects of BHPF:

1- BHPF behaves smoother than IHPF.

2- The boundaries are much less distorted than that of IHPF, even for the smallest value of cutoff frequency.

Gaussian Highpass Filter (GHPF)

The Gaussian Highpass Filter (GHPF) with cutoff frequency at distance D_0 is defined as:

$$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

<div align="center">(a)   (b)   (c)</div>
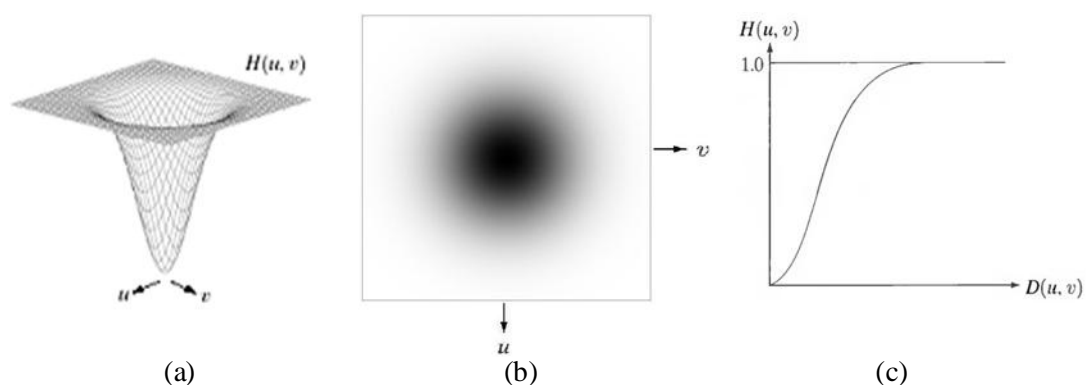
Figure 8.15 (a) GHPF transfer function. (b) GHPF as an image. (c) GHPF radial cross section

The figure below shows the results of applying GHPF with cutoff frequencies 15, 30 and 80.

<table>
<tr><td>(a)</td><td>(b)</td></tr>
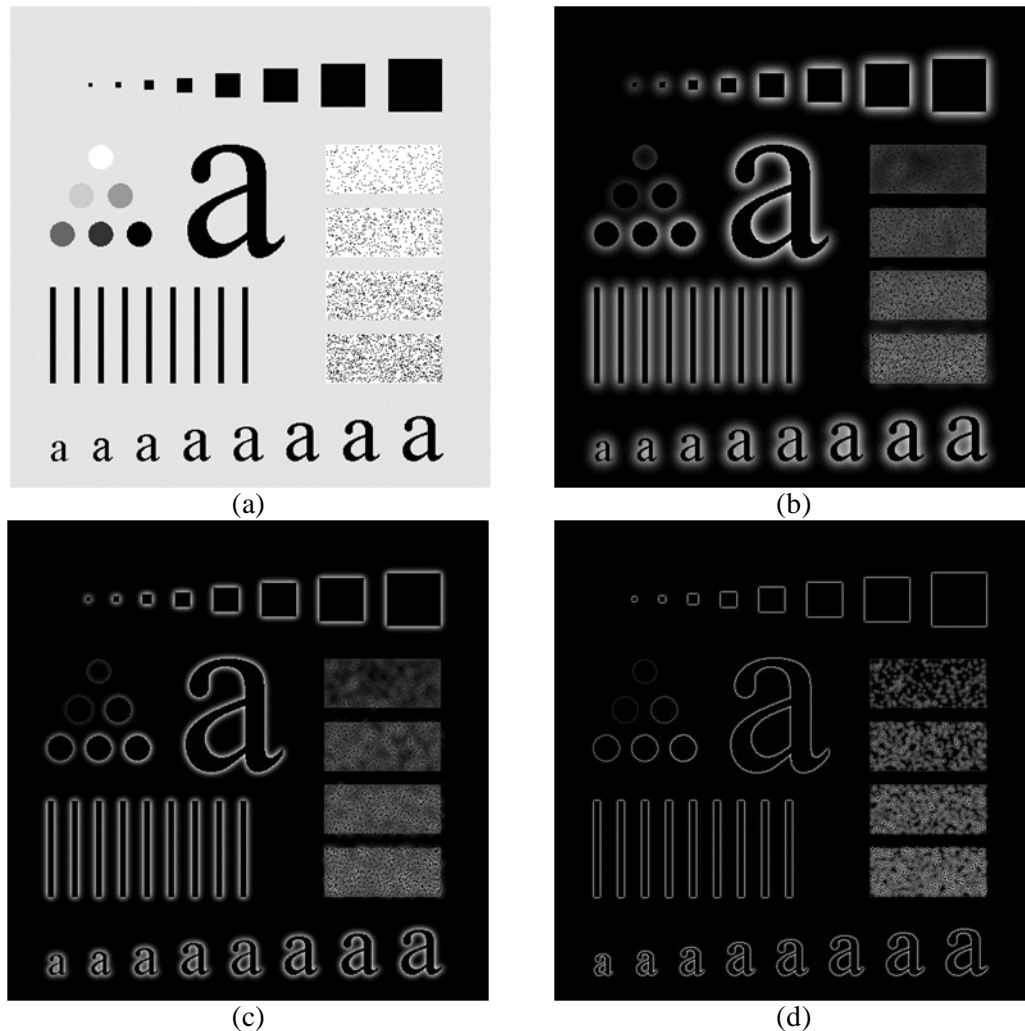<tr><td>(c)</td><td>(d)</td></tr>
</table>

Figure 8.16 (a) Original image. (b) - (d) Results of GHPF with cutoff frequencies 15, 30, and 80 respectively.

The effects of GHPF are:

1. No ringing effect.

2. Less edge distortion.

3. The results are smoother than those obtained by IHPF and BHPF.