29.04.2022 Friday

Model No-3.6 : Poisson Distribution

It's a Discrete Random Variable, when 'n' is Large, say >30 and 'p' is quite Small, say $p<0.1$, the Binomial Distribution can be approximated by the Poisson's Distribution With Mean 'np' and Variance also 'np'.

Note: $\boxed{n>30,\ p<0.1}$ $\boxed{\therefore \text{'q' is almost '1'}}$

The Probability of 'x' Successes in a Poisson's Distribution is Given by

$$P(X=x)=\begin{cases} \dfrac{e^{-\mu}\mu^x}{x!} & x=0,1,2,\cdots,n \\ 0 & \text{Otherwise} \end{cases}$$

Sol: We know that: $$P(X=x)=\begin{cases} \dfrac{e^{-\mu}\mu^x}{x!} & x=0,1,2,3,\cdots n \\ 0 & \text{Otherwise} \end{cases}$$

We know that $\boxed{e^x = 1+\dfrac{x^1}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots}$

Eg-1: The No-of Blind people born in a Town, In a particular Year.

ii) No-of Suicides reported in a particular day throughout the World.

Characteristics of a Poisson Distribution:

* Mean $\boxed{\mu=np}$
* Variance $\boxed{\sigma^2=np}$ $[\because q\to 1]$
* Standard Deviation $\boxed{\sigma=\sqrt{np}}$
* 'n' is Very large & 'p' is Very Small
* Mode:
  - i) If mean 'μ' is an Integer, then the Modes are $\boxed{\text{'}\mu\text{' and '}\mu-1\text{'}}$
  - ii) If 'μ' is not an Integer then the Integral part of μ is Called Mode(μ)

    [ ] → Integral

    Eg. $[5.9]=5$ (Integral Part) $[6.1]=6$

Mean $\Rightarrow \mu = \sum_{x=0}^{n} xp$

$$= \sum_{x=0}^{n} x \frac{e^{-\mu}\mu^x}{x!}$$

$$\sum_{x=1}^{n} \frac{e^{-\mu}\mu^x}{(x-1)!}$$

Put $x-1=y$

$x=y+1$

$$= \sum_{y=0}^{n} \frac{e^{-\mu}\mu^{y+1}}{y!} = \sum_{y=0}^{n} \frac{e^{-\mu}\mu^y.\mu}{y!} = e^{-\mu}\mu \sum_{y=0}^{n} \frac{\mu^y}{y!}$$

$$= e^{-\mu}\mu \sum_{y=0}^{n} \frac{\mu^y}{y!}$$

$$= e^{-\mu}\mu\left[1+\frac{\mu}{1!}+\frac{\mu^2}{2!}+\frac{\mu^3}{3!}+\cdots\right]$$

$$= e^{-\mu}\mu(e^{\mu}) = \boxed{\mu = np}$$

Variance $\Rightarrow \sigma^2 = \sum_{x=0}^{n}(x^2p) - \mu^2$

$$= \sum_{x=0}^{n} x^2\left(\frac{e^{-\mu}\mu^x}{x!}\right) - \mu^2$$

$$= \sum_{x=1}^{n} x \frac{e^{-\mu}\mu^x}{(x-1)!} - \mu^2$$

$$= \sum_{x=1}^{n} [(x-1)+1] \frac{e^{-\mu}\mu^x}{(x-1)!} - \mu^2$$

$$= \sum_{x=1}^{n} (x-1)\frac{e^{-\mu}\mu^x}{(x-1)!} + \sum_{x=1}^{n} \frac{e^{-\mu}\mu^x}{(x-1)!} - \mu^2$$

$$= \sum_{x=2}^{n} \frac{e^{-\mu}\mu^x}{(x-2)!} + \underbrace{\sum_{x=1}^{n} \frac{e^{-\mu}\mu^x}{(x-1)!}} - \mu^2$$

Put $x-2=y$ Put $x-1=y$

$$= \sum_{y=0}^{n} \frac{e^{-\mu}\mu^{y+2}}{y!} + \sum_{y=0}^{n} \frac{e^{-\mu}\mu^{y+1}}{y!} - \mu^2$$

$$= e^{-\mu}\mu^2\left[\sum_{y=0}^{n} \frac{\mu^y}{y!}\right] + e^{-\mu}\mu \sum_{y=0}^{n} \frac{\mu^y}{y!} - \mu^2$$

$$= e^{-\mu}\mu^2 e^{\mu} + e^{-\mu}\mu e^{\mu} - \mu^2 = \boxed{\mu = np}$$

The Other Name Of 'Mean' is Average

1, A Hospital Switch Board receives an Average Of 4 Emergency Calls in a 10 min Interval What's probability that: i,

i, There are atmost 2 Emergency Calls in 10 min Interval

ii, There are exactly 3 Emergency calls in 10 min Interval

Sol: i, Atmost 2 E.C: Given $\boxed{\mu = np = 4}$

$$P(X \le 2) = P(x=0) + P(x=1) + P(x=2) \qquad = \frac{e^{-\mu}\mu^x}{x!}$$

$$= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}(4^1)}{1!} + \frac{e^{-4}(4^2)}{2!} = e^{-4}[1+4+8] = 13e^{-4}$$

$$= 0.2381$$

ii, Exactly '3' E.C:

$$P(x=3) = \frac{e^{-4}4^3}{3!} = 0.1953$$

2, A Manufacturer Of Cotterpins knows that 5% of this product is Defective Pins are Sold in Boxes Of 100. He Guarantees that not more than 10 pins will be defective. What is the approximate probability of Box will fail meet the Guarantee Quality?

Sol: Given X = Random variable Of no.of defective Pins in a box.

$p = 5\% = \frac{5}{100} = 0.05$

$n = 100$

Mean $(\mu) = np = 0.05 \times 100 = 5$

* Not More than 10 Pins defective Of Guarantee $= P(X \le 10)$

= P(A Box will meet the Guarantee)

$= P(X > 10)$

$= 1 - P(X \le 10)$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)]$$

$$= 1 - \left[\frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!} + \frac{e^{-5}5^4}{4!} + \frac{e^{-5}5^5}{5!} + \frac{e^{-5}5^6}{6!} + \frac{e^{-5}5^7}{7!} + \frac{e^{-5}5^8}{8!} + \frac{e^{-5}5^9}{9!} + \frac{e^{-5}5^{10}}{10!}\right]$$

$= 0.0137$

3) If a Poisson distribution is such that $P(x=1)\frac{3}{2}=P(x=3)$ find i) $P(x\geq 1)$ ii) $P(x\leq 3)$ iii) $P(2\leq x\leq 5)$

Sol: Given the Relation is: $(P(x=1))\frac{3}{2}=P(x=3)$

$$3\frac{e^{-\mu}\mu^1}{1!}=2\frac{e^{-\mu}\mu^3}{3!}$$

$$3\cancel{\mu}=\frac{\cancel{2}\mu^{\cancel{3}2}}{3\times\cancel{2}} \Rightarrow \mu^2=9 \quad \mu=\pm 3$$

$$\boxed{\therefore \mu=3}$$

i) $P(x\geq 1)$

$= 1-P(x<1)$

$= 1-[P(x=0)]$

$= 1-\left[\frac{e^{-3}3^0}{0!}\right] = 1-e^{-3} = 0.9502$

ii) $P(x\leq 3) = P(x=0)+P(x=1)+P(x=2)+P(x=3)$

$$= \frac{e^{-3}3^0}{0!}+\frac{e^{-3}(3^1)}{1!}+\frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!} = 0.647$$

iii) $P(2\leq x\leq 5) = P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$$= \frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!}+\frac{e^{-3}3^4}{4!}+\frac{e^{-3}3^5}{5!} = 0.7169$$

4) If the Variance of a Poisson variance is 3, then find i) $P(x=0)$ ii) $P(0<x\leq 3)$ iii) $P(1\leq x<4)$

0.0497 0.5974 0.5974

Sol:- Given Variance $\sigma^2=np=3$ $\mu=np=3$

i) $P(x=0) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-3}3^0}{0!} = 0.04978$

ii) $P(0<x\leq 3) = P(x=1)+P(x=2)+P(x=3)$

$$= \frac{e^{-3}3^1}{1!}+\frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!} = 0.0497+0.1493+0.6721$$

$= 0.5974$

iii) $P(1\leq x<4) = P(x=1)+P(x=2)+P(x=3)$

$$= \frac{e^{-3}3^1}{1!}+\frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!}$$

$= 0.5974$

5) If 2% of Rifles are defective. Find:
i) Atleast 1 is defective
ii) Exactly 7 is defective
iii) $P(1<x<8)$ in a sample of 100.

Given that $n=100$, $p=2\% = \frac{2}{100} = 0.02$

Mean $\mu = np = 0.02 \times 100 = 2$

i) $P(x \geq 1) = 1 - P(x<1)$
$= 1 - P(x=0)$
$= 1 - \left[\frac{e^{-2}2^0}{0!}\right] = 1 - 0.1353 = 0.8646$

ii) $P(x=7)$ (Exactly) $= [P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)]$ ← wrong →
$=$

$P(x=7) = \frac{e^{-2}2^7}{7!} = \frac{0.1353 \times 128}{5040} = 0.0034$

iii) $P(1<x<8)$.
$= P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)$
$= \frac{e^{-2}2^2}{2!} + \frac{e^{-2}2^3}{3!} + \frac{e^{-2}2^4}{4!} + \frac{e^{-2}2^5}{5!} + \frac{e^{-2}2^6}{6!} + \frac{e^{-2}2^7}{7!}$
$= e^{-2}[2+1.33+0.66+0.266+0.088+0.025] = 0.1353 \times 4.369$
$= 0.59112$

6) After Correcting 50 Pages, the Proof Reader find that there are on the average of 2 errors on 5 Pages, How many Pages could one expect with
i) '0' error ii) '1' error iii) Atleast 3 in 1000 pages of 1st Print of book.

Sol: $N = 1000$ Mean $(\mu) = \frac{2}{5} = 0.4$

i) $P(x=0) = \frac{e^{-0.4}0.4^0}{0!} = 0.67032$

Now; $0.67032 \times 1000 = 670.32 \approx \boxed{670}$

Mac

ii) '1' Error

$$p(x=1) = \frac{e^{-0.4}(0.4)^1}{1!} = 0.268128 \times 100 = \boxed{268.12}$$

iii) Atleast 3

$$p(x \geq 3) = 1 - p(x<3)$$

$$= 1 - [p(x=0) + p(x=1) + p(x=2)]$$

$$= 1 - \left[\frac{e^{-0.4}(0.4)^0}{0!} + \frac{e^{-0.4}(0.4)^1}{1!} + \frac{e^{-0.4}(0.4)^2}{2!}\right]$$

$$= 1 - [0.67032 + 0.268128 + 0.05362]$$

$$= 1 - [0.9920]$$

$$= 0.0079 = 0.008$$

$$N \times 0.008 = 0.008 \times 1000$$

$$= \boxed{8}$$