

11.07.2022 80 Monday

## Speed, Distance And Time Concepts

\* The relation between Distance, Speed & Time is  $D \propto S \times T$

\* Hence  $\frac{S_1 T_1}{D_1} = \frac{S_2 T_2}{D_2}$  OR  $\frac{D_1}{S_1 T_1} = \frac{D_2}{S_2 T_2}$  AND

$$D = S \times T$$

$$S = \frac{D}{T}$$

$$T = \frac{D}{S}$$

\* To convert speed from km/h to m/sec we have to multiply with  $\frac{5}{18}$  and  $m/\text{sec} \rightarrow \text{km}/\text{hr} \Rightarrow$  multiply with  $\frac{18}{5}$

\* If the ratio of speeds of A and B is  $a:b$ , then the ratio of times taken by them to cover same distance is  $b:a$

18.07.2022

\* If speed & time both  $\uparrow$  by  $x\%$  &  $y\%$  respectively, then the distance travelled is  $\uparrow$  by  $(x+y + \frac{xy}{100})\%$

\* If speed & time both  $\downarrow$  by  $x\%$  &  $y\%$  respectively, then the distance travelled is  $\downarrow$  by  $(x+y - \frac{xy}{100})\%$

\* If speed  $\uparrow$  by  $x\%$  & time  $\downarrow$  by  $y\%$ , then the distance travelled is Changed by  $(x-y - \frac{xy}{100})\%$ .

- If +ve:  
there is  
Distance travelled  
Increases ( $\uparrow$ )
- If -ve:  
distance  
travelled ( $\downarrow$ )
- If Zero:  
then distance travelled  
doesn't change

If speed ↑ by  $x\%$ , then time ↓ by  $\left(\frac{x}{100+x}\right)100\%$ , so the distance travelled doesn't change or to Cover same distance.

If speed ↓ by  $x\%$ , then time ↑ by  $\left(\frac{x}{100-x}\right)100\%$ , to cover same distance.

If time ↑ by  $x\%$ , then speed ↓ by  $\left(\frac{x}{100+x}\right)100\%$ , to cover same distance.

If time ↓ by  $x\%$ , then speed ↑ by  $\left(\frac{x}{100-x}\right)100\%$ , to cover the same distance.

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total Time taken}}$$

The ratio of total distance to that of total time taken.

\* Suppose a man covers a certain distance at ' $x$ ' kmph and an equal distance at ' $y$ ' kmph. Then the average speed during Journey is  $\frac{2xy}{x+y}$  kmph.

If a person covers 2 equal distances with different speeds at ' $x$ ' kmph & ' $y$ ' kmph. Then the average speed of the person during the whole Journey is:  $\frac{2xy}{x+y}$  kmph.

\* Suppose a man covers ' $n$ ' equal distances with different speeds  $x_1, x_2, x_3, \dots, x_n$  then, the avg speed during Journey is  $\frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$  kmph

Relative Speed: Comparison of speed of one Person w.r.t another, is called Relative Speed.

Notes:

1. If 2 Persons travelled with different speeds ' $x$ ' kmph & ' $y$ ' kmph respectively, then the relative speed of:
  - (a) 1st Person w.r.t. 2nd Person is  $[x-y]$  kmph.
  - (b) 2nd Person w.r.t. 1st Person is  $[y-x]$  kmph.
2. If 2 Persons travelled with different speeds ' $x$ ' kmph & ' $y$ ' kmph respectively in opp. direction, then the relative speed of:
  - 1, 1st w.r.t. 2nd Person  $\Rightarrow [x+y]$  kmph
  - 2, 2nd w.r.t. 1st Person  $\Rightarrow [x+y]$  kmph

Points Regarding Problems based on Trains:

- \* If a Train Crosses a Stationary Object having Negligible length [Man, Tree or Pole or Signal Post], then it Travels a distance of Length of Train.
- \* If a Train Crosses a Stationary object having some length [Bridge, platform, Tunnel or Train] then it Travels a distance of Sum of the Length of Train & the length of Object which it crosses.
- \* If a Train Crosses a Moving Object, then also distances are same as above, but speeds are Considered as Relative Speeds.

Eg: ① If 2 Trains of lengths ' $x$ ' mts & ' $y$ ' mts are moving in opp. dir with speeds ' $u$ ' m/s & ' $v$ ' m/s respectively. Then the Time taken by the Trains to cross each other is  $\frac{x+y}{u+v}$  sec.

② If 2 Trains of length 'x' mts & 'y' mts respectively are moving in same dir. with speeds 'u' m/s & 'v' m/s respectively. Then the time taken by the faster train to cross slower train is

$$\boxed{\frac{x+y}{u-v} \text{ sec}}$$

\* If 2 Trains start at the same time in opp dir from 2 stations 'A' & 'B', after passing/crossing each other, they complete their remaining journeys in 'a' & 'b' hrs respectively, then the ratio of their speeds is  $\boxed{J_B : J_A}$

25.07.2022

### Circular Track:

Monday

① If two persons running on a Circular Track of length 'C' mts, with different speeds 'x' m/sec and 'y' m/sec respectively, then the first Meeting Time of two persons at any Point on the Circular Track is:

i, If they travel on opp directions

$$\text{is } \boxed{\frac{C}{x+y} \text{ sec}}$$

ii, If they travel in same dir.

$$\text{is } \boxed{\frac{C}{|x-y|} \text{ sec.}}$$

② If two persons running on a Circular Track of length 'C' mts with different speeds 'x' m/sec & 'y' m/sec respectively, then the first Meeting Time of two Persons at Starting Point:

$$\boxed{\text{L.C.M of } \left\{ \frac{C}{x}, \frac{C}{y} \right\} \text{ sec.}}$$

### 3) Boats & Streams:

#### i, Down Stream:

If the boat travels in the same dir of water flow, then speed of boat increases ( $\uparrow$ ), In this case we say that Boat Travels in Downstream or With Tide

#### ii, Up Stream:

If a boat travels in the opp dir of water flow, then speed of boat ( $\downarrow$ ), In this case we say that Boat Travels in Upstream or against Tide.

Note: If a boat Travels in Stationary Water with Speed

① of ' $x$ ' km/h and speed of water is ' $y$ ' km/h

i, The speed of boat in downstream is  $x+y$  km/h

ii, The speed of boat in Upstream is  $x-y$  km/h

② If a boat Travels in Downstream With a speed of ' $a$ ' km/h & In Upstream with a speed of ' $b$ ' km/h

$$\text{Let } a = x+y$$

$$\text{And } b = x-y \text{ then}$$

$$2x = a+b$$

$$x = \frac{a+b}{2}$$

$$2y = a-b$$

$$y = \frac{a-b}{2}$$

$$\begin{array}{r} a = x+y \\ -b = x-y \\ \hline \end{array}$$

i, The speed of boat in Stationary water is  $\frac{a+b}{2}$  kmph

The speed of water or Stream is  $\frac{a-b}{2}$  kmph

i) If the speed & time both ( $\uparrow$ ) by 20% & 30% respectively. Then what is the % increase in distance travelled?

Sol  $D = sxt \rightarrow \uparrow 30\%$

$$\uparrow 20\%, D = 20+30+\frac{20 \times 30}{100} = x+y+\frac{xy}{100} \%$$

$$50+6$$

$$56\% \uparrow \text{Distance}$$

ii, If speed & time both ( $\downarrow$ ) by 20% & 30% respectively, then distance travelled  $\downarrow$  by what %?

Sol  $D = 20+30-\frac{20 \times 30}{100} = 50-6 = 44\% \downarrow$

$$x+y-\frac{xy}{100} \%$$

iii) If the speed ( $\uparrow$ ) by 30% & time ( $\downarrow$ ) by  $x\%$ , due to that distance travelled is  ~~$\uparrow$~~  by 5%, then value of  $x$ ?

$$\text{Ex: } x - y - xy$$

$$30 - 5 = \frac{30 \times 5}{160} \quad (81)$$

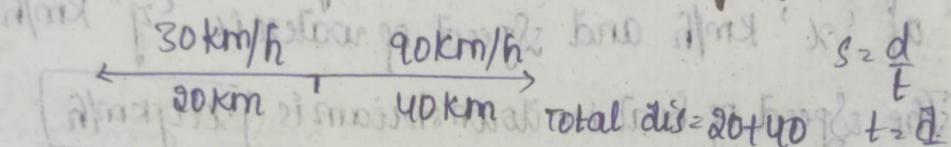
$$25 - \frac{3}{2} = 25 - 1.5$$

245

$$30 - x - \frac{30x}{100} = -5$$

$$30 - x = -5 + \frac{30x}{100}$$

2) If a Person travelled 20 km with Speed of 30 km/h.  
& another 40 km with Speed of 90 km/h. Then what is  
his avg. Speed during the whole Journey of 60 km.



$$\text{Avg Speed} = \frac{\text{Total distance}}{\text{total time taken}} = \frac{60 \text{ km}}{\frac{20}{30} \text{ hr}} = 90 \text{ km/hr}$$

$$= \underline{60 \text{ km}}$$

$$\frac{20}{30} + \frac{40}{90} = \frac{30}{90}$$

$$\begin{array}{r} 60 \\ \hline 60+60 \end{array}$$

90  
60x90 Film

$\frac{1}{100} = 54 \text{ km/h}$  p hingga sif

3) If a Person travelled  $\frac{1}{2}$  of distance with speed of 30 km/h &  $\frac{1}{2}$  with ~~10~~ km/h. & remaining at 40 km/h. Then what is Avg Speed of person during whole Journey ??

$$\begin{aligned}
 & \text{Let total distance} = 40 \text{ km} \\
 & \frac{x}{u} + \frac{2}{u} + \frac{2}{u} \\
 & \frac{2}{3} + 1 + \frac{1}{4} \\
 & \frac{4}{6} + \frac{12}{12} + \frac{3}{12} \\
 & \frac{12+10}{12} \\
 & \frac{22}{12} = 1 \frac{10}{12} \\
 & \frac{22}{12} = \frac{32}{12} \\
 & \frac{32}{12} = \frac{8}{3} \text{ hours} \\
 & S = \frac{d}{t} = \frac{40}{\frac{8}{3}} = 15 \text{ km/h}
 \end{aligned}$$

4) If a Person travelled 3 equal distances with different speeds 20 kmph, 40 kmph & 80 kmph. Then avg speed of a Person during whole Journey is?

$$\text{Speeds} \quad 20 \quad 40 \quad 80 \\ \frac{x}{3} \quad \frac{x}{3} \quad \frac{x}{3}$$

S

$$200 \\ 1600 \\ 900 \\ \hline 5600$$

$$\text{Avg Speed} = \frac{3xyz}{xy+yz+zx} = \frac{3(20)(40)(80)}{800 + 3200 + 1600} = \frac{3(20)(40)(80)}{5600}$$

$$7) 160 (22) \quad 7) 200 (34) \\ \frac{16}{14} \quad \frac{20}{18} \quad \frac{21}{20} \\ \frac{20}{14} \quad \frac{21}{20}$$

$$2 \frac{240}{7} = \boxed{34.2 \text{ kmph}}$$

5) If a Person travelled with a Speed of 20 km/h from his house to Office & return back in same route with a Speed of 60 km/h. Then what is his avg speed during whole Journey ??:

$$\text{Sol: } \text{Avg Speed} = \frac{2xy}{x+y} = \frac{2(20)(60)}{20+60} = \frac{2(20)(60)}{80+120} = \boxed{30 \text{ km/h}}$$

6) If a Person travelled <sup>4 equal</sup> same dist with different speeds 10, 20, 30 & 40 kmph. Then what is his avg speed during whole Journey ??:

$$\text{Sol: } \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}} = \frac{4}{\frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}} = \frac{4}{\frac{12+6+4+3}{120}} = \frac{4}{\frac{25}{120}} = \boxed{48 \text{ kmph}}$$

$$10, 20, 30, 40 \\ 1, 2, 3, 4 \\ \hline 10$$

$$240 \\ 4 \times 240 \\ \hline 50$$

$$\boxed{48 \text{ kmph}}$$

1,

$$\text{Avg Speed} = \frac{\text{tot dist}}{\text{tot time}} = \frac{320}{\frac{160}{90} + \frac{160}{54}} = \frac{320}{\frac{480+800}{270}} = \frac{320}{\frac{1160}{270}} = \frac{320 \times 27}{1160} = \frac{8640}{1160} = \frac{216}{29} \text{ km/h}$$

$$= \frac{216 \times 27}{29} = \frac{5832}{29} = 201.1 \text{ km/h}$$

$$(8) \quad \frac{2xy}{x+y} = \frac{6.75 \text{ km/h}}{2.5} = \frac{27}{10} = \frac{270}{100} = \frac{27}{10} \text{ km/h}$$

$$(8) \quad \frac{2}{60} + \frac{1}{30} + \frac{1}{10} = \frac{4}{60} = \frac{1}{15} \text{ hr}$$

Avg speed for 2nd half =  $\frac{2xy}{x+y} \Rightarrow x=30, y=10$

$$\text{Avg Speed for 1st half} = \frac{2x30 \times 10}{30+10} = \frac{2 \times 300}{40} = \frac{15}{2} = 7.5 \text{ km/h}$$

$$3) \quad \frac{S}{t} = 5 \text{ km/h} = \frac{5 \times 5}{18} = \frac{25}{18} \text{ m/s}$$

$$t = 15 \text{ s} \rightarrow \text{bridge length} = \frac{25}{18} \times 15 = \frac{25}{18} \times 100 = \frac{250}{18} \text{ m}$$

$$S = \frac{d}{t} \rightarrow d = S \times t = \frac{25}{18} \times 100 = \frac{250}{18} \text{ m}$$

length of bridge.

4)

$$\text{Original time} = \frac{80}{x-4} \quad (t_2)$$

$$\text{Increased speed} = x \text{ km/h}$$

$$\text{Hence } \frac{80}{x-4} - \frac{80}{x} = 1$$

$$\Rightarrow t_2 - t_1 = 1$$

$$\Rightarrow 80x - 80 + 320 = x(x-4)$$

$$80x - 80x + 320 = x^2 - 4x$$

$$x^2 - 4x - 320 = 0$$

$$x^2 - (-16+20)x + (-16 \times 20) = 0$$

$$x^2 + 16x - 20x + (-16 \times 20) = 0$$

$$x(x+16) - 20(x+16) = 0$$

$$(x+16)(x-20) = 0$$

$$\begin{matrix} x+16 \\ -16 \end{matrix} \quad \begin{matrix} x-20 \\ +20 \end{matrix}$$

$$\boxed{x = 20 \text{ km/h}}$$

5) Let time taken for they meet is 'x' hrs after 8 hrs.

1-60       $\frac{s_1}{t_1} = 60 \text{ kmph}$       1st train      Second train takes  $(x-1)$   
                  $s_1 = 60x \text{ kmph}$       train      because 2nd train starts  
                 after 1 hour when  
                 1st train starts.  
         And they covered total distance to cross each other 80  
 $\Rightarrow s_{1A} + s_{1B} = 80$   
 $\Rightarrow 60x + 75(x-1) = 80$   
 $\Rightarrow 135x - 75 = 80$   
 $\Rightarrow 135x = 155 \Rightarrow x = \frac{155}{135} = 3 \text{ hrs.}$

Required Time = 11 A.M.

6)  $d = s \times t$        $m =$   
 $d_1 = s_1 \times t_1$   
 $= 3 \times \frac{5}{18} \text{ m/s} \times \underline{30 \times 60 \text{ sec}}$   
 $= \frac{15 \times 400}{18} \times 200$   
 $= 1000 \text{ m}$

$$\begin{aligned} d_2 &= 4 \times \frac{5}{18} \text{ m/s} \times 30 \times 60 \\ &= \frac{4 \times 5 \times 18 \times 100}{18} \\ &= 2000 \text{ m} \\ &\boxed{= 10 \text{ km}} \end{aligned}$$

(Q)  
     Difference of Time  $\Rightarrow \frac{x}{3} - \frac{x}{4} = \frac{20+30}{60}$

$$\Rightarrow \frac{x}{3} - \frac{x}{4} = \frac{50}{60}$$

$$\Rightarrow \frac{4x - 3x}{12} = \frac{5}{6}$$

$$\Rightarrow \boxed{x = 10 \text{ km}}$$

$$l_1 = \frac{5x7}{3 \times 5} = \frac{35}{18} \times 36 = [30 \text{ m}]$$

$$l_2 = \frac{6x5}{18} \times 30 = [50 \text{ m}]$$

(8)

$$\frac{x}{y+3x5} = 36 \quad \text{I}$$

$$\frac{x}{y+6x5} = 30 \quad \text{II}$$

$$\Rightarrow x = 36y + 30$$

$$x = 30y + 50$$

$$36y + 30 = 30y + 50$$

$$6y = 20$$

$$y = \frac{20}{6}$$

$$x = 30 \left( \frac{20}{6} \right) + 50$$

$$x = 150 \text{ m}$$

8) Relative Speed =  $5.5 - 5 = 0.5 \text{ km/h}$

Distance = 8.5 km  
same direction (x-y relative speed)

$$\Rightarrow T = \frac{D}{S} = \frac{8.5}{0.5} = [17 \text{ hrs}]$$

9) Avg Speed =  $\frac{2xy}{x+y}$

Distance from Village  $\rightarrow$  Post office =  $x$  km

Total distance travelled =  $2x$  km

Total time is  $\leq 5 \text{ hrs } 48 \text{ min}$

$$\text{Avg Speed} = \frac{2x}{\frac{29}{5}} = \frac{10x}{29}$$

$$\frac{10x}{29} = \frac{2x25 \times 4}{25+4} \Rightarrow \frac{2xy}{x+y} \Rightarrow \frac{x}{y} = \frac{25}{4}$$

$$\frac{10x}{29} = \frac{2x25 \times 4}{29}$$

$$10x = 200$$

$$x = 20 \text{ km/h}$$

12.09.2022

Monday

Race & of Race: Race is a Contest of Speed in Running, Driving, Riding, Sailing, Rowing etc, Over a particular distance.

Race Course: Race Course is the Ground or path on which contests are conducted.

Starting Point: Starting Point is the point, where the race starts.

Winning Point(or) Goal: It is the point where race finishes or ends.

Dead-heat race: It is the race, If all the persons/participants contesting the race reaches the winning point exactly at the same time.

Winner: The Person who reaches the Goal first

1, A Game Of 100 Points means that:

The Person who Scores 100 points first is the Winner

General statements involved in Races & Games and their Mathematical Interpretation:

Let 'A' & 'B' be 2 participants in a race. Let examine some of the General statements & their Mathematical Interpretations:

1, 'A' beats 'B' by 't' seconds means that 'A' finishes the race 't' seconds before 'B' finishes. If  $T(A) = x \text{ sec}$  then  $T(B) = x + t$

2, 'A' gives 'B' a Start of 't' seconds means that 'A' Starts the race 't' seconds after 'B' from the same Starting point. If  $T(A) = y - t \text{ sec}$

If  $T(B) = y \text{ sec}$

3, If 'A' Gives 'B' a Start of 'x' mts means that while 'A' starts from Starting point, 'B' starts 'x' mts ahead from the Starting Point at the same time (81)

To cover a race of 'y' mts, 'A' will have covered 'y' mts, while 'B' will cover only ' $y-x$ ' mts

4, 'A' beats 'B' by 'x' mts means that, when 'A' reaches the Goal, 'B' is 'x' mts behind the Goal i.e. In a race of 'y' mts 'A' travels 'y' mts, In the same time 'B' travels only ' $y-x$ ' mts.

5, A Game of 100 points means that a Game in which the participant who scores 100 pts win Game

i) In a game of 100, 'A' can give 'B', 'x' points means that while 'A' Scores 100 points, 'B' Scores only  $100-x$  pts to win Game

ii) In a Game of 100 points, 'A' beats 'B' by 'x' points means that when 'A' Scores 100 points, in the same time 'B' Scores  $(100-x)$  points

iii) If 'A' is 'n' times as fast as 'B' and 'A' Gives 'B' a start of 'x' mts, then the length of race course, so that 'A' and 'B' reach the winning post at the same time is,  $x\left(\frac{n}{n-1}\right)$  mts.

$$D_1 = y$$

$$S_1 = nx$$

$$D_2 = y-x$$

$$S_2 = x$$

then

$$\frac{D_1}{S_1} = \frac{D_2}{S_2}$$

$$\frac{y}{nx} = \frac{y-x}{x}$$

$$y = \frac{nx}{n-1} \text{ mts}$$

$$y - ny = -nx$$

$$y(1-n) = -nx$$

IV, If 'A' can run/cover 'x' mts race distance in  $t_1$  sec & 'B' in  $t_2$  sec, where  $t_1 < t_2$ , then 'A' beats 'B' by a distance of  $\left[ \frac{x}{t_2} (t_2 - t_1) \right]$  mts.

$$\left| \begin{array}{l} D_1 = x \\ \text{in } t_1 \end{array} \right|$$

$$S_1 = \frac{x}{t_1}$$

$| D_2 = \frac{x-y}{t_2} |$  (A beats B by 'y' mts)

then:  $\frac{D_1}{S_1} > \frac{D_2}{S_2}$

$$S_2 = \frac{x-y}{t_2}$$

$$\Rightarrow \frac{D_1}{S_1} = \frac{D_2}{S_2}$$

$$\Rightarrow \frac{x}{t_1} = \frac{x-y}{t_2}$$

$$\Rightarrow \frac{x t_1}{t_1} = \frac{(x-y) t_2}{t_2}$$

$$y \Rightarrow \frac{x}{t_2} (t_2 - t_1)$$

$$\frac{y}{t_1} = \frac{x-y}{t_2}$$

$$\frac{y}{t_1} = \frac{x-y}{t_2}$$

$$x t_1 = x t_2 - y t_2$$

$$y t_2 = x (t_2 - t_1)$$

$$y = \frac{x}{(t_2 - t_1)}$$

Q

① 'A' runs  $1\frac{2}{3}$  times as fast as 'B'. If 'A' gives 'B' a start of 80m, how far must the winning post be so that 'A' & 'B' might reach at same time?

A)  $n = 1\frac{2}{3} = \frac{5}{3}$  then length of race course  $= \left( \frac{n}{n-1} \right) x$

$$x = 80 \text{ mts}$$

$$= \left( \frac{5}{3} \right) 80$$

$$= \frac{(5-3)}{3} 80$$

$$= \frac{5}{2} \times 80 = 200 \text{ m}$$

2) 'A' is  $2\frac{1}{3}$  times as fast as 'B'. If 'A' gives 'B' a start of 80m, how long should the race course be so that both of them reach at same time & Dead-heat race.

Sol:  $n = 2\frac{1}{3} = \frac{7}{3}$   $= \left( \frac{n}{n-1} \right) x$

$$x = 80$$

$$= \frac{7}{3} (80) = \frac{7}{4} \times 80 = 140 \text{ m}$$

3) 'A' runs  $1\frac{3}{8}$  times as fast as 'B'. If 'A' gives 'B' a start of 90m & they reach the goal at same time. The goal is at a distance of.

Sol  $n = 1\frac{3}{8} = \frac{11}{8}$

$$x = 90$$

$$\left( \frac{11}{8} \right) x 90 = \frac{11}{3} \times 90 = 330 \text{ m}$$

Q) 'A' can run 224 mts in 28 sec, & 'B' in 32 sec. By what distance 'A' beats 'B'?

Sol: 'A' beats 'B' by  $\frac{x}{72} (t_2 - t_1)$  mts

$$\frac{224}{32} (32 - 28) = \frac{224 \times 4}{32} = 28 \text{ mts}$$

5. In 100 mts race, 'A' can give 'B' 10 mts & 'C' 28 mts. In the same race 'B' can give 'C' :

Sol: ~~A=100 mts~~       $A_1 = 100 \text{ mts}$       In the same race:  
~~B=90 mts~~       $B_1 = 90 \text{ mts}$       'B' can give  
~~C=72 mts~~       $C_1 = 72 \text{ mts}$       ' $x$ ' mts to

then we know that,

$$\Rightarrow \frac{A_1}{B_1} = \frac{C_2}{B_2}$$

$$\Rightarrow \frac{72}{90} = \frac{100-x}{100} \quad \Rightarrow 9x = 900 - 720$$

$$720 = 900 - 9x \quad \therefore x = \frac{180}{9}$$

$$\boxed{x = 20 \text{ mts}}$$

26.09.2022

Monday

# SEQUENCE

- \* A Sequence is a Series of Numbers or Letters which maintain a Common Characteristic property throughout the Series.
- \* Generally Sequences are also called as Progressions. They are of 3 types:
  1. Arithmetic Progression [A.P]
  2. Geometric Progression [G.P]
  3. Harmonic Progression [H.P]

## Arithmetic Progression [A.P.]:

- \* The difference b/w any two consecutive

- \*  $n^{\text{th}}$  term of AP whose first term is 'a' & common difference is 'd', then  $T_n = a + (n-1)d$

- \* Sum of 'n' terms of AP is:

$$S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n}{2} [T_1 + T_n]$$

\*  $T_n - T_{n-1} = d$

\*  $T_n = S_n - S_{n-1}$

\* AM. of  $a$  &  $b$  is  $= \frac{a+b}{2}$

- \* To Insert 'n' arithmetic Means b/w '(a)' & '(b)' take them as:  $x_1, x_2, \dots, x_n$  where  $x_i = a + id$ ,  $i=1, 2, 3, \dots, n$
- &  $d = \frac{b-a}{n+1}$

\*  $S_n = n * \text{AM}$

- \* The reciprocals of terms of AP are in H.P.

- \* To Consider 3 terms in AP, we have to take them as:  $a-d, a, a+d$

- \* To Consider 4 terms in AP, we have to take them as:  $a-3d, a-d, a+d, a+3d$

2) Geometric Progression [G.P]: If the ratio of any two successive terms is same, then that series is called Geometric Progression [G.P] and the ratio is called Common Ratio.

\* The General terms of

\*  $n$ th term of GP whose 1st term is 'a' & common ratio 'r' is

$$T_n = ar^{n-1}$$

$$* \frac{T_n}{T_{n-1}} = r \quad * \text{Sum of } n \text{ terms of G.P} \quad S_n = \begin{cases} \frac{a(r^n - 1)}{r - 1} & \text{if } r > 1 \\ \frac{a(1 - r^n)}{1 - r} & \text{if } r < 1 \end{cases}$$

\* Sum of  $\infty$  terms of G.P.

$$S_\infty = \begin{cases} \infty & \text{if } r > 1 \\ \frac{a}{1 - r} & \text{if } r < 1 \end{cases}$$

\* GM of  $a$  &  $b$  is  $\sqrt{ab}$

\* GM of numbers which are in G.P. =  $\sqrt{T_1 * T_n}$

\* The reciprocals of terms of GP are again in G.P.

\* To consider 3 terms in GP, we have to take them as

$$\frac{a}{r}, a, ar$$

$r$  = Common Ratio.

\* To consider 4 terms in GP, we have to take them as:

$$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3, \quad \begin{cases} a = \text{first term} \\ r^2 = \text{common ratio.} \end{cases}$$

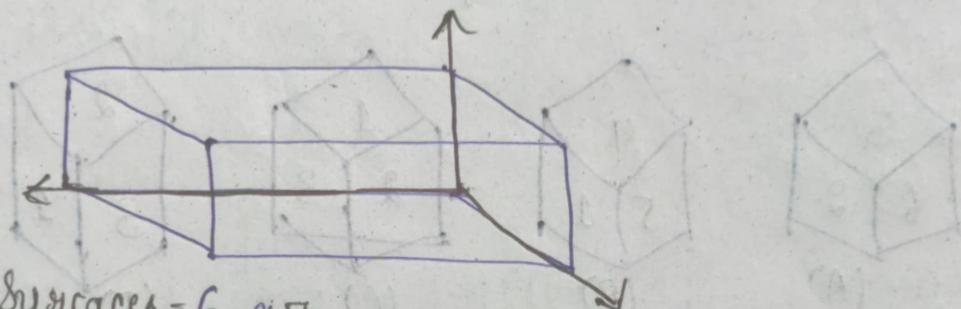
## Cube and Cuboid

\* Volume = Base Area  $\times$  Height

\* Lateral Surface Area (LSA) or Curved Surface Area  
 $=$  Base Perimeter  $\times$  Height

\* Total Surface Area = LSA + 2(Base Area)

## Cuboid:



Total surfaces = 6 & Faces = 6 [Rectangles]

Vertices / Corners = 8

Edges = 12

\* Volume = Base Area  $\times$  Height =  $(lb) \times h = [lwh]$  cubic units

\* (only side) Base Perimeter  $\times$  height =  $(2(l+b)) \times h = [2h(l+b)]$

\* TSA = LSA + 2(Base Area)

$$= 2h(l+b) + 2(lb)$$

$$= 2lh + 2bh + 2lb = [2(lb + bh + hl)]$$

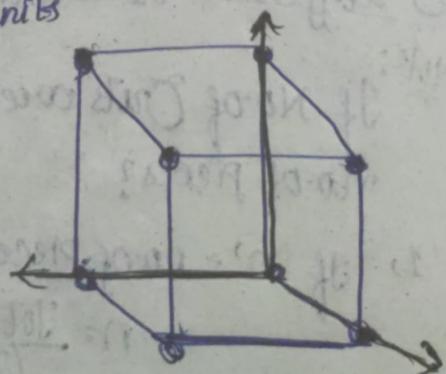
## Cube: $l=b=h=s$

Volume =  $[s^3]$  cu. units

LSA =  $2s(s+s) = 2s(2s) = [4s^2]$  sq. units

TSA =  $2(s^2 + s^2 + s^2)$

$$= [6s^2]$$
 sq. units.



Euler's Formula

F = Surfaces

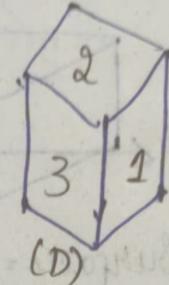
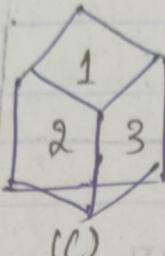
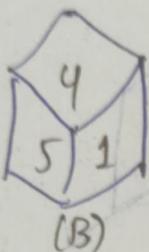
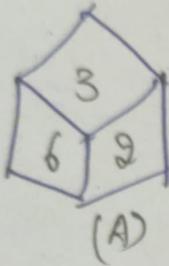
V = Vertices/Com.

E = Edges

$$F + V = E + 2$$

$$F + V = E + 2$$

$\Rightarrow$  2, 3, 4, 5 all are adjacent so remaining is '6'. So it is Opposite side of '1'.



If Top is (6)  $\rightarrow$  then opp of (8) is (1)

18/07/2022

Monday

\* If you want the Max No of Identical Pieces, then you should cut the Cube in all directions

\* If you want the minimum no of Identical Pieces, then you should cut the cube in same direction

\* In Order to obtain Max no of pieces, the Cuts Given to a large Cube must be divided as equally as possible in 3 different directions.

Note: If No of Cuts are Given. Then How to find Max no. of pieces?

1) If 'n' = no. of pieces along each Edge.

$$n = \frac{\text{Total Cuts}}{(3 \times n)} + 1 \quad | \quad n = \frac{\text{Total Cuts}}{3} + 1$$

$\rightarrow$  Max no. of Identical Pieces:  $n^3$

$\rightarrow$  No. of Pieces with '3' faces Visible = 8

No. of Pieces with '2' faces Visible =  $12(n-2)$

No. of Pieces with '1' face Visible =  $6(n-2)^2$

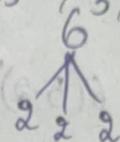
No. of Pieces with 'No' face Visible =  $(n-2)^3$

25.07.2022

Monday

① Min Pieces =  $n+1 = 5+1 = \boxed{6}$   $n = \text{no. of cuts}$

② Max Pieces =  $\frac{6}{3} \times 1 \times \frac{5}{3} \times \frac{5}{3} + 1 = \frac{6+3}{3} \times \frac{8}{3} = 3 \text{ on each edge}$   
 $12 \times 3 = \boxed{36 \text{ pieces}}$



$$\rightarrow (2+1)(2+1)(2+1)$$

$$= 3 \times 7 \times 3 = \boxed{27 \text{ pieces}}$$

③  $(2+1)(3+1)(4+1) = 3 \times 4 \times 5 = \boxed{60}$

④ Min = 216 pieces =  $6 \times 6 \times 6 = (6-1) + (6-1) + (6-1) = 5+5+5 = \boxed{15} \rightarrow \text{cuts factorization}$   
Cuts = 215

⑤ Min = 24 pieces =  $24-1 = 23$   
Cuts = 23

$$\begin{array}{c} 24 \\ 2 \quad 3 \quad 4 \\ \downarrow \quad \downarrow \quad \downarrow \\ 2 \times 3 \times 4 = 24 \end{array}$$

$$(2+1)(3+1)(4+1) = 1+2+3 = \boxed{6}$$

⑥ Cuts = 10  
Max Pieces =  $\frac{10}{3} \times 1 \text{ on each edge}$   
 $\rightarrow \frac{10+3}{3} = \frac{13}{3} \quad \frac{11}{3} \quad \frac{14}{3}$

$$\begin{array}{c} 10 \\ 3 \quad 3 \quad 4 \\ \downarrow \quad \downarrow \quad \downarrow \\ (3+1)(3+1)(4+1) = 4 \times 4 \times 5 = 80 \end{array}$$

Total Max Pieces =  $\boxed{80}$

7)  $27^3 = 343$   
 ~~$5^2 = 25$~~   
 ~~$128$~~

$$\begin{array}{c} 3 \times 3 \times 3 \\ \downarrow \quad \downarrow \quad \downarrow \\ (3+2) \quad (3+2) \quad (3+2) \\ 5 \quad 5 \quad 5 \\ 5 = 5^3 = 125 \end{array}$$

$$125 - 27$$

$$\boxed{98}$$

$$\begin{array}{r} 125 \\ - 27 \\ \hline 98 \end{array}$$

$$565 - 115$$

$$\begin{array}{r} 3 \quad 3 \quad 27 \\ \times \quad 27 \\ \hline 182 \end{array}$$

$$\textcircled{8} \quad 5 \times 5 \times 5 = 125$$

$$(5+1)(5+1)(5+1) = 216$$

$$= (5+1)^3 - (5)^3$$

$$= 216 - 125 = \textcircled{91}$$

\textcircled{9}: A cube placed on table, to cover cube how many small cubes are required?

$$\textcircled{10} \quad 125$$

$$5 \quad 5 \quad 5$$

$$(n-2)^3 = (5-2)^3 = 3^3 = 27 \rightarrow \boxed{\text{No face.}}$$

$$\textcircled{10} \quad \text{ii}, 6(n-2)^2 = 6(5-2)^2 = 6(3)^2 = 6 \times 9 = 54 \rightarrow \boxed{\text{One face.}}$$

$$\textcircled{11} \quad \text{iii}, 12(n-2) = 12(5-2) = 12 \times 3 = 36 \rightarrow \boxed{\text{Two faces exactly.}}$$

$$\textcircled{12} \quad 216$$

$$6 \times 6 \times 6 = 216$$

$$6 \quad 6 \quad 6$$

$$(6-2)(6-2)(6-2) = 4 \times 4 \times 4 = \textcircled{64} \rightarrow \boxed{\text{Inside blue cubes.}}$$

$$\textcircled{13} \quad 12(6-2) = 12(4) = 48$$

$$\textcircled{14} \quad n=4 \text{ (no need)}$$

$$3 \text{ faces Painted Blue} = 8 \rightarrow \text{Direct formula}$$

$$\textcircled{15} \quad 125 = 5 \times 5 \times 5$$

$$5 \quad 5 \quad 5$$

$$3 \text{ faces visible when painted white.}$$

$$at least 2 \& 3$$

$$= 5+4+4 = \textcircled{13}$$

\textcircled{16} No face painted:

$$(n-2)^3 = (5-2)^3 = (3)^3 = \textcircled{27}$$

$$(5-1)(5-1)(5-1)$$

$$4 \times 4 \times 4 = \textcircled{64}$$

$$8+12(n-2)^2$$

$$8+12(3^2)$$

$$8+12(9)$$

$$8+108$$

$$116$$

17)	20	6	6	6	Corners colors edges colors faces colors	$\rightarrow$	yyy-1 $\rightarrow$ 19th answer. YYG-2 YGB-1 GGB-1 YYB-1 YGB-2 $\rightarrow$ a cube having all three colors. GGY-1 $4+4=8$ $8(n-2) = 8(6-2) = 8(4) = 32$ + 5 <u>37</u>
20)	$2+2=4 \times 2$ $= 8+1$ $= ⑦$				YY-2 YG-4 YB-2 BG-2 GG-1		

01.08.2022

## Data Sufficiency

Monday

### Format:

Directions for questions: Each Problem consists of a Question and two statements labelled I and II. Decide whether the data given in the statements are sufficient or not to answer the question. Make an appropriate choice from (1) to (4) as follows:

- 1, If Statement-I is alone sufficient to answer the question.
- 2, If Statement-II alone is sufficient to answer the question.
- 3, If both statements I & II are required to answer the question, but neither statement alone is sufficient.
- 4, If statements I & II are not sufficient to answer the question and additional data are required to answer it.

(81)

Options	Statement-1	Statement-2
1	✓	✗
2	✗	✓
3	✓	✓
4	✗	✗