

11/7/22 Artificial Neural Network (ANN)

Introduction :-

① Artificial Intelligence

② Machine Learning

\* Sub domain of AI

\* data + Algorithms

Ex:- Loan approval systems.

Drawbacks :-

\* cannot handle huge amounts of data.

(Performance degrades when data is huge)

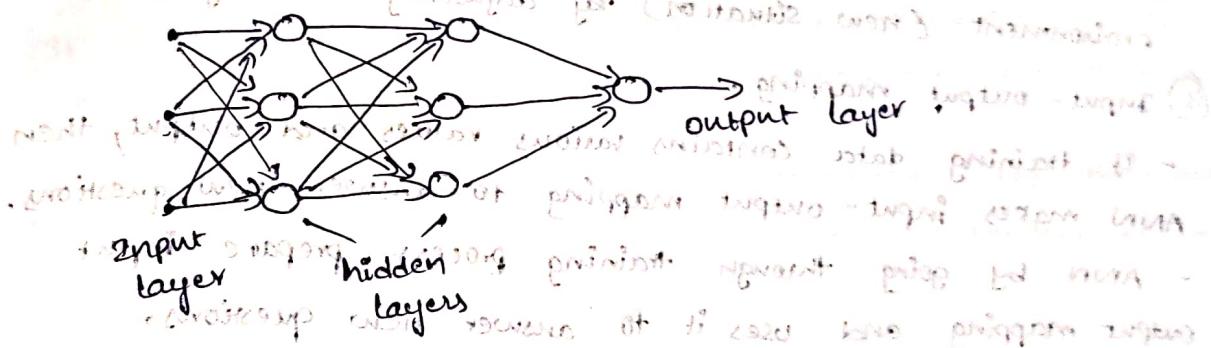
\* cannot answer the questions or tasks to which it is not trained.

③ Deep learning

\* Sub domain of ML

\* Artificial Neural Networks are building blocks

(Artificial neurons)



\* Advantages :-

\* can handle huge amounts of data.

\* can handle new situations for which it hasn't been trained.

\* can handle new situations for which it hasn't been trained.

\* Artificial Neural Networks

\* It is a massive distributed parallel processor.

\* Definition :- It is a massive distributed parallel processor.

\* History :-

1943 - McCulloch, Pitts developed first AN model.

1958 - Modified version of the above model is created by Rosenblatt and is named as perceptron

model.

1960's - Proposed modification for his model by introducing the back propagation algorithm.

\* Benefits :-

(1) Non-linearity.

- ANN has non-linear behaviour with the help of activation functions.
- It solves small problems using linear behaviour and also large or complex problems using non-linear behaviour.

(2) Adaptive.

- In ANN inputs have weights.
- Whenever ANN sees a new environment, then it adjusts inputs weights to handle it (or) answer to input.
- It can adopt new environment (or) it can work in new environment (new situation) by adjusting its weights.

(3) Input-output mapping.

- If training data contains various values and output, then ANN makes input-output mapping to answer new question.
- ANN by going through training process prepare input output mapping and uses it to answer new questions.

(4) Evidential Response :- Along with O/P ANN will also produce evidential response i.e., at what probability, the O/P is produced.

(5) Contextual Information :- Knowledge is represented by the structure of ANN & State of ANN.

(6) Uniformness :- Every neuron is equally capable of processing data.

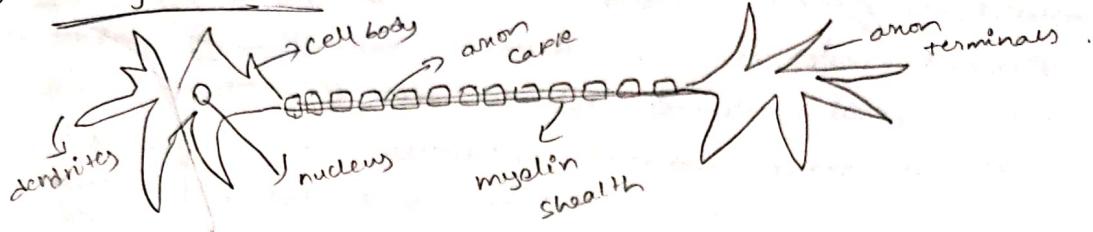
(7) VLSI compatibility :- As ANN consists of many processing elements (neurons), it is comparable with VLSI.

1 by introducing  
the concept of  
parallel processing  
and fault tolerance  
with the help of  
neurons.

8.) Fault tolerant :- Failure of one of the neuron won't affect the entire ANN.

9.) Neuro Biological Analogy :- Neurons are analogous to the neurons in the brain.

10.) Biological Neuron :- 10 neurons in human brain



Biological Neuron consists of

1.) dendrites act as I/P units providers to neurons.

2.) cell body

Cell nucleus - performs aggregation of electrical impulses received from the different neuron.

3.) axon cable

4.) axon terminals

It also performs thresholding operation. Aggregated sum is called induced local field value it compares with threshold value.

\* If induced local field values > threshold value then the info is transferred from one neuron to another neuron  
if  $FLFV < TV \Rightarrow$  stays idle.

3.) axon cable

i) Sheathed with myelin sheath.

ii) If thickness of myelin sheath is more than very less energy loss in electrical signal otherwise more energy loss.

iii) If thickness of myelin sheath is less than very less energy loss.

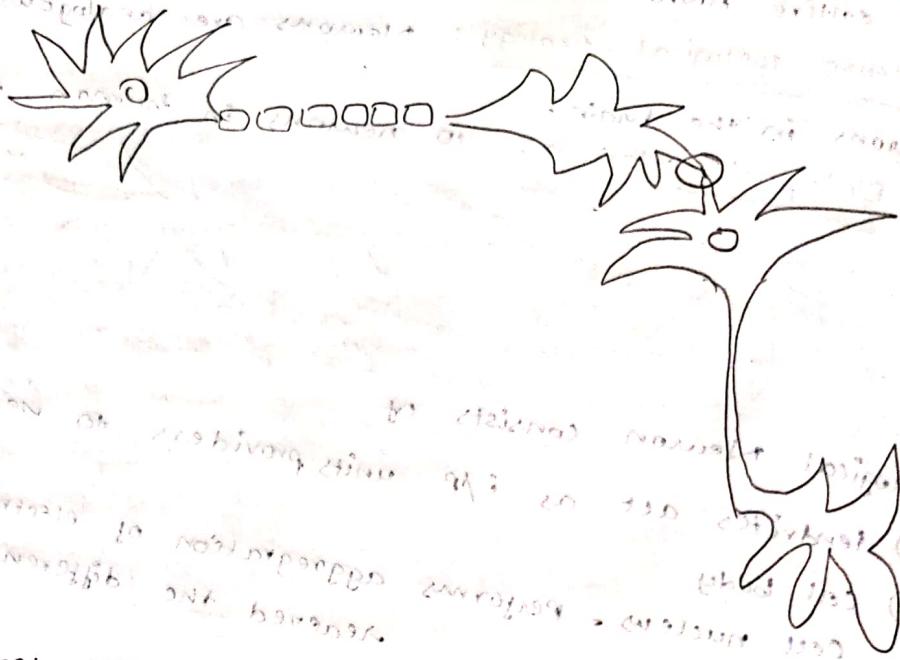
4.) axon terminals:

modification date of assignment Page

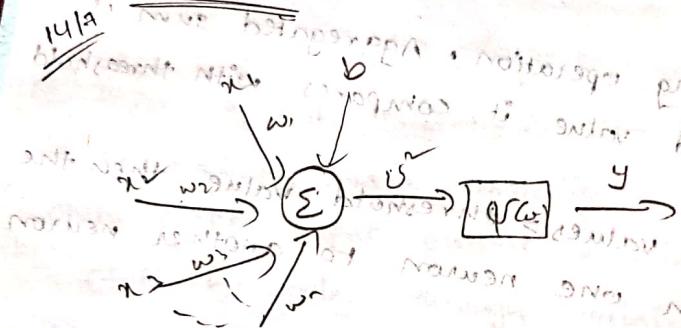
Continued

Mathematical definition of perceptron

If these are the output units.



### Artificial neurons



### Aggregator

$$v = \sum_{i=1}^n x_i w_i + b.$$

### \* Activation function

- mathematical function
- decides whether to activate (or) deactivate the neuron based on induced local field value i.e.,  $v$ .

### Benefits

- 1) Activation function add non-linearity to the artificial neuron.
- 2) Activation function makes neuron to handle complex problems.

- \* 3) It also produces evidences in the form of probability.
- \* 4) It makes neuron to perform multiclass classification
- \* 5) Map output to certain range of values along with evidence.
- \* If there is no activation function, neuron will become linear unit, linear unit can perform binary classification.
- \* If there is activation function, it gives non-linearity to the neuron. It can handle complex tasks.
- \* If there is no activation function there is no change in input and output.
- \* Sigmoid value of activation function produces probability value.

### Different types of Activation functions:

- \* Based on their classification and behaviour it was divided into 3 types.
  - 1) Linear activation function,
  - 2) Binary threshold activation function,
  - 3) Non-linear activation functions,

- 1) Linear activation function: Activation function is a mathematical function.
- \* Linear activation function, it produces output same as its input.  $f(v) = v$ . (Mathematical notation).

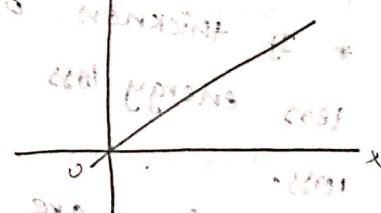
\* Range of the output is  $-\infty$  to  $+\infty$ .

#### Advantages:

- \* L.A.F is suitable for performing simple binary classification only.

#### Drawbacks:

- \* It is not suitable for error correction process as derivative is a constant.
- \* Difficult to implement if  $f(v)$  is constant.



Artificial neuron is a mathematical model of a biological neuron.

- \* It is a mathematical model of a biological neuron.
- \* It is inspired by biological neuron in many ways.
- \*  $v$  - induced local field.
- \* (Activation state) (transfer signals).
- \*  $v \geq \text{threshold} \Rightarrow y = 1$
- \*  $v < \text{threshold} \Rightarrow y = 0$  (idle)
- \*  $v$  is induced local field.
- \*  $x_1, x_2, \dots, x_n$  is input to neuron.
- \* Input is weighted sum  $w_1, w_2, \dots, w_n$  (multiplicative factor).
- \*  $b$  - bias (resting potential).
- \*  $y$  is output.
- $y = \psi(v)$  where  $\psi$  is activation function.
- $y = 1$  if  $v \geq \text{threshold}$ .
- $y = 0$  if  $v < \text{threshold}$ .

### Biological Neuron

- \* Dendrites acts as input providers to neurons.
- \* cell nucleus performs aggregation, thresholding.
- \* If thickness of myelin sheath is more than very less energy loss in electrical signal otherwise more energy loss.
- \* Dendrites are input units and axon terminates are output units of neuron.
- \* The connecting point of two neurons is called the synoptic structure.
- \* One neuron axon terminal is connected to dendrites of other neuron for transfer of signals. There is some gap between axon terminal of 1<sup>st</sup> neuron and dendrite of 2<sup>nd</sup> neuron.

- \* This is g
- 15/2
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- f(v)
- \* It
- \* Non-
- \* It is
- behavior
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- \* It is
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- \* we
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\* This gap is called synoptic structure.

15/a

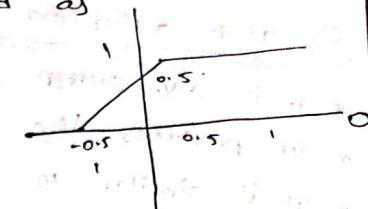
\* Piece wise linear activation function

\* It allows to have a neuron

\* It has real value boundaries.

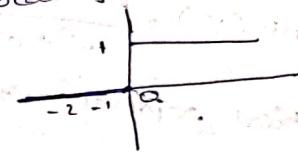
\* In mathematical notation it is defined as

$$f(v) = \begin{cases} 1 & v \geq +1/2 \\ v & -1/2 \leq v < +1/2 \\ 0 & v \leq -1/2 \end{cases}$$



\* Act Binary threshold

Binary threshold is a A.F it produces output as 1 or 0 based on input value (or) induced local field value.



$$f(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$

\* It is not suitable for using in back propagation.

\* Non-linear Activation function

\* It introduced and provide artificial neuron with non-linear behaviour or non-linear functionality.

### ① Sigmoid / Logistic activation function:

\* It is a probabilistic activation function.

\* It produces the output in the range 0 to 1 only.

\* It is most widely used activation function.

\* It is most widely used activation function in hidden layers (and output layers).

\* When we pass an input to this function it computes probability within the range 0 to 1.

\* Mathematical notation for sigmoid activation function is.

$$f(v) = \frac{1}{1 + e^{-v}}$$

\* The graph for sigmoid / logistic activation function is



### Adv

- \* And it has a gradient sigmoid ( $\sigma$ )  $\approx 1 - \text{sigmoid}(\sigma)$ , and it is used in the back propagation.

### \* Drawbacks

- \* It suffers with the vanishing gradient problem.

### (6) Tanh activation function:

- \* It is also called as hyperbolic tanh activation function.
- \* It produces the output within a range  $-1$  to  $1$ .
- \* It is similar to sigmoid activation function.

- \* It is zero centered.

- \* Mathematical function for tanh activation function is

$$\tanh(v) = \frac{e^v - e^{-v}}{e^v + e^{-v}}$$

- \* Graphical representation of tanh is

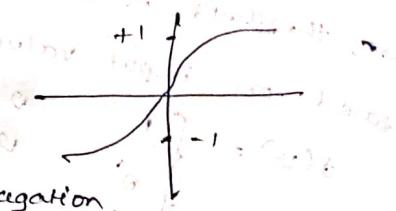
- \* has gradient  $\approx \tanh'(0)$ .

- \* And it is suitable for back propagation.

### \* Drawbacks:

- \* It is vanishing gradient problem.

- \* In this below  $-5$  and above  $+5$  the gradient is 0.



### \* Relu activation function:

- \* It is specified linear unit.

- \* It is most widely used in Hidden layers of Artificial Neural Networks.

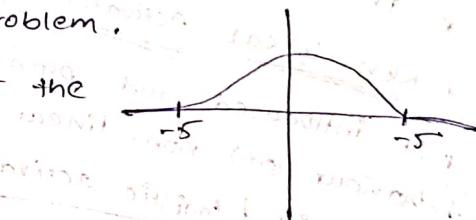
- \* Its mathematical function is

$$\text{relu}(v) = \max(0, v)$$

- \* Output ranges from  $0$  to  $+\infty$ .

- \* Its graphical representation is

- \* It has gradient hence used in back propagation for error correction.



### Drawbacks:

- \* since, it deactivates where all
- \* dying

### (7) Leaky

- \* similar is re.

- \* It's math

### LReLU ( $v$ )

- $a$  is c

$$a =$$

- \* It avoids multiplied

- \* output

- \* It is used

### \* Advantages:

- \* It doesn't

- \* It is siv

- \* It produce

- \* No dying

### (8) Softmax

- \* It is used

- \* It is most

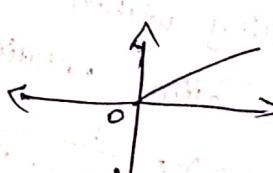
- \* Similar to

- \* It's math

### softmax

- \* It compu to 0 and 1.

- \* It has



### Drawbacks :-

- \* since, i/p is zero, when output is 0, neuron is in deactivation state so can't be used in solving a problem.
- \* where all neuron's should be in activation state.
- \* dying ReLU problem.

### (4) Leaky ReLU Activation function :-

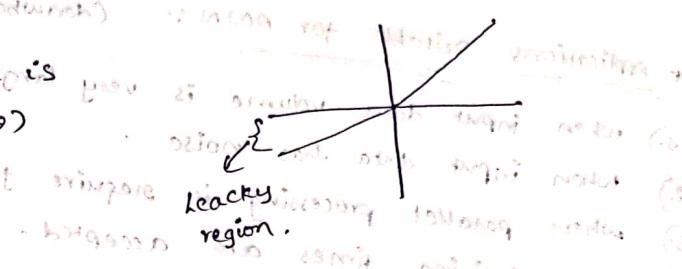
- \* similar to ReLU but it produces an o/p even when i/p is -ve.

- \* It's mathematical function is

$$LReLU(v) = \max(av, v)$$

-a is constant.

$$a = 0.01$$



- \* It avoids dying ReLU problem using a variable, a which is multiplied to input and max of av & i/p, is considered as o/p.
- \* output range is  $(-\infty, \infty)$
- \* It is used in hidden layers.

### \* Advantages :-

- \* It doesn't activate the neuron.
- \* It is simple activation function.
- \* It produces output for -ve values too.
- \* No dying ReLU Problem.

### (5) Softmax Activation function :-

- \* It is used at output layer.
- \* It is most widely used in A.F.
- \* Similar to sigmoid AF and called as sigmoid or sigmoids.
- \* It's mathematical function is

$$\text{softmax}(v) = \frac{\text{sigmoid}_i(v)}{\sum_{j=1}^k \text{sigmoid}_j(v)}$$

- \* It computes relative probability so its output range is between 0 and 1.
- \* It has gradient hence used in back propagation for the error correction.

\* Which function to use?

\* Simple problems Applications at all levels - sigmoidal (i.e., for binary classification)

\* If all neurons should be in active state - Leaky rule - use at hidden layers.

\* If deactivation of neuron's is needed use rectu.

\* use softmax for output layer - (complex applications) multi-class classification

\* Applications suitable for ANN :- (drawbacks of ML),

- 1) when input data volume is very large
- 2) when input data has noise
- 3) when parallel processing is required for applications
- 4) long training times are accepted
- 5) suitable for image processing applications, objects identification, language processing, voice synthesizing

[100 - 5]

for more info. refer slides

• Computer vision or handwriting

• Signal processing

• Data mining

• Machine learning

• Pattern recognition

• Decision support systems

• Intelligent agents

• Neural networks

• Expert systems

• Intelligent robots

• Intelligent sensors

• Intelligent manufacturing

• Intelligent transportation

• Intelligent instruments

- \* Which function to use?
- \* Simple problems Applications at all levels: sigmoidal (i.e., for binary classification) - Leaky rule (use at hidden layers)
- \* If all neurons should be in active state - use softmax
- \* If deactivation of neuron's is needed use rectu.
- \* use softmax for output layer - (complex applications) multi-class classification
- \* Applications suitable for ANN :- Drawbacks of ML

  - 1) When input data volume is very large.
  - 2) When input data has noise.
  - 3) When parallel processing is required for applications.
  - 4) Long training times are accepted.
  - 5) Suitable for image processing, applications, objects identification, language processing, voice synthesizing.

20/2/22

### \* Artificial Neuron Models :-

\* There are 3 different types.

1) McCulloch and Pitts Neuron Model.

2) Perception Model.

3) Adaline Model.

1) McCulloch and Pitts Neuron Model:

\* This is the first Artificial Neuron Model.

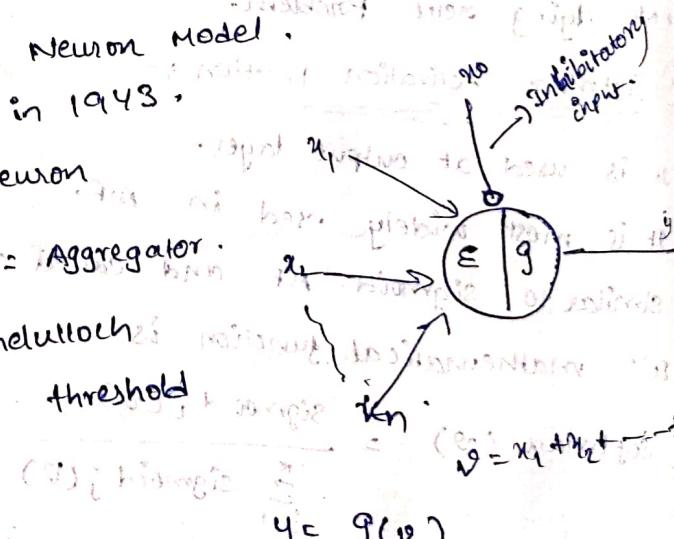
\* This model is invented in 1943.

\* Inspired by biological Neuron.

\*  $\theta$  - thresholding element,  $\Sigma$  = Aggregator.

\* Activation function in McCulloch and Pitts Neuron model is threshold function.

Activation function.



Light up神经系统 about  $\theta$  to threshold value  
where,  $\theta$  is threshold sum value.

- \* It accepts
- \* It is not
- \* It will accept
- 1) Excited
- 2) If it is
- respective
- \* If it is to
- output by
- 1) \* These
- excitatory
- \* Excitatory
- \* Inhibitory

### \* Uses :-

\* It is used

Drawbacks:

\* There is

\* It is

Perceptron

\* Rosenblatt

\* It was

\* Perceptron

\* Electrical

\* He is

\* He is

\* The

\* With

\* We

- \* It accepts only binary inputs only and produces binary outputs.
- \* It is not suitable for real value inputs.
- \* It will accept 2 types of inputs.

1) Excited Inputs

2) Inhibitory Inputs

- 2) \* If it is active then make neuron to produce output as 1 irrespective of value of excitatory inputs.
- \* If it is turned off then neuron produce 0 and computes output by passing it to g.

- 1) \* These are normal inputs which can make neuron go to excitatory state.
- \* Excitatory inputs are positively weighted.
- \* Inhibitory inputs are negatively weighted.

#### \* Uses :-

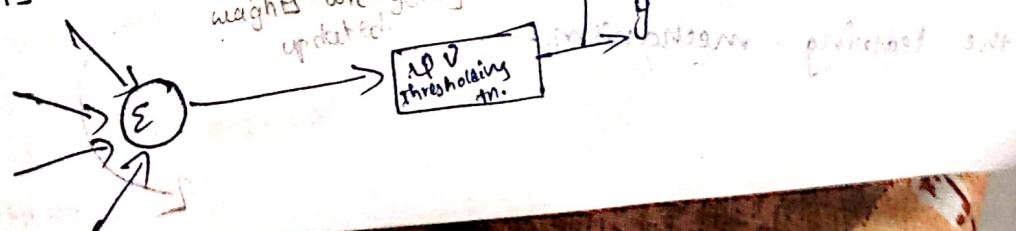
- \* It is used for primary classification only.

#### Drawbacks :-

- \* There is no learning mechanism.
- \* It is not suitable for complex problems like multi-class classification.

#### 7) Perception model :-

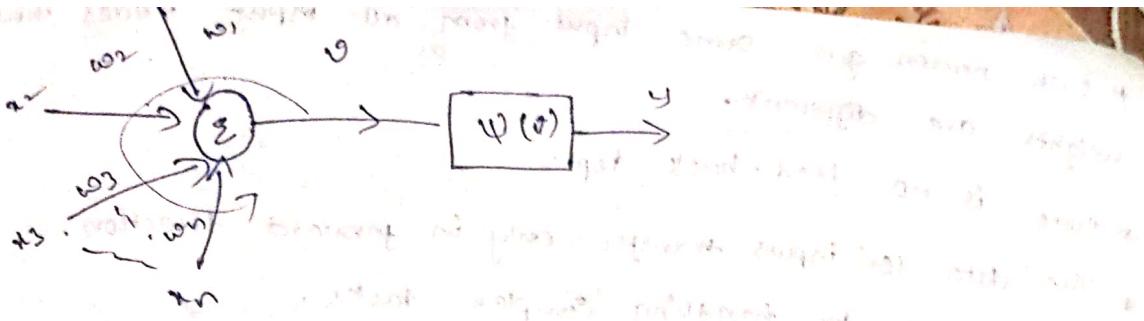
- \* Rosenblatt proposed perception.
- \* It was invented in 1958.
- \* Perception requires huge computation efficiency.
- \* Perceptron prepared perception model.
- \* Electrical circuits prepared perceptron model.
- \* He invented bias factor.
- \* He invented learning mechanism for neurons.
- \* He invented learning mechanism for inputs.
- \* The perceptron accepts the real values of inputs.
- \* With this changes the Rosenblatt invented perceptron.
- \* Inputs are weighted with random values.
- \* Weights are assigned with random values.
- \* Weights are getting updated.



- \*  $v = n_1 w_1 + n_2 w_2 + \dots + n_n w_n + b$
- \* In mathematical notation,
 
$$v = \sum_{i=1}^n n_i w_i \text{ with } i=1$$
- \* The output 'y' communicated as a function of v.
 
$$y = \psi(v)$$
- \* There are 2 types of Perceptron,
  - 1) Single layer perceptron.
  - 2) Multi-layer perceptron.

In this there are only one output layer.
- \* Advantages:
  - In this it contains real-valued inputs.
  - It contains multi-class classification.
  - Learning mechanism will be used.
- \* Adaline model:
  - It is also known as Widrow and Hoff model.
  - It was developed in Stanford University in 1960.
  - Adaline - Adaptive linear element.
  - It is the model and used to predict the transmission line (means).
  - Madaline, it is used to reduce the noise.
  - Madaline is induced.
  - These two models are prepared by the electrical circuits.
  - It is according the adaptive linear element.
  - It just look like perceptron but there is a change in the learning mechanism.

- \* Among
- 1) Perceptron
- 2) Multi-layer
- \* Network
- \* Network
- \* In ANN
- \* ANN neurons
- \* Neuron
- \* There
  - 1) Input
  - 2) Hidden
  - 3) Output
  - \* It can
  - \* It can
- \* There
  - 1) single
  - 2) Multi
  - 3) Rec
  - \* Single
  - \* In output



\* Among these three perception is very useful and the

- 1) perceptron has a single layer of artificial neurons.
- 2) perceptron has multi-layers of artificial neurons containing multi-layers.

#### \* Network Architectures (or) Network models :-

- \* Network is a interconnected structure of processing elements.
- \* Network is a interconnected structure of processing elements.
- \* In ANN, Processing element is artificial neuron.
- \* ANN is a interconnected structure of artificial neurons.
- \* Neurons are arranged in layers.
- \* There are 3 types of layers.

#### 1) Input layer

- \* It contains only input nodes.

#### 2) Hidden layer

- \* It contains artificial neurons.

#### 3) Output layer

- \* It contains artificial neurons.

- \* There are 3 different architectural models, they are

#### 1) single layer feed forward network

#### 2) Multi-layer feed forward network

#### 3) Recurrent network

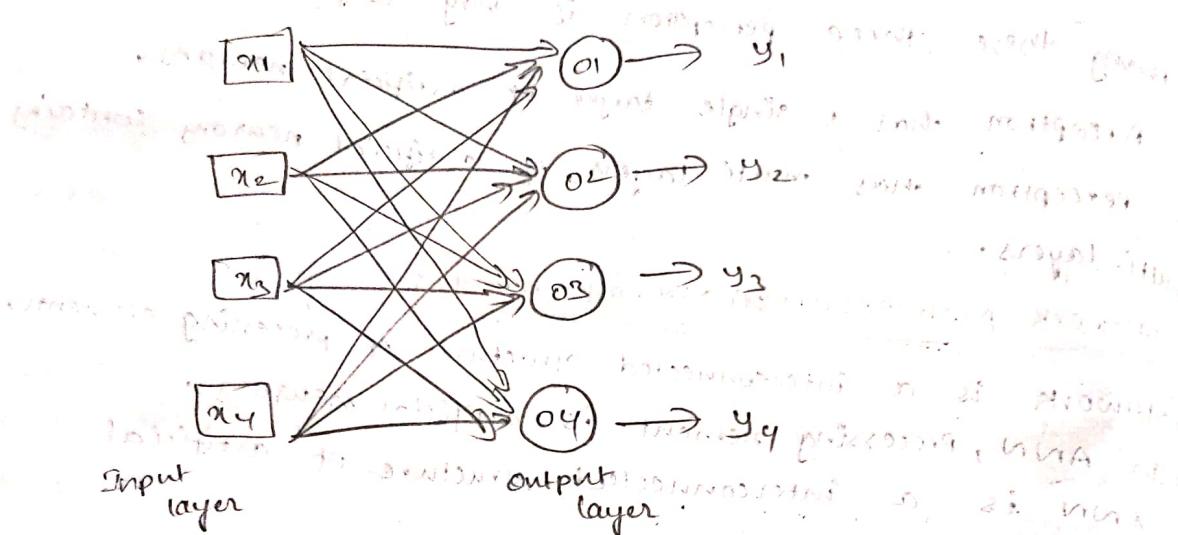
- \* single layer feed forward network:

- \* In this architectural model, ANN consists of 1 output layer.

- \* each neuron gets same input from all input nodes but weights are different.
- \* there is no feed-back loops.

\* The data (or) inputs transfer only in forward direction.

\* Not suitable for handling complex tasks.



\* Multi-layer feed forward Network

\* Along with 1 output layer, it also contain 1 or more hidden layers.

\* Hidden layers let ANN to handle complex tasks.

\* 1<sup>st</sup> level hidden layers can perform some low level tasks.

\* 2<sup>nd</sup> level hidden layers can perform a task higher than previous layer.

\* output layer neuron can handle high level tasks to produce the output.

\* Neurons in all layers are connected with previous layer neurons.

Input layer  $\rightarrow$  Hidden layer 1  $\rightarrow$  output layer  $\rightarrow$   $y$

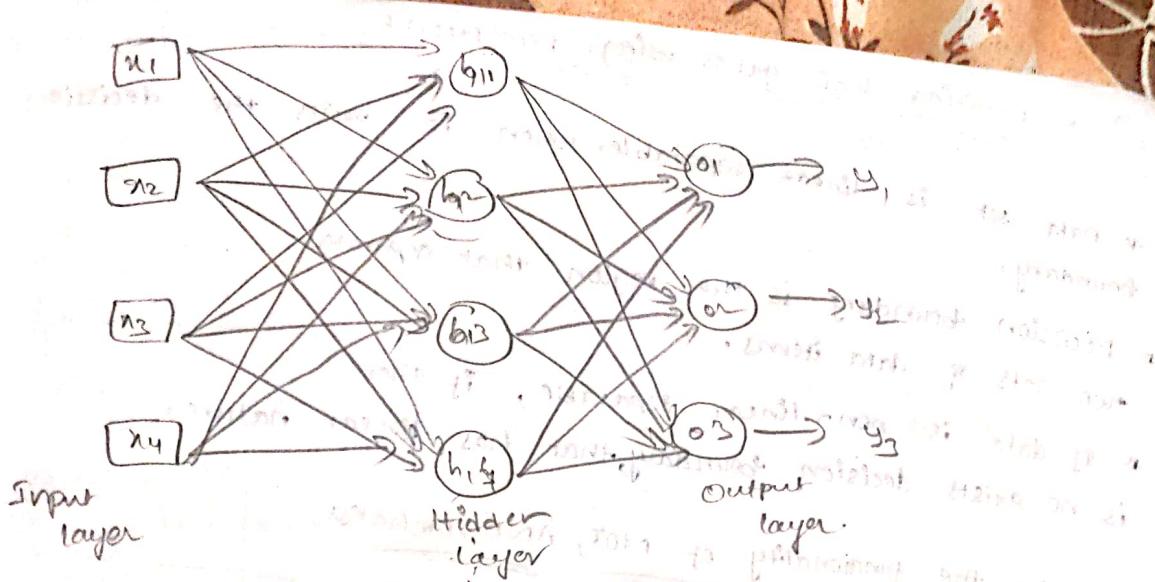
\* Information flows in forward direction only.

\* There exists no feed-back loops.

$x_1$   
 $x_2$   
 $x_3$   
 $x_4$   
Input layer

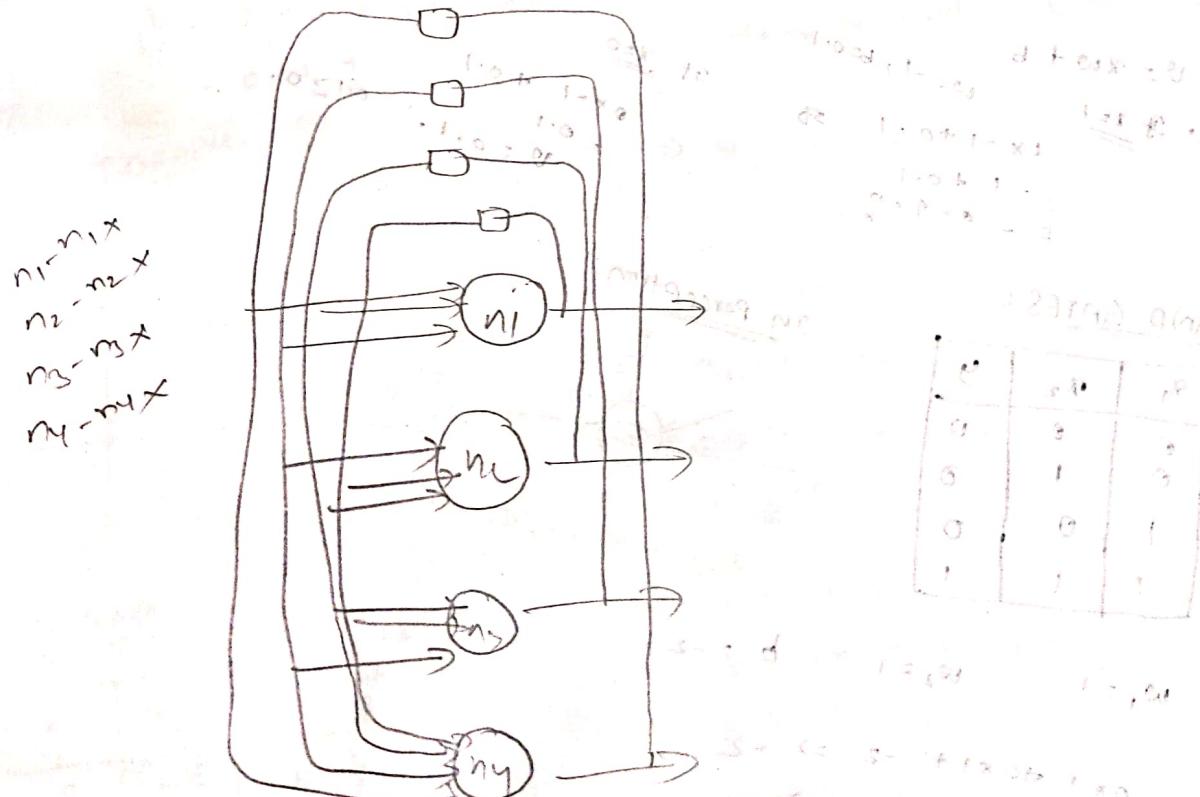
- \* Recurrent
- \* widely used in recognition
- \* There exists loops
- \* i.e., output of one layer becomes input of another layer
- \* consist of neurons to store information

$n_1 - m_1$   
 $n_2 - m_2$   
 $n_3 - m_3$   
 $n_4 - m_4$



### \* Recurrent network :-

- \* widely used in text and language processing , speech recognition etc.
- \* there exists feed - back loops.
- \* i.e., output of neuron fed as input .
- \* consist of unit delay operators (memory elements) to store previous output state of neurons .



## 22.1A \* Realizing logic gates using Perceptron

- \* Data set is linear separable when it exists the decision boundary.
- \* Decision boundary is the region that separates two sets of data items.
- \* If data is non-linear separable, if there is no exists decision boundary, that has linear nature.



### \* Realising the functionality of NOT, AND, OR gates

- \* Binary function is also called as medidile function.



### \* NOT GATES:

$$\begin{array}{c|c} x & y \\ \hline 0 & 1 \\ 1 & 0 \end{array}$$

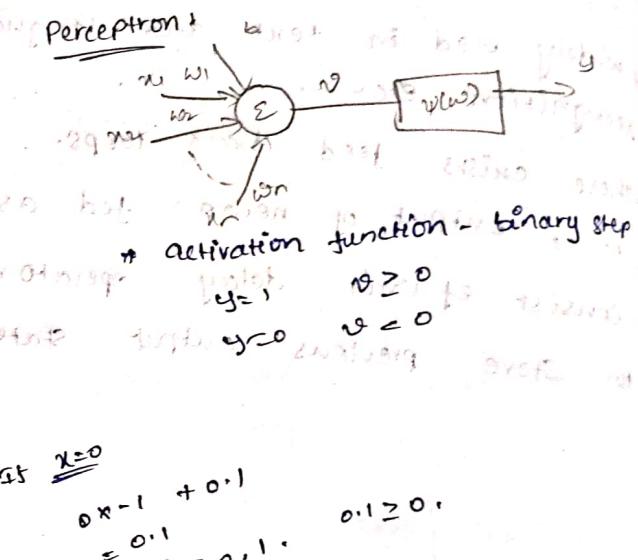
Output = 1

$v = \sum w_i x_i + b$

$\text{Let } x=1$

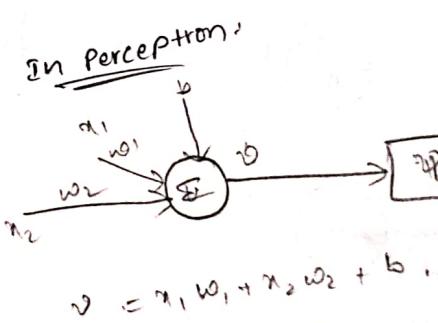
$w = -1, b = 0.1$

$x_1 w_1 + x_2 w_2 + b = -1 + 0.1 + 0.1 = -0.8 < 0$



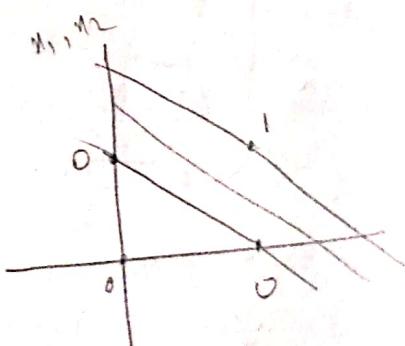
### \* AND GATES:

$x_1$	$x_2$	$y$
0	0	0
0	1	0
1	0	0
1	1	1



$$w_1 = 1 \quad w_2 = 1 \quad b = -2$$

$$\begin{aligned} 0 \times 1 + 0 \times 1 + -2 &\Rightarrow -2 \\ 0 \times 1 + 1 \times 1 + -2 &\Rightarrow -1 \\ 1 \times 1 + 0 \times 1 + -2 &\Rightarrow -1 \\ 1 \times 1 + 1 \times 1 - 2 &\Rightarrow 0 \end{aligned}$$



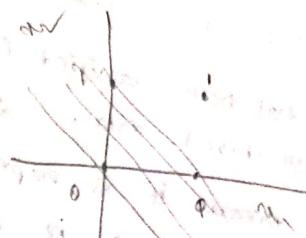
$x_1$	$x_2$	$y$
0	0	0
0	1	0
1	0	0
1	1	1

the decision

\*  $w_1 = 1, w_2 = 1, b = -2, x_1 = 0, x_2 = 0$   
 $\Rightarrow \theta = 0 \times 1 + 0 \times 1 - 2 = -2$   
 $y = 0$

$x_1 = 0, x_2 = 1$   
 $\theta = 0 \times 1 + 1 \times 1 - 2 = -1$   
 $y = 0$

### OR GRAPH

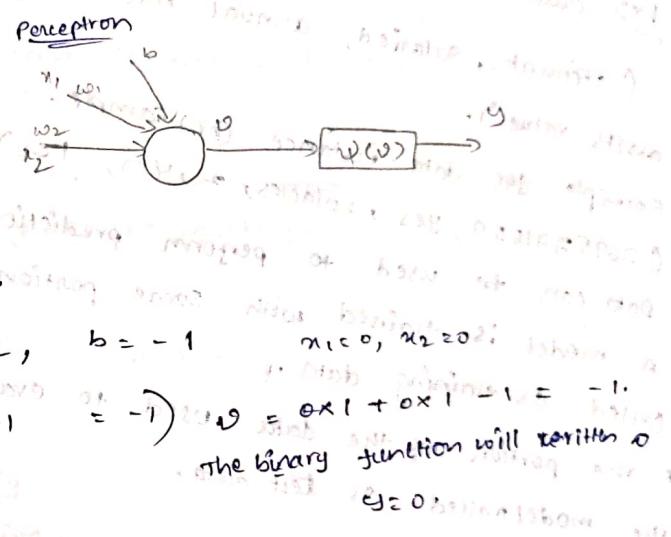


$x_1 = 1, x_2 = 0$   
 $\theta = 1 \times 1 + 0 \times 1 - 2 = -1$   
 $y = 0$

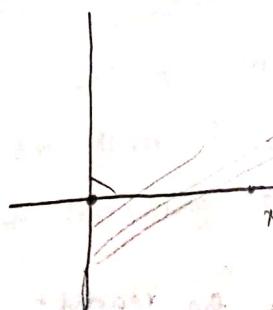
$x_1 = 1, x_2 = 1$   
 $\theta = 1 \times 1 + 1 \times 1 - 2 = 0$   
 $y = 1$

### OR GATES:

$x_1$	$x_2$	$y$
0	0	0
0	1	1
1	0	1
1	1	1



### NOT GRAPH



$\Rightarrow x_2 = 1, x_1 = 0$

$\theta = 0 \times 1 + 1 \times 1 - 1 = 0$

$y = 1$

$\Rightarrow x_1 = 1, x_2 = 0$

$\theta = 1 \times 1 + 0 \times 1 - 1 = 0$

$y = 0$

$\Rightarrow x_1 = 1, x_2 = 1$

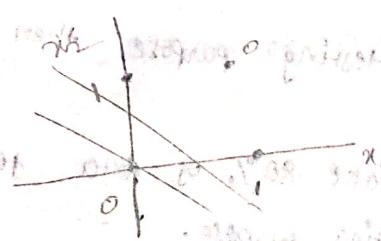
$\theta = 1 \times 1 + 1 \times 1 - 1 = 1$

$y = 1$

### XOR GATES

$x_1$	$x_2$	$y$
0	0	0
0	1	1
1	0	1
1	1	0

Principle of



Implement NOR gate functionality using perceptron.

NOTE: (use multiple perceptions to implement functionality).

## UNIT-1

\* Data has been classified into 2 types.

- 1) unstructured data.
- 2) structured data.

\* Data means it is represented using set of attributes (features).

\* Particular instance of all values for attributes we will represent data of (incident) instance, (or) sample (or) example.

\* Ex: customer information in loan management system.

(account, salaried, annual income, dependents, arrests, assets value).

\* Example for data instance (or) sample.

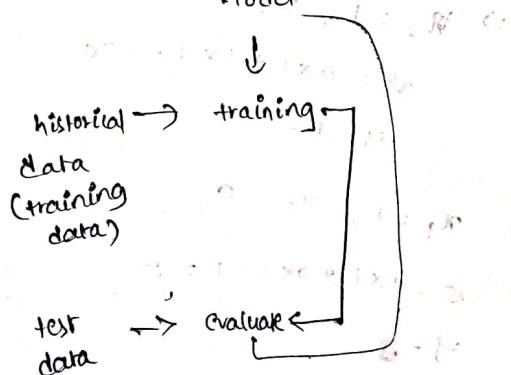
(3695771659, yes, 2 salaries, 2, yes, 1 cr).

\* Data can be used to perform predictions.

\* A model is trained with some portion of data and it is called as training data.

\* The portion of the data used to evaluate for performance of the model called as test data.

\* Training Model



\* After completion of this process the model will be ready.

\* 80:20.

when we have some data, and we have to use for training and testing purpose then we will use this 80:20 rule.

\* In this, take 80% of data for training purpose and 20% of data for testing purpose.

- \* we need to training data;
  - \* In labelled
  - \* Labelled data:
  - data instance
  - \* labelled data
  - \* data for associated with
  - \* customer in
- (3697675)

\* Learning process

the knowledge already known

\* ML / DL making correct

\* Types of learning

\* It was cl

1) supervised

2) unsupervi

3) Reinforce

4) Supervised

labelled da

on that tra

\* learning

2) unsupervised

\* learning

\* uses da

will be

reinforcement

\* learning ever

\* we need to randomize the selection of data members for training data, testing data to avoid bias.

\* In labelled data,

\* Labelled data: A label is nothing but target output for the data instance.

\* labelled data will be used for,

→ \* data for which every data sample has target output associated with it.

\* customer information

(369767513, Yes, 20Lakhs, 2, Yes, 109, Sanctioned)

Input labels

labelled output

\* Learning process: Learning process is a process for acquiring the knowledge, by making corrections to the information already known.

\* ML/DL models i.e., ANN learn from the data by making corrections to free parameters.

\* Types of learning:

\* It was classified into 3 types.

1) supervised learning.

2) unsupervised learning.

3) Reinforcement learning.

1) Supervised learning: In supervised learning, we use the labelled data set to train the machine and based on that training the machine will perform the outputs.

\* learning with a teacher

2) Unsupervised learning: uses data set

\* learning without a teacher means unlabelled data set

\* uses data without labels means unlabelled data set

will be used.

3) Reinforcement learning:

\* learning everything by interacting with environment.

## Int. to Linear Algebra &

- \* 3 types of values (i) objects for mathematical functions
- (1) Scalar (1D entity)
- (2) Matrices (2D entities)
- (3) Tensors.
- (4) vectors. (i) objects which have direction and magnitude  
(ii) collection of elements which are arranged in rows & columns.

2) we can define tensor of dimension two.

\* Relation between Neuron and linear algebra's mathematical operations of biological neuron.

\* Mathematical representation of biological neurons.

\* System of linear equations :-

$$\text{Ex: } \begin{cases} 2x + 3y = 6 \\ 4x - 2y = 4 \end{cases}$$

$\begin{bmatrix} 2 & 3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

$2x + 3y = 6$   
 $4x - 2y = 4$

$$\Rightarrow \begin{bmatrix} 2x + 3y \\ 4x - 2y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$



\* Artificial neuron performs mathematical operations.

\* Given a system of linear equations

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

it can be represented in terms of matrices and vectors as

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

\* In this case linear equations are solved by using operations on matrices and vectors.

\* In ANN, neuron can be represented as  $v$  which denotes

- \* ANN is made of linear functions
- \* inputs will be vectors, matrices.
- \* the computation involves matrices.
- \* operations
- \* scalar, matrix

\* In ANN we have

$$\text{Ex: } K$$

\* Bias, other

$$K +$$

\* operations

\* scalar

\* These

be

/ Num

\* Multipl

\* Divisi

indire

\* Add

\* Sub

- \* In ANN, the neurons computes  $v$  as linear computation i.e., neuron can produce linear equations. The input, weights represented as matrices and vectors.
- \*  $v$  is computed by performing operations on matrices, vectors which denote input, weight.

\* ANN is mathematical model.

↳ linear function.

\* Inputs will be scalar, vectors, matrices

\* The computation performed by neuron is in b/w vectors and matrices.

\* operations on matrices:

\* scalar, matrices: in b/w we can perform  $+$ ,  $*$ ,  $-$ ,  $\div$ .

\* In ANN we can use this in the bias.

\* Ex:  $K \times \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \Rightarrow \begin{bmatrix} ka_1 & kb_1 \\ ka_2 & kb_2 \end{bmatrix}$

$K + \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \Rightarrow \begin{bmatrix} k+a_1 & k+b_1 \\ k+a_2 & k+b_2 \end{bmatrix}$

\* operations on matrices:

\* scalar operations: we can perform  $+$ ,  $-$ ,  $*$ ,  $\div$ .

\* These are most frequently used in deep learning and ANN.

\* Numpy will support all operations indirectly.

\* Multiplication will directly allow numpy to support.

\* Division support on the matrices and it supports  $\frac{1}{K} A$ .

\* Indirectly. And if multiply  $\frac{1}{K} A$  are not directly allowed.

\* Addition and subtraction scalar value to  $A$ , then,

\* Suppose add (or) sub of scalar value to  $A$ , then  $[a_1 \ b_1] + 4 = [a_1+4 \ b_1]$

\* Atle,  $A - K$ .

\* Then the operations will be  $[a_1 \ b_1] - 4 = [a_1-4 \ b_1]$

\* These scalars are multiplied with some

unique matrix like '3'. It means dimensions of this unique matrix which is same as the columns. & scalars can be used in matrices.

\* Add, sub, not directly because, we want it - want requires two matrices of same size (or) dimensions, means  $2 \times 2$  or  $3 \times 3$ .

\* Matrices + (Addition).

two operations A, B:  $\Rightarrow A+B$ . If A and B are of same size, then we can perform '+' operation on same size only.

(Sub): If we want to subtract one matrix from other, then it will support when it has same size.

we want to sub one matrix to other, then it will support when it has same size.

Transpose: In this we can perform, convert rows into columns and columns into rows.

simply, we can write -

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \xrightarrow{\text{Transpose}} \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

\* diagonal won't change and remaining will change.

Inverse: Important

$A^{-1}$ ,  $\frac{\text{Adj } A}{\det A}$ , has 3 factors

Ex:  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det A = ad - bc$$

$$\text{adj. } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det A = a$$

$$\text{Adj } A =$$

$$* \text{Multiplication}$$

$$A,$$

$$B$$

\* It will

\* Inputs

\* weights

$$W$$

\* Bias

\* In

\* directly  
some

\* Vectors

contains

\* vector  
as

\* gen

\* Basic

\* Add

\* In

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det A = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\text{Adj } A = \begin{bmatrix} e & f & -h & i \\ -d & f & -g & h \\ -b & e & -g & i \\ a & c & -d & e \end{bmatrix}$$

### \* Multiplication:

$$A_{n_1 m_1} \quad B_{n_2 m_2}$$

$$\text{if } n_1 = m_2$$

- \* It will multiply just like, row of  $A$  with columns of  $B$ .
- \* Inputs are represented using vector.
- \* weighted are represented using matrix.
- \* Bias is a scalar.
- \* In multiplication, in programming languages it will directly support. But not supported in mathematics for some time.
- \* Vector vector is another important object.

$$W X + b$$

- \* contains :
- \* vector is a one dimensional unit and if it is called tensor means it is a part of matrix more than 2 dimensions.
- \* basic operations are having same rule.
- \* Add & sub are having per
- \* and we can multi operation.
  - a) Element wise

- \* Multiplication used in matrices, dot product.
- \* Mostly dot product will be used in vectors.
- \* vectors
- \* Input always represent in the form of vector.
- \* It is a 1-dimensional tensor.
- \* In mathematical terminology vectors means it has both magnitude and directions.
- \* we have 2 types of vectors;

1.) Row vector      2.) column vector

- 1.) Row vector
- 1.) Row wise representation is called row vector.
- 2.) Column wise representation is called column vector

Ex:  $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} [x_1, x_2, \dots, x_n]$  row vector.

column vector

### \* operations in vectors

- 1.) Addition
- 2.) Subtraction
- 3.) Multiplication

1.) Addition: When the dimensionality is same we can perform addition operation.

dimension  $x = \text{dimension } y$ .  
 $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} [x_1, x_2, \dots, x_n]$

2.) Subtraction: When the dimensionality is same we can perform subtraction operation.

dimension  $x = \text{dimension } y$ .

Ex:  $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$x + y$   
 $\begin{bmatrix} 2+4 \\ 1+3 \end{bmatrix}$

$x - y$   
 $\begin{bmatrix} 2-4 \\ 1-3 \end{bmatrix}$

\* Multiplication

In this two

- 1.) Element

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$x @$

$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

- 2.) Dot product

$x \cdot y$

\* The  $x$

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$= x_1$

this is

$* 85$

final

$$x + y$$

$$\begin{bmatrix} 2+4 \\ 1+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$x - y$$

$$\begin{bmatrix} 2-4 \\ 1-3 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

\* Multiplication:

In this two types of operations are allowed.

1) Element-wise multiplication,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$x \odot y$$

$$\begin{bmatrix} x_1 * y_1 \\ x_2 * y_2 \\ \vdots \\ x_n * y_n \end{bmatrix}$$

2) Dot product:- (mostly used in the multiplication)

$$x \cdot y$$

\* the result is between  $x^T y$ .

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$= x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$$

this is the result of the dot product of  $x$  and  $y$ .

\* if we apply a dot product scalar value will be

final result

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$w \cdot x \Rightarrow w^T x \Rightarrow w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$$

### \* Scalar operations on vectors :-

In mathematically, we can multiply all elements of scalars with vectors.

\* But in programming language, it supports by the python.  
\* only multiplication is allowed between scalar and a vector,

K - scalar.

x - vector.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$Kx = \begin{bmatrix} Kx_1 \\ Kx_2 \\ \vdots \\ Kx_n \end{bmatrix}$$

### \* Addition :-

K - scalar

x - vector

y - temporary with i's.

$$\Rightarrow x + Ky$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$y = \begin{bmatrix} : \\ : \\ \vdots \\ : \\ n \end{bmatrix}$$

$$x + Ky = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} K \\ K \\ \vdots \\ K \\ n \end{bmatrix}$$

$$= \begin{bmatrix} x_1 + K \\ x_2 + K \\ \vdots \\ x_n + K \end{bmatrix}$$

\* Norm: Distance between two vectors.

\* Length of a vector.

→ compute the length between two vectors.

\* 3 different types of norms.

1)  $\ell_1$  norm

2)  $\ell_2$  norm

3)  $\ell_{\infty}$  norm.

\*  $\ell_2$  norm is also called the Euclidean norm.

\*  $\ell_{\infty}$  norm

$\ell_1$  norm:

\* Representation

\*  $\ell_1$  norm values of

Ex: x =

$\ell_1$  norm

$\ell_2$  Norm:

\* The  $\ell_2$

$\ell_2$  nor

Ex:

$\ell_2$  norm

$\ell_{\infty}$  norm

\* we

$\ell_{\infty}$

\* Simple

$\ell_{\infty}$

$\ell_{\infty}$

B

\* L<sub>max</sub> norm is also known as infinity norm.

L<sub>1</sub> norm:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

\* Representation of L<sub>1</sub> norm = |x<sub>1</sub>| + |x<sub>2</sub>| + ... + |x<sub>n</sub>|.

\* L<sub>1</sub> norms means computing sum of the absolute values of individual squares vectors.

$$\text{Ex: } x = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$$

$$L_1 \text{ Norm} = |-2| + |3| + |6|$$

= 11.

L<sub>2</sub> Norm: It is also known as Euclidean norm.

\* The L<sub>2</sub> norm of vector is simply computed by taking

$$L_2 \text{ norm} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2}.$$

$$\text{Ex: } x = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}.$$

$$\begin{aligned} L_2 \text{ norm} &= \sqrt{(-4)^2 + (3)^2 + (2)^2} \\ &= \sqrt{16+9+4} \\ &= \sqrt{29} = 5.36 \end{aligned}$$

L<sub>max</sub> norm: it is also known as infinity norm.

\* we can compute L<sub>max</sub> norm as.

$$\text{Lmax norm} = \max(|x_1|, |x_2|, \dots, |x_n|).$$

\* Simply we can say that maximum of absolute of x.

$$\text{Lmax norm} = \max(|-4|, |3|, |2|).$$

L<sub>max</sub> norm

$$L_{\max} \text{ norm} = 4.$$

Because, it is largest among these.

Because,

\* Numpy: - module present in python with 2D arrays

\* Installation Numpy:

Pip install numpy

Pip - Python installation package

(OR)

conda installation numpy

\* Importing numpy

import numpy as np

\* Creating array

np.array (list of elements, dtype = type)

a = np.array ([1, 2, 3, 4, 5], dtype = np.int32)

a2 = np.array ([[1, 2, 3], [4, 5, 6]])

\* Array with zeroes

np.zeros (dimension, dtype)

- creates arrays with zero's

Ex: np.zeros ((1, 3))

= np.zeros ((3, 3))

\* Array with ones

np.ones (dimension, dtype)

- creates array with 1's

Ex: np.ones ((2, 2))

= np.ones ((1, 3))

\* Identity matrix

np.eye (dimension)

- returns identity matrix.

Ex: np.eye ((2, 2))

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

= np.eye ((3, 3))

\* Filling with

np.full (dimen-

- creates num-

Ex: np.full

np.full (c-

np.full (c-

finding max

min (Array

max (Array

\* Indexing

\* forward in-

\* Backward

\* Accessing

\* [ ] is used

- to access

A [i] to acc

\* 2D arra

at i<sup>th</sup> row

and j<sup>th</sup> col

\* Slicing:

\* To extract

array

Ex:

A[ :

]

### \* Filling with required value:

np.full (dimension , value)

- creates numpy array with value passed.

Ex: np.full (dimension, value)

np.full ((2,2), 0)

np.full ((2,2), 5).

### \* Finding max, min &

min (Array , axis)

axis = 0  $\Rightarrow$  row

axis = 1  $\Rightarrow$  column.

max (Array , axis)

### \* Indexing numpy arrays:

\* Forward indexing (0 to n-1) (left to right)

\* Backward indexing (last element to first element)

(-1 to -n) (right to left).

### \* Accessing elements of an array:

\* [ ] is used to access elements.

\* to access 1d array we have one level of indexing.

\* to access  $i^{th}$  element of array A. A[i] to access element

\* 2D array - 2 level indexing. A[i, j] to access element at  $i^{th}$  row,  $j^{th}$  column.

### \* Slicing:

\* To extract the portion of array ' : ' - slicing operator.

array [start : end]

A = [1, 2, 3, 4]

= A[1:3]  $\Rightarrow$  2, 3

A[1:4]

A[1:]

[1, 2, 3, 4]

[1, 2, 3, 4]

[1, 2, 3, 4]

Space separates the

start and end index.

## 3/8/02 \* Indexing and Slicing 3D Arrays

- \* There are 3 levels of indexing arrays.
- \* 3 levels of indexing required to access any element in 3D numpy array.

Ex:  $a[i, j, k]$  (3D array numpy)

$i \rightarrow$  refers to particular array.

$j \rightarrow$  refers to row in an array  $i$ .

$k \rightarrow$  refers to column in array  $i$ .

Ex:  $a = np.array([[[1, 2], [3, 4], [5, 6]], [[7, 8], [9, 10], [11, 12]], [[13, 14], [15, 16], [17, 18]]])$ ;

\* 3D array is an array of 2D arrays.

a =	<table border="1"> <tr><td>1</td><td>2</td></tr> <tr><td>3</td><td>4</td></tr> <tr><td>5</td><td>6</td></tr> </table>	1	2	3	4	5	6
1	2						
3	4						
5	6						
	<table border="1"> <tr><td>7</td><td>8</td></tr> <tr><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td></tr> </table>	7	8	9	10	11	12
7	8						
9	10						
11	12						
	<table border="1"> <tr><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td></tr> <tr><td>17</td><td>18</td></tr> </table>	13	14	15	16	17	18
13	14						
15	16						
17	18						

0	7	8
1	9	10
2	11	12

0	13	14
1	15	16
2	17	18

\* To access 1st array 2nd row 2nd elements.

$a[0, 1, 1]$  refers to 4.

refers to 4.

\* To access 2nd array 1st row 1st elements.

$a[1, 0, 0]$  refers to 3.

refers to 3.

\* Slicing: To extract the portion of 2D arrays.

arr [array range, row range, column range]

Ex: 1<sup>st</sup> and array's list two rows.

$a[0:2, 0:2, :]$

$\begin{bmatrix} [1, 2] & [3, 4] \end{bmatrix}$

$\begin{bmatrix} [7, 8] & [9, 10] \end{bmatrix}$

\* Fetch or extract last row 2<sup>nd</sup> column elements from 2<sup>nd</sup> and 3<sup>rd</sup> column arrays.

\*  $a[1:, 2:, 1:]$

\* Sparse Matrix:

\* In python we

\* Scipy :- (Scientific)

\* It is used to

+ to import sparse.

\* csc-matrix:

\* It is a function

\* we use function

+ It returns Sparse

Ex:  $b = np.array$

'sparse' \*

oprs (1, 2) 3

(2, 0) 4

\* Reshaping numpy

\* reshape(): It is

+ It takes target

array reshaped to

arr.reshape

Ex:  $a = np.array$

$b = a.reshape$

$b[1, 2, 3]$ ,

7

\* vectors and matrices

\* flattening numpy

\* flatten() - used

+ Rowvector [2, 4, 6]

\* Raven () - only

of original array

array.

$np.flatten()$  - uses

\*  $a[1:, 2:, 1:]$

\* Sparse Matrix :-

\* In python we have under package called as

\* scipy :- (Scientific python package)

\* It is used to represent the sparse matrix.

+ To import sparse matrix, we use scipy.

\* CSR-matrix :- compressed sparse row matrix.

\* It is a function to represent sparse row matrix.

\* We use function sparse.csr\_matrix (numpy\_ndarray).

\* It returns sparse matrix representation for numpy\_ndarray.

Ex:-  $b = np.array([[[0, 0, 0], [0, 0, 3], [4, 0, 0]]])$

$c = sparse.csr_matrix(b)$

Output :-

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 4 & 0 & 0 \end{bmatrix}$

\* Reshaping numpy Arrays :-

\* reshape() :- It is a method of numpy array object.

\* It takes target shape as an argument and returns numpy array reshaped to specific shape.

arr.reshape(shape) :- It has a default axis sent as -1.

Ex:-  $a = np.array([1, 2, 3, 4, 5, 6, 7, 8, 9])$

$b = a.reshape(3, 3)$

$b = [[1, 2, 3], [4, 5, 6], [7, 8, 9]]$

\* vectors and matrices :-

\* flattening numpy arrays :- It is a one-dimensional array.

\* flatten() - used to flatten numpy arrays.

\* Rowvector [2, 4, 6, 1]

\* flatten() - only difference is it has an duplicate copy.

\* flatten() - changes made to original array.

\* np.flatten() - used to flatten numpy array.

np.flatten() - used to flatten numpy array.

## \* Vectors and matrices Intro :-

\* Describing vectors and matrices :- Numpy array has shape attribute, size attribute.

shape - to know shape ; size - no. of elements present in numpy.

ndim - store dimensionality of numpy array.

\* shape, size, ndim are the numpy array objects.

\*  $a = \text{np.array}([[2, 3, 4], [5, 6, 7], [8, 9, 10]])$  ;

a - numpy array object. (Reference of numpy array).

\*  $a.\text{shape}()$  returns shape of numpy array. Shape is about rows and columns present in array.

\*  $a.\text{size}()$  - Contains no. of elements present in numpy array. size returns value.

\*  $a.\text{ndim}()$  - returns dimensionality of numpy array.

Ex 2.

\* Numpy array object has 3 attributes  $a.\text{shape}()$  ;

$a.\text{dim}()$ ,  $a.\text{size}()$ .

## \* Addition, subtraction and multiplication :-

\* we have two matrices A and B :-

Add -  $A+B$  Sub -  $A-B$ ,  $\text{Mul} - A \times B$ .

\* In case of multiplication, need to perform  $A @ B$ . It takes element wise multiplication.

\* The numpy package has functions add and subtract ( $\text{np.add}(arr1, arr2)$ ).

\* Numpy has the feature called broadcasting, it extends the size. It means it stretches the rows, hence it makes the size of the second operation.

Ex:-  $A = \text{np.array}([[2, 4], [1, 3]])$  ;

$B = \text{np.array}([[1, 6], [5, 1]])$  ;

$\text{np.add}(A, B) ; [[3, 10], [6, 9]]$

\* When shape is feature to make

$c = \text{np.array}([$

$\text{np.add}(A,$

$[5, 7],$

$c = [5, 7]$

\* c is stretched

to A.

$$c = \begin{bmatrix} 5 & 7 \\ 5 & 7 \end{bmatrix}$$

\* Addition using

\* Subtract operation

matrices.

\* Broadcasting is

\* If there is mismatch occurs

\* Subtraction :-

It means that

$\text{np.subtract}$

\* Multiplication

\* It can be used

\* It computes dot product

\* @-dot product

+ It can be used

to compute dot

\* Element wise

\* A - element

\*  $A^*B$  refers

- \* when shape is not equal to numpy uses broadcasting feature to make arrays equal in shape.

$c = np.array([5, 7])$ ;

$np.add(A, c)$ ;

$\begin{bmatrix} [2, 1], [4, 10] \end{bmatrix}$

$c = [5, 7]$

- \* c is stretched vertically to make a shape of c equal to A.

$c = \begin{bmatrix} 5 & 7 \\ 5 & 7 \end{bmatrix}$ .

- \* Addition using '+' is also allowed on numpy arrays.

- \* Subtract operations can be used in both vectors and matrices.

- \* Broadcasting is not done always.

- \* If there is more than 2 inf matrices, then broadcasting may not occurs.

- \* Subtraction: It can be implemented by using array.

- \* It means that it can subtract one from another.

$np.subtract(arr1, arr2)$

- \* Multiplication:  $np.dot$  (arr1, arr2) is used to multiply two matrices.

- \* It can be used to multiply two vectors or the two vectors of the matrices.

- \* It computes dot product of the two vectors or the two matrices.

- \* @-dot product operator is used to multiply two matrices or the two vectors.

- \* It can be used to compute dot product of two vectors.

- \* To compute dot product of two vectors.

- \* Element wise product: It is multiplication operator.

- \* A \* B represents that elements of the dot product.

- \* A \* B represents that elements of the dot product.

- \* transpose of a matrix is done by taking step by step transpose of each row.
  - \* Matrix (or) a vector.
  - \* transpose of numpy array can be computed by using 'T' operator.
  - \* Mostly we can use transpose operation.
  - Ex: arr.T or arr.T transpose of a matrix will return transpose of matrix.
  - A.T
  - \* Linear Algebra package.
  - \* It contains many functions used in linear algebra operations.
  - \* We can solve easily.
  - \* If  $A^{-1}$  is determined,  $b = A^{-1} \cdot b$
  - \* To compute the determinant and matrix simplicity.
  - (\*) Function of "math" det() is used here.
  - \* In numpy - we have linear Algebra package lin.alg(?)
  - \* Determinant, inverse to compute determinant of a matrix.
  - \* lin.alg has a function `det()`, usage is `np.linalg.det(arr)`.
  - `np.linalg.det(A)`
  - \* To compute inverse of a matrix use `lin.alg np.linalg.inv(arr)`
  - Ex: `np.linalg.inv(A)`
  - \* To compute eigen values, eigen vectors of a square matrix.
  - \* Use eigen function of lin.alg returns eigenvalues, vectors of a square matrix.
  - Ex: `np.linalg.eig(arr)`
  - `eval, ev = np.linalg.eig(A)`
  - \* To solve linear equations use and solve functions of lin.alg.
  - `lin.alg.solve(arr1, arr2)`
  - \* Scalar operator
  - \* using vec
  - \* That applies
  - \* It takes
  - \* A function
  - \* operation
  - \* It is lambda
  - \* we take
  - \* creates elements
  - Ex: `a =`  
`if =`  
Now, cr  
`if = np`  
`if - iterative`  
`if if(a) -`  
vectors we  
functions.
  - \* Numpy
  - \* working
  - \* panel data
  - \* It is all
  - \* It is v
  - \* It is us
  - \* Pandas
  - \* Pandas
  - \* Data- Str
  - \* Series
  - \* Series
  - \* In data
  - Columns

- \* Scalar operations :-
- \* using vectorize function in numpy library.
- \* That applies on all elements of matrix.
- \* It takes function as argument.
- \* A functions are used operation on variable.
- \* If : lambda  $a: a + 1$ .
- \* we take np. vector (Element function).
- \* creates iterative function that can be applied to all elements in ndarray.
- Ex:-  $a = \text{np.array}([2, 4, -1, 3])$
- $f = \text{lambda } b: b + 10$ .
- \* Now creating a function using vectorize,
- $vf = \text{np.vectorize}(f)$ .
- $vf$  - iterative function (or) vectorized function.
- \*  $vf(a)$  - If we add any arithmetic operations (or) matrix (or) vectors we will get the exact result by this vectorized functions.
- \* Numpy has broadcast casting feature.
- \* working with pandas :-
- \* panel data.
- \* It is also treated (as python and data analysis).
- \* It is used for built on top of numpy.
- \* It is used for data pre-processing operations.
- \* It is used for various data analytics operations.
- \* Pandas allows us to perform data visualizations.
- \* Pandas can perform data structures supported by Pandas.
- # Series
  - data frame.
  - organized only in the form of column.
  - series can be organized only in the form of column.
- \* In data frame it contains rows and columns, but in columns it contains similar data (same type of the value), rows (different types of values).

## 1) series

- \* It contains single column array.

## 2) Data Frame

- \* 2D array where information organized as rows & columns.

### \* Installing pandas :-

pip install pandas.

### \* Importing pandas :-

import pandas as pd

### \* Series :-

- \* It is a column in 2D array.

### \* Creating Series

pd.Series(list).

Ex:- s = pd.Series([20, 30, 40, 50]).

0	20
1	30
2	40
3	50

\* Index parameter of series method can be used to add user defined index.

Ex:- i = ['first', 'second', 'third', 'fourth']

s = pd.Series([20, 30, 40, 50], index=i).

### \* Creating from dictionary :-

\* Series object can be created from the dictionary.

\* Keys of dictionary used as indices.

\* Values are used as elements of series object.

Ex:- d = {'maths': 80, 'Physics': 75, 'chemistry': 100}.

s1 = pd.Series(d).

maths	80
physics	75
chemistry	100

## \* Accessing in

- \* Series obj

### 1) Index

### 2) Index attr

### 3) It contains

Ex:- s1. index

s1. values

### \* NAN (Not

### Point Comp

- \* It is a

### \* Series a

### 1) isnull

### \* s1.isnull

Ex:-

FA
FAL
FAL

### 2) It re

(not NULL)

- \* It a

### \* Handling

dropna

dropna -

jill n all -

jill n all -

Ex:- s1.

s1  
ma  
phy

## \* Accessing indices and elements :-

\* Series object can have 2 attributes.  
1.) index      2.) values.

- 1.) Index attribute contains list of indices.  
2.) It contains element values.

Ex:- `SI.index`

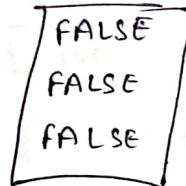
`SI.values`.

\* NAN (Not a Number): It is introduced as part of coding  
Point computation.

\* It is a floating point number.

\* Series and data-frames (objects) has 2 objects (methods)  
1.) `isnull()`    2.) `notnull()`.

\* `SI.isnull()` - It contains boolean list [ ]

Ex:-   
[  
FALSE  
FALSE  
FALSE]

It returns true when the element value is  
(`notnull()`) not NAN.

\* It returns false when the element value is NAN.

\* Handling NAN: There are two ways of handling NAN.

`dropna()`

`dropna` - It removes the element contains NAN value.

`fillna()` - It fills NAN elements with the value passed

as parameter.

Ex:- `SI.dropna()`.

	SI	maths	80
maths	80	physics	75
physics	75	chemistry	0

\* Working with pandas :-

\* Accessing of elements :-

\* Row index to access any element of series object.

\* [ ] used to access elements.

Ex:- s1 ["maths"]

Returns element, with index maths.

\* Multiple elements can be accessed by specifying row indices separated by commas.

Ex:- s1 ["maths", "chemistry"]

It returns elements associated with indices maths & chemistry.

\* Arithmetic operations :-

\* These are allowed on two series objects if two objects has same indices.

Ex:- s1 = pd.Series([1, 2, 3, 4, 5])

s2 = pd.Series([6, 7, 8, 9, 10])

s1 + s2. It adds each element of s1 with elements of s2.

\* Scalar operations :-

\* Scalar operations are allowed on series object directly.

Ex:- s1 + 3, s1 - 3, s1 \* 3, s1 / 3.

- add 3 to every element of series object s1.

\* Apply method &

apply(function[, arguments])

method in series object can be used to apply specific function to series data.

Ex:- lf = lambda x: x + 3

s1.apply(lf).

Ex:- s8 = pd.Series(['yes', 'no', 'no', 'yes', 'no'])

lf = lambda x: 1 if x == 'yes'  
else 0

s8.apply(lf)

[1, 0, 0, 1, 0]

\* Data Frame :-

\* Data frame

data frame

\* 2nd matrix

\* Creating data

using data

df = Data

Ex:-

dataframe = [[{"G":

[{"G":

\* Data Frame :- \* It is in the form of table.

\* Data frame constructor will be used to create new data frame.

\* 2nd matrix with row, column indices.

\* Creating DataFrame

using DataFrame Constructor,

→ pd • DataFrame (data-list [ , columns , index ] ).

Ex:-

country	population.
Germany	30,00,000
France	25,00,000
UK	45,00,000
USA	50,00,000

dataList = [ [ "Germany", "30,00,000" ],

[ "France", "25,00,000" ],

[ "UK", "45,00,000" ],

[ "USA", "50,00,000" ] ].

df = pd. DataFrame ( data-list , columns = [ "country" ,  
"population" ] ) , index = [ "Country1" , "Country2" , "Country3" , "Country4" ]

12/8/22 \* Summarizing DataFrame :-

\* Methods of DataFrame objects :-

\* Methods of DataFrame objects :-

→ head ( ) - it fetches first 5 rows of data.

→ head (n) - it fetches first n rows of data.

\* It can be used as df. head ( ).

→ tail ( ) - it fetches last 5 rows of data.

→ tail (n) - it fetches last n rows.

\* It can be used as df. tail ( ).

→ info ( ) - it writes information about DataFrame.

- returns the DataFrame information object which consists of rows, columns and their types.

- \* This can be used as `df.info()`.
- `describe()` - It returns the statistical related information like max, min, mean, 25% (first quartile), 50%, 75%.
- \* This can be used as `df.describe()`.
- `max, min`.
  - \* Returns the max and min of each and every column in the data frame.
  - \* This can be used as `df.max()`; `df.min()`.
- `count()`.
  - \* Returns the no. of non-NA values present in each and every column.
  - \* Returns count of non-NAN values for each column.
  - \* It can be used as `df.count()`.
- `mean()` - It returns the mean of each and every column present in the data frame.
- `var()`
  - \* We can compute variance using `var('group')`.
  - \* It returns variance.
  - \* It can be used as `df.var()`.
- `std()` - standard deviation
  - \* It returns the standard deviation of the data present in the data frame.
  - \* It can be used as `df.std()`.
- \* Transpose: It converts rows into columns and columns into rows.
- \* Numpy supports transpose.
- \* It can apply the operator T can be used `df.T`.
- \* concatenation and melting operation:-
- \* `concatenate()`: concatenate data frames either row wise or column wise.
- \* Axis parameters are used to specify axes.
  - 0 - row wise.
  - column wise.

melt();  
 columns;  
 columns +  
 sort(); w  
 sort\_value  
 one (or)  
 \* Parameter  
 1) by &  
 be applied  
 2) Ascending  
 ascending  
 \* when  
 \* when +  
 3) Find +  
 \* specify  
 merge so  
 \* quick  
 merge  
 heap  
 \* It can  
 be done  
 \* Extract  
 there  
 1)  
 2) [  
 3) to  
 (The  
 base  
 1) i  
 where  
 single  
 \* Sim

melt(): If we have some data frame containing rows & columns. This melt() will convert rows into columns and columns to rows.

sort(): We can sort the data based on specific column.  
sort\_values() - sort the data frame with respect to one or more columns.

\* Parameters: There are 3 important parameters:  
1) by - It specifies the column on which sorting needs to be applied.

2) ascending - It specifies whether the sorting is in ascending order.

\* When the statement is true - sorting in ascending order.

\* When the statement is false - sorting in descending order.

\* Find - It can be done by using quick sort method.

3) Specify kind of sorting means either quick sort, merge sort, heap sort.

merge sort, heap sort - to apply quick sort,

quick sort - to apply merge sort,

merge sort - to apply quick sort,

heap sort - to apply heap sort.

\* It can be used as `df.sort_values(by="column")`.

\* It can be used as `Dataframe`.

\* Extracting there are 3 different ways.

1) Notation

2) `[ ]`

3) `loc, iloc` which extracts rows and columns.

(These are 2 rows which extracts rows and columns based on index positions).

based on index positions).

1) Notation: When we want to access a data frame in single column, notation will be used.

simply extract specific column,

in simply extract specific column,

df	A	B	C
0	a	1	20
1	b	2	30
2	c	3	40

df.A → returns column A  
dataframe, column,

## 2. [ ] indexing

we can extract data from data frame by using [ ] (Index) operator.

\* In this we can extract row [ ] and column [ ].

\* used to access data by giving row (or) column index

→ dataframe [rowindex]

→ returns a row

→ dataframe [row : row]

returns row1 to row n + 1

→ we can access it by using columns also.

dataframe [col-name]

returns one single column,

\* To access list of columns pass the list of column names.

\* dataframe [[col1, col2, col3]]

\* dataframe [col1 : col n],

→ returns range of columns, starting from col1 to col n - 1.

\* "loc" is used to access the data from data frame by using labels, (labeled base operators).

dataframe . loc [row, column].

\* dataframe . loc [:, col1].

If we want to access more than 1 column.

dataframe . loc [:, [col1, col2, col3]].

\* Scikit - learn

\* Preprocessing

\* In preprocessor

\* Label Encoder

numerical

\* Encoder

- ```
* from sklearn.preprocessing import LabelEncoder  
* # create label encoder object.  
* l = LabelEncoder()  
* # fit data with label encoder  
* l.fit_transform(df[["Gender"]])
```
- (fit-transform takes input to convert numerical data and perform both scikit and transition),
- \* Data will be converted into numerical information.
  - \* Label encoder will help us to convert categorical data into numerical data.
- \* After converting the data into numerical data (by taking example) need to drop old gender column from data frame.
- ```
df.drop("Gender", axis=1, inplace=True).
```
- \* Now add encoded column to the dataset.
- ```
df[["Gender"]] = a
```
- \* Standardize the data (Data Standardization).
- \* It can be also called normalize the data.
  - \* In preprocessing method
  - \* Standard scalar, Min-Max scalar
  - \* These can be used in order to normalize the data.

- \* standard scalar is normalize the data, i.e. it performs unit variance.
- \* this will (or) it makes the data distribution as normal distribution. By this most of the data will be send to the min point.

\* And remaining most of data send around in the mean point.

- \* Inorder to do this, it removes the mean value if removed from each and every value.

Min-Max scalar: It makes use of min and max of values inorder to perform normalization. Once data is once, data is normalized, then the column values will be in between min & max.

# create standard scalar object

```
s = StandardScaler()
```

# fit the data

```
s.fit(data)
```

# transform

```
s.transform()
```

(transform returns an array to normalize the data).

### min-max scalar

from sklearn.preprocessing import MinMaxScaler

# create min max scalar object

```
m = MinMaxScaler()
```

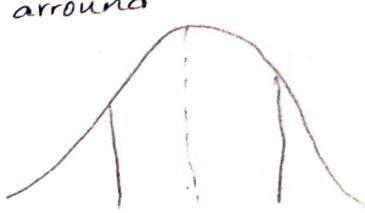
# fit the data

```
m.fit(data)
```

# transform

```
m.transform()
```

- \* preprocessing module has another function one hot encoder.
- \* Inorder to convert categorical data into numerical form.
- \* When we apply one hot encoder we get list of the data and we get the data in the form of vectors.



\* when we we get no

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

like this with 0's a

\* usage of from sklearn

# create or

c = OneH

# fit data

c.fit(d)

# transform

c.transform

(to transform

it return

\* Remaining

\* Data frame

rename

d = {"A":

Ex: df = P

df.rename

Train - +

sklearn

# splitting

from SK

# split

- \* when we apply one-hot encoder to the preprocessing we get no. of vectors.

for  $\{c_1, c_2, c_3\}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

like this the vectors are placed which are encoded with 0's and 1's.

- \* usage of one-hot encoder in scikit-learn,

from sklearn.preprocessing import OneHotEncoder

```
# Create one hot encoder object
```

```
c = OneHotEncoder() - (3-class) mapping
```

```
# fit data
```

```
c.fit(data)
```

```
# transform
```

```
c.transform()
```

(to transform the data into one hot encoder and it returns nd.array and no. of vectors will be produced).

- \* Remaining columns:

- \* Data frame object method

rename (column = dict)

$d = \{"A": [100, 200, 300], "B": [1, 2, 3]\}$

Ex: df = pd.DataFrame(d)

df.rename(columns = {"A": "first", "B": "Second"}, inplace=True)

- \* Train-test-Split:

- \* Selection

sklearn.model\_selection

train\_test\_split

# splitting data

from sklearn.model\_selection

import train\_test\_split

# split data by passing features data, response data, test\_size = 0.2, random\_state = True

train\_test\_split(x, y, test\_size=0.2, random\_state=True)

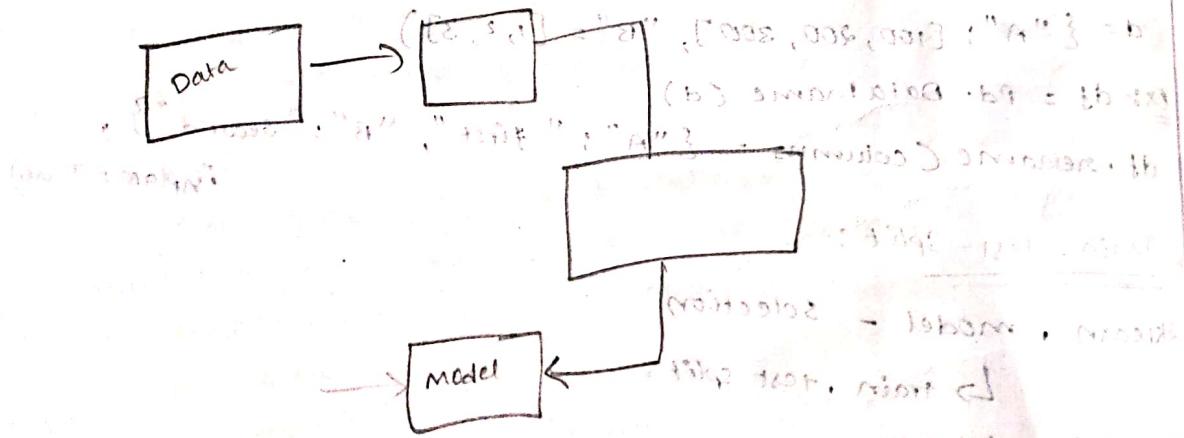
- \* Perceptron model: perceptron is a (neural) Artificial neural network models.
- \* In sklearn we have sub module called `sklearn.linear_model`.
- \* It is used to construct the perceptron model.
- \* This `linear model` has perceptron and this can be implemented the simple perceptron model.
- \* Perceptron classifier is used as learning algorithm and it provides when to stop the algorithm.
- # Create perceptron model
 

```
# create perceptron model
from sklearn.linear_model import Perceptron
PC = Perceptron(tol=1e-3)
```
- # fit the data (it passes input data to the training process).
 

```
# fit the data
PC.fit(X, Y)
```
- # check accuracy (score represents the accuracy of the model).
 

```
# check accuracy
PC.score(X, Y)
```
- # input new data to make predictions.
 

```
# input new data to
# make predictions.
PC.predict([X-new])
```



### Multi-layer Perceptron:-

Sklearn. neural - network  
↳ MLPClassifier

# Create MLPClassifier
 

```
# create MLPClassifier
from sklearn.neural_network import MLPClassifier
an = MLPClassifier()
```

```

# fit data
an.fit(X)
# prediction
an.predict(Y)

# Realising logic
def perception(x):
    # read x
    # convert x
    # read t
    # w = np.array([1, 2, 3])
    # v = np.array([1, 2, 3])
    # define C0 - initial
    # O = activation
    print(t)
    y
    def activation(z):
        if z > 0:
            return 1
        else:
            return 0
    z = np.dot(w, x) + v
    o = activation(z)
    print(o)
  
```

neural  
linear model  
(selection).

```
# fit data  
an.fit (Y-train, Y-train)
```

```
# prediction
```

```
an.predict(xnew)
```

## UNIT-II

\* Realising logic gates:

```
def perceptron_classifier ()
```

```
{
```

```
# read inputs (x, w)
```

```
# convert input into integer or float datatype & create  
list of inputs.
```

```
# read bias.
```

```
x = np.array (x)
```

```
w = np.array (w)
```

```
v = np.dot (x.T, w.T)
```

```
# define activation function & pass v as parameter.
```

```
(v - induced local field value).
```

```
o = activation (v)
```

```
print (o)
```

```
3
```

def activation (v) \* to implement AND, OR, ---  
\* to implement AND, OR, ---

```
{
```

if v >= 0.0: \* to implement AND, OR, ---  
 return 1.0  
else:

return 0.0

```
3
```

def AND\_gate (x1, x2): \* to implement AND, OR, ---  
 v = x1 \* x2  
 o = activation (v)  
 print ("AND gate output is", o)

def OR\_gate (x1, x2): \* to implement AND, OR, ---  
 v = x1 + x2  
 o = activation (v)

def NOT\_gate (x1): \* to implement AND, OR, ---  
 v = -x1  
 o = activation (v)

def XOR\_gate (x1, x2): \* to implement AND, OR, ---  
 v = x1 + x2  
 o = activation (v)

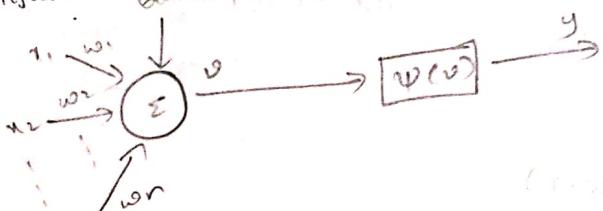
def XNOR\_gate (x1, x2): \* to implement AND, OR, ---  
 v = x1 \* x2  
 o = activation (v)

## UNIT - III

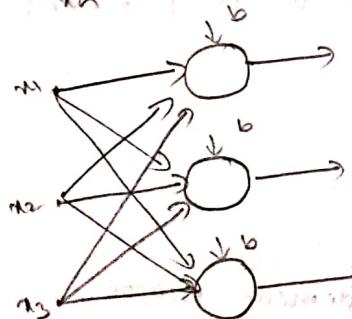
### Single layer perceptron :

- \* Perceptron is used to perform classification tasks.
- \* simple perceptron is used to perform binary classification only.
- \* Multilayer perceptron performs multiclass classification (complex classification)

\* single layer perceptron is a perceptron with 1 (or) more artificial neurons organised in a single layer.



single layer perceptron with 1 neuron.



single layer perceptron with n no. of neurons in output layer  
(n = number of classes)

- \* single layer perceptron has no hidden layer.
- \* simple perceptron can be used to perform binary classification.
- \* simple perceptron can classify data if it is linearly separable (Data is classified based on decision boundary).
- \* If perceptron produces actual output y that is different from target response D then this situation is called error.
- \* Reason for error is free parameters (these are initially initialised with random values).
- \* when there is an error, perceptron goes through learning mechanism (or) optimization procedure to minimize the error.

\* for this iterative  
\* unconstrained  
Generally,

$$\theta = \sum_{i=1}^n \theta_i w_i$$

$$= x^T \cdot w$$

$$y = \theta(w)$$

$$e = d - y$$

✓ target output

$$C(W) = \frac{1}{2} \sum_{i=1}^n e_i^2$$

graph to



\* Start with

$$\nabla = \left[ \begin{array}{c} \frac{\partial}{\partial w_1} \\ \vdots \\ \frac{\partial}{\partial w_n} \end{array} \right]$$

$$\nabla C(W)$$

$$\nabla C(W)$$

\* Gradient

\* It is

\* It is

\* It is

value

w(n+1)

gradient

information

- \* for this iterative learning / optimization procedure, is used.
- \* unconstrained optimization: no constraints on the variables.

Generally,  $x = [x_1, x_2, \dots, x_n]$

$w = [w_1, w_2, \dots, w_n]$

$b$  are inputs.

$$v = \sum_{i=1}^n x_i w_i + b$$

$$= x^T w + b$$

$$y = v(w)$$

$$e = d - y \rightarrow \begin{matrix} \text{actual} \\ \checkmark \\ \text{output} \end{matrix}$$

target output

$\Rightarrow$  minimizing errors results in minimizing

$$c(w) = \frac{1}{2} \sum_{i=1}^K e_i^2$$

graph looks like



global minimum error value position  
C position where error value is min

- \* Start with  $w^0, w^1, \dots, w^n$

$$\nabla = \left[ \frac{\partial}{\partial w_1}, \frac{\partial}{\partial w_2}, \dots, \frac{\partial}{\partial w_n} \right]$$

$$\nabla c(w) = \left[ \frac{\partial c(w)}{\partial w_1}, \frac{\partial c(w)}{\partial w_2}, \dots, \frac{\partial c(w)}{\partial w_n} \right]$$

$$\boxed{\nabla c(w^*) = 0}$$

- \* Gradient descent algorithm (or) steepest descent algorithm

- \* It is unconstrained optimization algorithm.

- \* It is an iterative algorithm.

- \* It uses gradient of cost function to determine update value for  $w$ .

$$w(n+1) = w(n) + \Delta w(n)$$

- \* gradient of cost function is  $g(w)$  used to get slope information and to calculate the new weight vectors.

$$w(n+1) = w(n) + \Delta w(n)$$

\* Gradient of cost function is  $g(n)$  used to get slope information and to calculate the new weight vectors.

$$w(n+1) = w(n) - \eta g(n)$$

$\eta$  = learning parameter - it defines a step size that needed to be taken.

- Its values is between 0 and 1  $\Rightarrow 0 \leq \eta \leq 1$ .

\* while computation, if error arises, then we calculate new weight function by subtracting  $g(n)$  from old weight function and new weight function is used for further computations that minimizes the errors.

#### \* Effect of $\eta$ :

\*  $\eta$  value is very small  $\rightarrow$  slow convergence to global minimum error

\* it takes more time to move to global minimum error value position and algorithm traps in local minimum error value position.

\*  $\eta$  value is large  $\rightarrow$  overshoot global minimum error points.

#### \* Advantages :-

\* convergence is very fast (movement from maximum error to minimum error value) we should check

$$\nabla C(w(n+1)) < \nabla C(w(n))$$

use first order taylor series

$$C(w(n)) + g(n)(w(n+1) - w(n))$$

$$w(n+1) = w(n) - \eta g(n) \quad \text{--- (1)}$$

$$w(n+1) - w(n) = -\eta g(n)$$

$$\Delta w(n) = -\eta g(n)$$

$$\Rightarrow C(w(n)) + g(n) \times -\eta g(n)$$

$$\Rightarrow C(w(n)) + \eta g(n) \times g(n)$$

$$\Rightarrow C(w(n)) - (1 - \eta) g(n)$$

$$\Rightarrow C(w(n+1)) = C(w(n)) - \eta (g(n))^2$$

$$C(w(n+1)) < C(w(n))$$

compute  $g$

$$\Rightarrow \frac{\partial C(w)}{\partial w}$$

$$\Rightarrow \frac{\partial C(w)}{\partial e}$$

$$\Rightarrow \frac{\partial e}{\partial w} =$$

$$e = d - y$$

$$= d - \hat{y}$$

$$= d - y$$

$$\Rightarrow \frac{\partial C(w)}{\partial w}$$

$$\Rightarrow -\sum_{i=1}^k e_i$$

$$w(n+1) =$$

$$g(n) - \eta g$$

\* we got  $\frac{\partial C(w)}{\partial w}$

$$\Rightarrow w(n+1)$$

$$w(n+1)$$

$$\text{Eg: } X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

iteration

$$w^0 =$$

$$v = \eta I$$

$$= 1$$

$$y = 4$$

$$e =$$

compute  $g(n) \Rightarrow$  derivative of  $C(w(n))$  w.r.t. weight vector;

$$\Rightarrow \frac{\partial C(w(n))}{\partial w} = \frac{\partial C(w(n))}{\partial e} \cdot \frac{\partial e}{\partial w}$$

$$\Rightarrow \frac{\partial C(w(n))}{\partial e} = \frac{1}{2} \sum_{i=1}^K e_i^2 = \sum_{i=1}^K e_i$$

$$\Rightarrow \frac{\partial e}{\partial w} = \frac{\partial}{\partial w} (d - \eta w) = -x$$

$$e = d - y$$

$$= d - \varphi(\eta w)$$

$$= d - \varphi(\eta w)$$

$$= d + \eta w \quad (\because \text{linear activation function})$$

$$\Rightarrow \frac{\partial C(w(n))}{\partial w} = \sum_{i=1}^K e_i x_i (-\eta)$$

$$\Rightarrow -\sum_{i=1}^K e_i x_i \quad \text{(ith input sample in a training set)} \\ \text{(denotes particular instance).}$$

$$w(n+1) = w(n) - \eta g(n)$$

$g(n) \sim$  gradient of cost function.

$$\star \text{we got } \frac{\partial C(w(n))}{\partial w} = -\sum_{i=1}^n e_i x_i$$

$$\Rightarrow w(n+1) = w(n) - \eta x - \sum_{i=1}^n w_i x_i$$

$$\boxed{w(n+1) = w(n) + \eta \sum_{i=1}^n e_i x_i}$$

$$\text{Eg: } x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad w = \begin{bmatrix} 1.0 \\ 0.8 \end{bmatrix} \quad \begin{array}{l} d = 1 \\ b = 0.3 \\ \eta = 0.03 \end{array}$$

$$\theta \geq 0 \Rightarrow 1$$

$$\theta < 0 \Rightarrow 0$$

Iteration 1:

$$w^0 = \begin{bmatrix} 1.0 \\ 0.8 \end{bmatrix}$$

$$\begin{aligned} v &= x_1 w_1 + x_2 w_2 + b \\ &= 1(1.0) + 2(-0.8) + 0.3 \\ &= 1 - 1.6 + 0.3 = -0.3 \end{aligned}$$

$$y = \varphi(v) = \varphi(-0.3) = 0$$

$$e = 1 - 0 = 1$$

### Iteration 2:

$$\begin{aligned}
 w_1 &= (w_1 + 1) = w_1 + \eta e^x \\
 &= \begin{bmatrix} 1.0 \\ -0.8 \end{bmatrix} + 0.03 [1] \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1.03 \\ -0.74 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 v &= w_1 x_1 + w_2 x_2 + b \\
 &= (1.03) 1 + (-0.74) 2 + 0.3 \\
 &= 1.03 - 1.48 + 0.3 \\
 &= -0.15
 \end{aligned}$$

$$\begin{aligned}
 \psi(-0.15) &\approx 0 \\
 \epsilon &= 1.0 \approx 1
 \end{aligned}$$

### Iteration 3:

$$\begin{aligned}
 w_2 &= w_2 + 1 = w_2 + \eta e^x \\
 &= \begin{bmatrix} 1.03 \\ -0.74 \end{bmatrix} + 0.03 [1] \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1.06 \\ -0.78 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 v &= x_1 w_1 + x_2 w_2 + b \\
 &= 1.06 + 2(-0.68) + 0.3 \\
 &= 1.36 - 1.36 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \psi(0) &\approx 1 \\
 \epsilon &= 0
 \end{aligned}$$

Q2: Now,  
 $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$      $w = \begin{bmatrix} 1.0 \\ -0.8 \end{bmatrix}$

$$\begin{aligned}
 d &= 1 \\
 \eta &= 0.5 \\
 b &= 0.3
 \end{aligned}$$

### Iteration 1:

$$w^0 = \begin{bmatrix} 1.0 \\ -0.8 \end{bmatrix}$$

$$\begin{aligned}
 v &= x_1 w_1 + x_2 w_2 + b \\
 &= 1.0 + 0.3 - 1.6 \\
 &= -0.3 \\
 \psi(-0.3) &= 0
 \end{aligned}$$

Iteration 2 :-

$$w^1 = w(0+1) = w(0) + \eta e x$$

$$= \begin{bmatrix} 1.09 \\ -0.8 \end{bmatrix} + 0.5 (1) \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 \\ -0.8 \end{bmatrix}$$

$$\vartheta = x_1 w_1 + x_2 w_2 + b$$

$$= 1.5 - 0.8(2) + 0.3$$

$$= 0.2$$

$$\varphi(0.2) = 1$$

$$c = 0$$

\* Drawbacks of gradient descent :-

\* Trap into the local minima point.

\* Convergence takes a long time when data size is large.

\* Stochastic gradient descent :-

\* Uses only one data sample to compute the update.

\* update rule

$$w(n+1) = w(n) - \eta e(n) x$$

\* Slowly converges to the global minimum error point.

\* Newton's method :-

\* Approximation of quadratic cost function, then computes gradient of the approximated cost function.

\* Uses second order taylor series expansion to approximate the cost function.

$$\Delta w(n) = w(n+1) - w(n)$$

$$\Delta f(x) = f(a) + f'(a)x + \frac{1}{2} f''(x) H(x) f(x) + \dots$$

$$\therefore \Delta f(x) = f(n) + f'(n) \Delta w(n) + \frac{1}{2} \Delta w(n)^T H(n) \Delta w(n)$$

$$\Delta w(n) = w(n+1) - w(n)$$

$$\approx g(n) \Delta w(n) + \frac{1}{2} \Delta w(n)^T H(n) \Delta w(n)$$

$H(n) \rightarrow$  Hessian matrix.

$$H = \begin{bmatrix} \frac{\partial c}{\partial w_1, w_2} & \frac{\partial c}{\partial w_1, \partial w_2} & \frac{\partial c}{\partial w_1, \partial w_3} & \cdots & \frac{\partial c}{\partial w_1, \partial w_n} \\ \frac{\partial c}{\partial w_2, \partial w_2} & \frac{\partial c}{\partial w_2, \partial w_2} & \frac{\partial c}{\partial w_2, \partial w_3} & \cdots & \frac{\partial c}{\partial w_2, \partial w_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial c}{\partial w_m, \partial w_1} & \frac{\partial c}{\partial w_m, \partial w_2} & \frac{\partial c}{\partial w_m, \partial w_3} & \cdots & \frac{\partial c}{\partial w_m, \partial w_n} \end{bmatrix}$$

$$\frac{\partial f}{\partial \Delta w(n)} \quad g(n) + H(n) \Delta w(n) = 0$$

$$H(n) = \frac{-g(n)}{\Delta w(n)} \Rightarrow \Delta w(n) = \frac{-g(n)}{H(n)}$$

$$\frac{\Delta w(n)}{w(n+1)} = -g(n) * t^{-1}(n)$$

$$w(n+1) = w(n) + \Delta w(n)$$

↳ update amount

$$w(n+1) = w(n) - \eta g(n) t^{-1}(n)$$

↳ start with old weight  
and subtract new gradient

\* Step size  $\eta$  is called learning rate. It's a parameter that controls how much we update our weights.

\* If step size is too small, it will take a long time to converge.  
\* If step size is too large, it may overshoot the minimum.

\* Learning rate is a hyperparameter that needs to be tuned.

\* Gradient descent is an iterative optimization algorithm that finds the minimum of a function by iteratively moving in the direction of steepest descent, which is the negative of the gradient.