Unit-5

Branch and Bound

Travelling Sales person problem using LCBB

1. Apply the LC Branch and Bound technique for the following graph adjacency matrix.

$$A = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

First we will find the minimum of each row

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline 21 \end{array} = \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

$\therefore$ row reduction cost $= 10+2+2+3+4 = 21$

Find out the minimum value in each & every columns sum those minimum value to get the column reduction cost

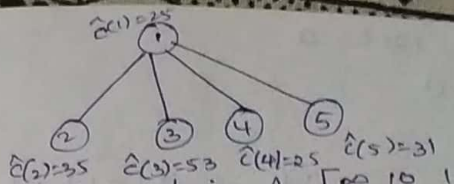
$$\begin{array}{c} \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \\ \begin{array}{ccccc} 1 & 0 & 3 & 0 & 0 \end{array} \end{array} = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$\therefore$ column reduction cost $= 1+0+3+0+0 = 4$

$\therefore$ Total Reduction cost = row reduction cost + column reduction cost

$= 21 + 4 = 25$

The total reduction cost is the cost of the first vertex i.e., $\hat{C}(1) = 25$. After selecting the first vertex, in the remaining vertices, we have to select the next minimum cost vertex.



Now the reduced matrix $A = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$

Consider the path (1,2), make first row and second column as infinity and set $A[2][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & \quad & \quad & 0 & 0 & 0 \end{matrix} \end{array}$$

Row reduction cost = 0

Column reduction cost = 0

Total reduction cost = 0+0 = 0

Total reduction cost

$\hat{c}(2) = \hat{c}(1) + A(1,2) + \text{Total reduction cost}$

$= 25 + 10 + 0$

$= 35$

Consider the path (1,3) as infinity and set $A[3][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \underline{0} \end{matrix} \\ \begin{matrix} 11 & 0 & \quad & 0 & 0 \end{matrix} \end{array} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

Total reduction cost = 0+11 = 11

$\hat{c}(3) = 25 + 17 + 11 = 53$

Consider the path (1,4) as infinity & set $A[4][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} \end{array}$$

Total reduction cost = 0

$\hat{c}(4) = 25 + 0 + 0$

$= 25$

Consider path (1, 5) as infinity & A[5][1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} \\ 2 \\ 0 \\ 3 \\ 0 \end{matrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$\underline{5}$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 7 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 7 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

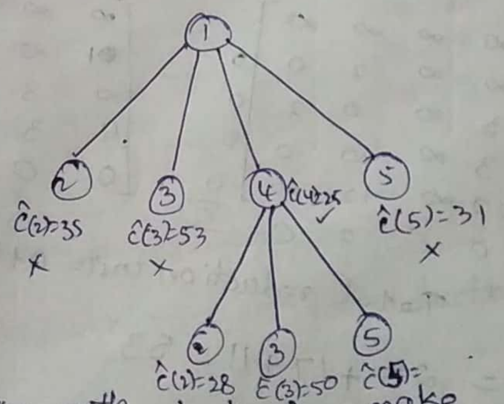
$\begin{matrix} 0 & 0 & 2 & 0 \end{matrix}$ $\quad \underline{2}$

Total reduction cost = 5 + 0 = 5

$\hat{c}(5) = 25 + 1 + 5$

$= 31$

Among all the nodes 2, 3, 4, 5, node 4 has the minimum value. We will set node 4 as E-Node & generate its childrens



Consider the path 1 - 4 - 2, make 1st row, 4th row & 2nd column to infinity, set A[4][1] = ∞, A[2][1] = ∞

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ \infty & \infty & \infty & \infty & 2 \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & 0 \end{bmatrix} \begin{matrix} 0 \\ \\ \\ 0 \end{matrix}$$

$0 \quad 0$

Total reduction cost $= 0 + 0 = 0$

$\hat{c}(2) = \hat{c}(4) + A[4][2]$ + Total reduction cost

$= 25 + 3 + 0$

$= 28$

Consider the path 1-4-3, make 1^{st} row, 4^{th} row & 3^{rd} column to infinity, set $A[3][1] = \infty$, $A[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 2 \\ \\ 0 \\ \hline 2 \end{matrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$11 \quad 0 \qquad 0 = \overline{11}$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

Total reduction cost $= 2 + 11 = 13$

$\therefore \hat{c}(3) = \hat{c}(4) + A[4][3]$ + Total reduction cost

$= 25 + 12 + 13$

$= 50$

Consider the path 1-4-5, make 1^{st} row, 4^{th} row & 5^{th} column to infinity, set $A[5][1] = \infty$, $A[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \\ 11 \\ 0 \\ \\ 0 \\ \hline 11 \end{matrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

$0 \quad 0 \quad 0$

Total reduction cost $= 0 + 11 = 11$

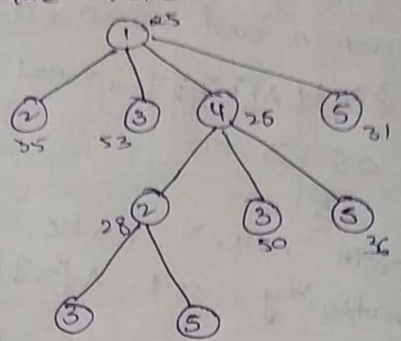
Total reduction cost

$\hat{c}(5) = \hat{c}(4) + A[4][5]$ + Total reduction cost

$= 25 + 0 + 11$

$= 36$

Among the cost of node 2, 3, 5, node 2 is the minimum. node 2 is the E-Node. node 2 generates the children node 3 and 5. The cost matrix obtained at node 2 is the reference matrix now.



$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

Consider the path 1-4-2-3 for node 3. Make 1st row, 4th row, 2nd row to infinity and also 3rd column to infinity. Set $A[4][1] = \infty$, $A[2][1] = \infty$, $A[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ \\ 2 \\ \\ 11 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\underline{13} \qquad \qquad 0 \quad \quad \quad \quad \quad \quad 0$$

Total reduction cost $= 13 + 0 = 13$

$\hat{c}(3) = \hat{c}(2) + A[2][3] + \text{Total reduction cost}$

$= 28 + 11 + 13$

$= 52$

Consider the path 1-4-2-5 for node 5. Make 1st row, 4th row, 2nd row, 5th column as infinity.

Set $A[4][1] = \infty$, $A[2][1] = \infty$, $A[5][1] = \infty$.

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ \\ 0 \\ \\ \end{array} \\ \begin{array}{ccccc} 0 & \quad & 0 & \quad & \quad \end{array} \end{array}$$

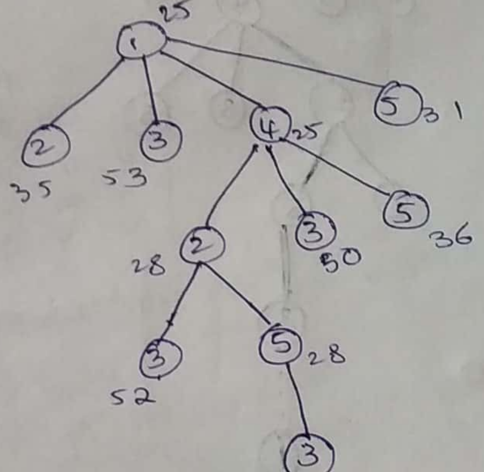
Total reduction cost $= 0 + 0 = 0$

$\hat{c}(5) = \hat{c}(2) + A[2][5] +$ Total reduction cost

$= 28 + 0 + 0$

$= 28$

Among the cost of node 3, 5 ~~mode~~ node 5 is the minimum. Node 5 is the E-node. Node 5 generates the children node 3. ~~The~~



The cost matrix obtained at node 5 is the reference matrix now.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

Consider the path 1-4-2-5-3 for node 3. Make 1st row, 4th row, 2nd row, 5th row & 3rd column as infinity and also $A[4][1] = \infty$, $A[2][1] = \infty$, $A[5][1] = \infty$, $A[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \; 0$$

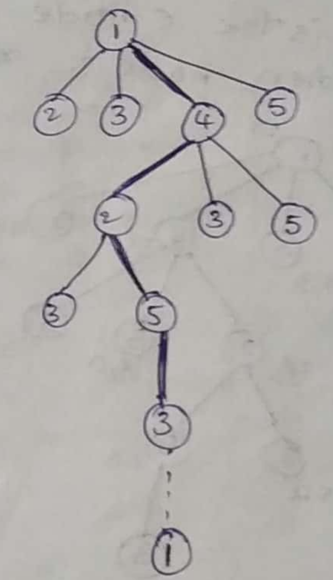
0

Total reduction cost = 0+0=0

$\hat{c}(3) = \hat{c}(5) + A[5][3] + \text{Total reduction cost}$

$= 28+0+0$

$= 28$



$\therefore$ Minimum tour cost = 1-4-2-5-3 =28

2. Consider the travelling sales person defined by the cost matrix

$$\begin{bmatrix} \infty & 7 & 3 & 12 & 8 \\ 3 & \infty & 6 & 14 & 9 \\ 5 & 8 & \infty & 6 & 18 \\ 9 & 3 & 5 & \infty & 11 \\ 18 & 14 & 9 & 8 & \infty \end{bmatrix}$$

(i) Obtain the reduced cost matrix

(ii) Obtain the portion of the stace space tree that will be generated by LCBB. Label each node by its $\hat{c}$ value. Write down the reduced matrices corresponding each of these nodes.

3. Apply the least cost Branch & Bound Algorithm to solve the travelling sales person problem for the following ~~set~~ cost matrix.

$$\begin{bmatrix} \infty & 11 & 10 & 9 & 6 \\ 8 & \infty & 7 & 3 & 4 \\ 8 & 4 & \infty & 4 & 8 \\ 11 & 10 & 5 & \infty & 5 \\ 6 & 9 & 5 & 5 & \infty \end{bmatrix}$$

3A.

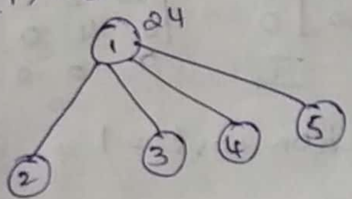
$$A = \begin{bmatrix} \infty & 11 & 10 & 9 & 6 \\ 8 & \infty & 7 & 3 & 4 \\ 8 & 4 & \infty & 4 & 8 \\ 11 & 10 & 5 & \infty & 5 \\ 6 & 9 & 5 & 5 & \infty \end{bmatrix} \begin{matrix} 6 \\ 3 \\ 4 \\ 5 \\ 5 \\ \hline 23 \end{matrix} \Rightarrow \begin{bmatrix} \infty & 5 & 4 & 3 & 0 \\ 5 & \infty & 4 & 0 & 1 \\ 4 & 0 & \infty & 0 & 4 \\ 6 & 5 & 0 & \infty & 0 \\ 1 & 4 & 0 & 0 & \infty \end{bmatrix}$$

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 \end{matrix} \quad \underline{1}$$

$$\begin{bmatrix} \infty & 5 & 4 & 3 & 0 \\ 4 & \infty & 4 & 0 & 1 \\ 3 & 0 & \infty & 0 & 4 \\ 5 & 5 & 0 & \infty & 0 \\ 0 & 4 & 0 & 0 & \infty \end{bmatrix}$$

Total reduction cost $= 23+1 = 24$

$\hat{c}(1) = 24$



$$A = \begin{bmatrix} \infty & 5 & 4 & 3 & 0 \\ 4 & \infty & 4 & 0 & 1 \\ 3 & 0 & \infty & 0 & 4 \\ 5 & 5 & 0 & \infty & 0 \\ 0 & 4 & 0 & 0 & \infty \end{bmatrix}$$

Consider 1-2, make 1st row & 2nd columns as infinity

$A[2][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 4 & 0 & 1 \\ 3 & \infty & \infty & 0 & 4 \\ 5 & \infty & 0 & \infty & 0 \\ 0 & \infty & 0 & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{matrix} 0 & & 0 & 0 & 0 \end{matrix}$$

Total reduction cost $= 0+0 = 0$

$\hat{c}(2) = 24 + 5 + 0$

$= 29$

Consider 1-3. Make 1st row & 3rd column as infinity.

$A[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & 1 \\ \infty & 0 & \infty & 0 & 4 \\ 5 & 5 & \infty & \infty & 0 \\ 0 & 4 & \infty & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & 0 & 1 \\ \infty & \infty & \infty & 0 & 4 \\ 5 & 1 & \infty & \infty & 0 \\ 0 & 0 & \infty & 0 & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & & 0 & 0 \end{matrix}$$

Total reduction cost = 0

$\hat{c}(3) = 24 + 4 + 0$

$= 28$

Consider 1-4. Make 1st row & 4th column as infinity

$A[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 4 & \infty & 1 \\ 3 & 0 & \infty & \infty & 4 \\ \infty & 0 & 0 & \infty & 0 \\ 0 & 4 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & 3 & \infty & 0 \\ 3 & 0 & \infty & \infty & 4 \\ \infty & 0 & 0 & \infty & 0 \\ 0 & 4 & 0 & \infty & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & & & \end{matrix} \qquad \underline{1} \qquad \begin{matrix} 0 & 0 & 0 & & 0 \end{matrix}$$

Total reduction cost = 1

$\hat{c}(4) = 24 + 3 + 1$

$= 28$

Consider 1-5. Make 1st row & 5th column as infinity

$A[5][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 4 & 0 & \infty \\ 3 & 0 & \infty & 0 & \infty \\ 5 & 5 & 0 & \infty & \infty \\ \infty & 4 & 0 & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 4 & 0 & \infty \\ 0 & 0 & \infty & 0 & \infty \\ 2 & 5 & 0 & \infty & \infty \\ \infty & 4 & 0 & 0 & \infty \end{bmatrix}$$

$$\begin{matrix} 3 & 0 & 0 & 0 \end{matrix} \qquad \underline{3} \qquad \begin{matrix} 0 & 0 & 0 & 0 \end{matrix}$$

Total reduction cost = 3

$\hat{c}(5) = 24 + 0 + 3 = 27$

consider node

Consider

2nd colum

Tot

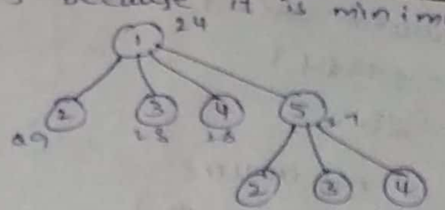
Conside

3rd col

Conside

4th

Consider node 5 because it is minimum.



$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 4 & 0 & \infty \\ 0 & 0 & \infty & 0 & \infty \\ 2 & 5 & 0 & \infty & \infty \\ \infty & 4 & 0 & 0 & \infty \end{bmatrix}$$

Consider 1-5-2. Make 1^{st} row, 5^{th} row & 2^{nd} column as infinity. $A[5][1] = \infty$, $A[2][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 4 & 0 & \infty \\ 0 & \infty & \infty & 0 & \infty \\ 2 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ \\ \\ \end{matrix}$$

$$\begin{matrix} 0 & & 0 & 0 & \end{matrix}$$

Total reduction cost $= 0 + 0 = 0$

$$\hat{c}(2) = 27 + 4 + 0$$
$$= 31$$

Consider 1-5-3. Make 1^{st} row, 5^{th} row. & 3^{rd} column as infinity. $A[5][1] = \infty$, $A[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty \\ \infty & 0 & \infty & 0 & \infty \\ 2 & 5 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 2 \\ \\ \\ \end{matrix} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty \\ \infty & 0 & \infty & 0 & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\underline{\quad 2 \quad}$$

$$\begin{matrix} 0 & 0 & & 0 & \end{matrix}$$

Total reduction cost $= 2 + 0 = 2$

$$\hat{c}(3) = 27 + 0 + 2$$
$$= 29$$

Consider 1-5-4. Make 1^{st} row, 5^{th} row & 4^{th} column as infinity. $A[5][1] = \infty$, $A[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 4 & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \\ \infty & 5 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \\ 1 \\ 0 \\ 0 \\ \\ \end{matrix} \qquad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \\ \infty & 5 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 0 & & \end{matrix}$$

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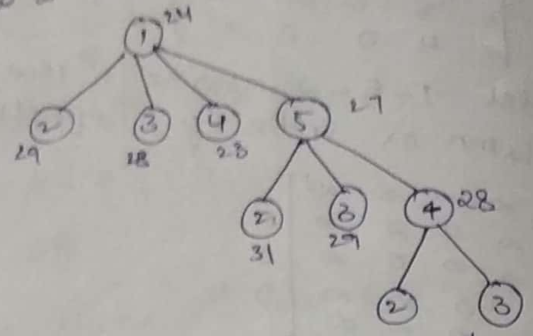
Total reduction cost = 1 + 0 = 1

$\hat{c}(4) = 27 + 0 + 1$

$= 28$

Node 4 is minimum

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$



Consider 1-5-4-2. Make 1st row, 5th row, 4th row, 2nd column as infinity. A[5][1] = ∞, A[4][1] = ∞, A[2][1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} 3 \\ 0 \\ \\ \\ \\ \end{matrix} \qquad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

0 ∞ 0

Total reduction cost = 3

$\hat{c}(2) = 28 + 5 + 3$

$= 36$

Consider 1-5-4-3. Make 1st, 5th, 4th row & 3rd column as infinity. A[5][1] = ∞, A[4][1] = ∞, A[3][1] = ∞

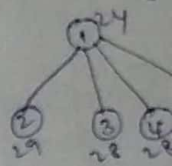
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(3) = 28 + 0 + 0$

$= 28$

Node 3 is minimum

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$



Consider 1-5-4-3-2 ...

3rd row & 2nd column ...

A[3][1] = ∞, A[2][1 ...

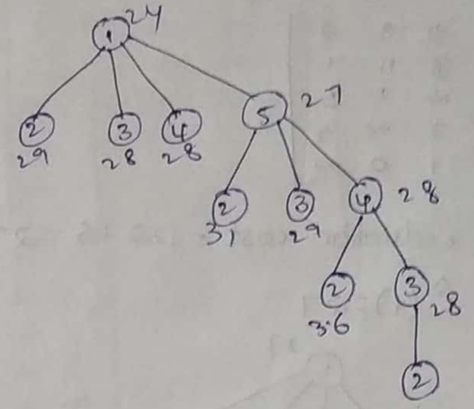
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$\hat{c}(2) =$ ...

∴ 1-5-4-3-2

Node 3 is minimum

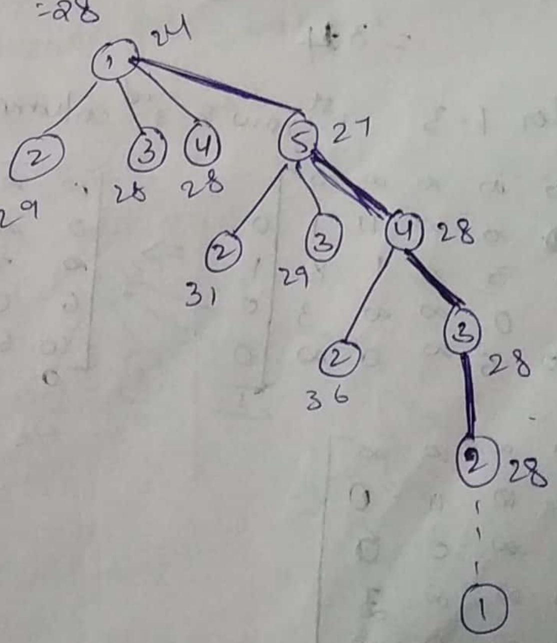
$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$



Consider 1-5-4-3-2. Make 1^{st} row, 5^{th} row, 4^{th} row, 3^{rd} row & 2^{nd} columns as infinity. $A[5][1]=\infty$, $A[4][1]=\infty$, $A[3][1]=\infty$, $A[2][1]=\infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\hat{c}(2) = 28 + 0 + 0$$
$$= 28$$



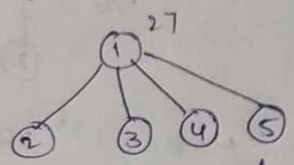
$\therefore 1-5-4-3-2 = 28$

2A. $A = \begin{bmatrix} \infty & 7 & 3 & 12 & 8 \\ 3 & \infty & 6 & 14 & 9 \\ 5 & 8 & \infty & 6 & 18 \\ 9 & 3 & 5 & \infty & 11 \\ 18 & 14 & 9 & 8 & \infty \end{bmatrix} \begin{matrix} 3 \\ 3 \\ 5 \\ 3 \\ 8 \\ \overline{22} \end{matrix} \Rightarrow \begin{bmatrix} \infty & 4 & 0 & 9 & 5 \\ 0 & \infty & 3 & 11 & 6 \\ 0 & 3 & \infty & 1 & 13 \\ 6 & 0 & 2 & \infty & 8 \\ 10 & 6 & 1 & 0 & \infty \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$

$$\begin{bmatrix} \infty & 4 & 0 & 9 & 0 \\ 0 & \infty & 3 & 11 & 1 \\ 0 & 3 & \infty & 1 & 8 \\ 6 & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & 0 & \infty \end{bmatrix}$$

Total reduction cost = 22+5 = 27

$\hat{C}(1) = 27$



Consider 1-2 . 1^{st} row & 2^{nd} column infinity. A[2][1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & 11 & 1 \\ 0 & \infty & \infty & 1 & 8 \\ 6 & \infty & 2 & \infty & 3 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ \overline{2} \end{matrix} \quad \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & 11 & 1 \\ 0 & \infty & \infty & 1 & 8 \\ 4 & \infty & 0 & \infty & 1 \\ 10 & \infty & 1 & 0 & \infty \\ 0 & & 0 & 0 & \phi \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 3 & 11 & 0 \\ 0 & \infty & \infty & 1 & 7 \\ 4 & \infty & 0 & \infty & 0 \\ 10 & \infty & 1 & 0 & \infty \end{bmatrix}$$

Total reduction cost = 2+ϕ = 3

$\hat{C}(2) = 27+4+3$

$= 34$

Consider 1-3 . 1^{st} row & 3^{rd} column infinity. A[3][1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 1 \\ \infty & 3 & \infty & 1 & 8 \\ 6 & 0 & \infty & \infty & 3 \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \overline{1} \end{matrix} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 1 \\ \infty & 2 & \infty & 0 & 7 \\ 6 & 0 & \infty & \infty & 3 \\ 10 & 6 & \infty & 0 & \infty \\ 0 & 0 & & 0 & 1 \end{bmatrix} \ \overline{1}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & 0 \\ \infty & 2 & \infty & 0 & 6 \\ 6 & 0 & \infty & \infty & 2 \\ 10 & 6 & \infty & 0 & \infty \end{bmatrix}$$

Total reduction cost $= 1 + 1 = 2$

$$\hat{c}(3) = 27 + 0 + 2$$
$$= 29$$

consider 1-4 . 1^{st} row & 4^{th} column infinity. $A[4][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 1 \\ 0 & 3 & \infty & \infty & 8 \\ \infty & 0 & 2 & \infty & 3 \\ 10 & 6 & 1 & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \\ \underline{1} \end{array} \Rightarrow \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 1 \\ 0 & 3 & \infty & \infty & 8 \\ \infty & 0 & 2 & \infty & 3 \\ 9 & 5 & 0 & \infty & \infty \end{bmatrix} \\ \begin{array}{ccccc} 0 & 0 & 0 & & 1 \end{array} \quad \underline{1} \end{array}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & \infty & 0 \\ 0 & 3 & \infty & \infty & 7 \\ \infty & 0 & 2 & \infty & 2 \\ 9 & 5 & 0 & \infty & \infty \end{bmatrix}$$

Total reduction cost $= 1 + 1 = 2$

$$\hat{c}(4) = 27 + 9 + 2$$
$$= 38$$

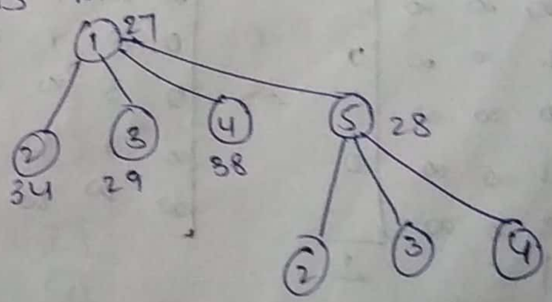
consider 1-5 . 1^{st} row & 5^{th} column infinity . $A[5][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 3 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 2 & \infty & \infty \\ \infty & 6 & 1 & 0 & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\ \begin{array}{ccccc} 0 & 0 & 1 & 0 & \end{array} \quad \underline{1} \end{array} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 1 & \infty & \infty \\ \infty & 6 & 0 & 0 & \infty \end{bmatrix}$$

Total reduction cost $= 0 + 1 = 1$

$$\hat{c}(5) = 27 + 0 + 1$$
$$= 28$$

~~fom~~ Node 5 is minimum.



$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & 1 & \infty & \infty \\ \infty & 6 & 0 & 0 & \infty \end{bmatrix}$$

Consider 1-5-2. Make 1^{st}, 5^{th} rows & 2^{nd} column as infinity

$A[5][1] = \infty$, $A[2][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 11 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 6 & \infty & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 2 \\ 0 \\ 1 \\ \\ \end{array} \\ \underline{3} \end{array} \Rightarrow \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 9 & \infty \\ 0 & \infty & \infty & 1 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \\ \begin{array}{ccccc} 0 & & 0 & 1 & \end{array} \quad \underline{1} \end{array} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 8 & \infty \\ 0 & \infty & \infty & 0 & \infty \\ 5 & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Total reduction cost $= 3 + 1 = 4$

$\hat{c}(2) = 28 + 6 + 4$

$= 38$

Consider 1-5-3. Make 1^{st}, 5^{th} rows & 3^{rd} column as infinity

$A[5][1] = \infty$, $A[3][1] = \infty$

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 11 & \infty \\ 0 & 3 & \infty & 1 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ \\ \end{array} \\ \begin{array}{ccccc} 0 & 0 & & 1 & \end{array} \quad \underline{1} \end{array} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Total reduction cost $= 0 + 1 = 1$

$\hat{c}(3) = 28 + 0 + 1$

$= 29$

Consider 1-5-4. Make 1^{st}, 5^{th} rows & 4^{th} column as infinity

$A[5][1] = \infty$, $A[4][1] = \infty$

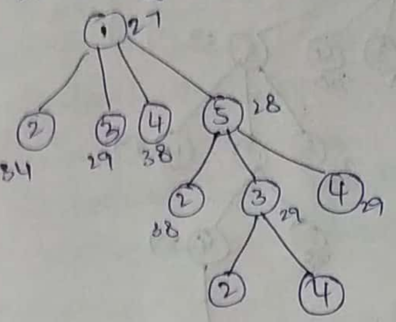
$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & 0 & 1 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ \\ \end{array} \\ \begin{array}{ccccc} 0 & 0 & 1 & & \end{array} \quad \underline{1} \end{array} \Rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & 1 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Total reduction cost $= 0 + 1 = 1$

$\hat{c}(4) = 28 + 0 + 1 = 29$

Node 3 is minimum.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 10 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 6 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$



Consider 1-5-3-2. Make $1^{st}, 5^{th}, 3^{rd}$ row & 2^{nd} column as infinity.

$A[5][1] = \infty$, $A[3][1] = \infty$, $A[2][1] = \infty$

$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 10 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 6 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} & \begin{matrix} \\ 10 \\ \\ 6 \\ \\ \hline 16 \end{matrix} \end{array} \Rightarrow \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \\ \begin{matrix} 0 & & & 0 & \end{matrix} \end{array}$$

Total reduction cost $= 16 + 0 = 16$

$$\hat{c}(2) = 29 + 3 + 16$$
$$= 48$$

Consider 1-5-3-4. Make $1^{st}, 5^{th}, 3^{rd}$ row & 4^{th} column as infinity.

$A[5][1] = \infty$, $A[3][1] = \infty$, $A[4][1] = \infty$

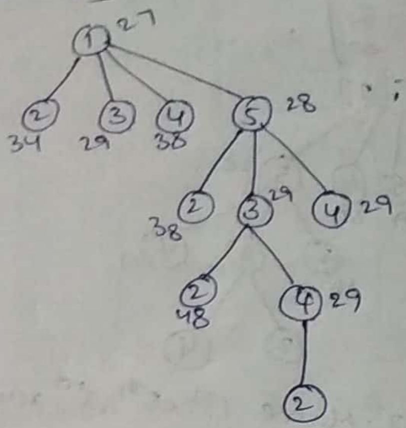
$$\begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} & \begin{matrix} 0 \\ \\ 0 \\ \\ \\ \hline 0 \end{matrix} \\ \begin{matrix} 0 & 0 & & & \end{matrix} & \end{array}$$

Total reduction cost $= 0 + 0 = 0$

$$\hat{c}(4) = 29 + 0 + 0$$
$$= 29$$

Node 4 is minimum

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$



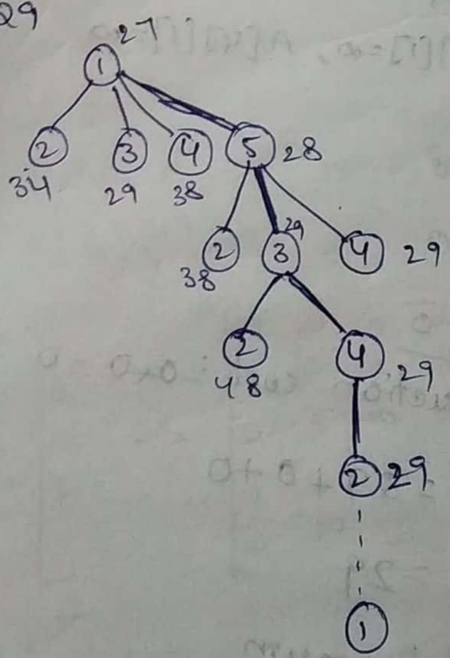
Consider 1-5-3-4-2. Make 1st, 5th, 3rd, 4th row & 2nd column as infinity. A[5][1]=∞, A[3][1]=∞, A[4][1]=∞, A[2][1]=∞.

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Total reduction cost = 0

$\hat{c}(2) = 29 + 0 + 0$

$= 29$



$\Rightarrow$ 1-5-3-4-2 = 29

Knapsack problem using Branch and Bound

Knapsack problem is a maximization problem because we all require the max profit from the knap sack problem.

But knapsack problem LCBB (Least cost Branch & Bound) is a minimization problem. By using minimization problem we have to findout the sol^n for maximization problem.

To convert maximization problem into minimization problem first convert all profits into negative profits by putting negative sign before each profit value.

In knapsack problem using LCBB, we have to calculate two bounds for each node.

One is lower bound ($\hat{c}$). Second one is upper bound ($\hat{U}$)

While calculating lower bound for a node it allows fraction

While calculating upper bound, we doesn't allow fractions

In branch & bound there are 2 variations of knapsack problem.

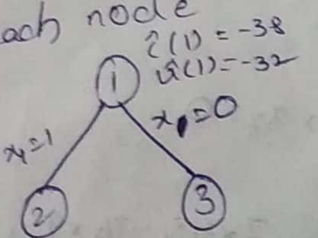
1. LCBB knapsack
2. FIFOBB knapsack

Q. Draw the portion of the state space tree generated by LCBB knapsack for the knapsack instances $n=4$, $M=15$, $(P_1,P_2,P_3,P_4)=(10,10,12,18)$, $(w_1,w_2,w_3,w_4)=(2,4,6,9)$

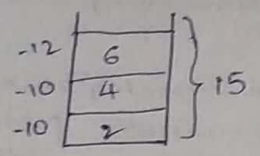
A. First convert the all profits into negative profits by putting negative sign infront of each value.

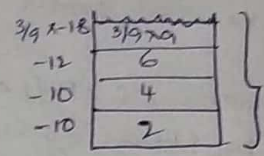
$\therefore (P_1,P_2,P_3,P_4)=(-10,-10,-12,-18)$

Next we have to calculate lower bound $\hat{c}$ & upper bound $\hat{U}$ for each node



For node 1:





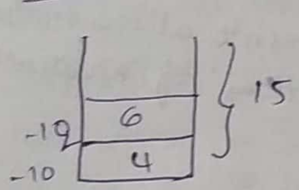
$\hat{u}(1) = -10-10-12$
$= -32$

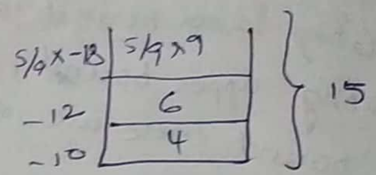
$\hat{c}(1) = -10-10-12+\left(\frac{3}{9} \times -18\right)$
$= -38$

For node 2: $x_1 = 1$

$\hat{c}(2) = -38$, $\hat{u}(2) = -32$

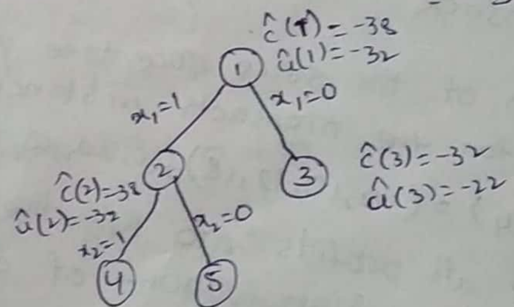
For node 3: $x_1 = 0$





$\hat{u}(3) = -10-12$
$= -22$

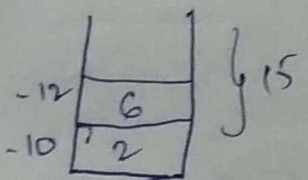
$\hat{c}(3) = -10-12+\left(\frac{5}{9} \times -18\right)$
$= -32$

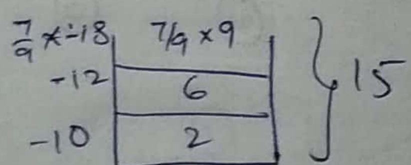


For node 4: $x_2 = 1$, $x_1 = 1$

$\hat{c}(4) = -38$, $\hat{u}(4) = -32$

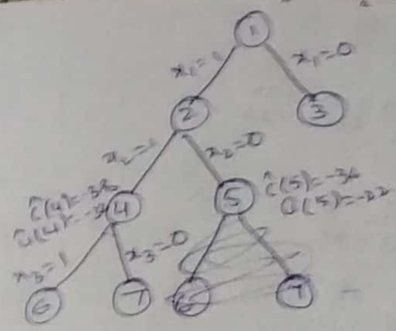
For node 5: $x_1 = 1$, $x_2 = 0$





$\hat{u}(5) = -10-12$
$= -22$

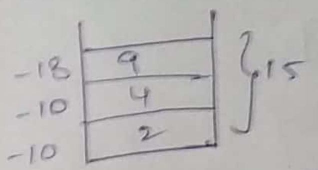
$\hat{c}(5) = -10-12+\left(\frac{7}{9} \times -18\right)$
$= -36$

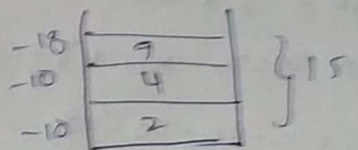


For node 6: $x_1=1, x_2=1, x_3=1$

$\hat{c}(6)=-38$, $\hat{u}(6)=-32$

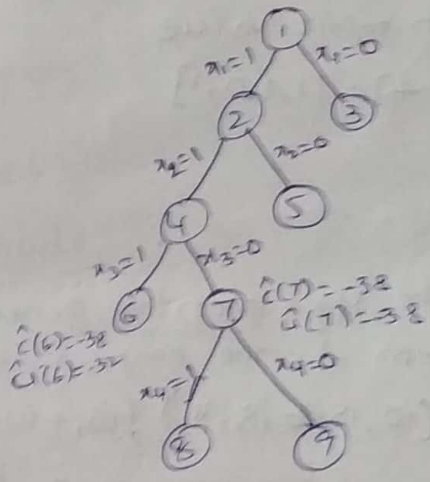
For node 7: $x_1=1, x_2=1, x_3=0$





$\hat{u}(7)=-10-10-18$
$=-38$

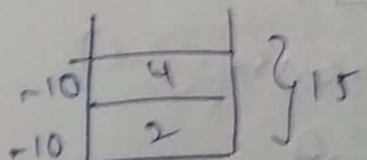
$\hat{c}(7)=-10-10-18$
$=-38$

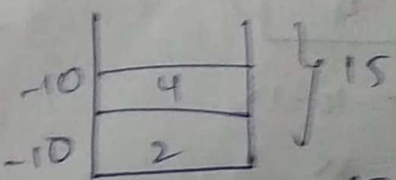


For node 8: $x_1=1, x_2=1, x_3=0, x_4=1$

~~$\hat{u}(7)=-38$~~ $\hat{c}(8)=-38$, $\hat{u}(8)=-38$

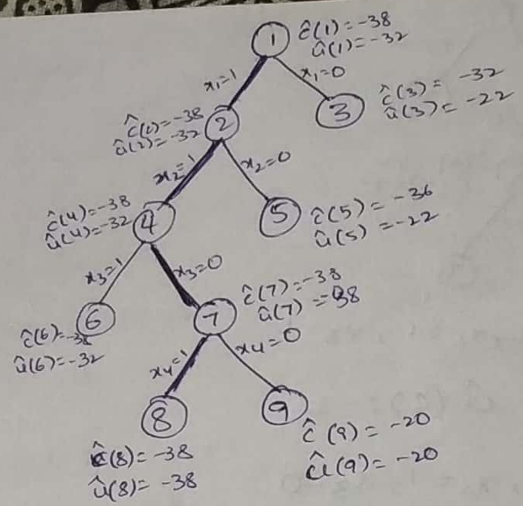
For node 9: $x_1=1, x_2=1, x_3=0, x_4=0$





$\hat{u}(9)=-10-10$
$=-20$

$\hat{c}(9)=-10-10$
$=-20$



Node 8 is E-node

Solution vector $x[1:4] = [1,1,0,1]$

$$\text{profit} = -10 - 10 - 18$$
$$= -38$$

To convert the minimization problem into maximization problem, we have to put '+' sign before each profit value

$\therefore$ Solution vector $x[1:4] = [1,1,0,1]$

$$\text{profit} = 10 + 10 + 18$$
$$= 38$$

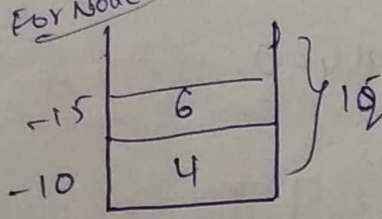
2. Draw the portion of the state space tree generated by LCBB knapsack problem for the knapsack instance $n = 5$, $M = 12$, $(P_1 - P_5) = (10, 15, 6, 8, 4)$, $(w_1 - w_5) = (4, 6, 3, 4, 2)$.

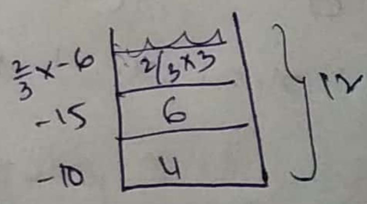
$(P_1 - P_5) = (-10, -15, -6, -8, -4)$

1

A.

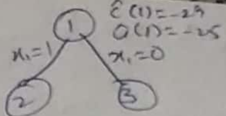
For Node 1:





$$\hat{u}(1) = -10 - 15$$
$$= -25$$

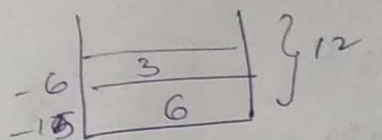
$$\hat{c}(1) = -10 - 15 + \left(\frac{2}{3} \times -6\right)$$
$$= -29$$

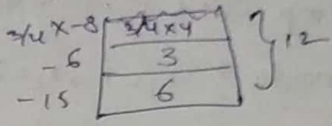


For node 2: $x_1 = 1$

$\hat{c}(2) = -29$, $\hat{u}(2) = -25$

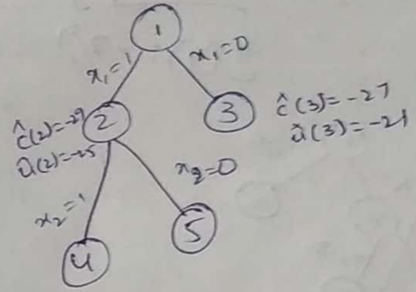
For node 3: $x_1 = 0$





$\hat{u}(3) = -15 - 6$
$= -21$

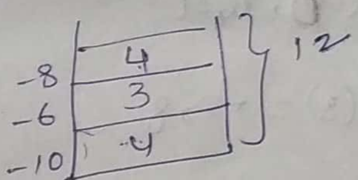
$\hat{c}(3) = -15 - 6 + \left(\frac{3}{4} \times -8\right)$
$= -27$

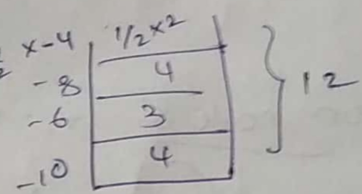


For node 4: $x_1 = 1$, $x_2 = 1$

$\hat{c}(4) = -29$, $\hat{u}(4) = -25$

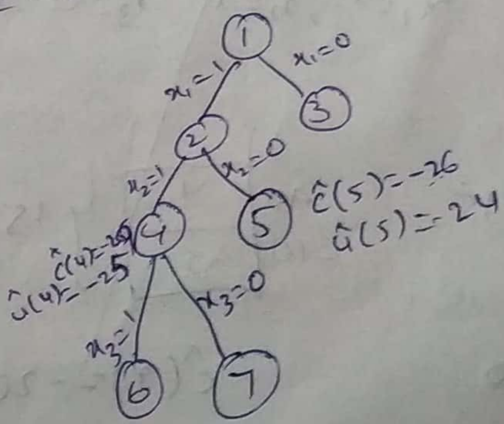
For node 5: $x_1 = 1$, $x_2 = 0$



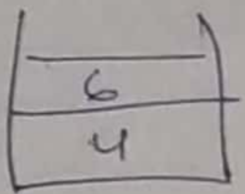


$\hat{u}(5) = -10 - 6 - 8$
$= -24$

$\hat{c}(5) = -10 - 6 - 8 + \frac{1}{2}(-4)$
$= -26$

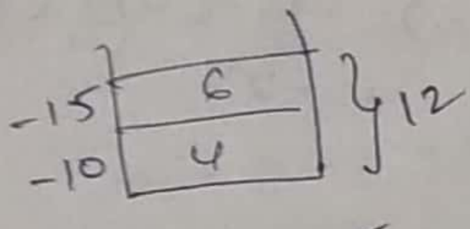


For node 6: $x_1 = 1 \quad x_2 = 1 \quad x_3 = 1$

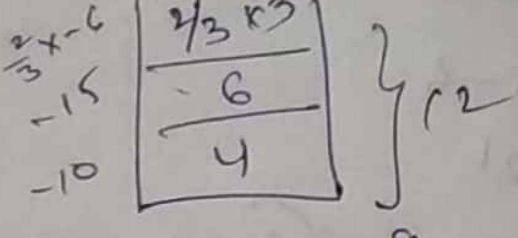


Since there is increase in capacity. So, we have to stop. This is infeasible sol^n

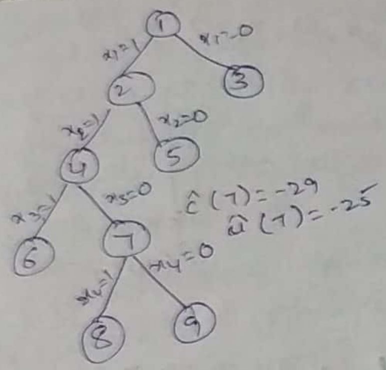
For node 7: $x_1 = 1, x_2 = 1, x_3 = 0$



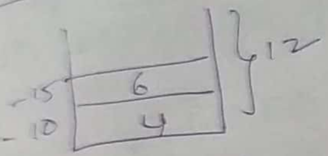
$\hat{u}(7) = -25$

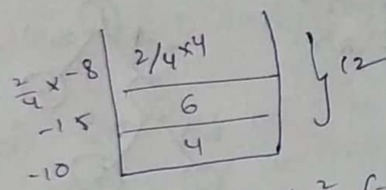


$\hat{c}(7) = -29$



for node 8: $x_1=1 \cdot x_2=1 \quad x_3=0 \quad x_4=1$

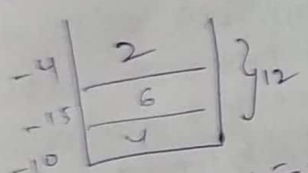


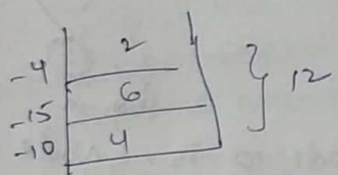


$\hat{u}(8) = -10-15$
$= -25$

$\hat{c}(8) = -10-15+\frac{2}{4}(-8)$
$= -29$

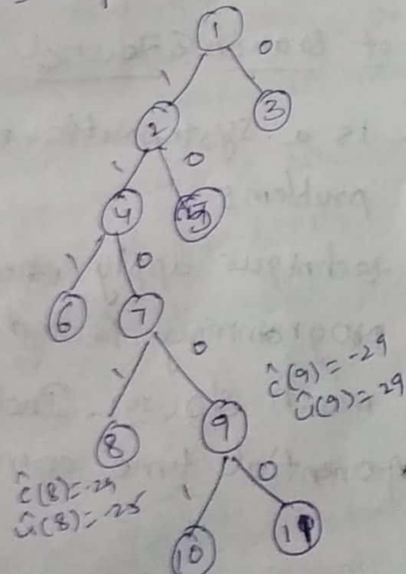
for node 9: $x_1=1, \; x_2=1 \quad x_3=0 \quad x_4=0$



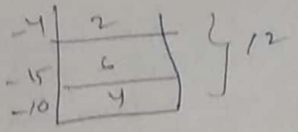


$\hat{u}(9) = -10-15-4$
$= -29$

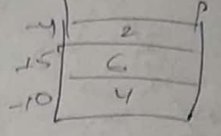
$\hat{c}(9) = -10-15-4$
$= -29$



For node 10: $x_1 = 1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 0$, $x_5 = 1$



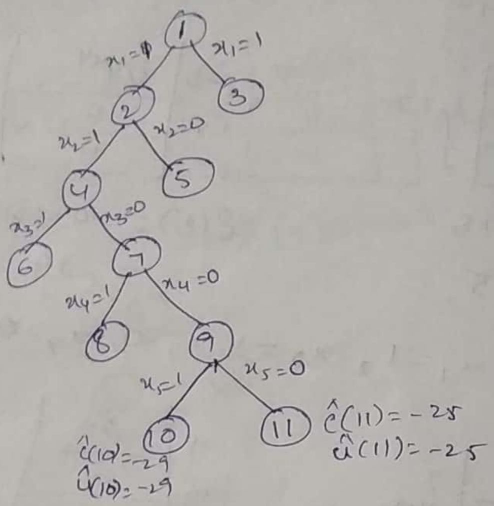
$\hat{u}(10) = -29$



$\hat{c}(10) = -29$

For node 11: $x_1 = 1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0$

$\hat{u}(11) = -25$
$\hat{c}(11) = -25$



Node 10 is E-Node

Profit = -29

$x[1:5] = [1, 1, 0, 0, 1]$

General Method of Branch & Bound

Branch and Bound is a systematic method for solving optimization problems.

Brach and Bound technique apply where greedy method & dynamic programming failed. However, Branch & Bound is much slower. Indeed, it often leads to exponential time complexities in the worst case.

On the other hand, if apply carefully, it can lead to algorithms that run reasonably fast on

average. The general id... like search for the optimal... expanded. Rather, a ... determines which n... another criterion te... solution has been...

Branch and bound ... methods in which ... generated before ... the E-Node

Both DFS & BFS ... strategies.

BFS is an FI... List of live n... a queue. DFS ... live nodes. Lis... the form of ...

Just like ba... functions to av... contain an a... of algorithm...

1. Tree organ...
2. Use of ... i.e., to k... that don...

Examples for...

1. 0/1 kna...
2. Travellin...
3. Search ...

1. BFS &...
2. Least ...

BFS & DF... and selec... nodes ba...

average. The general idea of branch & bound is a BFS like search for the optimal solution, but not all nodes get expanded. Rather, a carefully selected criterion determines which node to node and when, and another criterion tells the algorithm when a optimal solution has been found.

Branch and bound refers to all state space search methods in which all children of an E-Node are generated before any occur live node can become the E-Node

Both DFS & BFS generalize to branch & bound strategies.

BFS is an FIFO search in terms of live nodes. List of live nodes are represented in the form of a queue. DFS in an LIFO search in terms of live nodes. List of live nodes are represented in the form of a stack.

Just like backtracking, we will use bounding functions to avoid generating subtrees that donot contain an answer node. Branch & bound method involves 2 steps - of algorithm design

1. Tree organization of the sol^n search space.
2. Use of bounding functions to limit in the search. i.e., to help avoid the generation of subtrees that donot contain an answer node.

Examples for branch & bound

1. 0/1 knapsack problem
2. Travelling salesperson problem (TSP)

3. Search techiques used in Branch & Bond -

1. BFS & DFS
2. Least cost search techniques

BFS & DFS are used for storing the live node and selection of next E-Node among the live nodes based on FIFO (queue) and LIFO (stack) respectively in ~~bo~~.

In branch & bound method.

Branch - break down the problem into sub problems.

Bound - compute the bounds in every sub problem.

In the solution of every Branch and bound problem can be represented in the form of a tree called state space tree. In state space tree there are 3 types of nodes used

1. Live node
2. E-Node
3. Dead node

Live node:

A node which has been generated but children have not yet been generated is called a live node.

E-Node:

A live node whose children are currently being generated is called a E-node

Dead node:

A node which is already expanded and there is no use in future is called a dead node.

Bounding functions are used to kill live nodes without generating their children.

Differences b/w backtracking & branch and bound

Backtracking	Branch & bound
1. In backtracking, depth first search technique is used for tracking the sol^n for the given problem.	1. In branch & bound, breadth first technique is used for tracking the sol^n for the given problem.
2. Typically decision problems (Yes or No) can be solved using backtracking	2. Typically optimization problems can be solved using branch & bound.

3. While finding the given problem, bo... also evaluated.
4. The state space ... obtained until the ... obtained

5. Applications of ... are N-queens graph colouri... hamiltonian ... subsets proble...

3. While finding the solⁿ to the given problem, bad choices are also evaluated.

4. The state space tree is obtained until the solⁿ is obtained

5. Applications of backtracking are N-queens problem, graph colouring problem, hamiltonian cycle, sum of subsets problem.

3. Branch & Bound proceeds only on optimal or better solⁿs

4. The state space tree need to be searched completely as there be chances of being optimal solⁿ anywhere in the state space tree.

5. Applications of branch & bound are 0/1 knapsack problem, travelling sales person problem.

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