Rp &Rk

Model No 4.7: Unbiased Estimations
Model No 4.8: Maximum error of estimate

Formulae:

	Large sample $n \geq 30$	Small sample $n < 30$
Confidence interval for population mean $\bar{x}$ = Sample mean $z_{\frac{\alpha}{2}}$ = The confident coefficient α = Confidence level σ = Standard deviation n = Sample size s= standard deviation of the sample.	$\mu = \bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ (or) $\bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ (or) $\left(\bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$.	$\mu = \bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ or $\nu = n-1$ $\bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ or $\left(\bar{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}\right)$.
Confidence interval for Proportions		$p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{PQ}{n}}$
Limits for population parameter Proportion P for 99% or 1% only are given by Atmost		$p \pm 2.58\sqrt{\frac{pq}{n}}$ ~ $p \pm 3\sqrt{\frac{pq}{n}}$ depends on given data, replace p by 'P' in the formula.
Maximum error of the estimate E with $(1-\alpha)$ probability	$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$	$E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
Maximum error of the estimate for proportions Atmost (No Mean, no S.D)		$E = z_{\frac{\alpha}{2}} \sqrt{\frac{PQ}{n}}$
Sample size	$n = \left(\frac{z_{\frac{\alpha}{2}} \sigma}{E}\right)^2$	$n = \left(\frac{t_{\frac{\alpha}{2}} s}{E}\right)^2$

If the Proportions are not Given in the Table, Take $P = 1/2$.

Problem 22: If $x_1, x_2 \ldots\ldots x_n$ is random sample from a given population with mean μ and variance σ^2. Show that the sample mean is an unbiased estimator of population mean μ.

Solution: Now we have to show that $\bar{x}$ is an unbiased Estimator of Population Mean 'μ'. $E(\hat{\theta}) = \theta \Rightarrow \boxed{E(\bar{x}) = \mu}$

Proof: $E(\bar{x}) = \dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + \cdots + E(x_n)\right]$$

$$= \frac{1}{n}\ \mu + \mu + \mu + \cdots + \mu$$

$$= \frac{1}{n}(n\mu) = \mu \quad \boxed{\therefore E(\bar{x}) = \mu}$$

Problem 23: Show that the sample variance s^2 is an unbiased estimator of population variance σ^2.

Solution: $s^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n-1}$, Here we have to Prove $E(s^2) = \sigma^2$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$

$$= \frac{1}{n-1}\left[\Sigma\left[(x_i - \mu) - (\bar{x} - \mu)\right]^2\right]$$

$$= \frac{1}{n-1}\left[\Sigma (x_i - \mu)^2 + (\bar{x} - \mu)^2 - 2(x_i - \mu)(\bar{x} - \mu)\right] \left(\because (a-b)^2 = a^2 + b^2 - 2ab\right)$$

$$= \frac{1}{n-1}\left[\Sigma(x_i - \mu)^2 + (\bar{x} - \mu)^2(\Sigma 1) - 2(\bar{x} - \mu)\left[\Sigma x_i - \Sigma\mu\right]\right]$$

$$= \frac{1}{n-1}\left[\Sigma(x_i - \mu)^2 + n(\bar{x} - \mu)^2 - 2(\bar{x} - \mu)(n\bar{x} - n\mu)\right] \left(\because \bar{x} = \frac{\Sigma x_i}{n}\right)$$

$$s^2 = \frac{1}{n-1}\left[\Sigma(x_i - \mu)^2 + n(\bar{x} - \mu)^2 - 2n(\bar{x} - \mu)^2\right]$$

$$s^2 = \frac{1}{n-1}\left[\Sigma(x_i - \mu)^2 - n(\bar{x} - \mu)^2\right]$$

$$E[s^2] = \frac{1}{n-1}\left[\Sigma E(x_i - \mu)^2 - nE(\bar{x} - \mu)^2\right]$$

$$E(s^2) = \frac{1}{n-1}\left[\Sigma\sigma_{x_i}^2 - n\sigma_{\bar{x}}^2\right]$$

$$= \frac{1}{n-1}\left[n\sigma^2 - n\left(\frac{\sigma^2}{n}\right)\right]$$

$$= \frac{1}{\cancel{n-1}}\cancel{(n-1)}\sigma^2$$

$$= \sigma^2$$

$$\boxed{\therefore E(s^2) = \sigma^2}$$

Note: $\boxed{S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n}}$ Is not an Unbiased Estimator Of σ^2

Problem 24: In a study of an automobile insurance a random sample of $\overset{n}{80}$ body repair costs had a mean of ₹472.36 and the S.D of ₹62.35. If $\bar{x}$ is used as a point estimate to the true average repair costs, with what confidence we can assert that the maximum error doesn't exceed ₹10.

Solution:

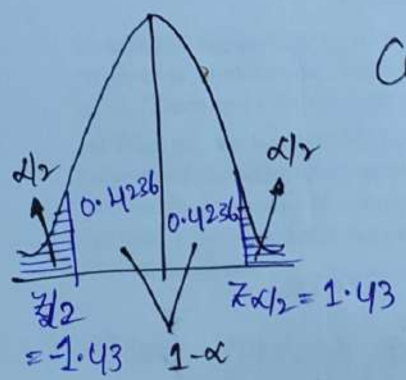
Sample Size $n = 80$, $\mu = 472.36$ & S.D $\sigma = 62.35$

Maximum Error $E = 10$

Now we have to find Confidence Level $= (1-\alpha)\times 100\%$

We know that Maximum Error $E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$$10 = Z_{\alpha/2}\frac{62.35}{\sqrt{80}} \Rightarrow 10 = Z_{\alpha/2}\frac{62.35}{8.94} \Rightarrow 10 = Z_{\alpha/2}\,6.974 \quad \boxed{Z_{\alpha/2} = 1.43}$$



Confidence Level $= (1-\alpha)\times 100\%$

$= (0.4236 + 0.4236)\times 100\%$

$= 0.8472 \times 100\%$

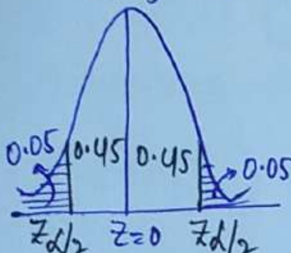
$$\boxed{\text{Confidence Level} = 84.72\%}$$

$\therefore$ Maximum Error doesn't Exceed with ₹10 with the Confidence Of 84.72%.

Problem 25: It is desired to estimate the mean number of hours of continuous use until a certain computer will first require repairs. If it can be assumed that $\sigma = 48$ hours, how large a sample be needed so that one will be able to assert with 90% confidence that the sample mean is off by at most 10 hours. Given $\sigma = 48$, Atmost 10 hours means Maximum Error $E = 10$;

Solution: How large a sample means is $n = ?$

90% Confidence Mean $(1-\alpha)100\% = 90\% \Rightarrow 1-\alpha = \frac{90}{100} \Rightarrow \boxed{\alpha = 0.1,\ \frac{\alpha}{2} = 0.05}$



$Z_{\alpha/2} = 1.65$, $E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \Rightarrow 10 = 1.65\left(\frac{48}{\sqrt{n}}\right) \Rightarrow \sqrt{n} = \frac{1.65 \times 48}{10}$

$\boxed{\therefore\ n \approx 63}$

$\sqrt{n} = 7.92 \quad n = 62.72$

$n \approx 63$

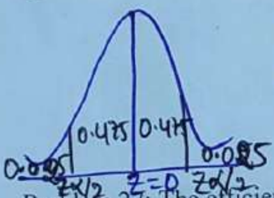
Problem 26: A random sample of size 100 has a standard deviation of 5. What can you say about the maximum error with 95% confidence.

(or)

If n = 100, $\sigma = 5$, find the maximum error with 95% confidence limits.

Solution: Given $n = 100$, $\sigma = 5$, Maximum Error with 95% confidence

$(1-\alpha)\times 100\% = 95\% \Rightarrow 1-\alpha = \frac{95}{100} = 0.95 \ \boxed{\alpha = 0.05,\ \alpha/2 = 0.025}$



$\boxed{Z_{\alpha/2} = 1.96}$

$E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.96\left(\frac{5}{\sqrt{100}}\right) = 1.96 \times 0.5$

$\boxed{\therefore E = 0.98}$

Problem 27: The efficiency expert of a computer company tested 40 engineers to estimate the average time it takes to assemble a certain computer component, getting a mean of 12.73 minutes and S.D of 2.06 minutes.

n

μ

(a) If $\bar{x} = 12.73$ is used as a point estimate of the actual average time required to perform the task, determine the maximum error with 99% confidence.

(b) Construct 98% confidence intervals for the true average time it takes to do the job.

(c) With what confidence can we assert that the sample mean does not differ from the true mean by more than 30 seconds. $\boxed{E = 30}$

Maximum Error

(or)

To estimate the average time it takes to assemble a certain computer component, the industrial engineer at an electronic firm timed 40 technicians in the performance of the task, getting a mean of 12.73 min and a S.D of 2.06 min.

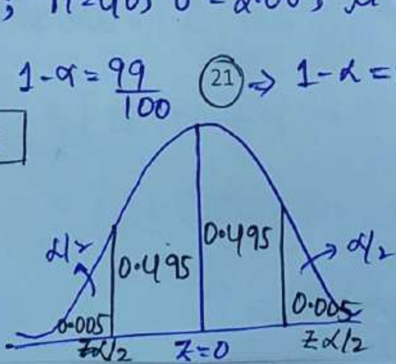
(a) What can we say with 99% confidence about the maximum error if $\bar{x} = 12.73$ is used a point estimate of the actual average time required to do the job ?

(b) Use the given data to construct 98% confidence interval.

(c) With what confidence we can assert that sample mean does not differ from the true mean by more than 30 sec. Here; $n = 40$, $\overset{s}{\sigma} = 2.06$; $\overset{\bar{x}}{\mu} = 12.73$

a) $(1-\alpha)100\% = 99\% \Rightarrow 1-\alpha = \frac{99}{100}$ (21) $\Rightarrow 1-\alpha = 0.99 \Rightarrow \boxed{\alpha = 0.01,\ \alpha/2 = 0.005}$

$\boxed{Z_{\alpha/2} = 2.575}$



$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 2.575 \times \frac{2.06}{\sqrt{40}}$

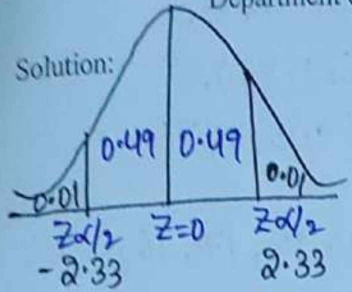
$= 2.575 \times 0.32$

$\boxed{E = 0.8387}$

b) $(1-\alpha)\times 100 = 98 \Rightarrow 1-\alpha = 0.98$ $\boxed{\alpha = 0.02, \alpha/2 = 0.01}$

$\boxed{Z_{\alpha/2} = 2.33}$

Solution:



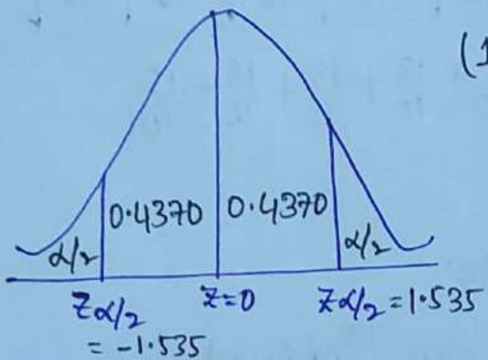
$$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 2.33 \times \frac{2.06}{\sqrt{40}} = 2.33 \times \frac{2.06}{6.32} = 2.33 \times 0.32$$

$$\boxed{E = 0.745}$$

c) With what Confidence $(1-\alpha)\times 100\% = ?$

True Mean = Maximum Error $(E) = 30$ sec $E = \frac{30}{60} = \frac{1}{2}$ $\boxed{E = 1/2}$

$$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \Rightarrow \frac{1}{2} = Z_{\alpha/2}\left(\frac{2.06}{\sqrt{40}}\right) \Rightarrow \frac{1}{2} = Z_{\alpha/2} \times 0.32 \quad \boxed{Z_{\alpha/2} = 1.535}$$



$$(1-\alpha)\times 100\% = (0.4370 + 0.4370) \times 100\%$$
$$= (0.874) 100\%$$
$$= 87.4\% \text{ is Confidence Level}$$

Problem 28: The mean and standard deviation of a population are 11.795 and 14.054 respectively. What can one assert with 95% confidence about the maximum error if $\bar{x} = 11.795$ and $n = 50$. And also construct 95% confidence interval for the true mean.

(or)

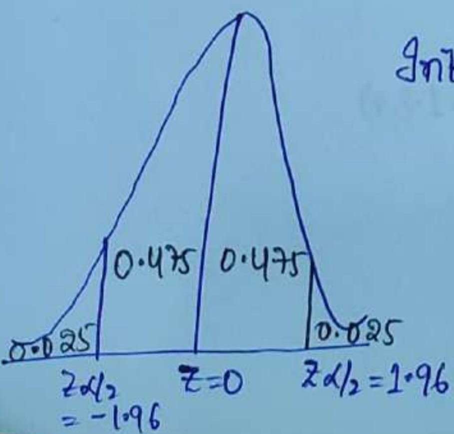
The mean and the standard deviation of a population are 11.795 and 14.054 respectively. If n = 50, find 95% confidence interval for the mean.

Solution: Population Mean $\mu = 11.795$, S.D Population $\sigma = 14.054$, $n = 50$.

Confidence Interval for Mean $= \left[\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$

Here Sample Mean $\bar{x}$ is Not Given. So we Consider Sample Mean as the population Mean i.e $\bar{x} = \mu = 11.795$

$(1-\alpha)100 = 95 \Rightarrow 1-\alpha = 0.95 \Rightarrow \boxed{\alpha = 0.05, \alpha/2 = 0.025}$ $\boxed{Z_{\alpha/2} = 1.96}$



$$\text{Interval} = \left[\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

$$= \left[11.975 - 1.96\left(\frac{14.054}{\sqrt{50}}\right), 11.975 + 1.96 \times \frac{14.054}{\sqrt{50}}\right]$$
$$= [11.795 - 3.895, 11.795 + 3.895]$$
$$= [7.9, 15.69]$$

Problem 29: The mean of random sample is an unbiased estimate of the mean of the population 3,6,9,15,27.

i) List of all possible samples of size 3 that can be taken without replacement from the finite population.

ii) Calculate the mean of each of the samples listed in (a) and assigning each sample a probability of 1/10. Verify that the mean of these $\bar{x}$ is equal to 12. Which is equal to the mean of the population θ i.e $E(\bar{x}) = \theta$ i.e., prove that $\bar{x}$ is an unbiased estimate of θ.

Solution:

Problem 30: A professor's feelings about the mean mark in the final examination in "Probability" of a large group of students is expressed subjectively by normal distribution with $\mu_0 = 67.2$ and $\sigma_0 = 1.5$.

(a) If the mean mark lies in the interval (65.0, 70.0) determine the prior probability the professor should assign to the mean mark.

(b) Find the professor mean μ_1 and the posterior S.D σ_1 if the examinations are conducted on a random sample of 40 students yielding mean 74.9 and S.D 7.4. Use S =7.4 as an estimate σ.

(c) Determine the posterior probability which he will thus assign to the mean mark being in the interval (65.0,70.0) using results obtained in (b).

(d) Construct a 95% Bayesian interval for μ.

Solution: Hints: $\mu_1 = \dfrac{n\,\bar{x}\sigma_0^{\,2} + \mu_0\sigma^2}{n\sigma_0^{\,2} + \sigma^2}$, $\sigma_1 = \sqrt{\dfrac{\sigma^2\sigma_0^{\,2}}{n\sigma_0^{\,2} + \sigma^2}}$

Problem-29: Population 3, 6, 9, 15, 27 ; $N=5$, $n=3$

i, List of all Possible Samples (3,6,9), (3,6,15), (3,6,27), ~~(3,9,15)~~,
(3,9,15), (3,9,27),
(3,15,27),
(6,9,15), (6,9,27),
(9,15,27)

Mean of Population:

$$\theta = \frac{3+6+9+15+27}{5} = 12$$

Mean of the Samples are 6, 8, 12, 10, 14, 9, 13, 17, 16, 15

Probability assigned to each one is $\boxed{\frac{1}{10} \text{ each}}$

$\bar{x}$ =	6	8	12	10	14	9	13	17	16	15
$P(\bar{x})$ =	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$$E(\bar{x}) = \Sigma x_i P_i = \frac{6}{10} + \frac{8}{10} + \frac{12}{10} + \frac{10}{10} + \frac{14}{10} + \frac{9}{10} + \frac{13}{10} + \frac{17}{10} + \frac{16}{10} + \frac{15}{10}$$

$$\boxed{E(\bar{x}) = \theta = 12}$$

$\therefore$ $\bar{x}$ is an Unbiased Estimate of 'θ'.

ie. The Mean of a Random Sample is an Unbiased Estimated of the Mean of the Population.

Problem-30: Given that $\mu_0 = 67.2$, $\sigma_0 = 1.5$

a, Now we have to find the Mean Mark lies in the Interval (65.0, 70.0)

i.e. $P(65 \le \bar{x} \le 70)$;

At $\bar{x}=65$, $z = \frac{x-\mu}{\sigma} = \frac{65-67.2}{1.5} = -1.4666 = -1.47$

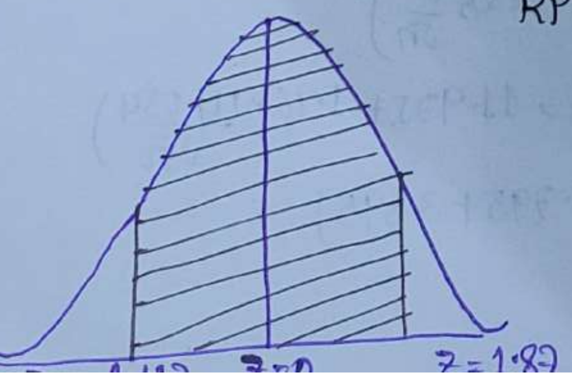
At $\bar{x}=70$, $z = \frac{x-\mu}{\sigma} = \frac{70-67.2}{1.5} = 1.8666 = 1.87$

$P(1.47 < Z < 1.87)$

RP = A(0 to 1.47) + A(0 to 1.87)

= 0.4292 + 0.4693

= 0.8965



b, Sample Space $n=40$; Sample Mean $\bar{x}=74.9$,
Sample S.D $\sigma = S =$ Population $= 7.4$

Now:
$$\mu_1 = \frac{n\bar{x}\sigma_0^2 + \mu_0\sigma^2}{n\sigma_0^2+\sigma^2} \Rightarrow \frac{40\times 74.9\times(1.5)^2 + 67.2(7.4)^2}{40\times(1.5)^2+(7.4)^2}$$

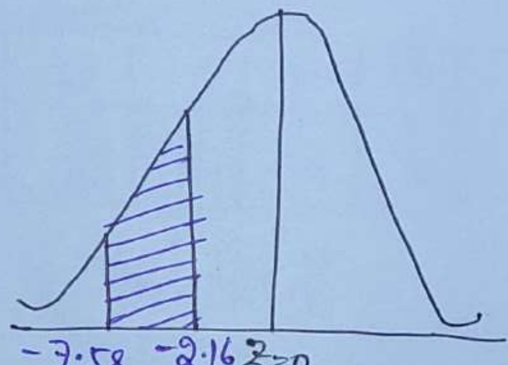
$$\Rightarrow \frac{6741+3679.872}{144.76} = 71.987 \Rightarrow \boxed{\mu_1 = 72}$$

$$\sigma_1 = \sqrt{\frac{\sigma^2\sigma_0^2}{n\sigma_0^2+\sigma^2}} = \sqrt{\frac{(1.5)^2(7.4)^2}{40(1.5)^2+(7.4)^2}} = \sqrt{\frac{123.21}{90+54.76}} = 0.9225 \quad \boxed{\sigma_1 = 0.923}$$

c, Here $\mu_1 = 72$; $\sigma_1 = 0.923$
Now; we have to Evaluate $P(65<\bar{x}<70)$

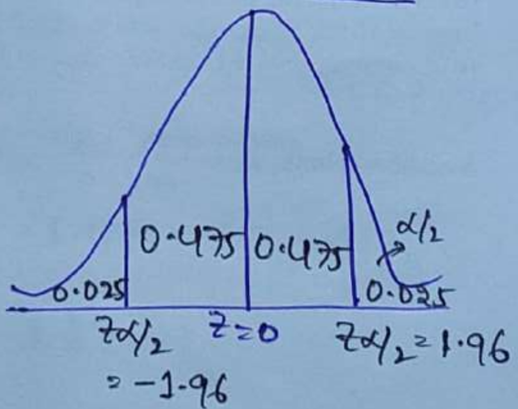
At $\bar{x}=65$; $Z = \frac{x-\mu}{\sigma} = \frac{65-72}{0.923} = -7.58$

At $\bar{x}=70$; $Z = \frac{x-\mu}{\sigma} = \frac{70-72}{0.923} = -2.166$



$RP = A(0 \text{ to } 7.58) - A(0 \text{ to } 2.16)$
$= 0.5 - 0.4846 = \boxed{0.0154}$

d, 95% Bayesian Interval Limits are $\mu_1 \pm Z_{\alpha/2}\sigma_1$
By Calculating $Z_{\alpha/2}$ for 95% Confidence Limit are 1.96
$(1-\alpha)100\% = 95\% \Rightarrow 1-\alpha = 0.95 \Rightarrow \boxed{\alpha = 0.05,\ \alpha/2 = 0.025}$
Search for 0.475 in Table, Gives
$$\boxed{Z_{\alpha/2} = 1.96}$$



$= \mu_1 \pm Z_{\alpha/2}(\sigma_1)$
$= 72 \pm 1.96(0.923)$
$= [70.190,\ 73.809]$

Problem31: A random sample of $\underset{}{\overset{n}{100}}$ teachers in a large metropolitan area revealed a mean weekly salary of Rs.487 with a standard deviation Rs.48 . With what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 to 502?

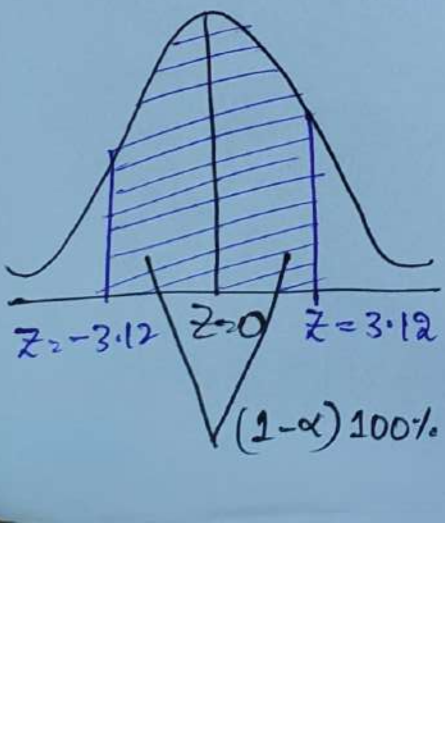
Solution: Given that Sample Space $n = 100$, $\mu = 487$, $\sigma = 48$,

Confidence $(1-\alpha)\times 100\% = ?$

$P(472 \leq \bar{x} \leq 502)$

At $x = 472$, $z = \dfrac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \dfrac{472-487}{\left(\frac{48}{\sqrt{100}}\right)} = -3.12$

At $x = 502$, $z = \dfrac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} = \dfrac{502-487}{\left(\frac{48}{\sqrt{100}}\right)} = 3.12$



$RP = A(0 \text{ to } 3.12) + A(0 \text{ to } 3.12)$

$= 0.4991 + 0.4991$

$= 0.9982 \times 100\%$

$\boxed{\therefore\ RP = 99.82\%}$ → Confidence Level.

Confidence Interval $CI = p \pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$

$$= 0.18 \pm 1.69\sqrt{\frac{0.18\times 0.82}{100}}$$

$$(0.2449, 0.1151)$$

Problem 32: Among 100 fish caught in a large lake, 18 were inedible due to the pollution of the environment. With what confidence can we assert the error of this estimate is at most 0.065? And also find Confidence Interval

Solution: Sample Space $n = 100$

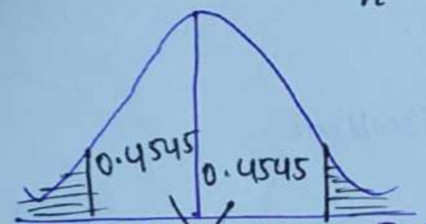
In the Given Problem, they didn't Mention about Mean & S.D ie No Mean & No S.D. So It always belongs to Proportions.

Now, Sample Proportion $p = \frac{18}{100} = 0.18$, $q = 1-p = 0.82$

Confidence Level = ? , Maximum Error $E = 0.065$

Now; we have to Evaluate Confidence Level $(1-\alpha)100\%$

$$\Rightarrow E = z_{\alpha/2}\sqrt{\frac{pq}{n}} \Rightarrow 0.065 = z_{\alpha/2}\sqrt{\frac{0.18\times 0.82}{100}} \Rightarrow \boxed{z_{\alpha/2} = 1.6919}$$



$z_{\alpha/2} = -1.69$ $(1-\alpha)100\%$ $z_{\alpha/2} = 1.69$

$$= (1-\alpha)100\%$$

$$= (0.4545 + 0.4545)100\%$$

$$= 0.9090 \times 100\% \qquad \boxed{(1-\alpha)100\% = 90.90\%}$$

Problem 33: The mean mark in mathematics in common entrance that will vary from year to year. If this variation of the mean mark is expressed subjectively by a normal distribution with mean $\mu_0 = 72$ and variance $\sigma_0^2 = 5.76$.

i) What probability can we assign to the actual mean mark being somewhere between 71.8 and 73.4 for the next years test ?

ii) Construct a 95% Bayesian interval for μ if the test is conducted for a random sample of 100 students from the next incoming class yielding a mean mark of 70 with S.D of 8.

iii) What posterior probability should we assign to the event of part (i).

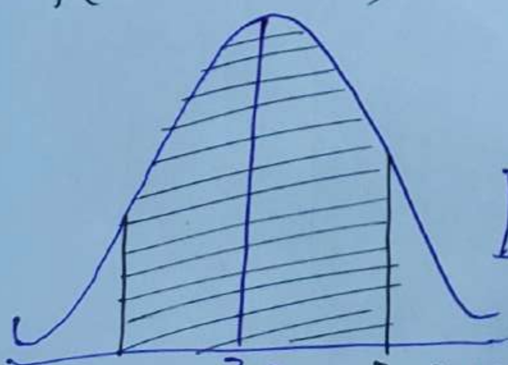
a) Given that $\mu_0 = 72$, $\sigma_0^2 = 5.76$; $\sigma_0 = 2.4$

Now, we have to find the Mean Mark lies In Interval (71.8, 73.4)

i.e. $p(71.8 < \bar{x} < 73.4)$: At $\bar{x} = 71.8$, $z = \frac{\bar{x}-\mu}{\sigma} = \frac{71.8-72}{2.4} = -0.08$

$$\bar{x} = 73.4, \quad z = \frac{\bar{x}-\mu}{\sigma} = \frac{73.4-72}{2.4} = 0.58$$

$P(0.08 < z < 0.58)$



$z = -0.08$ $z = 0$ $z = 0.58$

$$RP = A(0 \text{ to } 0.08) + A(0 \text{ to } 0.58)$$

$$= 0.0319 + 0.2190$$

$$\boxed{RP = 0.2509}$$

1) A Random Sample of 10 Ball bearings produced by a Company have a Mean diameter of 0.5060 cm with a Standard deviation of 0.004 cm. Find the Maximum error of estimate 'E' & also 95% Confidence level for actual Mean of diameter of ball bearings produced by a company assuming Normal Population & also find Confidence Interval.

Sol: Given that sample size $n = 10 < 30$, It is a Small Sample so we have to use t-distribution.

Sample Mean $\bar{x} = 0.5060$ cm, Sample S.D $(s) = 0.004$ cm

Now, We have to find Maximum Error:

Maximum Error $(E) = t_{\alpha/2} \frac{s}{\sqrt{n}}$, Confidence Level $(1-\alpha)100\% = 95\%$.

$\boxed{\alpha = 0.05,\ \alpha/2 = 0.025}$

This is t-distribution, so the degrees of Freedom $\nu = n-1 = 10-1 = 9$

$t_{\alpha/2} = t_{0.025} = 2.262$ (From table-2)

$E = t_{\alpha/2} \frac{S}{\sqrt{n}} = 2.262 \times \frac{0.004}{\sqrt{10}}$ $\boxed{E = 0.00286}$

Confidence Interval $= \bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} = 0.5060 \pm 0.00286$

$(0.5089, 0.5031)$

2) Find 95% Confidence in the limits for the Mean of the Normally distributed population from which the following Sample was taken
15, 17, 10, 18, 16, 9, 7, 11, 13, 14

Sol. Here Sample Size $n = 10\ (<30)$, So we use t-distribution, Confidence Limits are $\bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$

Here, we evaluate $\bar{x}$, s and $t_{\alpha/2}$ from Given Samples:

$$\bar{x} = \frac{\text{Sum of all Observations}}{\text{Total no. of Observations}} = \frac{15+17+10+18+16+9+7+11+13+14}{10} = 13$$

$$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(15-13)^2+(17-13)^2+\cdots+(13-13)^2+(14-13)^2}{9}} = 3.6514$$

[from this point onwards, we take $n-1$ Instead of 'n']

$$\boxed{S_x = 3.6514}$$