Rp &Rk

Model No 4.3: Sampling distribution of means and variances

Parameters of the Population: Population size-N

* Finite Population Means Without Replacement

1. Mean of the population $\mu = \frac{\sum x}{N}$
2. Variance of the population $\sigma^2 = \frac{\sum (x-\mu)^2}{N}$
3. Standard deviation of the population $\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$

* Infinite Population Means With Replacement

Statistics of the Sample: Sample size-n

$\boxed{\text{Correction factor} = \frac{N-n}{N-1}}$

1. The total number of samples with replacement (Infinite Population) is N^n
2. The total number of samples without replacement (Finite Population) is N_{C_n}.

Problem 1: What is the value of correction factor if n=5 and N=200.

Solution: Correction Factor $= \frac{N-n}{N-1} = \frac{200-5}{200-1} = \boxed{0.9799}$

Problem 2: The size of the population is 2000 and the size of the sample is 200. Find the correction factor in the population. Given $N=2000$, $n=200$

Solution: Correction factor $= \frac{N-n}{N-1} = \frac{2000-200}{2000-1} = \boxed{0.9005}$

Problem 3: How many different samples of size two can be chosen from a finite population of size 25. $N=25$, $n=2$ without Replacement

Solution: No. of different samples of size two from finite $N=25$ is $N_{C_2} = 25_{C_2} = \boxed{300}$

Problem 4: In a random sample of 1000 packages shipped by air freight 13 had some damage. Out of 100 Find the standard error proportions.

Solution: Here $P = \frac{13}{100} = 0.13$, $Q = 1-P = 1-0.13 = 0.87$, $n=1000$

Standard Error of Proportions $= \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.13 \times 0.87}{1000}} = \cancel{0.0314} = \boxed{0.0106}$

***** **Problem 5:** A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn from this population.

i) With replacement ii) Without replacement Find

(a) The mean of the population .

(b) The standard deviation of the population.

(c) The mean of the sampling distribution of means.

(d) The standard deviation of the sampling distribution of means.

06.05.2022 Friday

Problem-5 : Given Population 2, 3, 6, 8, 11

Population Size $N = 5$

a) Mean of the Population:

$$\mu = \frac{\Sigma x}{N} = \frac{\text{Sum of Observations}}{\text{Total Observations}} = \frac{2+3+6+8+11}{5} = 6$$

b) Standard deviation of population:

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} = \sqrt{\frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}} = 3.286$$

Samples: i) With Replacement

The no. of Samples $(n) = N^n = 5^2 = 25$

(2,2), (2,3), (2,6), (2,8), (2,11),
(3,2), (3,3), (3,6), (3,8), (3,11),
(6,2), (6,3), (6,6), (6,8), (6,11),
(8,2), (8,3), (8,6), (8,8), (8,11),
(11,2), (11,3), (11,6), (11,8), (11,11)

Sample Distribution Of Means:

$\frac{2+2}{2}$ 2	$\frac{2+3}{2}$ 2.5	4	5	6.5
2.5	3	4.5	5.5	7
4	4.5	6	7	8.5
5	5.5	7	8	9.5
6.5	7	8.5	9.5	11

c) Mean of the Sampling distribution Of Means

$$\mu_{\bar{x}} = \frac{\text{Sum of all Sample distribution Observations}}{25}$$

$$\boxed{\mu_{\bar{x}} = \frac{150}{25} = 6}$$

$$\boxed{\begin{array}{c}\text{Mean of the population = Mean Of the Sampling distribution Of Means} \\ \text{(Applicable For Every Problem)}\end{array}}$$

*

d, Standard deviation of the Sampling distribution Of Means

$$\sigma_{\bar{x}} = \sqrt{\frac{\Sigma(x-\mu_{\bar{x}})^2}{n}} = \sqrt{\frac{\Sigma(x-\mu_{\bar{x}})^2}{n}}$$

$$\sqrt{\frac{(2-6)^2+(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2+(2.5-6)^2+(3-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2+(4-6)^2+(4.5-6)^2+(6-6)^2+(7-6)^2+(8.5-6)^2+(5-6)^2+(5.5-6)^2+(7-6)^2+(8-6)^2+(9.5-6)^2+(6.5-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2+(11-6)^2}{25}}$$

$$\boxed{\sigma_{\bar{x}} = 2.323}$$

For Standard Error (SE) $= \frac{\sigma}{\sqrt{n}} = \frac{3.286}{\sqrt{2}} = 2.323$

Standard Error = Standard deviation of Sampling distribution of Means
(*)

ii, WITHOUT REPLACEMENT

No.of Samples $= {}^{N}C_{n} = {}^{5}C_{2} = 10$ 2, 3, 6, 8, 11

The Samples are : (2,3), (2,6), (2,8), (2,11),
(3,6), (3,8), (3,11),
(6,8), (6,11),
(8,11)

Sampling distribution Of Means =

2.5	4	5	6.5
	4.5	5.5	7
		7	8.5
			9.5

Mean of Sampling distribution Of Means = $\frac{\text{Sum of all Observations}}{10}$

$$\boxed{\mu_{\bar{x}} = \frac{60}{10} = 6}$$

Mean Of the Population = Mean Of the Sampling distribution Of Means d, Standard Deviation Of the Sampling distribution Of Means ⇒ $\sigma_{\bar{x}} = \sqrt{\frac{(x-\mu_{\bar{x}})^2}{n}}$

$$= \sqrt{\frac{(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2}{10}}$$

$$\boxed{\sigma_{\bar{x}} = 2.012}$$

: Standard Error S.E (without Replacement) $= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

$$= \frac{3.286}{\sqrt{2}}\sqrt{\frac{5-2}{5-1}} = 2.012$$

10.05.2022

Tuesday

If the population is 3, 6, 9, 15, 27 :

a, List all Possible Samples of Size '3', that can be taken as without replacement from the finite population.

b, Calculate the Mean Of each of the Sampling distribution of Means.

c, Find the S.D of Sampling distribution Of Means.

d, Mean Of the population

e, The Population Standard Deviation

Sol Population 3, 6, 9, 15, 27, Population Size $N = 5$

Sample Size $n = 3$

WITHOUT REPLACEMENT

N_{C_n} = No. of Samples

$= 5_{C_3} = 10$

(3,6,9) (3,6,15) (3,6,27)

(3,9,15) (3,9,27)

(3,15,27)

(6,9,15) (6,9,27)

(6,15,27)

(9,15,27)

d, Mean of the Population:

$$\mu = \frac{\Sigma x}{N} = \frac{3+6+9+15+27}{5} = 12$$

$$\boxed{\mu = 12}$$

e) Standard deviation Of the Population:

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} = \sqrt{\frac{(3-12)^2+(6-12)^2+(9-12)^2+(15-12)^2+(27-12)^2}{5}} = 8.4852$$

a) Sampling distribution Of Means:

6	8	12
9	13	15
10	14	16
	17	

b) Mean of Sampling distribution Of Means:

$$\mu_{\bar{x}} = \frac{\text{Sum of all Sample Observations}}{\text{No. of Observations}}$$

$$= \frac{6+8+12+9+13+15+10+14+16+17}{10} = 12$$

$$\boxed{\mu_{\bar{x}} = 12}$$

c) Standard Deviation Of the Sampling distribution Of Means:

$$\sigma_{\bar{x}} = \sqrt{\frac{(6-12)^2+(8-12)^2+(12-12)^2+(9-12)^2+(13-12)^2+(15-12)^2+(10-12)^2+(14-12)^2+(16-12)^2+(17-12)^2}{10}}$$

$$\boxed{\sigma_{\bar{x}} = 3.4641}$$

Standard Error (SE) $= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{8.4852}{\sqrt{3}}\sqrt{\frac{5-3}{5-1}} = 3.4641$

****Problem 6: A population consists of six numbers 4, 8, 12, 16, 20, 24. Consider all possible samples of size two that can be drawn without replacement and with replacement from this population. Find

(a) The mean of the population .

(b) The standard deviation of the population.

(c) The mean of the sampling distribution of means.

(d) The standard deviation of the sampling distribution of means.

Solution: Do Practice at note book

(6) Given 4, 8, 12, 16, 20, 24, $N = 6$, $n = 2$

a) Mean Of the Population:

$$\mu = \frac{4+8+12+16+20+24}{6} = 14 \qquad \boxed{\mu = 14}$$

b) S.D of population:

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} = \sqrt{\frac{(4-14)^2+(8-14)^2+(12-14)^2+(16-14)^2+(20-14)^2+(24-14)^2}{6}}$$

$$= \sqrt{\frac{280}{6}} = \sqrt{46.6667} = 6.8313 \qquad \boxed{\sigma = 6.8313}$$

c) i) WITH REPLACEMENT

No.of Samples $= N^n = 6^2 = 36$

Samples:

(4,4), (4,8), (4,12), (4,16), (4,20), (4,24),
(8,4), (8,8), (8,12), (8,16), (8,20), (8,24),
(12,4), (12,8), (12,12), (12,16), (12,20), (12,24),
(16,4), (16,8), (16,12), (16,16), (16,20), (16,24),
(20,4), (20,8), (20,12), (20,16), (20,20), (20,24),
(24,4), (24,8), (24,12), (24,16), (24,20), (24,24).

Mean Of Sampling distribution:

4	6	8	10	12	14
6	8	10	12	14	16
8	10	12	14	16	18
10	12	14	16	18	20
12	14	16	18	20	22
14	16	18	20	22	24

Mean Of Sampling distribution Of Means

$$= \frac{504}{36} = 14$$

$$\boxed{\mu_{\bar{x}} = 14}$$

4.8204

d) S.D of Sampling distribution Of Means:

$$\sigma_{\bar{x}} = \sqrt{\frac{\begin{array}{l}(4-14)^2+(6-14)^2+(8-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(6-14)^2+ \\ (8-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(16-14)^2+(8-14)^2+(10-14)^2+ \\ (12-14)^2+(14-14)^2+(16-14)^2+(18-14)^2+(10-14)^2+(12-14)^2+(14-14)^2 \\ +(16-14)^2+(18-14)^2+(20-14)^2+(12-14)^2+(14-14)^2+(16-14)^2+(18-14)^2 \\ +(20-14)^2+(22-14)^2+(14-14)^2+(16-14)^2+(18-14)^2+(20-14)^2+(22-14)^2 \\ +(24-14)^2\end{array}}{36}}$$

$$= \sqrt{\frac{23.3333}{36}} = \boxed{\sigma_{\bar{x}} = 4.8305}$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{6.8213}{\sqrt{2}}$$

$$\boxed{SE = 4.8304}$$

ii) WITHOUT REPLACEMENT:

No. of Samples = $N_{C_n} = 6C_2 = 15$.

a) Mean of Population $\boxed{\mu = 14}$

b) S.D of Population $\boxed{\sigma = 6.8313}$

c) Samples = 15   Pop = 4, 8, 12, 16, 20, 24

(4,8), (4,12), (4,16), (4,20), (4,24)
(8,12), (8,16), (8,20), (8,24)
(12,16), (12,20), (12,24)
(16,20), (16,24)
(20,24)

Means of Sampling distribution

6	8	10	12	14
	10	12	14	16
		14	16	18
			18	20
				22

Mean of S.D of Means $(\mu_{\bar{x}}) = \frac{210}{15} = 14$

$\boxed{\mu_{\bar{x}} = 14}$

d) S.D of Sampling distribution of Means $(\sigma_{\bar{x}})$ =

$$\sigma_{\bar{x}} = \sqrt{\frac{(6-14)^2+(8-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(10-14)^2+(12-14)^2+(14-14)^2+(16-14)^2+(14-14)^2+(16-14)^2+(18-14)^2+(20-14)^2+(22-14)^2}{15}}$$

$\boxed{\sigma_{\bar{x}} = 4.3205}$

$$SE = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{6.8313}{\sqrt{2}}\sqrt{\frac{6-2}{6-1}} = 4.3204$$

$\boxed{SE = 4.3204}$

****Problem 7: Find the mean and Standard deviation of sampling distribution of ~~variances~~ Means for the population 2, 3, 4, 5 by drawing samples of size two with replacement and without replacement.

Solution: Do Practice at note book

(7) Given 2,3,4,5

Size of Population $N=4$

$$\mu = \text{Mean} = \frac{\text{Sum of Observations}}{\text{Total}} = \frac{2+3+4+5}{4} = \frac{14}{4} = 3.5$$

i) WITH REPLACEMENT:

No. of Sample $= N^n = 4^2 = 16$

Samples				Means of Sampling Distribution			
(2,2)	(2,3)	(2,4)	(2,5)	2	2.5	3	3.5
(3,2)	(3,3)	(3,4)	(3,5)	2.5	3	3.5	4
(4,2)	(4,3)	(4,4)	(4,5)	3	3.5	4	4.5
(5,2)	(5,3)	(5,4)	(5,5)	3.5	4	4.5	5

Means of Sampling Distribution Of Means

$$\mu = \frac{2+2.5+3+3.5+2.5+3+3.5+4+3+3.5+4+4.5+3.5+4+4.5+5}{16}$$

$$\mu_{\bar{x}} = \frac{50}{16} = 3.5$$

Standard Deviation Of Sampling Distribution Of Means:

$$\sigma_{\bar{x}} = \sqrt{\frac{\Sigma(x-\mu_{\bar{x}})^2}{N}} =$$

$$\boxed{\sigma_{\bar{x}} = 0.79056}$$

ii) WITHOUT REPLACEMENT

No. of Samples $= N_{C_n} = {}^4C_2 = 6$

Samples are:

Samples	Means of Samples
(2,3), (2,4), (2,5)	2.5, 3, 3.5
(3,4), (3,5)	3.5, 4
(4,5)	4.5

$$\text{Mean} = \frac{2.5+3+3.5+3.5+4+4.5}{6} = 3.5$$

$$\boxed{\mu_{\bar{x}} = 3.5}$$

$$S.D(\sigma) = \sqrt{\frac{\Sigma(x-\mu_{\bar{x}})^2}{N}} = \sqrt{\frac{(2.5-3.5)^2+(3-3.5)^2+(3.5-3.5)^2+(3.5-3.5)^2+(4-3.5)^2+(4.5-3.5)^2}{6}}$$

$$\boxed{\sigma_{\bar{x}} = 0.6454}$$

Solution: Do Practice at note book

Problem 8: Let $u_1 = (3, 7, 8)$, $u_2 = (2, 4)$. Find

(a) $\mu_{u_1}, \mu_{u_2}, \mu_{u_1-u_2}$ (*Mean of the sampling distribution of means*)

(b) $\sigma_{u_1}, \sigma_{u_2}, \sigma_{u_1-u_2}$ (*Standard deviations of the sampling distribution of means*)

Sol: Given $(u_1) = (3, 7, 8)$, $u_2 = (2, 4)$, $(u_1 - u_2) = \{1, -1, 5, 3, 6, 4\}$

a) $\mu_{u_1} = \dfrac{3+7+8}{3} = \dfrac{18}{3} = 6$ $\quad \mu_{u_2} = \dfrac{2+4}{2} = 3$

$$\mu_{u_1-u_2} = \frac{1-1+5+3+6+4}{6} = 3$$

b) $$\sigma_{u_1} = \sqrt{\frac{(3-6)^2+(7-6)^2+(8-6)^2}{3}} = 2.1602$$

$$\sigma_{u_2} = \sqrt{\frac{(2-3)^2+(4-3)^2}{2}} = 1$$

9

$$\sigma_{u_1-u_2} = \sqrt{\frac{(1-3)^2+(-1-3)^2+(5-3)^2+(3-3)^2+(6-3)^2+(4-3)^2}{6}} = 2.3862$$

Problem 9: The variance of a population is 2. The size of the sample collected from the population is 169. What is the standard error of mean.

Solution: Here $n = 169 \rightarrow$ The size of the sample

σ = Standard Deviation Of Population $= \sqrt{\text{Variance}} = \sqrt{2}$

Standard Error of Mean:

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{2}}{\sqrt{169}} = 0.1087$$

Problem 10: When a sample is taken from an infinite population, what happens the standard error of the mean if the sample size is decreased from 800 to 200.

Solution: Infinite Population:

The Standard Error Of the Mean $= \frac{\sigma}{\sqrt{n}}$

Standard Error(1): $= \frac{\sigma}{\sqrt{n_1}} = \frac{\sigma}{\sqrt{800}} = \frac{\sigma}{2\sqrt{200}} = \frac{\sigma}{20\sqrt{2}}$

Standard Error(2): $\frac{\sigma}{\sqrt{n_2}} = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}} = 2\left[\frac{\sigma}{20\sqrt{2}}\right] = 2\,S.E(1)$

Hence, If Sample Size is reduced from 800 to 200, Standard Error Of Mean, Will be Multiplied by '2'.

Model No 4.4: Central limit theorem

Central Limit Theorem: If $\bar{x}$ be the mean of a sample size n drawn from a population mean μ and standard deviation σ then the standardized normal variate $z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$ is asymptotically normal.

Here $\bar{x}$ = Sample Mean

μ = Population Mean

σ = S.D of the population

n = Sample Size