Model No 5.4: Test of significance for difference of proportions:

(i) **Null Hypothesis** (H_0): $p_1 = p_2$ *or* $P_1 = P_2$ i.e., "there is no significance difference between the proportions of the samples or proportions of populations" or "the two samples have been drawn from the same population"

(ii) **Alternative Hypothesis** (H_1): $p_1 \neq p_2$ *or* $P_1 \neq P_2$

(iii) **Level of Significance** (α): Set a level of significance

(iv) **Test Statistic:**

Case(i): When the population proportions P_1 *and* P_2 are known

The test statistic $z = \dfrac{P_1 - P_2}{\sqrt{\dfrac{P_1Q_1}{n_1} + \dfrac{P_2Q_2}{n_2}}}$

Case(ii): When the population proportions P_1 *and* P_2 are unknown, and the sample proportions p_1 *and* p_2 are known

The test statistic $z = \dfrac{p_1 - p_2}{\sqrt{\dfrac{p_1q_1}{n_1} + \dfrac{p_2q_2}{n_2}}}$ or $z = \dfrac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ where $p = \dfrac{n_1p_1 + n_2p_2}{n_1 + n_2}$, $q = 1 - p$

(v) **Conclusion:** (i) If $|z| < z_\alpha$ we accept the Null Hypothesis H_0

(ii) If $|z| > z_\alpha$ we reject the Null Hypothesis H_0

i.e., we accept the Alternative Hypothesis H_1

Problem 22: In two large populations, there are 30% and 25% are fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?

Solution:

Sample Size $n_1 = 1200$ $n_2 = 900$

Population proportion $P_1 = 30\%$ $P_2 = 25\%$

$= 0.3$ $P_2 = 0.25$

Here only Population proportions are Given;

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(iii) Level of Significance (α): $\alpha = 0.05$ $\alpha/2 = 0.025$ $0.5 - 0.025 = 0.475$

$\boxed{z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

$$z_{cal} = \frac{P_1 - P_2}{\sqrt{\dfrac{P_1Q_1}{n_1} + \dfrac{P_2Q_2}{n_2}}} = \frac{0.3 - 0.25}{\sqrt{\dfrac{0.3 \times 0.7}{1200} + \dfrac{0.25 \times 0.75}{900}}}$$

$$= 2.5537$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$
Calculated value of $Z_{cal} = 2.5537$
Calculated value of $>$ Tabulated value of

$\therefore$ Null Hypothesis is Rejected

Problem 23: A cigarette manufacturing firm claims that its brand A line of cigarettes outsells its brand B by 8%. If it is found that 42 out of a sample of 200 smokers prefer brand A and 18 out of another samle of 100 smokers prefer brand B, test whether the 8% difference is a valid claim.

Solution: $n_1 = 200, \; n_2 = 100$

$$P_1 = \frac{42}{200} = 0.21 \;, \; P_2 = \frac{18}{100} = 0.18$$

(i) Null Hypothesis (H_0): $P_1 - P_2 = \cancel{0.21 - 0.18} = 0.08$

(ii) Alternative Hypothesis (H_1): $(P_1 - P_2) \neq 0.08$

(iii) Level of Significance (α): $\alpha = 0.05 \quad \alpha/2 = 0.025 \quad 0.5 - 0.025 = 0.475 \; \boxed{Z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

$$Z = \frac{(P_1 - P_2) - (P_1 - P_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.21 - 0.18) - (0.08)}{\sqrt{(0.2)(0.8)\left(\frac{1}{200} + \frac{1}{100}\right)}} = -1.0206$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$
Calculated value of $Z_{cal} = 1.0206$
Calculated value of $<$ Tabulated value of Null Hypothesis is Accepted

Problem 24: A machine puts out 9 imperfect articles ina sample of 200 articles. After the machine is over haulted it puts out 5 imperfect articles ina sample of 700 articles. Test at 5% level whether the machine is improved?

Solution: $n_1 = 200, \; n_2 = 700$

$$P_1 = \frac{9}{200} \;, \; P_2 = \frac{5}{700}$$

$$P_1 = 0.045, \; P_2 = 0.0071$$

$$p = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2} = \frac{(0.045)(200) + 700(0.0071)}{200 + 700}$$

$P = 0.0155 \quad q = 0.9845$

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$ (Two tailed Test)

(iii) Level of Significance (α): $\alpha = 0.05 \quad \alpha/2 = 0.025 \quad 0.5 - 0.025 = 0.475 \quad \boxed{Z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

$$Z_{cal} = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.045 - 0.0071}{\sqrt{(0.0155)(0.9845)\left(\frac{1}{200} + \frac{1}{700}\right)}} = 3.2865$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 3.2865$

Calculated value of $>$ Tabulated value of

Null Hypothesis is Rejected

Problem 25: **In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys has the same defect. Is the difference between the proportions significant at 005 level of significance.**

Solution: $n_1 = 900 \quad P_1 = \frac{20\% \text{ of } 900}{900} = \frac{180}{900} = 0.2$

$n_2 = 1600 \quad P_2 = \frac{18.5\% \text{ of } 1600}{1600} = \frac{296}{1600} = 0.185$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = 0.1904, \quad q = 0.8096$$

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(iii) Level of Significance (α): $\alpha = 0.05 \quad \alpha/2 = 0.025 \quad 0.5 - 0.025 = 0.475 \quad \boxed{Z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 0.916$

Calculated value of $<$ Tabulated value of

$$Z_{cal} = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.2 - 0.185}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.91692$$

Hence, Null Hypothesis is Accepted

Problem 26: Random samples of 400 men and 600 women asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% level.

Solution: $n_1 = 400$, $n_2 = 600$, $P_1 = \frac{200}{400} = 0.5$, $P_2 = \frac{325}{600} = 0.5416$

$$P = \frac{n_1P_1 + n_2P_2}{n_1+n_2} = \frac{(400)(0.5)+(600)(0.5416)}{400+600} = 0.52496, \quad q = 0.47504$$

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(iii) Level of Significance (α): $\alpha = 0.05$ $\alpha/2 = 0.025$ $0.5-0.025=0.475$ $\boxed{Z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.5-0.5416}{\sqrt{(0.52496)(0.47504)\left(\frac{1}{600}+\frac{1}{400}\right)}} = -1.2905$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 1.2905$

Calculated value of $<$ Tabulated value of

Null Hypothesis is Accepted.

Problem 27: A manufacturer of electronic equipment subjects sample of two completing brands of transistor to an accelerated performance test. If 45 of 180 transistors pf the first kind and 34 of 120 transistor of the second kind fail the test, what can he conclude at the level of significance a = 0.05 about the difference the corresponding sample proportions ?

Solution: $n_1 = 180$ $P_1 = \frac{45}{180} = 0.25$

$n_2 = 120$ $P_2 = \frac{34}{120} = 0.2833$

$$P = \frac{n_1P_1 + n_2P_2}{n_1+n_2} = \frac{180(0.25)+(120)(0.2833)}{120+180} = 0.26332 \quad q = 0.73668$$

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(iii) Level of Significance (α): $\alpha = 0.05$ $\alpha/2 = 0.025$ $0.5-0.025 = 0.475$ $\boxed{Z_{tab} = 1.96}$

(iv) Test Statistic: The test statistic

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 0.6451$ Null Hypothesis is Accepted

Calculated value of $<$ Tabulated value of

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.25-0.2833}{\sqrt{(0.26332)(0.73668)\left(\frac{1}{120}+\frac{1}{180}\right)}} = 0.6451$$

Problem 28: On the basis of their total scores, 200 candidates of a civil service examinations are divided into two groups, the upper 30% and the remaining 70%. Consider the first question of the examination. Among the first group, 40 had the correct answer, where as the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here ?

Solution: $n_1 = 30\%$ of $200 = \frac{30}{100}(200) = 60$ $\quad P_1 = \frac{40}{60} = 0.6666$

$n_2 = 70\%$ of $200 = \frac{70}{100}(200) = 140$ $\quad P_2 = \frac{80}{140} = 0.5714$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{(60)(0.6666) + (140)(0.5714)}{60+140} = 0.5999 \quad q = 0.4001$$

(i) Null Hypothesis (H_0): $P_1 = P_2$

(ii) Alternative Hypothesis (H_1): $P_1 \neq P_2$

(iii) Level of Significance (α): $\alpha = 0.05$ $\quad \alpha/2 = 0.025$ $\quad 0.5 - 0.025 = 0.475$ $\quad \boxed{Z\ tab = 1.96}$

(iv) Test Statistic: The test statistic

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.6666 - 0.5714}{\sqrt{(0.5999)(0.4001)\left(\frac{1}{60} + \frac{1}{140}\right)}} = 1.259$$

(v) Conclusion: Tabulated value of $Z_{tab} = 1.96$

Calculated value of $Z_{cal} = 1.259$

Calculated value of $<$ Tabulated value of

Null Hypothesis is Accepted.