



THE SEARCH FOR THE PERFECT ASSOCIATION RULE !

# NULL INVARIANCE MEASURE

DATA MINING

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The need for Mr. Perfect Association factor

## RECAP

NULL Invariance -  
Value does not change  
with the # of NULL  
transactions

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

$\chi^2$  tells also better than s & c

	B	$\neg B$	$\Sigma_{row}$
C	400 (450)	350 (300)	750
$\neg C$	200 (150)	50 (100)	250
$\Sigma_{col}$	600	400	1000

Expected value

Observed value

Lift is more telling than s & c

	B	$\neg B$	$\Sigma_{row}$
C	400	350	750
$\neg C$	200	50	250
$\Sigma_{col}$	600	400	1000

$$\text{Support (A)} = \frac{\text{Number of transaction in which A appears}}{\text{Total number of transactions}}$$

$$\text{Confidence (A} \rightarrow \text{B)} = \frac{\text{Support(A} \cup \text{B)}}{\text{Support(A)}}$$

Measure	Definition	Range	Null-Invariant
$\chi^2(A, B)$	$\sum_{i,j=0,1} \frac{(e(a_i b_j) - o(a_i b_j))^2}{e(a_i b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$AllConf(A, B)$	$\frac{s(A \cup B)}{\max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2} \left( \frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)} \right)$	$[0, 1]$	Yes
$MaxConf(A, B)$	$\max\left\{ \frac{s(A)}{s(A \cup B)}, \frac{s(B)}{s(A \cup B)} \right\}$	$[0, 1]$	Yes

	<i>milk</i>	$\neg milk$	$\Sigma_{row}$
<i>coffee</i>	<i>mc</i>	$\neg mc$	<i>c</i>
$\neg coffee$	$m\neg c$	$\neg m\neg c$	$\neg c$
$\Sigma_{col}$	<i>m</i>	$\neg m$	$\Sigma$

- Milk vs Coffee Contingency table
- Very low and very high Null values can be fatal and misleading !

Data set	<i>mc</i>	$\neg mc$	$m\neg c$	$\neg m\neg c$	$\chi^2$	<i>Lift</i>
$D_1$	10,000	1,000	1,000	100,000	90557	9.26
$D_2$	10,000	1,000	1,000	100	0	1
$D_3$	100	1,000	1,000	100,000	670	8.44
$D_4$	1,000	1,000	1,000	100,000	24740	25.75
$D_5$	1,000	100	10,000	100,000	8173	9.18
$D_6$	1,000	10	100,000	100,000	965	1.97



## Confidence

MAX conf - Picking the biggest value of confidence.

ALL conf - Minimum value of confidence

C

## Cosine

Lift function in disguise, but is null invariant.

Overcomes the deficiency of Lift function.

C

## Jaccard

Does not consider the NULL Values.

The formula is such that, it only considers the used area and eliminates the unused area.

J

## Kulczynski

The approach of averaging the 2 confidence together

$A \rightarrow B \text{ (\&) } B \rightarrow A$

K



# Confidence & Kulczynski

Variation of native  
Confidence

**Conf (A→B) =**

$S(A \text{ inter } B) / S(A)$

Confidence Measure	Definition
All Confidence	$\frac{s(A \cap B)}{\max\{s(A), s(B)\}}$
Kulczynski	$\frac{P(A B) + P(B A)}{2}$
Max Confidence	$\max\{P(A B), P(B A)\}$

“Biggest value of confidence  
between the 2 ways of looking  
at it”

**MAX Confidence**

“Average of 2 confidences”

**Kulczynski**

“By dividing by the maximum  
value of the support for A int B  
and B int A,

We're really just finding the  
minimum value of confidence  
between the 2 options.”

**ALL Confidence**



**DATA-MINE**  
NULL INVARIANCE

# Cosine

- Considering the grand total is a cause of concern.
- Lift can be simplified :-  
 $(AB)/(A \times B/G)$
- Cosine simplifies to :-  
 $AB/\sqrt{A \times B}$

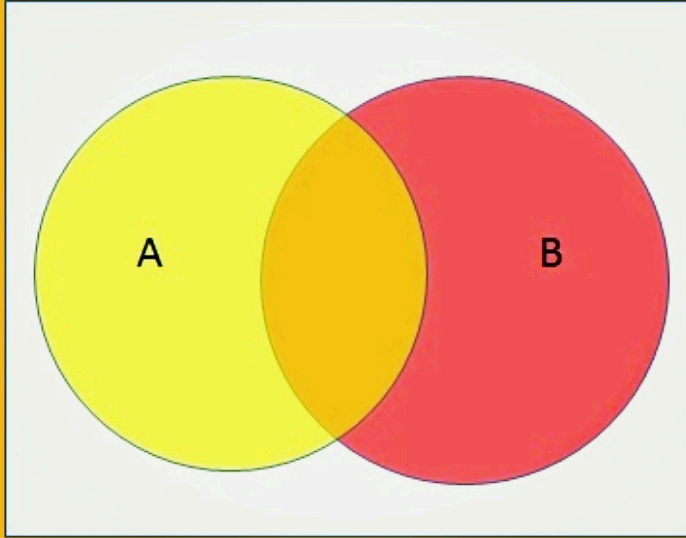
Confidence Measure	Definition
Lift	$\frac{s(A \cap B)}{s(A) \times s(B)}$
Cosine	$\frac{s(A \cap B)}{\sqrt{s(A) \times s(B)}}$

Confidence Measure	Fraction Explanation
Lift	$\frac{\text{Count of records with } A \& B / \text{grand total}}{(\text{count of records with } A / \text{grand total}) \times (\text{count of records with } B / \text{grand total})}$
Cosine	$\frac{\text{Count of records with } A \& B / \text{grand total}}{\sqrt{(\text{count of records with } A / \text{grand total}) \times (\text{count of records with } B / \text{grand total})}}$





# Jaccard



The Jaccard function is defined as :-

$$|A \cap B| / |A \cup B|$$

- The numerator of the Jaccard is the Orange area
- The denominator is the area of the yellow-orange-red area
- It doesn't use any data from the blank space

# Metrics

D4 is neutral and balanced; D5 is neutral but imbalanced; D6 is neutral but very imbalanced.

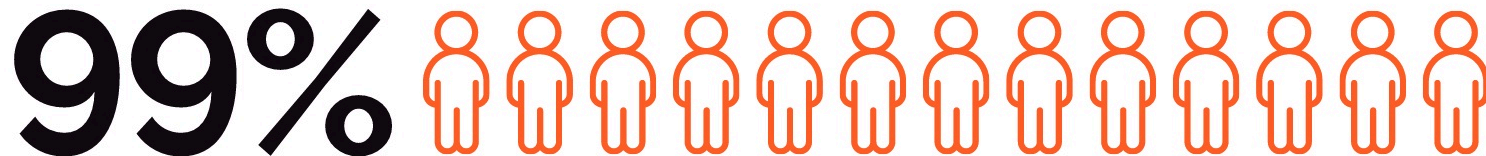
Data set	$mc$	$\neg mc$	$m\neg c$	$\neg m\neg c$	$AllConf$	Jaccard	Cosine	Kulc	MaxConf
$D_1$	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
$D_2$	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
$D_3$	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
$D_4$	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
$D_5$	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
$D_6$	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

# Comparison of measures

Neutral but very imbalanced



Kulc - It is proportional !



Max Conf - Always shows max !



# Case Study

ID	Author <i>A</i>	Author <i>B</i>	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	<i>Cosine</i>	<i>Kulc</i>
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)



Best NULL-invariance measure ?

**You decide !**



**DATA-MINE**  
NULL INVARIANCE

customer showcasing Feedback  
showcasing customer customer showcasing  
feedback feedback feedback Create  
customer feedback Feedback customer  
Create cloud Mining feedback cloud  
Create word cloud word  
Create feedback You Math Create  
feedback vector cloud  
customer cloud cloud  
Customer cloud Create customer  
showcasing Create word cloud  
word customer Data NULL word cloud  
showcasing Factors word cloud  
Customer showcasing showcasing  
word Create word cloud Create  
Customer Feedback customer feedback  
showcasing feedback showcasing

