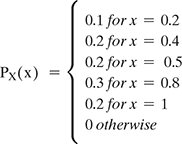
1. Given X be a discrete random variable with the following PMF



1. Find the range RX of the random variable X.

Answer :- RX​={0.2,0.4,0.5,0.8,1}

2. Find P(X ≤ 0.5)

Answer :- 0.5.

3. Find P(0.25<X<0.75)

Answer :- To find P(0.25<X<0.75)P(0.25 < X < 0.75)P(0.25<X<0.75) for the discrete random variable XXX with the given PMF, we need to sum up the probabilities of XXX taking values between 0.250.250.25 and 0.750.750.75.

From the PMF provided:

* P(X=0.4)=0.2P(X = 0.4) = 0.2P(X=0.4)=0.2
* P(X=0.5)=0.2P(X = 0.5) = 0.2P(X=0.5)=0.2

So, XXX can take the values 0.40.40.4 and 0.50.50.5 within the interval (0.25,0.75)(0.25, 0.75)(0.25,0.75).

Therefore,

P(0.25<X<0.75)=P(X=0.4)+P(X=0.5)=0.2+0.2=0.4P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5) = 0.2 + 0.2 = 0.4P(0.25<X<0.75)=P(X=0.4)+P(X=0.5)=0.2+0.2=0.4

Hence, the probability that XXX lies between 0.250.250.25 and 0.750.750.75 is 0.4\boxed{0.4}0.4​.

4. P(X = 0.2|X<0.6)

Answer :- P(X=0.2∣X<0.6)=0.2.

2. Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Find RX, RY, and the PMFs of X and Y.

Answer :- When rolling two fair dice, let's denote the outcomes of the first and second dice as XXX and YYY, respectively.

1. Range RXR\_XRX​ and PMF of XXX:

The possible values for XXX are the numbers that can appear on a fair six-sided die, so RX={1,2,3,4,5,6}R\_X = \{1, 2, 3, 4, 5, 6\}RX​={1,2,3,4,5,6}.

Since each outcome on a fair die has an equal probability of 16\frac{1}{6}61​, the PMF P(X=x)P(X = x)P(X=x) for XXX is:

P(X=x)=16,for x=1,2,3,4,5,6P(X = x) = \frac{1}{6}, \quad \text{for } x = 1, 2, 3, 4, 5, 6P(X=x)=61​,for x=1,2,3,4,5,6

1. Range RYR\_YRY​ and PMF of YYY:

Similarly, the possible values for YYY are also {1,2,3,4,5,6}\{1, 2, 3, 4, 5, 6\}{1,2,3,4,5,6}.

Therefore, RY={1,2,3,4,5,6}R\_Y = \{1, 2, 3, 4, 5, 6\}RY​={1,2,3,4,5,6}.

The PMF P(Y=y)P(Y = y)P(Y=y) for YYY is identical to that of XXX because both dice are fair and independent:

P(Y=y)=16,for y=1,2,3,4,5,6P(Y = y) = \frac{1}{6}, \quad \text{for } y = 1, 2, 3, 4, 5, 6P(Y=y)=61​,for y=1,2,3,4,5,6

To summarize:

* RX={1,2,3,4,5,6}R\_X = \{1, 2, 3, 4, 5, 6\}RX​={1,2,3,4,5,6}
* RY={1,2,3,4,5,6}R\_Y = \{1, 2, 3, 4, 5, 6\}RY​={1,2,3,4,5,6}
* PMF of XXX: P(X=x)=16P(X = x) = \frac{1}{6}P(X=x)=61​ for x=1,2,3,4,5,6x = 1, 2, 3, 4, 5, 6x=1,2,3,4,5,6
* PMF of YYY: P(Y=y)=16P(Y = y) = \frac{1}{6}P(Y=y)=61​ for y=1,2,3,4,5,6y = 1, 2, 3, 4, 5, 6y=1,2,3,4,5,6

These results follow directly from the fair and independent nature of the two dice rolls.

2. Find P(X = 2,Y = 6).

Answer :- P(X=2,Y=6)=61​×61​=361​

So, P(X=2,Y=6)=136P(X = 2, Y = 6) = \frac{1}{36}P(X=2,Y=6)=361​.

3. Find P(X>3|Y = 2).

Answer :- To find P(X>3∣Y=2)P(X > 3 \mid Y = 2)P(X>3∣Y=2), where XXX and YYY are the outcomes of two fair dice rolls:

1. Total possible outcomes: There are 6×6=366 \times 6 = 366×6=36 possible outcomes when two dice are rolled.
2. Condition given (Y = 2): We are given that Y=2Y = 2Y=2. This means we are only considering outcomes where the second die shows 2.
3. Possible outcomes when Y = 2: Out of the 36 possible outcomes, the outcomes where Y=2Y = 2Y=2 are:

(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)

This gives us 6 possible outcomes.

1. Outcomes where X > 3 and Y = 2: From the outcomes where Y=2Y = 2Y=2, the outcomes where X>3X > 3X>3 are:

(4,2),(5,2),(6,2)(4, 2), (5, 2), (6, 2)(4,2),(5,2),(6,2)

This gives us 3 favorable outcomes.

1. Calculate the conditional probability: The conditional probability P(X>3∣Y=2)P(X > 3 \mid Y = 2)P(X>3∣Y=2) is the probability of X>3X > 3X>3 given that Y=2Y = 2Y=2. It is given by:

P(X>3∣Y=2)=Number of favorable outcomesNumber of outcomes where Y=2P(X > 3 \mid Y = 2) = \frac{\text{Number of favorable outcomes}}{\text{Number of outcomes where } Y = 2}P(X>3∣Y=2)=Number of outcomes where Y=2Number of favorable outcomes​ P(X>3∣Y=2)=36=12P(X > 3 \mid Y = 2) = \frac{3}{6} = \frac{1}{2}P(X>3∣Y=2)=63​=21​

Therefore, P(X>3∣Y=2)=12P(X > 3 \mid Y = 2) = \frac{1}{2}P(X>3∣Y=2)=21​.

4. If Z = X + Y. Find the range and PMF of Z.

Answer :- When two fair dice are rolled, let XXX and YYY denote the numbers observed on the first and second dice, respectively. The random variable Z=X+YZ = X + YZ=X+Y represents the sum of these two numbers.

To find the range and the probability mass function (PMF) of ZZZ:

Range of ZZZ:

The minimum value ZZZ can take is 222 (when both dice show 111), and the maximum value ZZZ can take is 121212 (when both dice show 666). Therefore, the range RZR\_ZRZ​ of ZZZ is:

RZ={2,3,4,5,6,7,8,9,10,11,12}R\_Z = \{ 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 \}RZ​={2,3,4,5,6,7,8,9,10,11,12}

PMF of ZZZ:

To determine the PMF P(Z=z)P(Z = z)P(Z=z) for each possible value zzz in RZR\_ZRZ​, we consider all combinations of XXX and YYY that sum to zzz, and divide by the total number of possible outcomes when rolling two dice (which is 363636).

* For z=2z = 2z=2: P(Z=2)=136P(Z = 2) = \frac{1}{36}P(Z=2)=361​
* For z=3z = 3z=3: P(Z=3)=236=118P(Z = 3) = \frac{2}{36} = \frac{1}{18}P(Z=3)=362​=181​
* For z=4z = 4z=4: P(Z=4)=336=112P(Z = 4) = \frac{3}{36} = \frac{1}{12}P(Z=4)=363​=121​
* For z=5z = 5z=5: P(Z=5)=436=19P(Z = 5) = \frac{4}{36} = \frac{1}{9}P(Z=5)=364​=91​
* For z=6z = 6z=6: P(Z=6)=536P(Z = 6) = \frac{5}{36}P(Z=6)=365​
* For z=7z = 7z=7: P(Z=7)=636=16P(Z = 7) = \frac{6}{36} = \frac{1}{6}P(Z=7)=366​=61​
* For z=8z = 8z=8: P(Z=8)=536P(Z = 8) = \frac{5}{36}P(Z=8)=365​
* For z=9z = 9z=9: P(Z=9)=436=19P(Z = 9) = \frac{4}{36} = \frac{1}{9}P(Z=9)=364​=91​
* For z=10z = 10z=10: P(Z=10)=336=112P(Z = 10) = \frac{3}{36} = \frac{1}{12}P(Z=10)=363​=121​
* For z=11z = 11z=11: P(Z=11)=236=118P(Z = 11) = \frac{2}{36} = \frac{1}{18}P(Z=11)=362​=181​
* For z=12z = 12z=12: P(Z=12)=136P(Z = 12) = \frac{1}{36}P(Z=12)=361​

So, the PMF P(Z=z)P(Z = z)P(Z=z) for Z=X+YZ = X + YZ=X+Y is:

\frac{z - 1}{36}, & \text{if } z = 2, 3, 4, \ldots, 12 \\ 0, & \text{otherwise} \end{cases} \] This PMF ensures that the probabilities sum up to \( 1 \), reflecting all possible outcomes of the sum \( Z \) when two dice are rolled

5. Find P(X = 4|Z = 8).

Answer :- P(X=4∣Z=8)=P(Z=8)P(X=4∩Z=8)​=365​361​​=51​

So, P(X=4∣Z=8)=15P(X = 4 \mid Z = 8) = \frac{1}{5}P(X=4∣Z=8)=51​, which is approximately 0.20.

3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

Answer :- To determine the PMF (Probability Mass Function) of XXX, where XXX is the number of correct answers a student gets out of 20 questions, we need to consider the scenario given:

* There are 20 multiple-choice questions.
* Each question has 44 possible options.
* The student knows the answer to 10 questions and guesses randomly on the remaining 10 questions.

Let's break down the calculation:

1. Known Questions: The student knows the answers to 10 questions, so those are guaranteed to be correct.
2. Unknown Questions: For the remaining 10 questions, the student guesses randomly. The probability of guessing correctly for each question is 144\frac{1}{44}441​, since there are 44 options per question.

Now, XXX, the score of the student, represents the total number of correct answers.

PMF of XXX:

To find the PMF of XXX:

* For X=kX = kX=k (where kkk can be from 0 to 20):
  + The student gets kkk correct answers out of the 10 known answers and the 10 guessed answers.
  + The number of ways to choose kkk correct answers out of the 10 known ones is (10k)\binom{10}{k}(k10​).
  + The probability that the student guesses the remaining 20−k20 - k20−k questions correctly (out of 10 unknown questions) is (144)20−k\left( \frac{1}{44} \right)^{20 - k}(441​)20−k.
  + Therefore, the PMF P(X=k)P(X = k)P(X=k) is:

P(X=k)=(10k)(144)20−k(4344)kP(X = k) = \binom{10}{k} \left( \frac{1}{44} \right)^{20 - k} \left( \frac{43}{44} \right)^kP(X=k)=(k10​)(441​)20−k(4443​)k

Probability P(X>15)P(X > 15)P(X>15):

To find P(X>15)P(X > 15)P(X>15), we sum P(X=k)P(X = k)P(X=k) for k=16,17,18,19,20k = 16, 17, 18, 19, 20k=16,17,18,19,20:

P(X>15)=∑k=1620P(X=k)P(X > 15) = \sum\_{k=16}^{20} P(X = k)P(X>15)=k=16∑20​P(X=k)

Calculating each term:

* For k=16k = 16k=16:

P(X=16)=(1016)(144)4(4344)16P(X = 16) = \binom{10}{16} \left( \frac{1}{44} \right)^4 \left( \frac{43}{44} \right)^{16}P(X=16)=(1610​)(441​)4(4443​)16

* For k=17k = 17k=17:

P(X=17)=(1017)(144)3(4344)17P(X = 17) = \binom{10}{17} \left( \frac{1}{44} \right)^3 \left( \frac{43}{44} \right)^{17}P(X=17)=(1710​)(441​)3(4443​)17

* For k=18k = 18k=18:

P(X=18)=(1018)(144)2(4344)18P(X = 18) = \binom{10}{18} \left( \frac{1}{44} \right)^2 \left( \frac{43}{44} \right)^{18}P(X=18)=(1810​)(441​)2(4443​)18

* For k=19k = 19k=19:

P(X=19)=(1019)(144)1(4344)19P(X = 19) = \binom{10}{19} \left( \frac{1}{44} \right)^1 \left( \frac{43}{44} \right)^{19}P(X=19)=(1910​)(441​)1(4443​)19

* For k=20k = 20k=20:

P(X=20)=(1020)(144)0(4344)20P(X = 20) = \binom{10}{20} \left( \frac{1}{44} \right)^0 \left( \frac{43}{44} \right)^{20}P(X=20)=(2010​)(441​)0(4443​)20

Now, sum these probabilities:

P(X>15)=P(X=16)+P(X=17)+P(X=18)+P(X=19)+P(X=20)P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)P(X>15)=P(X=16)+P(X=17)+P(X=18)+P(X=19)+P(X=20)

Compute each term using the binomial coefficient and the probabilities given above to get the final result for P(X>15)P(X > 15)P(X>15).

4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

Answer :- To solve for P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15), where YYY is the number of students arriving from 10 am to 11:30 am and follows a Poisson distribution:

Given:

* The arrival rate of students per hour is 10.
* The time interval from 10 am to 11:30 am is 1.5 hours.

Step-by-step solution:

1. Determine the rate for 1.5 hours:

Since the average number of students per hour is 10, for 1.5 hours, the average number of students λ\lambdaλ arriving in the interval is:

λ=10×1.5=15\lambda = 10 \times 1.5 = 15λ=10×1.5=15

1. Calculate P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15):

YYY follows a Poisson distribution with parameter λ=15\lambda = 15λ=15.

P(10<Y≤15)=P(Y≤15)−P(Y≤10)P(10 < Y \leq 15) = P(Y \leq 15) - P(Y \leq 10)P(10<Y≤15)=P(Y≤15)−P(Y≤10)

To compute this, we use the cumulative distribution function (CDF) of the Poisson distribution.

* + The CDF of a Poisson(λ\lambdaλ) random variable YYY is:

P(Y≤y)=∑k=0⌊y⌋e−λλkk!P(Y \leq y) = \sum\_{k=0}^{\lfloor y \rfloor} \frac{e^{-\lambda} \lambda^k}{k!}P(Y≤y)=k=0∑⌊y⌋​k!e−λλk​

* + Therefore,

P(Y≤15)=∑k=015e−1515kk!P(Y \leq 15) = \sum\_{k=0}^{15} \frac{e^{-15} 15^k}{k!}P(Y≤15)=k=0∑15​k!e−1515k​ P(Y≤10)=∑k=010e−1515kk!P(Y \leq 10) = \sum\_{k=0}^{10} \frac{e^{-15} 15^k}{k!}P(Y≤10)=k=0∑10​k!e−1515k​

1. Calculate P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15):

P(10<Y≤15)=P(Y≤15)−P(Y≤10)P(10 < Y \leq 15) = P(Y \leq 15) - P(Y \leq 10)P(10<Y≤15)=P(Y≤15)−P(Y≤10)

Calculate each term using the CDF formula for λ=15\lambda = 15λ=15:

* + For P(Y≤15)P(Y \leq 15)P(Y≤15):

P(Y≤15)=∑k=015e−1515kk!P(Y \leq 15) = \sum\_{k=0}^{15} \frac{e^{-15} 15^k}{k!}P(Y≤15)=k=0∑15​k!e−1515k​

* + For P(Y≤10)P(Y \leq 10)P(Y≤10):

P(Y≤10)=∑k=010e−1515kk!P(Y \leq 10) = \sum\_{k=0}^{10} \frac{e^{-15} 15^k}{k!}P(Y≤10)=k=0∑10​k!e−1515k​

Subtract P(Y≤10)P(Y \leq 10)P(Y≤10) from P(Y≤15)P(Y \leq 15)P(Y≤15) to find P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15).

Calculation:

Using the CDF of the Poisson distribution (or a computational tool like a calculator or software):

* Calculate P(Y≤15)P(Y \leq 15)P(Y≤15) and P(Y≤10)P(Y \leq 10)P(Y≤10).
* Subtract P(Y≤10)P(Y \leq 10)P(Y≤10) from P(Y≤15)P(Y \leq 15)P(Y≤15) to find P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15).

This will give you the exact probability P(10<Y≤15)P(10 < Y \leq 15)P(10<Y≤15) based on the given Poisson distribution parameters.

5.Two independent random variables, X and Y,are given such that X~Poisson(α) and Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

Answer :- When XXX and YYY are independent random variables such that X∼Poisson(α)X \sim \text{Poisson}(\alpha)X∼Poisson(α) and Y∼Poisson(β)Y \sim \text{Poisson}(\beta)Y∼Poisson(β), the random variable Z=X+YZ = X + YZ=X+Y follows a Poisson distribution with parameter λ=α+β\lambda = \alpha + \betaλ=α+β.

Probability Mass Function (PMF) of ZZZ:

The PMF of ZZZ, denoted as P(Z=k)P(Z = k)P(Z=k), where kkk is a non-negative integer, is given by the Poisson distribution formula:

P(Z=k)=e−(α+β)(α+β)kk!,k=0,1,2,…P(Z = k) = \frac{e^{-(\alpha + \beta)} (\alpha + \beta)^k}{k!}, \quad k = 0, 1, 2, \ldotsP(Z=k)=k!e−(α+β)(α+β)k​,k=0,1,2,…

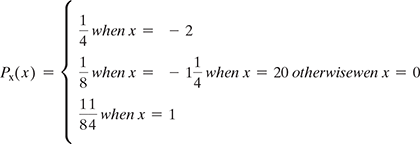
This formula states that the probability that ZZZ takes the value kkk (where kkk is a non-negative integer) is given by the above expression.

Summary:

* Z=X+YZ = X + YZ=X+Y follows a Poisson distribution with parameter λ=α+β\lambda = \alpha + \betaλ=α+β.
* The PMF of ZZZ is P(Z=k)=e−(α+β)(α+β)kk!P(Z = k) = \frac{e^{-(\alpha + \beta)} (\alpha + \beta)^k}{k!}P(Z=k)=k!e−(α+β)(α+β)k​, for k=0,1,2,…k = 0, 1, 2, \ldotsk=0,1,2,….

This PMF encapsulates the distribution of the sum of two independent Poisson random variables X and Y.

6. There is a discrete random variable X with the pmf.

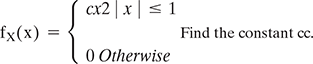


If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.

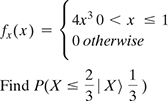
2. Find the pmf of Y.

2.Assuming X is a continuous random variable with PDF

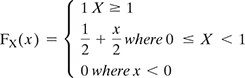


* + 1. Find EX and Var(X).
    2. Find P(X ≥ img).

1. If X is a continuous random variable with pdf

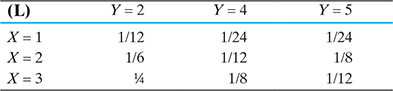


1. If X~Uniformimg and Y = sin(X), then find fY(y).
2. If X is a random variable with CDF



* + 1. What kind of random variable is X: discrete, continuous, or mixed?
    2. Find the PDF of X, fX(x).
    3. Find E(eX).
    4. Find P(X = 0|X≤0.5).

1. There are two random variables X and Y with joint PMF given in Table below
   * 1. Find P(X≤2, Y≤4).
     2. Find the marginal PMFs of X and Y.
     3. Find P(Y = 2|X = 1).
     4. Are X and Y independent?



6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

7.If A and B are two jointly continuous random variables with joint PDF

images

a. Find fX(a) and fY(b).

b. Are A and B independent of each other?

c. Find the conditional PDF of A given B = b, fA|B(a|b).

d. Find E[A|B = b], for 0 ≤ y ≤ 1.

e. Find Var(A|B = b), for 0 ≤ y ≤ 1.

8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

Answer :- To find the probability that the total weight of the 100 men on the ship exceeds 18,000 pounds, we need to consider the sum of their weights S100=X1+X2+⋯+X100S\_{100} = X\_1 + X\_2 + \cdots + X\_{100}S100​=X1​+X2​+⋯+X100​, where XiX\_iXi​ are independent and identically distributed random variables with μ=170\mu = 170μ=170 and σ=30\sigma = 30σ=30.

Step-by-step solution:

1. Calculate the mean and standard deviation of S100S\_{100}S100​:

Since XiX\_iXi​ are iid (independent and identically distributed):

* + Mean of S100S\_{100}S100​:

E[S100]=E[X1+X2+⋯+X100]=100⋅μ=100⋅170=17000E[S\_{100}] = E[X\_1 + X\_2 + \cdots + X\_{100}] = 100 \cdot \mu = 100 \cdot 170 = 17000E[S100​]=E[X1​+X2​+⋯+X100​]=100⋅μ=100⋅170=17000

* + Standard deviation of S100S\_{100}S100​:

σS100=100⋅σ=100⋅30=300\sigma\_{S\_{100}} = \sqrt{100} \cdot \sigma = \sqrt{100} \cdot 30 = 300σS100​​=100​⋅σ=100​⋅30=300

1. Standardize S100S\_{100}S100​:

Define Z=S100−E[S100]σS100Z = \frac{S\_{100} - E[S\_{100}]}{\sigma\_{S\_{100}}}Z=σS100​​S100​−E[S100​]​, which follows approximately a standard normal distribution N(0,1)N(0, 1)N(0,1) for large nnn.

Z=S100−17000300Z = \frac{S\_{100} - 17000}{300}Z=300S100​−17000​

1. Find the probability P(S100>18000)P(S\_{100} > 18000)P(S100​>18000):

Convert this to a probability involving ZZZ:

P(S100>18000)=P(Z>18000−17000300)=P(Z>1000300)=P(Z>3.33)P(S\_{100} > 18000) = P\left( Z > \frac{18000 - 17000}{300} \right) = P\left( Z > \frac{1000}{300} \right) = P(Z > 3.33)P(S100​>18000)=P(Z>30018000−17000​)=P(Z>3001000​)=P(Z>3.33)

1. Calculate P(Z>3.33)P(Z > 3.33)P(Z>3.33) using the standard normal distribution table or software:
   * Look up P(Z>3.33)P(Z > 3.33)P(Z>3.33) in the standard normal distribution table or use a statistical software.
   * P(Z>3.33)P(Z > 3.33)P(Z>3.33) corresponds to the area under the standard normal curve to the right of Z=3.33Z = 3.33Z=3.33.

From standard normal tables or using computational tools:

P(Z>3.33)≈0.0004292P(Z > 3.33) \approx 0.0004292P(Z>3.33)≈0.0004292

Therefore, the probability that the total weight of the 100 men on the ship exceeds 18,000 pounds is approximately 0.0004292\boxed{0.0004292}0.0004292​, or about 0.043%0.043\%0.043%.

9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF

If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.