Motor Selection Criteria:

In order to select motor first, we need to know what our system specifications are and how much load is distributed on one motor. After that we need to know how much force is being transmitted to motor and then calculating load torque of motor we can find the desired motor which meets our criteria.

System Specifications:

- Mass of robot = 6.3kg
- Wheel radius = 6.75×10^{-3} m
- Wheel mass = 0.27 kg

Load Distribution on wheels:

Consider a 4 wheeled robot is at equilibrium then at its center of gravitational pull of the earth is given by:

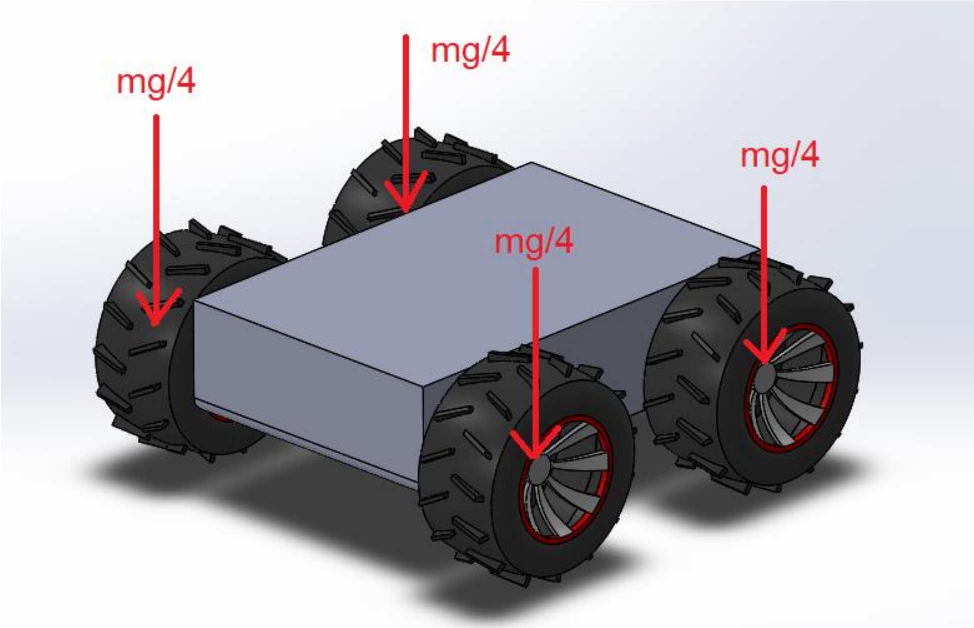
$$\boldsymbol{w = mg}$$

Where 'w' is the weight, 'm' is the mass and 'g' is the gravitational pull of the earth. Weight of the robot is given by:

$$w = 6.3\ x\ 9.8$$

$$w = 61.74\ N$$

If the robot is considered geometrically perfect (which is physically not possible) then its center of gravity would lie at the center of the whole robot. This assumption is made to ease in calculating the required load torque for the motor. If Center of gravity lies in the center of the robot, then upon each wheel ¼ of the whole weight acts on each wheel vertically downward.



This model is just used for conjecturing the concept for reader*

Considering the load is distributed evenly among 4 wheels:

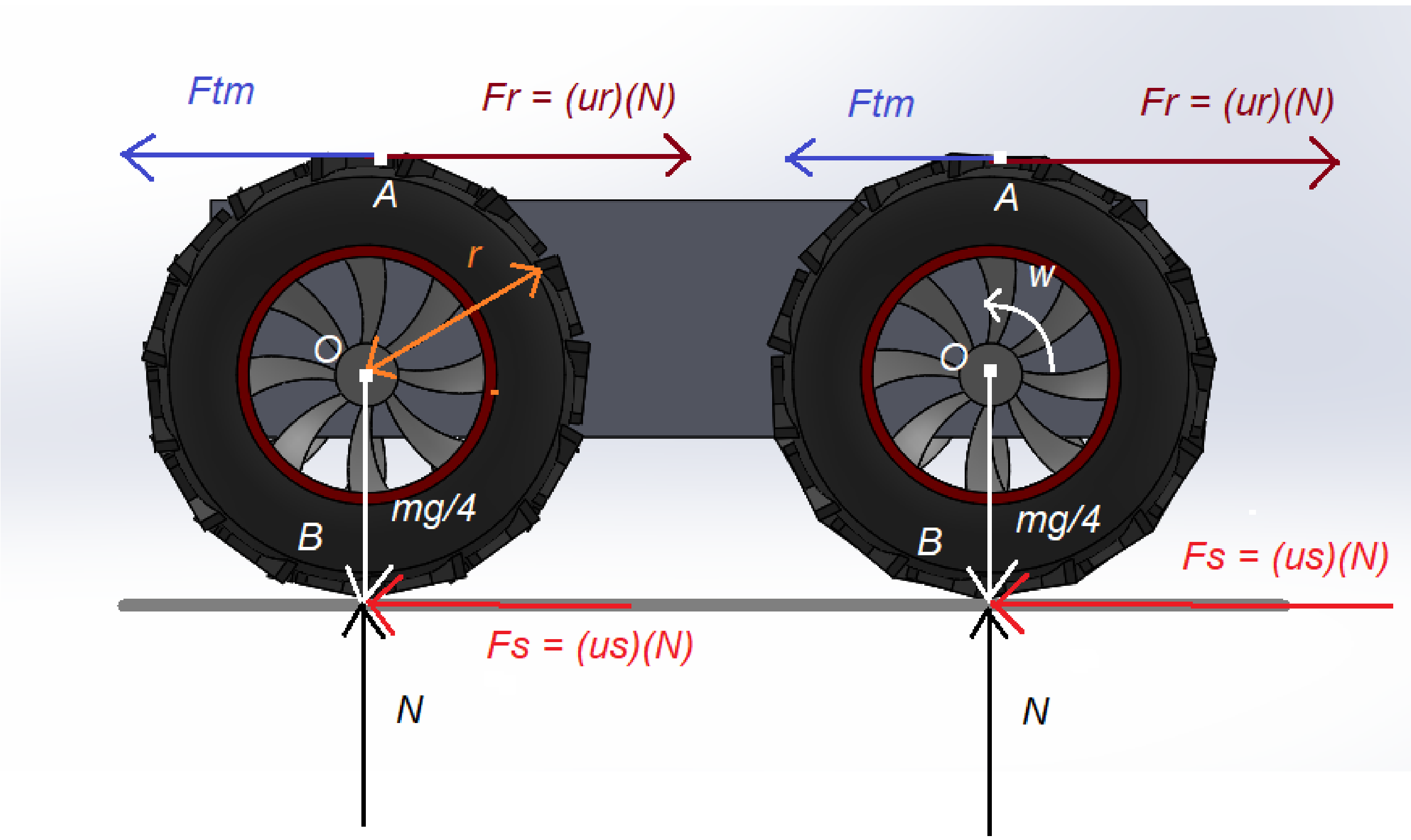
$$\boldsymbol{F_t} = \boldsymbol{mg}/\boldsymbol{4} = 61.74/4$$

$$F_t = 15.435\ N$$

This is the force acting downward on each wheel.

Robot Kinematics:

Consider a 4-wheeled robot (side-view). Actually two wheels will be rotated by one motor. At equilibrium position following forces would act on the robot:



*This model is used for delivering the concept to reader**

Here:

N = Normal force acting on point B of wheel

F_s = Static force of friction between wheel and floor

F_r = Rolling force at point A of wheel

F_{tm} = Force transmitted to motor

r = radius of wheel

Normal Force:

Normal force acting is the force that acts on the body in upward direction at point of contact. This is in accordance with the Newton's third law:

"*To every force acting on the body there is always a reaction force that is equal in magnitude but opposite in direction*"

Normal force acting on the wheel is given by:

$$\boldsymbol{N} = \boldsymbol{mg}/\boldsymbol{4} = 61.74/4 = 15.435N$$

$$\boldsymbol{N} = \boldsymbol{F_t}$$

Static Force:

Static force of friction is the force that exists when two bodies of different materials are in contact with eachother and oppose motion relative to one another. This force is given by:

$$\boldsymbol{F_s} = \boldsymbol{\mu_s N}$$

Here

$\boldsymbol{\mu_s}$ **=** Static frictional coefficient (For rubber and floor it is 0.85)

N = Normal force

$$F_s = \mu_s N = 0.85 \ x \ 15.435$$

$$F_s = 13.1197N$$

Rolling Force:

Rolling force is the resisting frictional force that acts on the body when the body rotates about its axis. This force is given by:

$$\boldsymbol{F_r} = \boldsymbol{\mu_r N}$$

Here

$\boldsymbol{\mu_r}$ **=** Rolling coefficient (Usually it is 0.01)

N = Normal force

$$F_r = \mu_r N = 0.01 \ x \ 15.435$$

$$F_r = 0.154N$$

Equilibrium Equation:

Since body is in equilibrium hence we can apply equilibrium equations in x and y directions as:

$$\boldsymbol{\Sigma F_x = 0}$$

$$\boldsymbol{\Sigma F_y = 0}$$

In y-direction we can apply:

$$\boldsymbol{\Sigma F_y = 0}$$

$$\boldsymbol{N = {}^{mg}/_{4}}$$

$$N = 15.435N$$

In x-direction we can apply:

$$\boldsymbol{\Sigma F_x = 0}$$

$$\boldsymbol{2F_{tm} + 2F_s = 2F_r}$$

$$\boldsymbol{2F_{tm} = 2F_r - 2F_s}$$

$$2F_{tm} = 2(0.154) - 2(13.1197)$$

$$F_{tm} = {}^{-25.931}/_{2}$$

$$F_{tm} = -12.95N$$

Since this force is acting in negative x direction hence there is negative sign. For only magnitude:

$$|F_{tm}| = |-12.95N| = 12.95N$$

Force Transmission and losses:

This force is transmitted via chain drive to the motor and bearings. One wheel is attached with one gear that has 25 teeth which is connected to the smaller one that has 17 teeth. The 17 teeth gear is connected via shaft and bearings to one inside gear that has 32 teeth. If the gear has teeth greater than or equal to 17 then there are mechanical vibrations induced in the drive which take part in 25% of loss in force transmission.

Since;

n_1 = Number of teeth on gear attached with wheel = $25 \geq 17$

n_2 = Number of teeth on gear attached with n_1 = $17 = 17$

n_3 = Number of teeth on gear attached with n_2 = $32 \geq 17$

Total loss in force transmission is:

$$25\% + 25\% + 25\% = 75\%$$

Hence net transmitted force is given by:

$$\boldsymbol{F'_{tm} = F_{tm} - (75\% \; of \; F_{tm})}$$

$$F'_{tm} = F_{tm} - (F_{tm} \; x \; {}^{75}/_{100})$$

$$F'_{tm} = 12.95 - (12.95 \; x \; {}^{75}/_{100})$$

$$F'_{tm} = 3.23N$$

Load Torque Required:

The load torque that is required is given by:

$$\boldsymbol{\tau = r \; x \; F}$$

$$\boldsymbol{\tau = rFsin\theta}$$

Here:

r = moment arm or distance between motor's shaft and the force transmitted

= Radius of wheel – Distance between motor's shaft and center of wheel

$= 6.75x10^{-2} - 4x10^{-2} = 2.75 \; x \; 10^{-2}m$

θ = angle between F and r which in this case is 90 degrees hence $sin\theta = 1$

$$\boldsymbol{\tau = rF} = (2.75 \; x \; 10^{-2})(3.23)$$

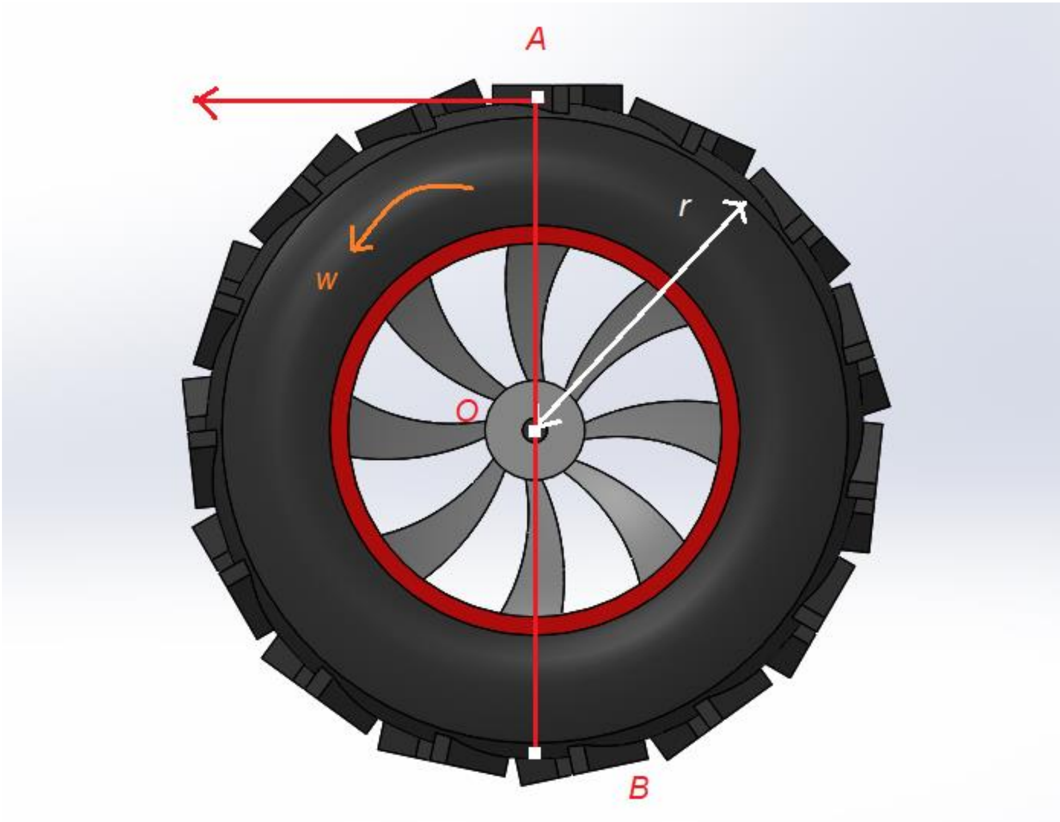
$$\tau = 0.089 \; Nm = 89 \; m - Nm$$

Conclusion:

Hence the torque required for a single motor to drive two wheels is 89 m-Nm.

Instantaneous Center of Zero Velocity Analysis:

Consider a wheel of radius ‘r’ is rotating with angular velocity ‘ω’ in an empty space. Point O is at the center of wheel whereas points A and B are at distance r from point O on wheel in opposite direction.



Tangential Velocities at points A and B:

The tangential velocity at point A is in negative x axis and tangential velocity at point B is in positive x direction. The relation between linear and angular velocity is given by:

$$\boldsymbol{v = \omega\ x\ r}$$

Velocity at point A is hence given by:

$$\boldsymbol{v_A = \omega\ x\ r_{A/O}}$$

$$\boldsymbol{v_A = [0, 0, \omega] x\ [0, r, 0]}$$

$$\boldsymbol{v_A = -\omega r\ \vec{\imath}}$$

Where $\boldsymbol{r_{A/O}}$ is distance of point A relative to O (center).

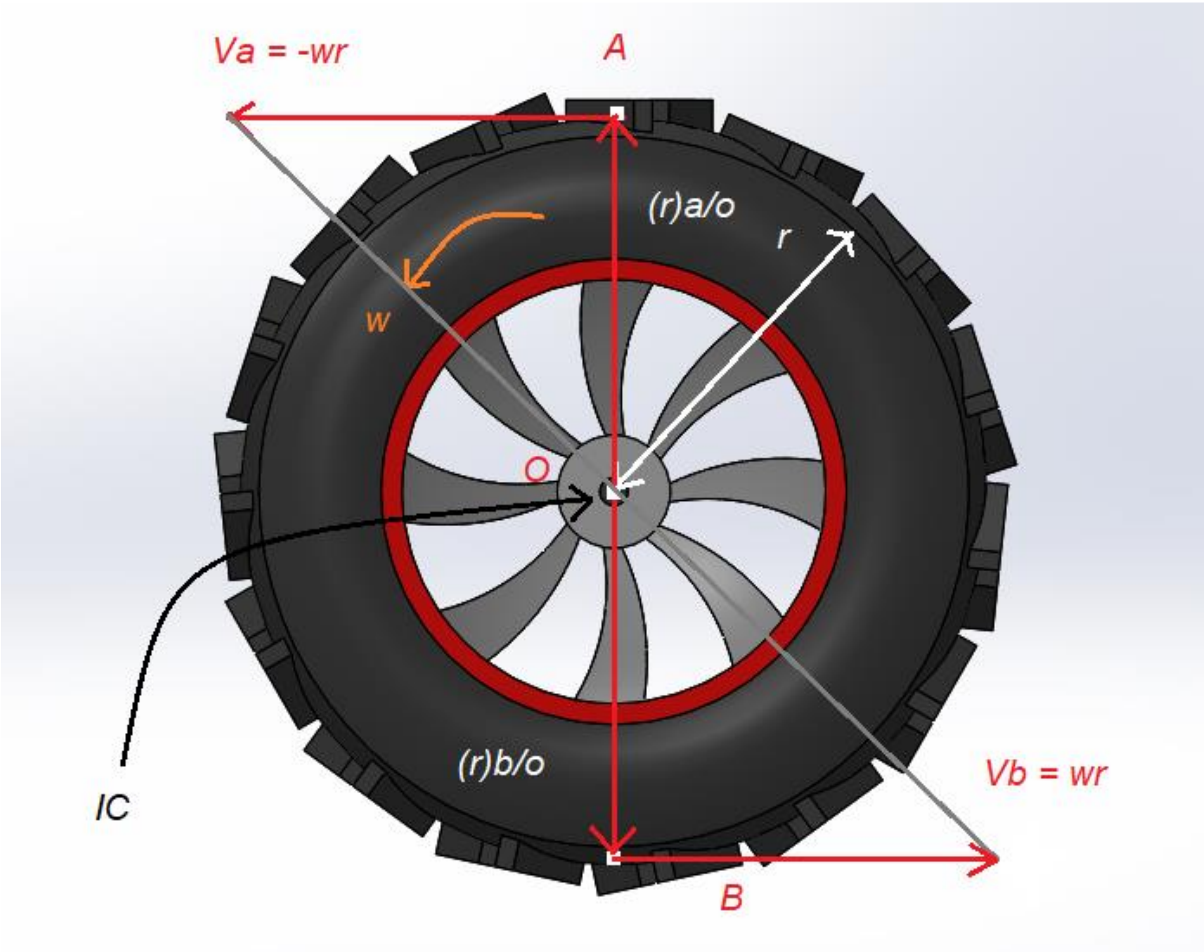
Velocity at point B is hence given by:

$$v_B = \omega \; x \; r_{B/O}$$

$$v_B = [0, 0, \omega] x \, [0, -r, 0]$$

$$v_B = \omega r \, \vec{\imath}$$

Where $r_{B/O}$ is distance of point B relative to O (center).



In this case the point of **Instantaneous center of Zero Velocity** is the center point O of wheel since this point has zero velocity at this instant.

Instantaneous Center of Zero Velocity for the wheel:

Now consider the wheel is on the ground and that point B touches the ground. Then apparently point O that is center of the wheel is moving to the left with:

$$\boldsymbol{v_O = \omega \; x \; r}$$

$$\boldsymbol{v_O = -\omega \; r \; \vec{\imath}}$$

Tangential Velocities:

Tangential velocity at point A in this case is given by:

$$\boldsymbol{v_A = \; v_O + v_{A/O}}$$

Where, $\boldsymbol{v_{A/O}}$ is the velocity of point A with respect to O and is given by:

$$\boldsymbol{v_{A/O} = \omega \; x \; r_{A/O} = \; -\omega r \; \vec{\imath}}$$

Hence $\boldsymbol{v_A}$ is given by:

$$\boldsymbol{v_A = -\omega r \; \vec{\imath} - \omega r \; \vec{\imath}}$$

$$\boldsymbol{v_A = \; -2\omega r \; \vec{\imath}}$$

$$\boldsymbol{|v_A| = |-2\omega r \; \vec{\imath} \,| = 2\omega r}$$

Tangential velocity at point B in this case is given by:

$$\boldsymbol{v_B = \; v_O + v_{B/O}}$$

Where, $\boldsymbol{v_{A/O}}$ is the velocity of point A with respect to O and is given by:

$$\boldsymbol{v_{B/O} = \omega \; x \; r_{B/O} = \; \omega r \; \vec{\imath}}$$

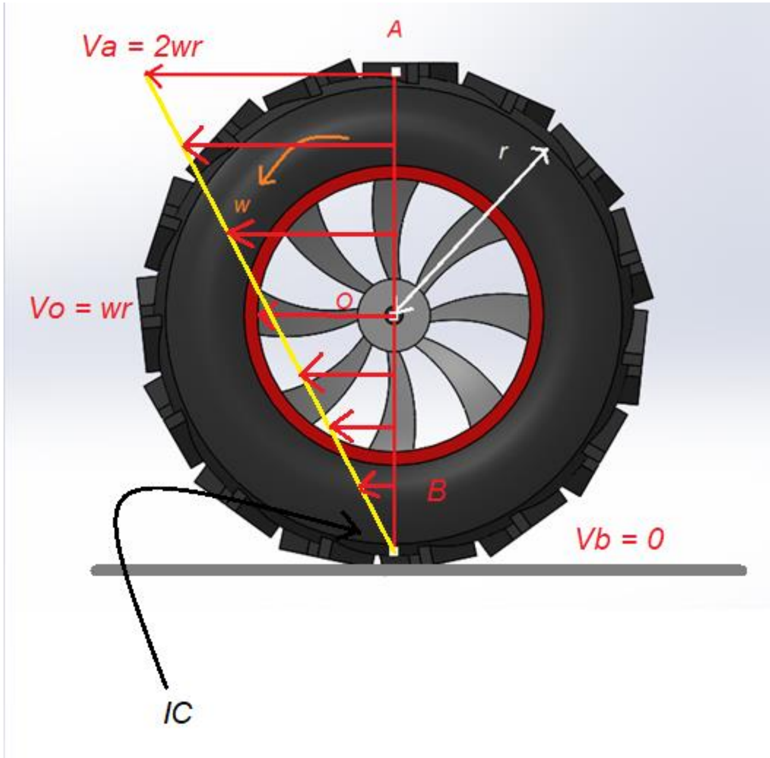
Hence $\boldsymbol{v_A}$ is given by:

$$\boldsymbol{v_B = -\omega r \; \vec{\imath} + \omega r \; \vec{\imath}}$$

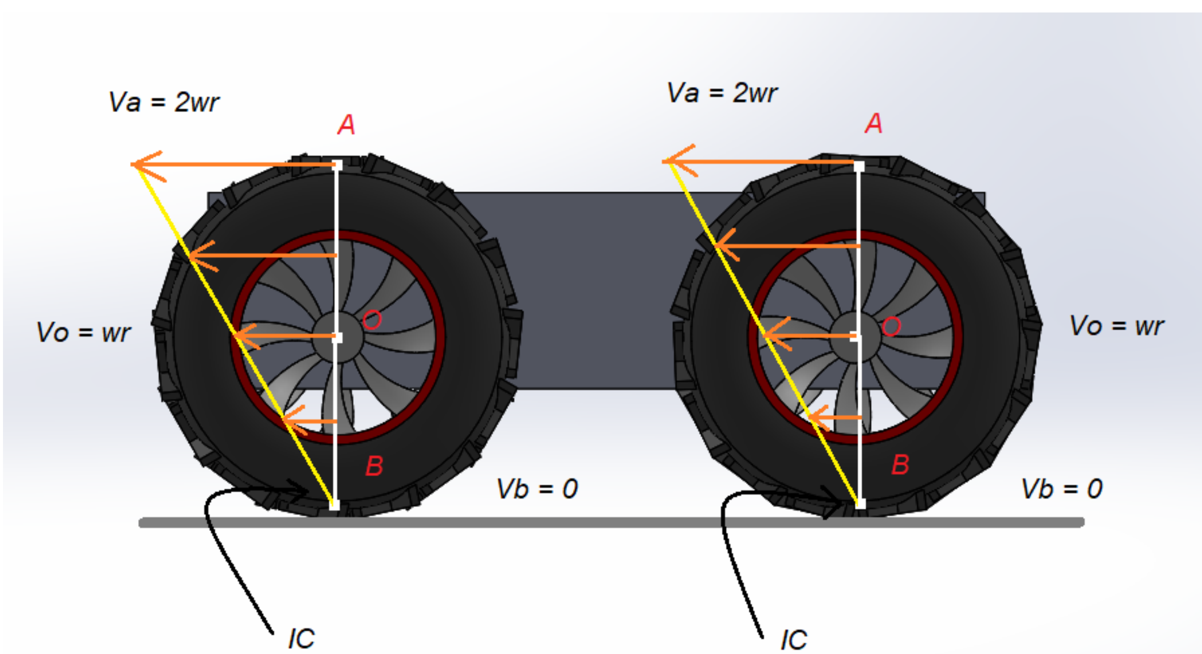
$$\boldsymbol{v_B = \; 0}$$

According to these calculations it is pretty obvious that velocity at point B is zero at current instant. On the other hand, velocity of point, A is 2 times the velocity of center.

The velocity profile of the wheel is given by:



- Point B is called the instantaneous Center of Zero Velocity.
- For instant velocity at point B is zero and we can find velocity at any point on wheel using B as instantaneous center.
- We treat B like whole system is moving around it.



Relative Acceleration Analysis:

In order to find force acting on point A we need to find the acceleration at point A. Now acceleration acting on point A has two components:

$$\boldsymbol{a_A} = \ (\boldsymbol{a_A})_t + (\boldsymbol{a_A})_n$$

Where;

$$(\boldsymbol{a_A})_t = \ \boldsymbol{\alpha} \ \boldsymbol{x} \ \boldsymbol{r} = \ -\boldsymbol{\alpha r} \ \vec{\boldsymbol{\imath}}$$

$$(\boldsymbol{a_A})_n = \ -\boldsymbol{\omega^2 r}$$

The acceleration of point O will be:

$$\boldsymbol{a_O} = \ \boldsymbol{\alpha} \ \boldsymbol{x} \ \boldsymbol{r}$$

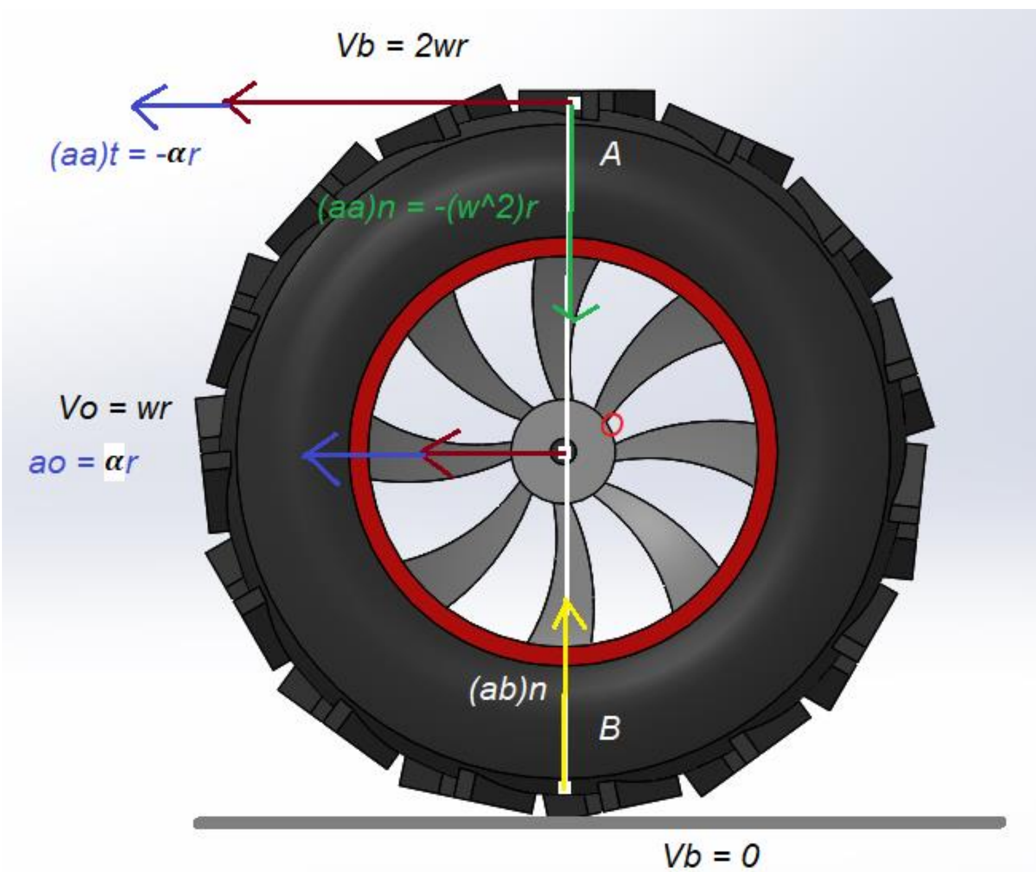
$$\boldsymbol{a_O} = \ -\boldsymbol{\alpha r} \ \vec{\boldsymbol{\imath}}$$

The acceleration of point B will be:

$$(\boldsymbol{a_B})_t = \ \boldsymbol{0}$$

$$(\boldsymbol{a_B})_n = \ \boldsymbol{\omega^2 r}$$

Now that all components of acceleration at points of interest have been computed so lets consider a wheel:



Acceleration of point A with respect to instantaneous center B is given as:

$$\boldsymbol{a_A} = (\boldsymbol{a_B})_{\boldsymbol{n}} + (\boldsymbol{\alpha}\ \boldsymbol{x}\ \boldsymbol{r_{AB}}) + (-\boldsymbol{\omega^2 r_{AB}})$$

$$\boldsymbol{a_A} = \boldsymbol{\omega^2 r_{AB}} + (\boldsymbol{\alpha}\ \boldsymbol{x}\ \boldsymbol{r_{AB}}) + (-\boldsymbol{\omega^2 r_{AB}})$$

$$\boldsymbol{a_A} = \boldsymbol{\alpha}\ \boldsymbol{x}\ \boldsymbol{r_{AB}}$$

$$\boldsymbol{a_A} = \boldsymbol{2\alpha r}$$

Now we have found acceleration at point A by finding angular acceleration of wheel we can estimate the value of $\boldsymbol{a_A}$**.** The angular acceleration is given by:

$$\boldsymbol{\alpha} = {}^{\boldsymbol{\tau}}/_{\boldsymbol{I}}$$

Where $\boldsymbol{\tau} = \boldsymbol{FR}$ (F is the net force acting on the wheel due from wheel kinematics and R is the radius)

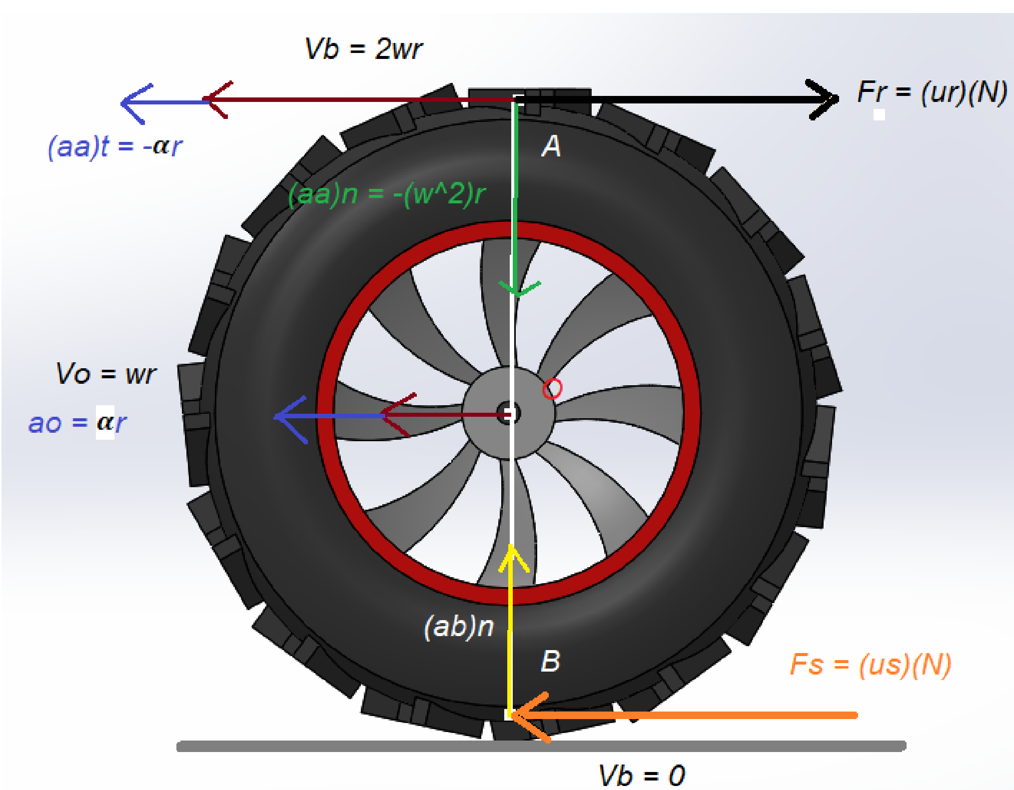
$\boldsymbol{I} = \boldsymbol{MR^2}$ (is the moment of inertia of wheel)

$$\boldsymbol{\alpha} = {}^{\boldsymbol{F}}/_{\boldsymbol{MR}}$$

$$\boldsymbol{\alpha} = {}^{|\boldsymbol{\mu_s N} - \boldsymbol{\mu_R N}|}/_{\boldsymbol{MR}}$$

$$\alpha = {}^{|13.1197 - 0.154|}/_{0.27\ x\ 6.72\ x\ 10^{-2}}$$

$$\alpha = 706.19\ rads^{-2}$$



Now the acceleration of point A is given as:

$$\boldsymbol{a_A = 2\alpha r}$$

$$a_A = 2x\ 706.19\ x\ 6.75\ x\ 10^{-2}$$

$$a_A = 95.33ms^{-2}$$

Force acting on Point A:

From Newton's Second Law:

$$\boldsymbol{F = ma}$$

For the wheel of mass M and acceleration associated at point A as a_A:

$$\boldsymbol{F_A = Ma_A}$$

$$F_A = 0.272\ x\ 95.33$$

$$F_A = 25.92\ N$$

Since this force is to be divided by 2 because motor is at mid distance from both wheels, hence;

$$\boldsymbol{F' = {}^{25.92}/_{2} = 12.96N}$$

Calculation for Load Torque:

The load torque that is required is given by:

$$\boldsymbol{\tau = r\ x\ F}$$

$$\boldsymbol{\tau = rF sin\theta}$$

Here:

r = moment arm or distance between motor's shaft and the force transmitted

= Radius of wheel – Distance between motor's shaft and center of wheel

= $6.75x10^{-2} - 4x10^{-2} = 2.75\ x\ 10^{-2}m$

θ = angle between F and r which in this case is 90 degrees hence $sin\theta = 1$

$$\boldsymbol{\tau = rF} = (2.75\ x\ 10^{-2})(3.23)$$

$$\tau = 0.089\ Nm = 89\ m - Nm$$

Motor Calculations:

Datasheet of Motor: 3863H024CR [1]

Series 3863 ... CR

	Values at 22°C and nominal voltage	3863 H		012 CR	018 CR	024 CR	036 CR	048 CR	
1	Nominal voltage	U_N		12	18	24	36	48	V
2	Terminal resistance	R		0,16	0,36	0,64	1,55	2,58	Ω
3	Efficiency, max.	$\eta_{max.}$		83	84	85	86	86	%
4	No-load speed	n_0		5 600	5 900	5 800	5 800	5 800	min^{-1}
5	No-load current, typ. (with shaft ø 6 mm)	I_0		0,335	0,232	0,168	0,112	0,084	A
6	Stall torque	M_H		1 424	1 394	1 455	1 363	1 461	mNm
7	Friction torque	M_R		6,5	6,5	6,5	6,5	6,5	mNm
8	Speed constant	k_n		480	332	240	160	120	min^{-1}/V
9	Back-EMF constant	k_E		2,08	3,01	4,17	6,25	8,33	mV/min^{-1}
10	Torque constant	k_M		19,9	28,8	39,8	59,8	79,7	mNm/A
11	Current constant	k_I		0,05	0,035	0,025	0,017	0,013	A/mNm
12	Slope of n-M curve	$\Delta n/\Delta M$		3,9	4,1	3,9	4,1	3,9	min^{-1}/mNm
13	Rotor inductance	L		45	90	180	400	700	µH
14	Mechanical time constant	τ_m		4,8	4,8	4,8	4,8	4,7	ms
15	Rotor inertia	J		120	110	120	110	115	gcm^2
16	Angular acceleration	$\alpha_{max.}$		119	127	121	124	127	$\cdot 10^3 rad/s^2$
17	Thermal resistance	R_{th1} / R_{th2}	2,5 / 6						K/W
18	Thermal time constant	τ_{w1} / τ_{w2}	50 / 900						s
19	Operating temperature range:								
	– motor		-30 ... +125						°C
	– winding, max. permissible		+155						°C
20	Shaft bearings		ball bearings, preloaded						
21	Shaft load max.:								
	– with shaft diameter		6						mm
	– radial at 3 000 min^{-1} (3 mm from bearing)		60						N
	– axial at 3 000 min^{-1}		6						N
	– axial at standstill		50						N
22	Shaft play:								
	– radial	≤	0,015						mm
	– axial	=	0						mm
23	Housing material		steel, black coated						
24	Mass		390						g
25	Direction of rotation		clockwise, viewed from the front face						
26	Speed up to	$n_{max.}$	7 000						min^{-1}
27	Number of pole pairs		1						
28	Magnet material		NdFeB						
	Rated values for continuous operation								
29	Rated torque	M_N		69	99	129	126	131	mNm
30	Rated current (thermal limit)	I_N		4	4	4	2,6	2	A
31	Rated speed	n_N		5 430	5 660	5 510	5 500	5 550	min^{-1}

Note: Rated values are calculated with nominal voltage and at a 22°C ambient temperature. The R_{th2} value has been reduced by 25%.

Important Features:

Values at 22°C and Nominal Voltage	**Symbols**	**Values**
Nominal Voltage	U_N	24V
Terminal Resistance	R	0.6Ω
Rated Current	I_N	4 Amp
Rated Speed	n_N	5510 rpm
Rated Torque	M_N	129 m-Nm
No Load Current	I_o	0.168 Amp
No Load Speed	n_o	5800 rpm
Stall Torque	M_H	1455 m-Nm
Friction Torque	M_R	6.5 m-Nm
Back emf Constant	k_E	4.17 mV/min^{-1}
Speed Constant	k_N	240 min^{-1}/V
Torque Constant	k_m	39.8 m-Nm/A
Current Constant	k_i	0.025 A/m-Nm
Angular Acceleration	α_{max}	121 x 10^3 rad/s^2
Efficiency	η_{max}	85%

Power Calculations for single motor:

Law of Conservation of Energy:

Law of conservation of energy says that:

"Energy can neither be created nor destroyed but it can be converted in another forms of energy"

Any practical device that exists in everyday life consumes certain amount of energy and converts it into some other form of energy while some of the energy is dissipated as heat or other forms. No device converts all one type of energy completely into another form some of the energy is always lost as heat or frictional losses etc. In other words, no practical device is 100% efficient.

A DC Motor converts electrical energy into mechanical energy. That means that it intakes electrical energy from source and converts that energy into rotational mechanical energy. According to law of conservation of energy the power consumed by motor and delivered is given by: [1]

$$\boldsymbol{P_{in} = P_{out} + P_{loss}}$$

Electrical Input Power:

The electrical power consumed by motor is simply the product of its nominal voltage and no load current (the current motor draws when no load is attached to it.) [2]

$$\boldsymbol{P_{in} = V\ x\ I\ (Watts)}$$

From data-sheet we have:

- Nominal Voltage = 24V
- No load Current = 4Amp

$$P_{in} = 24x4\ (Watts)$$

$$P_{in} = 96\ Watts$$

Mechanical Power Delivered by Motor:

The mechanical power delivered by motor is the power that it transmits to other parts or load. [3] It is simply the product of rated torque of motor multiplied by rated angular velocity of shaft. This is the output power hence;

$$\boldsymbol{P_{out} = \tau\ x\ \omega\ (Watts)}$$

From datasheet we have:

- Rated torque of motor = 129 m-Nm = 129 x 10^{-3} Nm
- Rated speed of motor = 5510 rpm

Now first we need to convert rpm (revolutions per minute) into rad-s^{-1}. This is done by using this formula:

$$\boldsymbol{\omega = \frac{2n\pi}{60}}$$

Where n is the rated speed of motor in revolutions per minute (rpm).

$$\omega = \frac{2\ x\ 3.141\ x\ 5510}{60}$$

$$\omega = 576.71\ rads^{-1}$$

Efficiency of Motor:

The efficiency of any device tells user how much that device is efficient in doing its work. Practically, no device has efficiency of 1 or percentage efficiency of 100%. That is some energy is always lost in carrying out work. [4]

Efficiency is given by:

$$\boldsymbol{Efficiency\ (\eta) = \frac{P_{out}}{P_{in}}}$$

$$\%\ \boldsymbol{Efficiency\ (\eta) = \frac{P_{out}}{P_{in}}\ x\ 100}$$

For our motor:

$$Efficiency\ (\eta) = \frac{74.4}{96}$$

$$Efficiency\ (\eta) = 0.775$$

Percentage efficiency is given by:

$$\%\ Efficiency\ (\eta) = 0.775\ x\ 100$$

$$\%\ Efficiency\ (\eta) = 77.5\ \%$$

Hence our motor is 77.5 % efficient which means that some energy losses are to be calculated.

Power losses of Motor:

Motors convert electrical power into mechanical power, but not all electrical energy is converted into mechanical energy some is dissipated as heat or other forms of energy. Normally such losses occur in DC motor: [5]

- Copper losses (Variable losses)
- Hysteresis & Eddy Current losses (20% of full load)
- Brush losses
- Mechanical losses or Bearing Frictional losses (very small)
- Stray losses (1% of full load)
- Thermal losses

Generally, these losses are calculated when some load is applied at motor shaft hence we have to calculate load current that motor draws when load is applied.

Load current calculation:

Load current is the current that motor draws from source when some load is applied at its rotating shaft. Usually load current is greater than operating current, because when load is applied the motor needs more energy to rotate its windings and develop enough flux hence it draws more current from source than normal operating current. As load is increased this load current is also increased.

The load current is given by the product of rated torque of motor and current constant. From datasheet we have: [4]

- Rated torque = 129 m-Nm
- Current constant = k_i = 0.025 A/m-Nm

$$\boldsymbol{I_{load} = \tau \; x \; k_i}$$

Inserting values in above equation:

$$I_{load} = 129 \; x \; 10^{-3} \; (Nm) \; x \; 25 \; (A/Nm)$$

$$I_{load} = 3.2 \; Amp$$

Total Current:

The total motor current now can be approximated by summing this value with no load current: [4]

$$\boldsymbol{I = I_{load} + I_o}$$

- No load current = 0.168 Amp

$$I = 3.2 + 0.168$$

$$I = 3.368 \text{ Amp}$$

Corrected Power Input: ($\boldsymbol{P_{in}'}$)

The corrected power input can be calculated by substituting the total current value in the input power formula:

$$\boldsymbol{P_{in}' = V\ x\ I\ (Watts)}$$

Where; V is the nominal voltage and I is the load current.

$$P_{in}' = 24\ x\ 3.4 = 81.6\ Watts$$

Power loss:

The power loss can be calculated using the equation:

$$\boldsymbol{P_{loss} = P_{in}' - P_{out}}$$

Inserting values:

$$P_{loss} = 81.6 - 74.4\ (watts)$$

$$P_{loss} = 7.2\ Watts$$

This means that almost 6.432 Watts of power losses occur every-time the motor delivers mechanical power. Now in order to calculate power losses we calculated major losses that occurred in motor.

1. **Copper Losses:** Copper loss is the power lost as heat in windings; it is caused by the flow of current through the coils of the DC armature. [3] This loss varies directly with the square of the current in the armature and the terminal resistance of motor.

$$\boldsymbol{P_c = I^2\ x\ R_t}$$

Where I is the total current consumption of motor and R_t is the terminal resistance of motor which is 0.6 Ω.

$$P_c = (3.3)^2 \ x \ (0.6)$$
$$P_c = 6.5 \ Watts$$

2. **Frictional Losses:** Frictional or mechanical losses occur when the motor shaft that when rotates between the bearings. In order to overcome the frictional forces motor losses some power in each revolution and drives the load at constant speed. [5]

   In order to calculate frictional loss first we need to calculate the current that motor draws in order to overcome the frictional forces. [4]

$$\boldsymbol{I_{lf} = M_R \ x \ k_i}$$

   Where $\boldsymbol{I_{lf}}$ is the frictional load current and $\boldsymbol{M_R}$ is the frictional torque of motor which is **6.5 m-Nm** and $\boldsymbol{k_i}$ is the current constant of motor which is **25 A/Nm**

$$I_{lf} = 6.5 \ x \ 25$$

$$I_{lf} = 0.1625 \ Amp$$

   Total current is hence: [4]

$$\boldsymbol{I = I_{lf} + I_o}$$

$$I = 0.1625 + 0.168$$

$$I = 0.33 \text{ Amps}$$

   Now the power dissipated due to friction is simply given by: [4]

$$\boldsymbol{P_f = I^2 \ x \ R_t}$$
$$P_f = (0.33)^2 \ x \ 0.64$$
$$P_f = 0.069 \text{ Watts}$$

3. **Stray Losses:** The combination of friction losses due to moving of bearing and shaft due to air gap are called stray losses. Usually such losses are 1% of full load delivered by motor. [4]

$$\boldsymbol{P_s = 1\% \ of \ full \ load = {}^{1}/_{100} \ x \ 74.4}$$
$$P_s = 0.74 \ Watts$$

Combining the losses:

Now so far major losses have been calculated. If now we calculate the power loss by adding $\boldsymbol{P_c}$, $\boldsymbol{P_f}$ and $\boldsymbol{P_s}$.

$$\boldsymbol{P_{loss} = P_c + P_f + P_s}$$

$$P_{loss} = 6.5 + 0.06 + 0.7$$

$$P_{loss} = 7.26\, Watts \approx 7.2\, Watts$$

The above losses show that their sum is approximately equal to the one calculated before by subtracting input and output powers.

Torque delivered at no-load:

The no load torque is the torque which motor delivers when no load is attached with the shaft of motor. This is given by modifying the efficiency equation for the motor as: [4]

$$\boldsymbol{P'_{in} x\, E = P_{out}}$$

$$\boldsymbol{P'_{in} x\, E = \tau\, x\, \omega}$$

$$\boldsymbol{\tau = {P'_{in} x\, E}/{\omega}}$$

$$\tau = {80.832\, x\, 0.92}/{576.71}$$

$$\tau = 0.1289\, Nm \approx 129\, m - Nm$$

Which is equal to the one mentioned in the data sheet.

References

[1] "FAULHABER DC MicroMotors," FAULHABER, 11 February 2021. [Online]. Available: https://www.faulhaber.com/fileadmin/Import/Media/EN_3863_CR_DFF.pdf. [Accessed 17 December 2021].

[2] "Electric Motor Power Measurement and Analysis," Yokogawa, 2014. [Online]. Available: https://www.yokogawa.com/library/resources/media-publications/electric-motor-power-measurement-and-analysis/. [Accessed 17 December 2021].

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Chain Drives:

A chain drive is a mechanically operating system where we used different types of chains to transmit the power or for the movement of systems. Generally, a chain drive is used where there is comparatively small distance between the source that produces power and the point where power is to be transmitted. Hence such systems are more efficient than those that use belt drive because in belt drives there is some amount of percentage slippage that occurs as the system moves towards dynamics. But that does not mean that in chain drives there is no power loss. Of course some frictional losses occur between gear-teeth and links of chain.

Characteristics:

In chain drives;

- The speed ratio remains constant, which is a major advantage of chain drive and there is no slippage.
- A chain is made by a number of links and those are connected with the help of pin.
- Chains run over a wheel named sprocket, which has several amount of teeth around the circumference of that to grip the chain.

Chain Types:

Chain drives are categorized into three types:

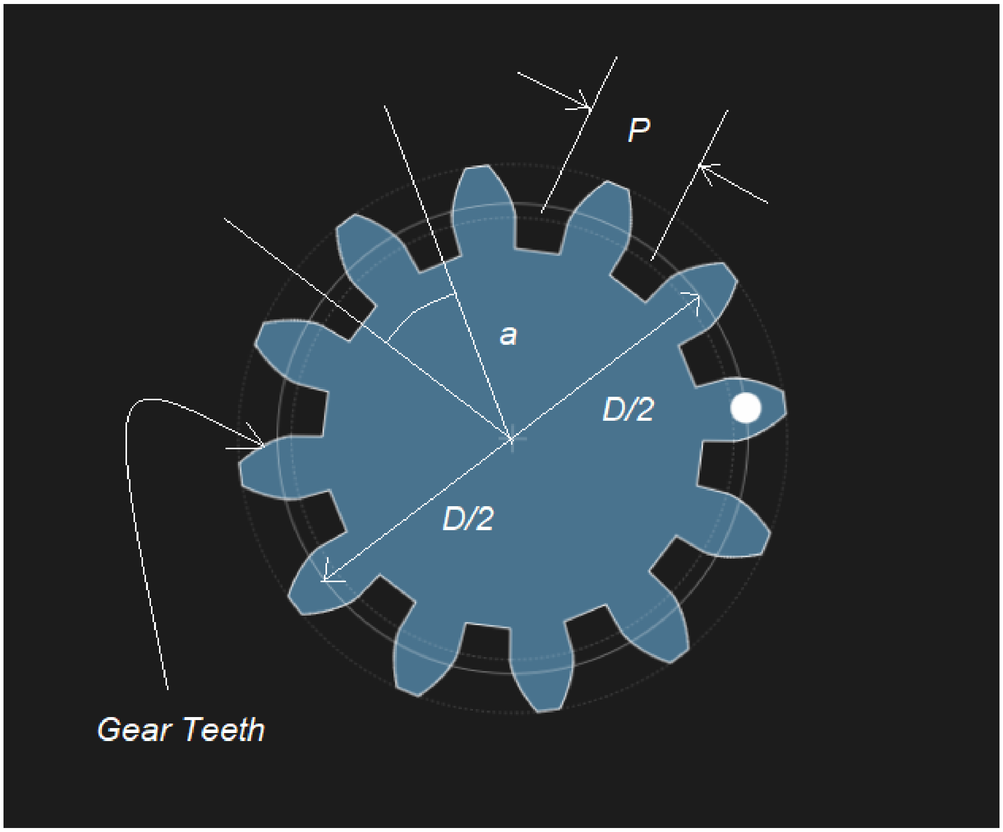
- Hoisting chains
- Conveyor Chains
- Power Transmission Chains

In our system we are using Power Transmission chain because we need to transmit mechanical power from the motor to the wheels. Power Transmission Chains are constructed by:

1. A bush
2. Inner Link
3. A pin
4. Outer Plate
5. Inner Plate
6. Rollers

In this type of chain, a bush along with the roller is fitted inside both plates, then a pin is passed through both ends of the roller to fasten it. The rollers are free to rotate inside the bush so that when it makes contact with the sprocket there is a minimal wear tear between them. Generally, such chains are made of steel.

Gear Nomenclature:



Consider a simple gear having n number of teeth and α as pitch angle. The diameter of sprocket can be represented as D. P is the pitch of the sprocket.

Pitch Angle:

Pitch angle α is given by:

$$\alpha = {}^{360}/_{n}$$

Where n is the number of teeth of sprocket.

Diameter of Sprocket:

The diameter of sprocket is given by:

$$\sin\left({}^{\alpha}/_{2}\right) = \frac{{}^{P}/_{2}}{{}^{D}/_{2}} = \frac{P}{D}$$

$$D = {}^{P}/_{\sin\left({}^{\alpha}/_{2}\right)}$$

Here P is the pitch (Distance between rollers) which is known in our case as **0.65cm = 0.0065m**

Gear Ratio:

For two sprockets there exists a velocity ratio between them which is equal to the ratio of teeth between them. This ratio is very useful in mechanics since it is used for velocity and power transmission between different gears. The gear ratio is given by:

$$\boldsymbol{n_1 v_1 = n_2 v_2}$$

$$\boldsymbol{\frac{v_1}{v_2} = \frac{n_2}{n_1}}$$

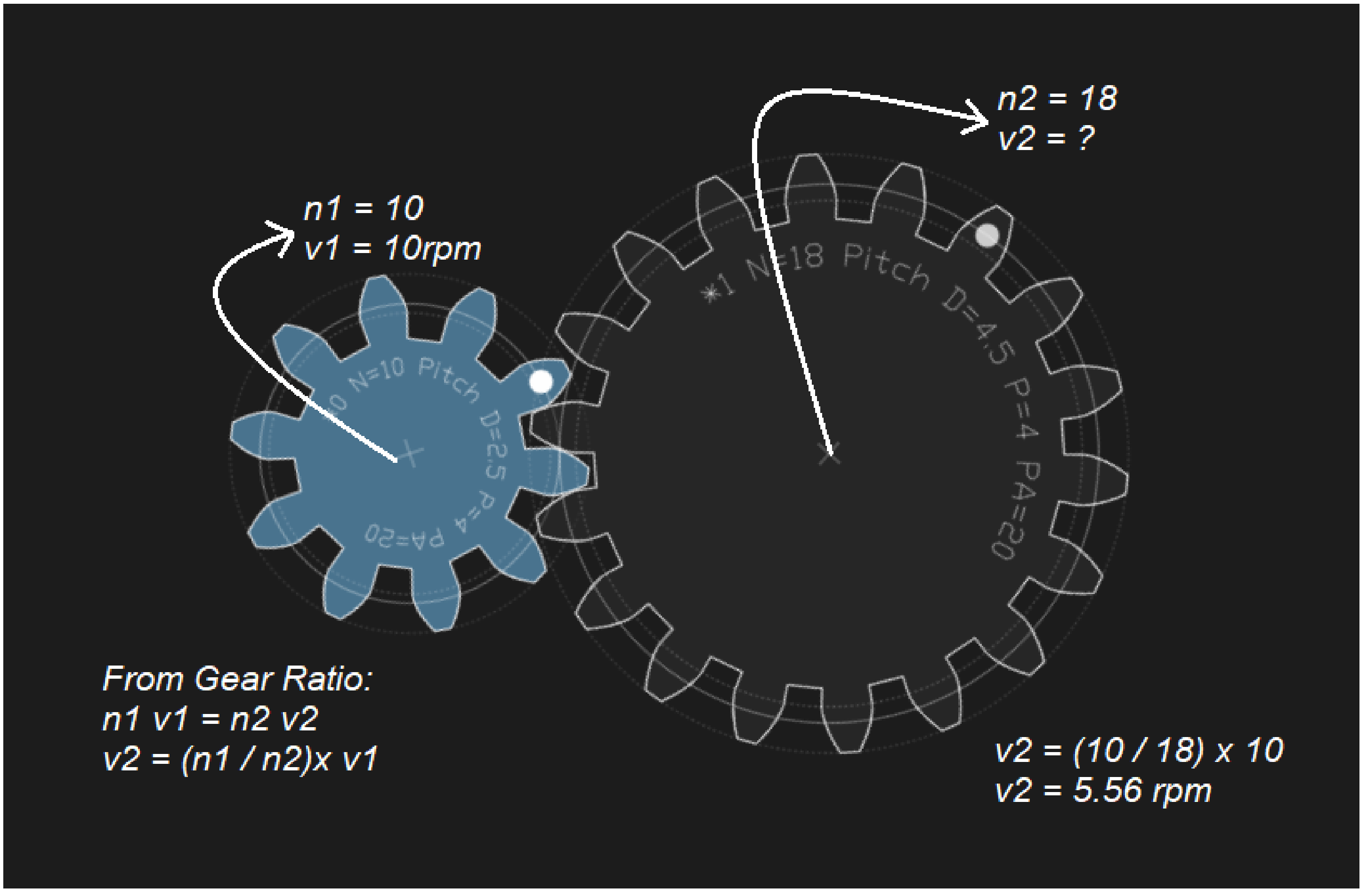
Here;

$n_1 = Number\ of\ teeth\ of\ Parent\ Gear$

$n_2 = Number\ of\ teeth\ of\ Child\ Gear$

$v_1 = Velocity\ (rpm)\ of\ Parent\ Gear$

$v_1 = Velocity\ (rpm)\ of\ Child\ Gear$



Gear Design:

Our robot uses 2 inside gears which are connected chain that passes over the motor gear. The motor is located at mid distance between these two gears so that same force is delivered to both gears. These 2 inside gears have same number of teeth. Now two small gears are mounted on the two inside gears with the help of shaft. These two gears also have same number of teeth. These two gears then make contact with the big gears that are connected with the shaft to wheels.

$n_1 = Number\ of\ teeths\ of\ left\ inside\ gear = 32$

$n_2 = Number\ of\ teeths\ of\ right\ inside\ gear = 32$

$n_m = Number\ of\ teeths\ of\ motor\ small\ gear = 12$

$n_3 = Number\ of\ teeths\ of\ left\ outside\ small\ gear = 17$

$n_4 = Number\ of\ teeths\ of\ right\ outside\ small\ gear = 17$

$n_5 = Number\ of\ teeths\ of\ left\ outside\ big\ gear = 25$

$n_6 = Number\ of\ teeths\ of\ right\ outside\ big\ gear = 25$

Pitch Angle:

The Pitch angle for the two inside gears is same and is given by:

$$\boldsymbol{\alpha_{1=2} = {}^{360}/_{n_{1=2}}}$$

$$\alpha_{1=2} = {}^{360}/_{32} = 11.25^o$$

The Pitch angle for motor gear is given by:

$$\boldsymbol{\alpha_m = {}^{360}/_{n_m}}$$

$$\alpha_m = {}^{360}/_{12} = 30^o$$

The Pitch angle for the two outside small gears is same and is given by:

$$\boldsymbol{\alpha_{3=4} = {}^{360}/_{n_{3=4}}}$$

$$\alpha_{3=4} = {}^{360}/_{17} = 21.27^o$$

The Pitch angle for the two outside big gears is same and is given by:

$$\boldsymbol{\alpha_{5=6} = {}^{360}/_{n_{1=2}}}$$

$$\alpha_{5=6} = {}^{360}/_{25} = 14.4^o$$

Gear Diameter:

Gear diameter is given by:

$$\boldsymbol{sin\left({}^{\alpha}/_{2}\right) = \frac{{}^{P}/_{2}}{{}^{D}/_{2}} = \frac{P}{D}}$$

$$\boldsymbol{D = {}^{P}\Big/_{sin\left({}^{\alpha}/_{2}\right)}}$$

Here P is the pitch (Distance between rollers) which is known in our case as **0.65cm = 0.0065m**

The diameter for the two inside gears is same and is given by:

$$\boldsymbol{D_{1=2} = {}^{P}\Big/_{sin\left({}^{\alpha_{1=2}}/_{2}\right)}}$$

$$D_{1=2} = {}^{0.0065}\Big/_{sin\left({}^{11.25}/_{2}\right)} = 0.06631m = 6.63cm$$

The diameter for motor gear is given by:

$$\boldsymbol{D_{1=2} = {}^{P}\Big/_{sin\left({}^{\alpha_{m}}/_{2}\right)}}$$

$$D_{m} = {}^{0.0065}\Big/_{sin\left({}^{30}/_{2}\right)} = 0.02511m = 2.51cm$$

The diameter for the two outside small gears is same and is given by:

$$\boldsymbol{D_{3=4} = {}^{P}\Big/_{sin\left({}^{\alpha_{3=4}}/_{2}\right)}}$$

$$D_{3=4} = {}^{0.0065}\Big/_{sin\left({}^{21.17}/_{2}\right)} = 0.0353m = 3.53cm$$

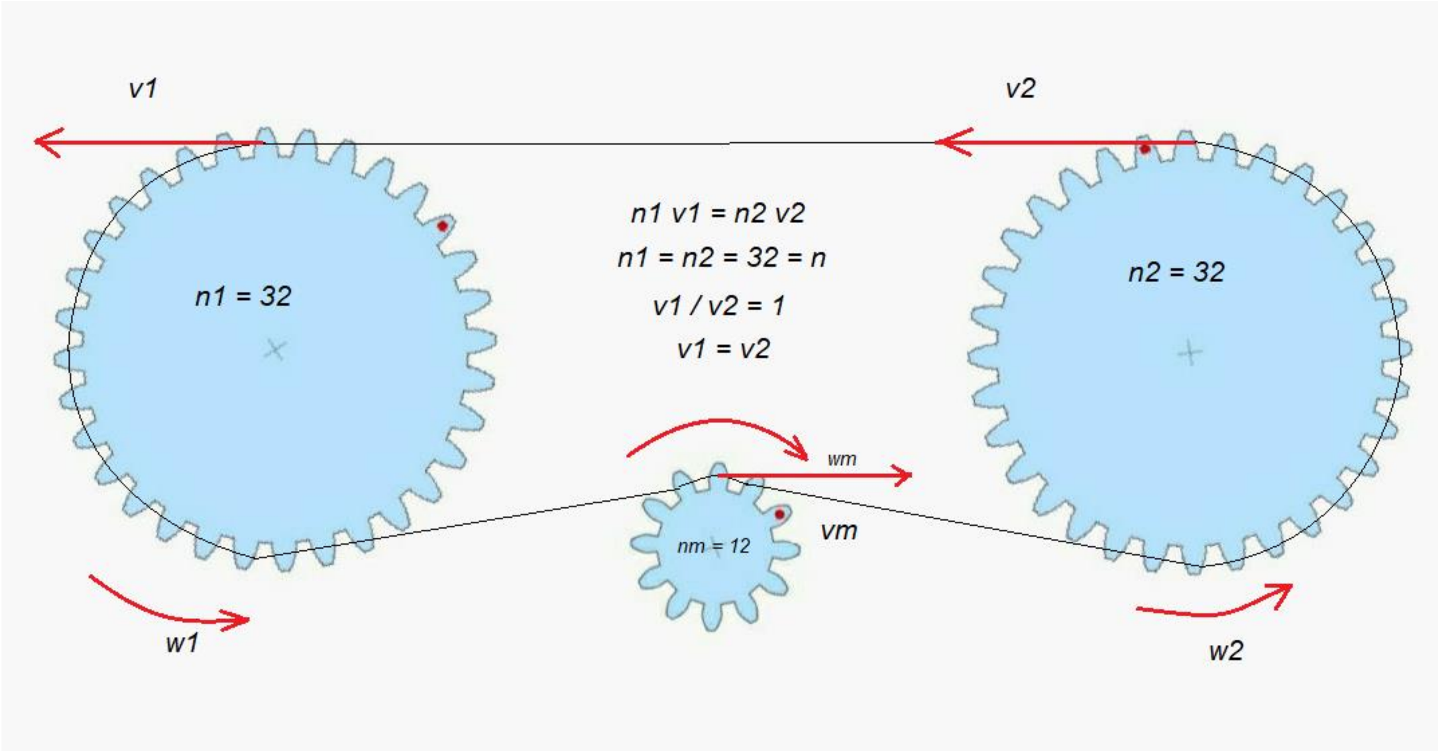
The diameter for the two outside big gears is same and is given by:

$$\boldsymbol{D_{5=6} = {}^{P}\Big/_{sin\left({}^{\alpha_{5=6}}/_{2}\right)}}$$

$$D_{5=6} = {}^{0.0065}\Big/_{sin\left({}^{14.4}/_{2}\right)} = 0.0518m = 5.18cm$$

Chain Drive of Robot:

Our robot uses 2 inside gears which are connected chain that passes over the motor gear. The motor is located at mid distance between these two gears so that same force is delivered to both gears. These 2 inside gears have same number of teeth so that both gears have velocity ratio of 1. The below figure shows the orientation of gears connected with motor:



$n_1 = Number\ of\ teeths\ of\ left\ inside\ gear = 32$

$n_2 = Number\ of\ teeths\ of\ right\ inside\ gear = 32$

$n_m = Number\ of\ teeths\ of\ motor\ small\ gear = 12$

$v_1 = Rpms\ associated\ with\ left\ inside\ gear$

$v_2 = Rpms\ associated\ with\ right\ inside\ gear$

$v_m = Rpms\ associated\ with\ motor = 5510\ rpm$

First we need to calculate gear ratio between the gears of same teeth:

$$\boldsymbol{n_1 v_1 = n_2 v_2}$$

$$\boldsymbol{\frac{v_1}{v_2} = \frac{n_2}{n_1}}$$

Since $\boldsymbol{n_1 = n_2 = n = 32}$ which implies:

$$\boldsymbol{\frac{v_1}{v_2} = 1 => v_1 = v_2}$$

Motor has small gear of 12 teeth. The gear ratio is given by:

$$\boldsymbol{n_1 v_1 = n_2 v_2 = nv = n_m v_m}$$

$$\boldsymbol{\frac{v}{v_m} = \frac{n_m}{n}}$$

$$\boldsymbol{v = \frac{n_m}{n}\, x\, v_m}$$

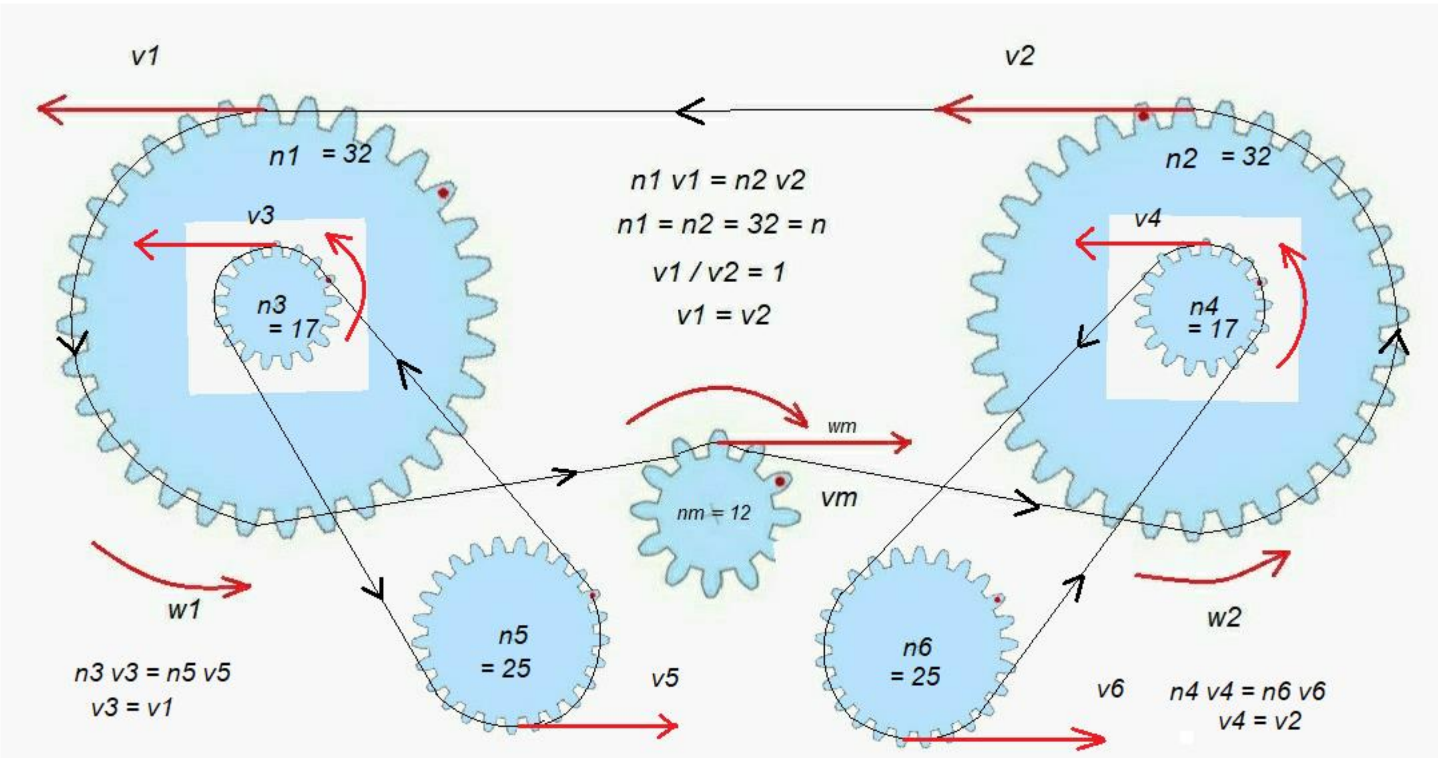
$$v = \frac{12}{32}\, x\, 5510 = 2066.25\, rpm$$

Hence;

$\boldsymbol{v_1 = 2066.25\, rpm}$

$\boldsymbol{v_2 = 2066.25\, rpm}$

Now two small gears are mounted on the two inside gears with the help of shaft. These two gears also have same number of teeth. These two gears then make contact with the big gears that are connected with the shaft to wheels.



Here;

$n_3 = Number\ of\ teeths\ of\ left\ outside\ small\ gear = 17$

$n_4 = Number\ of\ teeths\ of\ right\ outside\ small\ gear = 17$

$n_5 = Number\ of\ teeths\ of\ left\ outside\ big\ gear = 25$

$n_6 = Number\ of\ teeths\ of\ right\ outside\ big\ gear = 25$

$v_3 = Rpms\ associated\ with\ left\ outside\ small\ gear = \ v_1 = 2066.25\ rpm$

$v_4 = Rpms\ associated\ with\ right\ outside\ small\ gear = \ v_2 = 2066.25\ rpm$

$v_5 = Rpms\ associated\ with\ left\ outside\ small\ gear$

$v_6 = Rpms\ associated\ with\ right\ outside\ small\ gear$

Since n_3 and n_4 gears are mounted on n_1 and n_2 respectively hence both would have same velocities i.e. 2066.25 rpms. Now two gear ratios exist between n_3 / n_5 and n_4 /n_6. These can be given as:

$$\boldsymbol{n_3 v_3 = \ n_5 v_5}$$

$$\boldsymbol{\frac{v_3}{v_5} = \frac{n_5}{n_3}}$$

And;

$$\boldsymbol{n_4 v_4 = \ n_6 v_6}$$

$$\boldsymbol{\frac{v_4}{v_6} = \frac{n_6}{n_4}}$$

v_5 and v_6 can be given as:

$$\boldsymbol{v_5 = \ \frac{n_3}{n_5}\ x\ v_3} = \ \frac{17}{25}\ x\ 2066.25 = 1405.05\ rpm$$

$$\boldsymbol{v_6 = \ \frac{n_4}{n_6}\ x\ v_4} = \ \frac{17}{25}\ x\ 2066.25 = 1405.05\ rpm$$

Angular Velocities:

Angular velocity is a characteristic of rotating body. It tells about how much radians a body has covered per second. It is given by:

$$\boldsymbol{\omega = \frac{2\pi v}{60}}$$

Angular velocities of gears are given by:

$\boldsymbol{w_m = Angular\ velocity\ of\ motor}$

$$w_m = \frac{2\ x\ (3.14) x\ 5510}{60} = 576.71\ rad - s^{-1}$$

$\boldsymbol{w_1 = w_2 = Angular\ velocities\ of\ inside\ sprockets}$

$$w_1 = w_2 = \frac{2\ x\ (3.14) x\ 2066.25}{60} = 216.26\ rad - s^{-1}$$

$\boldsymbol{w_3 = w_4 = Angular\ velocities\ of\ outside\ small\ sprockets}$

$$w_3 = w_4 = \frac{2\ x\ (3.14) x\ 2066.25}{60} = 216.26\ rad - s^{-1}$$

$\boldsymbol{w_5 = w_6 = Angular\ velocities\ of\ outside\ big\ sprockets}$

$$w_3 = w_4 = \frac{2\ x\ (3.14) x\ 1405.45}{60} = 147.06\ rad - s^{-1}$$

Power Transmission:

The rated torque of the motor is:

$$\boldsymbol{\tau_m = 129\ m - Nm = 0.129\ Nm}$$

Power transmitted to the two inside gears is given by:

$$\boldsymbol{P_{1=2} = \tau_m w_{1=2}}$$

$$P_{1=2} = 0.129\ x\ 216.26$$

$$P_{1=2} = 27.8\ Watts$$

Some losses would occur due to bearing frictional forces. The rest would be transmitted to two outside small sprockets.

$$\boldsymbol{P_{3=4} = \tau_m w_{3=4}}$$

$$P_{3=4} = 0.129\ x\ 216.26$$

$$P_{3=4} = 27.8\ Watts$$

Power transmitted to the two outside big gears is given by:

$$\boldsymbol{P_{5=6} = \tau_m w_{5=6}}$$

$$P_{1=2} = 0.129\ x\ 147.06$$

$$P_{1=2} = 18.7\ Watts$$

Conservation of Energy:

Conservation of Energy says that total energy transmitted by source must be equal to the sum of all the individual powers of the consumer components. In our case the mechanical power transmitted by motor must be equal to the power transmission to the gears:

$$\boldsymbol{P_m = P_{1=2} + P_{3=4} + P_{5=6}}$$

Mechanical power transmitted by motor is calculated before which is given by:

$$\boldsymbol{P_m = 74.4 Watts}$$

Hence putting values in above equation we have;

$$\boldsymbol{P_m = P_{1=2} + P_{3=4} + P_{5=6}}$$

$$74.4 = 27.8 + 27.8 + 18.7$$

$$74.4 \approx 74.3\ Watts$$

Hence this also proves that the angular velocities calculated above are also true.

Battery Selection:

Battery selection or Electrical source selection is an important step in the design process of robot. It is obvious that one cannot wire his robot from a continuous source of electricity for the working of robot. It depends upon whether your robot is mobile or static. If the robot is to move in the environment and cover a wide range of distance, then wiring your robot from the socket or some other power source is not an efficient way. Hence Battery selection is an important step. Usually two types of electrical sources exist for the working of systems:

1. Voltage Source
2. Current Source

Voltage sources are the most commonly used sources whereas current sources are not so practical as compared to voltage sources. When dealing with circuit laws and analysis, electrical sources are often viewed as being "ideal", that is the source is ideal because it could theoretically deliver an infinite amount of energy without loss thereby having characteristics represented by a straight line. However, in real or practical sources there is always a resistance either connected in parallel for a current source, or series for a voltage source associated with the source affecting its output.

There are two approaches in selecting the Battery for the project:

- Multiple Sources
- Single Source

Multiple Sources:

The question is How do you know you have to choose Multiple Sources? The answer to this is to see how many components in your system consume same amount of voltage and how many consume high voltage source such that their nominal voltage doesn't lie in the nominal voltage ranges of other components. In this case you have to choose Multiple sources. One for the small consumption components and the other one for high voltage intake.

Advantages:

Advantages of choosing the multiple sources are:

- Requires Less Design time
- Can be more efficient

Disadvantages:

Disadvantages of choosing multiple sources are:

- Various parts of robot will stop working at different times
- Multiple batteries to recharge

Single Source:

Single source is used when the nominal voltage of every component have some common range and one can easily use a high voltage source and then by using the voltage regular to step down the voltage for the other components.

Advantages:

- One battery to recharge
- Less weight

Disadvantages:

- May require voltage regulator
- A bit more complex in designing and wiring the components

Basic Approach:

A basic approach is that consider the operating voltage of each component used in the project:

- Electronics (Microcontrollers, motor controller, power etc.) usually operate at 9V-12V. Some operate at low as 3.3V and 5V.
- Actuators (DC Gear Motors, Stepper motors, Servos etc.) usually operate at 6V to 12V. A few operate as low as 3V
- Sensors usually operate at 5V.

In our case we have:

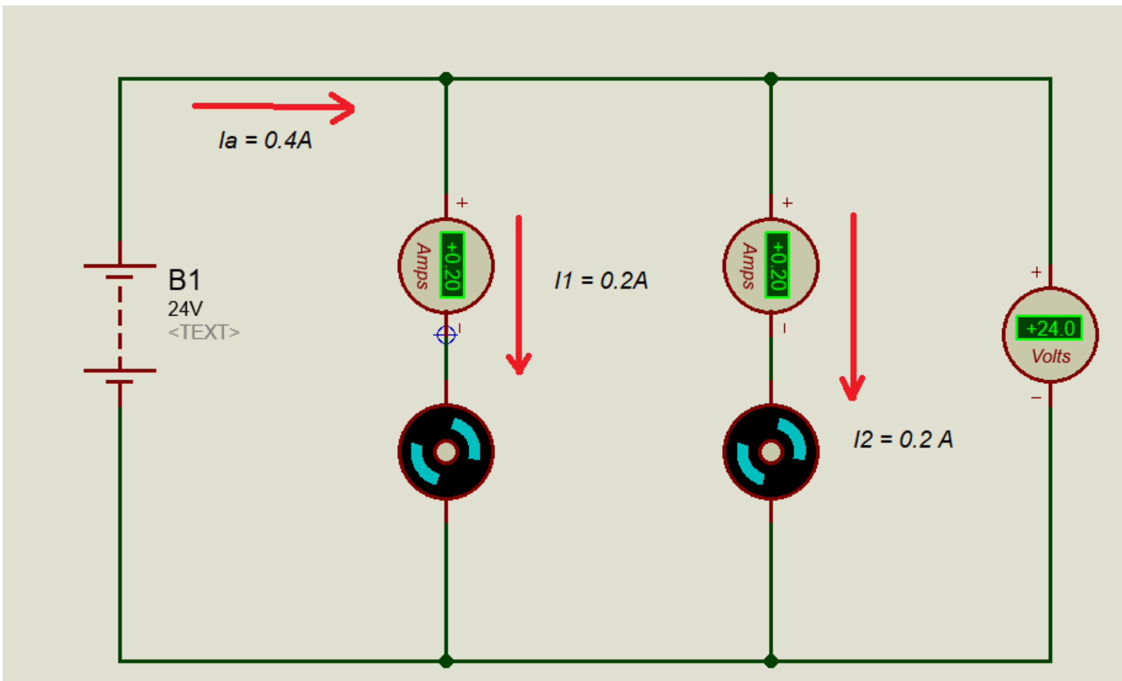
Components	Operating Voltage	Operating Current
Ultrasonic Sensors(HCSR-04)	5V	15 mA
Gyro-Accelerometer (MPU 6050)	2.37-3.46V	3.6mA
Voltage/ Current Sensors	3.3-5.5V	2-4mA
Motors (3868H024CR)	24V	@no-load = 0.168 A @rated = 4 A
Motor Driver (BTS7960)	6-27V	Can handle upto 43A
Microcontroller (Raspberry-Pi/ Jetson Nano)	Raspberry-pi @ 4.75-5.5V Jetson Nano @ 5V	Raspberry-pi @ 2A Jetson Nano @ 2A

Battery Selection Approach:

Since from above table it is clear that motors are the one who require high voltage comparative to other components in the list hence the approach that is suitable for this situation is selecting the multiple electrical sources.

- For Microcontroller a 5V 2Amp power bank can be used. (1850 Li-on Power bank)
- For Sensors 5Volts can be extracted from the output pins of controller
- The main voltage intake is for motors that use 24 Volts nominal voltage.

Consider motors connected in parallel with the source:



This is a general simulation of two motors whose nominal voltage is 24V and are in parallel with a 24V source. From the simulation we can deduce:

- Voltage in parallel acts equal for each motor.

$$\boldsymbol{E_a = E_{m1} = E_{m2} = 24V}$$

- Current gets divided for each motor:

$$\boldsymbol{I_a = I_{m1} + I_{m2}}$$

If at no load the current consumption of single motor is 0.168 A then for two motors it is:

$$\boldsymbol{I_{no-load} = (I_{m1})_{no-load} + (I_{m2})_{no-load}}$$

$$I_{no-load} = 0.168 + 0.168$$

$$I_{no-load} = 0.336\ Amps$$

Load current for single motor is done in motor calculations as:

$$\boldsymbol{I_{load} = 3.2\ Amp}$$

Total current drawn by motor under load condition is:

$$\boldsymbol{I = I_{load} + I_o}$$

$$I = 3.2 + 0.168$$

$$I = 3.368 \text{ Amp}$$

For two motors:

$$\boldsymbol{I_T = 2 \; x \; I}$$

$$I_T = 2 \; x \; 3.368 = 6.73 \; Amp$$

Hence we need a 24Volts supply that could deliver a constant current of 6.73 A (minimum).

Run time calculation:

For such a source shown above the run time of the battery could be calculated as:

$$\boldsymbol{\frac{Rated\ Ah\ of\ battery}{Required\ Current} = \frac{8\ Ah}{6.73\ A} = 1.18\ hours}$$