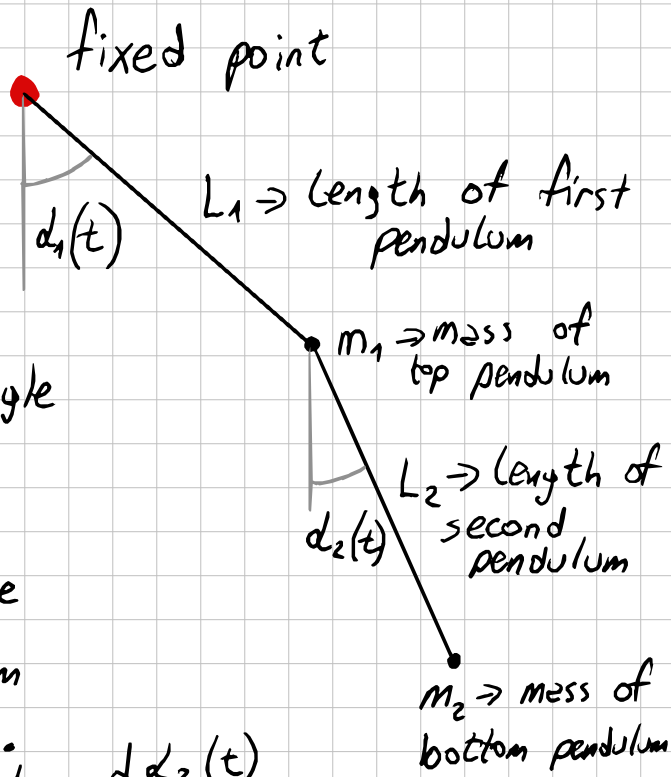
Double Pendulum Problem



$\alpha_1 = \alpha_1(t) \Rightarrow$ angle of top pendulum

$\alpha_2 = \alpha_2(t) \Rightarrow$ angle of bottom pendulum

$$\dot{\alpha}_1 = \frac{d\alpha_1(t)}{dt}, \quad \dot{\alpha}_2 = \frac{d\alpha_2(t)}{dt}$$

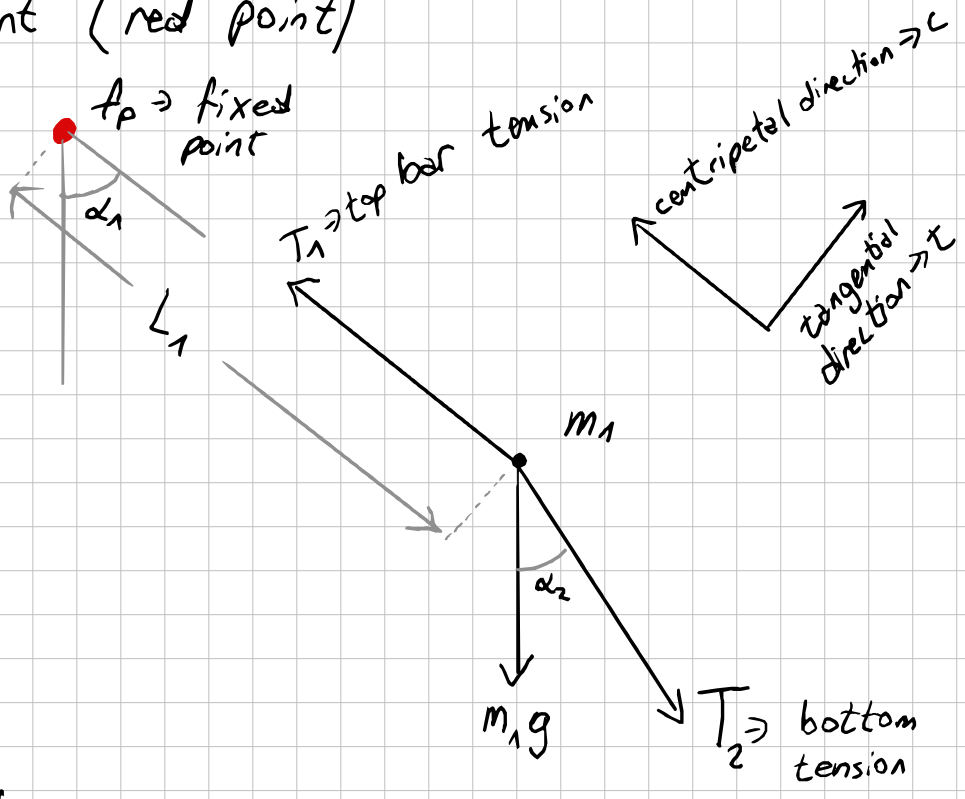
angular velocities

$$\ddot{\alpha}_1 = \frac{d^2\alpha_1(t)}{dt^2}, \quad \ddot{\alpha}_2 = \frac{d^2\alpha_2(t)}{dt^2}$$

angular accelerations

Free body diagram of top pendulum:

inertial reference frame fixed to fixed point (red point)



Newtons 2. law modified for rotation (not translation) will be used:

$$\tau = I\ddot{\alpha}$$

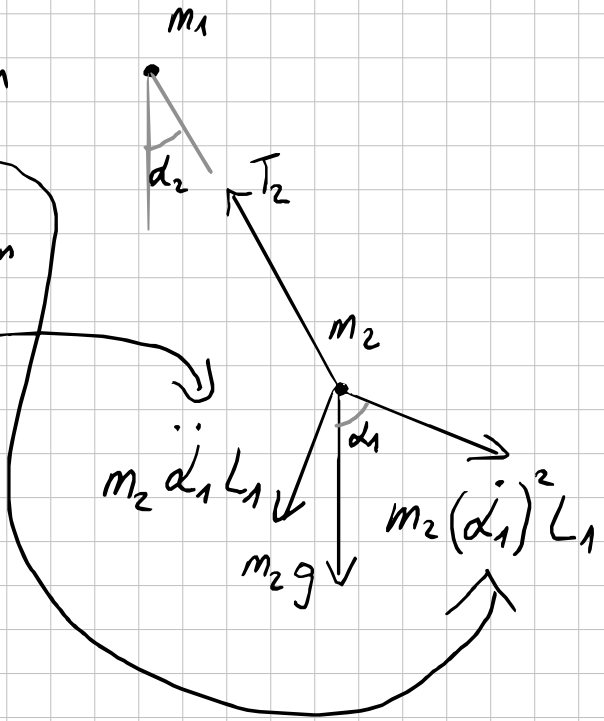
Torque is equal to moment of inertia times angular acceleration. Rotation around f_p will will be considered: ($I = m_1 \cdot L_1^2$)

1) $$\tau_1 = -m_1 g \sin(\alpha_1) L_1 + T_2 \sin(\alpha_2 - \alpha_1) L_1 = m_1 L_1^2 \ddot{\alpha}_1$$

Free body diagram for bottom pendulum: non-inertial reference frame fixed to m_1. Inertial forces due to m_1 centripetal and tangential accelerations will be added to the diagram:

centripetal acceleration of $m_1 \Rightarrow (\dot{\alpha}_1)^2 L_1$

tangential acceleration of $m_1 \Rightarrow \ddot{\alpha}_1 L_1$



2) $$T_2 = -m_2 g \sin(\alpha_2) L_2 - m_2 \ddot{\alpha}_1 L_1 \cos(\alpha_1 - \alpha_2) L_2 + m_2 (\dot{\alpha}_1)^2 L_1 \sin(\alpha_1 - \alpha_2) L_2 = m_2 L_2^2 \ddot{\alpha}_2$$

3) $$T_2 = m_2 (\dot{\alpha}_2)^2 L_2 + m_2 g \cos(\alpha_2) + m_2 (\dot{\alpha}_1)^2 L_1 \cos(\alpha_1 - \alpha_2) + m_2 \ddot{\alpha}_1 L_1 \sin(\alpha_1 - \alpha_2)$$

Plugging 3) into 1) and solving for $\ddot{\alpha}_1$:

4) $$\ddot{\alpha}_1 = \frac{-m_1 g \sin(\alpha_1) + m_2 (\dot{\alpha}_2)^2 L_2 \sin(\alpha_2 - \alpha_1) + \dots}{m_1 L_1 - m_2 \sin(\alpha_1 - \alpha_2) \sin(\alpha_2 - \alpha_1) L_1}$$

$$\dots \frac{+ m_2 g \cos(\alpha_2) \sin(\alpha_2 - \alpha_1) + m_2 (\dot{\alpha}_1)^2 \cos(\alpha_1 - \alpha_2) \sin(\alpha_2 - \alpha_1) L_1}{m_1 L_1 - m_2 \sin(\alpha_1 - \alpha_2) \sin(\alpha_2 - \alpha_1) L_1}$$

Solving for $\ddot{\alpha}_2$ from 2):

5) $$\ddot{\alpha}_2 = \frac{-g \sin(\alpha_2) - \ddot{\alpha}_1 L_1 \cos(\alpha_1 - \alpha_2) + (\dot{\alpha}_1)^2 L_1 \sin(\alpha_1 - \alpha_2)}{L_2}$$

Numerical integration: at t_0, $\alpha_1 = \alpha_{10}$, $\dot{\alpha}_1 = \dot{\alpha}_{10}$
$\alpha_2 = \alpha_{20}$, $\dot{\alpha}_2 = \alpha_{20}$

Compute $\ddot{\alpha}_1(t_0)$ and $\ddot{\alpha}_2(t_0)$ by 4) and 5)

$\dot{\alpha}_1(t_1) = \dot{\alpha}_{10} + \Delta t \ddot{\alpha}_1(t_0)$, $\alpha_1(t_1) = \alpha_{10} + \Delta t \dot{\alpha}_1(t_1)$

$\dot{\alpha}_2(t_1) = \dot{\alpha}_{20} + \Delta t \ddot{\alpha}_2(t_0)$, $\alpha_2(t_1) = \alpha_{20} + \Delta t \dot{\alpha}_2(t_1)$

Repeat for t_2