The Economics of Process Transparency

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We propose and analyze a novel framework to understand the role of *non-instrumental information disclosure* in service operations management, i.e., information that is shared by the firm not to affect consumers' actions, but to better manage their experience in the firm's process. To this end, we model the interactions between a service provider (firm) and a consumer. The operations of the firm are organized as a process that consists of a sequence of tasks, each of random duration. Consumers are delay-sensitive. In contrast to prior literature that exclusively considers the role of information sharing in influencing consumers' actions, our setting is one where consumers take no action – our goal is to study how the consumer-waiting experience is influenced by non-instrumental information disclosure, either directly or indirectly.

Our work draws upon the recent literature on belief-based utility in Economics. We analyze and compare information-disclosure strategies that are commonly observed in real-life service processes. Interestingly, complete transparency may not be optimal for the customer, even if available at no cost. Indeed, no disclosure, where the service process becomes a black-box, may sometimes be optimal. We also investigate the role of the number of tasks in the process and its effect on the choice of the firm's information disclosure strategy. Finally, we consider the impact of consumer-anxiety costs on the firm's choice of information disclosure.

Key words: Belief-Based Utility, Delay Disclosure, Process Analysis, Gain-Loss Utility, Information Design

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I was fine with the way pizza used to work . . . where they'd say it'd show up in 45 minutes and it would take an hour.

— Domino's Pizza customer about Domino's' real-time pizza tracker app (The Wall Street Journal 2017)

1. Introduction

A large body of research in the service operations management (OM) literature has studied the role of *instrumental* information sharing. At its core, the environment consists of a service provider (firm) and consumers. The service provider holds private information that is payoff-relevant to the consumers. In turn, consumers hold beliefs on the payoff-relevant variables and strategically choose whether to engage in trade with the firm. The firm chooses what information and how much of that

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information to share with the consumers to affect their beliefs (on the payoff-relevant variables), with the aim of incentivizing them to engage in trade. For example, in service operations, a service provider might provide information about consumers' wait-time to delay-sensitive consumers. The consumer might not have complete information either about the system parameters (e.g., service rate) or the state of the system (e.g., number of customers in the queue). Some recent examples that study the impact of information sharing about the state of the system include: Guo and Zipkin (2007), Allon et al. (2011) and Lingenbrink and Iyer (2019), that analyze a consumer's decision to join or balk a queue upon arrival; Armony and Maglaras (2004a) and Armony and Maglaras (2004b), that analyze a consumer's request for a call-back (in addition to joining or balking) at a call-center; Cui et al. (2019), that analyzes consumer retrials in a queue. Cui and Veeraraghavan (2016) consider the impact of information sharing in a setting where consumers make join/balk decisions and are uncertain about the system parameters (e.g., service rate), but the queue length is observable.

In contrast to the literature on instrumental information sharing, we study *non-instrumental* information sharing, where information is shared not to affect consumers' actions (in fact, in the settings we consider, consumers do not have any actions), but to better manage their experience in the firm's process. Specifically, consumers realize utility/disutility from the consumption of non-instrumental information while being part of the process. The firm fully internalizes the consumer's utility – consisting of *material* utility (from the completion of service) and *belief-based* utility (from the consumption of non-instrumental information during the process) – and shares information to maximize the consumer's utility.

Fundamental to our work is the notion of belief-based utility in Economics – we discuss this in detail in Section 3. Before proceeding further, we discuss some applications that motivate our work.

1.1. Waiting-Experience Management in Service Processes

We focus on applications commonly studied in OM – e.g., services, online retail, and on-demand mobility – where the outcome of interest to a consumer (agent) is the total amount of delay. We assume that, *ceteris paribus*, the consumer's material utility is decreasing in the amount of delay. We assume that the service is of sufficiently high value to ensure that the consumers participate, and study how information disclosure affects their experience *during* the process. Also, since our focus is on consumers' waiting experience during their sojourn through the process, we assume that they do not abandon the process before completion.

These settings are *processes*, consisting of a sequence of tasks, each of random duration. Consumers join the process (e.g., a queue with unobservable queue length) and await service. We focus on the consumer's waiting-experience: As consumers await service (while waiting in the queue, but before completion of service), the service provider could either provide no information, partial information, or complete information about the delay that will be realized. An agent may enjoy the waiting experience if he believes his total delay is low. Furthermore, during the waiting time, the consumer may derive positive (resp., negative) utility from signals that suggest a decrease (resp., increase) in his total delay. Under the complete information strategy, the firm resolves the uncertainty about the delay in one-shot, thereby providing the consumer with one piece of news – good or bad, small or large – at the start of the process. On the other hand, partial-information strategies provide information in small pieces over time, thereby resolving the consumer's uncertainty about the delay gradually (i.e., during the process). Key to our analysis is how consumers react to good and bad *news* (the shape of the belief-based utility). In particular, we will show that the complete information strategy may not be optimal for a consumer.

To fix ideas, consider the following examples of delay disclosure in service processes:

- **Order Fulfilment Process in Online Retail:** Consider a customer's post-sales shopping experience in online retail (i.e., after placing an order but before receipt of the item). The various tasks in this process are: item(s) picking and packing at the warehouse, (first mile) warehouse to a logistics provider, (middle-mile) transit by the logistics provider, (last-mile) delivery to the customer. The task-times, and therefore the total delivery lead time, are all uncertain; all else equal, the customer prefers a low delivery lead time. As the customer awaits the delivery of his order, the online retailer could either provide no shipping/delivery estimates, or partial information (e.g., current location of the item), or full/complete information (an accurate estimate of the delivery lead time, i.e., the time and date of delivery).

  In a similar vein, consider the post-ordering process of on-demand food delivery. For example, on Dominos.com, the various tasks after placing an order for a pizza are: preparation, bake, box and delivery (The Wall Street Journal 2017). The task-times, and therefore, the total delay, is uncertain. Again, the on-demand food delivery website can provide no information, partial information, or complete information about the total delay after a customer places an order.

- **Services in Call Centers:** Consider a customer's experience with a call-center, that serves customers in a FCFS manner. The total delay (waiting time) for the customer is equal to the sum of the service times of all the customers ahead of him. As the customer awaits service, the call-center can provide no information, partial information or complete information about his eventual delay, thus affecting his utility while being a part of the process. While services in call centers have also been used as examples to understand consumers' abandonment decisions while awaiting service, we assume that a participating consumer never abandons the process.

- **Review Process of Manuscripts:** Consider a researcher's post-submission experience of a manuscript (flow unit) in the review process of a journal. For example, on Manuscript Central, the various tasks after the submission of a manuscript include editorial office check, assessment of the referees, assessment of the associate editor, and decision of the department editor. All else equal, the researcher prefers a low delay for the review. The journal could provide no information, partial information, or complete information about the delay while the researcher awaits the completion of the review.

As these examples suggest, the firm's chosen information-disclosure strategy affects consumers' utility during their sojourn through the process. Further, it should be clear that the firm's preferred strategy depends on a variety of factors, including the number of tasks in the process, the distribution of the task durations, and consumers' utility from and sensitivity to information.

1.2. Our Contributions

We summarize our key contributions below:

- **One-Task Process:** We study two information disclosure strategies for a process consisting of one task: an *opaque* strategy, where the firm does not disclose any information (i.e., operate the service process as a black-box), or a *transparent* strategy, where the firm resolves all uncertainty by disclosing the delay at the start of the process.[1] We show that even if the information about the delay is available at no cost to the firm, the opaque strategy can be better than the transparent strategy for the consumer (Section 4; Proposition 1). This result is in stark contrast to the conventional wisdom in the strategic queueing literature that the full information disclosure strategy is optimal for the consumer; see e.g., Guo and Zipkin (2007), Lingenbrink and Iyer (2019) – these papers exclusively consider the instrumental value of information.

[1] The firm may have private information that enables it to make an accurate forecast about the delay. For instance, in the context of fulfilment in online retail, the firm may have an accurate estimate of the total load and available capacity at a fulfilment center, that enables it to accurately predict the delivery lead time.

- **Two-Task Process:** The presence of two (or in general, multiple) tasks allows us to study a richer set of information disclosure strategies. In addition to the opaque and transparent strategies, we analyze the *current-task-identity* strategy, where the firm discloses the identity of the task being performed at any time before the completion of the process, and the *next-task-information* strategy, where the firm resolves the uncertainty sequentially, i.e., disclose task durations at the beginning of each task.

  Through multiple pairwise comparisons between these information disclosure strategies, our main takeaway is that less informative disclosure strategies can be better for a consumer than more informative disclosure strategies (Section 5; Propositions 3 – 6). Further, for a specific choice of the belief-based utility (namely, the piecewise linear model), we obtain the firm's preferred choice across all these strategies (Proposition 7). We also present conditions under which the opaque or the current-task-identity strategy are the preferred choices (Section 5; Propositions 8 – 9).

- **Role of Number of Tasks in a Process:** For robustness, we extend our results for the two-task model to a process consisting of a sequence of n-tasks – this analysis is presented in Appendix B. We identify the impact of the number of tasks in a process on the firm's choice of disclosure. The key takeaway from this analysis is that the opaque strategy becomes more favorable as the number of tasks increases, and can be the preferred choice if the number of tasks is very high (Propositions B.2 – B.4).

- **Role of Anxiety Costs:** Anxiety arises as a result of variability in the value that the consumer derives from the process (Iyer and Zhong 2020). Consumer anxiety exacerbates the cost of waiting, thereby making waiting more painful. We identify the role of consumers' anxiety costs on the choice of disclosure. Our key finding here is that greater anxiety makes transparency more attractive to a consumer (Section 6; Propositions 10 – 11).

The rest of the paper is organized as follows. We review two streams of related literature, namely delay disclosure in service OM and process transparency, in Section 2. Some preliminaries related to belief-based utility and the canonical consumer utility model are discussed in Section 3. We present our main model for a process consisting of one task in Section 4. In Section 5, we analyze a process that consist of two tasks, that allows for a richer set of information disclosure policies. We relegate the analysis for an n-task process to Appendix B; however, we briefly discuss our key result – the impact of the number of tasks on the firm's information provision strategy (Proposition B.2) – in Remark 4 at the end of Section 5. Section 6 considers the role of consumers' anxiety costs on the choice of the firm's disclosure strategy. Section 7 concludes.

2. Related Literature

Our paper relates to two streams of work: Delay Disclosure in Service Operations Management (OM) and Process Transparency.

2.1. Delay Disclosure in Service OM

Prior literature in OM that studies information provision (by a firm) to consumers considers the instrumental role/value of information. For instance, in the context of a queue, the instrumental information could either be the exact delay that a consumer faces upon arrival, or queue length (thereby providing consumers with partial information about the delay). The literature on instrumental information sharing and delay disclosure in queues is extensive, starting from the pioneering work by Edelson and Hilderbrand (1975); we refer the readers to Hassin and Haviv (2003) for a comprehensive review.

Closer to our interest, Allon et al. (2011) and Lingenbrink and Iyer (2019) consider examples from service operations where information about the delay is shared with a consumer prior to joining a queue. Both papers consider an $M/M/1$ queue, where the queue length is unobservable. Allon et al. (2011) model a cheap-talk game between the firm (sender) and the consumer (receiver). The consumer updates his beliefs on the queue length based on the firm's messages and chooses whether to join the queue or balk. They show that partially-informative equilibria exist (i.e., the babbling equilibrium is not the only equilibrium). Lingenbrink and Iyer (2019) study the firm's optimal messaging strategy through a persuasion game. They show that the optimal (state-dependent) strategy involves the firm recommending the customers to join the queue if the queue length is below a threshold and not join the queue otherwise. The exact threshold is higher than the complete-information benchmark. In contrast to the cheap-talk model of Allon et al. (2011), Guo and Zipkin (2007) consider an $M/M/1$ queue where any information shared is truthful by assumption. They analyze three information provision strategies to consumers upon arrival: no information, full information, and queue-length information. In the aforementioned papers, if a consumer chooses to join a queue, it is rational for the consumer to wait until completion.

A separate stream of work analyzes the impact of information sharing on abandonment decisions after a consumer joins a queue; in these models, abandonment is an outcome of rational decision-making. We refer the readers to Chapter 5 of Hassin and Haviv (2003). Relatedly, Cui and Veeraraghavan (2016) consider *blind* queues, where consumers might not have complete information about the system parameters (e.g., the service rate), and analyze the role of (full) information revelation by the service provider.

Similar to us, Yuan et al. (2019) focus on consumers' waiting experience during their sojourn in the firm's process. Specifically, they analyze the role of entertainment options in waiting areas that reduce consumers' marginal disutility from waiting. They show that firms may engage in *coopetition*: an outcome where firms choose to jointly offer such options and compete on prices, instead of acting as a monopolist.

Yu et al. (2017) study the impact of delay announcements on customer behavior using a structural model of consumer behavior in a queue. They find that the cost of waiting (per unit time) decreases with waiting times associated with the announcements. Cui et al. (2020) empirically study how online retailers can tailor the promised delivery lead-time to incentivize customer-purchase.

In the cases discussed above, the firm uses information as a lever to incentivize consumers to participate in trade (e.g., join the queue). Our work complements this literature in that we consider the *non-instrumental* role of information. Specifically, we take the customers' participation as given, and study how information provided (by the firm) *in the interim* – i.e., while customers await service completion, but after the purchase/join decision (e.g., in a queue) – affects their utility.

2.2. Process Transparency

A second stream of research our work relates to is that of process transparency. The *process view* of a firm considers the firm as a process that transforms inputs into outputs, via a set of resource-consuming activities. The fundamental unit of transformation is defined to be a "flow unit" and basic measures of operational performance include flow time, inventory, and throughput (flow rate); see e.g., Anupindi et al. (2012), Cachon and Terwiesch (2013), Bo et al. (2019).

In this context, one aspect of processes that has received limited (theoretical) attention in the OM literature is that of process transparency. Consider a process that consists of a sequence of tasks. If the task durations are deterministic and the start time of the process is perfectly observed by the customer, then there is no uncertainty either in the flow time or in the identity of the task being performed at any instant before the completion of the process. In this case, process transparency is redundant. On the other hand, if task durations are random and the process is opaque, then the flow time and the task identities are uncertain. A fundamental question begets: How transparent should a process be?

The extant literature on process transparency explores the roles of behavioral motives, e.g., reciprocity through visual transparency between service providers and consumers, and its impact on perceived value by consumers (Buell and Norton 2011, Buell et al. 2017). In the context of delivery updates in online retail, Bray (2019) shows the presence of peak-end effect, i.e., the perceived value by customers is higher when the task completion updates occur closer to the completion of the process.

To our knowledge, ours is the first paper that proposes an analytical model to understand the incentives for process transparency. Specifically, we model the interactions between a firm (that provides a service) and a consumer. The firm's service is modeled as a process that consists of a fixed number of tasks, each of random duration. Consumers are delay-sensitive, where the delay is equal to the flow time of the customer.

We analyze commonly observed information-disclosure policies about flow time. At the one extreme, the firm can operate the process as a black-box (*opaque*) and reveal no information about the realized flow time while the customer awaits completion. On the other extreme, if the firm has perfect forecasting ability on the durations of each task, the firm can resolve all uncertainty about the flow time by truthful disclosure before the start of the process (*transparent*). The firm can also provide information passively by disclosing the identity of the task being performed at any instant before completion (commonly observed in many processes), or sequentially reveal the task duration before the start of each task. We model the utility that a consumer realizes from the consumption of information (about the flow time) while the flow unit is still in process.

Our analysis uses models of belief-based utility from the recent literature on information design in Economics. Before we proceed to our model in Section 4, we review some preliminaries on belief-based utility and describe the canonical model in the following section.

3. Preliminaries on Belief-Based Utility

Recent work in Economics suggest that agents realize utility from non-instrumental information. We provide the following illustrative quotes :

> It is both intuitive and well documented that beliefs about future consumption or life events directly affect well-being. For example, an individual may enjoy looking forward to an upcoming vacation and particularly so if the risk of severe weather conditions became very unlikely; on the other hand, the same individual may worry about a future medical procedure he determined to undertake . . . There is also widespread evidence from other fields discussing how anticipation of pain produces psychological-stress reactions.
>
> — Dillenberger and Raymond (2018)

> People are sometimes willing to pay a cost to change how they receive news over time, even though the information in question does not help them make better decisions.
>
> — Duraj and He (2019)

For concreteness, consider the following examples by Dillenberger and Raymond (2018) and Ely et al. (2015), respectively:

(a) An individual has a vacation upcoming in a few days. The weather on the day of the vacation – the outcome of interest – is uncertain and is realized only on the day of the vacation; the individual has preferences over the outcome (e.g., the individual prefers good weather over bad weather on the day of vacation). He monitors the weather forecast periodically and updates his belief (on the outcome) based on the forecast; the individual is a rational Bayesian. The individual may enjoy (resp., not enjoy) looking forward to the upcoming vacation if good (resp., bad) weather is more likely.

(b) An individual watches a sporting event/political debate. The identity of the eventual winner – the outcome of interest – is uncertain and is realized at the end of the game/debate. The individual has preferences over the outcome (e.g., the individual may have a favorite team/candidate and prefers his favorite team/candidate to win). He watches the game/debate; as it unfolds, the various events provide signals of the outcome (eventual winner). The individual periodically updates his belief on the outcome, based on these events. The individual may enjoy (resp., not enjoy) watching the game/debate if his favored team/player is more (resp., less) likely to win.[2]

In both these examples, observe that the agent realizes utility both from consumption and in the interim (before consumption). Below, we formally describe the canonical consumer-utility model from the recent literature in Economics, where the consumer realizes utility from consumption (material/consumption utility) and *news* (belief-based utility); Duraj and He (2019) and Dillenberger and Raymond (2018).

3.1. Canonical Consumer-Utility Model: Consumption and Belief-Based Utilities

The model described below is a discrete-time model. We employ a continuous time version in our main model in Sections 4 and 5.

Consider an environment that consists of one agent and nature. Time is discrete, consisting of T periods and indexed by $t \in [T]$.[3] At the start of period 1, nature chooses a state among a finite

[2] Figure 1 of Ely et al. (2015) refers to win probability plots – the likelihood of a player's win – during a game of tennis. Such in-game win probability plots are commonplace across sports (Lock 2016).

[3] For an integer $x > 0$, we denote the set $\{1, 2, \ldots, x\}$ by $[x]$.

number of states $\Omega = \{\omega_1, \omega_2, \ldots, \omega_N\}$ from a distribution $\boldsymbol{\pi}_0 = \{\pi_0(\omega_i)\}_{i=1}^N$, but does not inform the agent. Let $\omega \in \Omega$ denote a generic state; $\boldsymbol{\pi}_0$ is common knowledge. The agent does not have any actions. Consumption occurs at the end of period T, when the agent realizes a state-dependent consumption utility v_ω; consumption does not occur in any other periods. Without loss of generality, we assume that $\omega \neq \omega'$ for any $\omega, \omega' \in \Omega$, $\pi_0(\omega) > 0$ for all $\omega \in \Omega$, and $v_{\omega_1} < v_{\omega_2} < \ldots < v_{\omega_N}$. Therefore, ω_1 (resp., ω_N) is the worst (resp., best) state for the agent. Let $\boldsymbol{\Pi} = \Delta(\Omega)$ denote the set of possible distributions on Ω; $\boldsymbol{\pi}_0 \in \boldsymbol{\Pi}$.

In each period $t \in [T]$, nature provides the agent with *news*, i.e, messages (or signals) about ω through an (exogenous) messaging device. Let $\mathcal{M}$ denote the set of messages and $\mathbf{M} = \Delta(\mathcal{M})$ denote the set of all distributions over $\mathcal{M}$. The messaging device sends messages according to $\boldsymbol{\sigma} = \{\sigma_t\}_{t=1}^T$, where $\sigma_t(\cdot|h^{t-1}, \omega) \in \mathbf{M}$ is a distribution over messages in period t that depends on the state ω and the history $h^{t-1} \in H^{t-1} := (\mathcal{M})^{t-1}$, i.e., messages sent thus far; $\boldsymbol{\sigma}$ is common knowledge.

At the end of each period $t \in [T]$, the agent forms a Bayesian posterior belief, $\boldsymbol{\pi}_t$ about the state ω after the history h^t of messages; clearly $\boldsymbol{\pi}_t \in \boldsymbol{\Pi}$. This belief $\boldsymbol{\pi}_t$ is rational, and is calculated using $\boldsymbol{\pi}_0$ and the knowledge of the messaging device $(\mathcal{M}, \boldsymbol{\sigma})$. In period T, nature reveals the state (which it chose at the start of period 1), say $\tilde{\omega}$, to the agent. Then, the agent's belief is the degenerate distribution $\boldsymbol{\pi}_T(\tilde{\omega}) = 1$ and $\boldsymbol{\pi}_T(\omega) = 0$ for any $\omega \neq \tilde{\omega}$, $\omega \in \Omega$. We denote the degenerate belief by $\boldsymbol{\pi}_T = 1 \circ \tilde{\omega}$.

The agent derives utility based on the changes in his belief about the consumption in period T. Specifically, let $\boldsymbol{\mu} : \boldsymbol{\Pi} \times \boldsymbol{\Pi} \to \Re$ denote a mapping from his new and old beliefs into the real numbers. He realizes $\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1})$ at the end of period t, $t \in [T]$. Utility flow is undiscounted and $\boldsymbol{\mu}(\cdot|\cdot)$ is fixed across periods. The total expected utility of the agent is the sum of the consumption utility in period T and the belief-based utilities across all periods $t \in [T]$.

$$\begin{aligned} \text{Total Expected Utility } &= \mathbb{E}_{\omega \sim \boldsymbol{\pi}_0} \Bigg[\underbrace{v_\omega}_{\text{consumption utility in state } \omega} + \sum_{t \in [T]} \underbrace{\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1})}_{\text{belief-based utility in period } t} \Bigg] \\ &= \underbrace{\mathbb{E}_{\omega \sim \boldsymbol{\pi}_0}[v_\omega]}_{\text{Expected Consumption Utility, } U_M} + \underbrace{\sum_{t \in [T]} \mathbb{E}_{\omega \sim \boldsymbol{\pi}_0} \left[\boldsymbol{\mu}\left(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}\right)\right]}_{\text{Expected Belief-Based Utility, } U_B} \end{aligned} \tag{1}$$

The first term (the expected consumption utility, U_M) is independent of $(\mathcal{M}, \boldsymbol{\sigma})$. Hence, it is sufficient to consider only the second term (the expected belief-based utility, U_B) in our comparison of different information-provisioning strategies (of a firm to a consumer); we analyze such comparisons in Sections 4 and 5.

Broadly, papers that study *news* utility adopt one of two belief-based utility models (Dillenberger and Raymond 2018):

(a) Anticipatory Utility Models (AUM), and
(b) Changing Beliefs Models (CBM).

In both these models, the total utility is additively separable in the belief-based utility and the material utility; the difference lies in the model for belief-based utility. Consider a piece of news, that leads to a change in the agent's beliefs on ω from $\boldsymbol{\pi}$ to $\boldsymbol{\pi}'$. The agent's belief-based utility from consuming this piece of news under the two models is as follows:

- Under AUM (Caplin and Leahy 2001), the agent's belief-based utility depends on his absolute levels of beliefs:

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu(\boldsymbol{\pi}')$$

  where $\mu : \boldsymbol{\Pi} \to \Re$ maps beliefs on ω to the real line.
- Under CBM (Kőszegi and Rabin 2009), the agent's belief-based utility depends on the change in his beliefs on ω:

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu\left(F(\boldsymbol{\pi}') - F(\boldsymbol{\pi})\right)$$

  where $F : \boldsymbol{\Pi} \to \Re$. Some examples are:
  - Duraj and He (2019) employ a mean-based changing beliefs model, where $F(\boldsymbol{\pi}) = \mathbb{E}_{\omega\sim\boldsymbol{\pi}}[v_\omega]$. Therefore,

$$\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}) = \mu\left(\mathbb{E}_{\omega\sim\boldsymbol{\pi}_t}[v_\omega] - \mathbb{E}_{\omega\sim\boldsymbol{\pi}_{t-1}}[v_\omega]\right)$$

  - Kőszegi and Rabin (2009) briefly discuss the above model, but predominantly focus on a percentile-based changing beliefs model, where

$$\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}) = \int_{p=0}^{1} \mu\left(G^{-1}_{\omega\sim\boldsymbol{\pi}_t}(p) - G^{-1}_{\omega\sim\boldsymbol{\pi}_{t-1}}(p)\right) dp$$

    where $G_{\omega\sim\boldsymbol{\pi}}(\cdot)$ (resp., $G^{-1}_{\omega\sim\boldsymbol{\pi}}(p)$) denotes the CDF (resp., p^{th}-percentile) of ω with distribution $\boldsymbol{\pi}$.

In line with recent research that models belief-based utility from consumption of *news* (Duraj and He 2019), and for analytical tractability, we adopt the mean-based changing beliefs model. We formally state this below:

Assumption 1. *In our main model, we employ the mean-based changing beliefs model. Specifically, the agent realizes the following belief-based utility from consuming a piece of news that leads to a change in beliefs $\boldsymbol{\pi} \to \boldsymbol{\pi}'$:*

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu\left(\mathbb{E}_{\omega\sim\boldsymbol{\pi}'}[v_\omega] - \mathbb{E}_{\omega\sim\boldsymbol{\pi}}[v_\omega]\right), \tag{2}$$

where $\mu(\cdot)$ is increasing, differentiable everywhere except possibly at 0, and $\mu(0) = 0$.

Kőszegi and Rabin (2006) refer to (2) as the *reference-dependent universal gain-loss utility* model, where :

(a) the gain-loss utility is itself derived from consumption utility, and
(b) the *reference* is determined endogenously through the prior beliefs, i.e., rational expectations held in the recent past about outcomes.

We discuss further assumptions on $\mu(\cdot)$ (e.g., loss aversion, diminishing sensitivity) as required in Sections 4 and 5.

Using Assumption 1 and (1), the agent's total expected utility can be written as:

$$\text{Total Expected Belief-Based Utility} = \sum_{t\in[T]} \mathbb{E}_{\omega\sim\boldsymbol{\pi}_0}\left[\mu\left(\mathbb{E}_{\omega\sim\boldsymbol{\pi}_t}[v_\omega] - \mathbb{E}_{\omega\sim\boldsymbol{\pi}_{t-1}}[v_\omega]\right)\right]. \tag{3}$$

Remark 1. **(Consequentialist Model)** A *consequentialist* model is one where an agent does not incur any belief-based utility, i.e., $\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = 0$. In the standard consequentialist model, all information-disclosure strategies are identical. ■

Remark 2. **(Suspense and Surprise)** Ely et al. (2015) consider a model where the agent's belief-based utility from consuming a piece of news depends on the amount of *suspense* or *surprise*. In the suspense (resp., surprise) model, the agent has a preference for suspense (resp., surprise). Formally, consider the agent's belief-based utility in period t, where his beliefs are $\boldsymbol{\pi}_t$. Suspense is the amount of variability in period-$(t+1)$ beliefs (e.g., standard deviation of $\boldsymbol{\pi}_{t+1}$), while surprise is the difference in beliefs from period-$(t-1)$ to period-t (e.g., the Euclidean distance between $\boldsymbol{\pi}_{t-1}$ and $\boldsymbol{\pi}_t$). Formally,

- Under the suspense model,

$$\boldsymbol{\mu}(\boldsymbol{\pi}_t) = \mu\left(\mathbb{E}\left[\sum_{i\in[N]} (\pi_{t+1}(\omega_i) - \pi_t(\omega_i))^2\right]\right),$$

  where $\pi_{t+1}(\omega_i)$ is random and $\mathbb{E}[\pi_{t+1}(\omega_i)] = \pi_t(\omega_i)$, due to the Martingale property of Bayesian updating.

- Under the surprise model, the agent has a preference for surprise:

$$\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}) = \mu\left(||\boldsymbol{\pi}_t - \boldsymbol{\pi}_{t-1}||_2\right)$$
$$\text{where } ||\boldsymbol{\pi}' - \boldsymbol{\pi}||_2 = \sum_{i\in[N]} (\pi'(\omega_i) - \pi(\omega_i))^2 \text{ for any } \boldsymbol{\pi}, \boldsymbol{\pi}' \in \Pi.$$

Their focus is on the entertainment value of news (e.g., sporting events, movies), which is different from the value of news (e.g., delay estimate, ETA, etc.) to a consumer in our model. ■

Unlike prior work in Economics, where the payoff-relevant variable is an exogenous state of nature (e.g., the weather in Example (a); the winner of the sporting event/debate in Example (b)), the payoff-relevant variable for a consumer in our model is the length of the horizon (i.e., the delay). Furthermore, in prior work, the lack of information provision (i.e., no news) during a period does not affect the consumer's beliefs about the payoff-relevant variable. However, in our model, the mere passage of time provides information (*bad* news) to the consumer about the realized delay. We also remark that the messaging strategies we analyze are commonly observed in service processes.

4. One-Task Processes

Consider a process that comprises of one task. Let the task duration (equivalently, the total delay that a consumer experiences) be denoted by D. The consumer receives a material utility of

$$u_M = v - D \tag{4}$$

upon completion of the task (i.e., at time instant $t = D$). The term v denotes the material revenue that the consumer receives upon completion of the task and D is the cost due to delay.[4] The revenue v and the distribution of the task duration is common knowledge. Specifically, we assume that D is exponentially distributed with mean $\frac{1}{\lambda}$:

$$D \sim \mathsf{Exp}(\lambda) \equiv \boldsymbol{\pi}_0.$$

Since the focus of this paper is on sharing *non-instrumental information*, we assume the following:

ASSUMPTION 2. ***(Participation by Assumption)*** *We assume that consumers always participate in trade. Furthermore, given our focus on consumers' waiting-experience in the process, we assume that consumers do not abandon the process after participation.*

[4] The material revenue could be deterministic or random. If it is random, then v denotes the expected material revenue from the completion of the flow unit.

Consequently, any information about D *after purchase* is non-instrumental to the consumer.

We compare two information-provision (i.e., delay disclosure) strategies post-consumer-purchase for the firm: a one-shot resolution of uncertainty via disclosure at the start of the process (called transparent, denoted by TP) or no resolution (called opaque, denoted by OP). The transparent (resp., opaque) strategy in our model is identical to that of full information (resp., no information) in Guo and Zipkin (2007). We formally describe these two strategies below.

4.1. Sequence of Events

The sequence of events post-consumer-purchase are as follows:

- Stage 1: The firm commits to one of two information disclosure strategies – TP or OP. Let $\boldsymbol{\sigma}$ denote the firm's chosen strategy; $\boldsymbol{\sigma} \in \mathsf{M} = \{\mathsf{TP}, \mathsf{OP}\}$.
- Stage 2: Nature chooses $D \sim \mathsf{Exp}(\lambda)$ and privately informs the firm of D.
- Depending on the firm's choice in stage 1,
  - If $\boldsymbol{\sigma} = \mathsf{TP}$, then, at time instant $t = 0$, the firm truthfully reveals D to the consumer; the firm does not send any further message at time instant $t \in (0, D)$.
  - If $\boldsymbol{\sigma} = \mathsf{OP}$, then the firm does not send any message to the consumer at time instant $t \in [0, D)$.
- Time instant $t = D$: The consumer receives his flow unit and realizes his material utility.

From stages 1 and 2 in the sequence of events above, notice that the firm commits to an information-disclosure strategy *before* learning the realized delay. The firm's strategy can be viewed as a long-term decision that does not depend on the realized delay.

4.2. Belief-Based Utility

Consider a realized delay D. We model the belief-based utility that a consumer realizes from consuming information about D after purchase. Consider any instant $t \in [0, D]$. Let $\boldsymbol{\pi}_t$ denote the consumer's belief on D at time t, and consider an infinitesimal amount of time dt. The belief-based utility that the consumer realizes during the interval $[t, t + dt)$ under the information-disclosure strategy $\boldsymbol{\sigma}$ is:

$$u_B^{\boldsymbol{\sigma}}[t, t + dt) = \boldsymbol{\mu}(\boldsymbol{\pi}_{t+dt} | \boldsymbol{\pi}_t).$$

4.2.1. Belief Evolution

- Under TP, all uncertainty about D is fully resolved at time $t=0$. Therefore, the beliefs on D evolve as follows:

$$\boldsymbol{\pi}_0 \to 1\circ D \to 1\circ D \to \ldots \to 1\circ D$$

where $1\circ D$ denotes the degenerate belief on D.

- Under OP, the consumer's uncertainty about D is resolved *gradually*. The beliefs on D, in increments of an infinitesimal amount of time dt, evolve as follows:

$$\boldsymbol{\pi}_0 \to \boldsymbol{\pi}_{dt} \to \ldots \to \boldsymbol{\pi}_t \to \boldsymbol{\pi}_{t+dt} \to \ldots \to \boldsymbol{\pi}_D \to 1\circ D.$$

Combining the two statements above, we have that for any $t \in (0, D]$,

$$\boldsymbol{\pi}_t^{\boldsymbol{\sigma}} = \begin{cases} 1\circ D, & \text{if } \boldsymbol{\sigma} = \mathsf{TP}; \\ \mathsf{Exp}\left(\frac{1}{t+\frac{1}{\lambda}}\right) \equiv D|D>t, & \text{if } \boldsymbol{\sigma} = \mathsf{OP}. \end{cases} \tag{5}$$

Define the following:

$$\overline{D}_t = \mathbb{E}_{D\sim\boldsymbol{\pi}_t^{\mathsf{OP}}}[D] \equiv \mathbb{E}[D|D>t]. \tag{6}$$

The quantity $\overline{D}_t - t$ is referred to as the mean residual life of D at t, and is commonly employed in reliability analysis (Lai and Xie 2006). The belief-based utilities under the two information-disclosure strategies are as follows:

$$u_B^{\mathsf{TP}}[t, t+dt) = \begin{cases} \boldsymbol{\mu}(1\circ D|\boldsymbol{\pi}_0), & \text{if } t=0; \\ 0, & \text{if } t>0. \end{cases} \tag{7a}$$

$$u_B^{\mathsf{OP}}[t, t+dt) = \begin{cases} \boldsymbol{\mu}(\boldsymbol{\pi}_{t+dt}|\boldsymbol{\pi}_t), & \text{if } t<D; \\ \boldsymbol{\mu}(1\circ D|\boldsymbol{\pi}_D), & \text{if } t=D. \end{cases} \tag{7b}$$

Recall Assumption 1, where the belief-based utility is as prescribed by the mean-based changing beliefs model. Corresponding to a change in belief on D from $\boldsymbol{\pi}$ to $\boldsymbol{\pi}'$, the belief-based utility is:

$$\begin{aligned} \boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) &= \mu\left(\underbrace{\mathbb{E}_{D\sim\boldsymbol{\pi}'}[u_M] - \mathbb{E}_{D\sim\boldsymbol{\pi}}[u_M]}_{=\text{increase in the mean material payoff}}\right) \\ &= \mu\left(\underbrace{\mathbb{E}_{D\sim\boldsymbol{\pi}}[D] - \mathbb{E}_{D\sim\boldsymbol{\pi}'}[D]}_{=\text{decrease in mean delay}}\right), \end{aligned} \tag{8}$$

where $\mu(\cdot)$ is increasing, differentiable everywhere except (possibly) at 0, and $\mu(0)=0$. Also, let $\mu_+ : \Re_{++} \to \Re_+$ (resp., $\mu_- : \Re_{--} \to \Re_-$) be defined by $\mu_+(x) = \mu(x)$ for $x \in (0, \infty)$ (resp., $\mu_-(x) = \mu(x)$ for $x \in (-\infty, 0)$).

Consider a small interval $[t, t+dt)$ for any $t \in [0, D)$. Using (7) and (8), we have:

- Under TP:
$$u_B^{\mathsf{TP}}[t,t+dt) = \begin{cases} \mu\left(\frac{1}{\lambda} - D\right), & \text{if } t = 0; \\ 0, & \text{if } t > 0. \end{cases} \tag{9}$$
Therefore, $u_B^{\mathsf{TP}}[t,t+dt) = 0$ for any $t > 0$. Furthermore, $u_B^{\mathsf{TP}}[0,dt) > 0$ iff $D < \frac{1}{\lambda}$.[5]

- Under OP:
$$\begin{aligned} u_B^{\mathsf{OP}}[t,t+dt) &= \mu\left(\overline{D}_t - \overline{D}_{t+dt}\right) \\ &\approx -\mu'(0^-)\overline{D}_t' dt \quad (\text{if } \overline{D}_t \text{ is differentiable}). \end{aligned}$$
From (5), we have that $\overline{D}_t = t + \frac{1}{\lambda}$.
$$u_B^{\mathsf{TP}}[t,t+dt) = \begin{cases} -\mu'(0^-)dt, & \text{if } t < D; \\ \mu\left(\frac{1}{\lambda}\right), & \text{if } t = D. \end{cases} \tag{10}$$
From (10), $u_B^{\mathsf{OP}}[t,t+dt) < 0$ for all $t < D$, and $u_B^{\mathsf{OP}}[D,D+dt) > 0$.

4.3. Objectives

The consumer does not have any actions, but realizes utility (disutility). The firm fully internalizes the consumer's total utility, which is the sum of the belief-based utility and the consumption/material utility.

$$\textbf{(Firm's Objective)} \quad \max_{\boldsymbol{\sigma}\in\mathsf{M}} \quad U = \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[\underbrace{\int_{t=0}^{D} u_B^{\boldsymbol{\sigma}}[t,t+dt)}_{\text{Belief-Based Utility}} + \underbrace{u_M}_{\text{Material Utility}}\right]$$
$$= \underbrace{\mathbb{E}\left[\int_0^D u_B^{\boldsymbol{\sigma}}[t,t+dt)\right]}_{U_B^{\boldsymbol{\sigma}}} + \underbrace{\left(v - \frac{1}{\lambda}\right)}_{U_M}$$

We comment on this choice of the firm's objective. In general, the incentives of the firm and the consumers may not be perfectly aligned. In our model, we assume that the cost of adopting any information provision strategy to the firm is zero. Furthermore, the information provided to the consumers is after purchase, and does not affect the firm's revenues. Therefore, from the perspective of the firm, all information provision strategies are identical. Consequently, we choose to identify the consumer-preferred information provision strategy, i.e., one that maximizes the consumer's utility. In many sender-receiver information games in Economics, the assumption of a *benevolent* sender is often made to identify receiver-preferred outcomes, especially if the sender does not have any incentives (Duraj and He 2019, Lipnowski and Mathevet 2018).

As remarked in Section 3.1, the (expected) material utility is independent of the information provision strategy; therefore we restrict attention to $U_B^{\boldsymbol{\sigma}}$.

[5] Per Duraj and He (2019), we interpret $\mathcal{D}_G = \{D : D < \frac{1}{\lambda}\}$ as *good* states and $\mathcal{D}_B = \{D : D > \frac{1}{\lambda}\}$ as *bad* states.

4.4. Comparison

Consider a given realization of the delay, say D. The total stock of news is:

$$\text{Total stock of news for a given realization } D = \frac{1}{\lambda} - D$$

4.4.1. Transparent Process Under TP, all uncertainty is resolved at $t = 0$. Using (9), the (expected) belief-based utility under TP can be written as:

$$U_B^{\mathsf{TP}} = \mathbb{E}_{D \sim \boldsymbol{\pi}_0}\left[\mu\left(\frac{1}{\lambda} - D\right)\right]. \tag{11}$$

4.4.2. Opaque Process Corresponding to the realization D, the opaque process *reveals* the total stock of news over the time $[0, D]$ as follows:

$$\underbrace{\frac{1}{\lambda} - D}_{\text{total stock of news}} = \underbrace{\int_{t=0}^{D}(-dt)}_{\textit{bad}\text{ news released at a constant rate during } t \in [0,D)} + \underbrace{\frac{1}{\lambda}}_{\text{1 large piece of } \textit{good} \text{ news } \frac{1}{\lambda} \text{ released at } t=D}.$$

From (8), the belief-based utility of the consumer is:

$$\begin{aligned} u_B^{\mathsf{OP}}[0, D] &= \int_{t=0}^{D} \underbrace{\mu(-dt)}_{\approx -\mu'(0^-)dt} + \mu\left(\frac{1}{\lambda}\right) \\ &= -\mu'(0^-)D + \mu\left(\frac{1}{\lambda}\right). \end{aligned}$$

Therefore,

$$\begin{aligned} U_B^{\mathsf{OP}} &= \mathbb{E}_{D \sim \boldsymbol{\pi}_0}\left[u_B^{\mathsf{OP}}[0, D]\right] \\ &= -\mu'(0^-)\frac{1}{\lambda} + \mu\left(\frac{1}{\lambda}\right). \end{aligned} \tag{12}$$

Proposition 1, our main result for this section, characterizes the comparison of the (expected) belief-based utility under TP in (11) and OP in (12). We use the notation $\boldsymbol{\sigma} \succ \boldsymbol{\sigma}'$ (resp., $\boldsymbol{\sigma} \prec \boldsymbol{\sigma}'$) iff $U_B^{\boldsymbol{\sigma}} > U_B^{\boldsymbol{\sigma}'}$ (resp., $U_B^{\boldsymbol{\sigma}} < U_B^{\boldsymbol{\sigma}'}$).

Proposition 1. $\mathsf{OP} \succ \mathsf{TP}$ *iff* $\mu(\cdot)$ *satisfies* (13) *below.*

$$\mathbb{E}\left[\mu'\left(\frac{1}{\lambda} - D\right)\right] > \mu'(0^-). \tag{13}$$

The intuition behind Proposition 1 is as follows: If bad news is increasingly costly to the consumer, then he prefers to receive multiple pieces of small bad news (instead of one large piece of bad news). The term $\mu'(t)$ in the LHS of (13) for negative values of t is the marginal disutility of bad news at t. For (13) to hold, we require that $\mu'(t)$ for $t < \frac{1}{\lambda}$ be sufficiently large (i.e., the utility curve is sufficiently steep to the left of $\frac{1}{\lambda}$), while simultaneously $\mu'(0^-)$ not be too large.

4.5. Some Examples

Below, we present some examples for $\mu(\cdot)$ and the corresponding comparison of OP and TP.

EXAMPLE 1. (**Piecewise-Linear Utility Model**) Suppose

$$\mu(x) = \begin{cases} \rho_P x, & \text{if } x \geq 0; \\ \rho_N x, & \text{if } x < 0. \end{cases} \tag{14}$$

where $\rho_N > \rho_P > 0$. Then,

$$U_B^{\mathsf{OP}} = -\left(\frac{\rho_N - \rho_P}{\lambda}\right); \quad U_B^{\mathsf{TP}} = -\left(\frac{\rho_N - \rho_P}{e\lambda}\right).$$

Therefore, $\mathsf{OP} \prec \mathsf{TP}$. ■

EXAMPLE 2. (**CARA Utility Model**) Suppose

$$\mu(x) = \frac{1}{\rho}\left(1 - e^{-\rho x}\right), \tag{15}$$

where $\lambda > \rho > 0$. We provide an example of (15) in Figure 1. In this case, we have

$$U_B^{\mathsf{OP}} = \frac{1}{\rho}\left(1 - \frac{e^{-\frac{\rho}{\lambda}}}{1 - \frac{\rho}{\lambda}}\right); \quad U_B^{\mathsf{TP}} = \frac{1}{\rho}\left(1 - \frac{\rho}{\lambda} - e^{-\frac{\rho}{\lambda}}\right).$$

Therefore, $\mathsf{OP} \prec \mathsf{TP}$. ■

EXAMPLE 3. (**Linear-CARA Model**) Suppose

$$\mu(x) = \begin{cases} \frac{1}{\rho_N}\left(1 - e^{-\rho_N x}\right), & \text{if } x < 0; \\ \rho_P x, & \text{if } x \geq 0. \end{cases} \tag{16}$$

where $0 < \rho_N < \lambda$, $0 < \rho_P \leq 1$. We provide an example of (16) in Figure 1. Here, we have

$$U_B^{\mathsf{OP}} = \frac{\rho_P - 1}{\lambda}; \quad U_B^{\mathsf{TP}} = \frac{-\lambda + \lambda\rho_P - \rho_N\rho_P}{e\lambda(\lambda - \rho_N)}.$$

It is straightforward to show the following: $\mathsf{OP} \succ \mathsf{TP}$ iff the 3-tuple $(\lambda, \rho_P, \rho_N)$ lies in the following region: $\lambda > 0$, $\rho_P \in (0, 1]$, and $\rho_N \in (\overline{\rho}_N, \lambda)$, where

$$\overline{\rho}_N = \lambda\left(\frac{1 - \rho_P}{\underbrace{\left(\frac{e}{e-1}\right)}_{\approx 1.581} - \rho_P}\right).$$

■

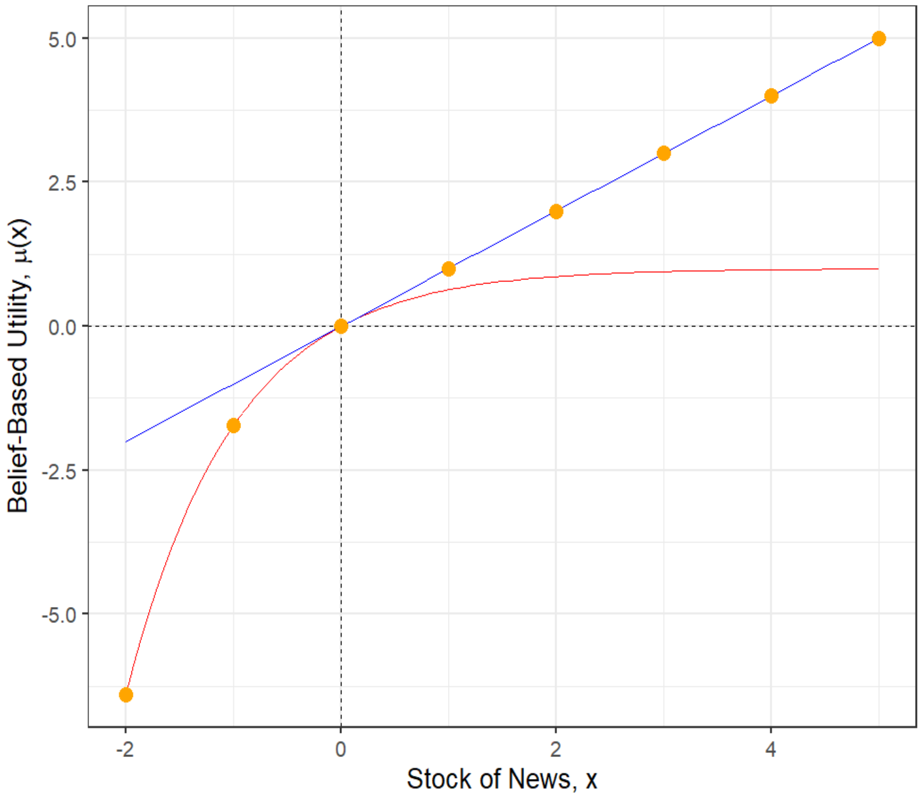

Figure 1 **The orange dotted line denotes the linear-CARA utility model in (16) with $\rho_P = \rho_N = 1$. The red-curve is the CARA utility function in (15) with $\rho = 1$**

4.6. Prospect Theory-Based Utility Model

Proposition 2 below compares the two information disclosure strategies – TP and OP – for a special class of belief-based utility models, obtained by making two additional assumptions on $\mu(\cdot)$.

PROPOSITION 2. *Consider $\mu(\cdot)$ that satisfies the following properties:*

1. *$\mu(\cdot)$ is increasing, continuous and differentiable everywhere except possibly at 0.*
2. *$\mu(0) = 0$.*
3. *$\mu(x) < -\mu(-x)$ for all $x > 0$.*
4. *$\mu''(x) < 0 < \mu''(-x)$ for all $x > 0$.*

Then, OP $\prec$ TP.

Properties 1 and 2 are as stated in Assumption 1. Properties 3 and 4 are commonly referred to as (strict) loss aversion and (strict) diminishing sensitivity, respectively.[6] Duraj and He (2019) refer to this class of belief-based utility models by *news-utility models*, inspired from prospect theory

[6] Properties 3 and 4 are referred to in the *weak* sense if the respective inequalities hold weakly.

(Kahneman and Tversky 1979). The intuition behind Proposition 2 is that under this class of belief-based utility models, the marginal disutility to a consumer from bad news is decreasing in the amount of bad news, i.e., $\mu'(\cdot)$ is highest at 0^-. Therefore, it is impossible that (13) holds.

5. Two-Task Processes

Consider two tasks in sequence, denoted by 1 and 2. Let X_1 and X_2 denote the processing times of these two tasks. We assume that:

$$X_1, X_2 \sim_{i.i.d} \mathsf{Exp}(\lambda).$$

The distribution of the task durations is common knowledge. The total delay is:

$$D = X_1 + X_2 \implies D \sim \mathsf{Erlang}(2, \lambda)\,(\equiv \boldsymbol{\pi}_0)\,.$$

The consumer utility model is the same as that in Section 4. We consider four possible information-provision strategies, in the increasing order of transparency (i.e., the information revealed):

- **Opaque (OP):** The firm does not disclose any information to the consumer at time instant $t \in [0, D)$. The consumer receives his flow unit at $t = D$ and realizes his material utility.
- **Current Task Identity (CTI):** The firm discloses the identity of the current task. That is, if $t \in [0, X_1]$, the firm sends a message to signal task 1; if $t \in [X_1, X_1 + X_2]$, the firm sends a message to signal task 2.
- **Next Task Information (NTI):** The firm discloses the time for a task just before the start of the task (e.g., information about a task is revealed in a just-in-time manner). Thus, the firm reveals X_1 at $t = 0$ and X_2 at $t = X_1$.
- **Transparent (TP):** The firm discloses the total delay $D = X_1 + X_2$ at the start of the process, i.e., $t = 0$.

The sequence of events is the same as in Section 4. The firm commits to one of OP, CTI, NTI, TP, before nature's draw of X_1, X_2. Let $\boldsymbol{\sigma}$ denote the firm's disclosure strategy; $\boldsymbol{\sigma} \in \mathsf{M} = \{\mathsf{OP}, \mathsf{CTI}, \mathsf{NTI}, \mathsf{TP}\}$.

Remark 3. For a one-task process, the strategies OP and CTI are identical, and the strategies NTI and TP are identical.

5.1. Analysis

Below, we analyze the four information-provision strategies of the firm – OP, CTI, NTI, TP. As shown in Sections 3 and 4, it is sufficient to restrict attention to the belief-based utility $U_B^{\boldsymbol{\sigma}}$, since the (expected) material utility is identical across all information provision strategies.

Consider a pair of realizations X_1, X_2; $D = X_1 + X_2$. The total stock of news corresponding to D is $\frac{2}{\lambda} - D$.

5.1.1. Opaque Process At any time t, we denote the belief on D by $\boldsymbol{\pi}_t$. Recall the definition of $\overline{D}_t$ from (6):

$$\overline{D}_t = \mathbb{E}_{D\sim\boldsymbol{\pi}_t}[D] \equiv \mathbb{E}\left[D|D>t\right].$$

If $D \sim \mathsf{Erlang}(2,\lambda)$, then $\overline{D}_t = t + \frac{2}{\lambda} - \frac{t}{1+t\lambda}$. Therefore, under OP, the total stock of news is revealed as follows:

$$\underbrace{\frac{2}{\lambda} - D}_{\text{total stock of news}} \quad = \quad \underbrace{\int_{t=0}^{D} \left(\overline{D}_t - \overline{D}_{t+dt}\right)}_{\text{news revealed between } t\in[0,D)} \quad + \quad \underbrace{\left(\overline{D}_D - D\right)}_{\text{news revealed at } t=D} \quad .$$

Consider any time $t \le D$ and a small interval $[t, t+dt)$. Using (8),

$$u_B[t,t+dt) = \begin{cases} \underbrace{\mu\left(\overline{D}_t - \overline{D}_{t+dt}\right)}_{=-\mu'(0^-)\overline{D}_t'dt}, \text{ if } t < D; \\ \\ \mu(\overline{D}_D - D), \qquad \text{if } t = D. \end{cases}$$

where $\overline{D}_t - t$ denotes the mean residual life of an $\mathsf{Erlang}(2,\lambda)$ at t. Therefore,

$$\begin{aligned} U_B^{\mathsf{OP}} &= \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[\int_{t=0}^{D} -\mu'(0^-)\overline{D}_t' dt + \mu(\overline{D}_D - D)\right] \\ &= \mathbb{E}\left[-\mu'(0^-)\left(\overline{D}_D - \overline{D}_0\right) + \mu(\overline{D}_D - D)\right] \\ &= \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[-\mu'(0^-)\frac{D^2\lambda}{1+D\lambda} + \mu\left(\frac{2+D\lambda}{\lambda(1+D\lambda)}\right)\right]. \end{aligned} \tag{17}$$

5.1.2. Current Task Identity We break the analysis into two intervals: (a) $t \in [0, X_1]$ and (b) $t \in [X_1, X_1 + X_2]$.

(a) At any time instant $t < X_1$, under CTI, we have:

$$\overline{D}_t = \mathbb{E}[D|X_1 > t] = \mathbb{E}\left[X_1|X_1>t\right] + \mathbb{E}\left[X_2|X_1>t\right].$$

Using the memorylessness property of the exponential distribution and the independence of X_1 and X_2, we have

$$\overline{D}_t = \left(t + \frac{1}{\lambda}\right) + \frac{1}{\lambda}.$$

Consider a small interval $[t, t+dt)$ for $t \leq X_1$. The belief-based utility in this interval is:

$$u_B[t, t+dt) = \begin{cases} \underbrace{\mu(\overline{D}_t - \overline{D}_{t+dt})}_{\approx -\mu'(0^-)dt}, & \text{if } t < X_1; \\ \mu\left(\frac{1}{\lambda}\right), & \text{if } t = X_1. \end{cases}$$

(b) Now, consider any time instant $t \in [X_1, X_2)$. Using the same arguments as above, we have:

$$\begin{aligned} \overline{D}_t = \mathbb{E}\left[D | X_1, X_2 > t - X_1\right] &= X_1 + \mathbb{E}\left[X_2 | X_2 > t - X_1\right] \\ &= t + \frac{1}{\lambda}. \end{aligned}$$

Consider any time $t \in [X_1, X_1 + X_2]$ and a small interval $[t, t+dt)$. The belief-based utility in this interval is:

$$u_B[t, t+dt) = \begin{cases} \underbrace{\mu(\overline{D}_t - \overline{D}_{t+dt})}_{\approx -\mu'(0^-)dt}, & \text{if } t \in [X_1, X_1 + X_2); \\ \mu\left(\frac{1}{\lambda}\right), & \text{if } t = X_1 + X_2. \end{cases}$$

That is, under CTI, the total stock of news is revealed as follows:

$$\begin{aligned} \underbrace{\frac{2}{\lambda} - D}_{\text{total stock of news}} = \underbrace{\int_0^{X_1} (-dt)}_{\textit{bad} \text{ news revealed between } t \in [0, X_1)} + \underbrace{\frac{1}{\lambda}}_{\textit{good} \text{ news revealed at } t = X_1} + \\ \underbrace{\int_{X_1}^{X_1+X_2} (-dt)}_{\textit{bad} \text{ news revealed between } t \in (X_1, X_1+X_2)} + \underbrace{\frac{1}{\lambda}}_{\textit{good} \text{ news revealed at } t = X_1+X_2}. \end{aligned}$$

Using (a) and (b), we have:

$$\begin{aligned} U_B^{\mathsf{CTI}} &= \mathbb{E}_{X_1, X_2 \sim \mathsf{Exp}(\lambda)}\left[u_B\left[0, X_1 + X_2\right]\right] \\ &= \mathbb{E}\left[\int_{t=0}^{X_1} -\mu'(0^-)dt + \mu\left(\frac{1}{\lambda}\right) + \int_{t=X_1}^{X_1+X_2} -\mu'(0^-)dt + \mu\left(\frac{1}{\lambda}\right)\right], \\ &= \mathbb{E}\left[-\mu'(0^-)\left(X_1 + X_2\right) + 2\mu\left(\frac{1}{\lambda}\right)\right] \\ &= -\mu'(0^-)\frac{2}{\lambda} + 2\mu\left(\frac{1}{\lambda}\right). \end{aligned} \tag{18}$$

Observe that the RHS in (18) is twice as that of the RHS in (12).

5.1.3. Next Task Information At any instant $t \in [0, X_1 + X_2]$, we have:

$$\overline{D}_t = \begin{cases} \frac{2}{\lambda}, & \text{if } t = 0; \\ X_1 + \frac{1}{\lambda}, & \text{if } t \in (0, X_1]; \\ X_1 + X_2, & \text{if } t > X_1. \end{cases}$$

That is, under NTI, the total stock of news is revealed as follows:

$$\underbrace{\frac{2}{\lambda} - D}_{\text{total stock of news}} = \underbrace{\left(\frac{1}{\lambda} - X_1\right)}_{\text{news revealed at } t=0} + \underbrace{\left(\frac{1}{\lambda} - X_2\right)}_{\text{news revealed at } t=X_1}.$$

Therefore, $u_B^{\mathsf{NTI}}[t, t+dt) = 0$ for all $t \neq \{0, X_1\}$. We can write the belief-based utility as follows:

$$\begin{aligned} U_B^{\mathsf{NTI}} &= \mathbb{E}_{X_1, X_2 \sim_{i.i.d} \mathsf{Exp}(\lambda)} \left[\mu\left(\frac{1}{\lambda} - X_1\right) + \mu\left(\frac{1}{\lambda} - X_2\right)\right] \\ &= \mathbb{E}\left[\mu\left(\frac{1}{\lambda} - X_1\right)\right] + \mathbb{E}\left[\mu\left(\frac{1}{\lambda} - X_2\right)\right] \\ &= 2\mathbb{E}\left[\mu\left(\frac{1}{\lambda} - X_1\right)\right]. \end{aligned} \tag{19}$$

where the last equality is because $X_1 =_d X_2$. Observe that the RHS in (19) is twice as that of the RHS in (11).

5.1.4. Transparent Process The firm resolves all uncertainty by disclosing D at $t = 0$ (i.e., the start of the process). The only instant where the consumer realizes utility is at $t = 0$. Therefore,

$$U_B^{\mathsf{TP}} = \mathbb{E}_{D \sim \boldsymbol{\pi}_0}\left[\mu\left(\frac{2}{\lambda} - D\right)\right]. \tag{20}$$

5.2. Pairwise Comparison

Arranged in the order of increasing transparency, our four strategies are: OP, CTI, NTI, and TP. We provide pairwise comparisons between successive pairs in this order. Specifically, we are interested in conditions under which a less-informative strategy is preferable to a more-informative strategy.

Definition 1. **(Strict Sub-Additivity)** $\mu(\cdot)$ is sub-additive if

$$\mu(x + y) < \mu(x) + \mu(y) \quad \forall x, y. \tag{21}$$

$\mu(\cdot)$ is weakly sub-additive if (21) holds as a weak inequality.

- OP **vs.** CTI: The comparison between OP and CTI is interesting because both these information-provisioning strategies can be adopted by the firm even if it does not learn the realized delay at $t=0$. On the contrary, the firm must learn the realized D (resp., X_1) at $t=0$ to adopt TP (resp., NTI).

PROPOSITION 3. $\mathsf{OP} \succ \mathsf{CTI}$ *iff* $\mu(\cdot)$ *satisfies:*

$$\mu'(0^-)\frac{0.596}{\lambda} > 2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\left[\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)\right]. \tag{22}$$

The piecewise-linear utility model in (14) *with* $\rho_N > \rho_P$ *satisfies* (22).

PROPOSITION 4. *Suppose* $\mu_+(x)$ *is concave. Then, a sufficient condition for* (22) *is:*

$$\mu'(0^-)\frac{0.596}{\lambda} > 2\mu\left(\frac{1}{\lambda}\right) - \mu\left(\frac{1.403}{\lambda}\right). \tag{23}$$

The CARA utility model in (15) *with* $\rho > 0$ *satisfies* (23).

- **CTI vs. NTI:** The following result compares CTI and NTI under the two task case; the comparison is straightforward from Proposition 1 and is therefore stated without proof.

PROPOSITION 5. $\mathsf{CTI} \succ \mathsf{NTI}$ *iff* $\mu(\cdot)$ *satisfies* (13).

- **NTI vs TP:** A sufficient condition for $\mathsf{NTI} \succ \mathsf{TP}$ is as follows.

PROPOSITION 6. *Suppose* $\mu(\cdot)$ *satisfies strict sub-additivity. Then, for any pair of realizations* X_1, X_2, *we have that*

$$U_B^{\mathsf{NTI}}(X_1, X_2) > U_B^{\mathsf{TP}}(X_1, X_2).$$

Therefore, $U_B^{\mathsf{NTI}} > U_B^{\mathsf{TP}}$ *and hence* $\mathsf{NTI} \succ \mathsf{TP}$.

5.3. Preferred Choice

We now identify the firm's preferred information-provision strategy across the four strategies, namely OP, CTI, NTI and TP. Proposition 7 shows the comparison among the four strategies for the piecewise-linear model for belief-based utility. For more-general models of belief-based utility too, one can obtain conditions under which each of these strategies are preferred. To illustrate, we identify conditions under which less-informative strategies, e.g., OP or CTI, are the preferred choices. Proposition 8 (resp., Proposition 9) presents a set of conditions under which CTI (resp., OP) is the preferred choice.

Our main results in this section extend to an n-task process, $n \geq 2$. Remark 4 summarizes our results for this generalization; the formal analysis is relegated to Appendix B. Table 4 presents the correspondence between the results in this section (for a two-task process) with those in Appendix B (for an n-task process).

5.3.1. Piecewise-Linear Model: Recall the piecewise-linear utility model in (14), where $0 < \rho_P < \rho_N$. Under this choice of $\mu(\cdot)$, the result below compares our four strategies.

PROPOSITION 7. *Consider the piecewise linear-utility model* (14) *for* $\mu(\cdot)$. *Then,*

$$\mathsf{CTI} \prec \mathsf{OP} \prec \mathsf{NTI} \prec \mathsf{TP}.$$

5.3.2. Can CTI be the preferred choice? Consider an arbitrary choice of $\mu(\cdot)$, as stated in Assumption 1. Using (17), (18), (19) and (20), the necessary and sufficient conditions for CTI to be the preferred choice are:

$$\mu'(0^-) < \frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{0.596}{\lambda}} \quad (\mathsf{CTI} \succ \mathsf{OP}) \tag{24a}$$

$$\mu'(0^-) < \mathbb{E}\mu'\left(\frac{1}{\lambda} - X_1\right) \quad (\mathsf{CTI} \succ \mathsf{NTI}) \tag{24b}$$

$$\mu'(0^-) < \frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - D\right)}{\frac{2}{\lambda}}. \quad (\mathsf{CTI} \succ \mathsf{TP}) \tag{24c}$$

Combining the inequalities in (24), we have:

$$\mathsf{CTI}\text{ is preferred} \Leftrightarrow \mu'(0^-) < L := \min\left\{\frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{0.596}{\lambda}}, \mathbb{E}\mu'\left(\frac{1}{\lambda} - X_1\right), \frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - D\right)}{\frac{2}{\lambda}}\right\}.$$

Intuitively, the conditions in (24) are more likely to be satisfied if μ_+ is not too steep (so that the RHS in both (24a) and (24c) is large), while μ_- is steep (so that the RHS in (24b) is large).

PROPOSITION 8. *Suppose* $\mu(\cdot)$ *is strictly increasing and* $\mu_+(\cdot)$ *is strictly concave. Then,* $L > 0$. *Consequently,* CTI *is the preferred choice iff* $\mu'(0^-) \in [0, L)$.

EXAMPLE 4. Suppose

$$\mu(x) = \begin{cases} \rho_P x, & \text{if } x \geq 0; \\ -\rho_N x^2, & \text{if } x < 0, \end{cases}$$

where $\rho_P > 0$ and $\rho_N > 0$. Then,

$$\mu'(0^-) = 0 < L = \min\left\{\rho_P, \frac{2}{e\lambda}\rho_N + \left(1 - \frac{1}{e}\right)\rho_P, \frac{5}{e^2\lambda}\rho_N + \left(1 - \frac{2}{e^2}\right)\rho_P\right\}.$$

Each of the three terms in the RHS above are strictly positive, and hence the RHS is strictly positive. Thus, under this choice of $\mu(\cdot)$, we have that CTI is the preferred choice. ■

5.3.3. Can OP be the preferred choice? Consider an arbitrary choice of $\mu(\cdot)$, as stated in Assumption 1. Using (17), (18), (19) and (20), the necessary and sufficient conditions for OP to be the preferred choice are:

$$\mu'(0^-) > \frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{0.596}{\lambda}} \quad (\mathsf{OP} \succ \mathsf{CTI}) \tag{25a}$$

$$\mu'(0^-) < \frac{\mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right) - \mathbb{E}\left(\frac{2}{\lambda} - D\right)}{\frac{1.403}{\lambda}} \quad (\mathsf{OP} \succ \mathsf{NTI}) \tag{25b}$$

$$\mu'(0^-) < \frac{\mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right) - 2\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right)}{\frac{1.403}{\lambda}}. \quad (\mathsf{OP} \succ \mathsf{TP}) \tag{25c}$$

Combining the inequalities in (25), OP is preferred iff the following condition holds:

$$\frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{0.596}{\lambda}} < \mu'(0^-) < \frac{\mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right) - \max\left\{\mathbb{E}\mu\left(\frac{2}{\lambda} - D\right), 2\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right)\right\}}{\frac{1.403}{\lambda}}. \tag{26}$$

Intuitively, the condition in (26) is likely to hold if $\mu(\cdot)$ is less steep in $(\frac{1}{\lambda}, \infty)$ and steep in $(-\infty, \frac{1}{\lambda})$. From (A.32), the term in the LHS of (26) is non-negative. Therefore, we require $\mu'(0^-) > 0$.

Proposition 9. *Suppose $\mu(\cdot)$ is strictly concave, and the following condition holds:*

$$\frac{2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{0.596}{\lambda}} < \mu'(0^-) < \frac{\mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)}{\frac{1.403}{\lambda}}. \tag{27}$$

Then, OP *is the preferred choice. A sufficient condition for (27) to hold is:*

$$1.408\mu\left(\frac{1}{\lambda}\right) < \mathbb{E}\left[\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)\right] \tag{28}$$

Example 5. Suppose

$$\mu(x) = \begin{cases} \rho_P x, & \text{if } x \geq 0; \\ -\rho_{NB}x^2 + \rho_{NA}x, & \text{if } x < 0, \end{cases}$$

where $0 < \rho_P < \rho_{NA}$ and $\rho_{NB} > 0$. The inequalities in (25) hold if $\rho_{NB} > 0.638\lambda(\rho_{NA} - \rho_P)$. ■

Remark 4. In Appendix B, we extend our model to a process with $n \geq 2$ tasks that are executed in sequence. For ease reference, Table 4 below provides the correspondence between the results in this section for the two-task model and the results in Appendix B for the n-task model. In particular, we highlight Proposition B.2, that compares the four information-provision strategies under the piecewise-linear utility model in (14). Observe that the exact ranking of these strategies depends on n. For values of $n \leq 125$, TP is the preferred choice, but for larger values of n, OP is the preferred choice.

Result	**Two-Task Process (Section 5)**	**n-Task Process (Appendix B)**
Comparison of OP and CTI	Proposition 3	Proposition B.1(a)
Comparison of CTI and NTI	Proposition 5	Proposition B.1(b)
Comparison of NTI and TP	Proposition 6	Proposition B.1(c)
Preferred Choice under Piecewise Linear Utility Model	Proposition 7	Proposition B.2
Sufficient Condition for CTI to be the Preferred Choice	Proposition 8	Proposition B.3
Sufficient Condition for OP to be the Preferred Choice	Proposition 9	Proposition B.4

Table 1 Generalization to n-task processes, $n \geq 2$: Correspondence between the results for the two-task model and the n-task model.

Finally, we study the role of consumers' anxiety on the firm's information-provision strategy. Anxiety is yet another behavioral force that affects consumers during their sojourn thorugh the process. Anxiety arises due to consumers' intolerance to uncertainty (Ladouceur et al. 2000, Dugas et al. 2001). Our model of consumer anxiety is motivated by the "flow disutility" model in Iyer and Zhong (2020).

6. Extension: Anxiety Costs

Consider the one-task model. Recall the consumer's material payoff in (4). Suppose that the material revenue v is random, with:

$$\mathbb{E}v = \mu_v; \quad \text{Var}\,(v) = {\sigma_v}^2.$$

Our extension of the one task-model below is motivated by the *anxiety costs* studied by Iyer and Zhong (2020). All else equal, a larger variance for v leads to greater anxiety to the receiver, thus making waiting more painful. We model the anxiety costs as follows. Corresponding to a realization v and delay D, the material payoff to the receiver is:

$$u_M = v - (1+\sigma_v)D. \text{ Thus,}$$

$$\mathbb{E}[u_M] = \mu_v - \frac{1}{\breve{\lambda}}, \text{ where } \frac{1}{\breve{\lambda}} = \frac{1+\sigma_v}{\lambda}. \tag{29}$$

If $\sigma_v = 0$, then the model above is identical the one task model in Section 4. All other assumptions, including the model for belief-based utility, are identical to that in Section 4. Following identical steps as in Section 4, we have:

$$U_B^{\boldsymbol{\sigma}} = \begin{cases} -\mu'(0^-)\frac{1}{\breve{\lambda}} + \mu\left(\frac{1}{\breve{\lambda}}\right), & \text{if } \boldsymbol{\sigma} = \mathsf{OP}; \\ \mathbb{E}\left[\mu\left(\frac{1}{\breve{\lambda}} - (1+\sigma_v)D\right)\right], & \text{if } \boldsymbol{\sigma} = \mathsf{TP}. \end{cases}$$

Analogous to Proposition 1, we have the following result that identifies the conditions under which OP is preferred to TP.

PROPOSITION 10. *Under the model above,* $\mathsf{OP} \succ \mathsf{TP}$ *iff the following condition holds:*

$$\mathbb{E}\mu'\left(\frac{1}{\breve{\lambda}} - D\right) > \mu'(0^-). \tag{30}$$

The condition above is identical to (13), except that λ is replaced with $\breve{\lambda}$. In the following result, we show that an increase in σ_v makes TP more preferable.

PROPOSITION 11. *Suppose $\mu(\cdot)$ is concave. Then, the scope of the result* $\mathsf{OP} \succ \mathsf{TP}$ *in Proposition 10 is decreasing in σ_v. Stated differently, consider two values of σ_v, say σ_{v1}, σ_{v2} s.t. $0 < \sigma_{v1} < \sigma_{v2}$. The following relationships hold:*

$$\mathsf{OP} \succ \mathsf{TP} \textit{ for } \sigma_{v2} \implies \mathsf{OP} \succ \mathsf{TP} \textit{ for } \sigma_{v1}$$
$$\mathsf{TP} \succ \mathsf{OP} \textit{ for } \sigma_{v1} \implies \mathsf{TP} \succ \mathsf{OP} \textit{ for } \sigma_{v2}$$

7. Concluding Remarks

The process view of a firm and operational metrics for process analysis are basic ideas in OM. Our work in this paper is an attempt to understand the economics behind post-sales process transparency, i.e., how transparent should a firm design its process, as customers await the completion of their service. Our work draws upon the notion of belief-based utility in Economics, that explains how consumers realize utility from news. Specifically, in our model, the firm provides news about the total delay that consumers realize as they await service.

The understanding of incentives for process transparency is of fundamental importance in OM, and has far-reaching theoretical and business implications. There are several directions in which future work can proceed; we offer some ideas below.

We assume that the firm commits to an information-provision strategy before learning the delay. That is, the firm's strategy does not depend on the realized delay. One could consider settings where the firm's strategy is delay-dependent. Furthermore, we assume that the firm's commitment is enforceable, e.g., through software algorithms. If commitment (to a strategy) is not enforceable, then future research could address the firm's strategy in the absence of a commitment. This is an important problem because the credibility of the firm's commitment to a strategy has been questioned in the popular press, leading to consumer mistrust. A recent example is that of Dominos Pizza's "tracker" – a real-time tracker of a consumer's order in the process – that has met a lot of criticism (The Huffington Post 2017). In the context of our model, consider, for example, the current-task-identity (CTI) strategy: We assume that the firm's messages about the identity of the task at any instant is truthful. However, this may not hold, and the firm may intentionally misrepresent using such messages.

We focus exclusively on non-instrumental information sharing (between a firm and its consumers) to understand consumers' waiting-experience in the process. Consequently, we assume that the consumers always participate in trade and do not abandon the process after joining. Future research could consider settings where the information is of both instrumental and non-instrumental value, where consumers make join/balk/abandon decisions, in addition to realizing belief-based utility while waiting in the process.

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Appendix A: Proofs of Technical Results

Proof of Proposition 1:

$$\begin{aligned}
U_B^{\mathsf{OP}} > U_B^{\mathsf{TP}} &\Leftrightarrow -\mu(0^-)\frac{1}{\lambda} + \mu\left(\frac{1}{\lambda}\right) > \mathbb{E}\left[\mu\left(\frac{1}{\lambda} - D\right)\right] \\
&\Leftrightarrow \frac{\mathbb{E}\left[\mu\left(\frac{1}{\lambda}\right) - \mu\left(\frac{1}{\lambda} - D\right)\right]}{\frac{1}{\lambda}} > \mu'(0^-) \\
&\Leftrightarrow \frac{\mathbb{E}\left[\int_{t=\frac{1}{\lambda}-D}^{\frac{1}{\lambda}} \mu'(t)dt\right]}{\frac{1}{\lambda}} > \mu'(0^-) \\
&\Leftrightarrow \frac{\int_{D=0}^{\infty}\left(\int_{t=\frac{1}{\lambda}-D}^{\frac{1}{\lambda}} \mu'(t)dt\right)\lambda e^{-\lambda D} dD}{\frac{1}{\lambda}} > \mu'(0^-) \\
&\Leftrightarrow \frac{\int_{t=-\infty}^{\frac{1}{\lambda}}\left(\int_{D=\frac{1}{\lambda}-t}^{\infty} \lambda e^{-\lambda D} dD\right)\mu'(t)dt}{\frac{1}{\lambda}} > \mu'(0^-) \\
&\Leftrightarrow \frac{\int_{t=-\infty}^{\frac{1}{\lambda}} e^{(t\lambda-1)}\mu'(t)dt}{\frac{1}{\lambda}} > \mu'(0^-) \\
&\Leftrightarrow \int_{t=-\infty}^{\frac{1}{\lambda}} \underbrace{\lambda e^{-\lambda\left(\frac{1}{\lambda}-t\right)}}_{w_\lambda(t)} \mu'(t)dt > \mu'(0^-)
\end{aligned}$$

Define

$$w_\lambda(t) = \lambda e^{-\lambda\left(\frac{1}{\lambda}-t\right)}.$$

$w_\lambda(\cdot)$ is the p.d.f corresponding to $\frac{1}{\lambda} - D$, where $D \sim \mathsf{Exp}(\lambda)$. Therefore, the last inequality can be written as:

$$U_B^{\mathsf{OP}} > U_B^{\mathsf{TP}} \Leftrightarrow \mathbb{E}\mu'\left(\frac{1}{\lambda} - D\right) > \mu'(0^-).$$

The last inequality is the required inequality.

Proof of Proposition 2: Since $\mu(\cdot)$ satisfies the condition 4 in the statement of the result, $\mu'(x)$ is decreasing in x if $x > 0$ and increasing in x if $x < 0$. Therefore,

$$\mu'(0^-) > \mu'(x) \quad \text{for all values of } x \neq 0.$$

Consider (13). An upper bound on the LHS is:

$$\underbrace{\int_{t=-\infty}^{\frac{1}{\lambda}} w_\lambda(t)\mu'(t)dt}_{=\text{LHS}} < \mu'(0^-).$$

Therefore, it is impossible that (13) holds.

Proof of Proposition 3: Using (17) and (18),

$$\begin{aligned}
U_B^{\mathsf{OP}} > U_B^{\mathsf{CTI}} &\Leftrightarrow -\mu'(0^-)\mathbb{E}\left[\frac{D^2\lambda}{1+D\lambda}\right] + \mathbb{E}\mu\left(\frac{2+D\lambda}{\lambda(1+D\lambda)}\right) > -\mu'(0^-)\underbrace{\frac{2}{\lambda}}_{=\mathbb{E}D} + 2\mu\left(\frac{1}{\lambda}\right) \\
&\Leftrightarrow \mu'(0^-)\mathbb{E}\left[\frac{D}{1+D\lambda}\right] > 2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\left[\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)\right]
\end{aligned}$$

Since $D \sim \mathsf{Erlang}(2,\lambda)$, the following holds:

$$\mathbb{E}\left[\frac{D}{\frac{1}{\lambda}+D}\right] = \int_{t=1}^{\infty} \frac{e^{1-t}}{t} dt \approxeq 0.596 \tag{A.31}$$

Substituting $\mu(\cdot)$ from (14) in the above equation simplifies to $\rho_N > \rho_P$, and therefore, the piecewise linear utility model in (14) satisfies (22).

Proof of 4: Suppose $\mu_+(\cdot)$ is concave. Then, for any random variable Y with support in $(0,\infty)$, we have:

$$\mathbb{E}[\mu_+(Y)] \le \mu_+(\mathbb{E}Y) \quad \text{(Jensen's Inequality)}$$

Since $D \sim \mathsf{Erlang}(2,\lambda)$, we have:

$$\frac{2}{\lambda} - \frac{D}{1+D\lambda} \in \left[\frac{1}{\lambda}, \frac{2}{\lambda}\right]. \tag{A.32}$$

Therefore,

$$\mu_+\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right) \le \mu_+\left(\underbrace{\mathbb{E}\left[\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right]}_{=\frac{1.403}{\lambda} \text{ from (A.31)}.}\right).$$

Therefore, we have that if $\mu_+(\cdot)$ is concave:

$$\mu'(0^-)\frac{0.596}{\lambda} > 2\mu\left(\frac{1}{\lambda}\right) - \mu\left(\frac{1.403}{\lambda}\right) \implies \mu'(0^-)\underbrace{\left(\frac{0.596}{\lambda}\right)}_{=\mathbb{E}\left[\frac{D}{1+D\lambda}\right] \text{ from (A.31)}} > 2\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\left[\mu\left(\frac{2}{\lambda} - \frac{D}{1+D\lambda}\right)\right].$$

Proof of Proposition 6: Consider a pair of realizations X_1, X_2; $D = X_1 + X_2$. If $\mu(\cdot)$ satisfies strict subadditivity, then,

$$\begin{aligned}
\mu\left(\frac{1}{\lambda} - X_1\right) + \mu\left(\frac{1}{\lambda} - X_2\right) &> \mu\left(\frac{2}{\lambda} - D\right) \\
\implies \underbrace{\mathbb{E}_{X_1,X_2}\left[\mu\left(\frac{1}{\lambda} - X_1\right) + \mu\left(\frac{1}{\lambda} - X_2\right)\right]}_{U_B^{\mathsf{NTI}}} &> \underbrace{\mathbb{E}_{X_1,X_2}\left[\mu\left(\frac{2}{\lambda} - D\right)\right]}_{U_B^{\mathsf{TP}}} \\
&\implies \mathsf{NTI} \succ \mathsf{TP}.
\end{aligned}$$

Proof of Proposition 7: Consider the linear utility model in (14). Below, we provide an analytical expression for the receiver's utility under each information provisioning strategy.

$$\begin{aligned}
U_B^{\mathsf{OP}} &= -\frac{2 + e\ \mathsf{EI}(-1)}{\lambda}(\rho_N - \rho_P) \approx \frac{-1.403}{\lambda}(\rho_N - \rho_P) \\
U_B^{\mathsf{CTI}} &= -\frac{2}{\lambda}(\rho_N - \rho_P) \\
U_B^{\mathsf{NTI}} &= -\frac{1}{e\lambda}(\rho_N - \rho_P) \approx \frac{-0.736}{\lambda}(\rho_N - \rho_P) \\
U_B^{\mathsf{TP}} &= -\frac{4}{e^2\lambda}(\rho_N - \rho_P) =\approx \frac{-0.541}{\lambda}(\rho_N - \rho_P)
\end{aligned}$$

where $\mathsf{EI}(\cdot)$ denotes the exponential integral function, i.e.,

$$\mathsf{EI}(z) = -\int_{-z}^{\infty} \frac{e^{-t}}{t} dt \tag{A.34}$$

Thus, for any $0 < \rho_P < \rho_N$, we have that

$$U_B^{\mathsf{CTI}} < U_B^{\mathsf{OP}} < U_B^{\mathsf{NTI}} < U_B^{\mathsf{TP}} \implies \mathsf{CTI} \prec \mathsf{OP} \prec \mathsf{NTI} \prec \mathsf{TP}$$

Proof of Proposition 8: Since $\mu(\cdot)$ is strictly increasing, we have that $\mu'(\cdot) > 0$. Therefore, the second term in L is strictly positive. Since $\mu(\cdot)$ is strictly increasing $\mu_+(\cdot)$ is strictly concave, there exists $c_1 > 0$ s.t.:

$$\mu\left(\frac{2}{\lambda} + c_1\right) = 2\mu\left(\frac{1}{\lambda}\right).$$

Since $D > 0$ (a.s), we have that

$$\mathbb{E}\mu\left(\frac{2}{\lambda} - D\right) < \mathbb{E}\mu\left(\frac{2}{\lambda} - \frac{D}{1 + D\lambda}\right) < \mu\left(\frac{2}{\lambda}\right)$$

the first and third terms are strictly positive.

Proof of Proposition 9: Consider (26) (the necessary and sufficient conditions for OP to be the preferred choice). If $\mu(\cdot)$ is strictly concave, from Jensens's inequality, we have:

$$\mathbb{E}\mu\left(\frac{2}{\lambda} - D\right) \leq 0$$
$$\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right) \leq 0$$

Therefore,

$$\max\left\{\mathbb{E}\mu\left(\frac{2}{\lambda} - D\right), 2\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right)\right\} \leq 0$$

Substituting the above inequality in (26), we have (27). Considering the first and last terms in (27), the inequality holds iff (28) holds.

Proof of Proposition 10: The proof below uses the same ideas as in Proposition 1.

$$\begin{aligned}
U_B^{\mathsf{OP}} > U_B^{\mathsf{TP}} &\Leftrightarrow \frac{\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu\left(\frac{1}{\lambda} - (1+\sigma_v)D\right)}{\frac{1}{\lambda}} > \mu'(0^-)\\
&\Leftrightarrow \frac{\mathbb{E}\left[\int_{\frac{1}{\lambda}-(1+\sigma_v)D}^{\frac{1}{\lambda}} \mu'(t)dt\right]}{\frac{1}{\lambda}} > \mu'(0^-)\\
&\Leftrightarrow \frac{\int_{D=0}^{\infty}\int_{t=\frac{1}{\lambda}-(1+\sigma_v)D}^{\frac{1}{\lambda}} \mu'(t)\lambda e^{-\lambda D} dt dD}{\frac{1}{\lambda}} > \mu'(0^-)\\
&\Leftrightarrow \frac{\int_{t=-\infty}^{\frac{1}{\lambda}}\int_{D=\frac{1}{\lambda}-\frac{t}{1+\sigma_v}}^{\infty} \mu'(t)\lambda e^{-\lambda D} dD dt}{\frac{1}{\lambda}} > \mu'(0^-)\\
&\Leftrightarrow \frac{\int_{t=-\infty}^{\frac{1}{\lambda}} e^{\check{\lambda}t-1}\mu'(t)dt}{\frac{1}{\lambda}} > \mu'(0^-)\\
&\Leftrightarrow \underbrace{\int_{t=-\infty}^{\frac{1}{\lambda}} w_{\check{\lambda}}(t)\mu'(t)dt}_{\mathbb{E}\mu'\left(\frac{1}{\lambda}-D\right)} > \mu'(0^-).
\end{aligned}$$

which is the required inequality.

Proof of Proposition 11: Consider $0 < \sigma_{v1} < \sigma_{v2}$. Consider the LHS in (30), and a realization of D, say D. Since $\mu(\cdot)$ is concave, i.e., $\mu'(\cdot)$ is decreasing, we have:

$$\mu'\left(\frac{1+\sigma_{v1}}{\lambda} - D\right) \geq \mu'\left(\frac{1+\sigma_{v2}}{\lambda} - D\right)$$

Since the above inequality hold for any D, we have:

$$\mathbb{E}\mu'\left(\frac{1+\sigma_{v1}}{\lambda}-D\right)\geq \mathbb{E}\mu'\left(\frac{1+\sigma_{v2}}{\lambda}-D\right)$$

Therefore, the LHS is decreasing in σ_v. Therefore,

$$\mathbb{E}\mu'\left(\frac{1+\sigma_{v2}}{\lambda}-D\right)>\mu'(0^-)\implies \mathbb{E}\mu'\left(\frac{1+\sigma_{v1}}{\lambda}-D\right)>\mu'(0^-),$$
$$\text{and } \mathbb{E}\mu'\left(\frac{1+\sigma_{v1}}{\lambda}-D\right)<\mu'(0^-)\implies \mathbb{E}\mu'\left(\frac{1+\sigma_{v2}}{\lambda}-D\right)<\mu'(0^-).$$

Appendix B: Generalization: n-Task Case

Consider a process that consists of a sequence of n tasks, indexed by $i \in [n]$.[7] Let X_i denote the processing time of task i; let $D = \sum_{i=1}^{n} X_i$ denote the total delay. We assume that:

$$\begin{aligned} X_i &\sim_{i.i.d} \mathsf{Exp}(\lambda) \\ \implies D &\sim \mathsf{Erlang}(n,\lambda) \quad (\equiv \boldsymbol{\pi}_0)\,. \end{aligned}$$

For convenience, we denote $X_0 = 0$. The distribution of the task durations is common knowledge.

We analyze the four information provisioning strategies OP, CTI, NTI and TP for this n-task process. The sequence of events and the messaging strategies are analogous to the two-task setting described in the main paper.

B.1. Opaque Process

Recall the definition of $\overline{D}_t = \mathbb{E}[D|D > t]$. Since $D \sim \mathsf{Erlang}(n,\lambda)$, we have:

$$\begin{aligned} \overline{D}_t &= \frac{\Gamma(1+n,\lambda t)}{\lambda\Gamma(n,\lambda t)} \\ &= \frac{n}{\lambda} + \frac{e^{-\lambda t}(\lambda t)^n}{\lambda\Gamma(n,\lambda t)} \end{aligned} \tag{B.35}$$

where $(\overline{D}_t - t)$ is the mean residual life (MRL) of $\mathsf{Erlang}(n,\lambda)$ and $\Gamma(\cdot,\cdot)$ is the (upper) incomplete gamma function,

$$\Gamma(a,z) = \int_z^\infty y^{a-1} e^{-y} dy.$$

The following result about the MRL of $\mathsf{Erlang}(n,\lambda)$ for $n \geq 2$ is available in Chapter 4 of Lai and Xie (2006), and is stated without a proof.

Lemma B.1. *$\overline{D}_t - t$ is decreasing in t.*

Following the identical steps as in the two task process, we have:

$$u_B^{\mathsf{OP}}[t,t+dt) = \begin{cases} \underbrace{\mu(\overline{D}_t - \overline{D}_{t+dt})}_{=-\mu'(0^-)\overline{D}_t' dt}, & \text{if } t < D; \\ \mu\left(\overline{D}_D - D\right), & \text{if } t = D. \end{cases}$$

The (expected) belief-based utility under OP is:

$$\begin{aligned} U_B^{\mathsf{OP}} &= \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[\int_{t=0}^{D} u_B^{\mathsf{OP}}[t,t+dt) + u_B^{\mathsf{OP}}[D,D+dt)\right] \\ &= \mathbb{E}\left[\int_0^D -\mu'(0^-)\overline{D}_t' dt + \mu\left(\overline{D}_D - D\right)\right] \\ &= -\mu'(0^-)\left(\mathbb{E}\overline{D}_D - \underbrace{\frac{n}{\lambda}}_{=\overline{D}_0}\right) + \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[\mu\left(\overline{D}_D - D\right)\right] \end{aligned}$$

[7] Here, $[n]$ denotes the set $\{1,2,\ldots,n\}$.

Since $\overline{D}_0 = \mathbb{E}D$, we have the following:

$$U_B^{\mathsf{OP}} = -\mu'(0^-)\mathbb{E}Y + \mathbb{E}\mu(Y) \tag{B.36}$$

where $Y = \overline{D}_D - D$ denotes the *MRL at the time of failure* for an $\mathsf{Erlang}(n,\lambda)$ random variable.

Corollary 1. *Since $D \sim \mathsf{Erlang}(n,\lambda)$, we have the following:*

$$\mathbb{E}[\overline{D}_D - D] < \underbrace{\overline{D}_0}_{\mathbb{E}D} \quad \textit{i.e.,} \quad \mathbb{E}Y < \frac{n}{\lambda}$$

B.2. Current Task Identity (CTI)

Consider an n-tuple of realizations $X_1, X_2, \ldots, X_n$; $D = \sum_{i\in[n]} X_i$. Consider any $i \in [n]$ and a time $t \in (\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j)$. The belief on D at time t is:

$$\begin{aligned}\overline{D}_t = \mathbb{E}\left[D \middle| X_1, X_2, \ldots, X_{i-1}, X_i > t - \sum_{j=0}^{i-1} X_j\right] &= \sum_{j=0}^{i-1} X_j + \left(t - \sum_{j=0}^{i-1} X_j + \frac{1}{\lambda}\right) + \frac{n-i}{\lambda} \\ &= t + \frac{n-i+1}{\lambda}\end{aligned}$$

Following identical steps as in the two task process, we have:

$$u_B^{\mathsf{CTI}}[t, t+dt) = \begin{cases} \underbrace{\mu(-dt)}_{=-\mu'(0^-)dt}, & \text{if } t \in \left(\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j\right) \text{ for some } i \in [n]; \\ \mu\left(\frac{1}{\lambda}\right), & \text{if } t = X_i \text{ for some } i \in [n]. \end{cases}$$

The (expected) belief-based utility under CTI is:

$$\begin{aligned} U_B^{\mathsf{CTI}} &= \mathbb{E}\left[-\mu'(0^-)\sum_{i\in[n]}\left(\int_{t=\sum_{j=0}^{i-1} X_j}^{\sum_{j=0}^{i} X_j} dt\right) + n\mu\left(\frac{1}{\lambda}\right)\right] \\ &= n\left(-\mu'(0^-)\frac{1}{\lambda} + \mu\left(\frac{1}{\lambda}\right)\right) \end{aligned} \tag{B.37}$$

Observe that the RHS above is n times the RHS in (12).

B.3. Next Task Information (NTI)

Consider an n-tuple of realizations $X_1, X_2, \ldots, X_n$; $D = \sum_{i\in[n]} X_i$. Consider any $i \in [n]$ and $t \in \left(\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j\right)$. The belief on D at time t is:

$$\overline{D}_t = \mathbb{E}\left[D \middle| X_1, X_2, \ldots, X_i\right] = \sum_{j=1}^{i} X_j + \frac{n-i}{\lambda}$$

Following identical steps as in the two task model, we have:

$$u_B^{\mathsf{NTI}}[t, t+dt) = \begin{cases} 0, & \text{if } t \in \left(\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j\right) \text{ for some } i \in [n]; \\ \mu\left(\frac{1}{\lambda} - X_i\right), & \text{if } t = X_i \text{ for some } i \in [n]. \end{cases}$$

The (expected) belief-based utility is:

$$\begin{aligned} U_B^{\mathsf{NTI}} &= \mathbb{E}\left[\sum_{i\in[n]} \mu\left(\frac{1}{\lambda} - X_i\right)\right] \\ &= n\mathbb{E}\left[\mu\left(\frac{1}{\lambda} - X_i\right)\right]. \end{aligned} \tag{B.38}$$

Observe that the RHS of the above equation is n times the RHS of (11).

B.4. Transparent Process (TP)

Consider an n-tuple of realizations $X_1, X_2, \ldots, X_n$; $D = \sum_{i\in[n]} X_i$. The belief-based utility is:

$$U_B^{\mathsf{TP}} = \mathbb{E}\mu\left(\frac{n}{\lambda} - D\right) \tag{B.39}$$

B.5. Pairwise Comparison

We first conduct a pairwise comparison of the four information provision strategies

Analogous to Propositions 3, 5, 6, we have the following result:

Proposition B.1. *(a)* $\mathsf{OP} \succ \mathsf{CTI}$ *iff the following condition holds:*

$$\mu'(0^-) > \frac{n\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu(Y)}{\frac{n}{\lambda} - \mathbb{E}Y}$$

The piecewise linear model in (14) satisfies this condition.

(b) $\mathsf{CTI} \succ \mathsf{NTI}$ *iff* $\mu(\cdot)$ *satisfies (13).*

(c) Suppose $\mu(\cdot)$ *satisfies the strict subadditivity condition in (21). Then, for any* n*-tuple of realizations* $X_1, X_2, \ldots, X_n$*, we have:*

$$U_B^{\mathsf{NTI}}(X_1, X_2, \ldots, X_n) > U_B^{\mathsf{TP}}(X_1, X_2, \ldots, X_n)$$

Therefore, we have $U_B^{\mathsf{NTI}} > U_B^{\mathsf{TP}}$ *and hence* $\mathsf{NTI} \succ \mathsf{TP}$.

B.6. Preferred Choice

B.6.1. Piecewise Linear $\mu(\cdot)$

Proposition B.2. *Consider the piecewise linear utility model shown in (14). Then,*

$$\mathsf{CTI} \prec \mathsf{OP} \quad \textit{and } \mathsf{CTI} \prec \mathsf{NTI} \prec \mathsf{TP}.$$

Furthermore,

- *If* $n \le 6$,

$$\mathsf{CTI} \prec \mathsf{OP} \prec \mathsf{NTI} \prec \mathsf{TP}$$

- *If* $7 \le n \le 125$,

$$\mathsf{CTI} \prec \mathsf{NTI} \prec \mathsf{OP} \prec \mathsf{TP}$$

- *If* $126 \le n \le 200$,[8]

$$\mathsf{CTI} \prec \mathsf{NTI} \prec \mathsf{TP} \prec \mathsf{OP}$$

[8] The absence of a closed-form expression for $r^{\mathsf{OP}}(n)$ makes an analytical comparison of $r^{\boldsymbol{\sigma}}$ for $\boldsymbol{\sigma} \in \mathsf{M}$ difficult. We compare these values by computing them. We conduct this comparison for $n \in \{1, 2, \ldots, 200\}$ – we believe that such a comparison covers most commonly occurring processes.

Proof: Substituting (14) in (B.36), (B.37), (B.38) and (B.39), we have the following:

$$\frac{U_B^{\boldsymbol{\sigma}}}{(\rho_N-\rho_P)\,\frac{n}{\lambda}} = -r^{\boldsymbol{\sigma}}(n) = \begin{cases} -\left(\frac{\mathbb{E}Y}{\frac{n}{\lambda}}\right), & \text{if } \boldsymbol{\sigma}=\mathsf{OP};\\ -1, & \text{if } \boldsymbol{\sigma}=\mathsf{CTI};\\ -\frac{1}{e}, & \text{if } \boldsymbol{\sigma}=\mathsf{NTI};\\ -\left(\frac{n}{e}\right)^n\frac{1}{n!}, & \text{if } \boldsymbol{\sigma}=\mathsf{TP}.\end{cases} \tag{B.40}$$

Using Corollary 1, we have that $\mathbb{E}Y < \frac{n}{\lambda}$ and hence $U_B^{\mathsf{CTI}} < U_B^{\mathsf{OP}}$. Therefore, $\mathsf{CTI} \prec \mathsf{OP}$. It is straightforward that $U_B^{\mathsf{CTI}} < U_B^{\mathsf{NTI}}$; therefore $\mathsf{CTI} \prec \mathsf{NTI}$. Next, we show that $U_B^{\mathsf{NTI}} < U_B^{\mathsf{TP}}$. We show that $r^{\mathsf{TP}}(n)$ is decreasing in n. To see this, define

$$\begin{aligned} \log r^{\mathsf{TP}}(n) \;&=\; \log\left(\left(\frac{n}{e}\right)^n \frac{1}{n!}\right) = n\left(\log n - 1\right) - \sum_{j\in[n]} \log n \\ \implies \log r^{\mathsf{TP}}(n+1) - \log r^{\mathsf{TP}}(n) \;&=\; n \log\left(1+\frac{1}{n}\right) - 1 = \log\left(\frac{\left(1+\frac{1}{n}\right)^n}{e}\right) \end{aligned}$$

Since $\left(1+\frac{1}{n}\right)^n$ is a strictly increasing sequence in $n>1$ and $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, the RHS in the above equation is strictly negative for any integer $n>1$. Since log is an affine transformation, we have that r^{TP} is decreasing in n. Therefore, U_B^{TP} is increasing in n. At $n=1$, we have that $r^{\mathsf{TP}}(1) = \frac{1}{e} = r^{\mathsf{NTI}}(1)$. Therefore, for any $n>1$, we have that $U_B^{\mathsf{NTI}} < U_B^{\mathsf{TP}}$, and therefore, $\mathsf{NTI} \prec \mathsf{TP}$.

Next, we show $\left(\frac{\mathbb{E}Y}{\frac{n}{\lambda}}\right)$ depends only n and is independent of λ. Using (B.35):

$$\begin{aligned} \mathbb{E}Y = \mathbb{E}[\overline{D}_D - D] \;&=\; \mathbb{E}\left[\frac{e^{-\lambda D}(\lambda D)^n}{\lambda \Gamma(n,\lambda D)}\right] \\ \implies \quad &=\; \int_{D=0}^{\infty} \frac{e^{-2\lambda D}(\lambda D)^{2n-1}}{\Gamma(n,\lambda D)(n-1)!} dD \\ \implies \left(\frac{\mathbb{E}Y}{\frac{n}{\lambda}}\right) \;&=\; r^{\mathsf{OP}}(n) = \int_0^{\infty} \frac{e^{-2t}t^{2n-1}}{\Gamma(n,t)n!} dt \end{aligned}$$

The RHS depends on n and independent of λ. For any given n, we calculate $r^{\mathsf{OP}}(n)$. We now compare $r^{\boldsymbol{\sigma}}(n)$ for each $\boldsymbol{\sigma} \in \mathsf{M}$. Please see Figures 2 and 3 below.

B.6.2. Can CTI be the preferred choice? Analogous to (24), we have the following: CTI is preferred iff the following condition holds

$$\mu'(0^-) < L$$

where $L = \min\left\{\frac{n\mu\left(\frac{1}{\lambda}\right)-\mathbb{E}\mu(Y)}{\frac{n}{\lambda}-\mathbb{E}Y}, \frac{\int_{t=-\infty}^{\frac{1}{\lambda}} e^{t\lambda-1}\mu'(t)dt}{\frac{1}{\lambda}}, \frac{n\mu\left(\frac{1}{\lambda}\right)-\mathbb{E}\mu\left(\frac{n}{\lambda}-D\right)}{\frac{n}{\lambda}}\right\}$.

Proposition B.3. *Suppose $\mu(\cdot)$ is strictly increasing and $\mu_+(\cdot)$ is concave, then $L>0$. Therefore, if $\mu'(0^-) \in [0, L)$, then CTI is the preferred choice.*

Proof: First, I demonstrate that each of the three terms in the above expression for L are positive. Since $\mu(\cdot)$ is strictly increasing and $\mu_+(\cdot)$ is concave, we have that

$$n\mu\left(\frac{1}{\lambda}\right) > \mu\left(\frac{n}{\lambda}\right) > \mathbb{E}\mu\left(\frac{n}{\lambda} - D\right) \quad \text{(the third term is strictly positive)}$$

From Lemma B.1, we have that for any $t>0$:

$$\overline{D}_t - t < \overline{D}_0 = \frac{n}{\lambda}$$

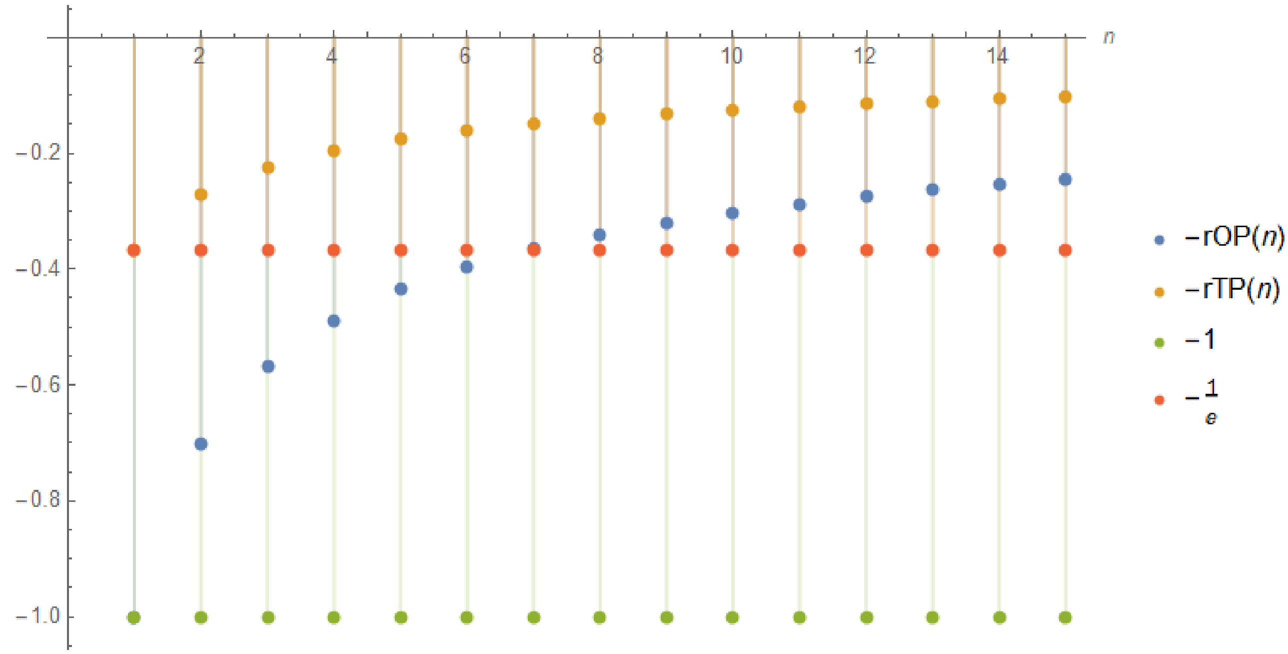

Figure 2 **Comparison of the RHS of** (B.40). $-r^{\mathsf{CTI}}(n) = -1$ **(green),** $-r^{\mathsf{NTI}}(n) = -\frac{1}{e}$ **(red),** $-r^{\mathsf{OP}}(n)$ **(blue) and** $-r^{\mathsf{TP}}(n)$ **(orange). Observe that at** $-r^{\mathsf{OP}}(7) = -0.364 > -\frac{1}{e}(\approx -0.367) = -r^{\mathsf{NTI}}(7)$**.**

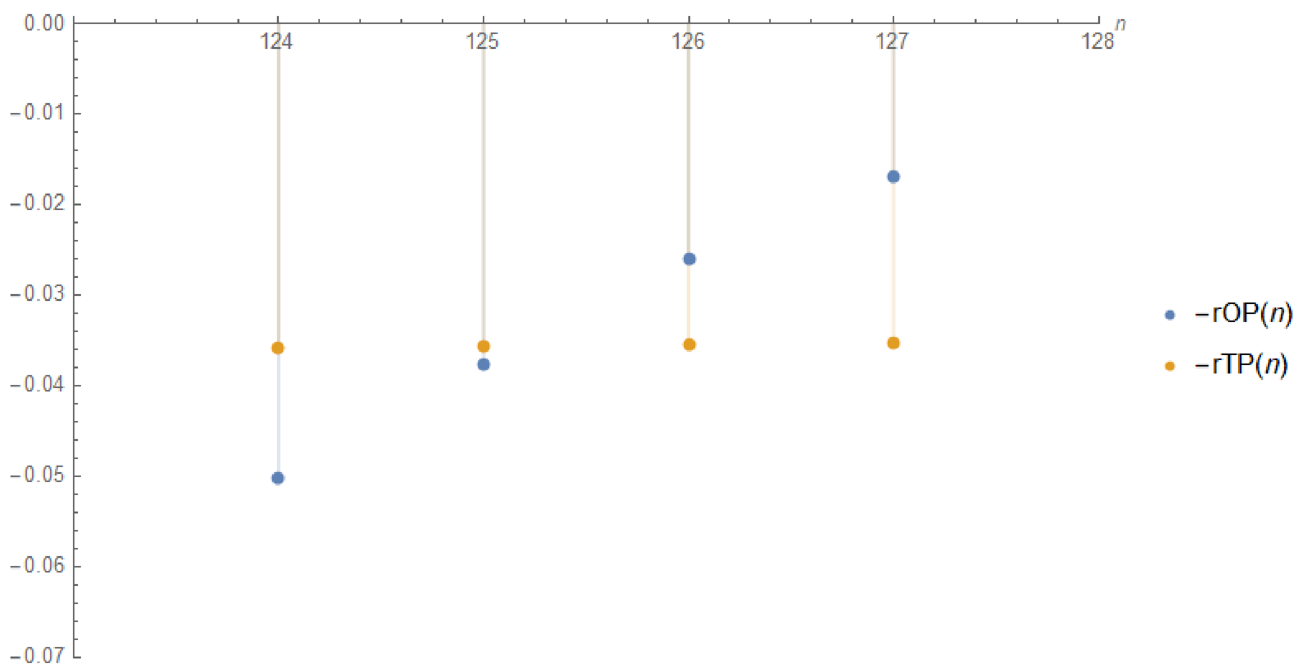

Figure 3 **Comparison of** $-r^{\mathsf{TP}}(n)$ **(orange) and** $-r^{\mathsf{OP}}(n)$ **(blue) of** (B.40)

Since $Y = \overline{D}_D - D$, we have:

$$\mathbb{E}Y < \frac{n}{\lambda} \quad \text{and} \quad \mathbb{E}\mu(Y) < \mu\left(\frac{n}{\lambda}\right) < n\mu\left(\frac{1}{\lambda}\right) \quad \text{(the first term is positive)}$$

Since $\mu(\cdot)$ is strictly increasing, the second term is positive. Thus, $L > 0$. Therefore, if $\mu'(0^-) \in [0, L)$, we have that CTI is the preferred choice.

B.6.3. Can OP be the preferred choice? Analogous to (26), we have the following: OP is preferred iff the following condition holds.

$$\frac{n\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu(Y)}{\frac{n}{\lambda} - \mathbb{E}Y} < \mu'(0^-) < \frac{\mathbb{E}\mu(Y) - \max\{n\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right), \mathbb{E}\mu\left(\frac{n}{\lambda} - D\right)\}}{\mathbb{E}Y} \tag{B.41}$$

Proposition B.4. *Suppose $\mu(\cdot)$ is concave, and the following condition holds:*

$$\frac{n\mu\left(\frac{1}{\lambda}\right) - \mathbb{E}\mu(Y)}{\frac{n}{\lambda} - \mathbb{E}Y} < \mu'(0^-) < \frac{\mathbb{E}\mu(Y)}{\mathbb{E}Y} \tag{B.42}$$

Then, OP *is the preferred choice. The above condition holds if:*

$$\frac{\mu\left(\frac{1}{\lambda}\right)}{\frac{1}{\lambda}} < \frac{\mathbb{E}\mu(Y)}{\mathbb{E}Y}$$

Proof: If $\mu(\cdot)$ is concave, then using Jensen's inequality, we have:

$$\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right) \leq 0$$
$$\mathbb{E}\mu\left(\frac{n}{\lambda} - D\right) \leq 0$$

Therefore,

$$\frac{\mathbb{E}\mu(Y)}{\mathbb{E}Y} \leq \frac{\mathbb{E}\mu(Y) - \max\{n\mathbb{E}\mu\left(\frac{1}{\lambda} - X_1\right), \mathbb{E}\mu\left(\frac{n}{\lambda} - D\right)\}}{\mathbb{E}Y}$$

Therefore, a sufficient condition for (B.41) is (B.42). Now, consider the first and third term in (B.42). For (B.42) to hold,

$$\frac{n\mu\left(\frac{1}{\lambda}\right) - E\mu(Y)}{\frac{n}{\lambda} - \mathbb{E}Y} < \frac{\mathbb{E}\mu(Y)}{\mathbb{E}Y}$$

Rearranging the terms, we have:

$$\frac{\mu\left(\frac{1}{\lambda}\right)}{\lambda} < \frac{\mathbb{E}\mu(Y)}{\mathbb{E}Y}$$