The Economics of Process Transparency

Harish Guda
Department of Supply Chain Management, W.P. Carey School of Business,
Arizona State University, Tempe, AZ 85281, hguda@asu.edu

Milind Dawande, Ganesh Janakiraman
Department of Operations Management, Naveen Jindal School of Management,
The University of Texas at Dallas, Richardson, TX 75080, {milind, ganesh}@utdallas.edu

We propose and analyze a novel framework to understand the role of non-instrumental information sharing in service operations management, i.e., information shared by the firm not to affect consumers' actions, but to better manage their experience in the firm's process. To this end, we model the interactions between a service provider (firm) and a consumer. The operations of the firm are organized as a *process*, consisting of a sequence of tasks, each of random duration. The firm shares real-time information with the consumer about the progress of their flow unit in the firm's process via a *process tracker*. We analyze when providing such real-time progress information via process trackers help, or can possibly hurt, a delay-sensitive consumer. Our work draws upon the recent literature on belief-based/news utility in Economics. Our findings inform a service firm's post-sales transparency strategy.

Key words: Belief-Based Utility, News Utility, Delay Disclosure, Process Analysis, Information Design.

I was fine with the way pizza used to work . . . where they'd say it'd show up in 45 minutes and it would take an hour.

— Domino's Pizza customer about Domino's' real-time pizza tracker (The Wall Street Journal 2017).

After I order, it's a black hole until I get my food at my door.

— Uber Eats' customer before the launch of Uber Eats' in-app order progress bar (Fast Company 2019).

1. Introduction

The process view of a firm – the consideration of the firm as a process that transforms inputs to outputs through a collection of value-adding tasks performed by resources – is arguably the most fundamental idea in operations management (OM). In this note, we consider the post-sales operations process of a service firm. Typically, this process consists of multiple tasks. Consider the following examples from on-demand food delivery. At Domino's Pizza, the post-sales process begins after a customer places an order online. The various tasks in this process are: preparation, bake, box, and delivery (The Wall Street Journal 2017). Similarly, at Uber Eats, the post-sales process begins after a customer places an order in-app and the various tasks in this process are: order confirmation, preparation, worker dispatch, pickup and delivery (Fast Company 2019).

An important practical question for a post-sales process is: how should a service firm inform a delay-sensitive consumer about the progress of their flow-unit in the firm's process while they await completion? That is, how transparent should the design of a firm's post-sales process be?

One popular tool that service firms use to provide information about the progress of a consumer's flow-unit in the firm's process is a *process tracker*. In the above example, Domino's' provides real-time information about the identity of the task being performed on a consumer's pizza as well as updates about the completion of tasks in the process at any time after they place an order but before receipt of their order. In Figure 1, we provide a snapshot of Domino's' tracker that it shares with its consumers. Similar process trackers are used by other firms, e.g., Uber Eats and the Internal Revenue Service (IRS); see Figure 2.

Figure 1 Domino's' Pizza Tracker (The Wall Street Journal 2017). The post-sales process at Domino's (i.e., after an order is placed) is as follows: Preparation → Bake → Box → Delivery. The tracker provides real-time information about a consumer's flow unit (pizza) at any time after a consumer places an order but before receipt of their flow-unit

Information about the progress of a consumer's flow-unit post-sales helps a consumer resolve uncertainty about their wait-time. Notice that such information about wait-time is *non-instrumental* to the consumer. That is, a firm's post-sales information disclosure strategy does not affect a consumer's actions, e.g., to engage in trade with the firm. Rather, firms provide such information to better manage their consumers' post-sales waiting experience.

Recent work in Behavioral Economics suggests that agents realize utility from non-instrumental information, called *belief-based/news* utility, in addition to their material/consumption utility. Beliefs create an *anticipation* of future consumption, and therefore directly affect an agent's utility.

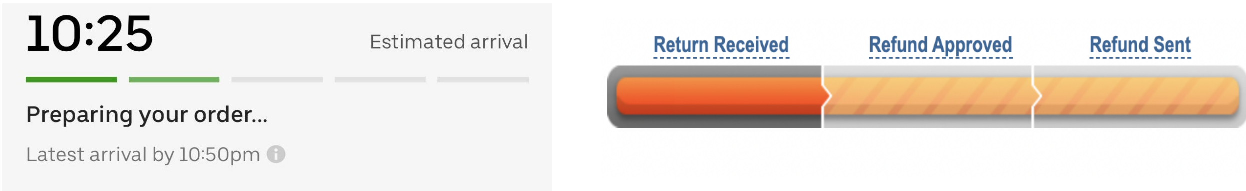


Figure 2 Process trackers in service processes.
Left: Uber Eats order progress bar (Fast Company 2019). The post-sales process at Uber Eats consists of five steps: Order confirmation → Order preparation → Worker enroute to pickup → Worker pickup and confirmation → Worker delivery.
Right: The Internal Revenue Service (IRS) refund status tracker (The Internal Revenue Service 2021). The post-sales process at IRS is: Return received → Refund approved → Refund sent.

Consequently, agents have informational preferences, i.e., preferences over timing and structure of information (Caplin and Leahy 2001, Kőszegi and Rabin 2009, Falk and Zimmermann 2016). In particular, theoretical and experimental research in Economics on non-instrumental information disclosure analyze an agent's preferences across two dimensions: (a) early vs. late resolution of uncertainty (sooner vs. later), and (b) one-shot vs. gradual resolution of uncertainty (clumped vs. piece-by-piece). Our work borrows the treatment of preferences towards non-instrumental information in Kőszegi and Rabin (2009), who formulate the first theory of reference-dependent utility from non-instrumental information. In their model, references are endogenous, i.e., rational beliefs at each point in time, and changes in beliefs (*news*) lead to belief-based/news utility. A central implication of their model of belief-based utility is that agents are averse to belief-fluctuations that arises due to loss aversion with respect to changes in beliefs. With regards to (a), the majority of the literature, both theoretical and experimental, finds that agents prefer earlier, rather than later, resolution of uncertainty, with a few exceptions; with regards to (b), the findings have been mixed with support for both clumped and piece-by-piece information (see Table 2 of Falk and Zimmermann (2016)).

In the context of our work, information disclosure via process trackers is related to gradual and early resolution of uncertainty, while providing no information during a consumer's sojourn while they await completion is related to one-shot and late resolution of uncertainty.[1] Furthermore, providing task completion updates leads to frequent fluctuations in beliefs (relative to the lack of such progress information), and therefore hurts a loss-averse consumer (akin to Kőszegi and Rabin (2009)). In light of the above observations, a natural question arises: Does providing non-instrumental information about delay via process trackers always help, or can possibly hurt, a delay-sensitive consumer?

[1] The two strategies we analyze, inspired from practice, are *related* to, but do not directly *correspond/map* to those analyzed in the Economics literature.

Beyond the theoretical motivation discussed above, our research question is of practical importance. On the one hand, firms such as Uber Eats strive to provide as much real-time information about a customer's order as possible, as evidenced by the second quote at the start of the paper and the following quote:

> *In the case of food delivery, people intuitively understand the difficulties that arise when you're trying to get hot food from a restaurant in the real world and drive it from point A to B. ...By acknowledging some of that complexity, and being transparent about it, we can increase people's confidence a lot.*
>
> — Andy Szybalski, Global Head of Product Design, Uber Eats (Fast Company 2019)

On the other hand, providing such information can also lead to consumers forming worse/pessimistic references about the total delay in the event that some interim task takes too long. In such cases, not providing such real-time progress information might seem better, leading to fewer fluctuations in beliefs. Customers often worry when an interim task takes too long (Reddit 2020, 2019).

Our analysis provides the following insights. We highlight the contrasting roles of two forces – *loss aversion* and *diminishing sensitivity* (to news) – that play a key role in understanding the value from sharing real-time progress and task completion information. Sharing such real-time information leads to greater fluctuation in the consumer's belief (relative to the absence of information sharing). In the presence of loss aversion but not diminishing sensitivity, sharing progress information hurts consumers (Theorems 1 and 2). In the presence of both loss aversion and diminishing sensitivity to news, consumers prefer multiple smaller pieces of good news over one large piece of good news, and one large piece of bad news over multiple pieces of bad news. If consumers are sufficiently insensitive to news further away from the reference, then sharing such information benefits consumers (Theorems 3 and 4).

Taken together, loss aversion and diminishing sensitivity (to news) move the result in opposite directions: loss aversion favors not sharing progress information, while diminishing sensitivity favors sharing such information. The firm's strategy depends on the dominant force. More broadly, within the service OM literature that study the reference-dependent behavior of consumers, researchers model loss aversion but not diminishing sensitivity, perhaps due to algebraic complexity. Our work draws caution to the predictions of models that incorporate only loss aversion but not diminishing sensitivity.

1.1. Related Literature

Our paper is broadly related to two streams of research in service OM. The first stream of research is on delay disclosure in service OM. The extant literature exclusively focuses on settings where information about wait-time is of *instrumental* value to the consumer, i.e., information provided *pre-sales* to incentivize consumers to engage in trade with the firm, see e.g., Allon et al. (2011), Lingenbrink and Iyer (2019). In our work, we consider information about wait-time provided *post-sales* and is therefore of *non-instrumental* value to the consumer. Due to this substantial distinction, we avoid a formal discussion of this literature and refer the readers to a recent review paper on instrumental information sharing in service operations by Ibrahim (2018) and the textbook by Hassin (2016) for a comprehensive review.

The second stream of research that is closer to our work is the design and management of consumers' post-sales waiting experience. While the firm's process (sequence of tasks performed) in the settings we study is fixed, Das Gupta et al. (2016) investigate the impact of memory decay and acclimation on the design of a firm's experiential service process in settings where the firm has discretion over the sequence in which the tasks are performed. Memory decay favors positioning high service level tasks closer to the end of the process, while acclimation favors maximizing the gradient of the service level. Li et al. (2020) extend the work of Das Gupta et al. (2016) under heterogeneous memory decay and find that the optimal sequence of tasks in a firm's experiential process may involve *interior peaks*. Similar to us, Yuan et al. (2021) focus on consumers' waiting experience during their sojourn in the firm's process. They analyze the role of entertainment options in waiting areas that reduce consumers' marginal disutility from waiting and show that firms may engage in *coopetition* instead of acting as a monopolist. In the context of online retail, Bray (2020) empirically analyzes the role of the timing of information disclosure (task completion updates) to non-Bayes rational consumers and demonstrates the presence of *peak-end* effect, i.e., the perceived value by consumers is higher when the task completion updates occur closer to the completion of the process. Unlike Bray (2020), in our model, we assume that the firm truthfully discloses task completion updates while consumers await completion. Kumar et al. (1997) analyze the role of an explicit wait-time guarantee as a signal of the firm's reliability. They show that the explicit provision of a wait-time guarantee enhances satisfaction both during as well as at the end of wait, if the realized wait is less than the guarantee. However, if the realized wait is higher than the guarantee, the provision of a wait-time guarantee decreases satisfaction at the end of the wait.

To our knowledge, our paper is the first to explore the implications of non-instrumental information sharing and the role of news utility in better managing a consumer's waiting experience in the service OM literature.

2. Model

Consider a service process comprising of n (>1) tasks in a sequence, indexed by $i \in [n]$.[2] The task durations, X_i, $i \in [n]$, are random and i.i.d. Let $f(\cdot)$ denote the p.d.f (resp., p.m.f) of X_i if X_i is continuous (resp., discrete), $F(\cdot)$ denote the CDF, and $\overline{x}$ denote the mean. That is,

$$X_i \sim f(\cdot), F(\cdot), \quad \mathbb{E}[X_i] = \overline{x}.$$

We assume that X_i has an increasing failure rate (IFR). Denote the p.d.f. (or p.m.f) (resp., CDF) of $\sum_{i\in[k]} X_i$ by $f^{(k)}$ (resp., $F^{(k)}$) for any $k \in \mathbb{I}^+$. The total delay imposed by the process, denoted by D, is the sum of the task durations. That is, $D = \sum_{i\in[n]} X_i$. Therefore,

$$D \sim f^{(n)}(\cdot), F^{(n)}(\cdot), \text{ and } \mathbb{E}[D] = n\overline{x}.$$

The consumer receives a material value of v from the completion of the service process, while the cost of waiting is normalized to 1 per unit time. Therefore, corresponding to a delay D, the consumer's material payoff (consumption utility) is:

$$U_M = v - D.$$

Therefore, a consumer rationally chooses to participate if $v > n\overline{x}$. We restrict attention to a participating consumer. Further, under the progress information sharing strategies we analyze, we show that abandonment by a participating customer before the completion of the process is irrational; see Remark 1 in Section 3.

In what follows, we first define the notion of belief-based/news utility under a progress disclosure strategy.[3] We then analyze and compare two progress disclosure strategies: the *opaque* strategy (denoted by OP), where the firm does not disclose any progress, and the *current-task identity* strategy (denoted by CTI), where the firm shares real-time progress of the flow-unit in the process by disclosing the identity of the task currently being performed.

2.1. Progress Disclosure Strategies

Let $\boldsymbol{\pi}_t$ denote the consumer's belief (p.d.f) on D at time t and let $\boldsymbol{\Pi}$ denote the set of all distributions over D; thus, $\boldsymbol{\pi}_0 = f^{(n)}$. A progress disclosure strategy $\boldsymbol{\sigma}$ and the prior $\boldsymbol{\pi}_0$ induces a

[2] For any $n \in \mathbb{I}^+$, we refer to the set $\{1, 2, \ldots, n\}$ by $[n]$.

[3] We use the term *progress* disclosure strategies, instead of *delay* disclosure strategies, to avoid any confusion about the nature of information being shared. In our setting, information about delay is of non-instrumental value to the consumer.

stochastic path of beliefs about the delay. We avoid a formal discussion about progress disclosure in general and restrict attention to the two progress disclosure strategies of our interest – CTI and OP – thereby simplifying our notation. Under the two progress disclosure strategies we analyze, we will show below that for a given $\boldsymbol{X} \triangleq (X_1, X_2, \ldots, X_n)$, the belief evolution is deterministic, i.e., the belief $\boldsymbol{\pi}_t^{\boldsymbol{\sigma}}$ at time t under $\boldsymbol{\sigma} \in \{\mathsf{OP}, \mathsf{CTI}\}$ for $t < D (= \sum_{i\in[n]} X_i)$ is fixed. Further, the consumer resolves all uncertainty at $t = D$ when they receive their flow unit. Thus, we require that the progress disclosure strategies resolve the uncertainty fully at $t = D$, i.e., $\boldsymbol{\pi}_D^{\boldsymbol{\sigma}} = 1 \circ D$.[4]

Define $\overline{D}_t^{\boldsymbol{\sigma}} = \mathbb{E}_{D\sim\boldsymbol{\pi}_t^{\boldsymbol{\sigma}}}[D]$. At any time $t \in (0, D)$, the consumer anticipates a material payoff of:

$$U_M\Big|_t = v - \overline{D}_t^{\boldsymbol{\sigma}}.$$

Consider a time instant t and a small interval dt. The consumer's anticipated material payoff changes by:

$$U_M\Big|_{t+dt} - U_M\Big|_t = \left(v - \overline{D}_{t+dt}^{\boldsymbol{\sigma}}\right) - \left(v - \overline{D}_t^{\boldsymbol{\sigma}}\right) = \underbrace{\overline{D}_t^{\boldsymbol{\sigma}} - \overline{D}_{t+dt}^{\boldsymbol{\sigma}}}_{\text{decrease in the } mean \text{ anticipated delay}}.$$

That is, *news* in the interval $[t, t+dt)$ corresponds to the reduction in the *mean* anticipated delay:

$$\text{News in } [t, t+dt) = \overline{D}_t^{\boldsymbol{\sigma}} - \overline{D}_{t+dt}^{\boldsymbol{\sigma}}.$$

We now proceed to define the belief-based utility (or news utility) under $\boldsymbol{\sigma}$. For readers unfamiliar with the notion of belief-based utility, we provide a detailed discussion in Appendix A, the canonical consumer utility model in Appendix A.1 and the main differences between our work and the extant literature in Economics in Appendix A.3. In short, belief-based/news utility corresponds to utility from *news* (non-instrumental information). Recall that the consumer is delay-sensitive. News about a decrease (resp., increase) in the anticipated delay, i.e., *good* news (resp., *bad* news) provides a positive (resp., negative) utility to the consumer at the time of provision of news in the interim while they await completion.

The belief-based utility (*news* utility), corresponding to the change in belief on D, due to the information (*news*) in the interval $[t, t+dt)$ is:

$$U_B[t, t+dt) = \mu\left(U_M\Big|_{t+dt} - U_M\Big|_t\right) = \mu\left(\underbrace{\overline{D}_t^{\boldsymbol{\sigma}} - \overline{D}_{t+dt}^{\boldsymbol{\sigma}}}_{\text{news in the interval } [t, t+dt)}\right), \tag{1}$$

[4] For any discrete random variable, say Y with support $\mathcal{Y}$, we denote the discrete distribution (p.m.f) with mass $p(y)$, $y \in \mathcal{Y}$ by $\sum_{y\in\mathcal{Y}} p(y) \circ y$. In particular, the degenerate distribution that places a probability mass of 1 on, say $y_1 \in \mathcal{Y}$, is denoted by $1 \circ y_1$.

where $\mu(\cdot)$ denotes the reference-dependent universal gain-loss utility model (Kőszegi and Rabin 2006, 2009, Duraj and He 2019).[5] That is, $\mu(\cdot)$ satisfies the following:

(A1) $\mu(\cdot)$ is continuous, increasing, differentiable (except, possibly at 0).

(A2) $\mu(0)=0$.

(A3) Loss Aversion: $-\mu(-x)>\mu(x)$ for all $x>0$.

(A4) Strict Diminishing Sensitivity: $\mu''(-x)>0>\mu''(x)$ for $x>0$. If this inequality holds in the weak sense, we refer to this property as weak diminishing sensitivity.

Observe from (1) that the consumer's reference about delay is *endogenous* in our model. At time t, the consumer's reference is the anticipated mean delay $\overline{D}_t^{\boldsymbol{\sigma}}$. An important implication of (A3) is that the consumer is belief-fluctuation averse.

Let $\mathscr{U}^{\mathsf{B}}$ denote the set of all functions $\mu(\cdot)$ that satisfy the above assumptions. A commonly-used example for $\mu(\cdot)$ is the piecewise linear utility model (Yu et al. 2021, Ho and Zheng 2004):

$$\mu(x)=\begin{cases}\rho_P x, \text{ if } x\geq 0;\\ \rho_N x, \text{ if } x<0.\end{cases} \tag{2}$$

where $0<\rho_P<\rho_N$; thus, (2) satisfies loss aversion but not (strict) diminishing sensitivity, i.e., (A1)-(A3), but not (A4).[6] Let $\mathscr{U}^{\mathsf{PL}}$ denote the set of all piecewise linear functions as shown in (2) above.

The consumer's total utility from the process under $\boldsymbol{\sigma}$ consists of the sum of their material payoff and their belief-based utility, i.e.,

$$\begin{aligned} U^{\boldsymbol{\sigma}} &= \mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[(v-D)+\int_{t=0}^{D}\mu\left(\overline{D}_t^{\boldsymbol{\sigma}}-\overline{D}_{t+dt}^{\boldsymbol{\sigma}}\right)\right],\\ &= \underbrace{(v-n\overline{x})}_{=U_M}+\underbrace{\mathbb{E}_{D\sim\boldsymbol{\pi}_0}\left[\int_{t=0}^{D}\mu\left(\overline{D}_t^{\boldsymbol{\sigma}}-\overline{D}_{t+dt}^{\boldsymbol{\sigma}}\right)\right]}_{=U_B^{\boldsymbol{\sigma}}}. \end{aligned}$$

The first term (the material payoff U_M) is a constant and independent of $\boldsymbol{\sigma}$, i.e., it does not depend on how the news about the realized delay is shared. The second term (the belief-based utility $U_B^{\boldsymbol{\sigma}}$) depends on $\boldsymbol{\sigma}$ via the evolution of $\overline{D}_t^{\boldsymbol{\sigma}}$, the consumer's references about delay at each point of time t until completion.[7]

[5] We adopt the changing mean-based beliefs model for the consumer's belief-based utility, consistent with recent literature on news utility (Duraj and He 2019). That is, in our model, *news* corresponds to a change in the mean anticipated delay. A more sophisticated utility model based on changing beliefs is as follows:

$$U_B[t,t+dt]=\mu\left(g(\boldsymbol{\pi}_{t+dt}^{\boldsymbol{\sigma}})-g(\boldsymbol{\pi}_t^{\boldsymbol{\sigma}})\right)$$

where $g:\boldsymbol{\Pi}\to\Re$. In this manner, one can incorporate a consumer's preferences over other quantities, e.g., mean and variance, in their belief-based utility. See Appendix A.2 for a more detailed discussion and other forms for $U_B[t,t+dt)$.

[6] In the rest of the paper, by diminishing sensitivity, we refer to the inequality in (A4) in the strict sense.

[7] While there are no contingent actions that await the consumer post-sales in our model, their post-sales waiting experience might impact the probability of future purchases. Consequently, the firm benefits from improving the

3. Analysis

We compare the two strategies – CTI and OP – under continuous and discrete distributions for the task durations separately; nevertheless, the intuition behind the comparison (and the dominance of one strategy over the other) remains identical. Under a piecewise linear model for the belief-based utility, we characterize the conditions under which each strategy is dominant. Surprisingly, we find that OP dominates CTI for many common distributions of task durations.

3.1. Continuous Distributions

Consider the case where $f(\cdot)$ is a continuous distribution.

Opaque Strategy (OP): Under OP, the firm does not provide any update in the interim $t \in [0, D)$. Consider any time $t < D$ and a small interval dt. The consumer updates their belief based on the information that they receive in $[t, t+dt)$, that their flow unit is still in the process. Thus, the consumer's mean belief evolution is as follows:[8]

$$\overline{D}_t = \mathbb{E}_{D\sim\boldsymbol{\pi}_t}[D] = \begin{cases} t + \mathsf{MRL}_D(t), & \text{if } t < D; \\ D, & \text{if } t = D. \end{cases}$$

where $\mathsf{MRL}_D(t)$ denotes the mean residual life of D at t, i.e.,

$$\mathsf{MRL}_D(t) = \mathbb{E}\left[D - t | D > t\right].$$

That is, under OP, corresponding to a realization of D, the total stock of news is resolved as follows:

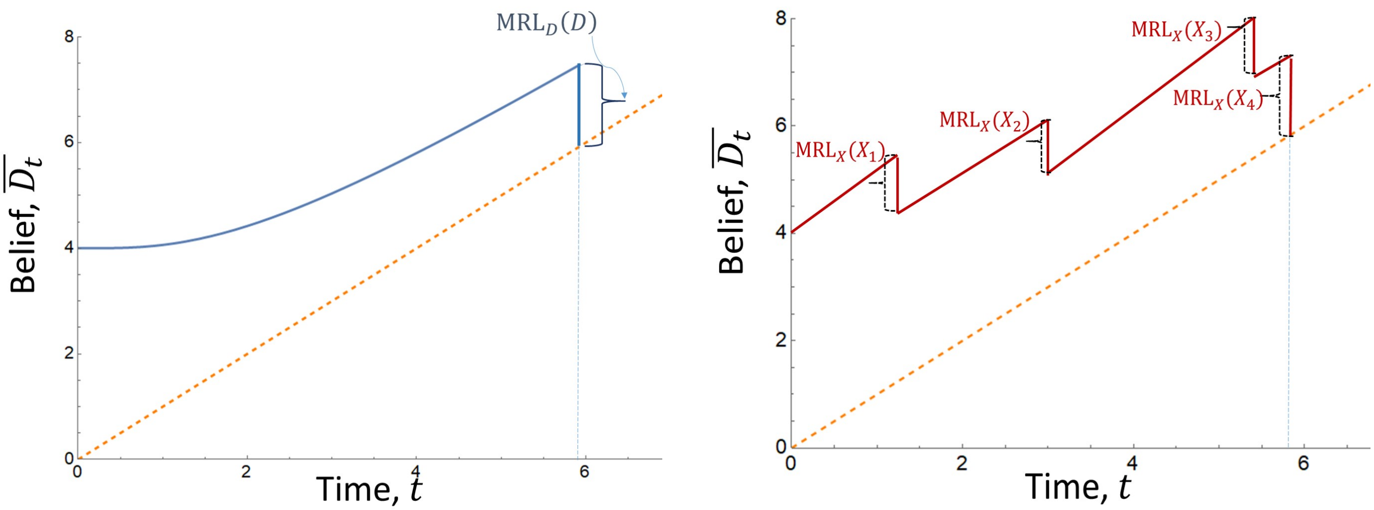
$$\underbrace{n\overline{x} - D}_{\text{total stock of news}} = \int_{t=0}^{D} \underbrace{-\left(1 + \mathsf{MRL}_D'(t)\right) dt}_{\textit{flow} \text{ of bad news in } t\in[0,D)} + \underbrace{\overline{D}_{D^-} - D}_{=\mathsf{MRL}_D(D),\ \textit{lump-sum} \text{ good news at } t=D} . \tag{3}$$

As an illustration, consider the example in the left panel of Figure 3. The consumer's mean belief, $\overline{D}_t$ increases in $t \in [0, D)$. At $t = D$, an instantaneous drop in the consumer's mean belief occurs. Stated differently, the consumer's mean belief continually worsens (a *flow of bad news*) in $t \in [0, D)$, and eventually receives a *lump-sum good news* at $t = D$. The magnitude of the good

consumer's post-sales waiting experience. Besides, the provision of information is costless to the firm. Therefore, we assume that the objective of the firm is to maximize the consumer's total utility.

[8] Precisely, the consumer's belief $\boldsymbol{\pi}_t^{\mathsf{OP}}$ (p.d.f), $t < D$ is as follows:

$$\boldsymbol{\pi}_t^{\mathsf{OP}}(x) = \begin{cases} \frac{f(x)}{\overline{F}(t)}, & \text{if } t < x < D; \\ 0, & \text{if } x \le t < D. \end{cases}, \text{ while } \boldsymbol{\pi}_D^{\mathsf{OP}} = 1 \circ D.$$



(a) OP (b) CTI

Figure 3 Belief Evolution under OP and CTI

Notes: In the above figure, we consider a process that consists of $n=4$ tasks. The task durations are drawn from an i.i.d. exponential distribution with mean 1, i.e., $X_i \sim \mathsf{Exp}(1) \equiv f$; thus $D \sim \mathsf{Erlang}(4,1) \equiv f^{(n)} = \boldsymbol{\pi}_0$ and $\overline{D}_0 = 4$. Let the realized task durations be: $X_1 = 1.25$, $X_2 = 1.25$, $X_3 = 2.75$, and $X_4 = 0.3$. The cumulative cancelled news under OP (resp., CTI) is the mean residual life at $t = D$ (resp., sum of the mean residual life at $t = X_1, X_2, \ldots, X_4$), and is equal to $\mathsf{MRL}_D(D)$ (resp., $\sum_{i\in[4]} \mathsf{MRL}_X(X_i)$).

news at $t = D$, is $\mathsf{MRL}_D(D)$, the mean residual lifetime at the time of failure D. We denote the mean of this quantity by $y_{(n)}$:

$$y_{(n)} = \mathbb{E}[\mathsf{MRL}_D(D)].$$

From Lemma C.1, it follows that $\overline{D}_t$ is increasing in t. Therefore, the expected belief-based utility under OP can be written as:

$$\begin{aligned} U_B^{\mathsf{OP}} &= \mathbb{E}_{\boldsymbol{X}}\left[\int_0^D \mu\left(-dt(1+\mathsf{MRL}'_D(t))\right) + \mu\left(\overline{D}_{D^-} - D\right)\right] \\ &= \underbrace{-\mu'(0^-)\mathbb{E}\left[\mathsf{MRL}_D(D)\right]}_{\text{Utility from the flow of bad news}} + \underbrace{\mathbb{E}\left[\mu(\mathsf{MRL}_D(D))\right]}_{\text{Utility from lump-sum good news}}. \end{aligned}$$

Suppose $\mu \in \mathscr{U}^{\mathsf{PL}}$ (i.e., μ is piecewise linear), then U_B^{OP} simplifies to

$$U_B^{\mathsf{OP}} = -\Delta_\rho y_{(n)}, \tag{4}$$

where $\Delta_\rho = \rho_N - \rho_P$. Intuitively, $y_{(n)}$ is the expected amount of *cancelled news*, i.e., the consumer's mean belief worsens, on average, by $y_{(n)}$, before improving by the same amount. Under the piecewise linear model, the consumer's belief-based utility is linear in the amount of cancelled news. Cancelled news leads to a net disutility for the consumer due to loss aversion (see (A3)). Thus, (4) follows.

Current-Task-Identity Strategy (CTI): Under CTI, the firm provides real-time updates to the consumer about the identity of the task currently being performed. In this manner, CTI resolves uncertainty about the total delay task-by-task.

For convenience, let $X_0 = 0$. Consider any time $t < D$, and a small interval dt. One of the following events occurs in the interval $[t, t + dt)$: Either the identity of the task remains the same, or an update about the completion of a task occurs. Formally, either $t \in \left(\sum_{j\in[i-1]} X_j, \sum_{j\in[i]} X_j\right)$ for some $i \in [n]$, or $t = \sum_{j\in[i]} X_j$. The consumer uses the information in this interval to update their belief on D. In particular, the information in this interval affects the consumer's belief on D only via X_i, the current task being performed. Consider the former, where $t \in \left(\sum_{j\in[i-1]} X_j, \sum_{j\in[i]} X_j\right)$ for some $i \in [n]$. The consumer's mean belief on D at t is:[9]

$$\overline{D}_t = t + \mathsf{MRL}_X\left(t - \sum_{j\in[i-1]} X_j\right) + (n-i)\overline{x}.$$

where $\mathsf{MRL}_X(t)$ denotes the mean residual life of X at t and $X \sim f$, i.e.,

$$\mathsf{MRL}_X(t) = \mathbb{E}\left[X - t | X > t\right].$$

Consider the latter, where $t = \sum_{j\in[i]} X_i$ for some $i \in [n]$. The consumer's mean belief on D at t is:

$$\overline{D}_t = t + (n-i)\overline{x}.$$

That is, under CTI, corresponding to a realization D, the total stock of news is resolved as follows:

$$n\overline{x} - D = \sum_{i=1}^{n}\left[\underbrace{\int_{t=\sum_{j=0}^{i-1} X_j}^{\sum_{j=0}^{i} X_j} -\left(1 + \mathsf{MRL}_X\left(t - \sum_{j=0}^{i-1} X_j\right)\right) dt}_{\text{flow of bad news in } t\in[\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j)} + \underbrace{\mathsf{MRL}_X(X_i)}_{\text{lump-sum good news at } t=X_i}\right].$$

As an illustration, consider the example in the right panel of Figure 3. The consumer's mean belief $\overline{D}_t$ is non-monotonic. Their mean belief increases continuously in $t \in [\sum_{j\in[i-1]} X_j, \sum_{j\in[i]} X_i)$ for $i \in [n]$, i.e., a *flow of bad news within each task.* At $t = \sum_{j\in[i]} X_j$, a drop in the consumer's mean

[9] Precisely, the consumer's belief $\boldsymbol{\pi}_t^{\mathsf{CTI}}$ (p.d.f), $t < D$ is as follows:

$$\boldsymbol{\pi}_t^{\mathsf{CTI}}(y) = \int_{\hat{x}_i=0}^{y-t} \underbrace{\frac{f(\hat{x}_i + (t - \sum_{j=0}^{i-1} X_j))}{\overline{F}(t - \sum_{j=0}^{i-1} X_j)}}_{=\mathbb{P}[X_i = t - (\sum_{j=0}^{i-1} X_j) + \hat{x}_i]} \underbrace{f^{(n-i)}(y - (t + \hat{x}_i))}_{\mathbb{P}[\sum_{j=i+1}^{n} X_j = y - (t + \hat{x}_i)]} d\hat{x}_i, \text{ where } t \in [\textstyle\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j), \text{ and } \boldsymbol{\pi}_D^{\mathsf{CTI}} = 1 \circ D.$$

belief occurs, i.e., a *lump-sum good news within each task*. The magnitude of each of these good news, on average, is $\mathbb{E}[\mathsf{MRL}_X(X)]$, the (expected) mean residual lifetime at the time of failure for X. We denote this quantity by $y_{(1)}$:

$$y_{(1)} = \mathbb{E}[\mathsf{MRL}_X(X)].$$

Therefore, the expected belief-based utility under CTI can be written as follows:

$$U_B^{\mathsf{CTI}} = \mathbb{E}_{\boldsymbol{X}}\left[\sum_{i=1}^{n}\left[\underbrace{\int_{t=\sum_{j=0}^{i-1}X_j}^{\sum_{j=0}^{i}X_j}\mu\left(-dt\left(1+\mathsf{MRL}'_X\left(t-\sum_{j=0}^{i-1}X_j\right)\right)\right)}_{\text{Utility from flow of bad news in } t\in[\sum_{j=0}^{i-1}X_j,\sum_{j=0}^{i}X_j)} + \underbrace{\mu(\mathsf{MRL}_X(X_i))}_{\text{Utility from lump-sum good news at } t=\sum_{j=0}^{i}X_j}\right]\right]$$
$$= n\left(\underbrace{-\mu'(0^-)\mathbb{E}[\mathsf{MRL}_X(X)]+\mathbb{E}\mu(\mathsf{MRL}_X(X))}_{\text{expected belief-based utility from each task}}\right).$$

Since task durations are i.i.d, the expected belief-based utility is n times the expected belief-based utility from each task.

Suppose $\mu \in \mathscr{U}^{\mathsf{PL}}$. Then, U_B^{CTI} simplifies to

$$U_B^{\mathsf{CTI}} = -\Delta_\rho\left(ny_{(1)}\right). \tag{5}$$

Observe than $ny_{(1)}$ is the cumulative amount of *cancelled news*: within each task, the consumer's belief worsens, on average, by $y_{(1)}$, before improving by the same amount. Under the piecewise linear model, the consumer receives a net disutility that is linear in the amount of cancelled news.

Comparison: The following result compares OP and CTI under the piecewise linear model of belief-based utility. Since the consumer's belief-based utility is linear in the amount of cancelled news under the piecewise linear model, it suffices to compare the cumulative expected cancelled news under the two strategies. We adopt the following notation: $\boldsymbol{\sigma} \succ \boldsymbol{\sigma}'$ (resp., $\boldsymbol{\sigma} \prec \boldsymbol{\sigma}'$) iff $U_B^{\boldsymbol{\sigma}} > U_B^{\boldsymbol{\sigma}'}$ (resp., $U_B^{\boldsymbol{\sigma}} < U_B^{\boldsymbol{\sigma}'}$).

THEOREM 1. *Suppose $\mu(\cdot) \in \mathscr{U}^{\mathsf{PL}}$. $\mathsf{OP} \succ \mathsf{CTI}$ iff the following holds:*

$$\underbrace{ny_{(1)}}_{\textit{Cancelled News under } \mathsf{CTI}} > \underbrace{y_{(n)}}_{\textit{Cancelled News under } \mathsf{OP}}. \tag{6}$$

The inequality (6) is satisfied by the following distributions for X_i:

(a) Exponential distribution.

(b) Normal distribution.

We show these in Lemma C.6 in Appendix. Furthermore, we verify that (6) is also satisfied by the uniform distribution: The expressions for $y_{(n)}$ are cumbersome, and hence we provide an analytical proof for $n=2$ and use Mathematica for higher values of n $(3 \leq n \leq 20)$; see Appendix D.

3.2. Discrete (Two-Point) Distributions

Consider the following two-point distribution for X_i with support $\{x_L, x_H\}$, $0 < x_L < x_H$:[10]

$$X_i = p \circ x_H + (1-p) \circ x_L, \text{ where } p \in (0,1). \tag{7}$$

Let $\overline{x} = \mathbb{E}[X_i] = p x_H + (1-p) x_L$. Therefore, $D = \sum_{i \in [n]} X_i$ is distributed according to the following binomial distribution:

$$D \sim \sum_i q_i \circ (x_H i + x_L (n-i)) \text{ where } q_i = \binom{n}{i} p^i (1-p)^{n-i}, \text{ for } i \in \{0\} \cup [n].$$

Denote $t_i = i x_H + (n-i) x_L$ for $i \in \{0, 1, \ldots, n\}$; thus, $D \in \{t_0, t_1, \ldots, t_n\}$.

Opaque Strategy: Under OP, the consumer does not receive any updates until $t = D$. Therefore, they update their belief at $t = t_i$, $i \in \{0, 1, \ldots\}$, $t \leq D$, at which instant they learn if $D = t_i$, or $D > t_i$; hence, they incur a belief-based utility only at these instants. Define the random variable $D_{\geq i} \triangleq D | D \geq t_i$ and its mean δ_i for $i \in \{0, 1, \ldots, n\}$:

$$\begin{aligned} D_{\geq i} \triangleq D | D \geq t_i &\sim \sum_{j=i}^{n} \left(\frac{q_j}{\sum_{k=i}^{n} q_k} \circ (j x_H + (n-j) x_L) \right), \text{ and} \\ \delta_i &= \mathbb{E}[D_{\geq i}] = t_i + \Delta_x \tau_i, \end{aligned}$$

where $\Delta_x = x_H - x_L$ and $\tau_i = \frac{\sum_{j=i}^{n} q_j (j-i)}{\sum_{j=i}^{n} q_j}$.[11] It is straightforward to verify that $D_{\geq i+1} \geq_{st} D_{\geq i}$; hence, δ_i is increasing in i.

Consider a time instant $t = t_i$. The consumer learns of one of the following two events that occurs: either $D = t_i$ (occurs w.p. q_i) or $D > t_i$ (occurs w.p. $\sum_{j=i+1}^{n} q_j$). Under the former, they update their mean belief on D from δ_i to t_i (*good news*, since $\delta_i > t_i$), while under the latter, they update

[10] For the purpose of exposition and use in subsequent analysis in Section 4, we restrict attention to a two-point distribution here. The analysis under a piecewise linear utility model and a general discrete distribution for the task durations is provided in Appendix E.

[11] Intuitively, τ_i is the *mean residual life* of a binomial random variable with parameters (n, p). Formally, let $I \sim \mathsf{Bin}(n, p)$ and $\tau_i = \mathsf{MRL}(i^-)$, i.e.,

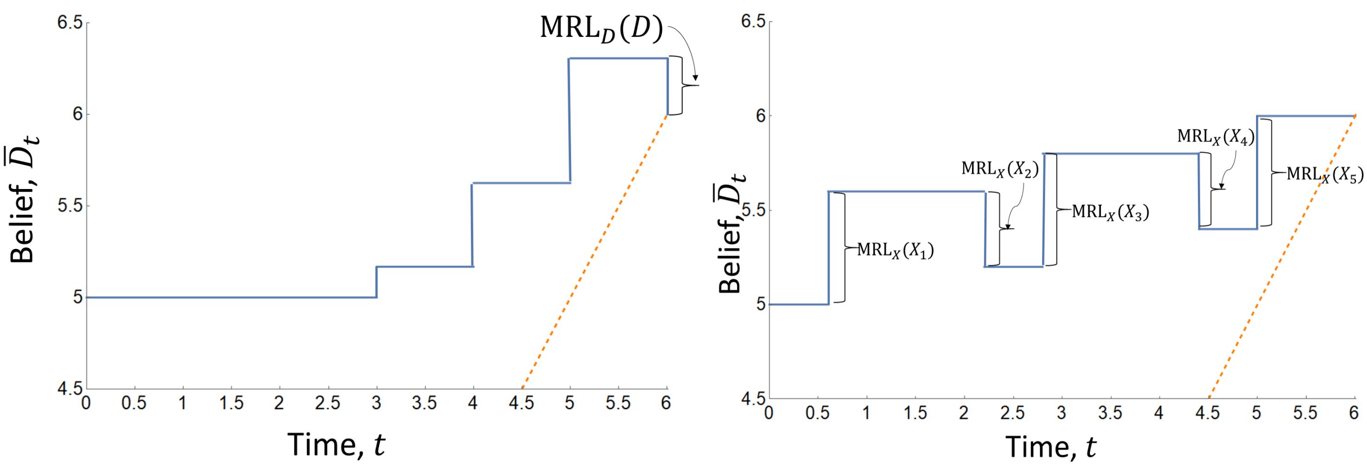
$$\tau_i = \mathbb{E}[I - i | I \geq i].$$

their mean belief from δ_i to δ_{i+1} (*bad news*, since $\delta_i < \delta_{i+1}$). The total stock of news corresponding to a realization of D, say t_{i^*}, is resolved as follows:

$$n\overline{x} - D = \underbrace{(\delta_0 - \delta_1)}_{\text{bad news at } t=t_0} + \underbrace{(\delta_1 - \delta_2)}_{\text{bad news at } t=t_1} + \ldots + \underbrace{(\delta_{i^*-1} - \delta_{i^*})}_{\text{bad news at } t=t_{i^*-1}} + \underbrace{(\delta_{i^*} - D)}_{\text{good news at } t=D(=t_{i^*})} . \quad (8)$$

Using straightforward algebra, the magnitude of good and bad news' at $t = t_i$ (depending on whether $D = t_i$, or $D > t_i$) can be further simplified as follows:

$$\text{Good news at } t = t_i \text{ (if } D = t_i\text{)}: \ \delta_i - t_i = \tau_i \Delta_x,$$
$$\text{Bad news at } t = t_i \text{ (if } D > t_i\text{)}: \ \delta_i - \delta_{i+1} = -\left(\frac{q_i}{\sum_{m=i+1}^{n} q_m}\right) \tau_i \Delta_x.$$



(a) OP (b) CTI

Figure 4 Belief Evolution under OP and CTI

Notes: In the above figure, we consider a process that consists of $n = 5$ tasks. The task durations are drawn from an i.i.d. two point distribution $X_i \sim 0.6 \circ 0.6 + 0.4 \circ 1.6$. Thus, the mean task duration is $\overline{x} = 1$ and $\Delta_x = 1$. Further, $D \sim \sum_{i=0}^{5} \binom{5}{i} 0.4^i 0.6^{5-i} \circ (1.6i + 0.6(5-i)) \equiv f^{(n)} = \boldsymbol{\pi}_0$ and $\overline{D}_0 = 5$. Let the realized task durations be: $X_1 = 1.6$, $X_2 = 0.6$, $X_3 = 1.6$, $X_4 = 0.6$, and $X_5 = 1.6$. Under OP, the belief updates occur at $t = 3, 4, 5, 6$, while under CTI, the belief updates occur at $t = 0.6, 2.2, 2.8, 4.4, 5$. The cumulative cancelled news under OP (resp., CTI) is equal to $\mathrm{MRL}_D(D)$ (resp., $\sum_{i=1}^{5} \mathrm{MRL}_X(X_i)$).

As an illustration, consider the example in the left panel of Figure 4. Under OP, the consumer's mean belief $\overline{D}_t$ increases (*worsens*) in $t \in [0, D)$, where the jumps in the mean belief occur at $t \in \{t_0, t_1, \ldots\}$. At $t = D$, the consumer experiences a decrease in their mean belief which is equal to the amount of cancelled news under OP; on average, this quantity is equal to $\mathbb{E}[\mathsf{MRL}_D(D)]$, the (expected) mean residual lifetime at the time of failure for D. We denote this quantity by $y_{(n)}$:

$$y_{(n)} = \mathbb{E}\left[\mathsf{MRL}_D(D)\right] = \sum_{i=0}^{n} q_i \underbrace{\mathsf{MRL}_D(t_i^-)}_{=\delta_i - t_i} = \sum_{i=0}^{n} q_i \tau_i \Delta_x.$$

Taken together, the consumer's expected belief-based utility from the information at $t = t_i$ is:

$$U_B^{\mathsf{OP}}[t, t+dt]\Big|_{t=t_i} = q_i \underbrace{\mu\left(\Delta_x \tau_i\right)}_{\text{Utility from } good \text{ news}} + \left(\sum_{j=i+1}^{n} q_j\right) \underbrace{\mu\left(-\Delta_x\left(\frac{q_i}{\sum_{m=i+1}^{n} q_m}\right)\tau_i\right)}_{\text{Utility from } bad \text{ news}}.$$

Therefore, the expected belief-based utility under OP, U_B^{OP}, can be written as:

$$U_B^{\mathsf{OP}} = \sum_{i=0}^{n}\left[q_i \mu\left(\Delta_x \tau_i\right) + \left(\sum_{j=i+1}^{n} q_j\right) \mu\left(-\Delta_x\left(\frac{q_i}{\sum_{m=i+1}^{n} q_m}\right)\tau_i\right)\right]. \tag{9}$$

Suppose $\mu \in \mathscr{U}^{\mathsf{PL}}$. Then, U_B^{OP} further simplifies to:

$$U_B^{\mathsf{OP}} = -\underbrace{\left(\sum_{0}^{n} q_i \tau_i\right)\Delta_x}_{=y_{(n)}} \Delta_\rho. \tag{10}$$

Current-Task Identity Strategy: Under CTI, the consumer receives an update upon each task completion, i.e., at time instants $t \in \left\{\sum_{j=0}^{i-1} X_i + x_L\right\}$, $i \in [n]$, where they learn if $X_i = x_L$ or x_H. Therefore, they incur a belief-based utility only at these time-instants.

Consider a time instant $t = \sum_{j=0}^{i-1} X_j + x_L$. The consumer learns whether $X_i = x_L$ or x_H, i.e., all uncertainty about task i's duration is resolved at t. Under the former, they update their mean belief on D from $\sum_{j=0}^{i-1} X_i + (n-i+1)\overline{x}$ to $\sum_{j=0}^{i-1} X_i + x_L + (n-i)\overline{x}$ (*good news*, since $\overline{x} > x_L$), while under the latter, they update their belief to $\sum_{j=0}^{i-1} X_i + x_H + (n-i)\overline{x}$ (*bad news*, since $\overline{x} < x_H$). Corresponding to a realization of D, the total stock of news is resolved as follows :

$$n\overline{x} - D = \underbrace{(\overline{x} - X_1)}_{\text{news resolved at } t = x_L} + \underbrace{(\overline{x} - X_2)}_{\text{news resolved at } t = X_1 + x_L} + \ldots + \underbrace{(\overline{x} - X_n)}_{\text{news resolved at } t = \sum_{j=1}^{n-1} X_j + x_L} \tag{11}$$

As an illustration, consider the right panel of Figure 4. The consumer's mean belief is non-monotonic, and depends on the uncertainty resolved in each task, i.e., the news at $t = \sum_{j=0}^{i-1} X_i + x_L$ may be good or bad, depending on the sign of $\overline{x} - X_i$. On average, the amount of cancelled news from each task is $\mathbb{E}[\mathsf{MRL}_X(X)]$, the (expected) mean residual life at the time of failure for X; we denote this quantity by $y_{(1)}$:

$$y_{(1)} = \mathbb{E}\left[\mathsf{MRL}_X(X)\right] = \overline{p}\mathsf{MRL}_X(x_L^-) + p\mathsf{MRL}_X(x_H^-) = \overline{p}p\Delta_x.$$

The expected belief-based utility (from the information on X_i at time $t=\sum_{j=0}^{i-1} X_i + x_L$) is:

$$U_B[t,t+dt)\Big|_{t=\sum_{j=0}^{i-1} X_i+x_L} = \overline{p}\mu\left(p\Delta_x\right)+p\mu\left(-\overline{p}\Delta_x\right)$$

Therefore, the total belief-based utility under CTI, U_B^{CTI}, is:

$$U_B^{\mathsf{CTI}} = n\left(\overline{p}\mu\left(p\Delta_x\right)+p\mu\left(-\overline{p}\Delta_x\right)\right) \tag{12}$$

Suppose $\mu \in \mathscr{U}^{\mathsf{PL}}$. Then, U_B^{CTI} simplifies to:

$$U_B^{\mathsf{CTI}} = -n\underbrace{p\overline{p}\Delta_x}_{y_{(1)}}\Delta_\rho. \tag{13}$$

Comparison: The following result compares OP and CTI under the piecewise linear model of belief-based utility and the two-point distribution for the task durations.

THEOREM 2. *Suppose $\mu \in \mathscr{U}^{\mathsf{PL}}$. Under the two-point distribution in (7), $\mathsf{OP} \succ \mathsf{CTI}$ iff:*

$$ny_{(1)} > y_{(n)}. \tag{14}$$

Observe that the conditions in (6) and (14) are identical – under discrete or continuous distributions, under the piecewise linear model, we compare the cumulative cancelled news under OP and CTI. In the Appendix, we analytically show that (14) holds for $n=2$ and 3 (Lemma C.7). For higher values of n, the algebraic expressions are cumbersome to analyze. Nevertheless, we show that (14) holds for higher values of n $(3 \le n \le 20)$ using Mathematica. That is, OP dominates CTI if the task durations are drawn from an i.i.d two-point distribution and the news-utility function is piecewise linear, regardless of the parameters $(p, x_L, x_H, \rho_N, \rho_P)$.

Taken together, the substantive implications of Theorems 1 and 2 are that sharing information about the progress of a flow-unit in a service process via updates of task completion creates greater fluctuations in the consumer's beliefs. Such a strategy hurts a consumer who exhibits loss aversion to news, i.e., they are belief-fluctuation averse.

REMARK 1. **(Abandonment by a Participating Consumer at $t>0$ is Irrational)** In the case of continuous distributions (Section 3.1), recall the assumption that X_i is IFR. Since the IFR property is closed under convolution (Lemma C.2), we have that the total delay D is IFR. Further, IFR implies DMRL (decreasing mean residual lifetime; Lemma C.2).

Consider OP. The consumer's expected delay at time t is $\overline{D}_t = t+\mathsf{MRL}_D(t)$. Therefore, their anticipated material payoff at time t is:

$$U_M\Big|_t = v - \mathsf{MRL}_D(t) - t.$$

However, the cost, $-t$, is sunk and does not affect their decision at time t. Since $v - \mathsf{MRL}_D(t)$ is increasing in t (since D has DMRL), it holds that

$$v - \underbrace{\mathsf{MRL}_D(0)}_{n\overline{x}} > 0 \implies v - \mathsf{MRL}_D(t) > 0.$$

The LHS of the first inequality above is the expected material payoff to a customer at the time of purchase; by rationality, we know that this quantity is positive and, thus, the first inequality is true. Therefore, the second inequality is also true. Therefore, abandonment by a participating consumer at time $t > 0$ is irrational under OP.

Consider CTI. The consumer's expected delay at time t, where $\sum_{j=0}^{i-1} X_i \leq t < \sum_{j=0}^{i} X_i$ for some $i \in [n]$ is

$$\overline{D}_t = t + \mathsf{MRL}_X\left(t - \sum_{j=0}^{i-1} X_i\right) + (n-i)\overline{x}.$$

Using an identical argument as above, it holds that

$$v - n\overline{x} > 0 \implies v - \mathsf{MRL}_X\left(t - \sum_{j=0}^{i-1} X_i\right) + (n-i)\overline{x} > 0,$$

where the LHS holds for any participating consumer.

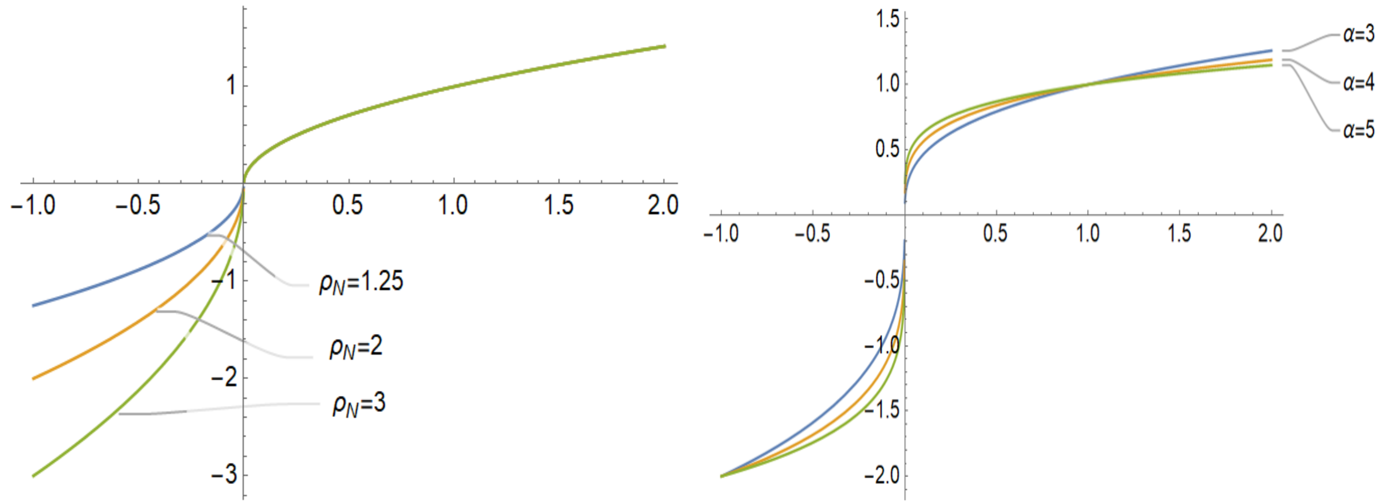
Now, consider the case of a two-point distribution for the task durations. Consider OP. First, observe that abandonment a consumer is irrational at $t \in (t_i, t_{i+1})$, $i \in \{0\} \cup [n]$ since no information arises in this interval. Second, the binomial distribution is IFR and hence has DMRL (Chapter 6 of Lai and Xie (2006)). Using an identical argument as before, it holds that at $t = t_i$, abandonment is irrational. Consider CTI. Abandonment is irrational at $t \neq \sum_{j=0}^{i} X_j + x_L$ since no new information arises, while at time $t = \sum_{j=0}^{i} X_j + x_L$, an identical argument as above holds. $\square$

4. Diminishing Sensitivity to News

Despite the prevalence of process trackers observed in practice, our two main results thus far, Theorems 1 and 2, show that under the piecewise linear model (i.e., loss aversion (A3) but not diminishing sensitivity (A4)) and the assumptions of the respective theorems, CTI is inferior to OP. We now consider a model where a consumer exhibits loss aversion and diminishing sensitivity to news (i.e, (A3) and (A4)). Due to algebraic complexity, we only study the case of a two-point distribution for the task durations.

Consider the following $\mu(\cdot)$ that satisfies (A1)-(A4) (in particular, (A3) and (A4) in the strict sense):

$$\mu(x) = \begin{cases} \rho_P \sqrt[\alpha]{x}, & \text{if } x \geq 0 \\ -\rho_N \sqrt[\alpha]{-x}, & \text{if } x < 0. \end{cases} \tag{15}$$



(a) ρ_N (b) α

Figure 5 Role of ρ_N and α

Notes: In the left figure, we consider the following form of $\mu(\cdot)$ as shown in (15), with $\alpha = 2$. The three curves correspond to $\rho_N = 1.25, 2, 3$ respectively. In the right figure, we consider $\mu(\cdot)$ as shown in (15) with $\rho_N = 2$. The three curves correspond to $\alpha = 3, 4, 5$ respectively.

where $\rho_N > \rho_P > 0$ and $\alpha > 1$. Let $\mathscr{U}^{\mathsf{DS}}$ denote the class of $\mu(\cdot)$ defined in (15). In Figure 5, we provide some examples of $\mu \in \mathscr{U}^{\mathsf{DS}}$. Without loss of generality, we assume that $\rho_P = 1 < \rho_N$. In the above model, ρ_N measures the degree of loss-aversion, while α measures the degree of diminishing sensitivity to news.

Substituting (15) in (9) and (12), the belief-based utility under OP and CTI is as follows:

$$U_B^{\mathsf{OP}} = \sum_{i=0}^{n} \left[q_i \sqrt[\alpha]{\Delta_x \tau_i} - \rho_N \left(\sqrt[\frac{\alpha}{\alpha-1}]{\sum_{j=i+1}^{n} q_j \sqrt[\alpha]{\Delta_x q_i \tau_i}} \right) \right] \tag{16}$$

$$U_B^{\mathsf{CTI}} = n \left(\overline{p} \sqrt[\alpha]{p \Delta_x} - \rho_N p \sqrt[\alpha]{\overline{p} \Delta_x} \right). \tag{17}$$

While both U_B^{OP} and U_B^{CTI} above are linear in ρ_N, further analysis and comparison are difficult due to algebraic complexity. Therefore, we first analyze the special case of a two-task process below.

4.1. Two-Task Process

Consider a two-task process ($n = 2$) and $\alpha = 2$. Define the following:

$$\overline{\rho}_N = \left(2 - \sqrt{2}\right) \frac{(1-p)\left(\sqrt{2-p} + \left(\frac{\sqrt{2}-\sqrt{2-p}}{\sqrt{2}-1}\right) p\right)}{\sqrt{p}\left(2(\sqrt{2} - \sqrt{(2-p)(1-p)}) - \sqrt{2}p(3 - p - \sqrt{1-p})\right)}. \tag{18}$$

Recall, from (16) and (17), that U_B^{OP} and U_B^{CTI} are linear in ρ_N. The following result characterizes the comparison of OP and CTI.

THEOREM 3. *CTI $\succ$ OP iff $\rho_N < \overline{\rho}_N$.*

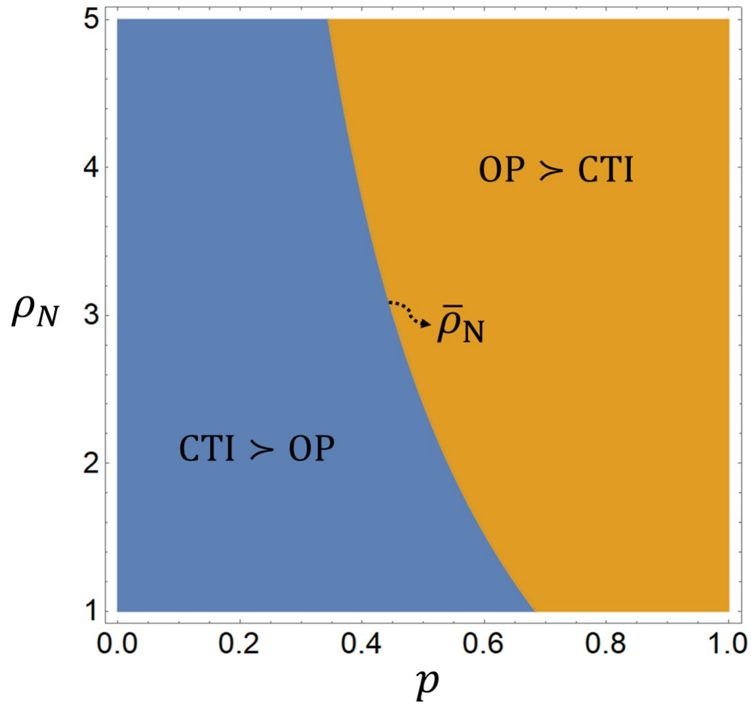


Figure 6 Comparison of OP and CTI under Loss Aversion and Diminishing Sensitivity: The x-axis denotes p, while the y-axis denotes ρ_N. Other parameters: $\alpha = n = 2$.

We illustrate Theorem 3 in Figure 6. Further, we have the following structural result.

THEOREM 4. *$\overline{\rho}_N$ in (18) is decreasing in p. That is, the following statements hold:*

- *If CTI $\succ$ OP for some $p' \in (0,1)$, then, CTI $\succ$ OP for all $p < p'$.*
- *If OP $\succ$ CTI for some $p' \in (0,1)$, then, OP $\succ$ CTI for all $p > p'$.*

The intuition behind Theorem 4 is as follows. Consider the following two outcomes:

- $D = 2x_L$: From (8) and (11), the total stock of news, $2\overline{x} - 2x_L$, is resolved as follows:

$$\text{Under OP: } 2\overline{x} - 2x_L = \underbrace{(2\overline{x} - 2x_L)}_{\textit{good}\text{ news resolved at } t=2x_L}$$
$$\text{Under CTI: } 2\overline{x} - 2x_L = \underbrace{(\overline{x} - x_L)}_{\textit{good}\text{ news resolved at } t=x_L} + \underbrace{(\overline{x} - x_L)}_{\textit{good}\text{ news resolved at } t=2x_L}$$

That is, OP resolves all uncertainty in one-shot (at $t = 2x_L$), while CTI resolves the uncertainty in two pieces – a piece $\overline{x} - x_L$ at $t = x_L$ and another piece $\overline{x} - x_L$ at $t = 2x_L$. In particular, observe that under both OP and CTI, the pieces of news are positive, i.e., *good* news (since $\overline{x} > x_L$). While both strategies provide the same stock of news, from the concavity of $\mu(\cdot)$ in

$x > 0$ (good news), we have that two small pieces of good news dominates one large piece of good news, i.e., $\mathsf{CTI} \succ \mathsf{OP}$.

- $D = 2x_H$: The total stock of news is, $2\overline{x} - 2x_H$, is resolved by OP and CTI as follows:

$$\text{Under } \mathsf{OP}\text{: } 2\overline{x} - 2x_H = \underbrace{(2\overline{x} - \delta_1)}_{bad \text{ news resolved at } t=2x_L} + \underbrace{(\delta_1 - 2x_H)}_{bad \text{ news resolved at } t=x_L+x_H}$$
$$\text{Under } \mathsf{CTI}\text{: } 2\overline{x} - 2x_H = \underbrace{(\overline{x} - x_H)}_{bad \text{ news resolved at } t=x_L} + \underbrace{(\overline{x} - x_H)}_{bad \text{ news resolved at } t=x_L+x_H}$$

That is, OP resolves uncertainty in two pieces – a piece $2\overline{x} - \delta_1$ at $t = 2x_L$ and another piece $\delta_1 - 2x_H$ at $t = x_L + x_H$, while CTI resolves uncertainty in two equal pieces of $\overline{x} - x_H$ at $t = x_L$ and at $t = x_H + x_L$. In particular, under both OP and CTI, the pieces of news are negative, i.e., *bad* news (since $\overline{x} < x_H$). While both strategies reveal the same stock of news, the convexity of $\mu(\cdot)$ in $x < 0$ shows that two equal pieces of bad news is dominated by a large and a small piece of bad news, i.e., $\mathsf{OP} \succ \mathsf{CTI}$.

If p is small, then the more likely outcome is $D = 2x_L$; thus, CTI is the preferred choice. If p is large, the more likely outcome is $D = 2x_H$; thus, OP is the preferred choice. Stated differently, if the eventual outcome is *good* (low delay), providing multiple pieces of good news is better. If the eventual outcome is *bad* (high delay), then providing multiple pieces of bad news hurts. Therefore, the scope of the result $\mathsf{CTI} \succ \mathsf{OP}$ is decreasing in p, the likelihood of the *bad* outcome.

Role of α: Consider a more general $\mu(\cdot)$ as defined in (15).

Define $\overline{\rho}_N(\alpha)$ as follows:

$$\overline{\rho}_N(\alpha) = \frac{(1-p)\sqrt[\alpha]{p}\left(2\sqrt[\alpha]{2-p} - (1-p)\sqrt[\alpha]{2(2-p)} - 2p\right)}{p\left(2\sqrt[\alpha]{(1-p)(2-p)} - (2-p)\sqrt[\alpha]{2(1-p)^2} - p\sqrt[\alpha]{2(1-p)}\right)}. \tag{19}$$

$\overline{\rho}_N$ defined in (18) is equal to $\overline{\rho}_N(2)$ defined above. Analogous to Theorem 3, we have the following characterization:

THEOREM 5. *$\mathsf{CTI} \succ \mathsf{OP}$ iff $\rho_N < \overline{\rho}_N(\alpha)$.*

Furthermore, we observe the following:

(a) Analogous to Theorem 4, we have that $\overline{\rho}_N(\alpha)$ is decreasing in p for a given α.

(b) Over the relevant range of p (where $\overline{\rho}_N(\alpha) > 1$), we have that $\overline{\rho}_N(\alpha)$ is increasing in α for a given p.

The intuition to (a) is identical to that of Theorem 4; thus, the scope of the result $\mathsf{CTI} \succ \mathsf{OP}$ is decreasing in p. The intuition to (b) is as follows: an increase in α makes the consumer less sensitive

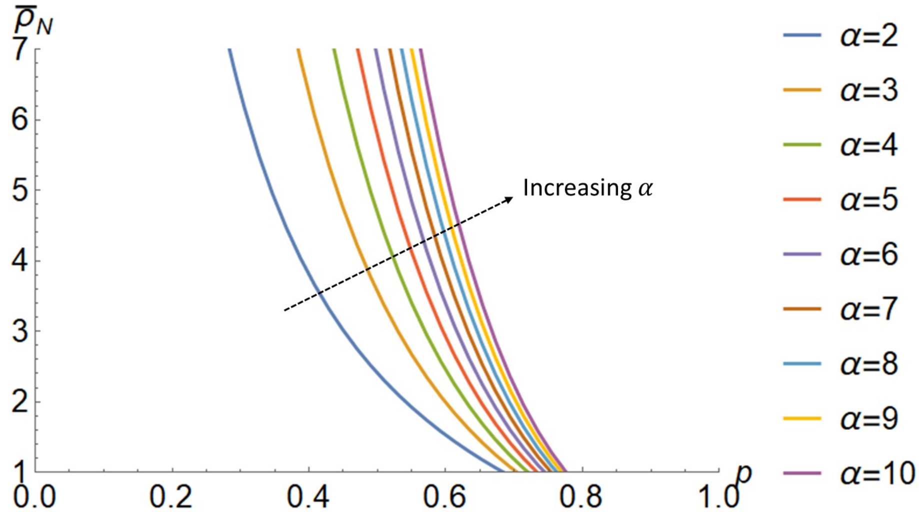


Figure 7 Effect of Higher α: $\overline{\rho}_N(\alpha)$ for $\alpha \in \{2,3,\ldots,10\}$. The x-axis denotes p, and the y-axis denotes $\overline{\rho}_N$. Other parameters: $n=2$. $\overline{\rho}_N(\alpha)$ is decreasing in p for given α (Observation (a)) and increasing in α for a given p (Observation (b)).

to large pieces of news (good or bad); consequently, the scope of the result $\mathsf{CTI} \succ \mathsf{OP}$ is increasing in α. Analytical proofs of the above observations elude us due to the algebraic complexity. We illustrate these observations in Figure 7.

Theorem 4 and the observations (a) and (b) above allude to the role of diminishing sensitivity to news in the consumer's belief-based utility that leads to greater value in sharing task completion updates in a service process. From a substantive standpoint, several papers in Behavioral Economics study the role of both loss aversion and diminishing sensitivity in the monetary domain since the seminal work by Kahneman and Tversky (1979) and in subsequent work (Kőszegi and Rabin 2009). However, almost all the work in service operations that explore reference-dependent behavior in the temporal domain consider the role of loss aversion but not diminishing sensitivity (Yu et al. 2021, Ho and Zheng 2004).

Our work highlights the contrasting roles of loss aversion and diminishing sensitivity to news in leading to opposite predictions. Loss aversion favors OP, while diminishing sensitivity favors CTI. The preferred strategy depends on the dominant economic force.

4.2. Robustness of Our Results: n-Task Processes

For robustness, we analyze processes with $n>2$ tasks. Define the following:

$$\overline{\rho}_N(n,\alpha) = \frac{n\overline{p}\,\sqrt[\alpha]{p} - \sum_{i=0}^{n}\left(q_i\,\sqrt[\alpha]{\tau_i}\right)}{np\,\sqrt[\alpha]{\overline{p}} - \sum_{i=0}^{n}\left(\sqrt[\alpha]{q_i\tau_i}\,\sqrt[\frac{\alpha}{\alpha-1}]{\sum_{j=i+1}^{n} q_j}\right)}. \tag{20}$$

$\overline{\rho}_N(\alpha)$ defined in (19) is equal to $\overline{\rho}_N(2,\alpha)$ defined above, while $\overline{\rho}_N$ defined in (18) is equal to $\overline{\rho}_N(2,2)$ defined above. From (9) and (12), we have that U_B^{OP} and U_B^{CTI} are linear in ρ_N. Analogous to Theorem 3 and 5, we have the following result from (16) and (17).

THEOREM 6. $\mathsf{CTI} \succ \mathsf{OP}$ *iff* $\rho_N < \overline{\rho}_N(n,\alpha)$.

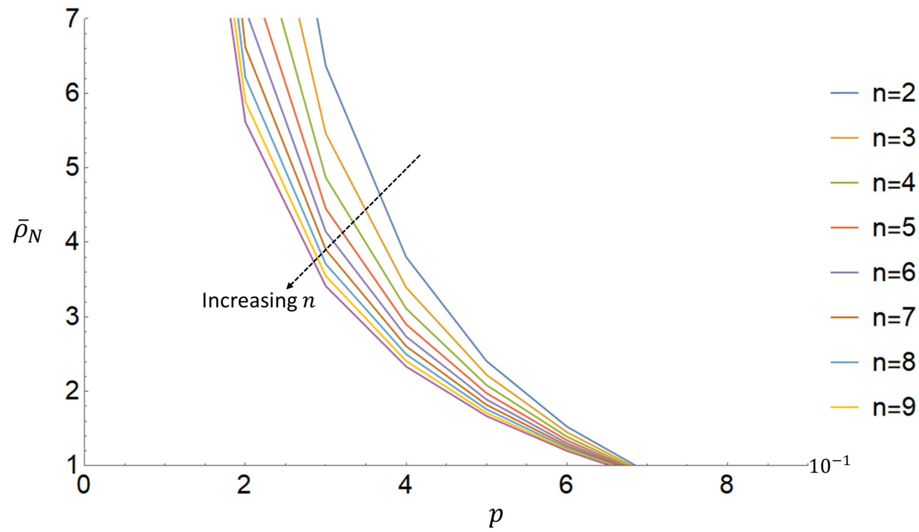


Figure 8 Effect of Higher n: $\overline{\rho}_N$ for $n \in \{2,3,\ldots,10\}$. The x-axis denotes p while the y-axis denotes $\overline{\rho}_N$. Other parameters: $\alpha = 2$.

An analytical investigation of $\overline{\rho}_N(n,\alpha)$ is difficult due to algebraic complexity; numerically, we observe the following:

(a) $\overline{\rho}_N(n,\alpha)$ is decreasing in p for a given n,α.

(b) $\overline{\rho}_N(n,\alpha)$ is decreasing in n for a given p,α.

Therefore, our analytical results for the special case of $n = 2$ in Theorems 3 and 4 are robust to higher values of n.

5. Substantive and Managerial Implications

From a substantive standpoint, our work demonstrates the roles of loss aversion and diminishing sensitivity to news in understanding consumers' reference-dependent utility from non-instrumental progress information. While several papers in OM study the role of loss aversion in understanding consumer behavior, diminishing sensitivity has received far less attention, perhaps due to algebraic complexity. Our work highlights the contrasting roles of loss aversion and diminishing sensitivity (to news), and how predictions of a model with loss aversion but not diminishing sensitivity can be different from those that consider both.

From a managerial standpoint, our work highlights the role of non-instrumental information about delay in better managing a consumer's post-sales waiting experience. While the extant literature in service OM focuses exclusively on whether and how a firm shares instrumental delay information pre-sales to the consumer, our work complements this literature in proposing the first model on non-instrumental delay information post-sales.

Some limitations of our work that future research can address are as follows. While our work considers two progress disclosure strategies commonly observed in practice, future research can consider other progress disclosure strategies, depending on the information available to the firm. The consumer's belief-based utility in our model arises due to good and bad news about delay (i.e., information that increases or decreases his mean belief on delay). Future research can consider more general forms of belief-based utility, e.g., anxiety costs (Iyer and Zhong 2021), suspense and surprise (Ely et al. 2015), etc. We assume that the consumer's value from the service process is independent of the delay. Future research can consider the case of customer-intensive processes where the value from the service may be correlated with delay (Anand et al. 2011).

References

Allon, G., A. Bassamboo, and I. Gurvich. 2011. "We will be right with you": Managing customer expectations with vague promises and cheap talk. *Operations Research* 59 (6): 1382–1394.

Anand, K. S., M. F. Paç, and S. Veeraraghavan. 2011. Quality–speed conundrum: Trade-offs in customer-intensive services. *Management Science* 57 (1): 40–56.

Bray, R. L. 2020. Operational transparency: Showing when work gets done. *Manufacturing & Service Operations Management*.

Caplin, A., and J. Leahy. 2001. Psychological expected utility theory and anticipatory feelings. *The Quarterly Journal of Economics* 116 (1): 55–79.

Das Gupta, A., U. S. Karmarkar, and G. Roels. 2016. The design of experiential services with acclimation and memory decay: Optimal sequence and duration. *Management Science* 62 (5): 1278–1296.

Dillenberger, D., and C. Raymond. 2020. Additive-belief-based preferences.

Duraj, J., and K. He. 2019. Dynamic information design with diminishing sensitivity over news.

Ely, J., A. Frankel, and E. Kamenica. 2015. Suspense and surprise. *Journal of Political Economy* 123 (1): 215–260.

Falk, A., and F. Zimmermann. 2016. Beliefs and utility: Experimental evidence on preferences for information.

Fast Company 2019. Uber Eats is going to stop gaslighting you with confusing design. https://www.fastcompany.com/90331468/uber-eats-is-going-to-stop-gaslighting-you-with-confusing-design. Online; accessed 23 June 2021.

Hassin, R. 2016. *Rational queueing*. CRC press.

Ho, T. H., and Y.-S. Zheng. 2004. Setting customer expectation in service delivery: An integrated marketing-operations perspective. *Management Science* 50 (4): 479–488.

Ibrahim, R. 2018. Sharing delay information in service systems: a literature survey. *Queueing Systems* 89 (1): 49–79.

Iyer, G., and Z. Z. Zhong. 2021. Pushing notifications as dynamic information design. *Marketing Science*.

Kahneman, D., and A. Tversky. 1979. Prospect Theory: An analysis of decision under risk. *Econometrica* 47 (2): 263–292.

Kőszegi, B., and M. Rabin. 2006. A model of reference-dependent preferences. *The Quarterly Journal of Economics* 121 (4): 1133–1165.

Kőszegi, B., and M. Rabin. 2009. Reference-dependent consumption plans. *American Economic Review* 99 (3): 909–36.

Kumar, P., M. U. Kalwani, and M. Dada. 1997. The impact of waiting time guarantees on customers' waiting experiences. *Marketing Science* 16 (4): 295–314.

Lai, C. D., and M. Xie. 2006. *Stochastic ageing and dependence for reliability*. Springer Science & Business Media.

Li, Y., T. Dai, and X. Qi. 2020. A theory of interior peaks: Activity sequencing and selection for service design. *Manufacturing & Service Operations Management*.

Lingenbrink, D., and K. Iyer. 2019. Optimal signaling mechanisms in unobservable queues. *Operations Research* 67 (5): 1397–1416.

Müller, A., and D. Stoyan. 2002. *Comparison methods for stochastic models and risks*, Volume 389. Wiley.

Reddit 2019. Why is this taking a year? https://www.reddit.com/r/UberEATS/comments/9l9bpu/why_is_this_taking_a_year/. Online; accessed 23 June 2021.

Reddit 2020. At what point does a restaurant actually start preparing an order? https://www.reddit.com/r/grubhub/comments/hcp7rf/at_what_point_does_a_restaurant_actually_start/. Online; accessed 23 June 2021.

Ross, S. M., J. G. Shanthikumar, and Z. Zhu. 2005. On increasing-failure-rate random variables. *Journal of Applied Probability* 42 (3): 797–809.

The Internal Revenue Service 2021. Here's how taxpayers can track the status of their refund. https://www.irs.gov/newsroom/heres-how-taxpayers-can-track-the-status-of-their-refund. Online; accessed 23 June 2021.

The Wall Street Journal 2017. Domino's Tracking App Tells You Who Made Your Pizza—Or Does It? https://www.wsj.com/articles/dominos-tracking-app-tells-you-who-made-your-pizzaor-does-it-1511889445. Online; accessed 23 June 2021.

Yu, Q., G. Allon, and A. Bassamboo. 2021. The reference effect of delay announcements: A field experiment. *Management Science*.

Yuan, X., T. Dai, L. G. Chen, and S. Gavirneni. 2021. Co-opetition in service clusters with waiting-area entertainment. *Manufacturing & Service Operations Management* 23 (1): 106–122.

Appendix A: Preliminaries on Belief-Based Utility in Economics

Recent work in Economics suggest that agents realize utility from non-instrumental information. Consider the following examples by Dillenberger and Raymond (2020) and Ely et al. (2015) to better understand the notion of belief-based utility.

(a) An individual has a vacation upcoming in a few days. The weather on the day of the vacation – the outcome of interest – is uncertain and is realized only on the day of the vacation; the individual has preferences over the outcome (e.g., the individual prefers good weather over bad weather on the day of vacation). They monitor the weather forecast periodically and updates their belief (on the outcome) based on the forecast; the individual is a rational Bayesian. The individual may enjoy (resp., not enjoy) looking forward to the upcoming vacation if good (resp., bad) weather is more likely.

(b) An individual watches a sporting event/political debate. The identity of the eventual winner – the outcome of interest – is uncertain and is realized at the end of the game/debate. The individual has preferences over the outcome (e.g., the individual may have a favorite team/candidate and prefers their favorite team/candidate to win). They watch the game/debate; as it unfolds, the various events provide signals of the outcome (eventual winner). The individual periodically updates their belief on the outcome, based on these events. The individual may enjoy (resp., not enjoy) watching the game/debate if their favored team/player is more (resp., less) likely to win.[12]

In both these examples, observe that the agent realizes utility both from consumption and in the interim (before consumption), while no contingent action awaits. Below, we formally describe the canonical consumer-utility model from the recent literature in Economics, where the consumer realizes utility from consumption (material payoff/consumption utility) and *news* (news/belief-based utility) (Duraj and He 2019, Dillenberger and Raymond 2020).

A.1. Canonical Consumer-Utility Model: Consumption and Belief-Based Utilities

The model described below is a discrete-time model. Consider an environment that consists of one agent and nature. Time is discrete, consisting of $T \in \mathbb{I}^+$ periods and indexed by $t \in [T]$. At the start of period 1, nature chooses a state among a finite number of states $\Omega = \{\omega_1, \omega_2, \ldots, \omega_N\}$ from a distribution $\boldsymbol{\pi}_0 = \{\pi_0(\omega_i)\}_{i=1}^N$, but does not inform the agent. Let $\omega \in \Omega$ denote a generic state; $\boldsymbol{\pi}_0$ is common knowledge. The agent does not have any action. Consumption occurs at the end of period T, when the agent realizes a (state-dependent) consumption utility v_ω; consumption does

[12] Figure 1 of Ely et al. (2015) refers to win probability plots – the likelihood of a player's win – during a game of tennis. Such in-game win probability plots are commonplace across sports.

not occur in any other periods. Without loss of generality, we assume that $\pi_0(\omega) > 0$ for all $\omega \in \Omega$, and $v_{\omega_1} < v_{\omega_2} < \ldots < v_{\omega_N}$. Therefore, ω_1 (resp., ω_N) is the worst (resp., best) state for the agent. Let $\mathbf{\Pi} = \Delta(\Omega)$ denote the set of possible distributions on Ω; $\boldsymbol{\pi}_0 \in \mathbf{\Pi}$.

In each period $t \in [T]$, nature provides the agent with *news*, i.e, messages (or signals) about ω through an (exogenous) messaging device. Let $\mathscr{M}$ denote the set of messages and $\mathbf{M} = \Delta(\mathscr{M})$ denote the set of all distributions over $\mathscr{M}$. The messaging device sends messages according to $\boldsymbol{\sigma} = \{\sigma_t\}_{t=1}^{T}$, where $\sigma_t(\cdot|h^{t-1}, \omega) \in \mathbf{M}$ is a distribution over messages in period t that depends on the state ω and the history $h^{t-1} \in H^{t-1} := (\mathscr{M})^{t-1}$, i.e., messages sent thus far; $\boldsymbol{\sigma}$ is common knowledge.

At the end of each period $t \in [T]$, the agent, a rational Bayesian, forms a posterior belief $\boldsymbol{\pi}_t$ about the state ω after the history h^t of messages; clearly $\boldsymbol{\pi}_t \in \mathbf{\Pi}$. This belief $\boldsymbol{\pi}_t$ is rational, and is calculated using $\boldsymbol{\pi}_0$ and the knowledge of the messaging device $(\mathscr{M}, \boldsymbol{\sigma})$. In period T, nature reveals the state (which it chose at the start of period 1), say $\tilde{\omega}$, to the agent. Then, the agent's belief is the degenerate distribution $\boldsymbol{\pi}_T(\tilde{\omega}) = 1$ and $\boldsymbol{\pi}_T(\omega) = 0$ for any $\omega \neq \tilde{\omega}$, $\omega \in \Omega$. We denote the degenerate belief by $\boldsymbol{\pi}_T = 1 \circ \tilde{\omega}$.

The agent derives utility based on the changes in their belief about the consumption in period T. Specifically, let $\boldsymbol{\mu} : \mathbf{\Pi} \times \mathbf{\Pi} \to \Re$ denote a mapping from their new and old beliefs into the real numbers. They realize $\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1})$ at the end of period t, $t \in [T]$. Utility flow is undiscounted and $\boldsymbol{\mu}(\cdot|\cdot)$ is fixed across periods. The total expected utility of the agent is the sum of the consumption utility in period T and the belief-based utilities across all periods $t \in [T]$.

$$
\begin{aligned}
\text{Total Expected Utility } &= \mathbb{E}_{\omega \sim \boldsymbol{\pi}_0}\left[\underbrace{v_\omega}_{\text{consumption utility in state } \omega} + \sum_{t \in [T]} \underbrace{\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1})}_{\text{belief-based utility in period } t} \right], \\
&= \underbrace{\mathbb{E}_{\omega \sim \boldsymbol{\pi}_0}[v_\omega]}_{\text{Expected Consumption Utility, } U_M} + \underbrace{\sum_{t \in [T]} \mathbb{E}_{\omega \sim \boldsymbol{\pi}_0}\left[\boldsymbol{\mu}\left(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}\right)\right]}_{\text{Expected Belief-Based Utility, } U_B} .
\end{aligned} \tag{A.21}
$$

The first term (the expected consumption utility, U_M) is independent of $(\mathscr{M}, \boldsymbol{\sigma})$. Hence, it is sufficient to consider only the second term (the expected belief-based utility, U_B) in our comparison of different information-provisioning strategies (of a firm to a consumer).

A.2. Models for $\boldsymbol{\mu}(\cdot|\cdot)$

Broadly, papers that study *news* utility adopt one of two belief-based utility models (Dillenberger and Raymond 2020):

(a) Anticipatory Utility Models (AUM), and

(b) Changing Beliefs Models (CBM).

In both these models, the total utility is additively separable in the belief-based utility and the consumption utility; the difference lies in the model for belief-based utility. Consider a piece of news, that leads to a change in the agent's beliefs on ω from $\boldsymbol{\pi}$ to $\boldsymbol{\pi}'$. The agent's belief-based utility from consuming this piece of news under the two models is as follows:

(a) Under AUM (Caplin and Leahy 2001), the agent's belief-based utility depends on their absolute levels of beliefs:

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu(\boldsymbol{\pi}'),$$

where $\mu : \boldsymbol{\Pi} \to \Re$ maps beliefs on ω to the real line.

(b) Under CBM (Kőszegi and Rabin 2009), the agent's belief-based utility depends on the change in their beliefs on ω:

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu\left(F(\boldsymbol{\pi}') - F(\boldsymbol{\pi})\right),$$

where $F : \boldsymbol{\Pi} \to \Re$. Some examples are:

- Duraj and He (2019) employ a mean-based changing beliefs model, where $F(\boldsymbol{\pi}) = \mathbb{E}_{\omega\sim\boldsymbol{\pi}}[v_\omega]$. Therefore,

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \mu\left(\mathbb{E}_{\omega\sim\boldsymbol{\pi}'}[v_\omega] - \mathbb{E}_{\omega\sim\boldsymbol{\pi}}[v_\omega]\right). \tag{A.22}$$

- Kőszegi and Rabin (2009) briefly discuss the above model, but predominantly focus on a percentile-based changing beliefs model, where

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = \int_{p=0}^{1} \mu\left(G^{-1}_{\omega\sim\boldsymbol{\pi}'}(p) - G^{-1}_{\omega\sim\boldsymbol{\pi}}(p)\right) dp,$$

where $G_{\omega\sim\boldsymbol{\pi}}(\cdot)$ (resp., $G^{-1}_{\omega\sim\boldsymbol{\pi}}(p)$) denotes the CDF (resp., p^{th}-percentile) of ω with distribution $\boldsymbol{\pi}$.

In line with recent research that models belief-based utility from consumption of *news* (Duraj and He 2019), and for analytical tractability, we adopt the mean-based changing beliefs model. Furthermore, $\mu(\cdot)$ is increasing, differentiable everywhere except possibly at 0, and $\mu(0) = 0$. Kőszegi and Rabin (2006) refer to the above model as the *reference-dependent universal gain-loss utility* model, where:

(a) the gain-loss utility is itself derived from consumption utility, and
(b) the *reference* is determined endogenously through the prior beliefs, i.e., rational expectations held in the recent past about outcomes.

Using (A.22) and (A.21), the agent's total expected utility can be written as:

$$\text{Total Expected Belief-Based Utility, } U_B = \sum_{t\in[T]} \mathbb{E}_{\omega\sim\boldsymbol{\pi}_0}\left[\mu\left(\mathbb{E}_{\omega\sim\boldsymbol{\pi}_t}[v_\omega] - \mathbb{E}_{\omega\sim\boldsymbol{\pi}_{t-1}}[v_\omega]\right)\right]. \tag{A.23}$$

REMARK A.1. **(Consequentialist Model)** A *consequentialist* model is one where an agent does not incur any belief-based utility, i.e.,

$$\boldsymbol{\mu}(\boldsymbol{\pi}'|\boldsymbol{\pi}) = 0.$$

In the standard consequentialist model, all information-disclosure strategies are identical. □

REMARK A.2. **(Suspense and Surprise)** Ely et al. (2015) consider a model where the agent's belief-based utility from consuming a piece of news depends on the amount of *suspense* or *surprise*. In the suspense (resp., surprise) model, the agent has a preference for suspense (resp., surprise). Formally, consider the agent's belief-based utility in period t, where their beliefs are $\boldsymbol{\pi}_t$. Suspense is the amount of variability in period-$(t+1)$ beliefs (e.g., standard deviation of $\boldsymbol{\pi}_{t+1}$), while surprise is the difference in beliefs from period-$(t-1)$ to period-t (e.g., the Euclidean distance between $\boldsymbol{\pi}_{t-1}$ and $\boldsymbol{\pi}_t$). Formally,

- Under the suspense model,

$$\boldsymbol{\mu}(\boldsymbol{\pi}_t) = \mu\left(\mathbb{E}\left[\sum_{i\in[N]}(\pi_{t+1}(\omega_i) - \pi_t(\omega_i))^2\right]\right),$$

  where $\pi_{t+1}(\omega_i)$ is random and $\mathbb{E}[\pi_{t+1}(\omega_i)] = \pi_t(\omega_i)$, due to the Martingale property of Bayesian updating.

- Under the surprise model, the agent has a preference for surprise:

$$\begin{aligned}\boldsymbol{\mu}(\boldsymbol{\pi}_t|\boldsymbol{\pi}_{t-1}) &= \mu\left(||\boldsymbol{\pi}_t - \boldsymbol{\pi}_{t-1}||_2\right),\\ \text{where } ||\boldsymbol{\pi}' - \boldsymbol{\pi}||_2 &= \sum_{i\in[N]}(\pi'(\omega_i) - \pi(\omega_i))^2 \text{ for any } \boldsymbol{\pi}, \boldsymbol{\pi}' \in \Pi.\end{aligned}$$

Their focus is on the entertainment value of news (e.g., sporting events, movies), which is different from the value of news (e.g., delay estimate, ETA, etc.) to a consumer in our model. □

A.3. Main Differences Between Our Work and Extant Literature in Economics

Unlike prior work in Economics, where the payoff-relevant variable is an exogenous state of nature (e.g., the weather in Example (a); the winner of the sporting event/debate in Example (b)), the payoff-relevant variable for a consumer in our model is the length of the horizon (i.e., the delay). Furthermore, in prior work, the lack of information provision (i.e., no news) during a period does not affect the consumer's beliefs about the payoff-relevant variable, and hence the agent does not realize any belief-based utility. However, in our model, the mere passage of time provides information (*bad* news) to the consumer about the realized delay. The sender's (firm) messaging strategies we analyze are commonly observed in service processes.

Appendix B: Proofs of Technical Results/Statements in the Paper

Proof of Theorem 1: From (4) and (5), we have that:

$$\mathsf{OP} \succ \mathsf{CTI} \Leftrightarrow U_B^{\mathsf{OP}} > U_B^{\mathsf{CTI}} \implies ny_{(1)} > y_{(n)}. \quad \square$$

Proof of Theorem 2: Recall that $U_B^{\mathsf{CTI}} = -np\overline{p}\Delta_\rho\Delta_x$, while $U_B^{\mathsf{OP}} = -\left(\sum_{i=0}^n q_i\tau_i\right)\Delta_\rho\Delta_x$. It suffices to compare the terms $np\overline{p}$ $(=ny_{(1)})$ and $\sum_{i=0}^n q_i\tau_i$ $(=y_{(n)})$. Thus, it suffices to show:

$$y_{(n)} < ny_{(1)}. \quad \square$$

Proof of Theorem 3: Recall the expressions of U_B^{CTI} and U_B^{OP}: Observe that both U_B^{CTI} and U_B^{OP} are linear in ρ_N. Therefore,

$$\begin{aligned} U_B^{\mathsf{CTI}} > U_B^{\mathsf{OP}} &\Leftrightarrow 2(1-p)\sqrt{p} - \rho_N\left(2p\sqrt{1-p}\right) > (1-p)\sqrt{p}\left(\sqrt{2}(1-p) + 2p\sqrt{\frac{1}{2-p}}\right) - \rho_N\left((2-p)\sqrt{\frac{2}{2-p} - 2p + p^2}\sqrt{2-\frac{2}{2-p}}\right) \\ &\Leftrightarrow \rho_N < \overline{\rho}_N. \quad \square \end{aligned}$$

Proof of Theorem 4: We calculate $\frac{d\overline{\rho}_N}{dp}$ and show that it is negative for $p \in (0,1)$. For convenience, denote the expression for $\overline{\rho}_N$ in (18) as $\frac{u}{v}$. Then, $\frac{d\overline{\rho}_N}{dp} = \frac{u'v - v'u}{v^2}$. It suffices to show that the numerator in the RHS, $u'v - v'u < 0$. The numerator of $\frac{d\overline{\rho}_N}{dp}$ is the sum of three terms:

$$\begin{aligned} \text{numerator} = &-\underbrace{\left[\frac{p}{\sqrt{2}}\left((\sqrt{2}-1)\sqrt{2-p} + (\sqrt{2-p}-\sqrt{2})p\right)\left(2\left(\sqrt{2(2-p)} + \sqrt{1-p} - \sqrt{2(2-p)(1-p)}\right) + \sqrt{2}p\left(-4\sqrt{2-p} + 2\sqrt{2-p}p + \sqrt{(2-p)(1-p)}\right)\right)\right]}_{\text{Ⓐ}} \\ &+ \underbrace{\left[(1-p)\left(2-\sqrt{2}+p\left(\sqrt{2}-\frac{2}{\sqrt{2-p}}\right)\right)\left(2\left(\sqrt{2}-\sqrt{(2-p)(1-p)}\right) - \sqrt{2}p\left(3-p-\sqrt{1-p}\right)\right)\right]}_{\text{Ⓑ}} \\ &- \underbrace{\left[((2-p)(1-p)p)\left(\sqrt{2}-\frac{2}{\sqrt{2-p}}-\frac{p}{(2-p)^{\frac{3}{2}}}\right)\left(2\left(\sqrt{2}-\sqrt{(2-p)(1-p)}\right) - \sqrt{2}p\left(3-p-\sqrt{1-p}\right)\right)\right]}_{\text{Ⓒ}}. \end{aligned}$$

We show that Ⓐ, Ⓒ are positive, while Ⓑ is negative; thus, the RHS is negative. Consider Ⓐ (resp., Ⓑ, Ⓒ), that comprises of the product of three terms. Below, we show that each of the terms are positive (resp., the first and second terms are positive, while the third term is negative, and the first term is positive, while the second and third term are negative).

Below, we consider Ⓐ, Ⓑ and Ⓒ and show that Ⓐ, Ⓒ > 0 while Ⓑ < 0.

- Ⓐ: Observe that term Ⓐ is the product of three terms. The first term $\frac{p}{\sqrt{2}} > 0$. The second term in Ⓐ is the product of the second term in Ⓑ and $\sqrt{1-\frac{p}{2}}$. Below, we show that the second term in Ⓑ is positive; thus, the second term in Ⓐ is positive. Consider the last term. Substitute $1-p = y^2$; hence, $p \in (0,1) \Rightarrow y \in (0,1)$. The last term can be written as:

$$\sqrt{2}y\left(\sqrt{2} - \sqrt{1+y^2}\left(1+y^2-2y^3\right)\right).$$

Since $y \in (0,1)$, we focus on the term inside the brackets, and show that this term is positive. To show that

$$\sqrt{2} > \sqrt{1+y^2}\left(1+y^2-2y^3\right),$$

we show that the expression in the RHS increases and then decreases. To see this, consider the first derivative w.r.t y:

$$\frac{d}{dy}\left(\sqrt{1+y^2}\left(1+y^2-2y^3\right)\right) = \frac{y}{\sqrt{1+y^2}}\left(3-6y+3y^2-8y^3\right).$$

Since $\frac{y}{\sqrt{1+y^2}} > 0$ for $y \in (0,1)$, we consider the cubic polynomial in the brackets in the RHS above. The discriminant of this cubic polynomial evaluates to -14688; hence, the cubic polynomial has exactly one real root. This root is:

$$\begin{aligned} y_R &= \frac{1}{24}\left(3 - 135\sqrt[3]{\frac{2}{3942+864\sqrt{34}}} + \sqrt[3]{\frac{3942+864\sqrt{34}}{2}}\right) \\ &\approx 0.471. \end{aligned}$$

Therefore, the first derivative is positive in $y \in (0, y_R)$ and is negative in $y \in (y_R, 1)$. Thus, the RHS attains a maximum at $y = y_R$. The maximum value attained (at $y = y_R$) is ≈ 1.119. Thus, the inequality holds.

- Ⓑ: Observe that Ⓑ is the product of three terms. The first term $1-p > 0$ for $p \in (0,1)$. The second term can be rewritten as follows:

$$\sqrt{2}\left((\sqrt{2}-1) - \underbrace{p\left(\frac{\sqrt{2}-\sqrt{2-p}}{\sqrt{2-p}}\right)}_{\text{increasing in } p}\right)$$

We focus on the term inside the brackets. Observe that $p\left(\frac{\sqrt{2}-\sqrt{2-p}}{\sqrt{2-p}}\right)$ is strictly increasing in p; besides, $p\left(\frac{\sqrt{2}-\sqrt{2-p}}{\sqrt{2-p}}\right)\Big|_{p=1} = \sqrt{2}-1$. Thus, the second term is positive. To show that the last term is negative, it suffices to show the following (by rearranging the last term):

$$\sqrt{2} > \sqrt{(2-p)(1-p)} + \frac{p}{\sqrt{2-p}}.$$

We show that the RHS is decreasing in p. The LHS $\sqrt{2} = \left(\sqrt{(2-p)(1-p)} + \frac{p}{\sqrt{2-p}}\right)\Big|_{p=0}$. Thus, the inequality holds. To see that the RHS is decreasing in p, consider the first derivative:

$$\frac{d}{dp}\left(\sqrt{(2-p)(1-p)} + \frac{p}{\sqrt{2-p}}\right) = \frac{1}{2\sqrt{2-p}}\left(2 + \frac{p}{2-p} - \frac{3-2p}{\sqrt{1-p}}\right).$$

We show that the term inside the bracket in the RHS above is negative. By rearranging these terms, it suffices to show that

$$(4-p)\sqrt{1-p} < (3-2p)(2-p).$$

Let $\sqrt{1-p} = z \Rightarrow p = 1-z^2$. To show the above inequality, it suffices to show that

$$2z^4 - z^3 + 3z^2 - 3z + 1 > 0 \text{ for } z \in (0,1).$$

The discriminant of the quartic polynomial above evaluates to 1940. Further, $8a_4a_3 - 3a_2^2$ evaluates to 45.[13] Thus, this quartic polynomial does not have any real roots; hence the quartic polynomial is always positive (in fact, for $z \in (0,1)$, the polynomial attains a minimum value of ≈ 0.246 at $z = y_R \approx 0.471$).

- C: Observe that term C is the product of three terms. The first term $(2-p)(1-p)p > 0$ for $p \in (0,1)$. The last term is identical to the last term in B which was shown to be negative. We show that the second term is negative. To see this, we rewrite the second term as:

$$-\frac{1}{\sqrt{2-p}}\left(2 + \frac{p}{2-p} - \sqrt{2}\sqrt{2-p}\right). \tag{B.24}$$

We show that the term inside the bracket is positive. Let $2-p = z^2$; thus, $p \in (0,1) \Rightarrow z \in (1,\sqrt{2})$. It suffices to show that

$$\sqrt{2}z^3 - z^2 - 2 < 0.$$

This cubic polynomial is increasing in z in the interval $(1,\sqrt{2})$: The first derivative is $3\sqrt{2}z\left(z - \frac{\sqrt{2}}{3}\right) > 0$ for $z \in (0,1)$. Besides, $\left(\sqrt{2}z^3 - z^2 - 2\right)\Big|_{z=\sqrt{2}} = 0$. Thus, the inequality holds.

Thus, $\overline{\rho}_N$ is decreasing in p. $\square$

Proof of Theorem 5: Both U_B^{CTI} and U_B^{OP} are linear in ρ_N. Thus, using straightforward algebra, we have that

$$\begin{aligned} U_B^{\mathsf{CTI}} > U_B^{\mathsf{OP}} &\Leftrightarrow 2\left((1-p)\sqrt[\alpha]{p} - \rho_N p\sqrt[\alpha]{1-p}\right) > (1-p)^2\sqrt[\alpha]{2p} + (2p-p^2)\left(-\rho_N\sqrt[\alpha]{2\left(\frac{(1-p)^2}{2-p}\right)}\right) + 2p(1-p)\sqrt[\alpha]{\frac{p}{2-p}} + p^2\left(-\rho_N\sqrt[\alpha]{2\left(\frac{1-p}{2-p}\right)}\right) \\ &\Leftrightarrow \rho_N < \overline{\rho}_N(\alpha). \quad \square \end{aligned}$$

Proof of Theorem 6: Using (9), (12) and (15), we have that

$$\begin{aligned} U_B^{\mathsf{OP}} &= \sum_{i=0}^{n}\left[q_i\sqrt[\alpha]{\Delta_x\tau_i} - \rho_N\left(\sqrt[\frac{\alpha}{\alpha-1}]{\sum_{j=i+1}^{n} q_j\sqrt[\alpha]{\Delta_x q_i\tau_i}}\right)\right] \\ U_B^{\mathsf{CTI}} &= n\left(\overline{p}\sqrt[\alpha]{p\Delta_x} - \rho_N p\sqrt[\alpha]{\overline{p}\Delta_x}\right). \end{aligned}$$

Both U_B^{OP} and U_B^{CTI} are linear in ρ_N. Therefore, $\mathsf{CTI} \succ \mathsf{OP}$ iff the following holds: $\rho_N < \overline{\rho}_N(n,\alpha)$.

$\square$

[13] We use a_k to denote the coefficient of x^k in a polynomial of degree K, $\sum_{k=0}^{K} a_k x^k$.

Appendix C: Supporting Results

Consider a random variable D with support $[0,\infty)$, p.d.f $f(\cdot)$, and CDF $F(\cdot)$. Suppose that $f(\cdot)\in\mathcal{C}^2$, $\mathbb{E}[D]$ is finite. Let $\mathsf{MRL}_D(t)$ for a random variable D denotes the mean residual lifetime at time t, i.e.,

$$\mathsf{MRL}_D(t)=\mathbb{E}[D-t|D>t].$$

Let D_t denote the random variable $D|D>t$, and $f_{D_t}(\cdot)$ (resp., $F_{D_t}(\cdot)$) its p.d.f (resp., CDF). Recall the definition of $\overline{D}_t$: $\overline{D}_t=\mathbb{E}[D|D>t]=t+\mathsf{MRL}_D(t)$.

Lemma C.1. *The following hold:*

(a) For any $t_2\geq t_1$, $D_{t_2}\geq_{st} D_{t_1}$.
(b) $\overline{D}_t$ is increasing in t.

Proof of Lemma C.1: (a) Consider any $d\in[t_1,t_2]$: It is straightforward that $F_{D_{t_1}}(d)\geq 0=F_{D_{t_2}}(d)$. Now, consider $d\in[t_2,\infty)$:

$$F_{D_{t_1}}(d)-F_{D_{t_2}}(d)=\frac{(F(t_2)-F(t_1))\,(1-F(d))}{(1-F(t_2))\,(1-F(t_1))}>0.$$

Thus, it follows that for any $d\in[t_1,\infty)$, $F_{D_{t_1}}(d)\geq F_{D_{t_2}}(d)$. Thus, $D_{t_2}\geq_{st} D_{t_1}$.

(b) Using the property of stochastic ordering, it follows that $\mathbb{E}[D_{t_2}]\geq\mathbb{E}[D_{t_1}]$ for any $t_2\geq t_1$. Thus, $\overline{D}_{t_2}\geq\overline{D}_{t_1}$; hence, $\overline{D}_t$ is increasing in t. Alternatively, $\overline{D}_t$ can be written as follows:

$$\overline{D}_t=\mathbb{E}[D|D>t]=\frac{\int_t^\infty xf(x)dx}{1-F(t)}.$$

Therefore,

$$\overline{D}_t'=\frac{1}{(1-F(t))^2}\left(-tf(t)\int_t^\infty f(x)dx+f(t)\int_t^\infty xf(x)dx\right)=\frac{f(t)}{\overline{F}(t)^2}\left(\int_{x=t}^\infty (x-t)f(x)dx\right)>0.$$

□

We state some useful results from the literature.

Lemma C.2. *(a) Suppose D is IFR. Then, $\mathsf{MRL}_D(t)$ is decreasing in t.*
(b) Let $X_i, i\in[n]$ denote independent random variables. Suppose $X_i, i\in[n]$ is IFR. Then, the convolution $\sum_{i=1}^n X_i$ is also IFR.

The first part – *IFR implies DMRL* – is a well-known result (Lai and Xie 2006). The second part – *IFR is closed under convolution* – can be found in Ross et al. (2005), Lai and Xie (2006).

Let $X_i, i\in[n]$ denote a collection of n i.i.d random variables, where

$$X_i\sim f(\cdot),F(\cdot),X_i\in[0,\infty).$$

For any n, let $X^{(n)} = \sum_{i\in[n]} X_i$; thus, $D =_d X^{(n)}$; let $f^{(n)}$ (resp., $F^{(n)}(\cdot)$) denote the p.d.f (resp., CDF) of D. Now, define

$$y_{(n)} = \mathbb{E}\left[\mathsf{MRL}_{X^{(n)}}(X^{(n)})\right]$$

Thus, $y_{(n)}$ (resp., $y_{(1)}$) denotes the expected mean residual lifetime at the time of failure for D (resp., X). Let $x_w^{(n)} = {F^{(n)}}^{-1}(w)$ denote the w^{th} quantile of $X^{(n)}$.

LEMMA C.3. *The following hold:*

(a) $X^{(n)} \geq_{st} X^{(n-1)}$.

(b) $x_w^{(n)} \geq x_w^{(n-1)}$.

Proof: (a) Let $\tilde{0}$ denote the random variable 0 w.p. 1. Then, $X^{(n-1)} = \sum_{i=1}^{n-1} X_i + \tilde{0}$, while $X^{(n)} = \sum_{i=1}^{n} X_i$. Since $X_n \geq_{st} \tilde{0}$, the result follows from Theorem 1.2.17 of Müller and Stoyan (2002).

(b) Since $X^{(n)} \geq_{st} X^{(n-1)}$, we have that:

$$\begin{aligned}
&\underbrace{\mathbb{P}[X^{n-1} \leq a]}_{F^{(n-1)}(a)} \geq \underbrace{\mathbb{P}[X^{(n)} \leq a]}_{F^{(n)}(a)} \text{ for all } a \in [0,\infty) \\
\implies &\underbrace{\mathbb{P}[X^{n-1} \leq x_w^{(n-1)}]}_{w} \geq \mathbb{P}[X^{(n)} \leq x_w^{(n-1)}] \text{ for some } w \in [0,1] \\
\implies &\mathbb{P}[X^{(n)} \leq x_w^{(n)}] \geq \mathbb{P}[X^{(n)} \leq x_w^{(n-1)}] \\
\implies &x_w^{(n)} \geq x_w^{(n-1)}.
\end{aligned}$$

□

LEMMA C.4. *$y_{(n)}$ is increasing in n.*

Proof:

$$\begin{aligned}
y_{(n)} &= \int_{t=0}^{\infty} \int_{z=t}^{\infty} \left(z \frac{f^{(n)}(z)}{\overline{F}^{(n)}(t)} dz \right) f^{(n)}(t) dt \\
&= \int_{z=0}^{\infty} \int_{t=0}^{z} \left(\frac{f^{(n)}(t)}{\overline{F}^{(n)}(t)} dt \right) z f^{(n)}(z) dz \\
&= \int_{z=0}^{\infty} -\ln(\overline{F}^{(n)}(z)) z f^{(n)}(z) dz \\
&= \int_{w=0}^{1} -\ln(w) {F^{(n)}}^{-1}(1-w) dw = \int_{w=0}^{1} \ln\left(\frac{1}{w}\right) {F^{(n)}}^{-1}(1-w) dw
\end{aligned}$$

Thus,

$$\delta_{(n)} = y_{(n)} - y_{(n-1)} = \int_{w=0}^{1} \ln\left(\frac{1}{w}\right) \underbrace{\left({F^{(n)}}^{-1}(1-w) - {F^{(n-1)}}^{-1}(1-w)\right)}_{x_{(1-w)}^{(n)} - x_{(1-w)}^{(n-1)}}$$

The term inside the brackets corresponds to the difference between $x^{(n)}_{(1-w)}$ and $x^{(n-1)}_{(1-w)}$; using part (b) of Lemma C.3, this is non-negative. Thus, $\delta_{(n)} \geq 0$. $\square$

LEMMA C.5. *$y_{(n)}$ is discrete concave in n if the following holds: $x^{(n)}_w (= {F^{(n)}}^{-1}(w))$ is discrete concave in n for all $w \in (0,1)$. This condition holds for the Normal distribution.*

Proof:

$$\delta_{(n)} - \delta_{(n-1)} = \int_0^1 \ln\left(\frac{1}{w}\right)\left(\left(\underbrace{{F^{(n)}}^{-1}(1-w) - {F^{(n-1)}}^{-1}(1-w)}_{x^{(n)}_{(1-w)} - x^{(n-1)}_{(1-w)}}\right) - \left(\underbrace{{F^{(n-1)}}^{-1}(1-w) - {F^{(n-2)}}^{-1}(1-w)}_{x^{(n-1)}_{(1-w)} - x^{(n-2)}_{(1-w)}}\right)\right).$$

Since $x^{(n)}_w$ is increasing and concave in n at all $w \in [0,1]$, the term inside the bracket is non-negative for all w. It follows that $\delta_{(n)} \leq \delta_{(n-1)}$.

Next, consider $X_i \sim \mathcal{N}(\mu, \sigma^2)$, with $\mu > 0$. For any $w \in [0,1]$, let $z_{(w)}$ denote the w^{th} quantile of the standard normal distribution, i.e.,

$$z_{(w)} = \Phi^{-1}(w),$$

where $\Phi(\cdot)$ denotes the CDF of the standard normal. Thus,

$${F^{(n)}}^{-1}(1-w) = n\mu + \sqrt{n}\sigma z_{(1-w)}.$$

Therefore, ${F^{(n)}}^{-1}(1-w) - {F^{(n-1)}}^{-1}(1-w) = \mu + \left(\sqrt{n} - \sqrt{n-1}\right)\sigma z_{(1-w)}$. Since $(\sqrt{n} - \sqrt{n-1})$ is (strictly) decreasing in n, it holds that ${F^{(n)}}^{-1}(1-w)$ is (strictly) discrete concave in n for all $w \in (0,1)$ $\square$

LEMMA C.6. (6) *holds for the exponential and normal distribution.*

Proof: First, it suffices to show the result for the exponential and normal distributions with means 1.

Consider $X_i \sim \mathsf{Exp}(1)$. Using the memorylessness property of exponential random variables, we have that $y_{(1)} = 1$. Since the sum of exponential distributions follows an Erlang distribution, which is DMRL, we have that $y_{(n)} < \mathsf{MRL}_D(0) = n$. Thus, (6) holds for the exponential distribution.

Consider $X_i \sim \mathcal{N}(1, \sigma^2)$. The proof follows in a straightforward manner from Lemma C.5. $\square$

LEMMA C.7. *Consider $X_i \sim p \circ x_H + (1-p) \circ x_L$.* (14) *holds for $n = 2$ and 3.*

Proof: We calculate the difference $y_{(n)} - ny_{(1)}$ for $n = 2$ and 3 and show that the difference is negative for $p \in (0,1)$. Further, w.l.o.g, we normalize $\Delta_x = 1$. For any n and $\Delta_x = 1$, from (10) we have the following:

$$y_{(n)} = \sum_{i=0}^{n}\sum_{j=i}^{n}(j-i)\frac{q_j q_i}{\sum_{k=i}^{n} q_k}.$$

- $n = 2$: Using straightforward algebra,

$$ny_{(1)} = 2p\overline{p} \text{ and } y_{(2)} = \frac{2\overline{p}p(p^2 + 2\overline{p})}{2-p}$$

  Therefore,

$$y_{(2)} - ny_{(1)} = -\frac{2(1-p)^2p^2}{2-p} < 0.$$

- $n = 3$: Using straightforward algebra,

$$ny_{(1)} = 3p\overline{p} \text{ and } y_{(3)} = \frac{3(1-p)p}{(3-2p)(3-(3-p)p)}\left(9 - 24p + 33p^2 - 27p^4 + 12p^5 - 2p^6\right).$$

  Therefore,

$$y_{(3)} - ny_{(1)} = -\left(\frac{3(3-p)(1-p)^2p^2}{(3-p(3-p))(3-2p)}\right)\left(3 - 4p + 2p^2\right) < 0.$$

  The first term is strictly positive in $p \in (0,1)$. The second term is decreasing in $p \in (0,1)$; at $p = 1$, the second term is 0. Thus, the second term is also positive in $p \in (0,1)$. □

REMARK C.1. In Lemma C.7, we show that $y_{(n)} < ny_{(1)}$ for $n = 2, 3$. For higher values of n, the expressions for $y_{(n)}$ are cumbersome. The details of the numerical verification of $y_{(n)} < ny_{(1)}$ for higher values of n using Mathematica are available upon request.

Appendix D: Expected Mean Residual Lifetime at the Time of Failure for the UniformSum$(n, \{0,2\})$ Distribution

Below, we show that (6) holds for the uniform distribution with positive support. It suffices to show this result for $X_i =_d \hat{X}_i \sim \mathsf{Uniform}[0,2]$. For $X_i \sim \mathsf{Uniform}[a,b]$, where $0 \le a < b$, we have that:

$$\mathbb{E}[\mathsf{MRL}_X(X)] = \left(\frac{b-a}{2}\right)\mathbb{E}[\mathsf{MRL}_{\hat{X}}(\hat{X})]$$
$$\mathbb{E}[\mathsf{MRL}_D(D)] = \left(\frac{b-a}{2}\right)\mathbb{E}[\mathsf{MRL}_{\hat{D}}(\hat{D})]$$

where $D = \sum_{i\in[n]} X_i$ and $\hat{D} = \sum_{i\in[n]} \hat{X}_i$. It is straightforward that $\hat{D}$ follows the $\mathsf{UniformSum}(n, \{0,2\})$ distribution (also called $\mathsf{IrwinHall}(n, \{0,2\})$ distribution).

Let

$$y_{(1)} = \mathbb{E}[\mathsf{MRL}_{\hat{X}}(\hat{X})] \text{ and } y_{(n)} = \mathbb{E}[\mathsf{MRL}_{\hat{D}}(\hat{D})].$$

For $n = 2$, we have the following:

- Since $\hat{X} \sim U[0,2]$, we have that $\mathsf{MRL}_{\hat{X}}(t) = 1 - \frac{t}{2}$. Therefore, $y_{(1)} = \frac{1}{2}$.
- Next, since $\hat{D} \sim \mathsf{UniformSum}(n, \{0,2\})$, we have that

$$\mathsf{MRL}_{\hat{D}}(t) = -t + \begin{cases} \frac{2}{3}(2+t), & \text{if } t \in [2,4]; \\ \frac{2}{3}\left(\frac{24-t^3}{8-t^2}\right), & \text{if } t < 2. \end{cases}$$

Therefore, $y_{(2)} = \mathbb{E}[\mathsf{MRL}_{\hat{D}}(\hat{D})] \approx 0.729$.

Combining these two, we have:

$$y_{(2)} < 2y_{(1)}.$$

For higher values of n ($n \le 20$), we provide the following table for $y_{(n)}$. It is straightforward to verify that $y_{(n)} < ny_{(1)}$.

n	y_n	n	y_n	n	y_n	n	y_n
1	0.5	6	1.27443	11	1.72755	16	2.08429
2	0.729093	7	1.37707	12	1.80456	17	2.14854
3	0.897992	8	1.47255	13	1.8784	18	2.21091
4	1.03896	9	1.56219	14	1.94945	19	2.27133
5	1.16272	10	1.64695	15	2.018	20	2.33039

Table D.1 Mean Residual Life at the Time of Failure for an $\mathsf{UniformSum}(n, \{0,2\})$ Random Variable

Appendix E: Analysis under General Discrete Distributions

Consider the following discrete distribution for the task durations with support $\{0,1,2,\ldots\}$ and the following p.m.f:

$$X_i \sim \sum_{m=0}^{\infty} p_m \circ m, \quad \mathbb{E}[X_i] = \overline{x}.$$

We assume that X_i is IFR. The total delay $D = \sum_{i\in[n]} X_i$ is as follows:

$$D = \sum_{i\in[n]} X_i \sim \sum_{m=0}^{\infty} r_m \circ m.$$

where $r_m = \sum_{i_1\in\{0\}\cup[m]} \sum_{i_2\in\{0\}\cup[m-i_1]} \cdots \sum_{i_n\in\{0\}\cup[m-\sum_{j\in[n-1]} i_j]} \left(\prod_{k=1}^{n} p_{i_k}\right)$.

- Consider OP: Consider a realization of D, say D. Let δ_t denote the consumer's mean belief on D at any time $t \le D$.

$$\overline{D}_t = \delta_t \triangleq \mathbb{E}[D|D \ge t] = t + \mathsf{MRL}_D(t^-).$$

For any $t_2 > t_1 \ge 0$, it is straightforward to verify that $(D|D \ge t_2) \ge_{st} (D|D \ge t_1)$; therefore, δ_t is increasing in t. OP resolves uncertainty on D as follows:

$$n\overline{x} - D = \underbrace{(\delta_0 - \delta_1)}_{\textit{bad news at } t=0^+} + \underbrace{(\delta_1 - \delta_2)}_{\textit{bad news at } t=1^+} + \ldots + \underbrace{(\delta_{D-1} - \delta_D)}_{\textit{bad news at } t=(D-1)^+} + \underbrace{(\delta_D - D)}_{\textit{good news at } t=D^+}$$

That is, under OP, the consumer receives bad news in $t \in \{0,1,2,\ldots,D-1\}$ (since δ_t is increasing in t) and good news at $t = D$. Now, define the expected mean residual life at the time of failure for D as follows:

$$y_{(n)} = \mathbb{E}[\mathsf{MRL}_D(D^-)].$$

The consumer's expected belief-based utility under OP under a piecewise-linear utility model is:

$$U_B^{\mathsf{OP}} = -\Delta_\rho y_{(n)}. \tag{E.25}$$

- Consider CTI: Consider realizations, say $X_1, X_2, \ldots, X_n$. Let

$$\tau_x = \mathbb{E}[X_i|X_i \ge x] = x + \mathsf{MRL}_X(x^-).$$

As before, τ_x is increasing in x. At any time $t \in [\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j)$, the consumer's belief on D at time t is as follows:

$$\overline{D}_t = t + \mathsf{MRL}_X\left(t - \sum_{j=0}^{i-1} X_j\right) + (n-i)\overline{x}.$$

CTI resolves uncertainty on D as follows:

$$n\overline{x} - D = \sum_{i=1}^{n}\left[\underbrace{(\tau_0 - \tau_1)}_{\textit{bad news} \text{ at } t=\sum_{j=0}^{i-1} X_j + 0^+} + \underbrace{(\tau_1 - \tau_2)}_{\textit{bad news} \text{ at } t=\sum_{j=0}^{i-1} X_j + 1^+} + \ldots + \underbrace{(\tau_{X_i - 1} - \tau_{X_i})}_{\textit{bad news} \text{ at } t=\sum_{j=0}^{i-1} X_j + (X_i - 1)^+} + \underbrace{(\tau_{X_i} - X_i)}_{\textit{good news} \text{ at } t=\sum_{j=0}^{i-1} X_j + X_i^+}\right].$$

That is, CTI resolves uncertainty task-by-task. Hence, in the interval $t \in [\sum_{j=0}^{i-1} X_j, \sum_{j=0}^{i} X_j)$ the consumer receives $(X_i - 1)$ pieces of bad news (since τ_x is increasing in x). At $t = \sum_{j=0}^{i} X_j$, the consumer receives good news. Define

$$y_{(1)} = \mathbb{E}[\mathsf{MRL}_X(X^-)].$$

The consumer's (expected) belief-based utility under CTI under a piecewise linear utility model can be written as:

$$U_B^{\mathsf{CTI}} = -\Delta_\rho n y_{(1)} \tag{E.26}$$

Therefore, we have the following result:

THEOREM E.1. OP *is preferred to* CTI *iff the following condition holds:*

$$n y_{(1)} > y_{(n)}.$$

The above condition holds under the geometric distribution for task durations.

Proof: The first part follows from (E.25) and (E.26).

For the second part, assume $X_i \sim \mathsf{Geometric}(p)$; thus, $\overline{x} = \frac{1}{p}$. The mean residual lifetime at any time $t \in \mathbb{I}^+$ of the geometric distribution is a constant and is equal to $\frac{1}{p}$, i.e, $\mathsf{MRL}_X(t^-) = \frac{1}{p}$. Since $X_i \sim \mathsf{Geometric}(p)$, it follows that $D \sim \mathsf{NegativeBinomial}(n, p)$. The negative binomial distribution is IFR and hence DMRL (Lai and Xie 2006). Hence,

$$y_{(n)} = \mathbb{E}[\mathsf{MRL}_D(D^-)] < \mathsf{MRL}_D(0) = \frac{n}{p} = n y_{(1)},$$

which is the required result. □

Further, analogous to Remark 1, it is straightforward to verify that under OP and CTI, abandonment by a participating consumer at $t > 0$ is irrational.