Solving Problems on FM

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Objective

1. Solving Problems related to Angle modulation-FM &PM

Q1. In a FM system, if the maximum value of deviation is 75 kHz and maximum modulating frequency is 10 kHz. Calculate the deviation ratio and find the bandwidth of the system using Carson's rule.

Given:

Maximum value of deviation =75 kHz

Maximum modulating frequency = 10 kHz

Answer:

Deviation ratio: $D = \frac{\Delta f_{mac}}{f_{m(\max)}}$

$$D = \frac{\mathbf{75}kHz}{\mathbf{10}kHz} = \mathbf{7.5}$$

Bandwidth using Carson's rule:

$$BW = \mathbf{2}[\Delta f + f_m]$$

$$BW = \mathbf{2}[\mathbf{75} + \mathbf{10}] = \mathbf{170}kHz$$

- *Q2. The carrier frequency of a broadcast signal is 50MHz. the maximum frequency deviation is 60 kHz. If the highest modulating frequency is limited to 15 kHz then what is the approximate bandwidth of the modulating signal?*

Given:

Carrier frequency- $f_c = 50MHz$

Message signal frequency- $f_m = 15kHz$

Maximum frequency deviation 60 kHz

Answer:

Bandwidth:

$$BW = \mathbf{2}\left[\Delta f + f_m\right]$$

$$BW = \mathbf{2}\left[\mathbf{60} + \mathbf{15}\right] = \mathbf{150}kHz$$

- Q3. *A 20MHz carrier is frequency modulated by a sinusoidal signal such that the maximum frequency deviation is 100 kHz. Find the modulation index and approximate bandwidth of FM signal, if the frequency of the modulating signal is 100 kHz.*

Given:

$$\Delta f_{\max} = \mathbf{100}kHz \;\; ; \; f_m = \mathbf{100}kHz$$

Answer:

Modulation index $\beta_{FM} = \dfrac{\Delta f}{f_m} = \mathbf{100/100 = 1}$

Bandwidth:

$$BW = \mathbf{2}\left[\Delta f + f_m\right]$$

$$BW = \mathbf{2}\left[\mathbf{100}kHz + \mathbf{100}kHz\right] = \mathbf{400}kHz$$

Q4. *Consider an angle modulated signal* $x_c(t) = 10\cos(\omega_c t + 3\sin\omega_m t)$ *.Consider FM modulation technique and* $f_m = \mathbf{1}kHz$ *, calculate modulation index and bandwidth.*

Given:

Modulating signal frequency, $f_m = \mathbf{1}kHz$ and

FM signal is, $x_c(t) = \mathbf{10}\cos(\omega_c t + \mathbf{3}\sin\omega_m t)$

Answer:

Compare given expression with generalized expression of FM signal.

Now generalized expression is, $x(t) = A\cos[\omega_c t + \beta\sin\omega_m t]$

Compared this equation it can be found, modulation index

$\beta = \mathbf{3}$

Bandwidth: $BW = \mathbf{2}[\Delta f + f_m]$

Now Δf is unknown, but we know the relation $\beta_{FM} = \frac{\Delta f}{f_m}$

$$\Delta f = \beta \times f_m = \mathbf{3} \times \mathbf{1}kHz = \mathbf{3}kHz$$

$$BW = \mathbf{2}[\mathbf{3}kHz + \mathbf{1}kHz] = \mathbf{8}kHz$$

- Q5. *Consider an angle modulated signal* $x_c(t) = \mathbf{10}\cos(\omega_c t + \mathbf{3}\sin\omega_m t)$*.Consider FM modulation technique and* $f_m = \mathbf{1}kHz$ *,calculate modulation index and bandwidth, when a)* f_m *is doubled b)* f_m *is decreased by half.*

Given:

Modulating signal frequency $f_m = 1kHz$, and

FM signal is, $x_c(t) = 10\cos(\omega_c t + 3\sin\omega_m t)$

Answer:

Compare given expression with generalized expression of FM signal.

Now generalized expression is, $x(t) = A\cos[\omega_c t + \beta\sin\omega_m t]$

Compared this equation we found, modulation index $\beta = 3$

Bandwidth: $BW = 2[\Delta f + f_m]$

Now Δf is unknown, but we know the relation $\beta_{FM} = \frac{\Delta f}{f_m}$

$$\Delta f = \beta \times f_m = 3 \times 1kHz = 3kHz$$

a) Bandwidth of the system, f_m when is doubled

$$BW = 2[3kHz + 2kHz] = 10kHz$$

b) Bandwidth of the system, when f_m is decreased by half

$$BW = 2\left[3kHz + \frac{1}{2}kHz\right] = 7kHz$$

Q6.The equation of angle modulated voltage is $e = 10\sin\left(10^8 t + 3\sin 10^4 t\right)$. Calculate the carrier and modulating frequencies, the modulation index and deviation and power dissipated in 100Ω resistor.

- Answer

The standard expression for FM is, $x(t) = A\cos[\omega_c t + \beta \sin \omega_m t]$

Compare this expression with the given expression,

a) Carrier frequency $2\pi f_c = 10^8$; $f_c = \frac{10^8}{2\pi} = 15.91\,MHz$

b) Modulating frequency $2\pi f_m = 10^4$; $f_m = \frac{10^4}{2\pi} = 1591.5\,Hz = 1.5915kHz$

c) Modulation index $\beta = 3$

d) Deviation: $\Delta f = \beta \times f_m = 3 \times 1591.5kHz = 4.774kHz$

e) Power dissipation in 100Ω resistor

$$P = \frac{A^2}{2R_L} = \frac{100}{2 \times 100} = 0.5\,W$$

Q7. Find the carrier and the modulating frequency, the modulation index and maximum deviation of the FM wave represented by the equation

$$e_{FM} = 12\sin\left(6\times10^{8}t + 5\sin1250t\right).$$

What power will FM wave dissipate in a 10Ω resistance?

- Answer

The standard expression for FM is, $x(t) = A\cos[\omega_c t + \beta \sin \omega_m t]$

Compare this expression with the given expression,

a) Carrier frequency $2\pi f_c = 6\times10^8$; $f_c = \dfrac{6\times10^8}{2\pi} = 95.49\ MHz$

b) Modulating frequency $2\pi f_m = 1250$; $f_m = 1250/2\pi = 198.94 Hz$

c) Modulation index $\beta = 5$

d) Deviation: $\Delta f = \beta \times f_m = 5\times198.94 Hz = 994.7 Hz$

e) Power dissipation in 10Ω resistor

$$P = \frac{A^2}{2R_L} = \frac{12^2}{2\times10} = 7.2\,W$$

Q8. Determine the carrier swing, the maximum and minimum frequency attained, and the modulation index of the FM signal generated by frequency modulation at 101.6 MHz carrier with an 8 kHz sine wave causing a frequency deviation of 40 kHz

- Answer

Given: $f_c = 101.6 MHz$; $f_m = 8\,kHz$; $\Delta f = 40\,kHz$

Maximum frequency: $f_{\max} = f_c + \Delta f = 101.6MHz + 40kHz = 101.64MHz$

Minimum frequency: $f_{\min} = f_c - \Delta f = 101.6MHz - 40kHz = 101.56MHz$

Carrier swing calculation: $carrier\ swing = f_{\max} - f_{\min}$

$carrier\ swing = 80kHz$

Modulation index: $\beta = \dfrac{\Delta f}{f_m} = 5$

- A baseband signal $x(t) = 5\cos\left(2\pi 15\times10^{3} t\right)$ angle modulates a carrier signal $A\cos w_c t$.
  - Determine the modulation index and bandwidth for the FM system and PM system.
  - Find the change in the bandwidth and modulation index for both FM and PM if the modulating frequency f_m is reduced to 5 kHz. Assume $k_f = k_p = kHz/volt$