EC8395 Communication Engineering

Unit-1-Infromation Theory and Coding

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Session Objectives

•Understand analog and digital communication techniques.

•Introduction to information theory-measure of information.

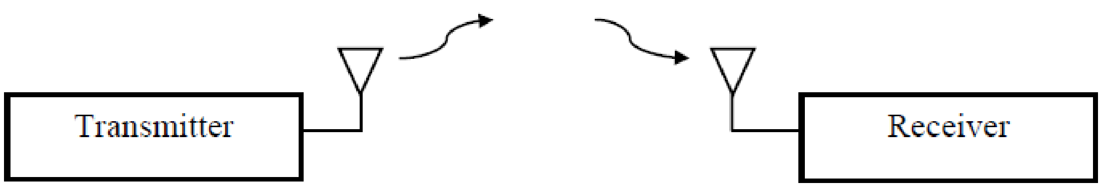
•Be familiarized with source and Error control coding.

Session Outcomes

- At the end of the session, students will be able to
  - Define signals and differentiate between continuous time and discrete time signals.
  - To identify different classes of signals by analyzing its characteristics.
  - Entropy-Definition
  - Source coding theorem

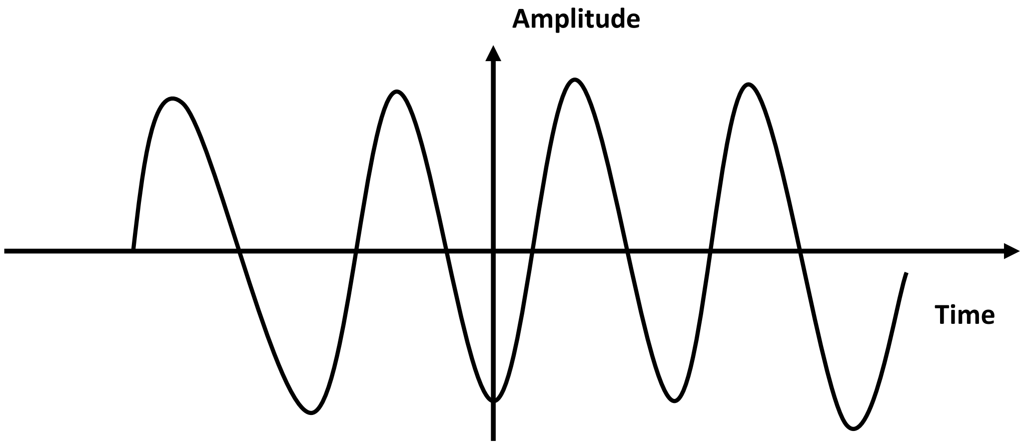
Communication

- Communication: It is the process of exchange of data or signal between two points.
- The two points are transmitter and a receiver.
- In both analog and digital communication, the transmitter conveys the information in the form of signals.
- Signal: Signal is a physical quantity that varies with time or space or any other independent variable. Signals are classified into two types.



Types of signals

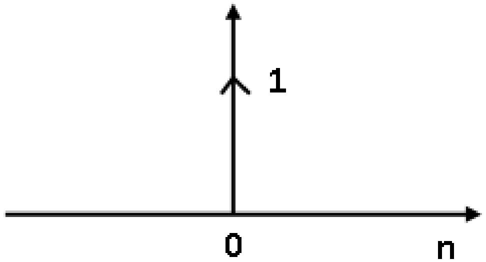
- Continuous time signal: The signal will be defined for all the time intervals. For example $x(t) = \sin \omega t$
- Here the continuous signal can be defined for all time intervals. The time period "t" varies from $-\propto < t < \propto$



- *Discrete time signal:* A discrete time signal can be defined as only for a specific time interval. For example

$$x(n) = \begin{cases} 1 & \textit{when} \quad n = 0 \\ 0 & \textit{otherwise} \end{cases}$$

- Here the discrete time "*n*" should be an integer. i.e. the amplitude of the signal can be defined as relevant to a time period *n=1 or 2* and so on. The signal cannot be defined for a time period n=0.5, 1.5, and so on.

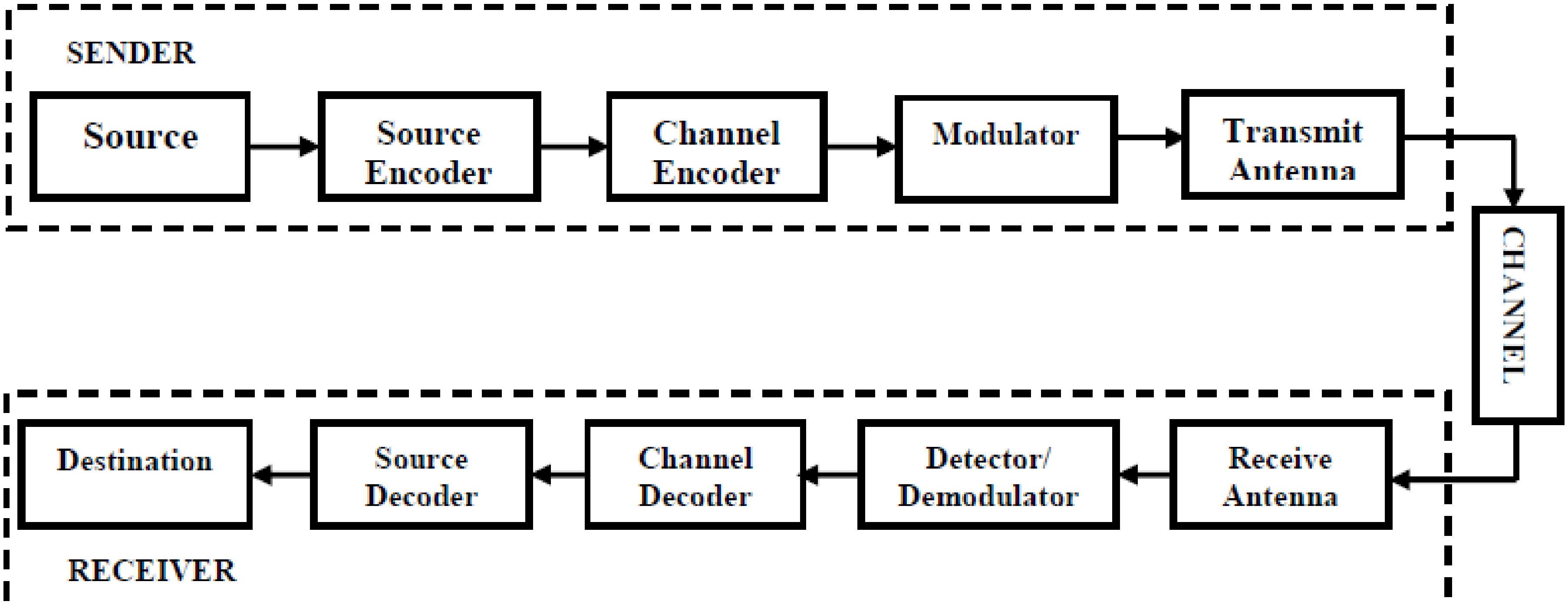


Types of communication

- Based on the classification of signals, the communication systems can be classified as *Analog and digital communication systems.*

- The analog communication system uses continuous time signal as input, where as digital communication uses discrete time signal as input.

Detailed view of communication systems



Discrete Memoryless Channel

The basic unit of digital communication system is "*Information source*" or simply *source*. Consider a source S, emitting different symbols $s_1, s_2, s_3 \ldots s_k$ for various time instants. i.e. at time period 1, source S emits symbol s_1, at time period 2 it emits symbol s_2 and so on. These symbols are statistically independent of each other. They are discrete in nature. Let p_1 be the probability of occurrence of symbol s_1, p_2 the probability of occurrence of symbol s_2, then p_k is the probability of occurrence of symbol s_k. These set of probabilities should satisfy the following condition

$$\sum_{k=0}^{K-1} p_k = 1.$$

When a source satisfies the above conditions, it is referred as *discrete memoryless source.*

Concept of Discrete Memoryless Source

The term discrete refers to a source emitting symbols $s_1, s_2, s_3 \ldots s_k$ for various time instants $t_1, t_2, t_3 \ldots t_k$ and these symbols are statistically independent. The term memoryless refers to the output of a source at any time instant depending on the present input and not with any past inputs/outputs.

Information

The amount of information gained by the user after observing symbol s_1 is given by

$$I(s_1) = \log\left(\frac{\mathbf{1}}{p_1}\right) \qquad (6.1)$$

where p_1 refers probability of occurrence of symbol s_1.

Similarly, the amount of information gained by the user from the symbol s_k is

$$I(s_k) = \log\left(\frac{\mathbf{1}}{p_k}\right) \qquad (6.2)$$

where p_k refers probability of occurrence of symbol s_k.

When the base of the logarithm is 2, the unit of information is a *bit*. It can be written as

$$I(s_k) = \log_2\left(\frac{\mathbf{1}}{p_k}\right) \text{bits.} \qquad k = \mathbf{0}, \mathbf{1}, \ldots K - \mathbf{1} \qquad (6.3)$$

Entropy

The mean value of $I(s_k)$ over the source alphabet S is given by

$$H(S) = E[I(s_k)]$$

$$= \sum_{k=0}^{K-1} p_k I(s_k)$$

$$= \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) \qquad (6.4)$$

The quantity $H(s)$ is called the *entropy* of a discrete memoryless source with source alphabet S.

Definition:

Entropy is a measure of the average information per source symbol. It depends only on the probabilities of the symbols.

Some results:

$H(S) = \mathbf{0}$, if and only if the probability $p_k = \mathbf{1}$.

Extension of discrete memoryless source:

$$H(S^n) = nH(S) \qquad (6.5)$$

Shannon Fano coding

- In Shannon Fano coding, a small number of bits are assigned to higher probable events and a large number of bits are assigned to smaller probable events.

- ***Steps involved in Shannon Fano coding:***

i) The source symbols are written in decreasing probabilities.

ii) The message set is partitioned into two most equi-probable groups [A1] and [A2]. The probability value of group [A1] should be approximately equal to the value of group [A2]. i.e. [A1]>=[A2] or [A1]<=[A2].

iii) After partitioning, “0” is assigned to each message contained in group [A1] and “1” to each message contained in group [A2].

iv) The same procedure is repeated for the groups [A1] and [A2]. i.e. Group [A1] will be divided into two equi-probable groups as [A11] and [A12] and group [A2] will be divided into two equi-probable groups as [A21] and [A22]. The code words in [A11] start with 00, [A12] starts with 01; [A21] start with 10 and [A22] start with 11.

v) The same procedure is repeated until each group contains only one message.

vi) The coding efficiency which for Shannon Fano coding is given by the expression

$$\eta = \frac{H(S)}{\overline{L}}$$

where $H(S)$ is entropy of the source S. $H(S) = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right)$

and $\overline{L}$ is average code word length.

$$\overline{L} = \sum_{k=0}^{K-1} p_k l_k$$

;where l_k is length of binary code word assigned to symbol.

vi) Redundancy can be calculated by following formula.

Redundancy= $1-\eta$.

Solved Problems

- 1. Find out the Shannon Fano code for a discrete memory less source with probability statistics

$$\{0.1, 0.1, 0.2, 0.2, 0.4\}.$$

<u>Answer</u>

- Step 1:

Arrange the probabilities in the descending order

0.4, 0.2, 0.2, 0.1, 0.1

- Step 2:

i) Divide the total symbol or message into two equi-probable groups as and It can be

$A_1 > A_2$ or $A_1 = A_2$ or $A_1 < A_2$

$$Group\,A_1 \begin{cases} \mathbf{0.4} \\ \mathbf{0.2} \end{cases}$$

$$Group\,A_2 \begin{cases} \mathbf{0.2} \\ \mathbf{0.1} \\ \mathbf{0.1} \end{cases}$$

ii) Assign "0" to group A_1 symbols and assign "1" to group A_2 symbols.

$$Group\,A_1 \begin{cases} \mathbf{0.4} & \mathbf{0} \\ \mathbf{0.2} & \mathbf{0} \end{cases}$$

$$Group\,A_2 \begin{cases} \mathbf{0.2} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \end{cases}$$

iii) Divide the group A_1 into two equi-probable groups as A_{11} and A_{12}

$$
\begin{array}{ll}
Group\,A_1 \left\{ \begin{array}{lll} \mathbf{0.4} & \mathbf{0} & Group\,A_{\mathbf{11}} \\ \hline \mathbf{0.2} & \mathbf{0} & Group\,A_{\mathbf{12}} \end{array} \right. \\
\hline
Group\,A_2 \left\{ \begin{array}{ll} \mathbf{0.2} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \end{array} \right.
\end{array}
$$

iv) Assign "0" to symbols in group A_{11} and assign "1" to symbols in group A_{12}

$$
\begin{array}{ll}
Group\,A_1 \left\{ \begin{array}{llll} \mathbf{0.4} & \mathbf{0} & \mathbf{0} & Group\,A_{\mathbf{11}} \\ \hline \mathbf{0.2} & \mathbf{0} & \mathbf{1} & Group\,A_{\mathbf{12}} \end{array} \right. \\
\hline
Group\,A_2 \left\{ \begin{array}{ll} \mathbf{0.2} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \\ \mathbf{0.1} & \mathbf{1} \end{array} \right.
\end{array}
$$

v) Further division of group A_{11} and A_{12} is not possible. Hence divide the group A_2 into A_{21} and A_{22} by two equi-probable groups.

$$
\begin{array}{l}
Group\,A_1\left\{\begin{array}{llll}
\mathbf{0.4} & \mathbf{0} & \mathbf{0} & Group\,A_{11} \\ \hline
\mathbf{0.2} & \mathbf{0} & \mathbf{1} & Group\,A_{12}
\end{array}\right. \\ \hline
Group\,A_2\left\{\begin{array}{lll}
\mathbf{0.2} & \mathbf{1} & Group\,A_{21} \\ \hline
\mathbf{0.1} & \mathbf{1} & \multirow{2}{*}{$\left.\right\}Group\,A_{22}$} \\
\mathbf{0.1} & \mathbf{1} &
\end{array}\right.
\end{array}
$$

vi) Assign "0" to symbols in group and assign "1" to symbols in group

$$
\begin{array}{l}
Group\,A_1\left\{\begin{array}{llll}
\mathbf{0.4} & \mathbf{0} & \mathbf{0} & Group\,A_{11} \\ \hline
\mathbf{0.2} & \mathbf{0} & \mathbf{1} & Group\,A_{12}
\end{array}\right. \\ \hline
Group\,A_2\left\{\begin{array}{llll}
\mathbf{0.2} & \mathbf{1} & \mathbf{0} & Group\,A_{21} \\ \hline
\mathbf{0.1} & \mathbf{1} & \mathbf{1} & \multirow{2}{*}{$\left.\right\}Group\,A_{22}$} \\
\mathbf{0.1} & \mathbf{1} & \mathbf{1} &
\end{array}\right.
\end{array}
$$

vii) Further division of group A_{21} is not possible. Hence divide group A_{22} into two equi-probable groups A_{221} and A_{222}

$$
\begin{array}{l}
Group\,A_1\left\{\begin{array}{llll}
\mathbf{0.4} & \mathbf{0} & \mathbf{0} & Group\,A_{\mathbf{11}} \\ \hline
\mathbf{0.2} & \mathbf{0} & \mathbf{1} & Group\,A_{\mathbf{12}}
\end{array}\right. \\ \hline
Group\,A_2\left\{\begin{array}{lllll}
\mathbf{0.2} & \mathbf{1} & \mathbf{0} & Group\,A_{\mathbf{21}} & \\ \hline
\mathbf{0.1} & \mathbf{1} & \mathbf{1} & \multirow{2}{*}{$\left.\begin{array}{l} \\ \\ \end{array}\right\} Group\,A_{\mathbf{22}}$} & Group\,A_{\mathbf{221}} \\ \cline{1-3} \cline{5-5}
\mathbf{0.1} & \mathbf{1} & \mathbf{1} & & Group\,A_{\mathbf{222}}
\end{array}\right.
\end{array}
$$

viii) Assign "0" to symbols in group A_{221} and assign "1" to the symbols in group A_{222} .

$$
\begin{array}{l}
Group\,A_1\left\{\begin{array}{llll}
\mathbf{0.4} & \mathbf{0} & \mathbf{0} & Group\,A_{\mathbf{11}} \\ \hline
\mathbf{0.2} & \mathbf{0} & \mathbf{1} & Group\,A_{\mathbf{12}}
\end{array}\right. \\ \hline
Group\,A_2\left\{\begin{array}{llllll}
\mathbf{0.2} & \mathbf{1} & \mathbf{0} & & Group\,A_{\mathbf{21}} & \\ \hline
\mathbf{0.1} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \multirow{2}{*}{$\left.\begin{array}{l} \\ \\ \end{array}\right\} Group\,A_{\mathbf{22}}$} & Group\,A_{\mathbf{221}} \\ \cline{1-4} \cline{6-6}
\mathbf{0.1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & & Group\,A_{\mathbf{222}}
\end{array}\right.
\end{array}
$$

- Step 3:

Symbol	Probability	Codeword	Length (l)
S_0	0.4	0 0	2
S_1	0.2	0 1	2
S_2	0.2	1 0	2
S_3	0.1	1 1 0	3
S_4	0.1	1 1 1	3

- Step 4:

To find out efficiency (η), we must calculate the average codeword length $(\overline{L})$ and entropy $H(S)$.

$$\eta = \frac{H(S)}{\overline{L}}$$

- Where $\overline{L} = \sum_{k=0}^{4} p_k l_k$

$$= p_0 l_0 + p_1 l_1 + p_2 l_2 + p_3 l_3 + p_4 l_4$$

= (0.4)(2) + (0.2)(2) + (0.2)(2) + (0.1)(3) + (0.1)(3)

=0.8 + 0.4+ 0.4+ 0.3+0.3

$$\overline{L} = 2.2\ bits / symbol$$

$$H(S) = \sum_{k=0}^{4} p_k \log_2\left(\frac{1}{p_k} \right)$$

$$= p_0 \log_2\left(\frac{1}{p_0} \right) + p_1 \log_2\left(\frac{1}{p_1} \right) + p_2 \log_2\left(\frac{1}{p_2} \right) + p_3 \log_2\left(\frac{1}{p_3} \right) + p_4 \log_2\left(\frac{1}{p_4} \right)$$

$$= 0.4 \log_2\left(\frac{1}{0.4} \right) + 0.2 \log_2\left(\frac{1}{0.2} \right) + 0.2 \log_2\left(\frac{1}{0.2} \right) + 0.1 \log_2\left(\frac{1}{0.1} \right) + 0.1 \log_2\left(\frac{1}{0.1} \right)$$

$$= 0.4\frac{\log_{10}\left(1/0.4\right)}{\log_{10}2} + 0.2\frac{\log_{10}\left(1/0.2\right)}{\log_{10}2} + 0.2\frac{\log_{10}\left(1/0.2\right)}{\log_{10}2} + 0.1\frac{\log_{10}\left(1/0.1\right)}{\log_{10}2} + 0.1\frac{\log_{10}\left(1/0.1\right)}{\log_{10}2}$$

$$= \left(0.4\times 1.3219\right) + \left(0.2\times 2.3219\right) + \left(0.2\times 2.3219\right) + \left(0.1\times 3.3219\right) + \left(0.1\times 3.3219\right)$$

$$= 0.5287 + 0.4643 + 0.4643 + 0.33219 + 0.33219$$

$$= 2.12\ bits/symbol$$

$$H\left(S\right) = 2.12\ bits/symbol$$

$$\eta = \frac{H\left(S\right)}{\overline{L}}$$

$$= \frac{2.12}{2.2} = 0.96$$

$$\eta = 96\%$$