FREQUENCY MODULATION

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Objective

1. To introduce and understand Angle modulation .

2. Mathematical representation of Frequency modulation (FM)-Time domain and frequency domain concepts

Introduction

The angle modulated signal is represented as

$$x(t) = A\cos[\omega_c t + \varphi(t)] \qquad (2.1)$$

Where *A* is amplitude of angle modulated signal, $\omega_c = 2\pi f_c$, f_c is carrier frequency and is constant, $\varphi(t)$ is phase information and it is a time variant quantity. In angle modulation $\varphi(t)$ carries the information to be conveyed from the transmitter to the receiver. i.e. modulating signal modifies the in appropriate manner.

Now let us take the argument of cosine function as,

$$\theta_i(t) = \omega_c t + \varphi(t) \qquad (2.2)$$

where $\theta_i(t)$ is the instantaneous phase of the carrier.

The question arises as what is instantaneous frequency. It is only a derivative of instantaneous phase.

Hence, instantaneous frequency $\omega_i(t)$ is,

$$\omega_i(t) = \frac{d\vartheta_i(t)}{dt} = \omega_c + \frac{d\varphi(t)}{dt} \qquad (2.3)$$

Here is instantaneous phase deviation. Then instantaneous frequency deviation is a derivative of instantaneous phase deviation.

Therefore, instantaneous frequency deviation $\frac{d\varphi(t)}{dt}$.

These definitions, lead us to those of what is PM and FM?

PM is,

$$\varphi(t) = k_p m(t) \qquad (2.4)$$

where is k_p phase modulation constant.

Similarly, FM is

$$\frac{d\varphi(t)}{dt} = k_f m(t) \qquad (2.5)$$

where is k_f frequency modulation constant. So, phase $\varphi(t)$ for FM is found through integration on both sides of the above equation.

Hence, $$\varphi(t) = k_f \int_{-\infty}^{t} m(s)ds \qquad (2.6)$$

Therefore, PM signal is given as

$$x_{PM}(t) = A\cos[\omega_c t + k_p m(t)] \qquad (2.7)$$

For FM,

$$x_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(s)ds\right] \qquad (2.8)$$

Where the message signal or modulating signal is $m(t)$, .

$$m(t) = A_m \cos\omega_m t$$

So, integration of $m(t)$ is, $\frac{A_m}{\omega_m}\sin\omega_m t$. Substituting this value in to generalized expression of FM,

$$x(t) = A\cos\left[\omega_c t + k_f \frac{A_m}{\omega_m}\sin\omega_m t\right] \qquad (2.9)$$

Now, the modulation index of FM as and it is given as,

$$\beta = k_f \frac{A_m}{\omega_m} \qquad (2.10)$$

Another way to express is,

$$\beta = \frac{\Delta f}{f_m} \qquad (2.11)$$

Where Δf is maximum frequency deviation and f_m is maximum frequency of modulating signal frequency.

Hence, the finalized expression of FM is,

$$x(t) = A\cos\left[\omega_c t + \beta\sin\omega_m t\right] \qquad (2.12)$$

$$x(t) = A[\cos\omega_c t \cos\beta\sin\omega_m - \sin\omega_c t \sin\beta\sin\omega_m t]$$

The above expression is in the form of cos [. .. .sin] format. i.e. periodic function (….periodic function). This type of expressions are solved with the help of Bessel functions. Where

$$\cos\beta\sin\omega_m t = J_0(\beta) + 2J_2(\beta)\cos 2\omega_m t + 2J_4(\beta)\cos 4\omega_m t + \ldots\ldots + 2J_{2n}(\beta)\cos 2n\omega_m t + \ldots$$

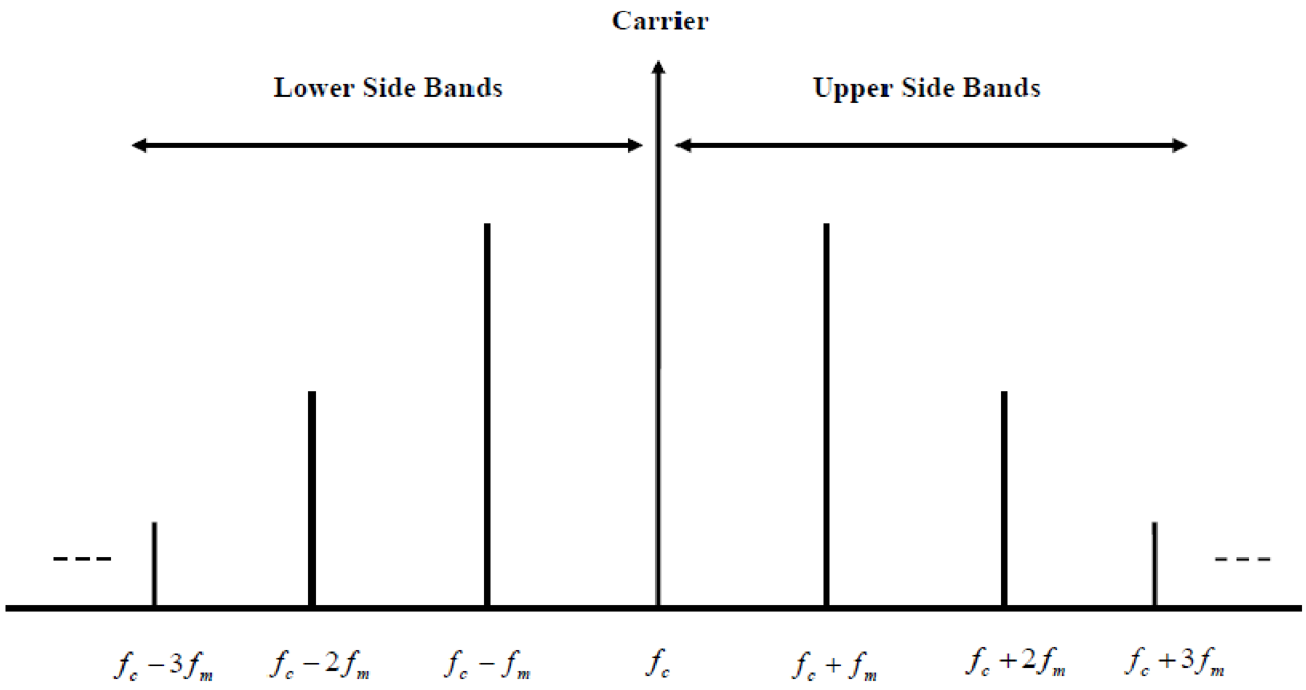
$$\sin\beta\sin\omega_m t = 2J_1(\beta)\sin\omega_m t + 2J_3(\beta)\sin 3\omega_m t + \ldots\ldots + 2J_{2n-1}(\beta)\sin(2n-1)\omega_m t + \ldots$$

Hence, the final expression is,

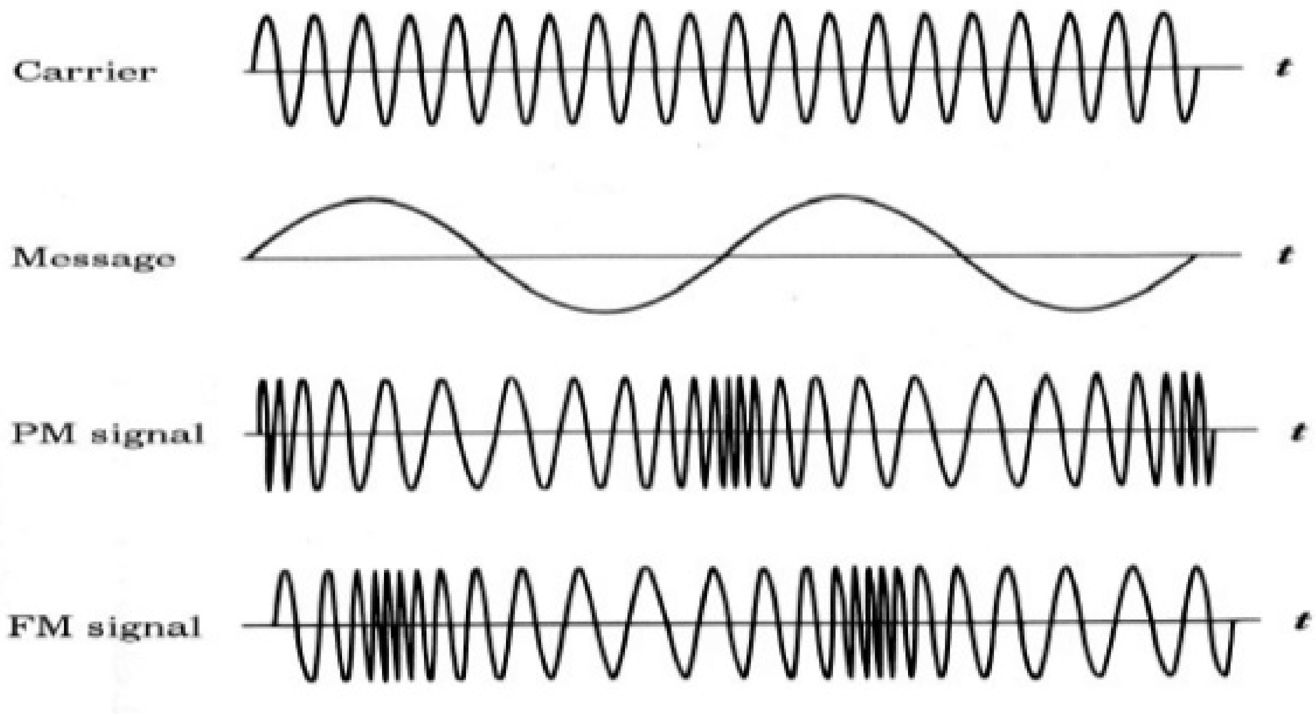
$$x_{FM}(t) = A\left\{\begin{array}{l}[J_0(\beta)\cos\omega_c t] + J_1(\beta)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t] + J_2(\beta)[\cos(\omega_c + 2\omega_m)t - \cos(\omega_c - 2\omega_m)t] + \\ J_3(\beta)[\cos(\omega_c + 3\omega_m)t - \cos(\omega_c - 3\omega_m)t] + \ldots\ldots\ldots\ldots \\ \end{array}\right\}$$

i.e. FM signal frequency spectrum consists of carrier component + infinite number of side bands. The same can be identified from the figure, frequency spectrum of FM signal.

Frequency spectrum of FM signal



Pictorial Representation (Time domain) of FM&PM



Mathematical Analysis for PM

$$x_{PM}(t) = A\cos\left[\omega_c t + k_p m(t)\right] \qquad (2.18)$$

Where $m(t) = A_m \cos\omega_m t$.

Hence,

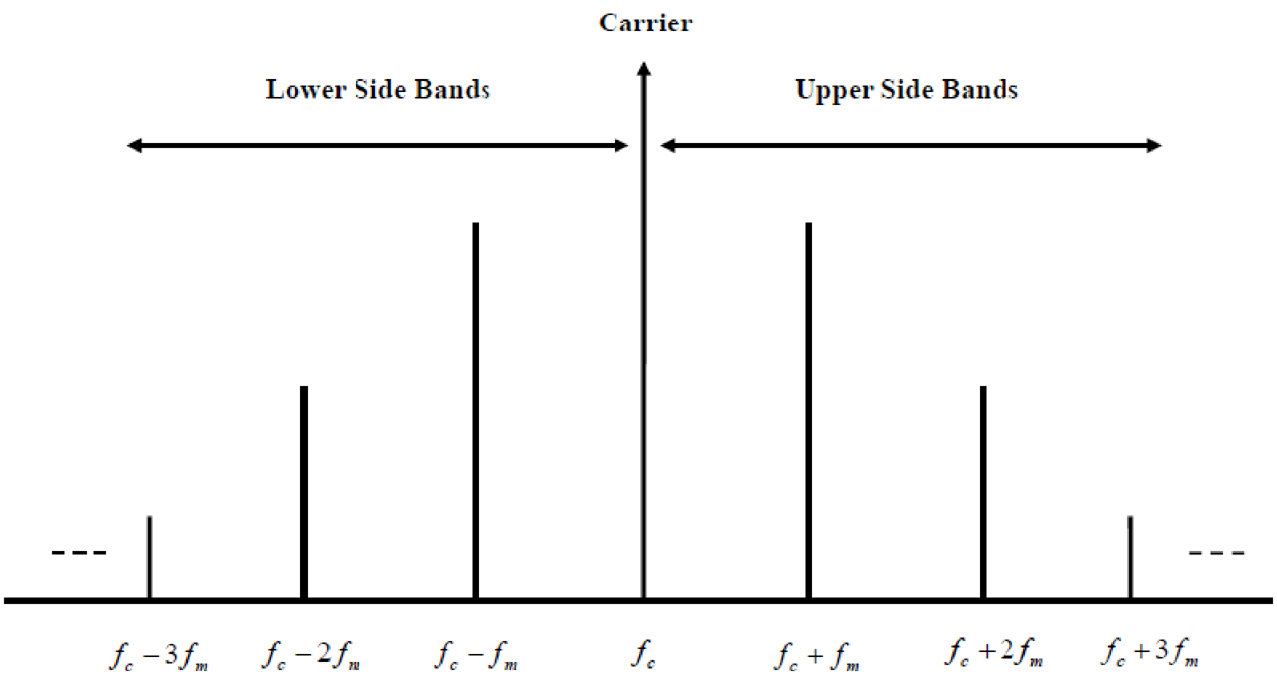
$$x_{PM}(t) = A\cos\left[\omega_c t + k_p A_m \cos\omega_m t\right] \qquad (2.19)$$

Now modulation index of PM is,

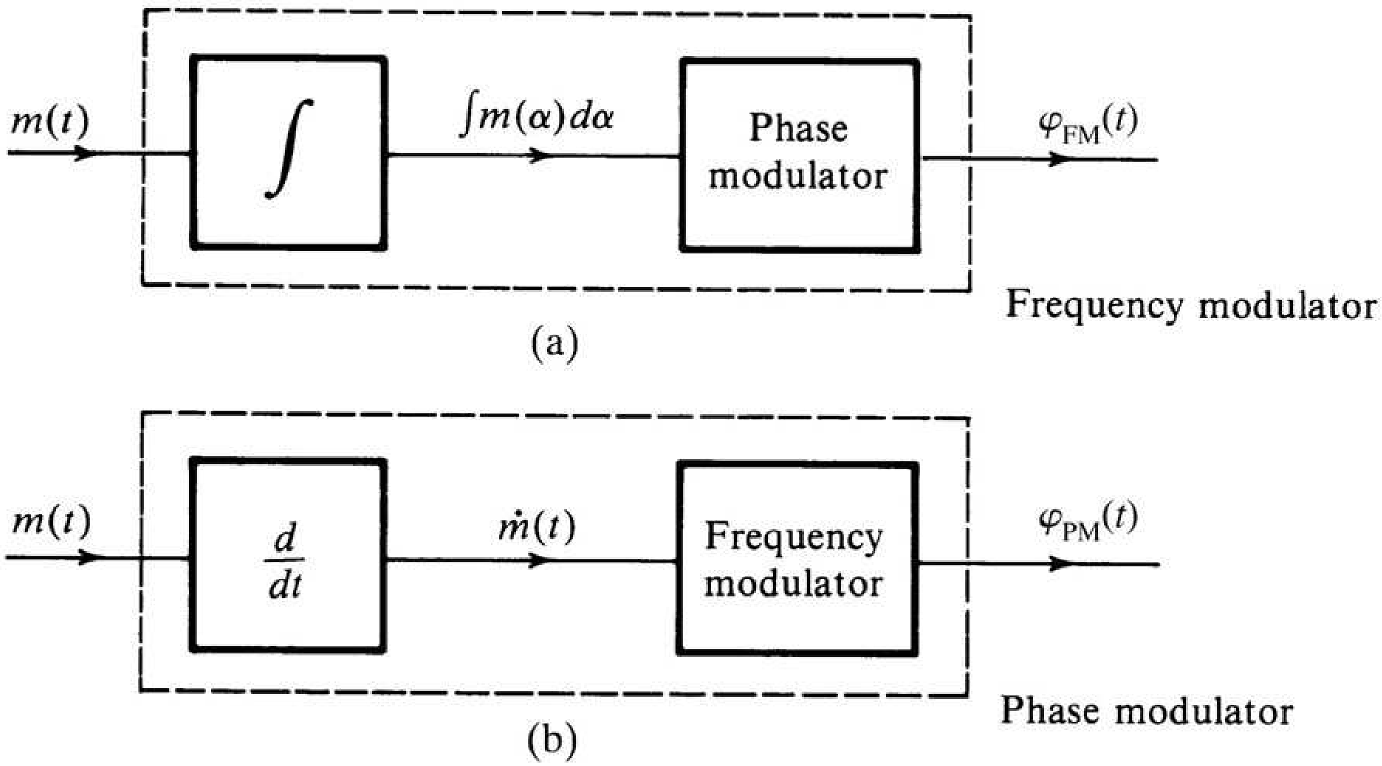
$$\beta_{PM} = k_p A_m \qquad (2.20)$$

Once again for PM signal also, cos (….cos) function, i.e. periodic function (periodic function). Hence apply the Bessel function expression is applied to the above equation and the final spectrum also consists of an infinite number of sidebands.

Frequency spectrum of PM signal



Relation between FM and PM



a) Generation of FM using PM b) Generation of PM using FM modulation

Modulation index	Sideband																
	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.00	1.00																
0.25	0.98	0.12															
0.5	0.94	0.24	0.03														
1.0	0.77	0.44	0.11	0.02													
1.5	0.51	0.56	0.23	0.06	0.01												
2.0	0.22	0.58	0.35	0.13	0.03												
2.41	0	0.52	0.43	0.20	0.06	0.02											
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01										
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01										
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02									
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02								
5.53	0	−0.34	−0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01							
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02							
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02						
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03					
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02				
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01			
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01		
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

Bandwidth of FM

Theoretically the bandwidth occupied by FM is infinite, since, it has infinite number of side bands. But there is a thumb rule available for finding bandwidth of FM, it is

Bandwidth (BW) $= 2\times f_m \times number\ of\ significant\ sidebands$

(2.14)

i.e. all the infinite number of sidebands carry the useful information. Only the finite number of sidebands carry the useful information.

Carson's Rule for Bandwidth calculation

Definition: Bandwidth is twice the sum of the maximum frequency deviation and the modulating frequency.

$$BW = 2[\Delta f + f_m]$$

This is the usual method for finding bandwidth of FM signal.

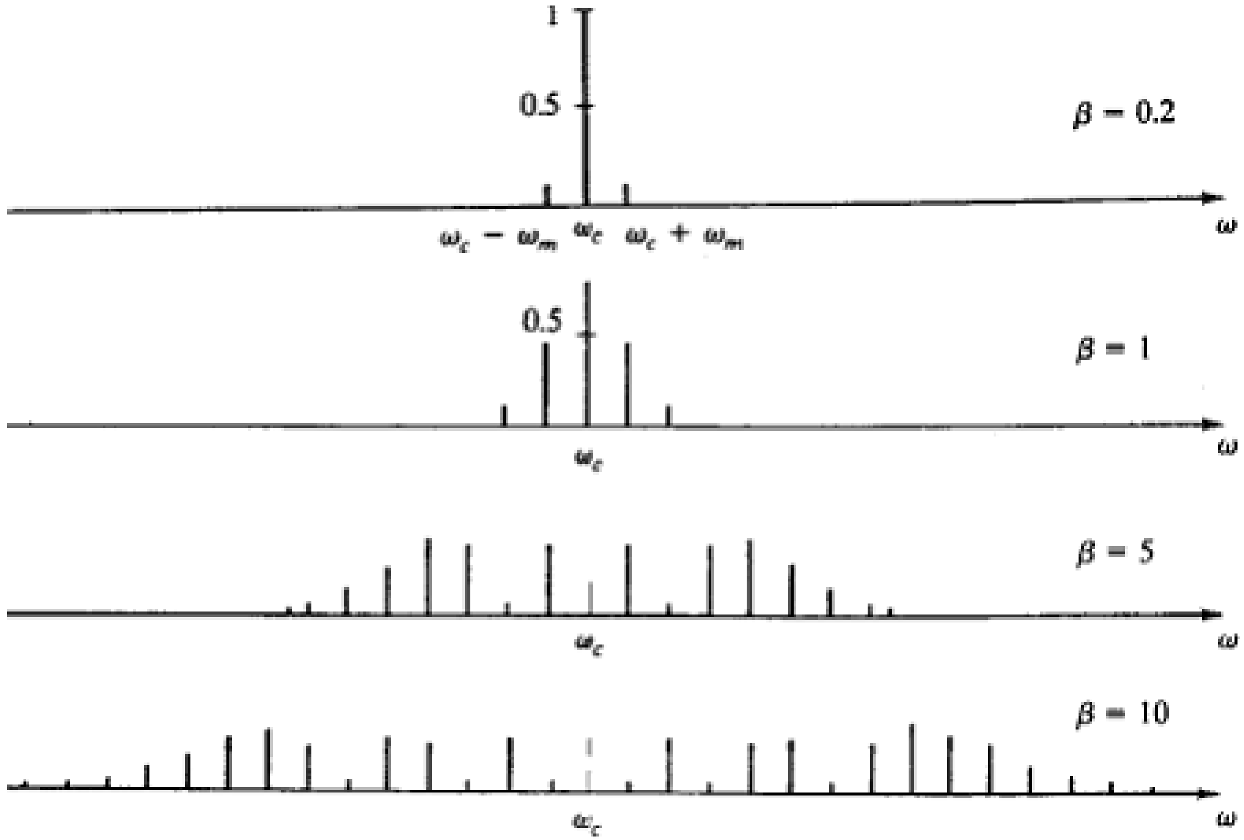
Now we need to recall the modulation index β of FM,

$$\beta = \frac{\Delta f}{f_m}$$

Since both β and BW is depend on Δf and f_m .

$$BW = 2(\beta + 1) f_m$$

The spectra of sinusoidally modulated FM signals for various values of β



Types of FM

When β is less than 1,it is a narrow band FM and when β >>1, it is wideband FM.

S.No.	Parameter	Narrowband FM	Wideband FM
1	Modulation index (β)	Less than one	Greater than one
2	Maximum frequency deviation (Δf)	5 kHz	75 kHz
3	Range of Modulating signal Frequency	30 Hz to 3 kHz	30 Hz to 15 kHz
4	Maximum modulation index	1	5 to 2500
5	Bandwidth	Small, approximately same as AM	Large, it is around 15 times higher than narrowband FM.

Total Power requirement of FM

$$P = \frac{A^2}{2R_L}$$

where R_L – load resistance.

Deviation ratio (D) or percentage of modulation of FM

This is only the ratio of maximum frequency deviation to maximum modulating signal frequency.

$$D = \frac{\Delta f_{\max}}{f_{m(\max)}}$$