Unit-4
QPSK

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Objective

1. To discuss about Quadrature Phase shift Keying modulation (QPSK) technique

2. Discuss about Mathematical representation

3. Discuss about probability of error

4. Discuss about generation and reception of QPSK signal

Quadrature Phase Shift Keying QPSK

- In the binary PSK system, symbols 0 and 1 are differentiated by phase angles $0°$ and $180°$. In QPSK, two bits can be transmitted simultaneously (or we can transmit 2 bits per symbol interval). When the symbol size is two, transmission of four different information's is possible. The information may be 00 or 01 or 10 or 11. In QPSK, four information's are differentiated by four phase angles. Hence, the phase angle for different points of information are $\pi/4 = 45°, 3\pi/4 = 135°, 5\pi/4 = 225°$ and $7\pi/4 = 315°.$

Mathematical Representation

- QPSK can be represented as follows,

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] \quad 0 \le t \le T \qquad (1)$$

$$0 \qquad \text{elsewhere}$$

where $i = 1,2,3,4$, E –symbol energy which is $E = 2E_b$, E_b is the bit energy, and *T* is the symbol duration.

Hence, when *i* = 1, signal representation for symbol 1 or data *1* = 00

$$S_1(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \frac{\pi}{4}\right]$$

at *i* = 2; signal representation for symbol 2 or data 2 = 01

$$S_2(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + 3\times\frac{\pi}{4}\right]$$

at *i* = 3; signal representation for symbol 3 or data 3 = 10

$$S_3(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + 5 \times \frac{\pi}{4}\right]$$

and at *i* = 4; signal representation for symbol 4 or data 4 = 11

$$S_4(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + 7 \times \frac{\pi}{4}\right]$$

Basis signal

- Finding of basis function of QPSK is the next step, which needs expansion of Equation (1) by cos[A + B] formula. (cos [*A* + *B*] = cos *A* cos B – sin *A* sin *B*)

$$S_i(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t)\cos\left[(2i-1)\frac{\pi}{4}\right] - \sqrt{\frac{2E}{T}}\sin(2\pi f_c t)\sin\left[(2i-1)\frac{\pi}{4}\right] \quad 0 \le t \le T$$
$$0 \qquad \text{elsewhere}$$

The above equation shows that there are two basis functions of QPSK, and they are

$$\varphi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t \qquad 0 \le t \le T$$

$$\varphi_2(t) = \sqrt{\frac{2}{T}}\sin 2\pi f_c t \qquad 0 \le t \le T$$

Now the four message points and the associated signal vector are defined by

$$S_i = \begin{bmatrix} \sqrt{E}\cos(2i-1)\dfrac{\pi}{4} \\ -\sqrt{E}\sin(2i-1)\dfrac{\pi}{4} \end{bmatrix} \quad \text{where } i = 1, 2, 3, 4.$$

The elements of the signal vector S_i are s_{i1} and s_{i2}

For example, if i = 1, phase angle = $\pi/4$; input dibit 00;

$$S_1 = \begin{bmatrix} +\sqrt{\dfrac{E}{2}} \\ -\sqrt{\dfrac{E}{2}} \end{bmatrix}$$

$$\left(\because \cos \pi/4 = 45^\circ = 1/\sqrt{2} = 0.707\right)$$

$$\left(\because \sin \pi/4 = 45^\circ = 1/\sqrt{2} = 0.707\right)$$

if i = 2; phase angle = $3\times(\pi/4)$; input dibit = 01;

$$S_2 = \begin{bmatrix} -\sqrt{\frac{E}{2}} \\ -\sqrt{\frac{E}{2}} \end{bmatrix}$$

$$\left(\because \cos 3\pi/4 = 135^\circ = -1/\sqrt{2} = -0.707\right)$$

$$\left(\because \sin 3\pi/4 = 135^\circ = 1/\sqrt{2} = 0.707\right)$$

if i = 3; phase angle = $5\times(\pi/4)$; input dibit = 10;

$$S_3 = \begin{bmatrix} -\sqrt{\frac{E}{2}} \\ +\sqrt{\frac{E}{2}} \end{bmatrix}$$

$$\left(\because \cos 5\pi/4 = 225^\circ = -1/\sqrt{2} = -0.707\right)$$

$$\left(\because \sin 5\pi/4 = 225^\circ = -1/\sqrt{2} = -0.707\right)$$

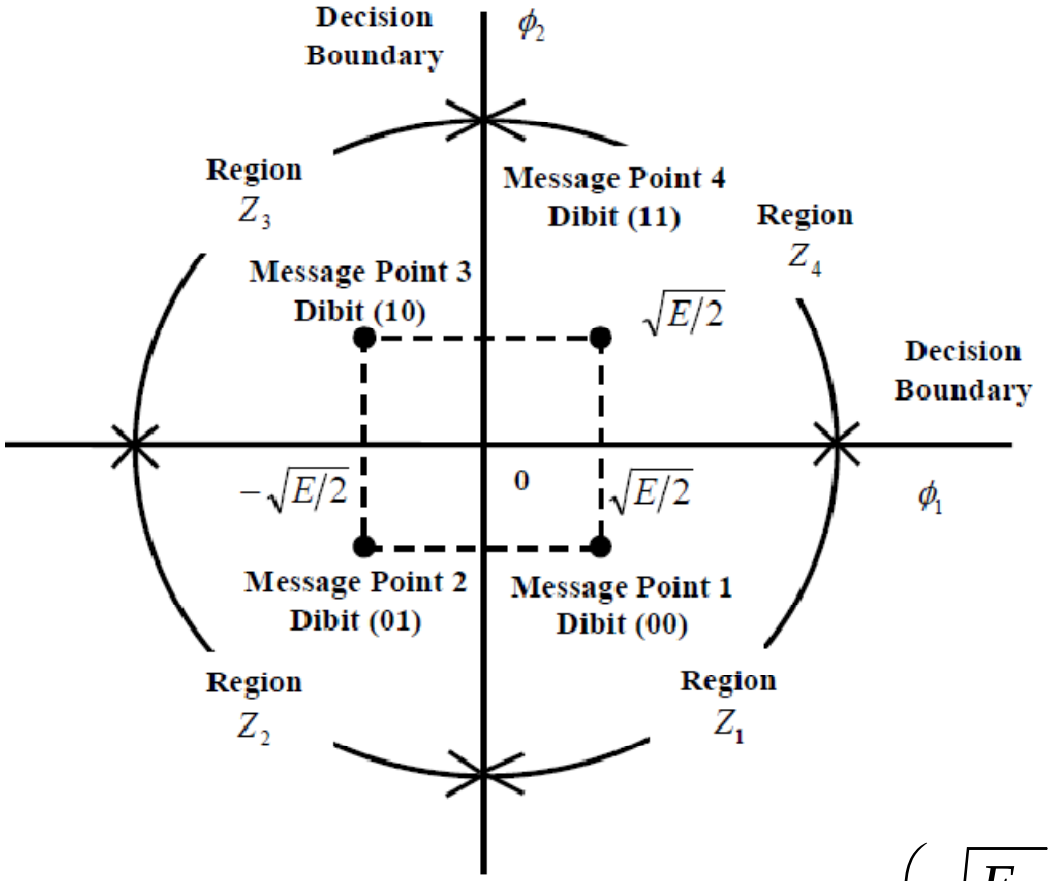
and if i = 4; phase angle = $7\times(\pi/4)$; input dibit = 11;

$$S_4 = \begin{bmatrix} +\sqrt{\frac{E}{2}} \\ +\sqrt{\frac{E}{2}} \end{bmatrix}.$$

$$\left(\because \cos 7\pi/4 = 315^\circ = 1/\sqrt{2} = 0.707\right)$$

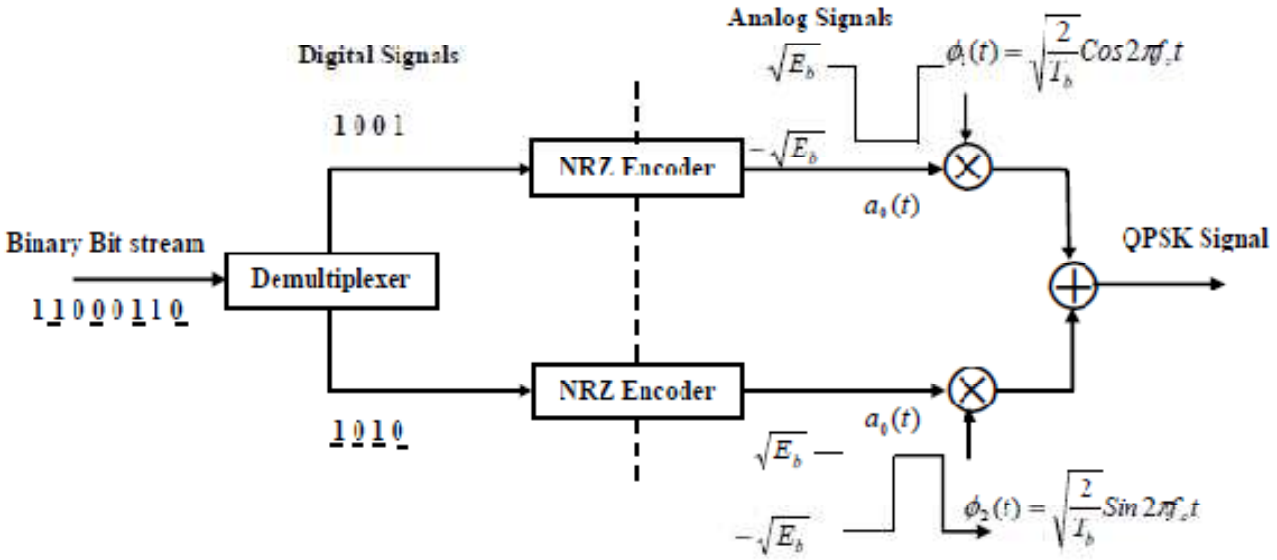
$$\left(\because \sin 7\pi/4 = 315^\circ = -1/\sqrt{2} = -0.707\right)$$

Signal Space Diagram



- Probability of Error $P_e = \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

QPSK Generation



QPSK Reception