# EC8395 Communication Engineering

## Unit-1-Infromation Theory and Coding

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# Session Objectives

- To understand about Shannon’s Theorem's I, II and III

- Know the fundamental limit of channel capacity

# Session Outcomes

- At the end of the session, students will be able to
  - Understand effect of Bandwidth and Noise in channel capacity.
  - To understand the difference and importance of Shannon's algorithm in information theory.

# Shannon's First Theorem-Source coding Theorem

Coding efficiency is given by,

$$\eta = \frac{L_{\min}}{\overline{L}} \qquad (7.1)$$

'where $\overline{L}$ represents the average number of bits per symbol and called code length. $L_{\min}$ denotes the minimum possible value of $\overline{L}$.

Given a discrete memoryless source of entropy $H(s)$, the average code word length $\overline{L}$ for any source coding is bounded as,

$$\overline{L} \geq H(S) \qquad (7.2)$$

Here $H(S)$ represents the fundamental limit on the average number of bits per symbol. Hence we can make $H(S) \approx L_{\min}$; therefore efficiency is given by

$$\eta = \frac{H(S)}{\overline{L}} \qquad (7.3)$$

# Shannon's Second Theorem-Channel coding theorem

Consider a bandlimited channel, whose bandwidth is *'B'*, and which carries a signal having $M$ number of levels. Then the maximum data rate *'R'* of the channel is

$$R = 2B\log_2 M \qquad (7.4)$$

For noiseless channel data rate, $R$ should satisfy the following condition $R << C$. This condition is referred to as channel coding theorem or Shannon's second theorem. Here $C$ represents the capacity of the channel.

# Shannon's Channel Capacity Theorem

The channel capacity of a white, bandlimited Gaussian channel is given by

$$C = B\log_2\left(\mathbf{1} + S/N\right)\text{bits/sec} \qquad (7.5)$$

Where B-is channel bandwidth, S-is signal power, N-is noise within the channel bandwidth and it is referred to as noise power. $N = N_o B$ .

$\left(N_o/\mathbf{2}\right)$ is the power spectral density of white noise.

Capacity $C$ depends on two factors $B$ and $S/N$ ratio. We have to find out the maximum possible value of $C$.

# Effect of *S/N* on *C*

Let us assume the communication channel as noiseless, then $N=0$, therefore $S/N \rightarrow \infty$. Hence capacity $C$ also tends to $\infty$. Thus a noiseless channel has *infinite capacity*.

# Effect of *B* on Channel Capacity *C*

Let us consider some noise is present in the channel. Then *S/N* is not infinite.

Now if the bandwidth approaches to $\infty$ (infinity), the channel capacity does not become infinite, since $N = N_o B$, i.e. noise power also increases with the channel bandwidth *B*. This reduces the value of *(S/N)*, with increase in *B*, assuming signal power *(S)* as constant.

Therefore we can conclude that a channel with infinite bandwidth has a finite channel capacity. It is denoted by $C_\infty$.

$$C_\infty = \mathbf{1.44}\, {}^{S}\!/_{N_o}$$

# Proof

*Channel with infinite bandwidth has finite capacity:*

Channel capacity $C$ is given by,

$$C = B\log_2\left(1 + S/N\right) \tag{7.6}$$

Noise power $N$ is given by,

$$N = N_o B \tag{7.7}$$

Substitute equ. (7.7) in (7.6),

$$C = B\log_2\left(1 + S/N_oB\right)$$

$$= \left(\frac{S}{N_o}\right)\left(\frac{N_o}{S}\right)B\log_2\left(1 + S/N_oB\right)$$

$$= \left(\frac{S}{N_o}\right)\left(\frac{N_oB}{S}\right)\log_2\left(1 + S/N_oB\right)$$

$$= \left(\frac{S}{N_o}\right)\log_2\left[\left(1 + S/N_oB\right)\right]^{N_oB/S}$$

When $B \rightarrow \infty$;

$$C_\infty = \left(\frac{S}{N_o}\right)\log_2 e \qquad \because \underset{x\to 0}{Lt}\left(1+x\right)^{(1/x)} = e$$

$$C_\infty = \left(\frac{S}{N_o}\right)1.44 \tag{7.8}$$

*Summary:*

Channel capacity $C$ depends on bandwidth $B$ and $S/N$.

- Noiseless channel has infinite capacity.
- Channel with infinite bandwidth has a finite channel capacity.

This theorem is applicable to Gaussian noise channel.