Reg. No.

Question Paper Code : 80609

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Electronics and Communication Engineering

MA 6451 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A random variable X is known to have a distribution function $F(x) = u(x)\left[1 - e^{-x^2/b}\right]$, where $b > 0$ is a constant. Determine its density function.

2. Find the expected value of the discrete random variable X with the probability mass function $p(x) = \begin{cases} \frac{1}{3} & ; x = 0 \\ \frac{2}{3} & ; x = 2 \end{cases}$.

3. Can $Y = 5 + 2.8x$ and $x = 3 - 0.5y$ be the estimated regression equations of y on x respectively explain your answer.

4. The joint probability density function of the random variable x and y is defined as $f(x,y) = \begin{cases} 25e^{-5y}; & 0 < x < 0.2, y > 0 \\ 0 & \text{otherwise} \end{cases}$. Find the marginal PDFs of x and y.

5. Let $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ be a stochastic matrix. Check whether it is regular.

6. Prove that random telegraph process $\{Y(t)\}$ is a wide sense stationary process.

7. Prove that auto correlation function is an even function of τ.

8. Find the power spectral density of the random process $\{X(t)\}$ whose auto correlation is $R(\tau)=\begin{cases}-1; & -3<\tau<3\\ 0; & \text{otherwise}\end{cases}$.

9. When a system is said to be stable?

10. Assume that the input $X(t)$ to a linear time – invariant system is white noise. What is the power spectral density of the output process $Y(t)$ if the system response $H(w)$ is $H(w)=\begin{cases}1 & w_1<|w|<w_2\\ 0 & \text{otherwise}\end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the probability of success is $\frac{1}{100}$, how many trials are necessary in order that the probability of atleast one success is greater than $\frac{1}{2}$? (8)

(ii) Find the moment generating function of Gamma distribution, with one parameter K and hence find its mean and variance. (8)

Or

(b) (i) A and B shoot independently until each has his own target. The probability of their hitting the target at each shot is $\frac{3}{5}$ and $\frac{5}{7}$ respectively? Find the probability that B will require more shots than A. (8)

(ii) If $\log_e^x$ is normally distributed with mean 1 and variance 4, find $P(\frac{1}{2}<x<2)$ given that $\log_e^2=0.693$. (8)

12. (a) (i) Given the following bivariate probability distribution obtain

(1) Marginal distributions of x and y

(2) Conditional distribution of x given $y=2$. (8)

(ii) Find the coefficient of correlation between industrial production and export using the following data. (8)

Production (x):	55	56	58	59	60	60	62
Export (y):	35	38	37	39	44	43	44

Or

(b) Given the joint density function of x and y as $f(x,y)=\begin{cases}\frac{1}{2}xe^{-y}; & 0<x<2, \quad y>0\\ 0 & elsewhere\end{cases}$. Find the distribution $X+Y$. (16)

13. (a) (i) The process $X(t)$ whose probability distribution under certain condition is given by $P[X(t)=n]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}; & n=1,2,3...\\ \dfrac{at}{1+at}, & n=0\end{cases}$. Show that it is not a stationary process. (8)

(ii) Customers arrive at a grocery store in a Poission manner at an average rate of 10 customers per hour. The amount of money that each customer spends is uniformly distributed between \$ 8.00 and \$ 20.00. What is the average total amount of money that customers who arrive over a two-hour interval spend in the store? What is the variance of this total amount? (8)

Or

(b) (i) The transition probability matrix of the Markov chain $\{X_n\}$ with $n=1,2,3..$ having 3 states 1,2,3 is $P=\begin{pmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{pmatrix}$ and the initial distribution is $P^{(0)}=(0.7\ 0.2\ 0.1)$. Find $P(x_2=3)$ and $P(x_3=2, x_2=3, x_1=3, x_0=2)$. (8)

(ii) Find the aute correlation function of random telegraph process. (8)

14. (a) (i) If $X(t) = 5\sin(\omega t+\phi)$, $y(t)=2\cos(\omega t+\phi)$ and ϕ is a random variable distibuted in $(0,2\pi)$ where ω is a constant and $0+\phi=\dfrac{\pi}{2}$ find $R_{xx}(\tau), R_{yy}(\tau)$ and verify the property that autocorrelation function is an even function of τ. (8)

(ii) Find the spectral density of WSS random process $\{X(t)\}$ whose auto correlation function is $e^{\frac{-\alpha^2\tau^2}{2}}$. (8)

Or

(b) (i) If $X(t)$ and $Y(t)$ are WSS random processes then prove that $|R_{xy}(\tau)| \le \sqrt{R_{xx}(0).R_{yy}(0)}$. (8)

(ii) If the power spectral density of a WSS is given by $S(\omega)=\begin{cases}\frac{b}{a}(a-|w|) & |w|\le a\\ 0 & |w|>a\end{cases}$, find the autocorrelation function of the process. (8)

15\. (a) (i) A random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}; t \geq 0$. If the autocorrelation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$, find the power spectral density of the output process $y(t)$. (8)

(ii) If the input to a time invariant stable line system is a WSS process then prove that the output will also be a WSS process. (8)

Or

(b) (i) If $y(t)$ is the output process when an input process $x(t)$ is applied to the linear time invariant system with impulse response. The autocorrelation function of the output system is $S_{yy}(w) = |H(w)|^2 S_{xx}(w)$, where $H(w)$ is the system transfer function. (8)

(ii) A linear time invariant system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{yy}(\tau)$ corresponding to an input $x(t)$. (8)
