

	Contract of the last	200			CONTRACTOR OF THE PARTY OF THE		
D N.							
Reg. No.							760

Question Paper Code: 80609

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Electronics and Communication Engineering

MA 6451 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulations 2013)

Time: Three hours

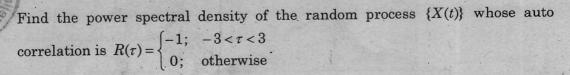
Maximum: 100 marks

Answer ALL questions.

PART A
$$-(10 \times 2 = 20 \text{ marks})$$

- 1. A random variable X is known to have a distribution function $F(x) = u(x) \left[1 e^{-x^2/b} \right]$, where b > 0 is a constant. Determine its density function.
- 2. Find the expected value of the discrete random variable X with the probability mass function $p(x) = \begin{cases} \frac{1}{3} & ; x = 0 \\ \frac{2}{3} & ; x = 2 \end{cases}$
- 3. Can Y = 5 + 2.8x and x = 3 0.5y be the estimated regression equations of y on x respectively explain your answer.
- 4. The joint probability density function of the random variable x and y is defined as $f(x,y) = \begin{cases} 25e^{-5y}; & 0 < x < 0.2, y > 0 \\ 0 & \text{otherwise} \end{cases}$. Find the marginal PDFs of x and y.
- 5. Let $A = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ be a stochastic matrix. Check whether it is regular.
- 6. Prove that random telegraph process $\{Y(t)\}$ is a wide sense stationary process.
- 7. Prove that auto correlation function is an even function of

www.rejinpaul.com Get All Study Material From Rejinpaul.com



- 9. When a system is said to be stable?
- 10. Assume that the input X(t) to a linear time invariant system is white noise. What is the power spectral density of the output process Y(t) if the system response H(w) is $H(w) = \begin{cases} 1 & w_1 < |w| < w_2 \\ 0 & \text{otherwise} \end{cases}$

PART B
$$-$$
 (5 × 16 = 80 marks)

- 11. (a) (i) If the probability of success is ½00, how many trials are necessary in order that the probability of atleast one success is greater than ½?
 - (ii) Find the moment generating function of Gamma distribution, with one parameter K and hence find its mean and variance. (8)

Or

- (b) (i) A and B shoot independently until each has his own target. The probability of their hitting the target at each shot is $\frac{3}{5}$ and $\frac{5}{7}$ respectively? Find the probability that B will require more shots than A.
 - (ii) If \log_e^x is normally distributed with mean 1 and variance 4, find $P(\frac{1}{2} < x < 2)$ given that $\log_e^2 = 0.693$. (8)
- 12. (a) (i) Given the following bivariate probability distribution obtain

 (1) Marginal distributions of x and y
 - (2) Conditional distribution of x given y = 2. (8)
 - (ii) Find the coefficient of correlation between industrial production and export using the following data. (8)

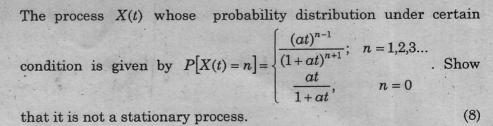
Production (x): 55 56 58 59 60 60 62 Export (y): 35 38 37 39 44 43 44

Or

(b) Given the joint density function of x and y as $f(x,y) = \begin{cases} \frac{1}{2}xe^{-y}; & 0 < x < 2, & y > 0 \\ 0 & elsewhere \end{cases}$. Find the distribution X + Y. (16)



www.rejinpaul.com Get All Study Material From Rejinpaul.com



(ii) Customers arrive at a grocery store in a Poission manner at an average rate of 10 customers per hour. The amount of money that each customer spends is uniformly distributed between \$ 8.00 and \$ 20.00. What is the average total amount of money that customers who arrive over a two-hour interval spend in the store? What is the variance of this total amount?

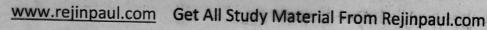
Or

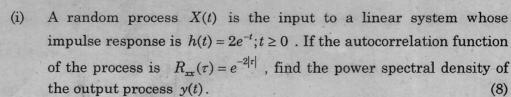
(i)

- (b) (i) The transition probability matrix of the Markov chain $\{X_n\}$ with n=1,2,3. having 3 states 1,2,3 is $P=\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)}=(0.7\ 0.2\ 0.1)$. Find $P(x_2=3)$ and $P(x_3=2,x_2=3,x_1=3,x_0=2)$.
 - (ii) Find the aute correlation function of random telegraph process. (8)
- 14. (a) (i) If $X(t) = 5\sin(\omega t + \phi)$, $y(t) = 2\cos(\omega t + \phi)$ and ϕ is a random variable distibuted in $(0,2\pi)$ where ω is a constant and $0 + \phi = \frac{\pi}{2}$ find $R_{xx}(\tau), R_{yy}(\tau)$ and verify the property that autocorrelation function is an even function of τ .
 - (ii) Find the spectral density of WSS random process $\{X(t)\}$ whose auto correlation function is $e^{-\frac{\alpha^2 r^2}{2}}$. (8)

Or

- (b) (i) If X(t) and Y(t) are WSS random processes then prove that $\left|R_{xy}(\tau)\right| \leq \sqrt{R_{xx}(0).R_{yy}(0)}$. (8)
 - (ii) If the power spectral density of a WSS is given by $S(\omega) = \begin{cases} \frac{b}{a}(a |w|) & |w| \le a \\ 0 & |w| > a \end{cases}$, find the autocorrelation function of the process.





(ii) If the input to a time invariant stable line system is a WSS process then prove that the output will also be a WSS process. (8)

Or

- (b) (i) If y(t) is the output process when an input process x(t) is applied to the linear time invariant system with impulse response. The autocorrelation function of the output system is $S_{yy}(w) = |H(w)|^2 S_{xx}(w)$, where H(w) is the system transfer function.(8)
 - (ii) A linear time invariant system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{yy}(\tau)$ corresponding to an input x(t).