# Analysis of a Power Grid using Kuramoto Model

Harishankar M EE21BTECH11022
Kartik Agrawal EE21BTECH11030
Hiya Mehta EE21BTECH11024

EP3227: Non Linear Dynamics

## Background

Power grids are complex systems that must maintain synchronization between various components—primarily generators and loads—to ensure a stable and reliable energy supply. Synchronization failures can result in significant events such as large-scale blackouts, as observed in New York in 1965. The Kuramoto model, which describes the synchronization of coupled nonlinear oscillators, provides a robust theoretical framework for understanding such dynamic phenomena. This project explores the use of a generalized Kuramoto model to analyze the dynamics of a simplified power grid.

## Introduction

Synchronization is a widespread phenomenon observed in systems of coupled oscillators, ranging from fireflies blinking in unison to the electrical activity of cardiac cells. The Kuramoto model formalizes this behavior by describing the phase dynamics of oscillators with individual natural frequencies and sinusoidal coupling. The classical Kuramoto model assumes global coupling and has been applied to various natural and technological systems.

In power grids, both generators and loads can be modeled as oscillators. The synchronized operation of these units is critical for stable grid performance. Breakdowns in synchronization can be modeled as transitions from coherent to incoherent states. Traditional models often lack the ability to capture component-level interactions and dynamics, motivating the need for a Kuramoto-like approach to better analyze and simulate grid stability.

## 1 Kuramoto Model: Basic Formulation

The system modelling and dynamics is derived from [1], [2] and [3]. The dynamics of each oscillator (representing a grid component) is governed by:

$$
\dot{\theta}_j = \omega_j
$$

$$
\dot{\omega}_j = -\alpha\omega_j + W_j + K\sum_{k=1}^{N} A_{jk}\sin(\theta_k - \theta_j)
$$

Where:

- $\theta_j$ is the phase angle of node $j$
- $\alpha$ is the damping coefficient
- $W_j$ is the power injection (positive) or consumption (negative) at node $j$
- $A_{jk}$ is the adjacency matrix element (1 if nodes $j$ and $k$ are connected, 0 otherwise)

Behavior varies based on the coupling constant $K$:

- **Weak Coupling**: No synchronization; oscillators behave independently.
- **Critical Coupling**: System transitions from disorder to partial/full synchronization.
- **Strong Coupling**: Oscillators exhibit phase-locking behavior.

## From power grid dynamics to Kuramoto model

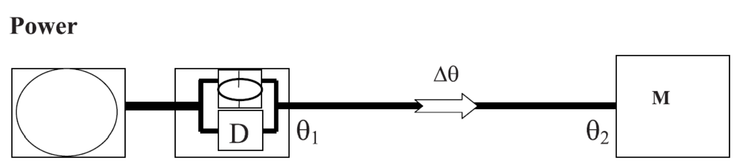


Figure 1: Equivalent diagram of generator and machine connected by a transmission line. The turbine consists of a flywheel and dissipation D

The power grid can be abstracted into an equivalent model consisting of generators and machines linked via transmission lines as shown in Fig. 1. A generator with a turbine (modeled as a flywheel) has a phase:

$$
\theta_1 = \Omega t + \tilde{\theta}_1 \tag{1}
$$

Balancing power yields:

$$P_{\text{source}} = P_{\text{diss}} + P_{\text{acc}} + P_{\text{trans}} \tag{2}$$

$$= K_D(\dot{\theta}_1)^2 + \frac{1}{2} I \frac{d}{dt}(\dot{\theta}_1)^2 - P_{\max}\sin(\Delta\theta) \tag{3}$$

Assuming small deviations from $\Omega$, equation (5) simplifies to::

$$I\Omega\ddot{\tilde{\theta}}_1 = P_{\text{source}} - K_D\Omega^2 - 2K_D\Omega\dot{\tilde{\theta}}_1 + P_{\max}\sin(\Delta\theta) \tag{4}$$

The generalised form becomes:

$$\ddot{\tilde{\theta}}_1 = P - \alpha\dot{\tilde{\theta}}_1 + P_{\max}\sin(\Delta\theta) \tag{5}$$

Extending it to a network with N oscillators (generators + machines) with local connectivity:

$$\ddot{\tilde{\theta}}_i = W_i - \alpha\dot{\tilde{\theta}}_i + K\sum_{j\neq i} A_{ij}\sin(\tilde{\theta}_j - \tilde{\theta}_i) \tag{6}$$

This resembles a second-order Kuramoto model with inertia. The power injections $W_i$ show a bimodal distribution, reflecting the roles of generators and loads. The model thus links synchronization theory with power grid stability.

# Simulation and Results

A simple three-node grid was simulated with two generators and one machine as shown in Fig 2: We choose $P_1 = P_2 = 1, P_0 = -2, \alpha = 0.1$

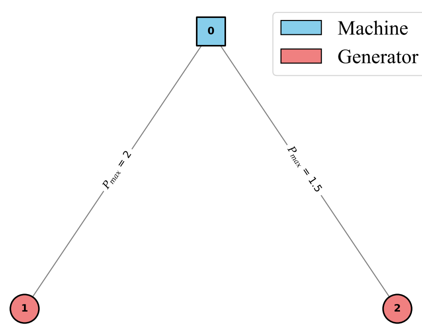


Figure 2: Diagram of network simulated

Along with the phase difference between the nodes and the power delivered to the machine, we also look at the order parameter (r(t)) given by :

$$r(t) = \left| \frac{1}{N} \sum_{i=1}^{N} e^{j\theta(t)} \right| \tag{7}$$

The order parameter is an indication of the phase synchronisation in the system with $r(t) = 1$ showing full synchronisation and $r(t) = 0$ indicating incoherence among the states. Under regular conditions, the system reaches a synchronized state as shown in Fig. 3

$$\lim_{t\to\infty} (\dot{\theta}_i(t) - \dot{\theta}_j(t)) = 0 \qquad \forall i, j \tag{8}$$

The power delivered to the machine stabilises over time, phase differences

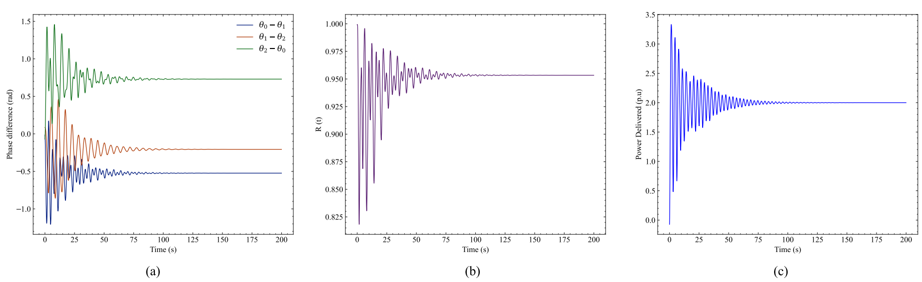


Figure 3: (a) Phase difference between the machine and the generators, (b) Order parameter (c) Power delivered to the machine

reach a steady value, and the order parameter settles close to 1 in steady state, confirming synchronisation and effective power distribution.

## Perturbation Analysis

In order to study the robustness of the network to the perturbation in the network (such as variations in power demand), we look at the dynamics of the network subjected to perturbation sin power requirement in the machine. A sudden perturbation ($P = P_0 + \Delta P_{max}$) (shown in Fig. 4 ) was introduced, simulating a short-term demand spike (such as due to a short circuit in the network). The additional power was drawn from the rotors' kinetic energy, reducing their speeds. If the system eventually recovered, it would return to the original steady state, it demonstrates the system's robustness.

## Case 1: Synchronisation Recovery

When the perturbation remains below a critical threshold, the system restores synchronization. Steady-state operation is reestablished post-perturbation, and phase locking and power delivery resume normal behaviour as shown in Fig. 5. This threshold is determined by network topology and system parameters.

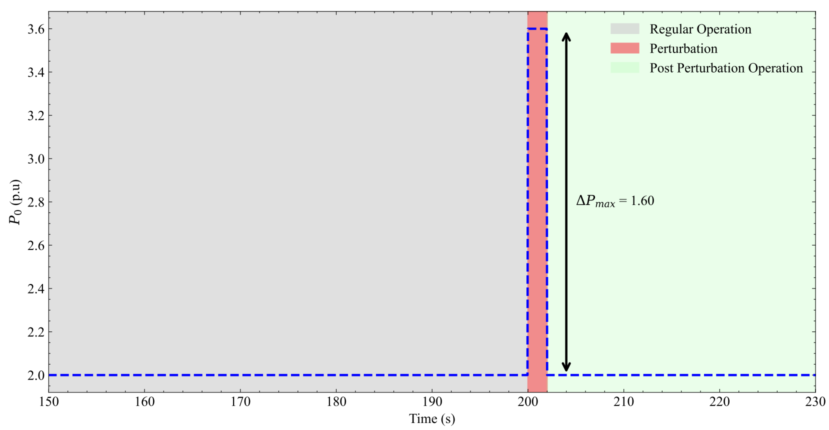


Figure 4: Pertubation in power requirement of machine

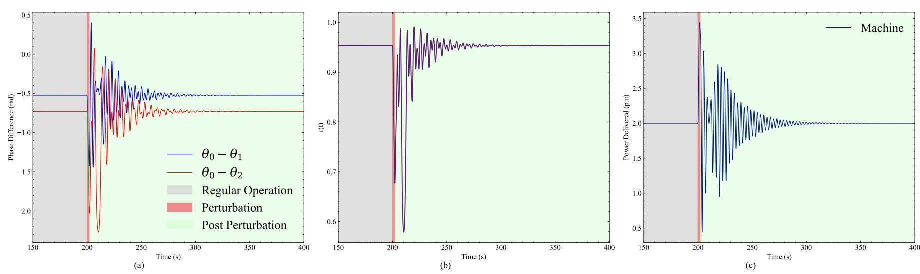


Figure 5: $\Delta P_{max} = 1.6$: System dynamics and properties is recovered to the synchronous region of operation

### 1.1 Case 2: Transition

In the transition region, when the perturbation is close to the threshold, the system dynamics and properties show highly oscillatory behaviour and do not settle as shown in Fig. 6. The phase difference of the generators with the machine shows both continuously increasing and constant trends leading to the oscillatory behavior.

### 1.2 Case 3: Loss of synchronisation

With the perturbation above the threshold, the system is not able to come back to its synchronous region of operation as shown in Fig. 7. The phase differences of the machine with the generator continuously increase, and the order parameter comes close to zero. The same is the case for the power delivered, as we will have very little power delivered, which also has oscillatory behaviour.

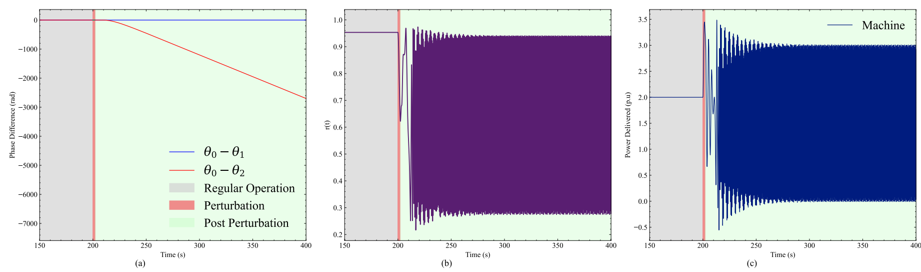


Figure 6: $\Delta P_{max} = 1.7$: System dynamics and properties is in transition region

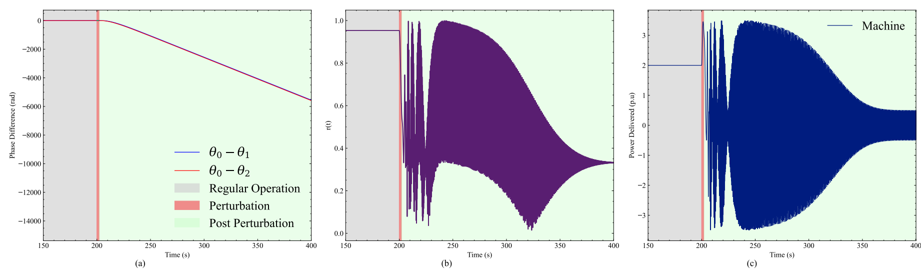


Figure 7: $\Delta P_{max} = 1.8$: System dynamics and properties is incoherent

# Discussion

The application of the Kuramoto model to power grid dynamics offers a compelling way to understand synchronization and stability in complex energy networks. In this study, we adopted a second-order generalized Kuramoto model to represent a simplified power grid composed of two generators and one machine. By modeling grid components as coupled nonlinear oscillators, we captured essential features of synchronization, phase locking, and dynamic response to perturbations.

The simulation results demonstrate that under normal operating conditions and appropriate coupling strength, the system evolves toward a synchronized steady state. This is indicated by constant phase differences, stable power delivery, and an order parameter $r(t)$ approaching 1. However, when the system is subjected to perturbations—represented by sudden increases in power demand at the load node—the dynamics shift significantly depending on the perturbation magnitude.

A small perturbation (e.g., $\Delta P_{\text{max}} = 1.6$) causes a temporary disturbance, but the system gradually returns to the synchronized state. This recovery highlights the robustness of the network to moderate fluctuations. However, a marginal increase in the perturbation (e.g., $\Delta P_{\text{max}} = 1.7$) places the system in a transitional regime, where synchronization is neither completely lost nor fully restored. The system exhibits persistent oscillations and unstable phase differences, indicating sensitivity to initial conditions and system parameters.

When the perturbation exceeds a critical threshold (e.g., $\Delta P_{\max} = 1.8$), the system fails to recover. The phase differences diverge over time, the order parameter decays toward zero, and power delivery becomes erratic. This behavior signifies a loss of global coherence among the oscillators, effectively modeling a failure in grid synchronization akin to a blackout.

Thus, a key insight is that even a small incremental change in perturbation strength can lead to qualitatively different system dynamics, transitioning the grid from a stable to a chaotic regime. This critical threshold behavior is a hallmark of nonlinear systems and underscores the importance of carefully managing system parameters and external disturbances in real-world power grids.

# Conclusion

This study demonstrates the effectiveness of using a Kuramoto-inspired model to analyze and predict power grid behavior under varying operating conditions. By abstracting the grid as a network of coupled oscillators, we explored how synchronization arises and how it is disrupted by perturbations in power demand. The simulations revealed a critical transition point: below a certain perturbation threshold, the system is able to recover and maintain synchrony, while above this threshold, the network becomes unstable and incoherent.

These findings not only validate the Kuramoto model's relevance in power systems analysis but also provide a framework to assess grid resilience. Understanding how small changes in external conditions can lead to large-scale desynchronization is crucial for designing more robust and adaptive energy infrastructures. Future work could involve scaling the model to larger, more realistic grid topologies and incorporating stochastic disturbances to further evaluate real-world applicability.

# References


[1] G. Filatrella, A. H. Nielsen, and N. F. Pedersen, "Analysis of a power grid using a kuramoto-like model," *The European Physical Journal B*, vol. 61, pp. 485–491, 2008.

[2] J. Grzybowski, E. Macau, and T. Yoneyama, "On synchronization in power-grids modelled as networks of second-order kuramoto oscillators," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 26, no. 11, 2016.

[3] Y. Guo, D. Zhang, Z. Li, Q. Wang, and D. Yu, "Overviews on the applications of the kuramoto model in modern power system analysis," *International Journal of Electrical Power & Energy Systems*, vol. 129, p. 106804, 2021.