Analysis of a Power Grid using a Kuramoto model

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Background

- Power grids must maintain synchronization to ensure stable energy supply.
- Loss of synchronization can lead to large-scale blackouts.
- **Kuramoto model**: A powerful framework to study synchronization in coupled oscillator systems.
- This work explores how a Kuramoto-like model can describe and analyze power grid dynamics.

Introduction — Synchronization & Kuramoto Model

- Synchronization of coupled nonlinear oscillators is a common phenomenon in nature & technology.
- The **Kuramoto model** offers a theoretical framework to study synchronization in such systems.
- In the standard model:
  - Each oscillator has its own natural frequency ω_i
  - Coupling is global and sinusoidal in nature.
- Real-world examples include fireflies blinking in sync, Josephson junctions, circadian rhythm, electrical activity of cardiac cells etc.

Introduction — Motivation for Power Grid Modeling

- **Power grids** consist of generators and consumers which can be modelled as coupled oscillators operating in synchronisation.
- A breakdown in this synchronization can lead to **blackouts** (e.g., New York, 1965).
- Such events can be modeled as transitions from synchronized to incoherent states in a Kuramoto-like system.
- However, traditional models lack detailed component-level representation of the grid.

Kuramoto Model

- **Basic Formulation**

$$\dot{\theta}_j = \omega_j \tag{1}$$

$$\dot{\omega}_j = -\alpha\omega_j + W_j + K\sum_{k=1}^{N} A_{jk}\sin(\theta_k - \theta_j) \tag{2}$$

where:

  - θ_j is the phase angle of node j
  - α is the damping coefficient
  - W_j is the power injection (positive) or consumption (negative) at node j
  - A_{jk} is the adjacency matrix element (1 if nodes j and k are connected, 0 otherwise)

- **Weak Coupling** (K small):
  - Oscillators behave independently $\rightarrow$ **no synchronization**
- **Critical Coupling** (K_0):
  - Transition from disorder $\rightarrow$ partial/full synchronization
- **Strong Coupling** (K large):
  - Oscillators synchronize (**phase-lock** ($\theta_j - \theta_i \approx$ constant))

Power Grid Dynamics — Toward a Kuramoto Model

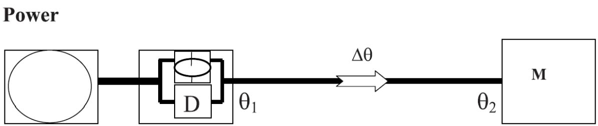


Figure 1: Equivalent diagram of generator and machine connected by a transmission line. The turbine consists of a flywheel and dissipation D.

- A power plant consists of a turbine (with inertia) generating AC power at frequency Ω.
- Phase of the generator is: $\theta_1 = \Omega t + \tilde{\theta}_1$ (3)
- Total power balance of the generator:

$$\begin{aligned} P_{\text{source}} &= P_{\text{diss}} + P_{\text{acc}} + P_{\text{trans}} \\ &= K_D(\dot{\theta}_1)^2 + \frac{1}{2} I \frac{d}{dt}(\dot{\theta}_1)^2 - P_{\max}\sin(\Delta\theta) \end{aligned} \tag{4}$$

- Assuming small deviations from Ω, the equation simplifies to:

$$I\Omega\ddot{\tilde{\theta}}_1 = P_{\text{source}} - K_D\Omega^2 - 2K_D\Omega\dot{\tilde{\theta}}_1 + P_{\max}\sin(\Delta\theta) \tag{5}$$

Generalized Kuramoto-like Model for Power Grids

- In normalized form:

$$\ddot{\tilde{\theta}}_1 = P - \alpha\dot{\tilde{\theta}}_1 + P_{\max}\sin(\Delta\theta) \tag{6}$$

- Extend to N oscillators (generators + machines) with local connectivity:

$$\ddot{\tilde{\theta}}_i = W_i - \alpha\dot{\tilde{\theta}}_i + K\sum_{j\neq i} A_{ij}\sin(\tilde{\theta}_j - \tilde{\theta}_i) \tag{7}$$

- The model resembles a second-order Kuramoto model (with inertia).
- The frequency distribution W_i is bimodal due to generators and loads.
- This formulation connects power grid stability to synchronization theory.

Simulation: Network topology

- We simulate the phase dynamics for a simple grid topology consisting of two generators and one machine connected as shown in Fig. 2
- We choose $P_1 = P_2 = 1, P_0 = -2, \alpha = 0.1$

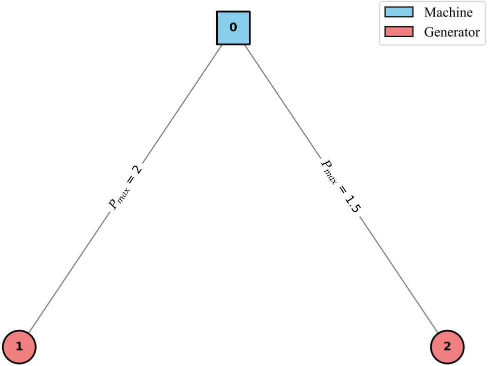


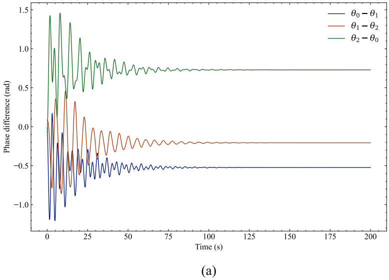
Figure 2: A simple network topology of generators and machines

Phase dynamics under regular operation

- Under regular grid operation, oscillators show synchronisation in phase (phase locking) as shown in Fig. 3a

$$\lim_{t\to\infty} (\dot{\theta}_i(t) - \dot{\theta}_j(t)) = 0 \quad \forall i, j \tag{8}$$

- Starting from a random initial distribution of phases, the phase difference and power delivered to the machine achieve a steady state (Fig. 3a)
- Machine receives the required power supply in steady state (Fig. 3b)



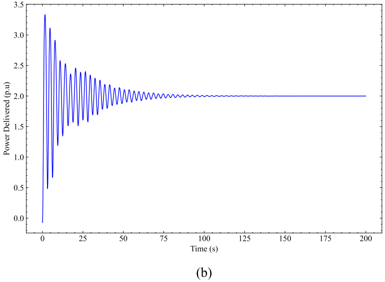


Figure 3: (a) Phase difference between the machine and the generators, (b) Power delivered to machine

Pertubation Analysis

- We analyse the behaviour of the system under a sudden short perturbation ($\Delta P = 2.5$) as shown in Fig.4.
- The machine for a short while requires more power than under standard operation (e.g.short circuit)
- This extra energy is taken from the kinetic energy of the rotators that after few time units restores normal operation.

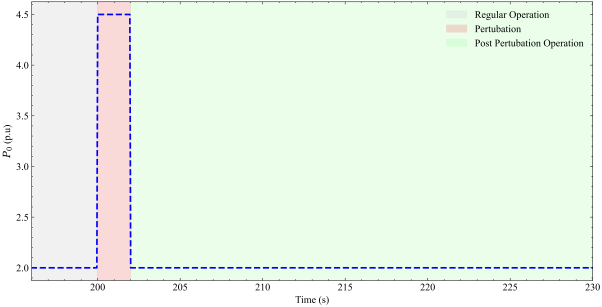


Figure 4: Pertubation in power requirement of Machine

Case 1: Synchronisation Restored

- For perturbation strength below a particular threshold ($\Delta P = 2.5$), the system is restored to its synchronised state of phases (Fig. 5a).
- Threshold depends upon the oscillator parameters and network topology.
- Upon reaching steady state after perturbation, the synchronisation & stable power delivery to the machine in the system are restored.(Fig. 5b,c)

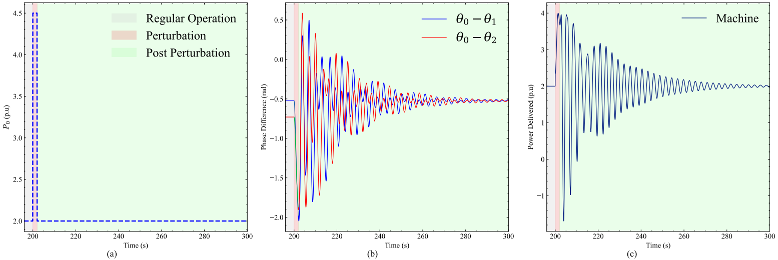


Figure 5: (a) Perturbation vs time, (b) Phase dynamics between generators and machine, (c) Power delivered to the machine

Case 2: Synchronisation Breakdown

- For perturbation strength above the threshold ($\Delta P = 3$), the system fails to revert to its synchronised phase dynamics (Fig. 6a).
- The phase difference between the machines and the generators continuously increases (Fig. 6b).
- The power delivered to the machine is minimal and shows oscillating behaviour (may cause system damage or stop working) (Fig. 6c).

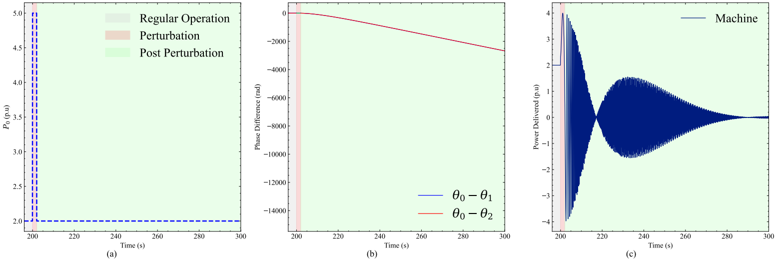


Figure 6: (a) Perturbation vs time, (b) Phase dynamics between generators and machine, (c) Power delivered to the machine

Observation and Conclusions

- **Observations**
  - Under normal operation, oscillators (generators and machine) synchronize in phase (phase locking).
  - Small perturbations temporarily disrupt the system, but synchronization is restored due to system inertia.
  - Synchronization is re-established if the perturbation remains below a critical threshold.
  - Large perturbations cause phase desynchronisation leading to, minimal and unstable power delivery.
- **Conclusion**
  - Power grid dynamics can be effectively modeled using a second-order Kuramoto-like system.
  - Synchronisation is crucial for maintaining stable power delivery across the grid.
  - The model can help in identifying conditions under which synchronisation is lost and design of more robust and resilient power grids.

References

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