

# On flocking in the presence of noise

Harishankar M., Vishnu R. Unni  
Indian Institute of Technology, Hyderabad

## Collective motion

The phenomenon of flocking is ubiquitous in nature. Noise can significantly influence the dynamics of flocking. Our study explores how noise can aid the process of flocking in different conditions. In particular, we explore the effect of noise on transient flocking.



Flock of Starlings - Credit: Ashley Cooper / The Image Bank / Getty

## Multi-agent model for flocking

We study flocking using an agent-based model inspired by Jia and Vicsek [1]. The position and velocity of each agent is updated as follows:

$$\begin{aligned}\vec{x}_i(t+1) &= \vec{x}_i(t) + \vec{v}_i(t)\Delta t \\ \vec{v}_i(t+1) &= \vec{v}_i^{\text{align}}(t) + \vec{v}_i^{\text{ra}}(t)\end{aligned}$$

$\vec{v}_i^{\text{align}}(t)$  represents the tendency of the agent to align with its neighbors  $\vec{v}_i^{\text{ra}}(t)$  represents the combined term for the repulsion between agents and the tendency to move towards the center of flock.

$$\begin{aligned}\vec{v}_i^{\text{align}}(t) &= c_1(\cos \theta \hat{x} + \sin \theta \hat{y}) \\ \theta &= \text{Arg} \left( \sum_{j \in N_c} e^{j \text{Arg}(\vec{v}_j(t) + \vec{U}_{ji})} \right) \\ \vec{U}_{ji} &= U_x \hat{x} + U_y \hat{y}\end{aligned}$$

Where  $(U_x, U_y)$  belongs to a Normal distribution  $N(0, \sigma)$  and  $N_c$  refers to the neighbourhood of  $i^{\text{th}}$  agent.

$$\vec{v}_i^{\text{ra}}(t) = \sum_{j \in N_c} \vec{r}_{ij} \left( \frac{A}{|\vec{r}_{ij}|^3} - B \right)$$

Where (A, B) are constants and  $\vec{r}_{ij}$  is the vector from  $j^{\text{th}}$  agent to  $i^{\text{th}}$  and  $\vec{U}_{ji}$  represents the noise interference in estimating the velocity of the  $j^{\text{th}}$  agent by the  $i^{\text{th}}$  agent. At each instance, the flocking efficiency is understood by examining two parameters:

**Flocking coefficient (F) and Correlation coefficient (C)**

$$\begin{aligned}F &= \frac{1}{N} \sum_{i=1}^N \frac{|\vec{V}_i|}{|\vec{V}|} \\ C &= \frac{1}{N^2} \sum_{(i,j) \in (1,N)} \frac{\vec{V}_i \cdot \vec{V}_j}{|\vec{V}_i| |\vec{V}_j|}\end{aligned}$$

**References:**  
1)Yongnan and Tamas Vicsek. Modelling hierarchical flocking. The New Journal of Physics, 21(9):093048, 2019

## Influence of noise on flocking

Noise can influence flocking in many ways. We explore two aspects. a) Dynamics of formation of flocks, b) division and regroup of flocks.

**Noise can delay or suppress flocking.**

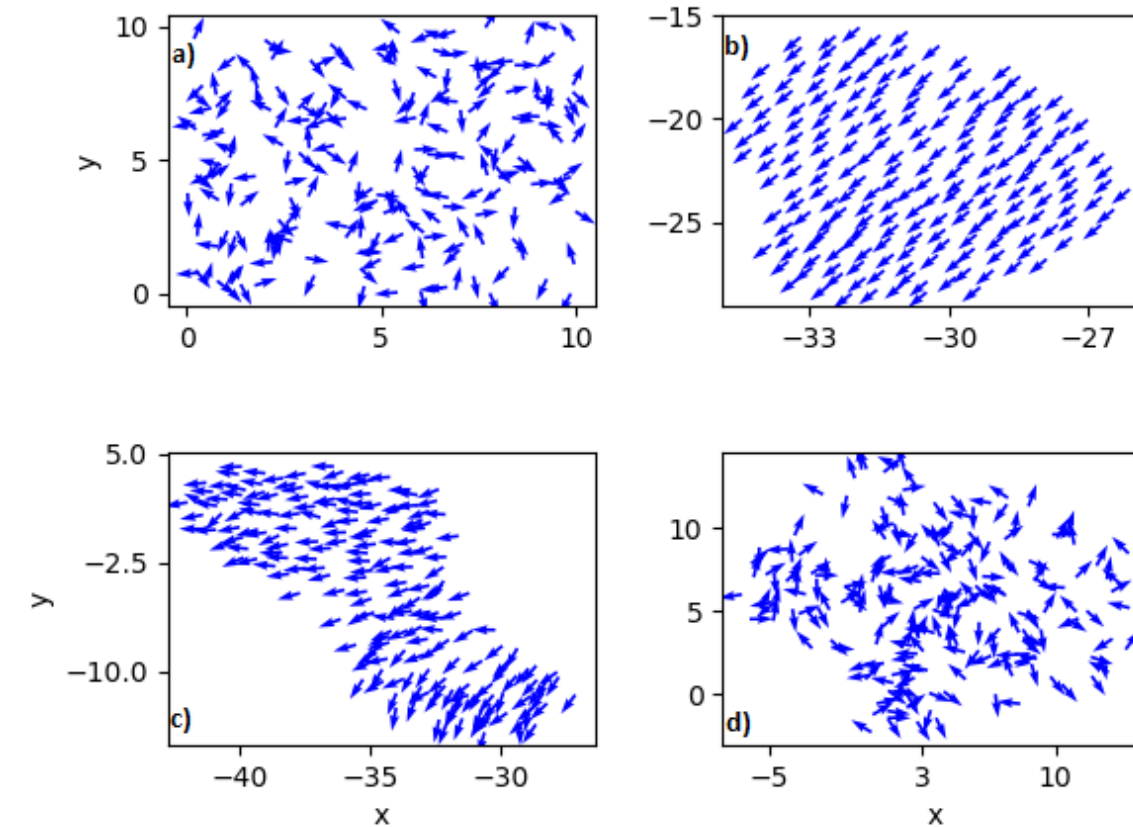


Figure 1: System comprising of 200 agents at time step: a) t=0 a.u., b) t=35 a.u. and Gaussian noise of std dev=0.0075, c) t=99 a.u. and Gaussian noise of std dev=0.36, d) t=99 a.u. and Gaussian noise of std dev=1.25. Initial conditions are the same across all sub-figures.

**Presence of noise can cause generation of disjointed flocks with multiple groups.**

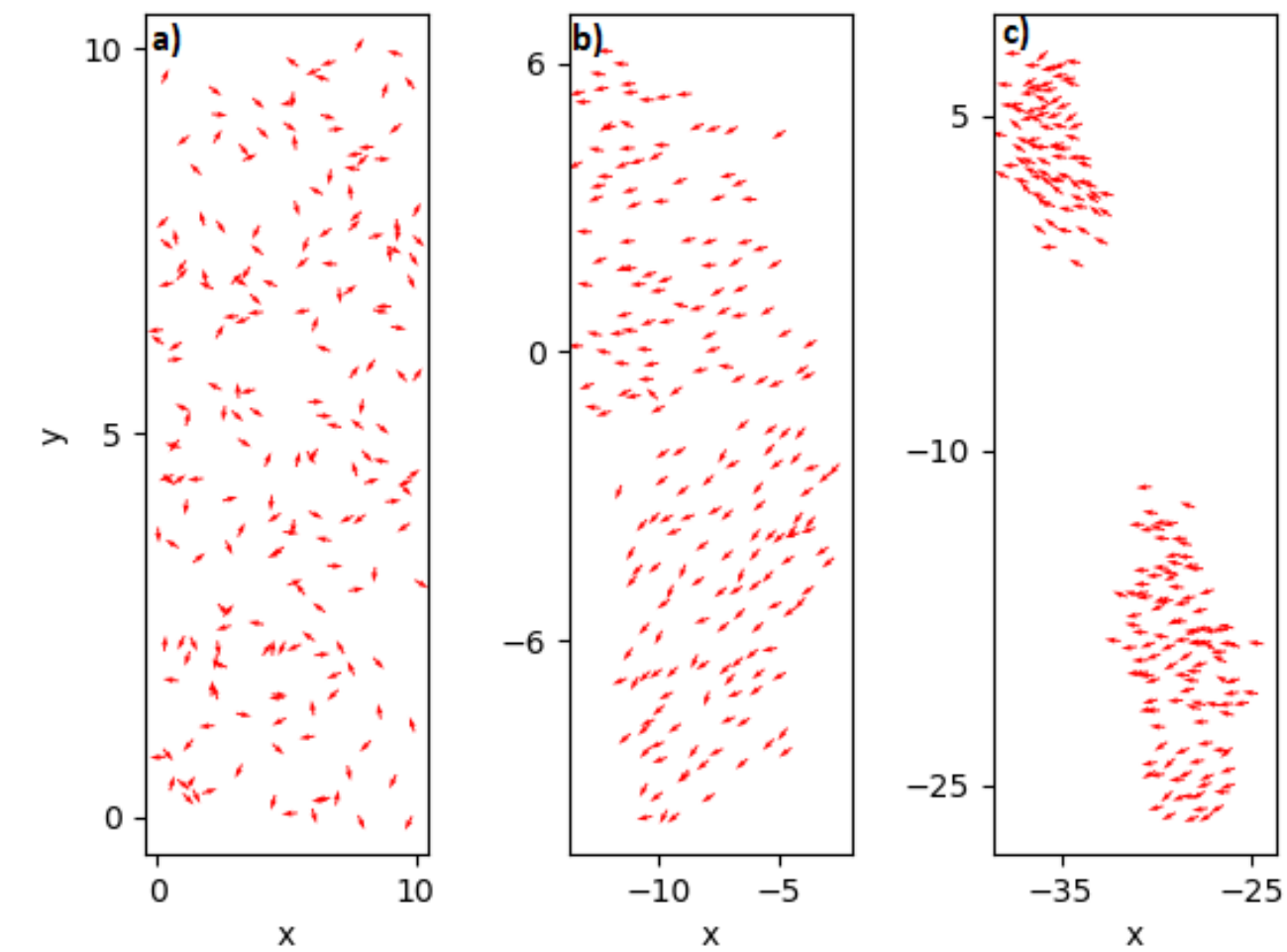


Figure 2: System with Gaussian noise of std dev = 0.3975 comprising of 200 agents at time step: a) t=0 a.u. b) t=35 a.u. c) t=99 a.u.

**Noise improves transient flocking.**

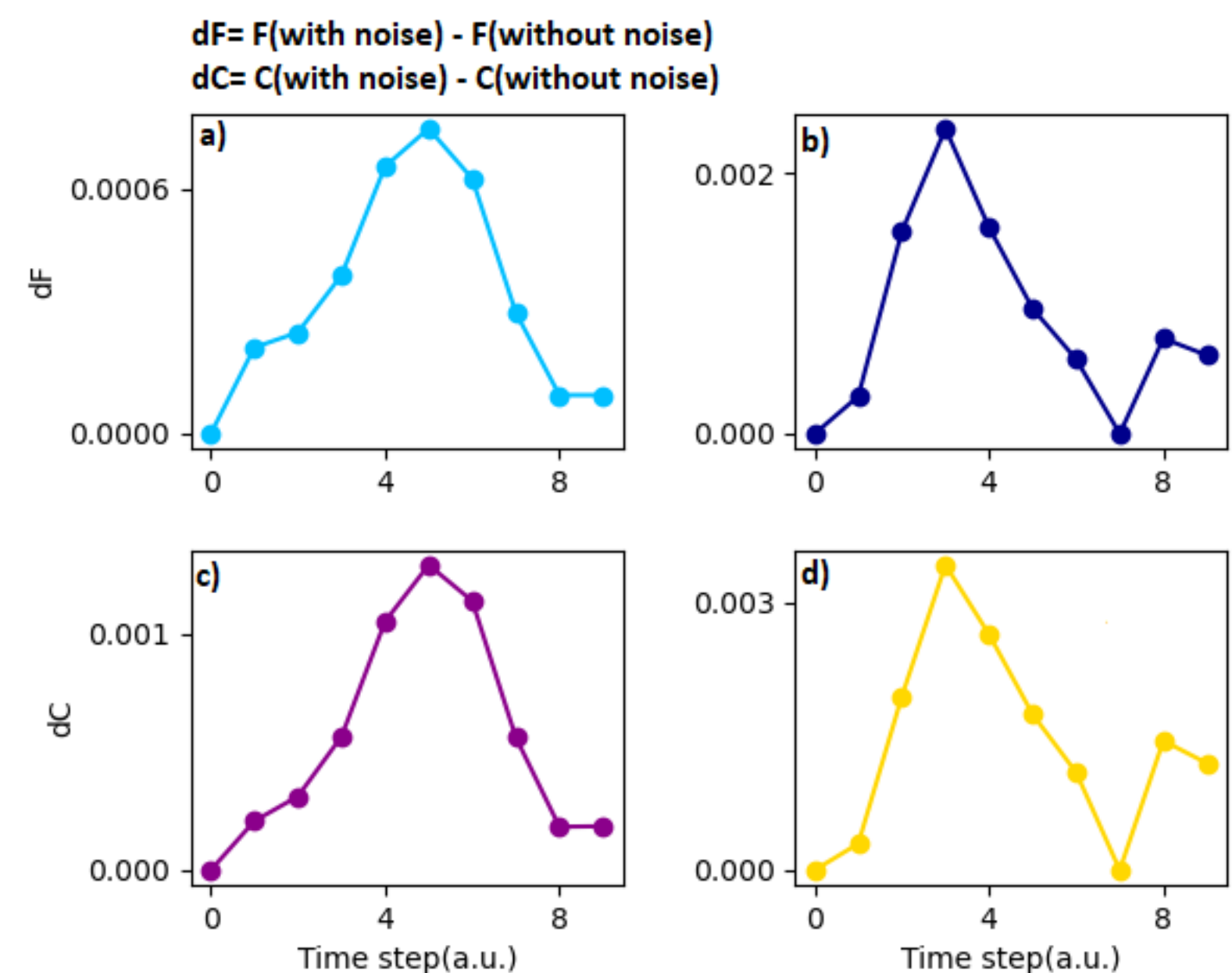


Figure 3: (a) and (b) shows dF for two different initial states. (c) and (d) shows dC for two different initial states. Plotted dF and dC are evaluated by averaging over 40 realisations with Gaussian noise of std dev=0.006

**Transient flocking and its applications are important for engineering systems that would rely on temporally varying collective interaction of agents. Improvement of transient flocking using noise in such context will be the focus of future study.**