

Project

ISEN 622 – Linear Programming

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Section A

Indices:

$$N = 1, \dots, n$$

Parameters:

p_t : unit production cost in period t , $t \in N$.

h_t : unit storage cost in period t , $t \in N$.

d_t : demand in period t , $t \in N$.

C : maximum production capacity in any period (all periods have identical production capacity)

s_0 : initial inventory

Variables:

x_t : number of product units produced in period t

s_t : number of product units carried in inventory from period t to $t+1$

Formulations:

$$\text{Min } \sum_{t=1}^n (p_t x_t + h_t s_t)$$

s.t

$$s_t = s_{t-1} + x_t - d_t \quad t=1, \dots, n$$

$$s_0 = 0$$

$$0 \leq x_t \leq C \quad t=1, \dots, n$$

$$s_t \geq 0 \quad t=1, \dots, n$$

Section B

Model File

```
# AMPL model file for LSC

# No. of periods
param n;

set N := {1..n};

# Demand in period  $t, t \in N$ 
param d {t in N};

# Unit production cost in period  $t, t \in N$ 
param p {t in N};

# Unit storage cost in period  $t, t \in N$ 
param h {t in N};

# Maximum production capacity in any period
param C;

# No. units produced in period  $t, t \in N$ 
var x {t in N} >= 0;

# Inventory at the end of each period
var s {t in 0..n} >= 0;

# Sum of cost of producing  $x[t]$  units and storing  $s[t]$  units in time  $t, t \in N$ 
minimize z: sum {t in N} (p[t]*x[t] + h[t]*s[t]);

# Initial inventory is 0
subject to initial_inventory: s[0] = 0;

# Inventory at time  $t, t \in N$ 
subject to inventory {t in N}: s[t] = s[t-1] + x[t] - d[t];

# Production Limit at time  $t, t \in N$  (which is constant for all  $t, t \in N$ )
subject to production_limit {t in N}: x[t] <= C;
```

Data File

param n := 90;

param p :=

1	92
2	92
3	104
4	81
5	108
6	98
7	99
8	88
9	113
10	83
11	93
12	88
13	101
14	118
15	85
16	88
17	89
18	109
19	113
20	104
21	90
22	94
23	99
24	102
25	112
26	91
27	93
28	102
29	113
30	103
31	89
32	95
33	110
34	105
35	110
36	87
37	113
38	101
39	118
40	118
41	90
42	113
43	82
44	104
45	91

46	89
47	108
48	84
49	88
50	107
51	84
52	99
53	114
54	112
55	104
56	91
57	83
58	107
59	85
60	116
61	101
62	106
63	104
64	91
65	101
66	98
67	86
68	112
69	102
70	115
71	83
72	99
73	118
74	83
75	108
76	93
77	91
78	113
79	84
80	96
81	87
82	96
83	111
84	86
85	85
86	104
87	103
88	94
89	96
90	99;

param h :=
1 10
2 10
3 10
4 10

5	10
6	10
7	10
8	10
9	10
10	10
11	10
12	10
13	10
14	10
15	10
16	10
17	10
18	10
19	10
20	10
21	10
22	10
23	10
24	10
25	10
26	10
27	10
28	10
29	10
30	10
31	10
32	10
33	10
34	10
35	10
36	10
37	10
38	10
39	10
40	10
41	10
42	10
43	10
44	10
45	10
46	10
47	10
48	10
49	10
50	10
51	10
52	10
53	10
54	10
55	10

56	10
57	10
58	10
59	10
60	10
61	10
62	10
63	10
64	10
65	10
66	10
67	10
68	10
69	10
70	10
71	10
72	10
73	10
74	10
75	10
76	10
77	10
78	10
79	10
80	10
81	10
82	10
83	10
84	10
85	10
86	10
87	10
88	10
89	10
90	10;

param d :=

1	134
2	118
3	33
4	185
5	147
6	144
7	123
8	181
9	47
10	74
11	16
12	115
13	109
14	107

15	109
16	126
17	175
18	144
19	122
20	70
21	135
22	176
23	174
24	31
25	115
26	170
27	108
28	153
29	17
30	149
31	148
32	47
33	73
34	149
35	161
36	46
37	179
38	17
39	40
40	121
41	40
42	155
43	116
44	89
45	46
46	37
47	80
48	38
49	86
50	94
51	114
52	94
53	81
54	76
55	165
56	100
57	149
58	72
59	61
60	102
61	26
62	100
63	24
64	131
65	99


```

66      91
67      20
68      76
69      104
70      112
71      160
72      37
73      105
74      63
75      159
76      39
77      144
78      62
79      171
80      56
81      68
82      46
83      104
84      40
85      15
86      61
87      96
88      185
89      133
90      22;

```

```
param C := 180;
```

Run File

```

reset;

option solver cplex;

#option cplex_options 'timing 4';      # For displaying the Input, Output and
                                     #Solve Times in Ticks. Put 'timing 1'
                                     #for getting the value in seconds

option cplex_options 'sensitivity'; # For performing sensitivity analysis

#option 'timelimit= 0.00000023728752'; # For terminating the search after the
                                     #given amount of time

option presolve 0;

model lsc.mod;

data lsc.dat;

```

```
#expand z,initial_inventory,inventory,production_limit;

solve;

display z,x,s>'lsc.out';

display production_limit.up, production_limit.current,
production_limit.down>>'lsc.out';
```

Output

$z = 853945$

t	x	s
1	134	0
2	156	38
3	0	5
4	180	0
5	147	0
6	144	0
7	124	1
8	180	0
9	47	0
10	74	0
11	16	0
12	180	65
13	151	107
14	0	0
15	109	0
16	180	54
17	180	59
18	85	0
19	122	0
20	70	0
21	135	0
22	176	0
23	174	0
24	31	0
25	115	0
26	170	0
27	108	0
28	170	17
29	0	0
30	149	0

31	148	0
32	120	73
33	0	0
34	149	0
35	161	0
36	180	134
37	45	0
38	57	40
39	0	0
40	121	0
41	180	140
42	15	0
43	180	64
44	25	0
45	46	0
46	117	80
47	0	0
48	38	0
49	180	94
50	0	0
51	180	66
52	109	81
53	0	0
54	76	0
55	165	0
56	100	0
57	180	31
58	41	0
59	163	102
60	0	0
61	26	0
62	100	0
63	24	0
64	180	49
65	50	0
66	91	0
67	96	76
68	0	0
69	180	76
70	36	0
71	180	20
72	122	105
73	0	0

74	180	117
75	42	0
76	39	0
77	180	36
78	26	0
79	180	9
80	47	0
81	68	0
82	150	104
83	0	0
84	40	0
85	76	61
86	0	0
87	101	5
88	180	0
89	133	0
90	22	0

t	production_limit.up	production_limit.current	production_limit.down
1	1e+20	180	134
2	1e+20	180	156
3	1e+20	180	0
4	185	180	156
5	1e+20	180	147
6	1e+20	180	144
7	1e+20	180	124
8	181	180	124
9	1e+20	180	47
10	1e+20	180	74
11	1e+20	180	16
12	331	180	151
13	1e+20	180	151
14	1e+20	180	0
15	1e+20	180	109
16	265	180	126
17	265	180	121
18	1e+20	180	85
19	1e+20	180	122
20	1e+20	180	70
21	1e+20	180	135
22	1e+20	180	176

23	1e+20	180	174
24	1e+20	180	31
25	1e+20	180	115
26	1e+20	180	170
27	1e+20	180	108
28	1e+20	180	170
29	1e+20	180	0
30	1e+20	180	149
31	1e+20	180	148
32	1e+20	180	120
33	1e+20	180	0
34	1e+20	180	149
35	1e+20	180	161
36	225	180	46
37	1e+20	180	45
38	1e+20	180	57
39	1e+20	180	0
40	1e+20	180	121
41	195	180	40
42	1e+20	180	15
43	205	180	116
44	1e+20	180	25
45	1e+20	180	46
46	1e+20	180	117
47	1e+20	180	0
48	1e+20	180	38
49	180	180	38
50	1e+20	180	0
51	289	180	114
52	1e+20	180	109
53	1e+20	180	0
54	1e+20	180	76
55	1e+20	180	165
56	1e+20	180	100
57	221	180	149
58	1e+20	180	41
59	1e+20	180	163
60	1e+20	180	0
61	1e+20	180	26
62	1e+20	180	100
63	1e+20	180	24
64	230	180	131
65	1e+20	180	50

66	1e+20	180	91
67	1e+20	180	96
68	1e+20	180	0
69	216	180	104
70	1e+20	180	36
71	302	180	160
72	1e+20	180	122
73	1e+20	180	0
74	222	180	63
75	1e+20	180	42
76	1e+20	180	39
77	206	180	144
78	1e+20	180	26
79	227	180	171
80	1e+20	180	47
81	1e+20	180	68
82	1e+20	180	150
83	1e+20	180	0
84	1e+20	180	40
85	1e+20	180	76
86	1e+20	180	0
87	1e+20	180	101
88	185	180	101
89	1e+20	180	133
90	1e+20	180	22

Section C

1. What was the version of CPLEX you used in solving LSC problem?

CPLEX 12.7.1.0

2. What was the specifications of the computer you used to run LSC AMPL model?

Intel Core i7-7th generation, 2.81GHz

8 GB DDR-4 RAM

4 GB DDR-5 NVIDIA GEFORCE GTX 1050

3. What was the input time, solve time, and output time to solve LSC on this computer? What was the total time?

Times (seconds):

Input = 0

Solve = 0

Output = 0

Total = 0

Times (ticks);

Input = 0.00516224

Solve = 0.468297

Output = 0.0011158

Total = 0.47457504

1 tick = 10^{-6} seconds

4. What is the optimal objective function value for LSC?

$z = 853945$

5. Use the sensitivity ranging capability of AMPLCPLEX to determine for what range of parameter CC , the optimal solution remains optimal.

```

option cplex_options 'sensitivity'; # For performing sensitivity
                                   #analysis

option presolve 0;

display production_limit.up, production_limit.current,
production_limit.down>>'lsc.out';

```

Output for the same

t	production_limit.up	production_limit.current	production_limit.down
1	1e+20	180	134
2	1e+20	180	156
3	1e+20	180	0
4	185	180	156
5	1e+20	180	147
6	1e+20	180	144
7	1e+20	180	124
8	181	180	124
9	1e+20	180	47
10	1e+20	180	74
11	1e+20	180	16
12	331	180	151
13	1e+20	180	151
14	1e+20	180	0
15	1e+20	180	109
16	265	180	126
17	265	180	121
18	1e+20	180	85
19	1e+20	180	122
20	1e+20	180	70
21	1e+20	180	135
22	1e+20	180	176
23	1e+20	180	174
24	1e+20	180	31
25	1e+20	180	115
26	1e+20	180	170
27	1e+20	180	108
28	1e+20	180	170
29	1e+20	180	0
30	1e+20	180	149

31	1e+20	180	148
32	1e+20	180	120
33	1e+20	180	0
34	1e+20	180	149
35	1e+20	180	161
36	225	180	46
37	1e+20	180	45
38	1e+20	180	57
39	1e+20	180	0
40	1e+20	180	121
41	195	180	40
42	1e+20	180	15
43	205	180	116
44	1e+20	180	25
45	1e+20	180	46
46	1e+20	180	117
47	1e+20	180	0
48	1e+20	180	38
49	180	180	38
50	1e+20	180	0
51	289	180	114
52	1e+20	180	109
53	1e+20	180	0
54	1e+20	180	76
55	1e+20	180	165
56	1e+20	180	100
57	221	180	149
58	1e+20	180	41
59	1e+20	180	163
60	1e+20	180	0
61	1e+20	180	26
62	1e+20	180	100
63	1e+20	180	24
64	230	180	131
65	1e+20	180	50
66	1e+20	180	91
67	1e+20	180	96
68	1e+20	180	0
69	216	180	104
70	1e+20	180	36
71	302	180	160
72	1e+20	180	122
73	1e+20	180	0

74	222	180	63
75	1e+20	180	42
76	1e+20	180	39
77	206	180	144
78	1e+20	180	26
79	227	180	171
80	1e+20	180	47
81	1e+20	180	68
82	1e+20	180	150
83	1e+20	180	0
84	1e+20	180	40
85	1e+20	180	76
86	1e+20	180	0
87	1e+20	180	101
88	185	180	101
89	1e+20	180	133
90	1e+20	180	22

- 6. Set the time limit of CPLEX to half the total run time you report in part 3 and re-solve your model. Report the objective value for the best solution found within this time limit.**

Objective value found within this time limit is same as in question C.3, i.e. $z=853945$

Section D

Indices:

$$N = 1, \dots, n$$

Parameters:

p_t : unit production cost in period t , $t \in N$.

h_t : unit storage cost in period t , $t \in N$.

d_t : demand in period t , $t \in N$.

C : maximum production capacity of each module

f_t : unit module cost in each period t , $t \in N$

s_0 : initial inventory

Variables:

x_t : number of product units produced in period t

s_t : number of product units carried in inventory from period t to $t+1$

y_t : number of modules in each period (takes only integer values)

Formulations:

$$\text{Min } \sum_{t=1}^n (p_t x_t + h_t s_t + y_t f_t)$$

s.t

$$s_t = s_{t-1} + x_t - d_t \quad t=1, \dots, n$$

$$s_0 = 0$$

$$0 \leq x_t \leq y_t * C \quad t=1, \dots, n$$

$$S_t \geq 0 \quad t=1, \dots, n$$

Section E

Model File

```
# AMPL model file for LSCM
# No. of periods
param n;
set N := {1..n};
# Demand in period  $t, t \in N$ 
param d {t in N};
# Unit production cost in period  $t, t \in N$ 
param p {t in N};
# Unit module cost in period  $t, t \in N$ 
param f {t in N};
# Unit storage cost in period  $t, t \in N$ 
param h {t in N};
# Maximum production capacity in any period
param C;
# No. units produced in period  $t, t \in N$ 
var x {t in N} >= 0;
# Inventory at the end of each period
var s {t in 0..n} >= 0;
# Number of capacity modules installed in period  $t, t \in N$ 
var y {t in N} >= 0 integer;
# Sum of cost of producing  $x[t]$  units, storing  $s[t]$  units and cost of  $y[t]$  in
time  $t, t \in N$ 
minimize z: sum {t in N} (p[t]*x[t] + h[t]*s[t] + y[t]*f[t]);
# Initial inventory is 0
subject to initial_inventory: s[0] = 0;
# Inventory at time  $t, t \in N$ 
subject to inventory {t in N}: s[t] = s[t-1] + x[t] - d[t];
# Production Limit at time  $t, t \in N$ 
subject to production_limit {t in N}: x[t] <= y[t]*C;
```

Data File

param n := 90;

param p :=

1	92
2	92
3	104
4	81
5	108
6	98
7	99
8	88
9	113
10	83
11	93
12	88
13	101
14	118
15	85
16	88
17	89
18	109
19	113
20	104
21	90
22	94
23	99
24	102
25	112
26	91
27	93
28	102
29	113
30	103
31	89
32	95
33	110
34	105
35	110
36	87
37	113
38	101
39	118
40	118
41	90
42	113
43	82
44	104
45	91

46	89
47	108
48	84
49	88
50	107
51	84
52	99
53	114
54	112
55	104
56	91
57	83
58	107
59	85
60	116
61	101
62	106
63	104
64	91
65	101
66	98
67	86
68	112
69	102
70	115
71	83
72	99
73	118
74	83
75	108
76	93
77	91
78	113
79	84
80	96
81	87
82	96
83	111
84	86
85	85
86	104
87	103
88	94
89	96
90	99;

param h :=
1 10
2 10
3 10
4 10

5	10
6	10
7	10
8	10
9	10
10	10
11	10
12	10
13	10
14	10
15	10
16	10
17	10
18	10
19	10
20	10
21	10
22	10
23	10
24	10
25	10
26	10
27	10
28	10
29	10
30	10
31	10
32	10
33	10
34	10
35	10
36	10
37	10
38	10
39	10
40	10
41	10
42	10
43	10
44	10
45	10
46	10
47	10
48	10
49	10
50	10
51	10
52	10
53	10
54	10
55	10

56	10
57	10
58	10
59	10
60	10
61	10
62	10
63	10
64	10
65	10
66	10
67	10
68	10
69	10
70	10
71	10
72	10
73	10
74	10
75	10
76	10
77	10
78	10
79	10
80	10
81	10
82	10
83	10
84	10
85	10
86	10
87	10
88	10
89	10
90	10;

param d :=

1	134
2	118
3	33
4	185
5	147
6	144
7	123
8	181
9	47
10	74
11	16
12	115
13	109
14	107

15	109
16	126
17	175
18	144
19	122
20	70
21	135
22	176
23	174
24	31
25	115
26	170
27	108
28	153
29	17
30	149
31	148
32	47
33	73
34	149
35	161
36	46
37	179
38	17
39	40
40	121
41	40
42	155
43	116
44	89
45	46
46	37
47	80
48	38
49	86
50	94
51	114
52	94
53	81
54	76
55	165
56	100
57	149
58	72
59	61
60	102
61	26
62	100
63	24
64	131
65	99

66	91
67	20
68	76
69	104
70	112
71	160
72	37
73	105
74	63
75	159
76	39
77	144
78	62
79	171
80	56
81	68
82	46
83	104
84	40
85	15
86	61
87	96
88	185
89	133
90	22;

param C := 180;

param f :=

1	5000
2	5000
3	5000
4	5000
5	5000
6	5000
7	5000
8	5000
9	5000
10	5000
11	5000
12	5000
13	5000
14	5000
15	5000
16	5000
17	5000
18	5000
19	5000
20	5000
21	5000
22	5000

23	5000
24	5000
25	5000
26	5000
27	5000
28	5000
29	5000
30	5000
31	5000
32	5000
33	5000
34	5000
35	5000
36	5000
37	5000
38	5000
39	5000
40	5000
41	5000
42	5000
43	5000
44	5000
45	5000
46	5000
47	5000
48	5000
49	5000
50	5000
51	5000
52	5000
53	5000
54	5000
55	5000
56	5000
57	5000
58	5000
59	5000
60	5000
61	5000
62	5000
63	5000
64	5000
65	5000
66	5000
67	5000
68	5000
69	5000
70	5000
71	5000
72	5000
73	5000

```

74    5000
75    5000
76    5000
77    5000
78    5000
79    5000
80    5000
81    5000
82    5000
83    5000
84    5000
85    5000
86    5000
87    5000
88    5000
89    5000
90    5000;

```

Run File

```

reset;

option solver cplex;

option cplex_options 'timing 1'; # For displaying the Input, Output and Solve
Times

model lscm.mod;

data lscm.dat;

#expand z,initial_inventory,inventory,production_limit;

solve;

display z,x,s,y>'lscm.out';

```

Output

z = 1134640

t	x	s	y
1	164	30	1
2	180	92	1
3	0	59	0
4	360	234	2
5	0	87	0
6	180	123	1

7	0	0	0
8	318	137	2
9	0	90	0
10	0	16	0
11	0	0	0
12	357	242	2
13	0	133	0
14	0	26	0
15	180	97	1
16	180	151	1
17	360	336	2
18	0	192	0
19	0	70	0
20	0	0	0
21	135	0	1
22	176	0	1
23	174	0	1
24	146	115	1
25	0	0	0
26	170	0	1
27	163	55	1
28	180	82	1
29	0	65	0
30	180	96	1
31	180	128	1
32	0	81	0
33	0	8	0
34	180	39	1
35	180	58	1
36	180	192	1
37	0	13	0
38	180	176	1
39	0	136	0
40	0	15	0
41	180	155	1
42	0	0	0
43	251	135	2
44	0	46	0
45	0	0	0
46	160	123	1
47	0	43	0
48	0	5	0
49	180	99	1

50	0	5	0
51	360	251	2
52	0	157	0
53	0	76	0
54	0	0	0
55	165	0	1
56	150	50	1
57	180	81	1
58	0	9	0
59	180	128	1
60	0	26	0
61	0	0	0
62	124	24	1
63	0	0	0
64	321	190	2
65	0	91	0
66	0	0	0
67	132	112	1
68	0	36	0
69	180	112	1
70	0	0	0
71	302	142	2
72	0	105	0
73	0	0	0
74	287	224	2
75	0	65	0
76	0	26	0
77	180	62	1
78	0	0	0
79	305	134	2
80	0	78	0
81	0	10	0
82	180	144	1
83	0	40	0
84	0	0	0
85	177	162	1
86	0	101	0
87	0	5	0
88	180	0	1
89	155	22	1
90	0	0	0

Section F

1. Solve LSCM on the same computer on which you solved LSC and compare its total running time with that of LSC.

Input = 0

Solve = 4.78125

Output = 0

2. What is the optimal objective function value for LSCM? Compare this optimal objective value with the optimal objective value of LSC plus $5000n$ (since in LSC every period has a capacity of C , or equivalent to one module, the total cost of capacity installation would be $5000n$ if we assume each module in LSC also costs 5000). Theoretically argue why the optimal objective of LSCM is larger or smaller than that of LSC plus $5000n$.

The optimal objective value of LSCM is 1134640

The optimal objective value of LSC + $5000 \cdot n$ is 1303940

The optimal objective of LSCM will always be equal to or smaller than the optimal objective cost of LSC plus $5000n$. The LSC after adding $5000n$ can be represented as the current LSCM with an additional constraint, $y_t=1$. Adding a constraint to an LP either reduces or doesn't change the optimal objective value. In the current case (LSC plus $5000n$ /LSCM with $y_t=1$) since the feasible region reduces (when compared with LSCM) and the optimal solution of LSCM is not a part of the current feasible region of LSC (LSCM with $y_t=1$) and hence the objective value of LSC (LSCM with $y_t=1$) is larger than LSCM.

When comparing LSC with LSCM for the current data, we can see that it is advantageous to produce the items in periods with lower production costs and hold it as inventory for satisfying demand in subsequent periods rather than setting up production and producing items in every period. Therefore the cost of LSCM is lower than the cost of LSC.