Flow measurement in closed conduits

Hydraulic Engineering Experiment H6

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1 Introduction

An experiment was conducted in a closed conduit hydraulic apparatus setup. It consisted a venturimeter, an orifice plate meter, a rotameter, a wide-angled diffuser, and a right-angled bend. The head loss across each flow meter was evaluated. The coefficient of discharge, C_d, for the venturimeter and the orifice plate meter was also determined. The head losses and coefficients of discharge were then corelated with the actual discharge, Q_A.

2 Objectives

1. To determine the coefficient of discharge, C_d, for the
   (a) Venturimeter, and
   (b) Orifice plate meter.
2. To evaluate and corelate the head losses across the
   (a) Venturimeter,
   (b) Orifice plate meter,
   (c) Rotameter,
   (d) Wide angled diffuser, and
   (e) Right angled bend

   with the actual discharge, Q_A.

3 Procedure

1. Ensure that the delivery valve is closed and start the pump.
2. Ensure that there is no air trapped in the apparatus.
3. Adjust the flow rate to the chosen level, regulating only the delivery value. Ensure that the exit valve from the rotameter is fully open.
4. Allow flow to stabilise before taking the manometer readings.
5. Measure the time taken for 10.0 kg of water to be collected in the weighing tank.
6. Repeat the procedure for eight different rates of flow.
7. At the end of the experiment close the delivery valve before switching off the pump.

4 Experimental theory

The measurements of discharge and head loss in this experiment are based on the Principle of Conservation of Energy. The total energy of the flowing fluid is assumed to be constant at every cross section along the pipe. This principle can be expressed by Bernoulli's equation,

$$\frac{P_{\mathrm{X}}}{\rho g} + z_{\mathrm{X}} + \frac{v_{\mathrm{X}}^2}{2g} = \frac{P_{\mathrm{Y}}}{\rho g} + z_{\mathrm{Y}} + \frac{v_{\mathrm{Y}}^2}{2g}, \tag{1}$$

where P is the pressure, and
v is the mean velocity.

The discharge is taken to be constant at every cross-section, giving the continuity equation,

$$v_X A_X = v_Y A_Y = Q, \tag{2}$$

where A is the area of the cross-section in consideration.

Equations for the theoretical discharge, Q_T, and head loss through the various flow meters are derived from equations (1) & (2).

5 Results of the experiment

5.1 Sample calculations

Below are sample calculations for the flow rate corresponding to a rotameter reading of 18.0 cm. All values listed are in the cgs system of units.

$$\begin{aligned} Q_A &= \frac{\text{mass of water}}{\text{density of water}} \times \frac{1}{\text{time taken}} \\ &= \frac{10000}{1} \times \frac{1}{23.08} \\ &= 433 \end{aligned}$$

$$\begin{aligned} Q_T \text{ Venturi} &= A_B \sqrt{\frac{2g(h_A - h_B)}{1 - (A_B/A_A)^2}} \\ &= 2.01 \sqrt{\frac{2 \times 981(35.4 - 14.4)}{1 - 2.01/5.31)^2}} \\ &= 441 \end{aligned}$$

$$\begin{aligned} C_d \text{ Venturi} &= \frac{Q_A}{Q_T} \\ &= \frac{433}{441} \\ &= 0.981 \end{aligned}$$

$$
\begin{aligned}
Q_T \text{ Orifice} &= A_F\sqrt{\frac{2g(h_E-h_F)}{1-(A_F/A_E)^2}} \\
&= 3.14\sqrt{\frac{2\times 981(34.2-8.0)}{1-3.14/20.43)^2}} \\
&= 720 \\
C_d \text{ Orifice} &= \frac{Q_A}{Q_T} \\
&= \frac{433}{720} \\
&= 0.601
\end{aligned}
$$

$$
\begin{aligned}
H_V &= h_A - h_C \\
&= 35.4 - 32.6 \\
&= 2.8
\end{aligned}
$$

$$
\begin{aligned}
H_O &= (h_E - h_F)(1 - {C_d}^2) \\
&= (34.2 - 8.0)(1 - 0.60^2) \\
&= 16.7
\end{aligned}
$$

$$
\begin{aligned}
H_R &= h_H - h_I \\
&= 11.6 - 0.6 \\
&= 11.0
\end{aligned}
$$

$$
\begin{aligned}
H_D &= (h_C - h_D) + \frac{{Q_A}^2}{2g}\left[\frac{1}{{A_C}^2} - \frac{1}{{A_D}^2}\right] \\
&= (32.6 - 33.2) + \frac{433^2}{2\times 981}\left[\frac{1}{5.31^2} - \frac{1}{20.43^2}\right] \\
&= 2.56
\end{aligned}
$$

$$
\begin{aligned}
H_B &= (h_G - h_H) + \frac{{Q_A}^2}{2g}\left[\frac{1}{{A_G}^2} - \frac{1}{{A_H}^2}\right] \\
&= (12.2 - 11.6) + \frac{433^2}{2\times 981}\left[\frac{1}{20.43^2} - \frac{1}{5.11^2}\right] \\
&= -2.84
\end{aligned}
$$

5.2 Tables

Rotameter (cm)	Manometer Readings (mm)									Weighing Tank	
	A	B	C	D	E	F	G	H	I	Weight (kg)	Time (s)
18.0	354	144	326	332	342	80	122	116	6	10	23.08
16.0	312	152	290	294	304	104	136	132	26	10	26.45
14.0	282	159	264	268	276	122	146	144	40	10	30.21
12.0	254	166	242	244	249	140	156	154	49	10	36.02
10.0	232	170	222	223	228	153	164	162	56	10	44.05
8.0	214	174	208	208	212	162	170	169	67	10	54.47

Table 1: Measurement results

Rotameter (cm)	Q_A (cm^3/s)	Q_T Venturi (cm^3/s)	Q_T Orifice (cm^3/s)	Venturi Loss, H_V (cm)	Orifice Loss, H_O (cm)	Rotameter Loss, H_R (cm)	Wide-ang Loss, H_D (cm)	Right-ang Loss, H_B (cm)	Coeff. of discharge, C_d Venturi	Coeff. of discharge, C_d Orifice
18.0	433	441	720	21.0	16.7	11.0	2.56	-2.84	0.98	0.60
16.0	378	385	629	16.0	12.8	10.6	2.01	-2.22	0.98	0.60
14.0	331	337	552	12.3	9.87	10.4	1.45	-1.80	0.98	0.60
12.0	278	285	465	8.80	7.01	10.5	1.10	-1.21	0.97	0.60
10.0	227	240	385	6.20	4.90	10.6	0.77	-0.74	0.95	0.59
8.0	184	192	315	4.00	3.30	10.2	0.57	-0.52	0.95	0.58

Table 2: Results from calculations

5.3 Determination of C_d

5.3.1 Venturimeter

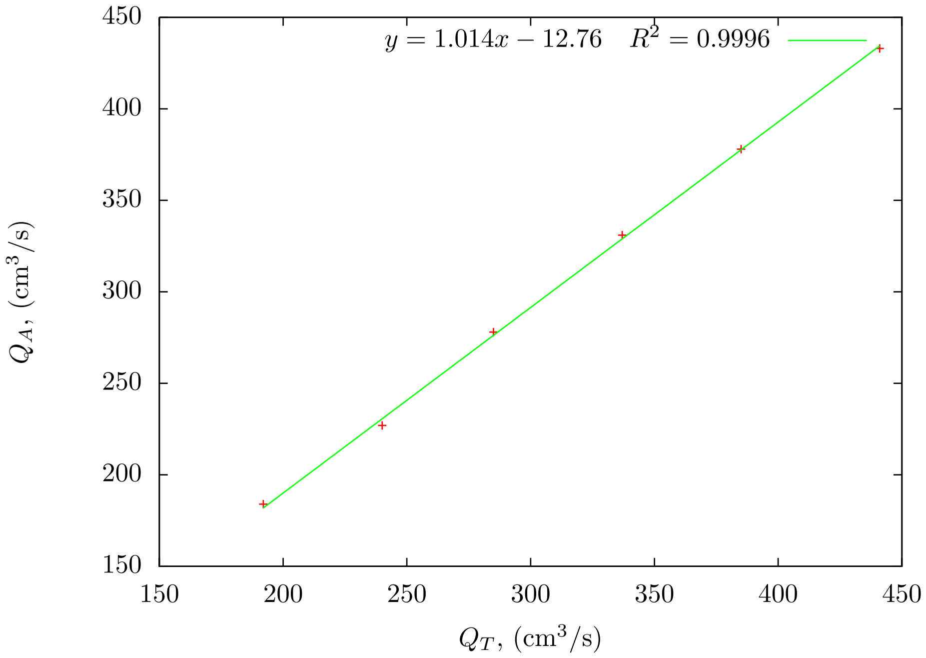

Figure 1: Q_A versus Q_T for the venturimeter.

From figure 1, the following values can be determined.

$$
\begin{aligned}
n &= 6 \\
\alpha &= 0.05 \\
SS_e &= 16.50 \\
S_{xx} &= 42790 \\
t_{\alpha,n-2} &= 2.132
\end{aligned}
$$

Given the above data, the 90% confidence interval for the slope, s is 1.014 ± 0.021 and for the intercept, I is -12.76 ± 1.81. Removal of point 5 (240,227) gives the best improvement in the confidence intervals. The error for s reduces to 0.015 and the error for I reduces to 1.18.

5.3.2 Orifice plate meter

From figure 2, the following values can be determined.

$$
\begin{aligned}
n &= 6 \\
\alpha &= 0.05 \\
SS_e &= 2.872 \\
S_{xx} &= 11600 \\
t_{\alpha,n-2} &= 2.132
\end{aligned}
$$

Given the above data, the 90% confidence interval for the slope, s is 0.6160 ± 0.0053 and for the intercept, I is -9.829 ± 0.747. Removal of point 4 (465,278) gives the best improvement in the confidence intervals. The error for s reduces to 0.0038 and the error for I reduces to 0.532.

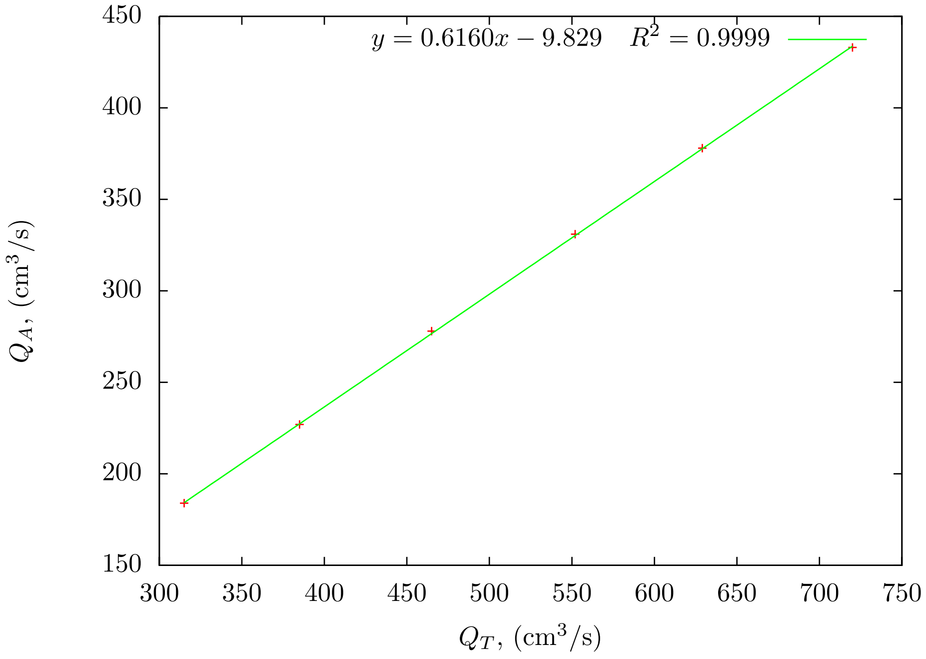

Figure 2: Q_A versus Q_T for the orifice plate meter.

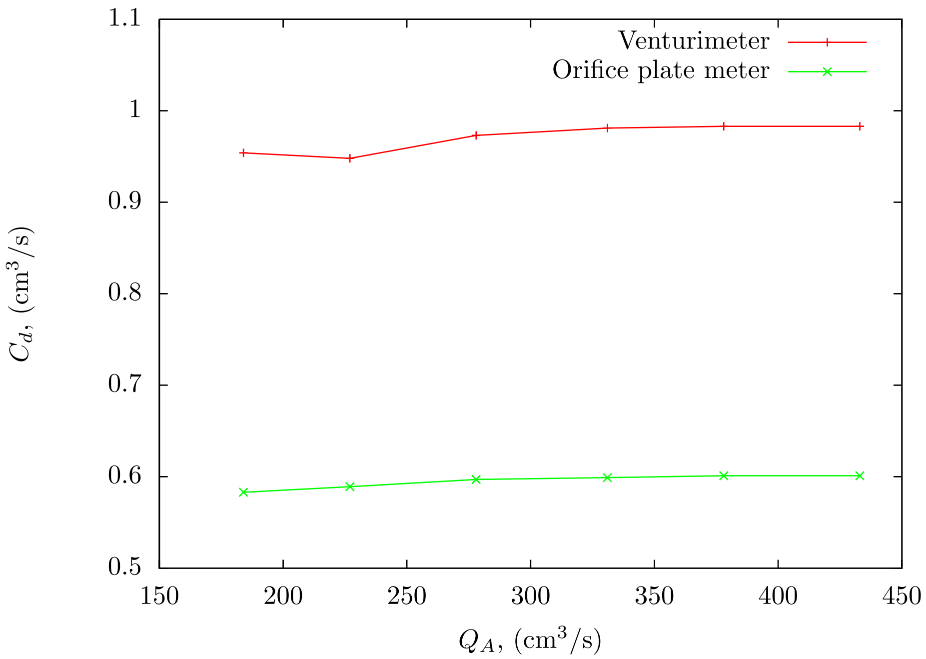

Figure 3: C_d versus Q_A for the venturimeter and the orifice plate meter.

5.3.3 Comments on C_d values

The differing values of C_d at different rates of flow, Q_A, are shown in figure 3. The values of C_d for both the venturimeter and the orifice plate meter seem to stabilise at higher rates of flow. Thus, the correct value of C_d should be 0.983 for the venturimeter and 0.601 for the orifice plate meter. This is attributed to the fact that the flow is more stable at higer values of Q_A.

5.4 Corelation between head losses and Q_A

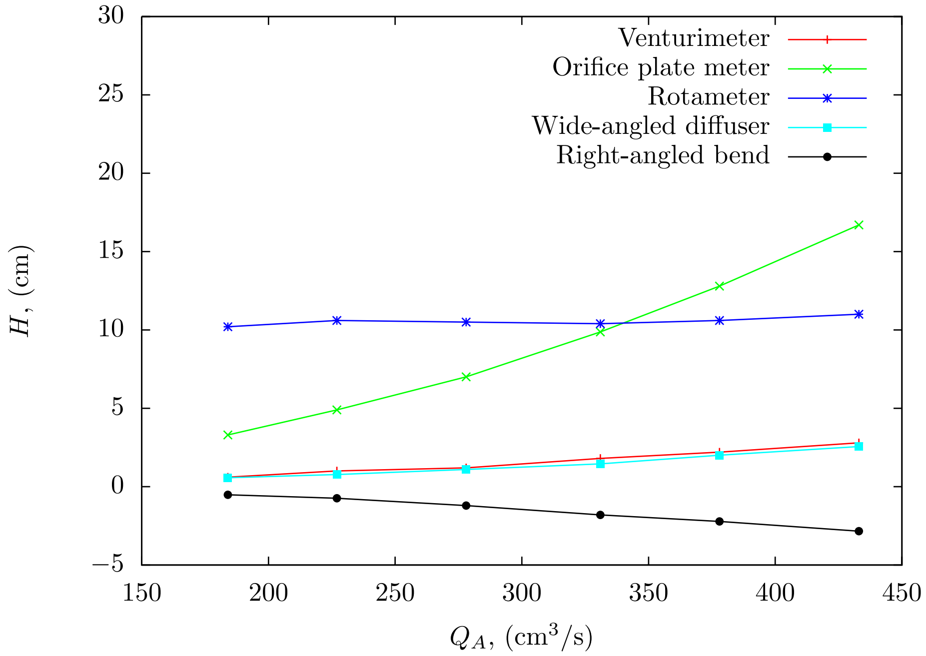

Figure 4: The various head losses versus Q_A.

Values of the head losses across the various flow meters are given in figure 4.

For the venturimeter, the head losses increase slightly with an increase in Q_A. The corelation appears to be fairly linear.

For the orifice plate meter, the head losses increase significantly with an increase in Q_A. The rate of this increase is in turn greater at higer values of Q_A. Thus, there appears to be a higher order corelation.

For the rotameter, the head losses are fairly stable at all values of Q_A. There appears to be no significant corelation.

For the wide-angled diffuser, the head losses increase slightly with an increase in Q_A. The corelation appears to be fairly linear.

For the right-angled bend, the head losses decrease slightly with an increase in Q_A. The corelation appears to be fairly linear.

5.5 Proportionality between Q and position of rotameter float

The discharge through the rotameter float,

$$Q = 2\pi R_f L\theta v, \tag{3}$$

where R_f is the float radius,
L is the height from the datum to the top of the float,
θ is the semi-angle of the tube taper, and
v is the mean velocity.

R_f is a property of the float and is thus constant. θ is a property of the rotameter tube and is thus constant. Now, the weight of the float in water is balanced by the force due to the water moving at a certain velocity v. The float will rise to the level at which this value of v is attained. This leaves L as the only independent variable in equation (3). Thus, Q is proportional to L.

6 Conclusion

The coefficient of discharge, C_d was determined to be 0.981 for the venturimeter and 0.601 fo the orifice plate meter. The linear regression line with a 90% confidence interval worked out to be $y = (1.014 \pm 0.021)x - (12.76 \pm 1.81)$ for the venturimeter and $y = (0.6160 \pm 0.0053)x - (9.829 \pm 0.747)$ for the orifice plate meter.

There appears to be a positive linear corelation between the head losses across the venturimeter and across the wide-angled diffuser with actual discharge, Q_A. There appears to be a negative linear corelation between the head losses across the right-angled bend with Q_A. There appears to be a positive higher order corelation between the head losses across the orifice plate meter with Q_A. Lastly, there appears to be no significant corelation between the head losses across the rotameter with Q_A.

Further analyses over a larger range of discharge values can be performed to ascertain more general corelations between the head losses and Q_A.