Sensitivity behaviour of the reliability index against initiation of supercritical flow in triangular channels

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1 Summary

Flow in open channels can be classified as subcritical or supercritical. For the same flow rate, subcritical flow is characterised by a higher water depth and lower water velocity than supercritical flow. In practice, it is often desirable to ensure that the flow remains within the subcritical range. This may be for reasons such as to control flow from downstream locations, to ensure smooth uniform flow under conditions of varying channel bottom slopes, or to keep velocity low so as to reduce erosion.

In this study, the sensitivity of the reliability index against initiation of supercritical flow was investigated in triangular sections. The FORM method of reliability analysis was adopted in the calculation of reliability indexes, and a verification study was conducted for a subset of the results using Monte Carlo simulations.

At the end of the study, the results and findings were compared to norms in civil engineering practice and back-compared to the performance function so as to make design recommedations.

2 Objectives and scope

The objectives of the study were to qualitatively investigate the sensitivity of the reliability index under variations in

- Channel bed slope, S
- Manning's roughness coefficient, n
- Flow rate, Q
- Section slope, m

so that design recommendations could be made.

To keep the mathematics simple, only triangular sections were analyzed in the current study. m was taken to be the ratio between the horizontal distance to the vertical distance of the side walls in the triangular sections.

Four different sections (A–D) were investigated in the study. The baseline mean values for the parameters of each section are given in Table 1, where $\bar{X}_b$ refers to the baseline mean value of parameter X. The value of $\bar{S}_b = 0.00100$ corresponded to a 1 m drop in elevation for every 1 km in channel length and the value of $\bar{n}_b = 0.0120$ corresponded to that of smooth concrete. For each channel section, $\bar{Q}_b$ was set to the value which resulted in a reliability index of $\beta = 2.5$ for that section.

Section	$\bar{m_b}$	$\bar{S_b}$	$\bar{n_b}$	$\bar{Q_b}$	β_b
A	0.75	0.00100	0.0120	250	2.5
B	1.0	0.00100	0.0120	67.5	2.5
C	1.5	0.00100	0.0120	20	2.5
D	2.0	0.00100	0.0120	12	2.5

Table 1: Baseline mean values of variables under investigation. $\bar{X}_b$ refers to the baseline mean value of parameter X.

3 Theory/method

The performance function for the investigation was given by

$$\begin{aligned} g(\mathbf{X}) &= y - y_c \\ &= 4^{1/8} Q^{3/8} n^{3/8} m^{-5/8} (1+m)^{1/8} S^{-3/16} - 2^{1/5} Q^{2/5} m^{-2/5} g^{-1/5} \end{aligned}$$

where y = flow depth; y_c = critical flow depth; and g = acceleration due to gravity.

The reliability indexes of the various scenarios was determined through FORM. In particular, the gradient projection method of FORM was used to calculate the reliability index values. Monte Carlo simulations (with 50,000 trials each) were used for verification of a subset of the results.

In this study, all the parameters under investigation (S, n, Q and m) were assumed to normally distributed with a constant coefficient of variation as given in Table 2. g was taken to be constant at 9.81 m/s. Only the mean values of the variables under investigation were varied. The range of variation in the mean values was chosen to roughly cover a reliability index range of $1 \leq \beta \leq 4$ for each investigation.

	m	S	n	Q
COV	0.01	0.05	0.1	0.2

Table 2: Coefficient of variation of variables under investigation.

4 Results

4.1 Verification study

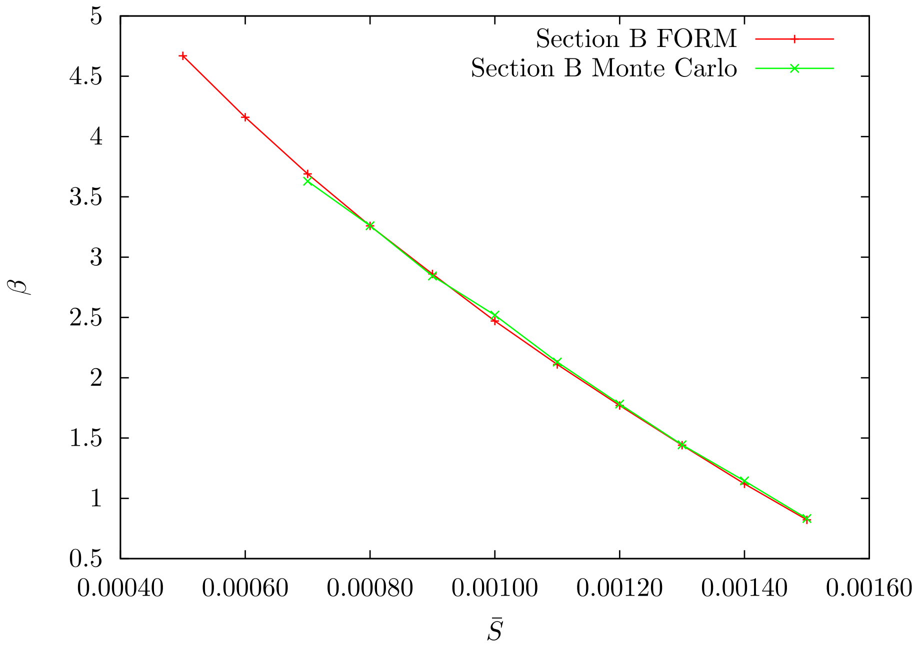

Figure 1: Plot of verification study on results from variations in channel bed slope for Section B. There was good agreement between results from FORM and Monte Carlo simulations.

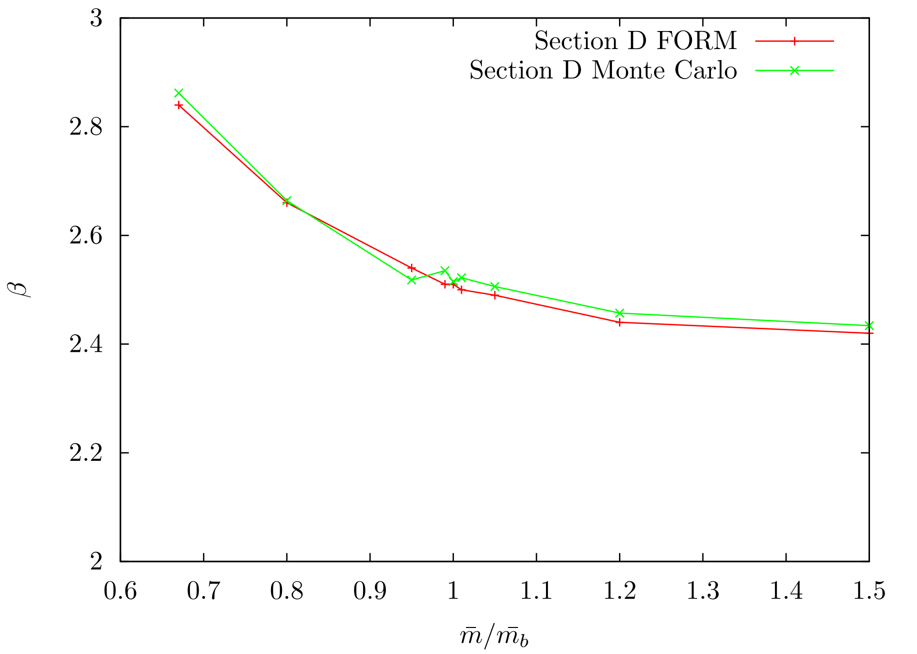

Figure 2: Plot of verification study on results from variations in section slope for Section D. There was good agreement between results from FORM and Monte Carlo simulations.

4.2 Sensitivity of reliability index to variations in channel bed slope

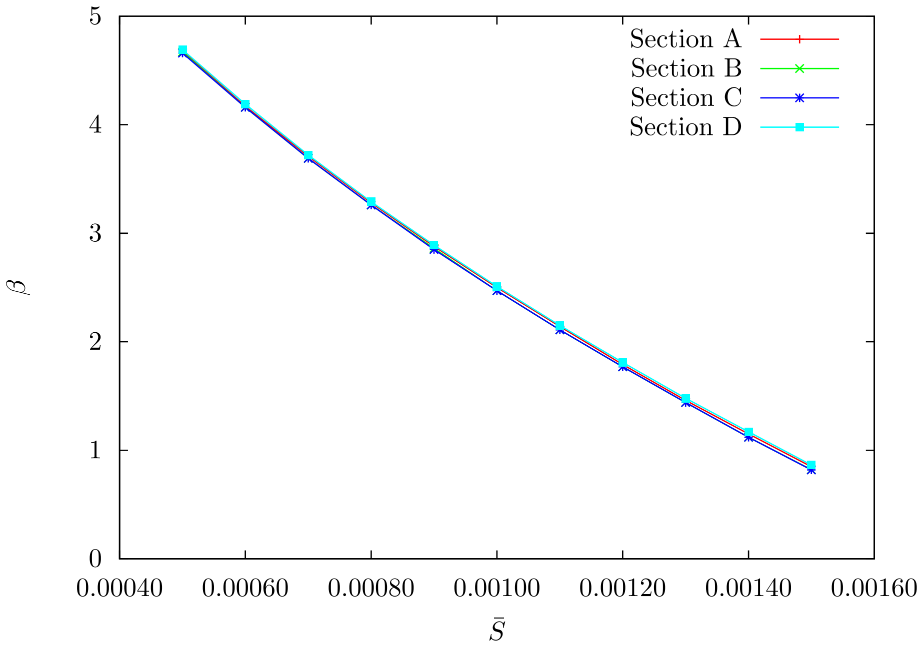

Figure 3: Plot of reliability index versus channel bed slope for all sections. All sections showed very similar responses.

4.3 Sensitivity of reliability index to variations in Manning's roughness coefficient

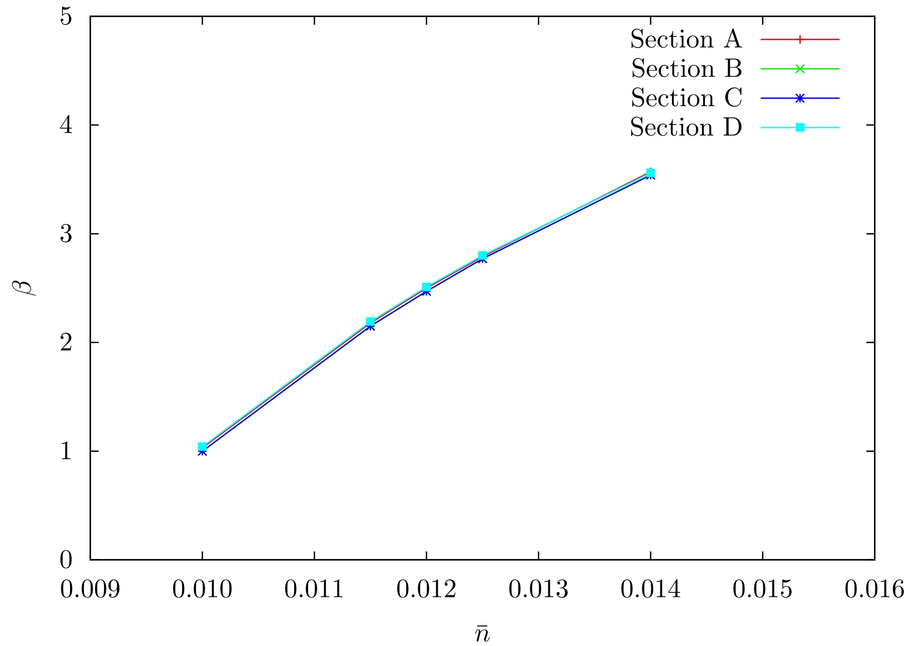

Figure 4: Plot of reliability index versus Manning's roughness coefficient for all sections. All sections showed very similar responses.

4.4 Sensitivity of reliability index to variations in flow rate

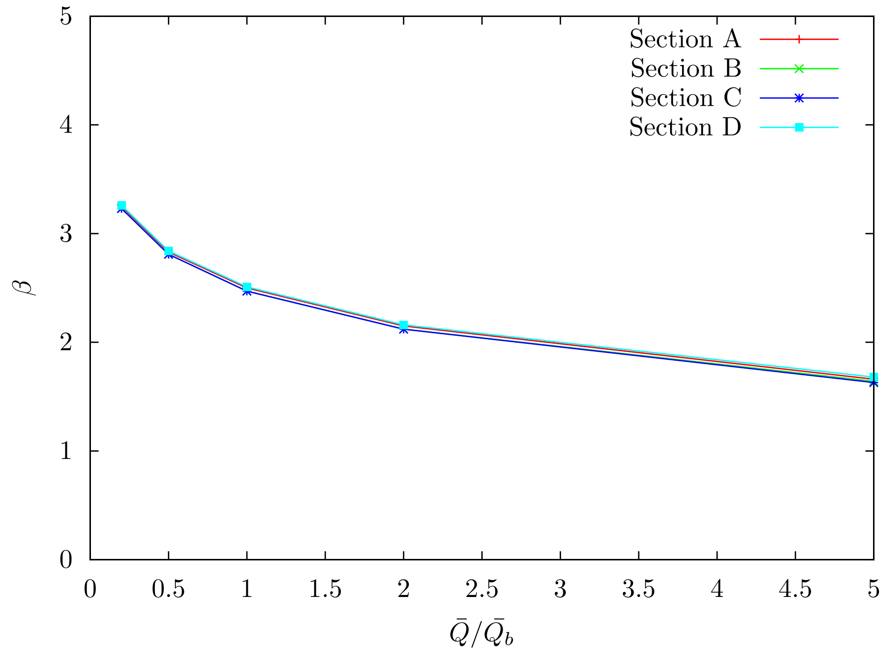

Figure 5: Plot of reliability index versus flow rate for all sections. All sections showed very similar responses.

4.5 Sensitivity of reliability index to variations in section slope

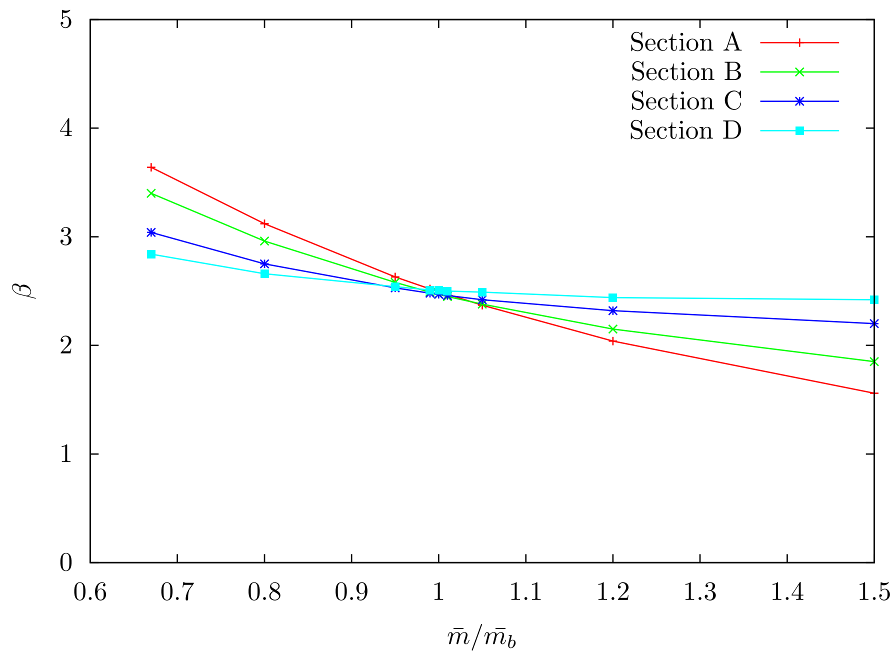

Figure 6: Plot of reliability index versus the modified section slope for all sections.

5 Discussion

5.1 Verification study

Results of the verification study are given in Figure 1 and Figure 2.

Verification was performed on results from varying $\bar{S}$ in section B and on results from varying $\bar{m}$ in section D. These two cases were semi-randomly chosen, while ensuring that different sections under variation of different parameters were represented. The sensitivity behaviour of the reliability index in these two cases was also markedly different.

Both verifications showed a good agreement between reliability index values from FORM and from Monte Carlo simulations with 50 000 trials.

5.2 Sensitivity of reliability index to variations in $\bar{S}$

Results from variations in $\bar{S}$ are given in Figure 3.

Section slope did not affect the sensitivity behaviour of the reliability index to variations in $\bar{S}$. The sentitivity of the reliability index to variations in $\bar{S}$ was slightly greater at higher β values.

5.3 Sensitivity of reliability index to variations in $\bar{n}$

Results from variations in $\bar{n}$ are given in Figure 4.

Section slope did not affect the sensitivity behaviour of the reliability index to variations in $\bar{n}$. The sensitivity of the reliability index was slightly greater at lower β values.

5.4 Sensitivity of reliability index to variations in $\bar{Q}$

Results from variations in $\bar{Q}$ are given in Figure 5.

Section slope did not affect the sensitivity behaviour of the reliability index to variations in $\bar{Q}$. The sensitivity of the reliability index was significantly greater at higher β values.

5.5 Sensitivity of reliability index to variations in $\bar{m}$

Results from variations in section slope are given in Figure 6.

Steeper section slopes (lower m) resulted in a significantly greater sensitivity of the reliability index to variations in $\bar{m}$. The sensitivity of the reliability index was also significantly greater at higher β values.

5.6 General observations

Generally, the sensitivity of the reliability index was seen to increase at higher β values. This is significant for civil engineering design, which usually requires high levels of safety, corresponding to high β values. Only for variations in $\bar{n}$ did the sensititvity of the reliability index decrease (but only slightly) at higher β values.

For most of the parameters investigated, the sensitivity behaviour of the reliability index did not differ among the different sections considered. Only under variations in $\bar{m}$ itself was there a difference among the different sections considered. Checking back with the performance function, this was attributed to the $(1+m)^{1/8}$ term in the expression for y.

The increasing sensitivity of the reliability index to variations in $\bar{Q}$ and $\bar{m}$ at higher reliability index was attributed to Q and m being present in both terms (y and y_c) of the performance function.

6 Conclusion

Sensitivity of the reliability index against initiation of supercritical flow in triangular channels generally increased at higher β values. This is significant for civil engineering design which usually requires high β values. The sensitivity behaviour was generally not altered by the section slope of the triangular channel. Results from FORM were found to agree well with results from Monte Carlo simulations.