prompt

I am teaching a course in control theory. This is a senior undergrad/lower-level grad course.

I want to autogenerate questions on the topic I am teaching.

The topic is: "State feedback control".

Learning outcome: Given the control problem context in natural language, students should be able to formulate the state space equation, and design a state feedback controller using the LQR method.

The question should have the following parts:

1. A natural language description of a control problem to control the position of the rotor of a DC motor.
2. physics equations governing the DC motor.
3. explicit requirements for the control problem.
4. A list of specific outputs which will be graded
5. A rubic to grade the problem

Control Problem Description

You are tasked with designing a control system for a DC motor to precisely control the position of its rotor. The DC motor is part of a larger system that requires accurate positioning for correct operation. Rapid and precise adjustments to the rotor position are crucial for the system's performance. Your goal is to develop a state feedback controller that meets the specified performance criteria.

Physics Equations Governing the DC Motor

The dynamics of the DC motor can be represented by the following equations:

1. **Electrical Equation**: $V = L\frac{di}{dt} + Ri + K_e\omega$
   - V is the input voltage.
   - L is the inductance.
   - R is the resistance.
   - i is the armature current.
   - K_e is the electromotive force constant.
   - ω is the angular velocity of the rotor.
2. **Mechanical Equation**: $J\frac{d\omega}{dt} = K_t i - b\omega - T_l$
   - J is the moment of inertia of the rotor.
   - K_t is the motor torque constant.
   - b is the viscous friction coefficient.
   - T_l is the load torque.

Control Problem Requirements

1. **State Space Formulation**: Formulate the state space representation of the DC motor using the rotor position and velocity as state variables.

2. **Controller Design**: Design a state feedback controller using the Linear Quadratic Regulator (LQR) method to meet the following requirements:

   - Minimize the settling time for position changes.
   - Minimize the overshoot in the rotor's position.
   - Ensure robustness to variations in load torque T_l.

Specific Outputs for Grading

1. **State Space Model**: A clear and correct formulation of the state space model of the DC motor.

2. **LQR Design**: The design process of the LQR controller, including the selection of weighting matrices Q and R.

3. **Controller Performance**: Analysis and simulation results demonstrating the controller's performance relative to the specified requirements. Include plots of the rotor's position over time for step changes in the desired position.

Rubric for Grading

Criteria	**Excellent (5)**	**Good (4)**	**Satisfactory (3)**	**Needs Improvement (1-2)**	**Unacceptable (0)**
State Space Model	Accurately formulated with clear, detailed explanations.	Accurately formulated with minor omissions or errors.	Correctly formulated but with insufficient explanation.	Incorrect formulation with major errors or omissions.	Completely incorrect or missing.
LQR Design	Correctly designed with optimal selection of Q and R, and thorough justification.	Correctly designed with minor issues in the selection of Q and R.	Correct design but with inadequate justification for choices.	Incorrect design with major flaws in the selection of Q and R.	Completely incorrect or missing.
Controller Performance	Performance exceeds requirements with excellent analysis and simulation results.	Meets and slightly exceeds requirements with good analysis.	Meets basic requirements with satisfactory analysis and results.	Fails to meet one or more requirements with insufficient analysis.	Performance is poor or analysis is completely missing.