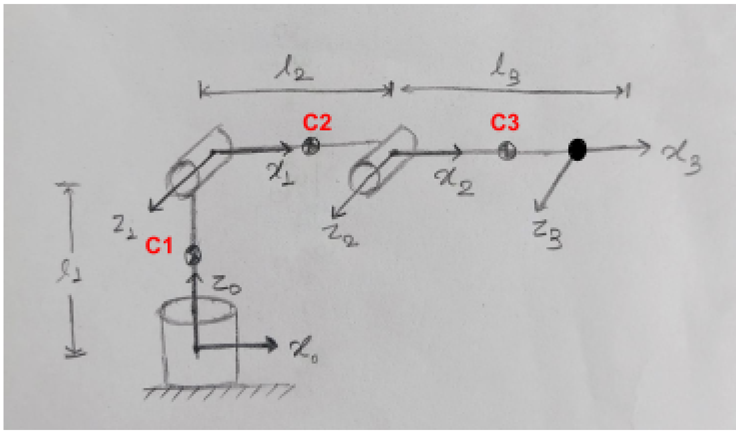
Q6.
Ans:

PUMA (RRR) manipulator



$DH =$

link	d	θ	a	α
1	l_1	q_1	0	90
2	0	q_2	l_2	0
3	0	q_3	l_3	0

For link 1:

$$H_0^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J_{V_{c1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad J_{w_{c1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

For link 2 :

$$H_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2.c_2 \\ s_2 & c_2 & 0 & l_2.s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_0^2 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & l_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & l_2s_1c_2 \\ s_2 & c_2 & 0 & l_1 + l_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_{V_{c2}} = \begin{bmatrix} -\left(\frac{l_2}{2}\right)s_1c_2 & -\left(\frac{l_2}{2}\right)c_1s_2 & 0 \\ \left(\frac{l_2}{2}\right)c_1c_2 & -\left(\frac{l_2}{2}\right)s_1s_2 & 0 \\ 0 & \left(\frac{l_2}{2}\right)c_2 & 0 \end{bmatrix} \quad J_{w_{c2}} = \begin{bmatrix} 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

For link 3 :

$$H_2^3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3c_3 \\ s_3 & c_3 & 0 & l_3c_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_0^3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & l_3c_1c_{23} + l_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & l_3s_1c_{23} + l_2s_1c_2 \\ s_{23} & c_{23} & 0 & l_3s_{23} + l_2s_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_{V_{c3}} = \begin{bmatrix} -\left[\left(\frac{l_3}{2}\right)s_1c_{23} + l_2s_1c_2\right] & -\left[\left(\frac{l_3}{2}\right)c_1s_{23} + l_2c_1s_2\right] & -\left[\left(\frac{l_3}{2}\right)c_1s_{23}\right] \\ \left[\left(\frac{l_3}{2}\right)c_1c_{23} + l_2c_1c_2\right] & -\left[\left(\frac{l_3}{2}\right)s_1s_{23} + l_2s_1s_2\right] & -\left[\left(\frac{l_3}{2}\right)s_1s_{23}\right] \\ 0 & \left[\left(\frac{l_3}{2}\right)c_{23} + l_2c_2\right] & \left[\left(\frac{l_3}{2}\right)c_{23}\right] \end{bmatrix}$$

$$J_{w_{c3}} = \begin{bmatrix} 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

The Inertia matrix is given as:

$$D(q) = \sum_{i=1}^{3}\left(m_iJ_{V_{ci}}^T . J_{V_{ci}} + I_iJ_{w_{ci}}^T . J_{w_{ci}}\right)$$

$$= \begin{bmatrix} \left[\frac{m_2l_2^2c_2}{4} + \frac{m_3(l_3c_{23}+l_2c_2)^2}{2}\right] & 0 & 0 \\ 0 & \left[\frac{m_2l_2^2s_2^2}{4} + m_3\left(\frac{l_3^2}{4} + l_2^2 + l_2l_3c_3\right)\right] & \left[m_3\left(\frac{l_3^2}{4} + \frac{l_2l_3c_3}{2}\right)\right] \\ 0 & \left[m_3\left(\frac{l_3^2}{4} + \frac{l_2l_3c_3}{2}\right)\right] & \left[\frac{m_3l_3^2}{4}\right] \end{bmatrix}$$

$$+ \begin{bmatrix} I_1 + I_2 + I_3 & 0 & 0 \\ 0 & I_2 + I_3 & I_3 \\ 0 & I_3 & I_3 \end{bmatrix}$$

The potential terms is given as:

$$V(q) = \frac{m_1gl_1}{2} + m_2g\left(l_1 + \frac{l_2s_2}{2}\right) + m_3g\left(l_1 + l_2s_2 + \frac{l_3s_3}{2}\right)$$