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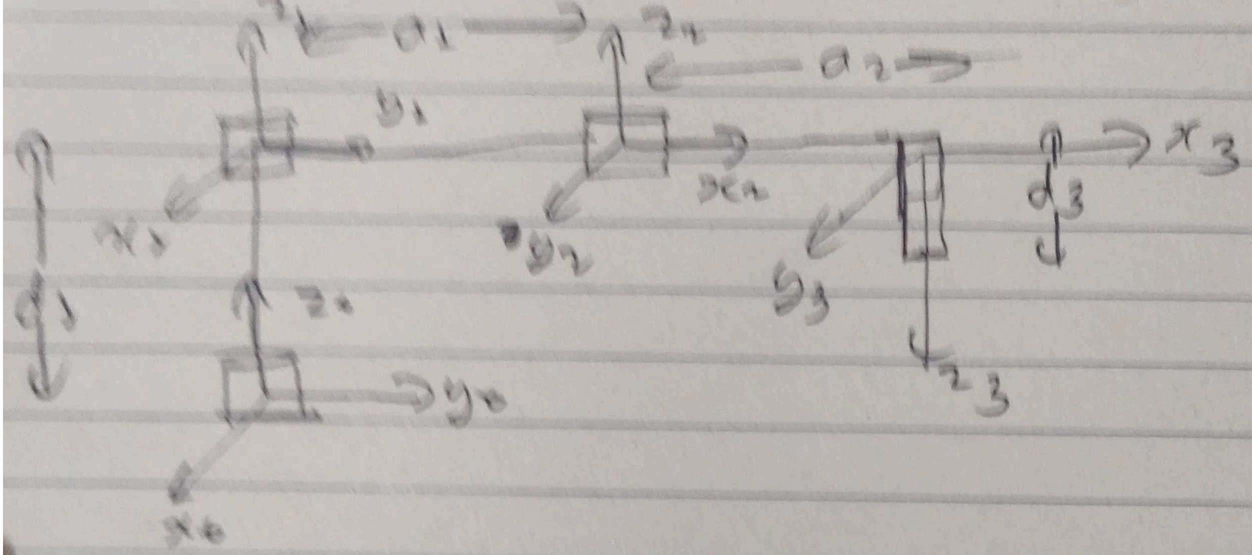
Assignment - 2

2] SCARA configuration (RRP)



Joint variables :-

Joint (i)	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	θ_1
2	π	a_2	0	θ_2
3	0	0	d_3	0

Homogeneous transformation :-

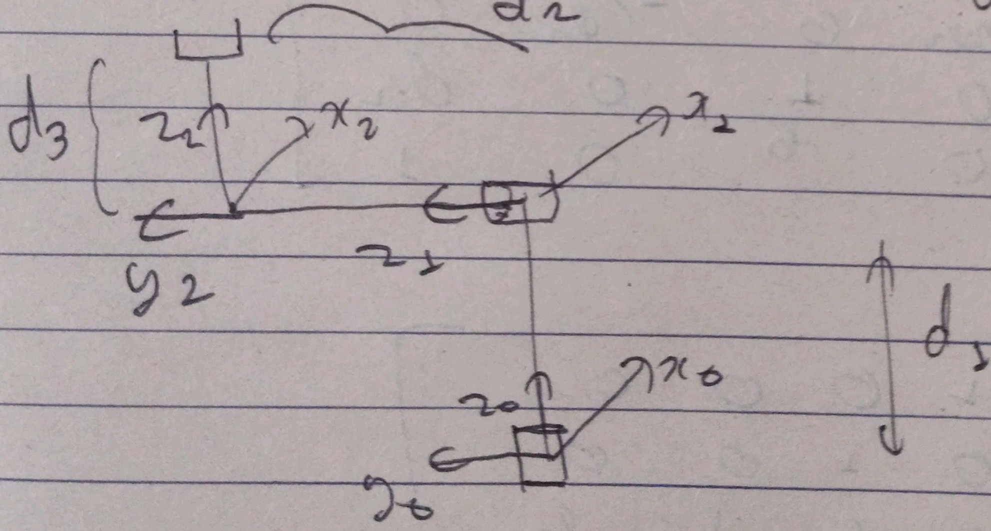
$$A_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A_3^0 = A_1^0 \; A_2^1 \; A_3^2}$$

$$A_3^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1 C\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & S\theta_2 & 0 & a_2 C\theta_2 \\ \sin\theta_2 & -C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans

4] Stanford style RRP configuration



Joint parameters 2i

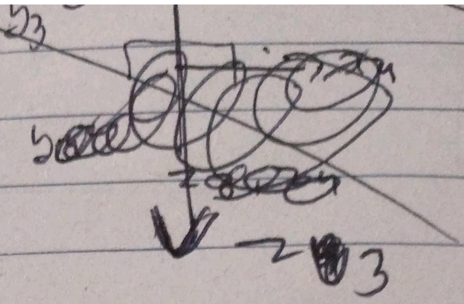
Joint	θ_i	d_i	a_i	α_i
1	θ_1	d_1	0	-90
2	θ_2	d_2	0	90
3	0	d_3	0	0

$$T_1^0 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

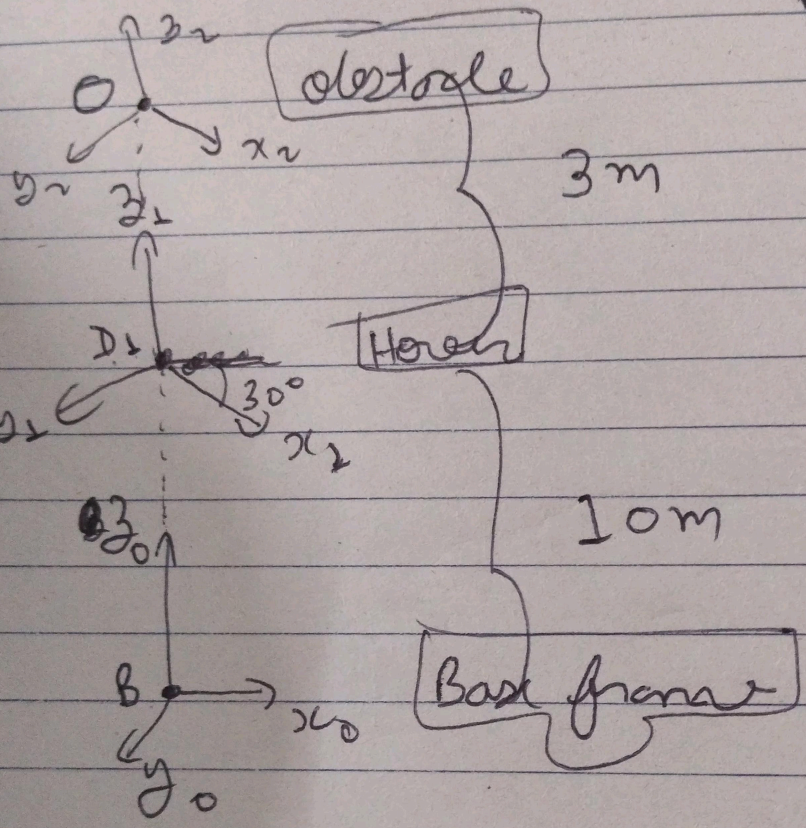
$$T_2^1 = \begin{bmatrix} C\theta_2 & 0 & S\theta_2 & 0 \\ \sin\theta_2 & 0 & -C\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_3^0 = T_1^0 \, T_2^1 \, T_3^2}$$



5]



Using Homogeneous Transformations ⇓

From B to D_1, translation = 10m (z dirⁿ)
Rotation = 30° (about x axis)

$\therefore T_B^{D_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

now from D_1 to O
$\Downarrow$

$T_{D_1}^{O} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

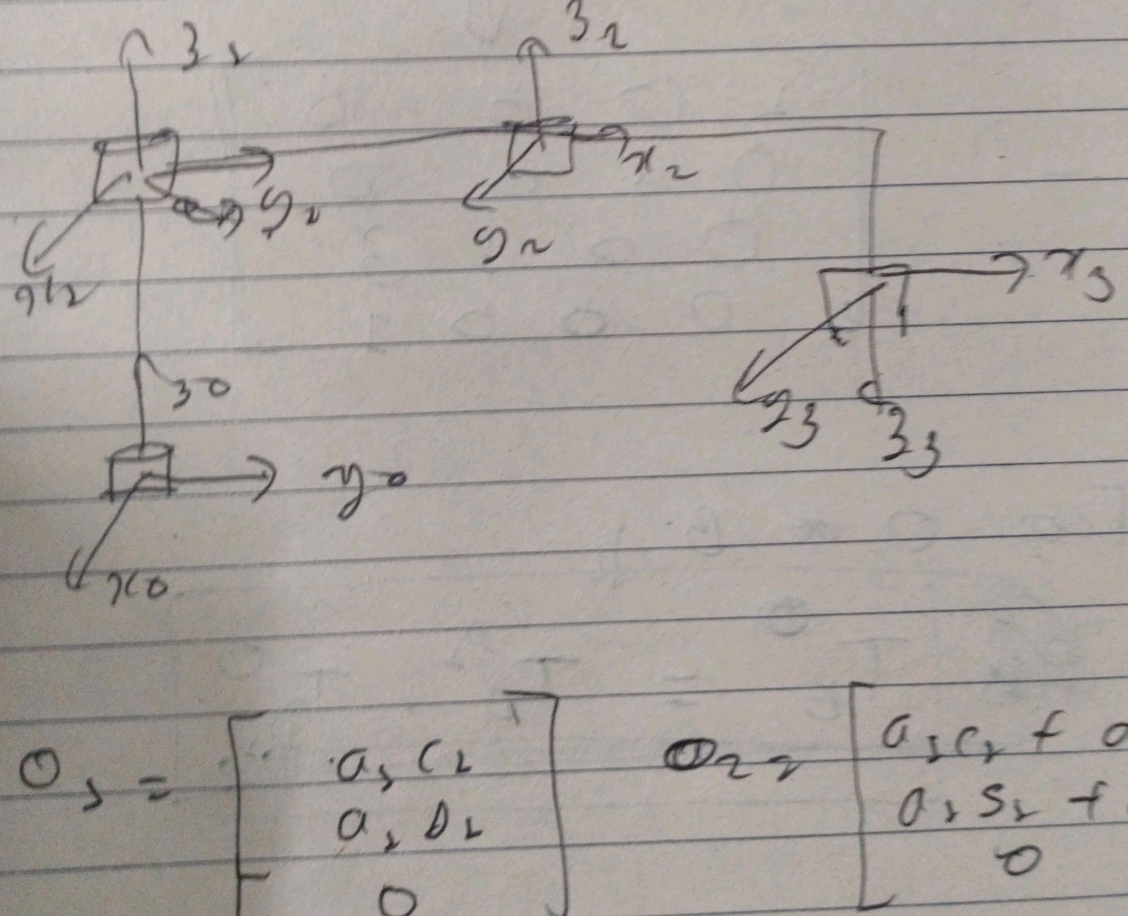
$\therefore$ From O to B $\Downarrow$

$$\boxed{T_B^{O} = T_B^{D_1} \cdot T_{D_1}^{O}}$$

7] SCARA configuration

Jacobian for above is of the form is

$$J = \begin{bmatrix} z_0 \times (o_4 - o_0) & z_1 \times (o_4 - o_1) & z_2 & 0 \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$



$$O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_4 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

$z_0 = z_1 = z_2 = \hat{k}$

$z_3 = -\hat{k}$

$\therefore$ Jacobian Matrix is

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

1] To prove :→ $R S(\alpha) R^T = S(R\alpha)$

$LHS = R S(\alpha) R^T$

WKT : $S(\alpha) p = \alpha \times p$

$$\therefore \; R S(\alpha) R^T b = R(\alpha \times R^T b)$$
$$= (R\alpha) \times (R R^T) b$$
$$= (R\alpha) \times b$$
$$= \underline{S(R\alpha)\, b}$$

$$\boxed{\text{Hence, } R S(\alpha) R^T = S(R\alpha)}$$