ASSIGNMENT 2

1) S.T $RS(a)R^T = S(Ra)$

Let a, b be vectors of R^3

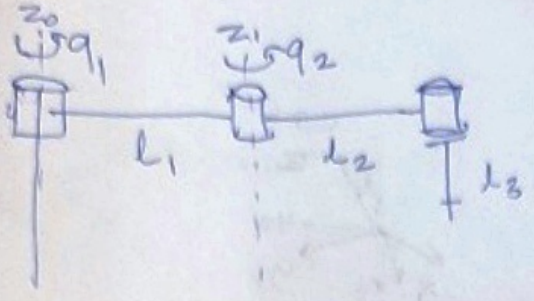
$$RS(a)R^T b = R(a \times R^T b) \quad [S(a)p = a \times p]$$

$$= (Ra) \times (RR^T b) \quad [R(a \times b) = Ra \times Rb]$$

$$= (Ra) \times b$$

$$= S(Ra)b$$

2)



- $R_0^1 = R_{z,q_1}$
- $R_1^2 = R_{z,q_2}$, $d_1^2 = [l_1, 0, 0]^T$
- $R_2^3 = R_{z,0}$, $d_2^3 = [l_2, 0, d_3]^T$

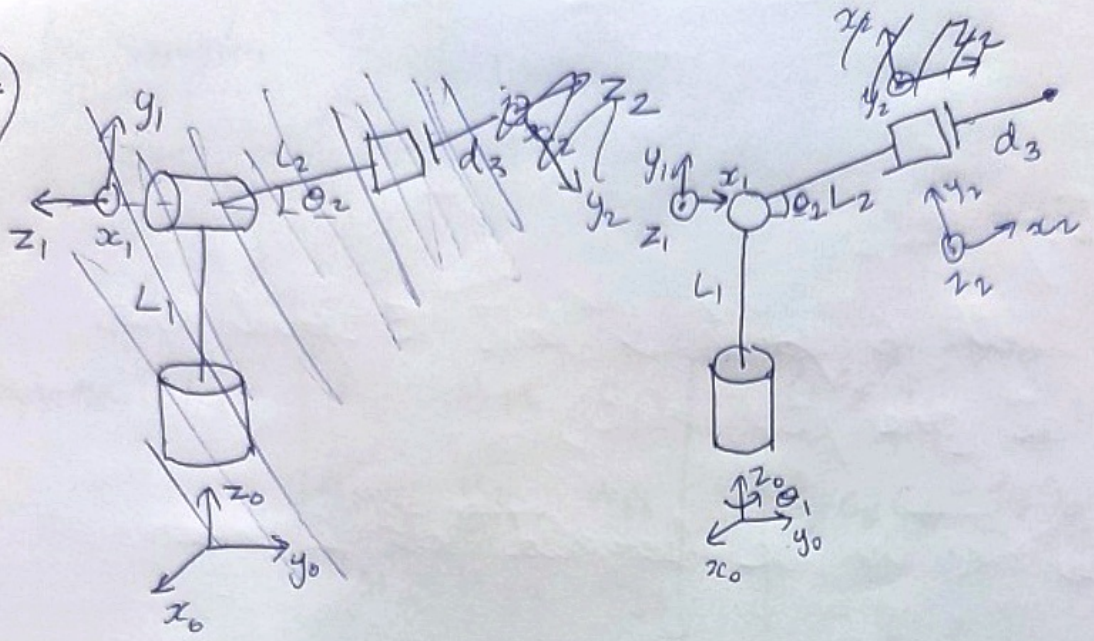
$$H_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & 0 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

4)



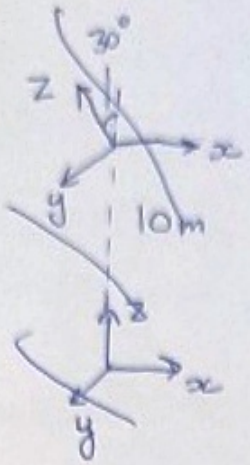
$$T_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

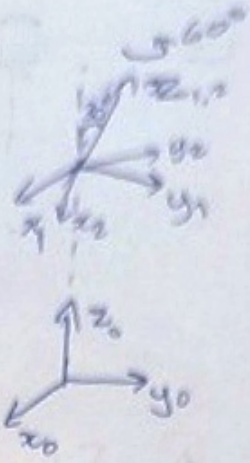
$$T_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & L_2 \\ S_{\theta_2} & C_{\theta_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = [d_3 \; 0 \; 0 \; 1]^T$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = T_1 \, T_2 \begin{bmatrix} P \\ 1 \end{bmatrix}$$

5)





$$P_0 = H_{z,0,10} H_{x,30} H_{z,60} P_1$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{30} & -S_{30} & 0 \\ 0 & S_{30} & C_{30} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{60} & -S_{60} & 0 & 0 \\ S_{60} & C_{60} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{30} & -S_{30} & 0 \\ 0 & S_{30} & C_{30} & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{60} & -S_{60} & 0 & 0 \\ S_{60} & C_{60} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{60} & -S_{60} & 0 & 0 \\ C_{30}S_{60} & C_{30}C_{60} & -S_{30} & 0 \\ S_{30}S_{60} & S_{30}C_{60} & C_{30} & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3S_{30} \\ 3C_{30}+10 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ \frac{3\sqrt{3}}{2}+10 \\ 1 \end{bmatrix}$$

$$P_0 = \frac{-3}{2}\hat{j} + \left(\frac{3\sqrt{3}}{2}+10\right)\hat{k}$$