Q:-1 Ans Singularity.

A Singularity in a robotics manipulator is a Configration in which the robot losses one or more DOF. This means that robot can not move its end effector in certain directions, even if all of its joints are moving.

Two main types of Singularities

Work space interior Singularity

These Singularities occurs within the robot's Workspace, and are typically caused by two or more of the robots joints axes lining up with each other. for example :- a 6 DOF robot arm Can experience a Workspace interior singularity when its 3 wrist joints become Coplanar. This is known as Gimble lock.

Boundary Singularities.

These Singularities occur at the edges of the robot's Workspace, and are typically caused by the robot's end effector reaching the limits of its reach. For example, a Six-axis robot arm can experience a boundary Singularity when its end-effector reaches the edge of its workspace.

Decoupling of Singularity.

Decoupling of Singularity is a technique that can be used to reduce the impact of Singularities on the performance of a robotics manipulator. This is done by reconfiguring the robot's joints so that it can avoid the singularity. For eg. a 6-axis robot arm can avoid a gimble lock Singularity by rotating its wrist joint so that they are no Longer Coplanar.

Eg. of Singularities and Singular Configration.

Gimble lock - A gimble lock is a Singularity that occurs when the 3-axis of rotation of gimble become aligned. This prevent the gimble from rotating about any axis that is normal to the axes of the three gimbles. Gimbal locks are Common problem in robotic arm manipulators, and can caused by the robot's end-effector reaching certain positions.

Wrist Singularity-

It occurs when the 3-wrist joints of a 6-axis robot arm become coplanar. This prevents the robot from rotating its end effector about any axis that is normal to the axes of the 3-wrist joints.

Shoulder Singularity-

It occurs when 2-shoulder joints ~~axis~~ of a 6-axis robot arm become aligned. This prevents the robot from moving its end effector in certain directions within the Robot's workspace.

Boundary Singularity-

It occurs when the end effector of a robotic manipulator reaches the edge of the robot's workspace. This prevents the robot from moving its end effector any further in that dir^n.

How do we find Singular Configration

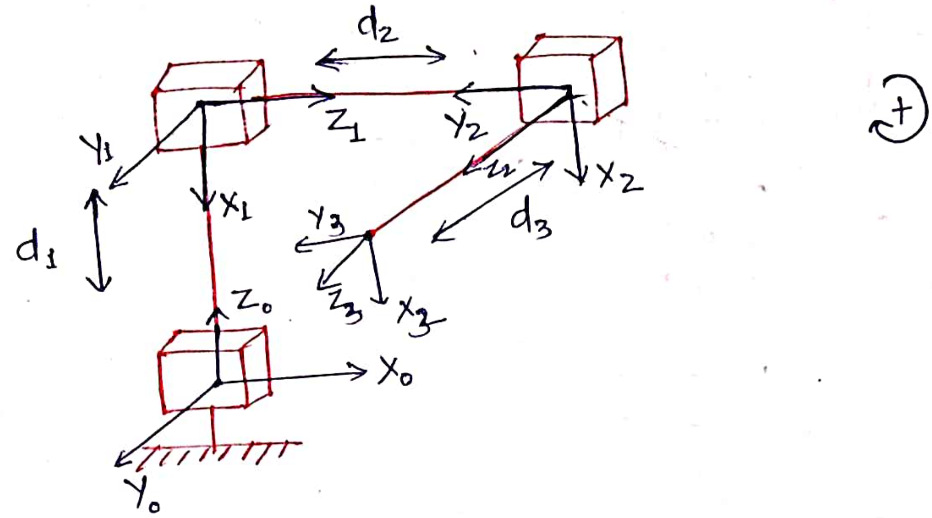
These configrations are typically indentified by analyzing the robot's Jacobian matrix, which describes the relationship b/w the robot's joint velocities and velocities of its end effector. A singularity occurs when the jacobian matrix becomes singular, indicating a loss of control over some of the robot's motion parameters.

Can you detect if a particular Configuration is close to a Singular Configration using the manipulator jacobian?

Yes, we can use the manipulator jacobian Matrix to detect if a particular Configration is close to a Singular Conf. When robot is near a singularity, the,

$$\boxed{|J| = 0}$$

Q: 5



D-H Parameter

d	θ	a	α
d_1^*	0	0	+90
d_2^*	0	0	-90
d_3^*	0	0	0

$$H_0^3 = H_0^1 H_1^2 H_2^3$$

$$H_0^1 = Tran_{z,d} \; Rot_{z,\theta} \; Tran_{x,a} \; Rot_{x,\alpha}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c90 & -s90 & 0 \\ 0 & s90 & c90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = Tran_{z,d} \; Rot_{z,\theta} \; Tran_{x,a} \; Rot_{x,\alpha}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(-90) & -S(-90) & 0 \\ 0 & S(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = Tran_{z,d} \; Rot_{z,\theta} \; Tran_{x,a} \; Rot_{x,\alpha}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(0) & -S(0) & 0 \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

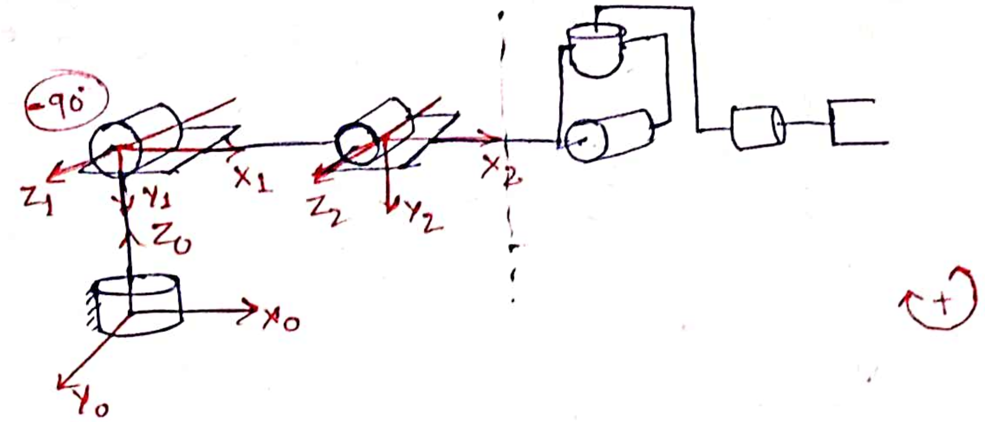
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_2 \\ 0 & 0 & 1 & d_3+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q :- 6



d	θ	a	α
d_1	θ_1^*	0	-90
0	θ_2^*	l_2	0
0	θ_3^*	l_3	0

{Three Link Articulated Manipulator}

D-H parameter.

$$H_0^3 = H_0^1 H_1^2 H_2^3$$

$$H_0^1 = \text{Trans}_{z,d}\ \text{Rot}_{z,\theta}\ \text{Trans}_{x,a}\ \text{Rot}_{x,\alpha}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & l_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

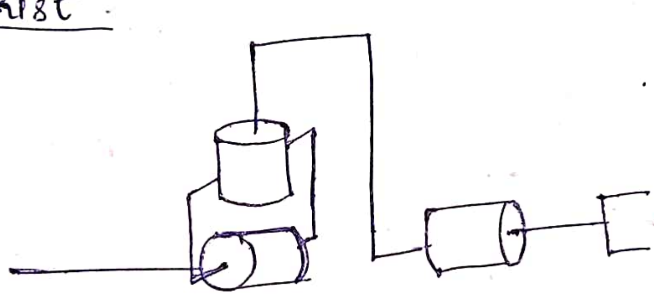
$$H_2^3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & l_2C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & l_2S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & S\theta_2 & 0 & l_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & l_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & l_2 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & l_2 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} C\theta_1 C\theta_2 & C\theta_1 S\theta_2 & -S\theta_1 & C\theta_1 l_2 C\theta_2 \\ S\theta_1 C\theta_2 & S\theta_1 S\theta_2 & C\theta_1 & l_2 S\theta_1 C\theta_2 \\ -S\theta_2 & -C\theta_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & l_2 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & l_2 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1 C\theta_2 C\theta_3 + C\theta_1 S\theta_2 S\theta_3 & -S\theta_3 C\theta_1 C\theta_2 + C\theta_1 S\theta_2 C\theta_3 & -S\theta_1 & l_2 C\theta_3 C\theta_1 C\theta_2 + l_2 C\theta_1 S\theta_2 S\theta_3 + C\theta_1 l_2 C\theta_2 \\ S\theta_1 C\theta_2 C\theta_3 + S\theta_1 S\theta_2 S\theta_3 & -S\theta_1 S\theta_3 C\theta_2 + S\theta_1 S\theta_2 C\theta_3 & C\theta_1 & l_2 C\theta_3 S\theta_1 C\theta_2 + l_2 S\theta_3 S\theta_1 S\theta_2 + l_2 S\theta_1 C\theta_2 \\ -S\theta_2 C\theta_3 - C\theta_2 S\theta_3 & + S\theta_2 S\theta_3 - C\theta_2 + C\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical Wrist.



D-H parameter

d	θ	a	α
0	θ_4^*	0	$-\pi/2$
0	θ_5^*	0	$\pi/2$
l_6	θ_6^*	0	0

After applying the Homogeneous Transformation,

$$H = Tran_{z,d}\ Rot_{z,\theta}\ Tran_{x,a}\ Rot_{x,\alpha}$$

$$H_3^4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & -C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5^6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^6 = H_3^4 H_4^5 H_5^6 \Rightarrow \begin{bmatrix} R_3^6 & d_3^6 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_4 C\theta_5 C\theta_6 - S_4 S_6 & -C_4 C_5 S_6 - S_4 C_6 & C_4 S_5 & C_4 S_5 l_6 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & S_4 S_5 & S_4 S_5 l_6 \\ -S_5 C_6 & S_5 S_6 & C_5 & C_5 l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{H_0^6 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6}$$

Final Homogeneous Transformation.

Q :-7 Ans.

Direct drive 2R Manipulator.

Configration :- In direct drive 2R manipulator, each of the two revolute joints is directly actuated by a motor. This means that there is a motor attached to each joint. providing direct control over their motion.

Advantages :- (1) precise control of joint angles.
(ii) Simple in design.
(iii) Low backlash

Disadvantages :- (i) Multiple motor can increase the cost of the manipulator.
(ii) The additional motor may add bulk to the manipulator.

Remotely Driven 2R Manipulator :

Configration :- In a remotely driven 2R Manipulator, one or both joints are actuated by motors located at a distance from the joints. Actuation is typically achieve through mechanisms like belts, cables or gears.

Advantages :- (i) Compact design
(ii) Reduced motor count.
(iii) Versatility

Disadvantages :- (i) Increase Complexity
(ii) Potential for backlash
(iii) Remote components may require more maintenance

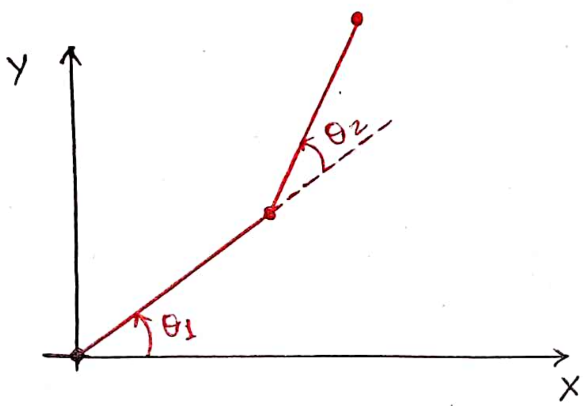
5- Bar Parallelogram Arrangement

Configration :- In a 5- bar parallelogram arrangement, two of the robot's links are fixed in the parallel to maintain a const. Orientation of the end effector. The remaining two links are connected with two revolute joints.

Advantages :- (i) Rigidity
(ii) Simplicity
(iii) Backlash reduction

Disadvantages :- (i) Limited motion
(ii) Less versatility
(iii) Reduced flexibility

Q:- 8 Ans



[2R- Manipulator]

Deriving the Equation of Motion-

Compute position and velocities

$$\begin{bmatrix} P_{1,x} \\ P_{1,y} \end{bmatrix} = \begin{bmatrix} l_1 \cos(\theta_1) \\ l_1 \sin(\theta_1) \end{bmatrix}$$ ⇒ Position of Link-1

and,

$$\begin{bmatrix} P_{2,x} \\ P_{2,y} \end{bmatrix} = \begin{bmatrix} l_1 \cos(\theta_1) + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$ ⇒ Position of Link-2

We next Compute Velocities above using.

$$v = \frac{dP}{dt} = \frac{\partial P}{\partial \theta_1} \cdot \frac{\partial \theta_1}{\partial t} + \frac{\partial P}{\partial \theta_2} \cdot \frac{\partial \theta_2}{\partial t}$$

$$= \frac{\partial P}{\partial \theta_1} \dot{\theta}_1 + \frac{\partial P}{\partial \theta_2} \cdot \dot{\theta}_2$$

$$\boxed{\begin{aligned} v_x &= -l_1 \sin(\theta_1) \cdot \dot{\theta}_1 - l_2 \sin(\theta_1+\theta_2) \cdot \dot{\theta}_2 \\ v_y &= l_1 \cos(\theta_1) \dot{\theta}_1 + l_2 \cos(\theta_1+\theta_2) \cdot \dot{\theta}_2 \end{aligned}}$$ ⇒ End effector velocity

Compute Kinetic energy and potential energies of the System.

$$\boxed{KE = \frac{1}{2} m_1 v_1^T v_1 + \frac{1}{2} m_2 v_2^T v_2} \Rightarrow \text{Kinetic Energy}$$

$$\boxed{PE = m_1 g P_{1,y} + m_2 g P_{2,y}}$$

Derive equ^ns of Motion.

We derive equations of Motions by first settingup a Lagrangian L as.

$$L = KE - PE$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau$$

Where. $q = [\theta_1, \theta_2]^T$ is the vector of angular position and velocities. and τ is the vector of torques applied by motors at the two joints. After grouping terms appropriately. the equ^ns of motion can be written as.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

$$\ddot{q} = D(q)^{-1}(\tau - C(q,\dot{q})\dot{q} - G(q))$$

We can rewrite the equ^n above as.

$$\ddot{q} = \alpha(q,\dot{q}) + \beta(q)\tau$$

where, $\alpha(q,\dot{q}) = D(q)^{-1}(-C(q,\dot{q})\dot{q} - G(q))$

and $\beta(q) = D(q)^{-1}$

This form of equ^n is very common in control of Many Nonlinear dynamic systems

Q:-9 Ans: There are two other Configrations of 2R. manipulator.

(1) Configration 1 :- The upper arm is horizontal and lower arm is verticle.

(2) Configration 2 :- The upper arm is verticle and lower arm is horizontal.

The Euler's langrange equns for a system with 2DOF are given by.

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_1}\right) - \frac{dL}{dq_1} = \tau_1$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_2}\right) - \frac{dL}{dq_2} = \tau_2$$

Configration (1)

The lagrange of the system is.

$$\boxed{L = \frac{1}{2}(m_1+m_2)\, l_2^2 \dot{q}_2^2 + m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 - m_2 g l_2 \cos(q_1+q_2)} \quad \text{--- (1)}$$

Where m_1, m_2 :- masses of the links

l_1, l_2 :- Length of the links

Configration (2)

The lagrange of the system is -

$$\boxed{L = \frac{1}{2}(m_1+m_2)\, l_1^2 \dot{q}_1^2 + m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 - m_2 g l_1 \cos(q_1+q_2)} \quad \text{--- (2)}$$

Substituting equ (1) & (2) into the Euler-Lagrangian Equn.

$$\boxed{(m_1+m_2) L_1^2 \ddot{q}_1 + m_2 l_1 l_2 \ddot{q}_2 - m_2 l_1 \sin(q_1+q_2)\dot{q}_1^2 - m_2 g l_1 \sin(q_1+q_2) = \tau_1}$$

$$\boxed{m_2 l_1 l_2 \ddot{q}_1 + m_2 l_2^2 \ddot{q}_2 = \tau_2}$$

Q:-10 Ans When we have D(q) and V(q), then the key steps for finding the Equations of Motions.

① Lagrangian of Mechanical System is defined as follows-

$$\boxed{L(q, \dot{q}) = T(q, \dot{q}) - V(q)}$$

Where $L \Rightarrow$ Lagrangian
q = vector of generalized Co-ordinate
$\dot{q}$ = ,, ,, ,, velocity
$T(q, \dot{q})$ = K.E.
$V(q)$ = P.E.

② Euler-Lagrange Equations are as follows-

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q}$$

Where, Q = Vector of generalized forces

③ Equation of Motion.

$$KE = (T) = \frac{1}{2} m l^2 \dot{q}^2$$

$$PE = V = -mgl\cos(q)$$

$$\text{Lagrangian} = L = \frac{1}{2} m l^2 \dot{q}^2 + mgl\cos(q)$$

Euler-Lagrangian equns.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q$$

$$\Rightarrow m l^2 \ddot{q} + mgl\sin(q) = Q$$