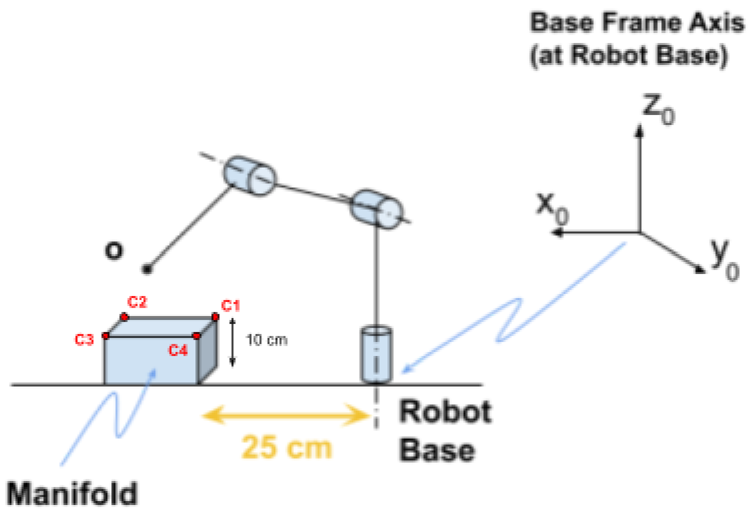
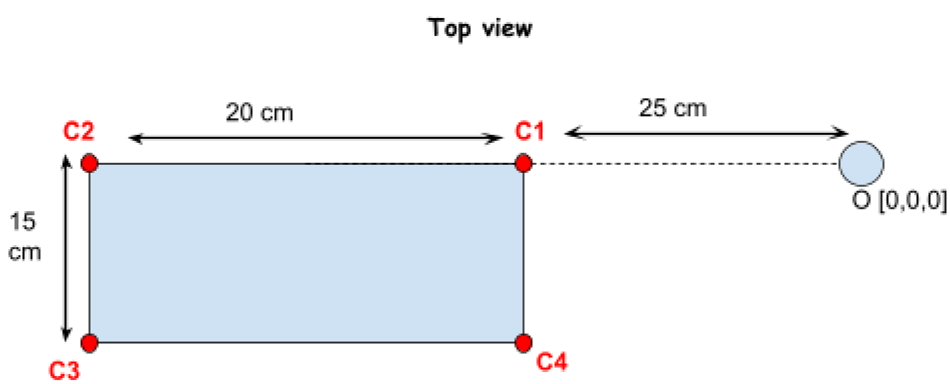
Q2.a]
Ans: PUMA





Corner points:

$$C_1 = [0.25, 0, 0.1]$$
$$C_2 = [0.45, 0, 0.1]$$
$$C_3 = [0.45, 0.15, 0.1]$$
$$C_4 = [0.25, 0.15, 0.1]$$

- ❖ <u>**Inverse kinematics for PUMA**</u>

Let the position of end effector is $P = [P_x,\ P_y,\ P_z]$

Let the angles at each joint are $[q_1, q_2, q_3]$

Let Link lengths are $[l_1, l_2, l_3]$

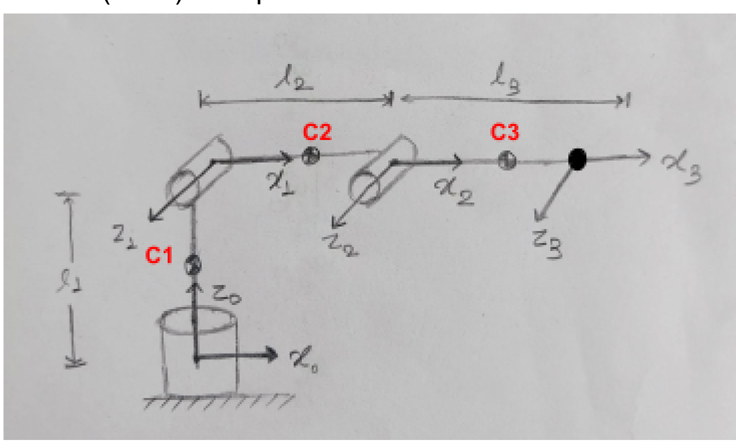
$$q_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$$

$$q_3 = \cos^{-1}\left(\frac{P_x^2 + P_y^2 + (P_z - l_1)^2 - l_2^2 - l_3^2}{2.l_2 \cdot l_3}\right)$$

$$q_2 = \tan^{-1}\left(\frac{P_z - l_1}{\sqrt[2]{P_x^2 + P_y^2}}\right) - \tan^{-1}\left(\frac{l_3 . \sin(q_2)}{l_2 + l_3 \cdot \cos(q_2)}\right)$$

❖ **Forward kinematics for PUMA**

PUMA (RRR) manipulator



$$DH = \begin{array}{|c|c|c|c|c|} \hline \text{link} & d & \theta & a & \alpha \\ \hline 1 & l_1 & q_1 & 0 & 90 \\ \hline 2 & 0 & q_2 & l_2 & 0 \\ \hline 3 & 0 & q_3 & l_3 & 0 \\ \hline \end{array}$$

Let the position of end effector is $P = [P_x, P_y, P_z]$

Let the angles at each joint are $[q_1, q_2, q_3]$

Let Link lengths are $[l_1, l_2, l_3]$

For link 1:

$$H_0^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For link 2 :

$$H_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2.\,c_2 \\ s_2 & c_2 & 0 & l_2.\,s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_0^2 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & l_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & l_2s_1c_2 \\ s_2 & c_2 & 0 & l_1 + l_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 3 :

$$H_2^3 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3c_3 \\ s_3 & c_3 & 0 & l_3c_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_0^3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & l_3c_1c_{23} + l_2c_1c_2 \\ s_1c_{23} & -s_1s_{23} & -c_1 & l_3s_1c_{23} + l_2s_1c_2 \\ s_{23} & c_{23} & 0 & l_3s_{23} + l_2s_2 + l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence, The end effector position is given as:

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} l_3 \cos(q_1)\cos(q_2+q_3) \ + \ l_2 \cos(q_1)\cos(q_2) \\ l_3 \sin(q_1)\cos(q_2+q_3) \ + \ l_2 \sin(q_1)\cos(q_2) \\ l_3 \sin(q_2+q_3) \ + \ l_2 \sin(q_2) \ + \ l_1 \end{bmatrix}$$

So Let plug position of corners of Manifold in inverse kinematics:

```
Px = 0.25
Py = 0
Pz = 0.1

Inverse kinematics:
The angles (degrees) are:
q1 =  0.0
q2 =  -85.29521897454477
q3 =  108.66292488494248

Using same angle values in forward kinematics
Position of end effector:
Px = 0.25
Py = 0.0
Pz = 0.1
```

```
Px = 0.45
Py = 0
Pz = 0.1

Inverse kinematics:
The angles (degrees) are:
q1 =  0.0
q2 =  -36.86989764584401
q3 =  36.86989764584401

Using same angle values in forward kinematics
Position of end effector:
Px = 0.45
Py = 0.0
Pz = 0.1
```

```
Px = 0.45
Py = 0.15
Pz = 0.1

Inverse kinematics:
The angles (degrees) are:
q1 =  18.43494882292201
q2 =  -23.28757109105909
q3 =  11.478340954533579

Using same angle values in forward kinematics
Position of end effector:
Px = 0.45
Py = 0.15
Pz = 0.1
```

```
Px = 0.25
Py = 0.15
Pz = 0.1

Inverse kinematics:
The angles (degrees) are:
q1 =  30.96375653207352
q2 =  -76.24955112221971
q3 =  98.04784624731153

Using same angle values in forward kinematics
Position of end effector:
Px = 0.25
Py = 0.15
Pz = 0.1
```

Hence, the entire top surface of the manifold lies within the robot workspace

Q2.b]
Ans:

Jacobian for PUMA is

$$J = \begin{bmatrix} -[l_3 s_1 c_{23} + l_2 s_1 c_2] & -[l_3 c_1 s_{23} + l_2 c_1 s_2] & -l_3 c_1 s_{23} \\ l_3 c_1 c_{23} + l_2 c_1 c_2 & -[l_3 s_1 s_{23} + l_2 s_1 s_2] & -l_3 s_1 s_{23} \\ 0 & l_3 c_{23} + l_2 c_2 & l_3 c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

The corner points of rectangle are:

$$A = [0.40,\ 0.06,\ 0.1]$$
$$B = [0.40,\ 0.01,\ 0.1]$$
$$C = [0.35,\ 0.01,\ 0.1]$$
$$D = [0.35,\ 0.06,\ 0.1]$$

The trajectory is created by linearly interpolating between consecutive points in the list points. The linear interpolation formula for a single dimension (let's call it dim) is given by:

$$\text{Interpolated\_point[dim]} = (1-t) \times \text{start\_point [dim]} + t \times \text{end\_point[dim]}$$

This formula calculates the interpolated value for each dimension (dim) based on the parameter ' t ', which varies from 0 to 1. It starts from the start_point and gradually moves towards the end_point as ' t ' increases.

For calculating the joint angles, Used inverse kinematics.

$$q[k+1] = q[k] + J^{-1}\dot{X}dt$$

Then plotted in 3D space.