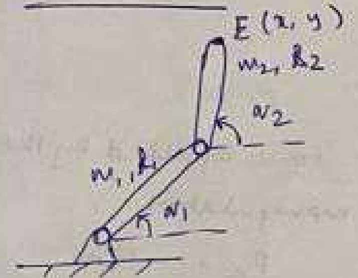
INTRODUCTION TO ROBOTICS

MINIPROJECT:



2R manipulator.

Tasks: & Assumptions

① We assume we have actuators at each joint connected to each link.

② We assume we can provide τ_1 & τ_2 @ each joint to control q_1 & q_2 as we require.

T1: Make the robot follow an arbitrary trajectory of the end-effector [given (x, y) as a f^n of t).

T2: Given a location & orientation on a wall, make the robot touch the wall & apply a pre-specified constant force at the wall.

T3: Fixed point spring feedback control.

Now,

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \cos q_2$$

or

$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} - ①$$

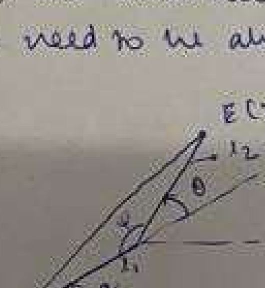
Differentiating ①, we get :

$$\left.\begin{aligned} \dot{x} &= - l_1 s q_1 \dot{q}_1 - l_2 c q_2 \dot{q}_2 \\ \dot{y} &= l_1 c q_1 \dot{q}_1 + l_2 c q_2 \dot{q}_2 \end{aligned}\right. - ②$$

→ End effector velocity

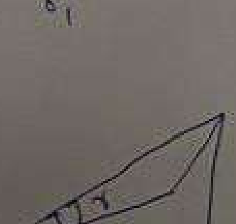
$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Now, we will need the inverse of the above relationship, i.e given x, y, we need to be able to find q_1, q_2.



cosine rule:

$$\left.\begin{aligned} & x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta \\ & \text{or} \quad \theta = \cos^{-1}\left[\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right] \\ & q_2 = q_1 + \theta \\ & q_1 = \beta - \gamma = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_1 \sin\theta}{l_1 + l_2 \cos\theta}\right) \end{aligned}\right] - ③$$

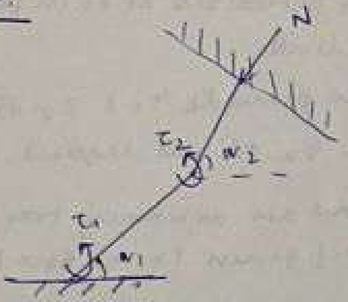


→ This is the first level answer to Task 1.

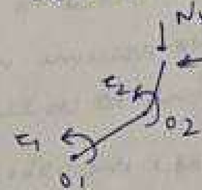
NOTE:

We will later start using the notation x_d & y_d and $q_{1,d}$ & $q_{2,d}$ here for desired values.
(They are not necessarily actual values).

TASK 2:



FBD of entire robot:



Force applied by manipulator.

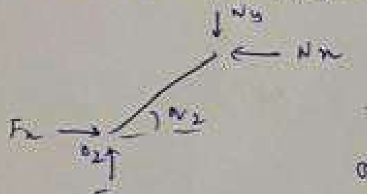
$F_x = -N_x$

$F_y = -N_y$

(Neglect gravity)

State of static Equilibrium. $[\Sigma F = 0, \Sigma M = 0]$

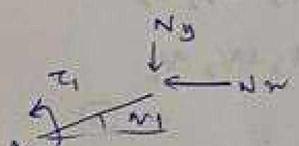
Now, FBD for each link:



$\therefore \Sigma M_{O_2} = 0$ ⊛

$\therefore -N_y l_2 c q_2 + N_x l_2 s q_2 + \tau_2 = 0$

or $\tau_2 = N_y l_2 c q_2 - N_x l_2 s q_2$



$\Sigma M_{O_1} = 0$

$\therefore N_y l_1 c q_1 - N_x l_1 s q_1 = \tau_1$

$\Bigg\}$ ④

$$\therefore \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix}$$

Now, equation ③ along with ④ solves T_2.

Task 3 & next level answer to Task-1:

Lagrange's Equations: [Lagrangian equation]

$$L = K - V$$

$$\therefore \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \right]$$

Q_i - generalized forces derived using principle of virtual work.

Kinetic Energy:

$$KE = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

where

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

Potential Energy:

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

Now using L-E for q_1 & q_2 we get:

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2}\ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2}\dot{q}_2(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4}\ddot{q}_2 + m_2 \frac{l_1 l_2}{2}\ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2}\dot{q}_1(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1)$$

$$+ m_2 g \frac{l_2}{2} s q_2 = \tau_2$$

Next, we note that eqn (4) is valid for any forces F_x, F_y at end effector. (Not just wall forces).

$\therefore$ We want:

$$F_x = kx \quad \left[F_x = k_x(x - x_0) \right.$$
$$F_y = ky \quad \left. F_y = k_y(y - y_0) \right]$$

$\therefore$ From (1):

$$F_x = K(l_1 c q_1 + l_2 c q_2)$$
$$F_y = K(l_1 s q_1 + l_2 s q_2)$$

From ④:

$$K(l_1 s q_1 + l_2 s q_2) l_2 c q_2 - K(l_1 s q_1 + l_2 c q_2) l_2 s q_2 = \tau_{2s}$$

$$K(l_1 s q_1 + l_2 s q_2) l_1 c q_1 - K(l_1 c q_1 + l_2 c q_2) l_1 s q_1 = \tau_{1s}$$

$\therefore$ set motor torques to be $\tau_1 + \tau_{2s}$ & $\tau_2 + \tau_{1s}$ respectively.

→ Another way to solve T1:

is to solve for $q_1 d$ & $q_2 d$ from ③.

$\therefore$ i.e $\dot{q}_1 d, \dot{q}_2 d, \ddot{q}_1 d, \ddot{q}_2 d \Rightarrow \tau_1, \tau_2$ from

This method works when dynamic effects are significant. But still needs feedback control.

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CONTROL BLOCK