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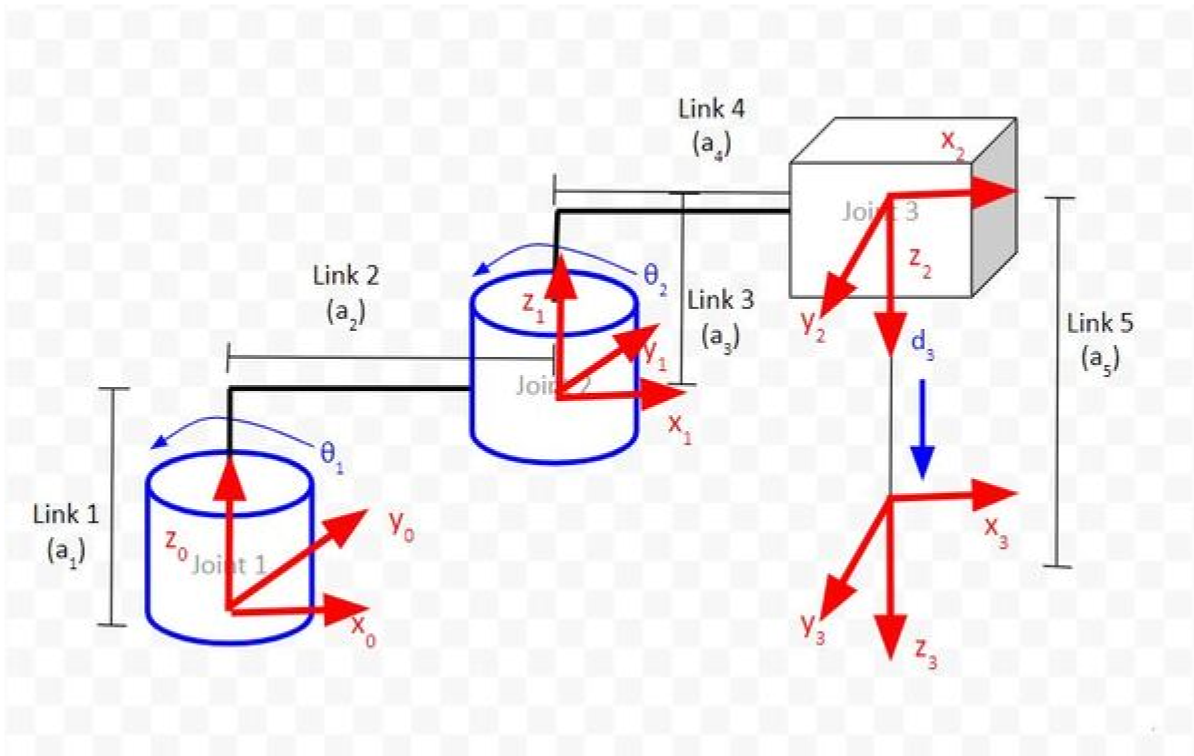
Assignment 3&4

Q 1)

A singular configuration in robotics occurs when a specific arrangement of a robot's joints leads to a loss of one or more degrees of freedom, signifying a decrease in the manipulator's Jacobian matrix. Configurations for which the rank J(q) is less than its maximum value are called singularities or singular configurations. This reduction in degrees of freedom can pose challenges in executing certain tasks. Identification of singular configurations involves analyzing the Jacobian matrix, which delineates the relationship between joint velocities and end-effector velocities. The singularity is manifested in the Jacobian becoming singular, indicating a loss of invertibility and potentially causing unpredictable behavior. Proximity to a singular configuration can be assessed by examining the condition number of the Jacobian matrix; a high condition number implies closeness to a singularity, suggesting that slight changes in joint velocities may result in substantial changes in end-effector velocities. To determine if a specific configuration is singular, the determinant of the Jacobian matrix is calculated, and if it equals zero, the configuration is deemed singular. The condition number is further computed as the ratio of the largest singular value to the smallest singular value of the Jacobian matrix.

Qn 4)

<u>SCARA Manipulator</u>



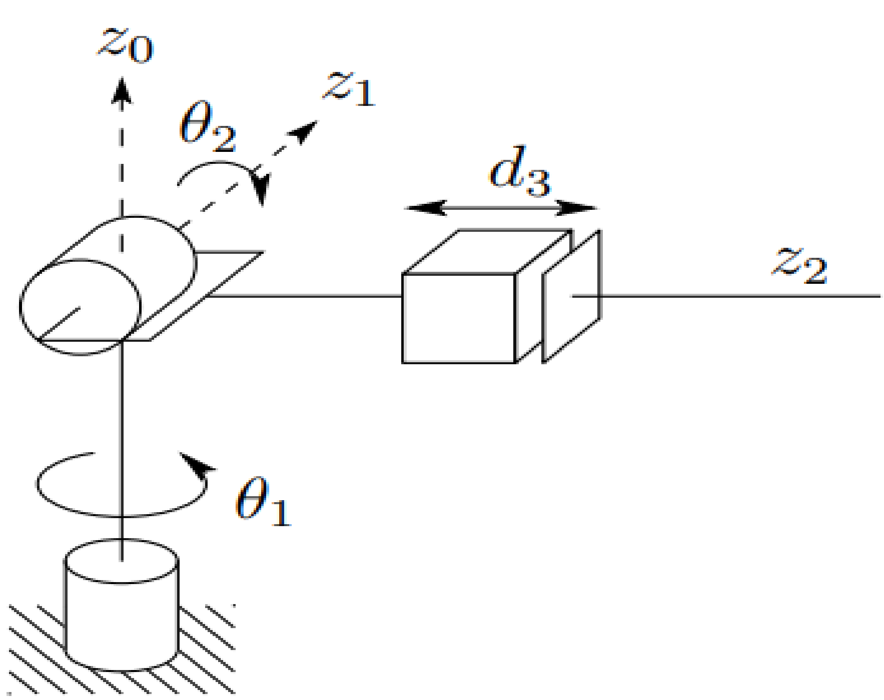
$DH =$

$Link$	d	θ	a	α
1	1	0	1	0
2	2	90	1	180
3	1	-90	0	0

```
Manipulator Jacobian:
[[-1. -1.  0.]
 [ 1.  0. -0.]
 [ 0.  0. -1.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 1.  1.  0.]]

End-effector Position:
[[1.]
 [1.]
 [2.]]
```

```
Enter joint velocity(q_dot) for joint 1: 10
Enter joint velocity(q_dot) for joint 2: 10
Enter joint velocity(q_dot) for joint 3: 10

End-effector Velocity:
[[-20.]
 [ 10.]
 [-10.]
 [  0.]
 [  0.]
 [ 20.]]
```

Stanford Manipulator



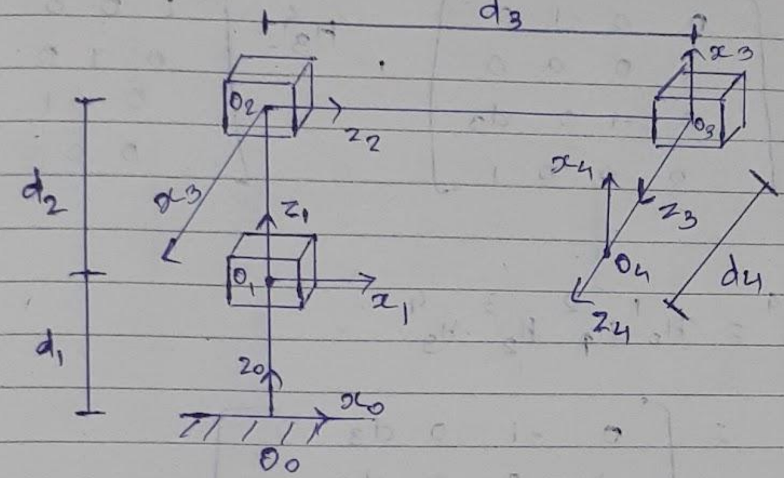
$$DH = \begin{array}{|c|c|c|c|c|} \hline Link & d & \theta & a & \alpha \\ \hline 1 & 2 & 90 & 0 & -90 \\ \hline 2 & 1 & -90 & 0 & 90 \\ \hline 3 & 1.5 & 0 & 0 & 0 \\ \hline \end{array}$$

```
Manipulator Jacobian:
[[ 1.5  0.  -0. ]
 [-1.  -0.  -1. ]
 [ 0.   1.5  0. ]
 [ 0.  -1.   0. ]
 [ 0.   0.   0. ]
 [ 1.   0.   0. ]]

End-effector Position:
[[-1. ]
 [-1.5]
 [ 2. ]]
```

```
Enter joint velocity(q_dot) for joint 1: 10
Enter joint velocity(q_dot) for joint 2: 10
Enter joint velocity(q_dot) for joint 3: 10

End-effector Velocity:
[[ 15.]
 [-20.]
 [ 15.]
 [-10.]
 [  0.]
 [ 10.]]
```

Q5)

5



d1 is fixed
here, $d_1 = 0$

DH Parameters:

Link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	d_1	0
2	0	$-\pi/2$	d_2	$-\pi/2$
3	0	$-\pi/2$	d_3	$\pi/2$
4	0	0	d_4	0

$$H_{i-1}^{i} = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$H_0^4 = H_0^1 H_1^2 H_2^3 H_3^4$$

$$= \begin{bmatrix} 0 & -1 & 0 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & 0 & -1 & d_1 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad O_2 = \begin{bmatrix} 0 \\ d_2 \\ d_1 \end{bmatrix}, \quad O_3 = \begin{bmatrix} d_3 \\ d_2 \\ d_1 \end{bmatrix}$$

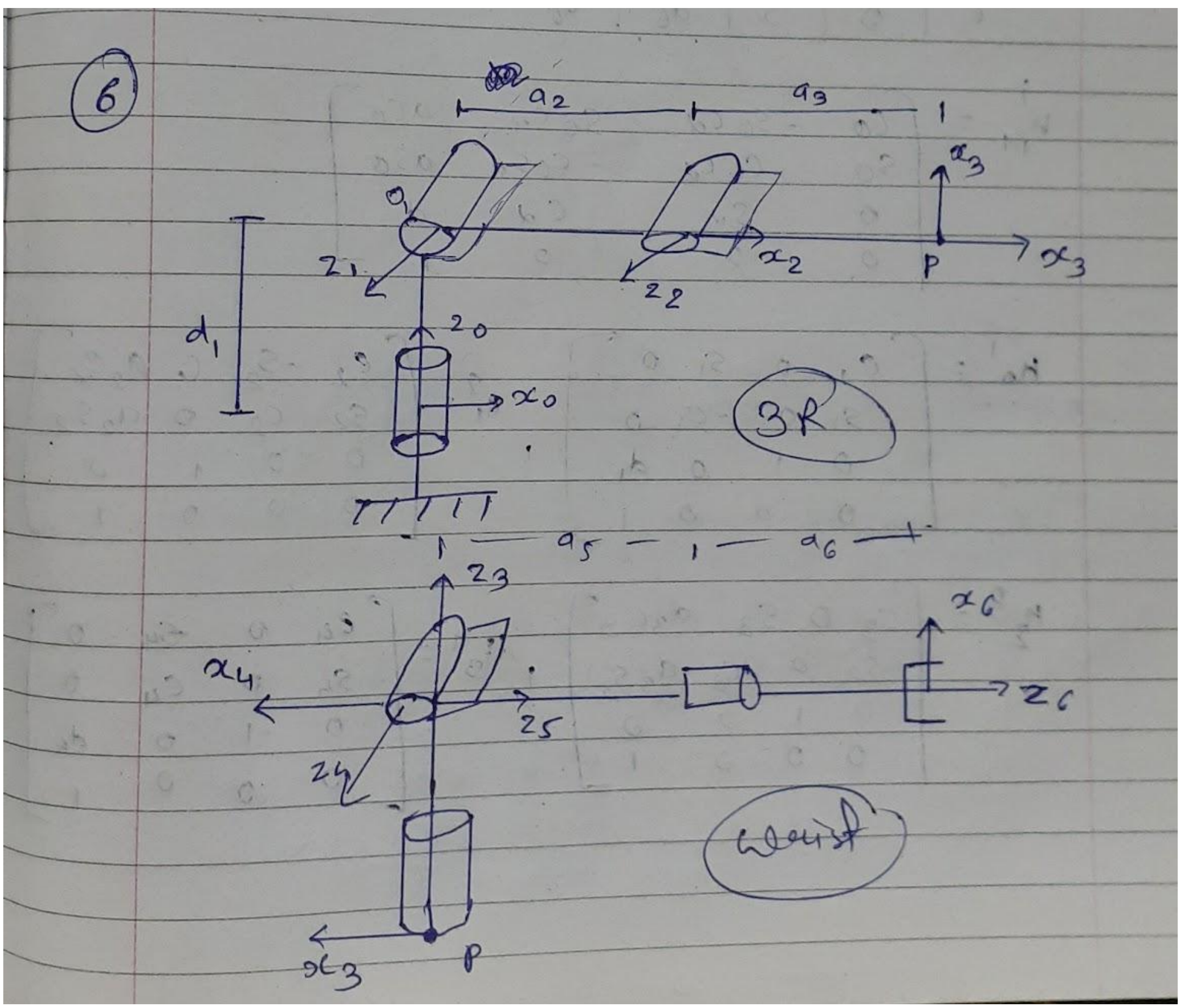
$$O_4 = \begin{bmatrix} d_3 \\ d_2 \\ d_1 - d_4 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 & z_1 & z_2 & z_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q 6)



DH Parameters

Link	a	α	d	θ
1	0	$\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	$\pi/2$	0	θ_3
4	0	$-\pi/2$	d_4	θ_4
5	a_5	$\pi/2$	0	θ_5
6	0	0	d_6	θ_6

$$H_{i-1}^{i} = \begin{bmatrix} C\theta & -S\theta C\alpha & S\theta S\alpha & aC\theta \\ S\theta & C\theta C\alpha & -C\theta S\alpha & aS\theta \\ 0 & S\alpha & C\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} C_3 & 0 & S_3 & a_3C_3 \\ S_3 & 0 & -C_3 & a_3S_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_3^4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^5 = \begin{bmatrix} C_5 & 0 & S_5 & a_5C_5 \\ S_5 & 0 & -C_5 & a_5S_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_5^6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^6 = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot H_3^4 \cdot H_4^5 \cdot H_5^6$$

Q7)

The 2R manipulator configurations are showed below

Direct Drive Configuration:

- This setup involves attaching the end-effector directly to the two rotational joints.
- The design is straightforward with minimal components.
- This configuration is typically more compact due to the elimination of additional linkages.
- The motors need to be constrained in such a way that we get 2^{nd} angle relative to 1^{st}.

Advantages:

- The streamlined design simplifies both construction and maintenance.
- Well-suited for applications where space is limited.

Remotely-Driven Configuration:

- This configuration introduces an additional link or arm between the actuators and the end-effector.
- A separate linkage connects the actuators to the end-effector, providing increased flexibility.
- The added link enhances the manipulator's reach.
- The motors need to be attached at the base and both of the are driven separately.

Advantages:

- Well-suited for scenarios where extended reach is a crucial factor.
- The additional link allows for more versatile movements.

5-Bar Parallelogram Arrangement:

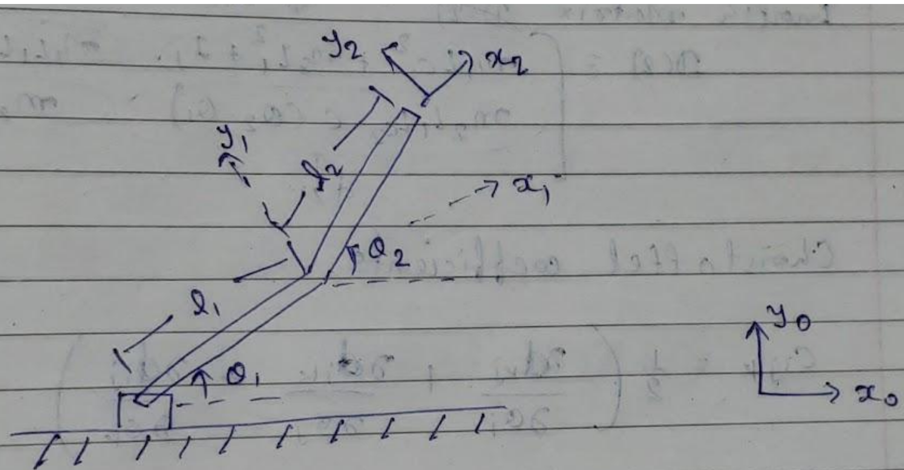
- This configuration includes a parallelogram linkage in addition to the two rotational joints.
- The incorporation of a parallelogram mechanism helps maintain the orientation of the end-effector.
- The parallelogram linkage contributes to stability during motion.
- The motors should be independently controlled but need to be at the base as well as at the same line.

Advantages:

- Improved precision in controlling the orientation of the end-effector.
- Particularly beneficial when stability is critical for precise tasks.

Qn 8)

8



Taking I_1, I_2 as moment of inertia at centre of individual links Lc_1 & Lc_2 are com

L_1 & L_2 are lengths of links

m_1 & m_2 are mass of links

$$V_{c_1} = \begin{bmatrix} -Lc_1 \, s\theta_1 \\ Lc_1 \, c\theta_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$V_{c_2} = \begin{bmatrix} -L_1 S\theta_1 & -Lc_2 S\theta_2 \\ L_1 C\theta_1 & Lc_2 C\theta_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix}$$

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 \end{bmatrix}, \quad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_2 \end{bmatrix}$$

Inertia matrix ~~$D(q)$~~

$$D(q) = \begin{bmatrix} \underbrace{m_1 Lc_1^2 + m_2 L_1^2 + I_1}_{d_{11}} & \underbrace{m_1 L_1 Lc_2 \, c(\theta_2 - \theta_1)}_{d_{12}} \\ \underbrace{m_2 L_1 Lc_2 \, c(\theta_2 - \theta_1)}_{d_{21}} & \underbrace{m_2 Lc_2^2 + I_2}_{d_{22}} \end{bmatrix}$$

Christoffel coefficients

$$c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial \theta_i} + \frac{\partial d_{ik}}{\partial \theta_j} - \frac{\partial d_{ij}}{\partial \theta_k} \right)$$

$$c_{111} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial \theta_1} \right) = 0$$

$$c_{121} = c_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial \theta_2} = 0$$

$$c_{221} = \frac{1}{2}\left(\frac{\partial d_{12}}{\partial \theta_2} + \frac{\partial d_{21}}{\partial \theta_2} - \frac{\partial d_{22}}{\partial \theta_1} \right) = -m_2 L_1 Lc_2 \, s(\theta_2 - \theta_1)$$

$$c_{112} = \frac{1}{2}\left(\frac{\partial d_{21}}{\partial \theta_1} + \frac{\partial d_{12}}{\partial \theta_1} - \frac{\partial d_{11}}{\partial q_2}\right) = m_2 L_1 L_{c2}\, S(\theta_2 - \theta_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial \theta_1} + \frac{\partial d_{12}}{\partial \theta_2} - \frac{\partial d_{12}}{\partial \theta_2}\right) = 0$$

$$C_{222} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial q_2}\right) = 0$$

Potential energy

$$V = m_1 g L_{c1}\, S\theta_1 + m_2 g\,(l_1 S\theta_1 + L_{c2} S\theta_2)$$

Potential terms

$$\phi_1 = \frac{\partial V}{\partial \theta_1} = (m_1 L_{c1} + m_2 L_1)\, g\, C\theta_1$$

$$\phi_2 = \frac{\partial V}{\partial \theta_2} = m_2 g L_{c2}\, C\theta_2$$

$\theta_i = q_i$

$$\text{Torque } \tau_K = \sum_i^n d_{ik} \cdot \ddot{q}_i + \sum_{i,j}^n C_{ijk}\, \dot{q}_i \dot{q}_j + \phi_k$$

$$\tau_1 = \sum_i^2 d_{i1} \cdot \ddot{q}_i + \sum_{i,j}^2 C_{ij1} \cdot \dot{q}_i \dot{q}_j + \phi_1$$

$$= d_{11}\ddot{q}_1 + d_{21}\ddot{q}_2 + C_{221}\dot{q}_2^2 + \phi_1$$

$$\tau_2 = \sum_i^2 d_{i2} \cdot \ddot{q}_i + \sum_{i,j}^2 C_{ij2} \cdot \dot{q}_i \dot{q}_j + \phi_2$$

$$\tau_2 = d_{12}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2$$

here, $L_{c1} = L_1/2$ & $L_{c2} = L_2/2$

Qn 10)

(10) → Given $D(q)$ & $V(q)$

Langrangiam eq^n is $\mathcal{L} = K - V$

Where, $K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$

$$V = V(q)$$

$$\tau = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q}$$

where $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \sum_i d_{ik}(\ddot{q}_i) + \sum_{i,j} \frac{\partial d_{ik}}{\partial q_j}\dot{q}_i \dot{q}_j$

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i \dot{q}_j - \frac{\partial V(q)}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + \frac{1}{2}\sum_{i,j}\left[\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k}$$

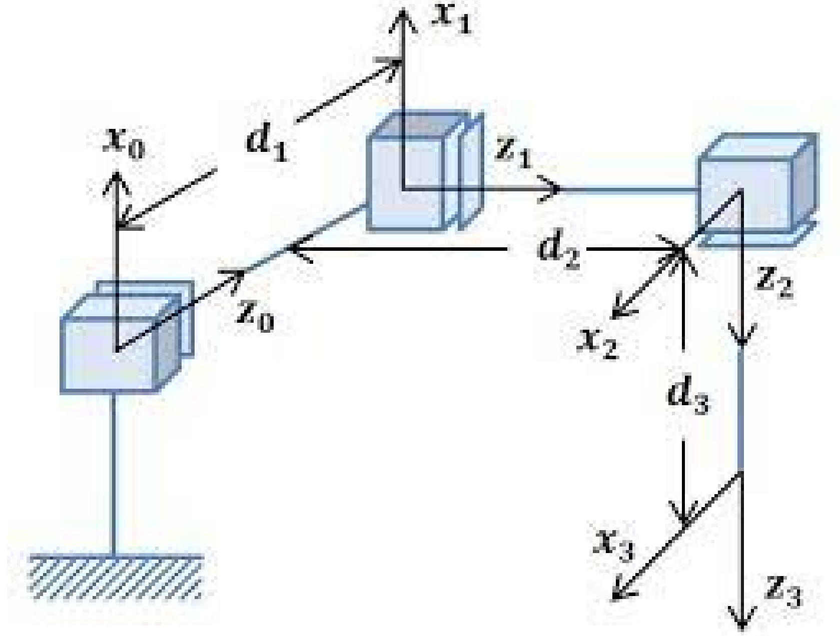
$$\tau_k = \sum_i d_{ik}\ddot{q}_i + c_{ijk}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + c_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q)$$

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q), \quad \left(\because g(q) = \frac{\partial V}{\partial q_k}\right)$$

So, when we given $D(q)$ we will find $C(q,\dot{q})$ using $\sum_{i,j} c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right)$

Qn 18)



$$DH =$$

$Link$	d	θ	a	α
1	l_1	0	0	-90
2	l_2	90	0	-90
3	l_3	0	0	0

Ex 1)

for, $l_1 = 2$, $l_2 = 2$ and $l_3 = 1$

$DH =$

$Link$	d	θ	a	α
1	2	0	0	-90
2	2	90	0	-90
3	1	0	0	0

Let's put this value in Qn 3 code.

```
Manipulator Jacobian:
[[ 0.  0. -1.]
 [ 0.  1.  0.]
 [ 1.  0. -0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]

End-effector Position:
[[-1.]
 [ 2.]
 [ 2.]]
```

```
Enter joint velocity(q_dot) for joint 1: 2
Enter joint velocity(q_dot) for joint 2: 5
Enter joint velocity(q_dot) for joint 3: 8

End-effector Velocity:
[[-8.]
 [ 5.]
 [ 2.]
 [ 0.]
 [ 0.]
 [ 0.]]
```

Ex 2)

for, $l_1 = 1$, $l_2 = 1.5$ and $l_3 = 2$

$DH =$

$Link$	d	θ	a	α
1	1	0	0	-90
2	1.5	90	0	-90
3	2	0	0	0

Let's put this value in Qn 3 code.

```
Manipulator Jacobian:
[[ 0.  0. -1.]
 [ 0.  1.  0.]
 [ 1.  0. -0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]

End-effector Position:
[[-2. ]
 [ 1.5]
 [ 1. ]]
```

```
Enter joint velocity(q_dot) for joint 1: 2
Enter joint velocity(q_dot) for joint 2: 5
Enter joint velocity(q_dot) for joint 3: 8

End-effector Velocity:
[[-8.]
 [ 5.]
 [ 2.]
 [ 0.]
 [ 0.]
 [ 0.]]
```