Q 1

A singular configuration in robotics refers to a specific state or position of a robotic manipulator where it loses its full range of motion and becomes unable to move in certain direction or perform certain tasks. It occurs when the manipulator's joint align in such a way that it cannot change its end-effector's position or orientation without reaching a limit or encountering discontinuities in its movement.

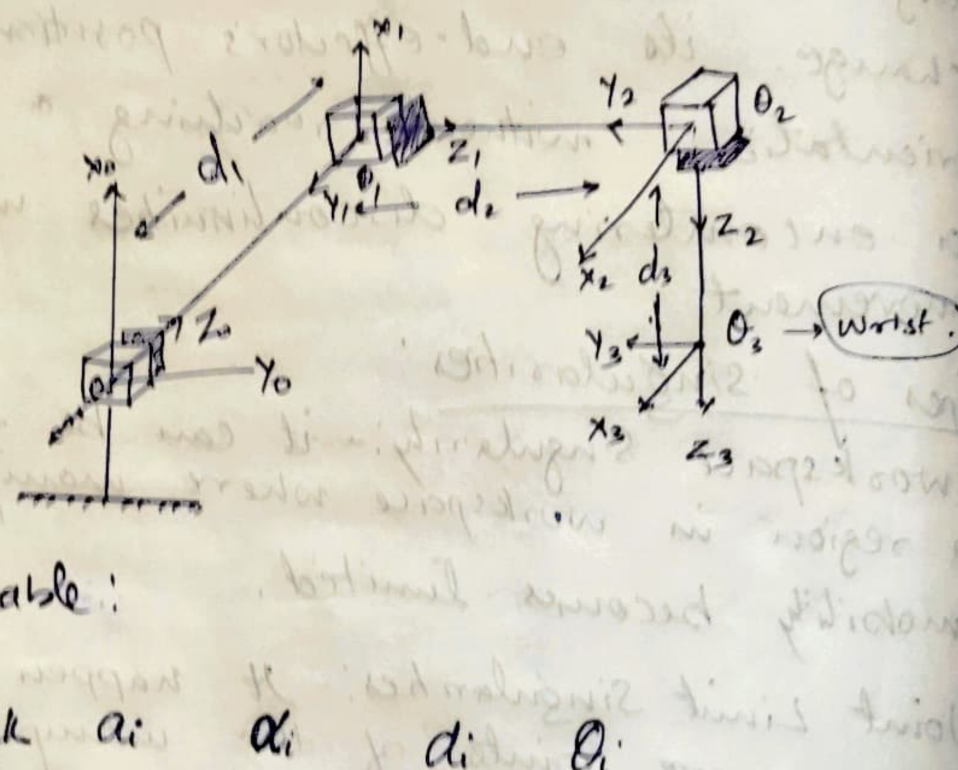
Types of singularities:

- workspace Singularity:- it can be visualised as a region in workspace where manipulator's mobility becomes limited.
- Joint Limit Singularities:- It happen when one or more joints of the manipulator reach their mechanical limits.
- wrist Singularity - It happens when there is redundancy in the wrist's orientation.

To find the singular configuration, we need to analyse the robot's Jacobian matrix. A singularity occurs when the determinant of the Jacobian becomes zero or very close to zero.

Detecting if a particular configuration is close to a singularity involves examining the eigenvalues of the Jacobian matrix. If any of the eigenvalues are approaching zero or becomes very small it indicates that the robot is nearing a singular configuration.

Q5




DH table :

| link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | -90 | $d_1$ | 0 |
| 2 | 0 | 90 | $d_2$ | -90° |
| 3 | 0 | 0 | $d_3$ | 90° |

using DH table.

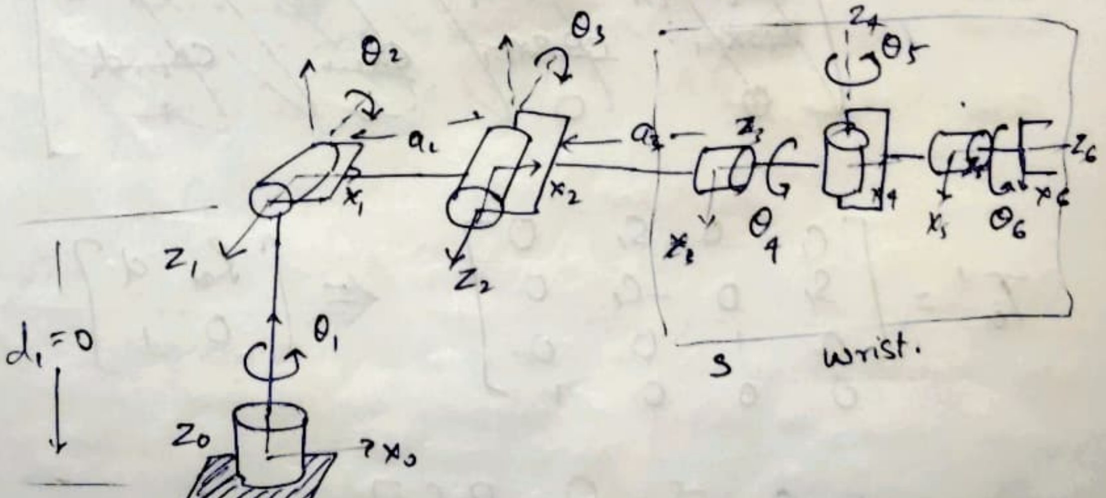
$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T_0^3 = H_0^1 H_1^2 H_2^3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Q6




$d_1$ is assumed to be zero for easing calculation.
$d_4$ & $d_5$ are also zero as it is spherical wrist.

DH Table for the given manipulator

| link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 90 | $d_1 = 0$ | $\theta_1$ |
| 2 | $a_2$ | 0 | 0 | $\theta_2$ |
| 3 | $a_3$ | 0 | 0 | $\theta_3$ |
| 4 | 0 | -90 | 0 | $\theta_4$ |
| 5 | 0 | 0 | 0 | $\theta_5$ |
| 6 | 0 | 0 | $d_6$ | $\theta_6$ |

calculating transformation matrix using data from dh table.

$$\cancel{T_i^{i-1}} = \begin{bmatrix} \cancel{c\theta_i} & \cancel{-s\theta_i} & 0 & a_{i-1} \\ \cancel{s\theta_i c\alpha_{i-1}} & \cancel{c\theta_i c\alpha_{i-1}} & \cancel{-s\alpha_{i-1}} & \cancel{-s\alpha_{i-1} d_i} \\ \cancel{s\theta_i s\alpha_{i-1}} & \cancel{c\theta_i s\alpha_{i-1}} & \cancel{c\alpha_{i-1}} & \cancel{c\alpha_{i-1} d_i} \\ \cancel{0} & 0 & \cancel{0} & \cancel{1} \end{bmatrix}$$

$$T_0^{\,1} = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Leftarrow \begin{bmatrix} R_{ot} & d \\ 0 & 1 \end{bmatrix}$$

$$T_1^{\,2} = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 C_2 \\ S_2 & C_2 & 0 & a_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_1 & C_3 & 0 & a_3 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_3^4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_5^6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Thus. $T_0^6 = T_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6$

$\cancel{[C_1[C_5C_6C_{234} - S_6S_{234}] - S_1S_5S_6]}$

$$T_0^6 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & d_x \\ a_{21} & a_{22} & a_{23} & d_y \\ a_{31} & a_{32} & a_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$a_{11} = C_1 [C_5 C_6 C_{234} - S_6 S_{234}] - S_1 S_5 S_6$

$a_{12} = -C_1 [C_5 S_6 C_{234} - C_6 S_{234}] + S_1 S_5 S_6$

$a_{13} = C_1 S_5 C_{234} + S_1 C_5$

$a_{21} = C_1 S_5 S_6 + S_1 C_5 C_6 C_{234} - S_1 S_6 S_{234}$

$a_{22} = -C_1 S_5 S_6 - S_1 C_5 S_6 C_{234}$

$a_{23} = -C_1 C_5 + S_1 S_5 C_{234}$

$a_{31} = S_6 C_{234} + C_5 S_6 S_{234}$

$a_{32} = C_6 S_{234} - C_5 S_6 S_{234}$

$a_{33} = S_5 S_{234}$

$d_x = a_2 C_1 C_2 + a_3 C_1 C_{23} + d_6 [C_1 S_5 C_{234} + S_1 C_5]$

$d_y = a_2 S_1 C_2 + a_3 S_1 C_{23} - d_6 [C_1 C_5 + S_1 S_5 C_{234}]$

Q7. The three distinct configuration for 2R manipulators are

1. Direct Drive : in direct drive 2R manipulator the two revolute joints are aligned in series, meaning that the second joint follows the first one directly.
2. Remotely Driven :- here the revolute joints are not in direct series but instead connected via mechanical linkage often using additional intermediate links.
3. 5-bar Parallelogram :- here a llgm linkage is formed by two additional links connected to the base and end effector, is added to the basic 2R-Setup.

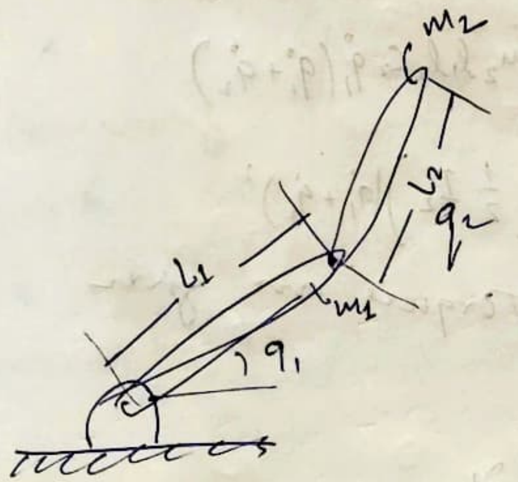
Advantages

| Direct Drive | Remote Drive | llgm |
|---|---|---|
| • Easy operation and Control due to its simple nature. • well suited for Point to Point. • use few mechanical components & therefore reliable | • Slightly Complex but offers broader range of motion • flexible for Complex trajectory and orientation • Ideal for Precise movements. | • well suited for task where end-effector orientation is important. • Provides high rigidity for maintaining orientation • reduce need for Complex Calculation. |

## Key differences.

| Direct Drive | Remote Drive | ‖gm. |
|---|---|---|
| • Straight forward & compact design. • minimal intermediate linkages. • Joint movement directly translate to end-effector motion • Simple Kinematics | • Extra links and elements introduce kinematic complexities • End-Effector motion is influenced by intricate linkage and geometries. • Sometimes the kinematics involved is non-trivial | • ‖gm linkage maintain the orientation of the end-effector while the manipulator is in motion. • End-effector remains parallel to base • Planar motor of End-Effector wrt. base. |

Q8




To compute the dynamics of 2R Manipulator (planar) we will use the Langrangian equations.

Langrangian L, is defined as :

$L = k - V$ where $k$ - kinetic energy
$V$ - Potential energy.

for 2R case
$V = V_1 + V_2$ $V_1$ = P.E. of link 1
$V_2$ = P.E of link 2.

Thus $V = m_1 g \frac{l_1}{2} s_1 + m_2 g \left( l_1 s_1 + \frac{l_2}{2} s_2 \right)$

[Note: if the 2R manipulator is horizontal then it can be zero.]

Now $K = K_1 + K_2$ (sum of K.E of both arms)

where

$$K_1 = \underbrace{\frac{1}{2} I_1 \dot{q}_1^2}_{\text{rotational K.E}} \qquad K_2 = \underbrace{\frac{1}{2} m_2 V_c^2}_{\text{Translational K.E}} + \underbrace{\frac{1}{2} I_2 \dot{q}_2^2}_{\text{rotational K.E.}}$$

where

$$V_c = \dot{x}_c^2 + \dot{y}_c^2 \; ; \quad x_c = l_1 c_1 + \frac{l_2}{2} c_{12}, \quad y_c = l_1 s_1 + \frac{l_2}{2} s_{12} \quad \{\text{from geometry}\}$$

Thus,

$$K = \frac{1}{2}(I_1 + m_2 l_1^2)\dot{q}_1^2 + \frac{1}{2} m_2 l_1 l_2 c_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2) + \left(\frac{1}{2} m_2 l_2^2 + \frac{1}{2} I_2\right)(\dot{q}_1 + \dot{q}_2)^2$$

The general forces/Torques are given by:

$$J_i = \frac{d}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

Computing the differentials.

$$\frac{\partial L}{\partial \dot{q}_1} = (I_1 + m_2 l_1^2)\dot{q}_1 + m_2 l_1 l_2 (\dot{q}_1 + \frac{1}{2}\dot{q}_2) + \left(\frac{1}{4} m_2 l_2^2 + I_2\right)(\dot{q}_1 + \dot{q}_2)$$

$$\frac{\partial L}{\partial \dot{q}_2} = \frac{1}{2} m_2 l_1 l_2 c_2 \dot{q}_1 + \left(\frac{1}{4} m_2 l_2^2 + I_2\right)(\dot{q}_1 + \dot{q}_2)$$

$$\frac{\partial L}{\partial q_1} = -m_1 g \frac{l_1}{2} c_1 - m_2 g \left( l_1 c_1 + \frac{l_2}{2} c_{12} \right)$$

$$\frac{\partial L}{\partial q_2} = -m_2 g \frac{l_2}{2} c_{12} - \frac{1}{2} m_2 l_1 l_2 S_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2)$$

Substituting the above we get

$$\tau_1 = \left( \frac{1}{4} m_1 l_1^2 + m_2 l_1^2 + I_1 \right) \ddot{q}_1 + m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \ddot{q}_2 - m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1) \dot{q}_2^2 + \boxed{\left( m_1 \frac{l_1}{2} + m_2 l_1 \right) g \cos q_1}$$

$$\tau_2 = \left( m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \right) \ddot{q}_1 + \left( m_2 \frac{l_2^2}{4} + I_2 \right) \ddot{q}_2 + m l_1 \frac{l_2}{2} \sin(q_2 - q_1) \dot{q}_1^2 + \boxed{\left( m_2 \frac{l_2}{2} \right) g \cos q_2}$$

On comparing the above equation with our case the term in box (i.e Potential energy due to gravity) was taken to be zero.

**Q10.** if we are given $D(q)$ & $V(q)$

a) we will compute the Christoffel symbols of first kind.

$$c_{ijk} = \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{ij}}{\partial q_k} \right\}$$

b) Find gravitational (Potential Energy) term as:

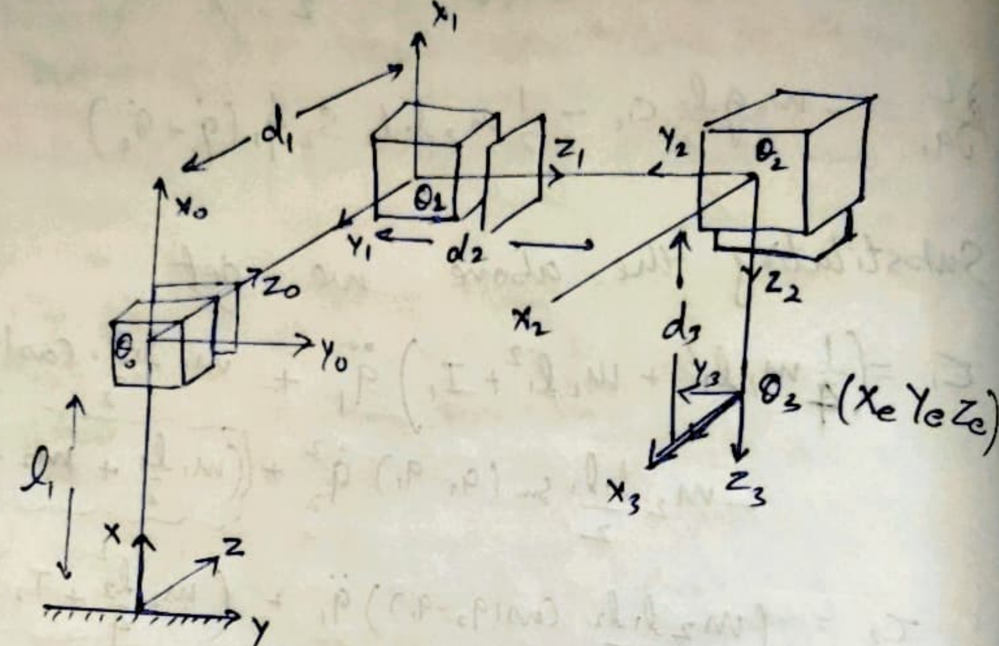
$$\phi_k = \frac{\partial V}{\partial q_k}$$

c) Substitute the above in Standard Dynamic Equation

$$\Rightarrow \sum_j d_{kj}(q)\ddot{q}_j + \sum_{ij} c_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k$$

where $k = 1, 2, 3 \dots n$.

Q18. Schematic of 3D Printer with PPP configuration




DH Parameter

| link | d | $\theta$ | a | $\alpha$ |
|---|---|---|---|---|
| 1 | $d_1$ | 0 | 0 | $-\pi/2$ |
| 2 | $d_2$ | $\pi/2$ | 0 | $\pi/2$ |
| 3 | $d_3$ | $-\pi/2$ | 0 | 0 |

Q19.

using the figure in Q18. the cordinates of end-effector $(x_e, y_e, z_e)$ and $\{d_1, d_2 \& d_3\}$ can be related as:

$$d_1 = z_e$$

$$d_2 = y_e$$

$$d_3 = l_1 - x_e.$$

Q2 - reading task

Q3 - Coding

Q4 - Coding

Q9 - ~~reweiw~~ reviewing task.

Q11 - Coding

Q12 - "

Q13 - "

Q14 - "

Q15 - reading task.

Q16 "

Q17 - Coding