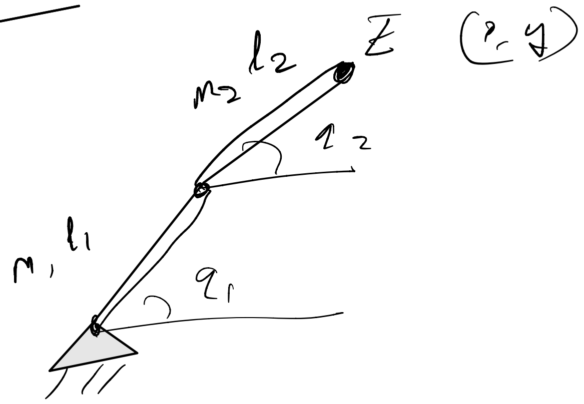
7-8-23



2R Manipulator

T_1 - Trajectory following

T_2 - Apply a force on a wall

T_3 - Act like a spring.

$\underline{T_1}$

$$x = l_1 \cos q_1 + l_2 \cos q_2 \quad -(1)$$

$$y = l_1 \sin q_1 + l_2 \sin q_2$$

or

$$x = l_1 cq_1 + l_2 cq_2$$

$$y = l_1 sq_1 + l_2 sq_2$$

differentiating

$$v_x = \dot{x} = -l_1 sq_1 \dot{q}_1 - l_2 sq_2 \dot{q}_2$$

$$v_y = \dot{y} = l_1 cq_1 \dot{q}_1 + l_2 cq_2 \dot{q}_2$$

[Transformation from joints → end effectors]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad -(2)$$

Forward kinematics

we need the reverse relationship

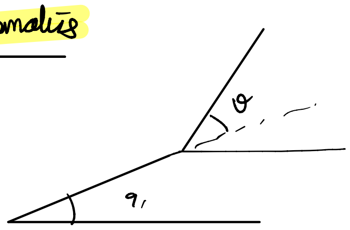
ie we need to find values of q_1 q_2 given x, y

option - 1 solve numerically.

option - 2 - derive a closed form expression. ←
(substitute knowns, we get unknown)

- easier to handle but hard to derive.
- multiple solutions.

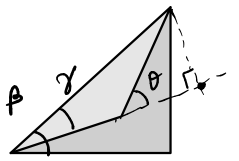
Inverse kinematics



$$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

(3)



$$q_1 = \beta - \gamma$$
$$= \tan^{-1}\frac{y}{x} - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

1st level ans for task 1

(d → desired)

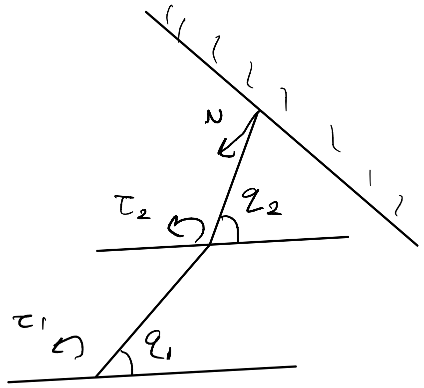
$x \to x_d$

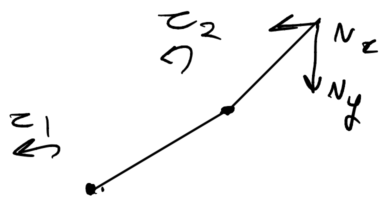
$y \to y_d$

$q_1 \to q_{1d}$

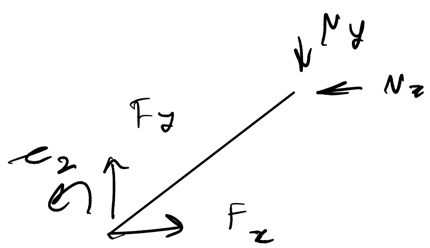
$q_2 \to q_{2d}$

Task-2





[Static equilibrium]

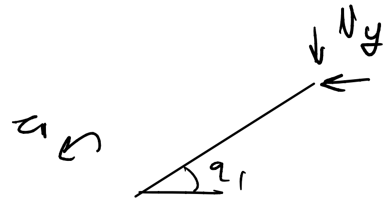


$N_x = F_x$

$N_y = F_y$

$\sum M = 0 \Rightarrow \tau_2 - l_2\cos q_2 N_y + N_x l_2 \sin q_2 = 0$

$\tau_2 = N_y l_2 c q_2 - N_x l_2 s q_2$



(No τ_2 because τ_2 is applied relative to gnd)

$\sum M_{O_1} = 0$

$\tau_1 = N_y l_1 \cos q_1 - N_x l_1 s q_1$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_2 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad - \ (4)$$

eq (3) & (4) solves T_2

Task - 3

Lagrange's Equation.

$$\underbrace{L = K}_{\text{Kinetic energy}} - \underbrace{V}_{\text{Potential energy}}$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}_i}\right) - \frac{\delta L}{\delta q_i} = Q_i$$

[Q_i → are generalized forces derived using principle of virtual work]

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2 + \frac{1}{2} m_2 V_{c2}^2$$

$$V_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1 = \tau_1 \quad - (1)$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 c(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_2 g \frac{l_2}{2} s q_2 = \tau_2$$

We want

$F_x = kx$

$F_y = ky$

more generally

$F_x = k_x(x - x_0)$

$F_y = k_y\,(y - y_0)$

From (1)

$F_x = k(l_1 cq_1 + l_2 cq_2)$

$F_y = k(l_1 sq_1 + l_2 sq_2)$

from (4)

$$k(l_1 sq_1 + l_2 sq_2)\, l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2)\, l_2 sq_2 = \tau_{2s}$$

$$k(l_1 sq_1 + l_2 sq_2)\, l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2)\, l_2 sq_2 = \tau_{1s}$$

Set motor torques to be

$\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$ (Ans to T3)