Assignment 3 & 4
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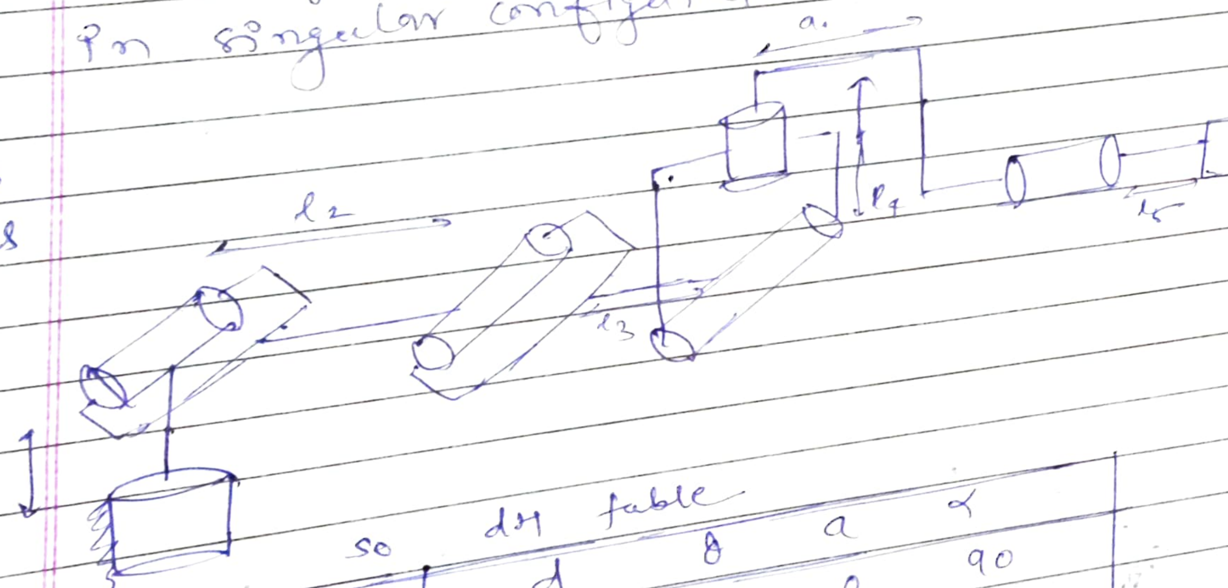
Ques 1
Ans when a parallel robot losses its control and rigidity and the end-effector degree of freedom is uncontrolable at that configuration.

→ we can calculate the singular configuration by determining the rank of matrix J^{-1}

→ when the rank of matrix J^{-1} is smaller then the degree of freedom then manipulator is in singular configuration.

Ques 6
Ans



so dH table

Link	d	θ	a	α
1	l_1	0	0	90
2	0	0	l_2	0
3	0	0	l_3	90
4	0	90	l_4	-90
		90	a_1	90
				0

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad H_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -l_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

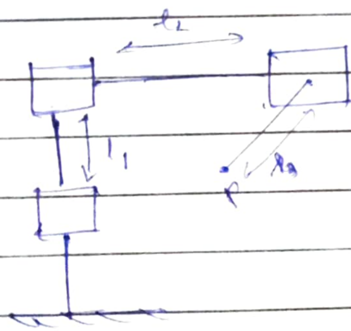
$$H_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad H_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

also we can write $H_0^6 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6$
which ~~beame~~ became

$$H_0^6 = \begin{bmatrix} 0 & 1 & 0 & l_2+l_3-l_4 \\ 0 & 0 & 1 & l_4+l_5 \\ 1 & 0 & 0 & l_1+a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and also writen as, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^6 \begin{bmatrix} P_6 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} P_0^6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & l_2+l_3-l_4 \\ 0 & 0 & 1 & l_4+l_5 \\ 1 & 0 & 0 & l_1+a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Ques



Ans

DH table

link	d	θ	a	α
1	l_1	0	0	-90
2	l_2	-90	0	-90
3	l_3	0	0	0

similarly we can write. $H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ and we can write

$$H_0^3 = H_0^1 H_1^2 H_2^3$$

so, by multiplying we get

$$H_0^3 = \begin{bmatrix} 0 & 0 & 1 & l_3 \\ 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

also,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0^3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & l_3 \\ 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ l_3 \\ 1 \end{bmatrix}$$

→ Inverse Kinematics.

$v = J$

Ques 10

Ans so for the given $D(q)$ and $V(q)$

$\mathcal{L} = K - V$ $\left\{ \begin{array}{l} K = \frac{1}{2}\{\dot{q}^T \cdot (D(q)) \cdot \dot{q}\} \\ V = V(q) \end{array} \right.$

So,

$$\tau = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q}$$

where $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \sum_i d_{ik}\ddot{q}_i + \sum_{ij} \frac{\partial d_{ik}}{\partial q_j}\dot{q}_i \dot{q}_j$

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{1}{2}\sum_{ij}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_k - \frac{\partial V(q)}{\partial q_k}$$

$$\therefore \tau_k = \sum_i d_{ik}\ddot{q}_i + \frac{1}{2}\sum_{ij}\left[\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + C_{ij}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + C_{ijk}\dot{q}_i\dot{q}_k + \phi_k(q)$$

More commonly.

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

where $D(q) = \begin{bmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{21} & - & - & & \\ d_{31} & - & - & & \vdots \\ \vdots & - & & & d_{nn} \end{bmatrix}$

so for $D(q)$ we can find $C(q,\dot{q})$ by

$$\sum_{ij} C_{ij} = \frac{1}{2}\left(\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right) \quad ; \quad g(q) = \frac{\partial V}{\partial q_k}$$

Ques 18

Ans 3D printer (PPP)

DH =

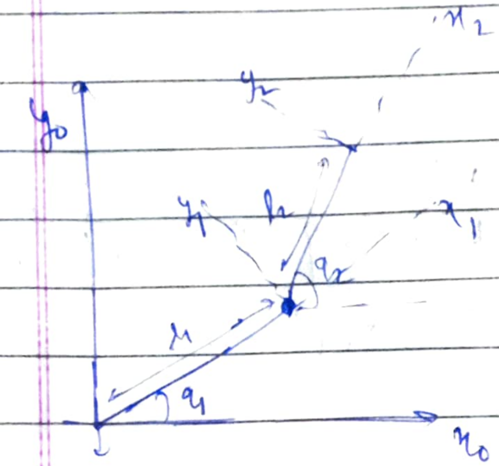
Link	d	θ	a	α
1	l_1	0	0	-90
2	l_2	-90	0	-90
3	l_3	0	0	0

By putting value $l_1 = l_2 = l_3 = 1$ and effector position [1, 1, 1]

Teacher's Signature

Ques 8

Ans



using Jacobian

$$V_{c1} = J_{c1} \cdot \dot{q} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 & 0 \\ \frac{l_1}{2}\cos q_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$V_{c2} = J_{c2} \cdot \dot{q} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Inertia matrix $D(q)$ is,

$$D(q) = m_1 J_{c1}^T \cdot J_{c1} + m_2 J_{c2}^T \cdot J_2 + \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$$

so, $$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & \frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2} \\ \frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2} & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

christoffels symbols.

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ik}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

so by putting values

$C_{111} = 0$, $C_{121} = C_{211} = 0$, $C_{221} = -\frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}$

$C_{112} = \frac{m_2 l_1 l_2 \sin(q_2 - q_1)}{2}$; $C_{212} = C_{122} = 0$, $C_{222} = 0$

also, $$V = \frac{m_1 g l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = \left(\frac{m_1 g l_1}{2} + m_2 g l_1\right)\cos q_1$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = \frac{m_2 g l_2 \cos q_1}{2}$$

so, $\tau_k = \sum_i d_{ik} \cdot \ddot{q}_i + \sum_{i,j} C_{ijk} \cdot \dot{q}_i \dot{q}_j + \phi_k$

so, $\tau_1 = \sum_i^2 d_{i1} \cdot \ddot{q}_i + \sum_{ij}^2 C_{ij1} \cdot \dot{q}_i \dot{q}_j + \phi_1$

$= d_{11} \ddot{q}_1 + d_{21} \ddot{q}_2 + C_{221} \dot{q}_2^2 + \phi_1$

$\tau_2 = \sum_i^2 d_{i2} \cdot \ddot{q}_i + \sum_{ij}^2 C_{ij2} \dot{q}_i \dot{q}_j + \phi_2$

$= d_{12} \ddot{q}_1 + d_{22} \ddot{q}_2 + C_{112} \dot{q}_1^2 + \phi_2$

Que 7

Ans i) Direct drive → the actuator is directly connected to the joints of the manipulator without intermidiate mechani

→ often provide high precision and accuracy

→ fast response time, suitable for quick movements.

→ relative simple in terms of construction & design

ii) Remotely ~~contr~~ driven → actuator is placed remotely, and connected through linkage or other transmission element

→ more compact design, benifical in constrained space.

→ reduced inertia and improved dynamic movement.

iii) 5-Bar parallelogram bar → linkage mechanism to drive. the manipulator joint

→ can provide increased movement stability.

→ force and torques are distributed more evenly