Assignment - 3

2011 00 ZP
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Q5)

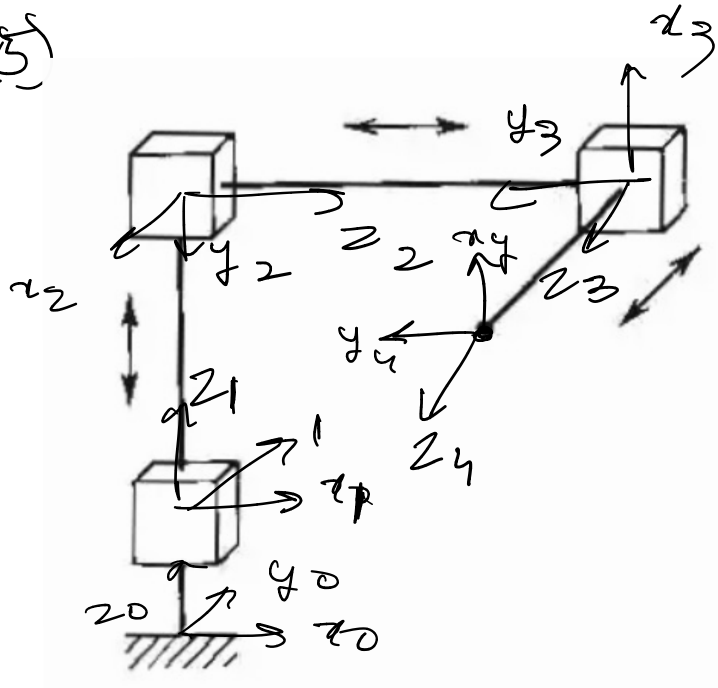


FIGURE 3-17
Three-link cartesian robot.

DH parameters

	d	θ	a	α
1	d_1	0	0	0
2	d_2	$-\pi/2$	0	$-\pi/2$
3	d_3	$-\frac{\pi}{2}$	0	$-\pi/2$
4	d_4	0	0	0

$$A_i = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & a c\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & a s\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = A_1 A_2 A_3 A_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & d_4 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & d_3 \\ 0 & 0 & -1 & d_4 \\ 1 & 0 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q1)
In a singular configuration, the rank of the Jacobian matrix reduces. It may also be observed that the manipulator's DOF will reduce, making it impossible to attain some configurations. To determine if the configuration is singular, we can take the matrix's determinant; if that becomes zero, we have a Singular configuration.

Q7)
1. Direct Drive 2R Manipulator:
    - Both robot arms are directly connected to motors at the base, moving the arms precisely.
    - Advantage: Simple and precise.
    - Disadvantage: Limited reach and not suitable for heavy loads.
2. Remotely-Driven 2R Manipulator:
    - One arm is driven by a motor away from the base, connected through belts or gears, allowing for extended reach.
    - Advantage: Can reach farther.
    - Disadvantage: More complex and less precise due to the added transmission elements.
3. 5-Bar Parallelogram Arrangement 2R Manipulator:
    - Two extra bars create a stable structure, and one arm connects to the end of one of these bars, providing stability and reducing up-and-down movement.
    - Advantage: Stable and reduces vertical motion.
    - Disadvantage: Limited flexibility and more complex to design.

26)

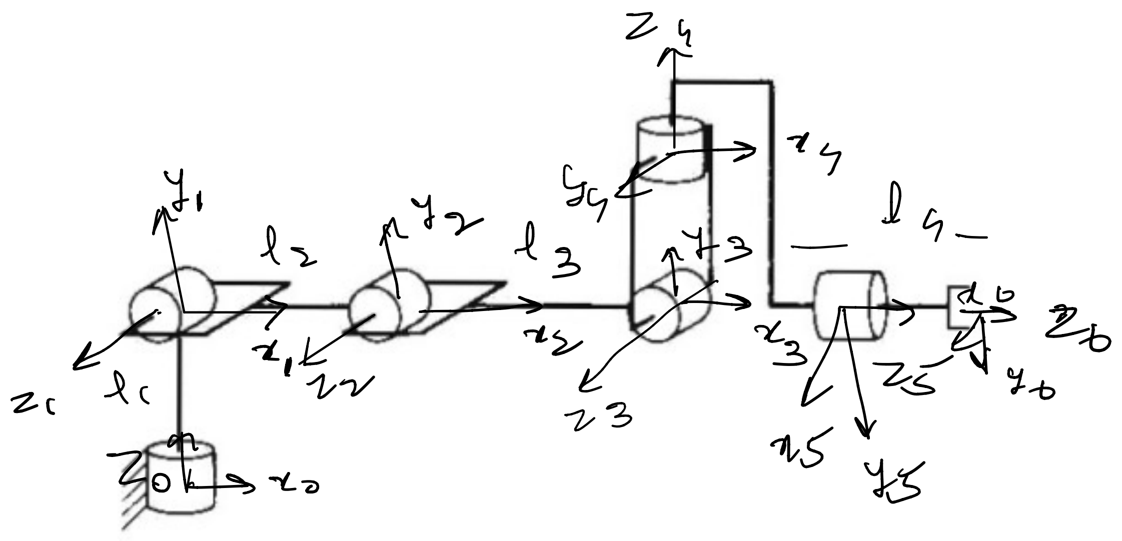


FIGURE 3-18
Elbow manipulator with spherical wrist.

Assuming $O_3 O_4 O_5$ coincide.

	d	θ	a	α
1	d_1	θ_1^*	0	$\pi/2$
2	0	θ_2^*	a_2	0
3	0	θ_3^*	l_3	0
4	0	θ_4^*	0	$-\pi/2$
5	0	θ_5^*	0	$\pi/2$
6	d_6	θ_6^*	0	0

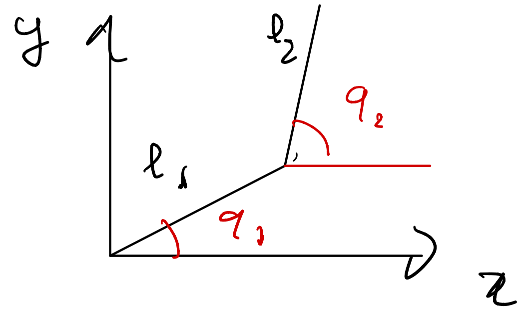
$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & a_3 C\theta_3 \\ S\theta_3 & 0 & -C\theta_3 & a_3 S\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} C\theta_4 & 0 & -S\theta_4 & 0 \\ S\theta_4 & 0 & C\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C\theta_5 & 0 & S\theta_5 & 0 \\ S\theta_5 & 0 & C\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

Q8)



$$v_{c_1} = \begin{bmatrix} -l_{c_1} s q_1 \\ l_{c_1} c q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$v_{c_2} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

$$D(q) = \begin{bmatrix} m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 & m_1 l_1 l_{c_2} c(q_2 - q_1) \\ m_2 l_1 l_{c_2} c(q_2 - q_1) & M_2 l_{c_2}^2 \; I_2 \end{bmatrix}$$

Cristoffel coefficients

$$C_{111} = \frac{1}{2} \frac{\delta d_{11}}{\delta q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \frac{\delta d_{11}}{\delta q_2} = 0$$

$$c_{221} = \frac{\delta d_{12}}{\delta q_2} - \frac{1}{2}\frac{\delta d_{21}}{\delta q_1} = -m_2 l_1 l_{c_2} \, s(q_2 - q_1)$$

$$c_{112} = \frac{\delta d_{21}}{\delta q_1} - \frac{1}{2}\frac{\delta d_{11}}{\delta q_2} = -m_2 l_1 l_{c_2} \, s(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\delta d_{22}}{\delta q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\delta d_{22}}{\delta q_2} = 0$$

$$V = m_1 g l_{c_1} sq_1 + m_2 g (l_1 sq_1 + l_{c_2} sq_2$$

$$\phi_1 = \frac{\delta V}{\delta q_1} = (m_1 l_{c_1} + m_2 l_1) g \, cq_1$$

$$\phi_2 = \frac{\delta V}{\delta q_2} = m_2 l_2 g \, c(q_2)$$

$$\tau_1 = d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1$$

$$\tau_2 = d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2$$

$d_{ij} \in D$ matrix

Q 10)

Eqns of motion

Given $D(q)$ & $V(q)$

$$L = K - V$$

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

$$V = V(q)$$

$$\tau = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$$\Rightarrow \tau_k = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k}\right] \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$\phi_k = \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{ij} c_{ijk} \dot{q}_i \dot{q}_j + \phi_k(q)$$

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \phi(q)$$