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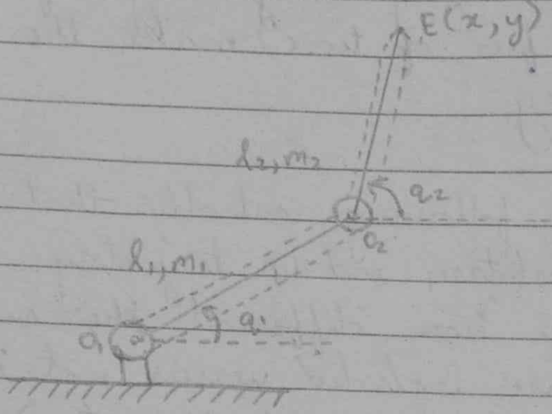
* Problem Statement :-

* 2R elbow manipulator



Consider the 2R elbow manipulator with two links of masses m_1 & m_2, length l_1 & l_2 & moments of inertia I_1 & I_2, End Effector E[x, y], joint angle (q_1, q_2), origins O_1 & O_2 connected to each motor respectively (Assume).

Let us assume we have a way to control either the torque τ_1 & τ_2 applied to two joint or control angles q_1 & q_2 directly.

↳ How we can control τ_1 & τ_2.

NOTE:- Angles are sometimes θ_1 & θ_2 or P_1 & P_2 in various textbook.

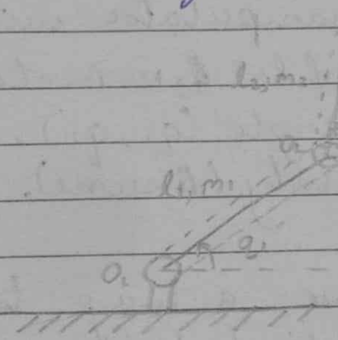
* Let us consider 3 tasks :-

Task - I :-

Given an arbitary trajectory of end-effector (given $E(x,y)$ as a fun^n of time) make the robot follow the trajectory.

A trajectory following controller that makes the robot follow an arbitary end-tip trajectory.

a) Also, analyze how different the results are when dynamics are included versus not included (try both low-speed and high-speed trajectory).



Now,

$x = l_1 \cos q_1 + l_2 \cos q_2$

$y = l_1 \sin q_1 + l_2 \sin q_2$

or Simplified Notation

$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2 \end{aligned}\right\} \text{———} \quad (1)$$

Differentiating eq^n ①, we get

$x' = -l_1 \sin q_1 \, q_1' - l_2 \sin q_2 \, q_2'$

$y' = l_1 \cos q_1 \, q_1' + l_2 \cos q_2 \, q_2'$

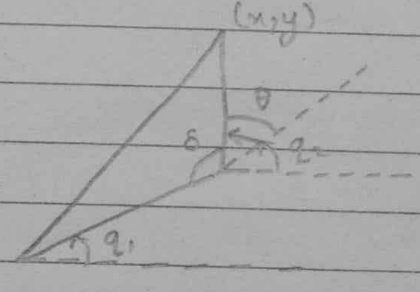
⇒ End-Effector velocity

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ q_2' \end{bmatrix} \quad \text{—— ②}$$ [Imp for Robotic]

we will need the reverse relationship.
Given x & y, we need to be able to find q_1 & q_2.

Option 1:- Solve numerically
Option 2:- Derive a closed form expression (easy method)
↳ We get answer directly substituting known value.

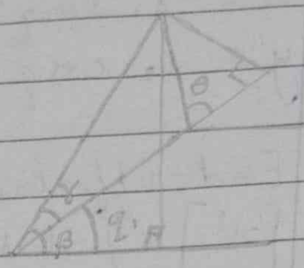
$q_1 = ----$, $q_2 = ----$
↳ Hard in general
↳ Multiple Solution



we get

$x^2 + y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$ (By cosine rule $c^2 = a^2 + b^2 + 2ab\cos\theta$.)

$$\therefore \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$



$$q_1 = \beta - \gamma$$

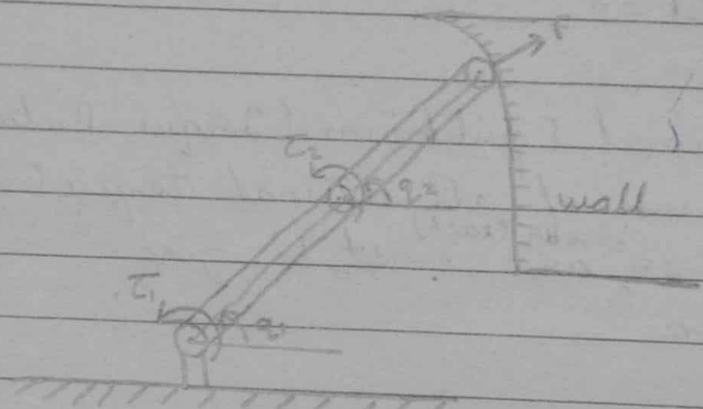
$$\therefore q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

$$q_2 = q_1 + \theta$$

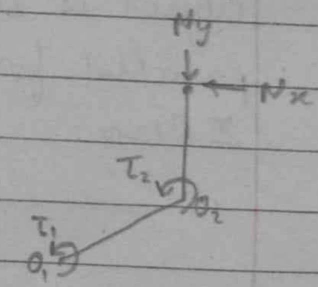
* Task - 2

Given a location on a wall (any solid surface), make a robot touch the wall and apply the pre-specified (constant) force at that location.

Assume a wall location and orientation, develop a robot reaching the wall and then applying a prespecified force in the direction normal to the wall.



Free-Body Diagram of entire robot



Force applied by the manipulator

$F_x = -N_x$ (neglect gravity)

$F_y = -N_y$

* (Concept - Static & Dynamic Equilibrium

① Static Equilibrium

↳ Refers to state in which an object ~~or~~ is at rest and its net force and net torque are both zero.

↳ Eq^n of Static Equilibrium:-

① Translational Equilibrium (Force Balance)
The vector sum of external forces acting in each dir^n must be 0. $\Sigma F = 0$.

② Rotational Equilibrium (Torque Balance)
The sum of all external torque (moments) acting on an object about an chosen (each) axis must be zero.
$\Sigma T = 0$.

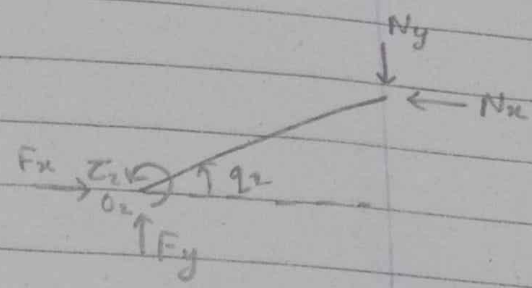
③ Couple or Moment Equilibrium
Object is subject to a system of couples (equal & opposite parallel forces), the net moment (torque) about any point = 0
$\Sigma M = 0$.

② Dynamic Equilibrium

↳ Refers to state in which an object is moving at a constant velocity (not accelerating) and its net force and net torque both zero.)

⇒ FBD of each link

↳ link (2)

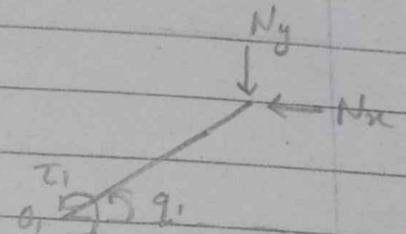


$\Sigma M_{O_2} = 0$ (By equilibrium)

Direction :- c.clockwise ⇒ +ve

$\therefore +N_y l_2 cq_2 - N_x l_2 sq_2 = \tau_2$

↳ link (1)



$\Sigma M_{O_2} = 0$

$\therefore N_y l_1 cq_1 - N_x l_1 sq_1 = \tau_1$

$$\therefore \left.\begin{array}{l} N_y l_1 cq_1 - N_x l_1 sq_1 = \tau_1 \\ N_y l_2 cq_2 - N_x l_2 sq_2 = \tau_2 \end{array}\right\} \text{— (4)}$$

$$\therefore \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & l_1 cq_1 \\ -l_2 sq_2 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix}$$

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* Task-3
Make a robot behave like a spring connected from E to given point (x_0, y_0)

Develop and implement a control scheme that makes the robot end-tip act like a virtual spring centred at mean position x_0 and y_0.

⇒ Task 3 is next level answer to Task I.

* Concept:-
Lagrange's Equation:-
↳ Lagrangian :- $\mathcal{L} = K - V$
here, K - Kinetic Energy, V = Potential energy.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i$$

Q_i are generalized forces derived using principle of virtual work.

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2}_{\text{pure rotation of } \mathcal{L}_1} + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2 + \frac{1}{2} m_2 v_{C_2}^2$$

$$v_{C_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_3\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

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$$v = m_1 g \frac{l_1}{2} sq_1 + m_2 g \left(l_1 sq_1 + \frac{l_2}{2} sq_2 \right)$$

(6)

$$\therefore \frac{1}{3} m_1 l_1^2 q_1'' + m_2 l_2^2 q_1'' + m_2 \frac{l_1 l_2}{2} q_2'' \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} q_2' (q_2' - q_1') \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} cq_1 + m_2 g l_1 cq_2 = \tau_1$$

$$\therefore \frac{1}{3} m_2 l_2^2 q_2'' + \frac{m_2 l_2^2}{4} q_2'' + m_2 \frac{l_1 l_2}{2} q_1'' (\cos(q_2 - q_1)) - m_2 \frac{l_1 l_2}{2} q_1' (q_2' - q_1') \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} sq_2 = \tau_2$$

Eq^n (6) describe dynamic motion of robot.

Next, we note that eq^n (4) is valid for any forces F_x, F_y (not just wall).

Eq^n (4) :- $N_y l_1 cq_1 - N_x l_1 sq_1 = \tau_1$

$$N_y l_2 cq_2 - N_x l_2 sq_2 = \tau_2$$

Want

$F_x = Kx$ { more generally, $F_x = k_x(x - x_0)$ }

$F_y = ky$ { $F_y = k_y(y - y_0)$ }

From eq^n (1)

$F_x = k(l_1 cq_1 + l_2 cq_2)$

$F_y = k(l_1 sq_1 + l_2 sq_2)$

From eq^n (4)

$k(l_1 sq_1 + l_2 sq_2) l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) l_2 sq_2 = \tau_{2s}$

$k(l_1 sq_1 + l_2 sq_2) l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) l_1 sq_1 = \tau_{1s}$

(τ_{1s} & τ_{2s} are τ_1 & τ_2 in multiverse)

Get motor torques to be $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$, resp.

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⇒ High motion of inertia will not to able to make movement in robot.

Another way to takle Task-I.
Solve $q_1 d$ & $q_2 d$ from eq^n ③

$$\downarrow$$

$\dot{q}_1 d_1$, $\ddot{q}_1 d$, $\dot{q}_2 d$, $\ddot{q}_2 d$.

$$\downarrow$$

τ_1 & $\tau_2 \longrightarrow$ ①

work better when dynamic effect are significants still need feedback control.

* Investigate with simulation :-
① What goes wrong with no feedback control?
② What goes wrong with no dynamic & only static?
③ What goes wrong with trying to achieve force & only position control ~~simulation~~ simultaneous.