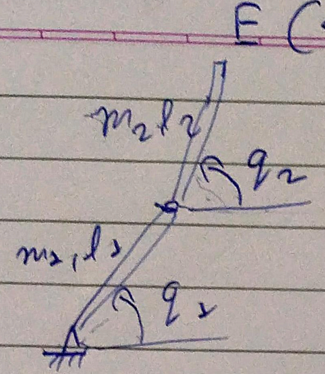
$\Rightarrow$



2 R Manipulator

T1 - Trajectory following.
T2 - Apply a force on a wall.
T3 → Act like a spring.

Sol^n $\Rightarrow$

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

or using simplified notation.

$$\left.\begin{aligned} x &= l_1 Cq_1 + l_2 Cq_2 \\ y &= l_1 Sq_1 + l_2 Sq_2 \end{aligned}\right\} \text{—— (1)}$$

Differentiating (1)

$$\dot{x} = -l_1 (Sq_1)\dot{q}_1 - l_2 Sq_2 (\dot{q}_2)$$
$$\dot{y} = l_1 Cq_1 (\dot{q}_1) + l_2 Cq_2 (\dot{q}_2)$$

$\Rightarrow$ End effector's velocity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 Sq_1 & -l_2 Sq_2 \\ l_1 Cq_1 & l_2 Cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

extremely imp q^n

(2)

Actual ques / Task 4

T_1 :→ Given an arbitary trajectory of the end effector (given (x,y) as fn of time), make the robot follow the trajectory.

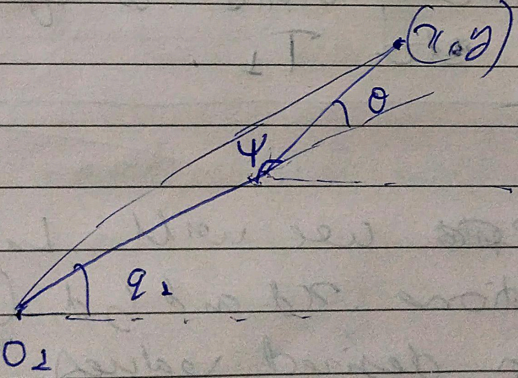
So, we will need the reverse relationship. i.e. given (x,y), we need to find q_1, q_2.

Option 1 → Solve numerically

Option 2 → Derive a closed form expression.

⇓

- Hard in general to derive
- Multiple solutions.



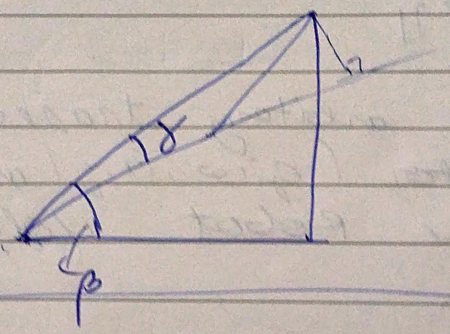
$\psi = 180 - \theta$

Using cosine rule 4

$$x^2+y^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\psi$$

$$x^2+y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

$$\therefore \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$



$$q_1 = \beta - \gamma$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2\cos\theta}\right)$$

$$q_2 = q_1 + \theta$$

(3)

So if we know what is (x,y), we can reach there by ~~q1 and q2~~ q_1 and q_2
This is inverse kinematics.

So till now, this is first black consumer to T_1.

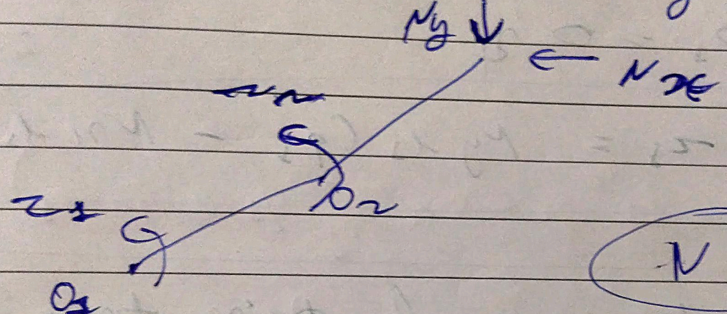
So, ~~some~~ we will later start using notations x_d and y_d (and $q_1 d$ and $q_2 d$) for desired values

{Bcoz they are not necessarily actual values.}

Task 2 ⇒ Given a location on a wall make the robot touch the wall at that location and apply a prespecified (constant) force at that location.

Solⁿ ⇒

Let forces applied by the manipulator are N_x and N_y.



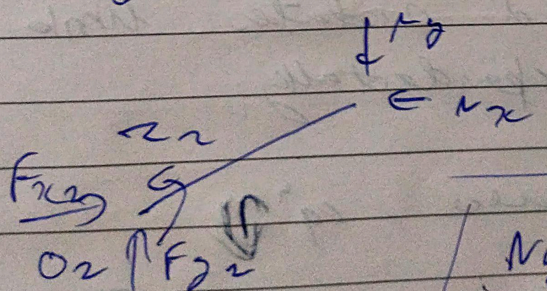
!!! Neglect Gravity !!!

N is given

Unknowns are τ_1, τ_2 !!

Approach ① !! Static equⁿ eqⁿ

→ FBD of each link separately

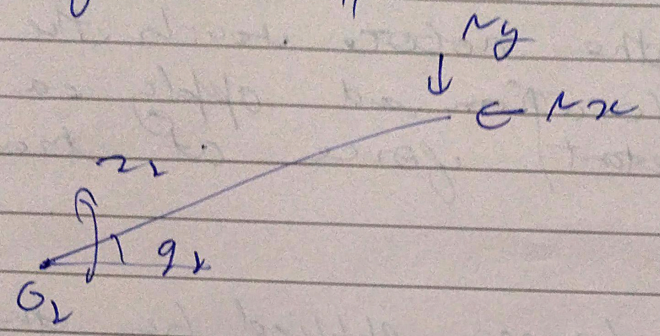


$$\sum M_{O_2} = 0$$

$$\boxed{N_y l_2 C_{q_2} - N_x l_2 S_{q_2} = \tau_2}$$

$$F_{x_2} = -N_x$$
$$F_{y_2} = -N_y$$

→ FBD of link 2



$$\Sigma M_{O_2} = 0 \quad \#$$

$$\boxed{\therefore \tau_2 = N_y l_2 C q_2 - N_x l_2 S q_2}$$

So my motors need this torque

NOTE: Above all is done when there are not joint motors i.e. ~~motor~~ there is no motor at O_2, one another motor at ground rotates link 2 independently.

From above two eqn (2)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 S q_1 & l_1 C q_1 \\ -l_2 S q_2 & l_2 C q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix} \quad \#$$

(4)

Same Relation as Position velocity ???

Task-3 => Make the robot behave like a virtual spring connected from E to a given point (x_0, y_0).

Sol^n => For T_3 and next need answer to T_1. Need to understand Dynamics

→ Lagrange's equations

Lagrangian :-> $L = \underset{K.E}{K} - \underset{P.E}{V}$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i}$$

Q_i are generalised forces derived using principle of virtual work.

here, $K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2$

$+ \cancel{\frac{1}{2} m_2 v_{c2}^2}$

$+ \frac{1}{2} m_2 v_{c2}^2$

where $v_{c2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$

$P.E\,(V) = m_1 g \frac{l_1}{2} Sq_1 + m_2 g \left(l_1 Sq_1 + \frac{l_2}{2} Sq_2 \right)$

By Langrange eq^n ∴

$$\left\{ \begin{array}{l} \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) \\ - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} Cq_1 + m_2 g l_1 Cq_1 \\ = \tau_1 \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - \\ m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} Sq_2 = \tau_2 \end{array} \right.$$

6

eq^n

Date
Page

These eq^n is like Janm kundli of the robot. i.e everything about the Robot !!

Next, we note that (4) is valid for any forces F_x, F_y (not just wall forces)

And we want these forces to be spring forces ⇒

$$F_x = kx \qquad F_y = ky$$

more generally ⇒
$$F_x = k_x(x - x_d)$$
$$F_y = k_y(y - y_d)$$

From (1) ⇒

$$F_x = k(l_1 Cq_1 + l_2 Cq_2)$$
$$F_y = k(l_1 Sq_1 + l_2 Sq_2)$$

From (4) ⇒

$$k(l_1 Sq_1 + l_2 Sq_2) l_2 Cq_2 - k(l_1 Cq_1 + l_2 Cq_2) l_2 Sq_2 = \tau_{2s}$$

$$k(l_1 Sq_1 + l_2 Sq_2) l_1 Cq_1 - k(l_1 Cq_1 + l_2 Cq_2) l_1 Sq_1 = \tau_{1s}$$

— (7)

means if at this τ_1 and τ_2, end effectors feels like behaving as 'Spring' !!

Set motor torques to be $\tau_1 + \tau_{1s}$ and $\tau_2 + \tau_{2s}$, respectively

Answer to T_3