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# Assignment 3-4

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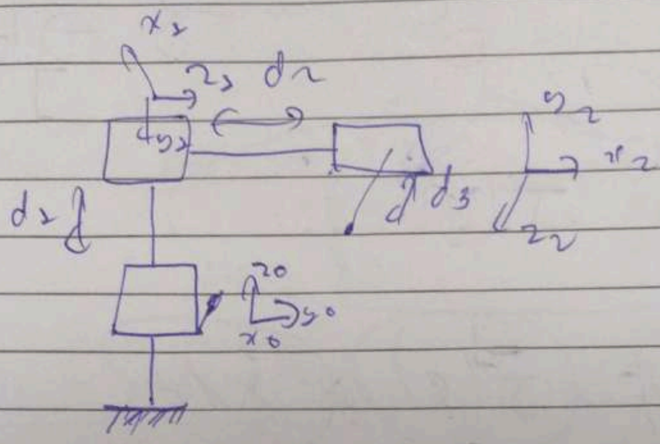


DH Parameters

| $n$ | $\theta$ | $a$ | $d$ | $\alpha$ |
|---|---|---|---|---|
| 1 | 0 | 0 | $d_1$ | -90 |
| 2 | 0 | 0 | $d_2$ | 90 |
| 3 | 0 | 0 | $d_3$ | 0 |

$$\therefore A_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A = A_0^1 \, A_1^2 \, A_2^3$$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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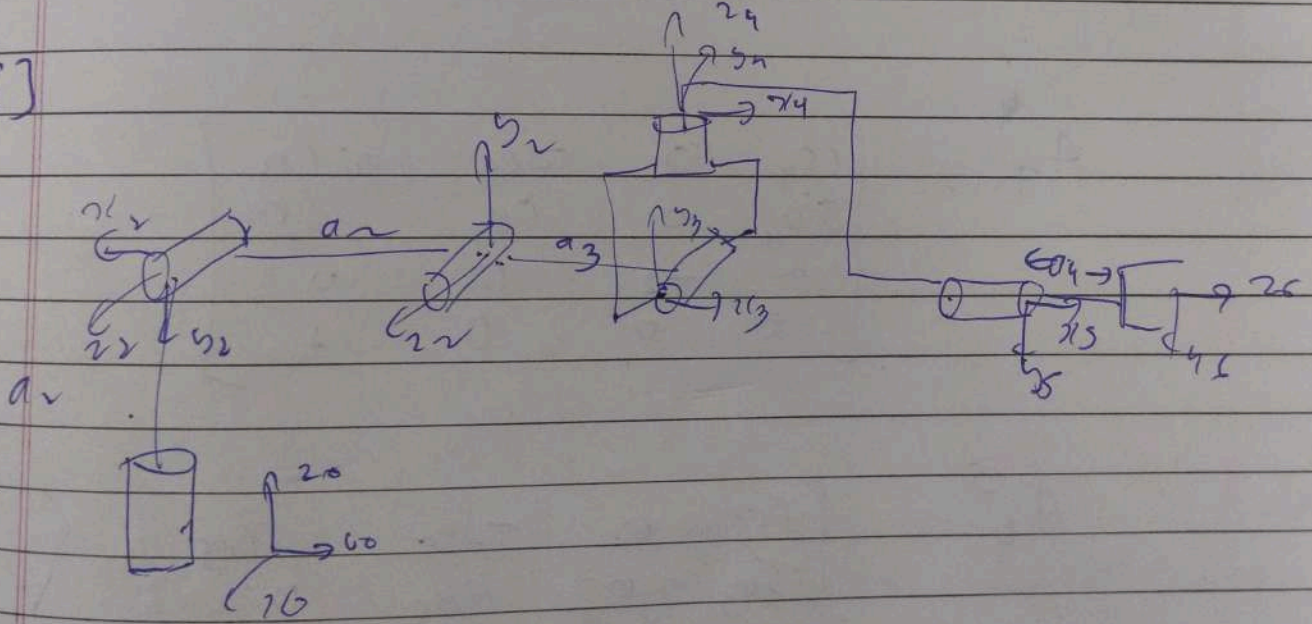


| n | θ | d | a | α |
|---|---|---|---|---|
| 1 | * | $a_1$ | 0 | −90 |
| 2 | * | 0 | $a_2$ | 0 |
| 3 | * | 0 | $a_3$ | 0 |
| 4 | * | 0 | 0 | −90 |
| 5 | * | 0 | 0 | −90 |
| 6 | * | $a_4$ | 0 | 0 |

$$\therefore A_0^1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} c\theta_2 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_2 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & a_4 c\theta_4 \\ s\theta_4 & 0 & c\theta_4 & a_4 s\theta_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^5 = \begin{bmatrix} c\theta_5 & 0 & -s\theta_5 & a_5 c\theta_5 \\ s\theta_5 & 0 & c\theta_5 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$a_5^6 = \begin{bmatrix} c\theta_6 & \overset{-\sin q}{0} & 0 & a_6 c\theta_6 \\ s\theta_6 & -c\theta_6 & 0 & a_6 s\theta_6 \\ 0 & 0 & 1 & a_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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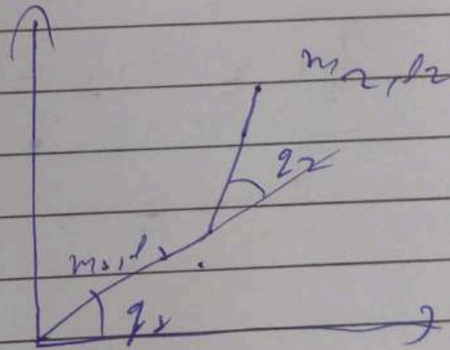
$$\boxed{\text{Hence, } A = A_0^1 A_1^2 A_2^3 A_3^4 A_4^5 A_5^6}$$

8] Lagrangian

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k$$

K.E $= \frac{1}{2} \dot{q}^T D \dot{q}$




$$J_{cv_1} = \begin{bmatrix} -l_1 S q_1 / 2 & 0 \\ l_1 C q_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{cv_2} = \begin{bmatrix} -l_1 \sin q_1 - \frac{l_2}{2} S(q_1+q_2) & -\frac{l_2}{2} S(q_1+q_2) \\ l_1 C q_1 + \frac{l_2}{2} C(q_1+q_2) & \frac{l_2}{2} C(q_1+q_2) \\ 0 & 0 \end{bmatrix}$$

$$V_{trans} = \frac{1}{2} \dot{q}^T \left\{ m_1 J_{cv_1}^T J_{cv_1} + m_2 J_{cv_2}^T J_{cv_2} \right\} \dot{q}$$

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$\omega_1 = \dot{q}_1 \hat{k}$ $\qquad\qquad$ $\omega_2 = (\dot{q}_1 + \dot{q}_2)\hat{k}$

$$K_{Rot} = \frac{1}{2}\dot{q}^T \left[ I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right] \dot{q}$$

Let $D(q) = m_1 J_{cv_1}^T J_{cv_1} + m_2 J_{cv_2}^T J_{cv_2} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$

$\therefore\ d_{11} = m_1 l_1^2/4 + m_2\left(l_1^2 + \frac{l_2^2}{4} + l_1 l_2 \cos q_2\right) + I_1 + I_2$

$d_{12} = d_{21} = m_2\left(\frac{l_2^2}{4} + \frac{l_1 l_2}{2}\cos q_2\right) + I_2$

$d_{22} = \frac{m_2 l_2^2}{4} + I_2$

$\therefore\ c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$

$c_{121} = \frac{\partial d_{12}}{\partial q_2} = h$

$c_{211} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$

$c_{122} = c_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$

$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$

For Potential energy ?

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$$V_1 = m_1 g \frac{l_1}{2} S q_1$$

$$V_2 = m_2 g \left( l_1 \sin q_1 + \frac{l_2}{2} \sin(q_1 + q_2) \right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = \left( \frac{m_1 l_1}{2} + m_2 l_1 \right) g \, c q_1 + m_2 \frac{l_2}{2} g \cos(q_1 + q_2)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 \frac{l_2}{2} \cos(q_1 + q_2)$$

∴ Now .k

$$\sum_j d_{kj}(q) \ddot{q}_j + \sum_{i,j} c_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k$$

$$k = 1, 2$$

∴

$$d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 + c_{121} \dot{q}_1 \dot{q}_2 + c_{211} \dot{q}_2 \dot{q}_1 + c_{221} \dot{q}_2^2 = \tau_1$$

$$d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + c_{112} \dot{q}_1^2 + \phi_2 = \tau_2$$

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(1) <u>Direct Drive Configuration</u>

⇓

End effector is directly connected to motors at the joint.

<u>No gear boxes in the motors.</u>

Advantages :→ (1) Simplicity
(2) Reduced mechanical compliance.

(2) <u>Remotely driven</u>

⇓

Here, end effector is not directly connected to the motors.

Advantages :→ (1) Increased workspace
(2) Increased Reach.
(4) Exterior applications.

(3) <u>5 Bar - Parallelogram</u>

⇓

Two base joints forms one side of parallelogram and two end effector joints forms opposite side of it.

Advantages :→ (1) Improved Structure
(2) Enhanced control.

1] Singular Configurations

a) It refers to specific arrangement where it loses some degree of freedom and become unstable to move in certain directions.

Singularity arises when J (Jacobian) loses Rank.

b) Robot configuration is singular when,

$$|J(q)| = 0$$

i.e. $\underline{Det(J(q)) = 0}$

c) If determinant of Jacobian tends to zero, we can say that the configuration is close to a singular config.