Q 5

⇒ Problem 3.7

Consider the three link cartesian manipulator of figure. Derive the forward kinematics equations using DH convention.

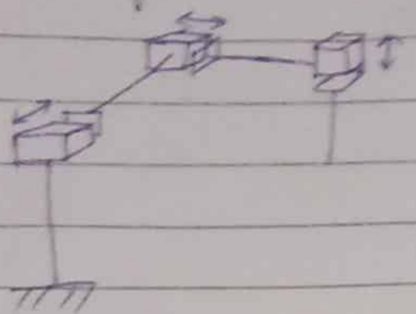

Fig :-

* DH - Parameter Table

link	d	θ	a	α
1	d_1	0	0	$-\pi/2$
2	d_2	0	0	$-\pi/2$
3	d_3	0	0	0

We know

from 0 to n links

$$A_{i-1}^{i} = Trans_{z,d}\ Rot_{z,\theta}\ Trans_{x,a}\ Rot_{x,\alpha}$$

$$\therefore A_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$\therefore$ $A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ~~, A_3~~

$\therefore$ $A = A_0 \times A_1 \times A_2$

$\Rightarrow$ $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) ~~Further conclusion.~~

Q-6

Problem 3.8
Attach a spherical wrist to the three-link articulated manipulator of Problem 3.6 as shown in fig. 3.29
Derive the forward kinematics equations for this manipulator.

⇒ Problem 3.6
Consider the three-link articulated robot of fig 3.27
Derive the forward kinematics eq^n using DH-convention.

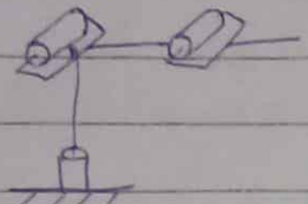

Fig 3.27

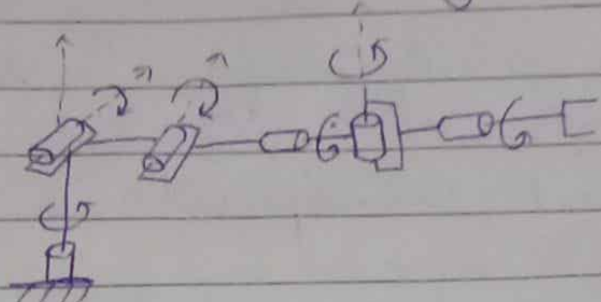

Fig 3.29

⇒ DH - Parameters Tables

links	d	θ	a	α
1	d_1	0	$\pi/2$	θ
2	0	a_2	0	θ
3	0	a_3	0	θ
4	0	$-\pi/2$	0	θ
5	0	$-\pi/2$	0	θ
6	d_6	0	0	θ

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we know

$$A_{n+1} = \begin{bmatrix} c\theta_n & -s\theta_n c\theta_n & s\theta_n s\alpha_n & a c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_1 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_1 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A_6^0 = A_1 . A_2 . A_3 . A_4 . A_5 . A_6$$

$$\Rightarrow A_6^0 = \begin{bmatrix} R_{3\times3} & t_{1\times3} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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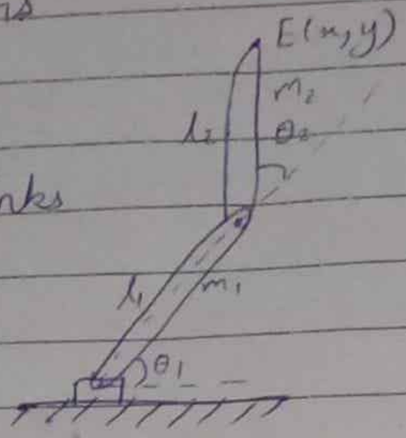
Q-8

Complete the derivation of the dynamic eq^ns of 2R manipulator discussed in class and compare your results with those in the miniproject. Remark on any discrepancies or observations



θ_1, θ_2: Joint angles
l_1, l_2: lengths of the two links
m_1, m_2: masses of the links
I_1, I_2: moment of inertia of each link

=> First Kinematics Energy

$$\boxed{K_i = \frac{1}{2} m_i V_{c_i}^2 + \frac{1}{2} I_c \dot{\theta}_i^2}$$

using this the kinematics energy for link 1 & link 2 can be calculated

$\therefore K_i = K_1 + K_2$

=> Potential Energy
$V = V_1 + V_2$

=> Lagrangian => $\boxed{\mathcal{L} = K - V}$

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Lagrangian Equation

$$\boxed{\frac{d}{dt}\left(\frac{dL}{d\theta_i} - \frac{\partial L}{\partial \theta_i}\right) = \tau_n} \quad \text{— (1)}$$

Finally solving (1) we get

$$\sum_i d_{in}(\theta)\ddot{\theta}_i + \sum_{i,j} \frac{\partial}{\partial \theta_i} d_{in}(\theta)\dot{\theta}_i\dot{\theta}_j - \frac{1}{2}\sum_{i,j}\frac{\partial_{id}(\theta_i)}{\partial \theta_n}\dot{\theta}_i\dot{\theta}_j + \frac{\partial V(\theta)}{\partial \theta_n} = \tau_n$$

$$\boxed{C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{in}(\theta)}{\partial \theta_j} + \frac{\partial d_{ij}(\theta)}{\partial \theta_i} - \frac{\partial d_{ij}(\theta)}{\partial \theta_k}\right]} \rightarrow \text{Christoffel's symbols}$$

$$\therefore \sum d_{in}(\theta)\ddot{\theta}_i + \sum C_{ijk}(\theta,\dot{\theta})\dot{\theta}_i\dot{\theta}_j + d_n(\theta) = \tau_n$$

$$\boxed{D(\theta)\ddot{\theta} + C(\theta\dot{\theta}) + \phi(q) = \tau}$$

⇒ Hence ~~Pr~~ Derived.

In miniproject, a brute force approach was used ~~at~~ without ~~code~~ considering Christoffel symbols, making calculation challenging.