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ME639 - Assignment 3 & 4

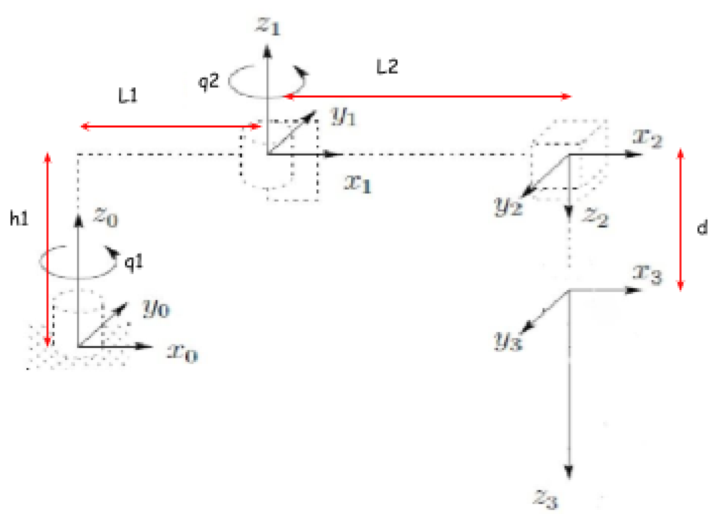
Q1.

Ans: A singular configuration in robotics refers to a specific pose of a robotic arm where the manipulator loses one or more degrees of freedom, making certain motions impossible or highly sensitive. Singular configurations are identified by examining the determinant of the Jacobian matrix, which describes the relationship between joint velocities and end-effector velocities. A singular configuration occurs when the determinant of the Jacobian becomes zero, indicating that the robot is at risk of becoming stuck or experiencing infinite joint velocities. The proximity to a singular configuration can be inferred by analyzing the condition number of the Jacobian i.e. how sensitive a robotic manipulator is to small changes in joint velocities. It is calculated as the ratio of the largest singular value to the smallest singular value of the Jacobian matrix. where a high condition number suggests the manipulator is close to a singularity.

Q4.

Ans:

- **SCARA Robot**



$DH =$

Link	d	θ	a	α
1	h_1	q_1	l_1	0
2	0	q_2	l_2	180
3	d	0	0	0

Jacobian for SCARA robot is:

$$\begin{bmatrix} -l_1 \cdot \sin(q_1) - l_2 \cdot \sin(q_1 + q_2) & -l_2 \cdot \sin(q_1 + q_2) & 0 \\ l_1 \cdot \cos(q_1) + l_2 \cdot \cos(q_1 + q_2) & l_2 \cdot \cos(q_1 + q_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Let's check the output of code with manually calculated jacobian matrix

Example1:

$DH =$

Link	d	θ	a	α
1	1	0	1	0
2	0	0	1	180
3	1	0	0	0

$$J = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[ 0. -0.  0.]
 [ 2.  1.  0.]
 [ 0.  0. -1.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 1.  1.  0.]]
```

Example2:

$DH =$

Link	d	θ	a	α
1	1	90	1	0
2	0	90	1	180
3	1	0	0	0

$$J = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[-1. -0.  0.]
 [-1. -1.  0.]
 [ 0.  0. -1.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 1.  1.  0.]]
```

Example3:

$DH =$

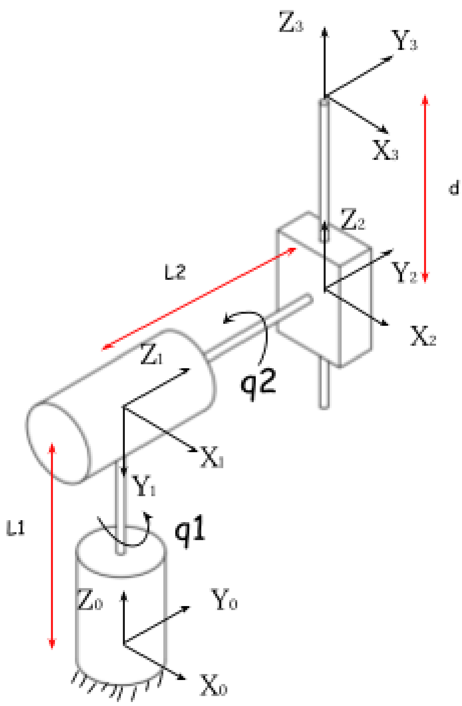
Link	d	θ	a	α
1	1	30	1	0
2	0	60	1	180
3	1	0	0	0

$$J = \begin{bmatrix} -1.5 & -1 & 0 \\ 0.865 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[-1.51 -1.01  0.  ]
 [ 0.87  0.    0.  ]
 [ 0.    0.   -1.  ]
 [ 0.    0.    0.  ]
 [ 0.    0.    0.  ]
 [ 1.    1.    0.  ]]
```

Hence the manually calculated results are the same as results obtained from code.

- ## **Stanford Manipulator**



DH parameters:

Link	d	θ	a	α
1	l_1	q_1	0	-90
2	l_2	q_2	0	90
3	d	0	0	0

Jacobian for SCARA robot is:

$$\begin{bmatrix} -d\cdot\sin(q_1)\cdot\sin(q_2)-l_2\cdot\cos(q_1) & d\cdot\cos(q_1)\cdot\cos(q_2) & \cos(q_1)\cdot\sin(q_2) \\ d\cdot\cos(q_1)\cdot\sin(q_2)-l_2\cdot\sin(q_1) & d\cdot\sin(q_1)\cdot\cos(q_2) & \sin(q_1)\cdot\sin(q_2) \\ 0 & -d\cdot\sin(q_2) & \cos(q_2) \\ 0 & -\sin(q_1) & 0 \\ 0 & \cos(q_1) & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Let’s check the output of code with manually calculated jacobian matrix

Example1:

$DH =$

Link	d	θ	a	α
1	1	0	0	-90
2	1	0	0	90
3	1	0	0	0

$$J = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[-1.  1.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  1.]
 [ 0.  0.  0.]
 [ 0.  1.  0.]
 [ 1.  0.  0.]]
```

Example2:

$DH =$

Link	d	θ	a	α
1	1	90	0	-90
2	1	90	0	90
3	1	0	0	0

$$J = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[-1.  0.  0.]
 [-1.  0.  1.]
 [ 0. -1.  0.]
 [ 0. -1.  0.]
 [ 0.  0.  0.]
 [ 1.  0.  0.]]
```

Output of code:

Example3:

$DH =$

Link	d	θ	a	α
1	1	30	0	-90
2	1	60	0	90
3	1	0	0	0

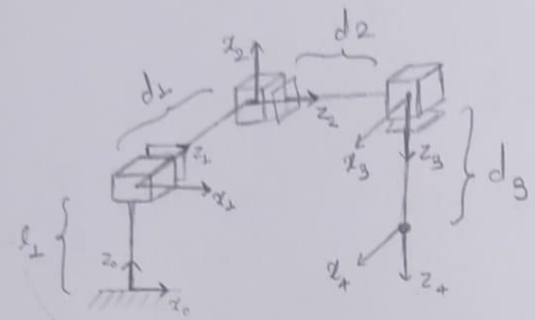
$$J = \begin{bmatrix} -1.3 & 0.433 & 0.75 \\ 0.25 & 0.25 & 0.433 \\ 0 & -0.866 & 0.5 \\ 0 & -0.5 & 0 \\ 0 & 0.866 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

```
Manipulator Jacobian:
[[-1.3   0.44  0.76]
 [ 0.26  0.25  0.44]
 [ 0.   -0.88  0.5 ]
 [ 0.   -0.5   0.  ]
 [ 0.    0.87  0.  ]
 [ 1.    0.    0.  ]]
```

Hence the manually calculated results are the same as results obtained from code.

Q5.
Ans: problem 3-7 in the textbook

Q.5.



l_1 is fixed

$\dot{l}_1 = 0$

DH parameters are:

Link	d	θ	a	α
1	l_1	0	0	$-\pi/2$
2	d_1	$-\pi/2$	0	$-\pi/2$
3	d_2	$\pi/2$	0	$-\pi/2$
4	d_3	0	0	0

$$H_{i-1}^{i} = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 \cdot H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_0^3 = H_0^2 \cdot H_2^3 = \begin{bmatrix} 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & -1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^4 = H_0^3 \cdot H_3^4 = \begin{bmatrix} 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & -1 & l_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore\ o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \quad o_2 = \begin{bmatrix} 0 \\ d_1 \\ l_1 \end{bmatrix} \quad o_3 = \begin{bmatrix} d_2 \\ d_1 \\ l_1 \end{bmatrix} \quad o_4 = \begin{bmatrix} d_2 \\ d_1 \\ l_1 - d_3 \end{bmatrix}$$

$$R_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \Rightarrow z_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow z_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 & z_1 & z_2 & z_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{Jacobian}$$

If let's $l_1 = 1$, $d_1 = 2.5$, $d_2 = 3.7$, $d_3 = 5.7$
then end effector position should be = $\begin{bmatrix} 3.7 \\ 2.5 \\ -4.7 \end{bmatrix}$

Running code from Task 3 (Q3) for above values gives:

```
Provide DH parameters

For Link 1:
d: 1
θ (degrees): 0
a: 0
α (degrees) : -90

For Link 2:
d: 2.5
θ (degrees): -90
a: 0
α (degrees) : -90

For Link 3:
d: 3.7
θ (degrees): 90
a: 0
α (degrees) : -90

For Link 4:
d: 5.7
θ (degrees): 0
a: 0
α (degrees) : 0
```

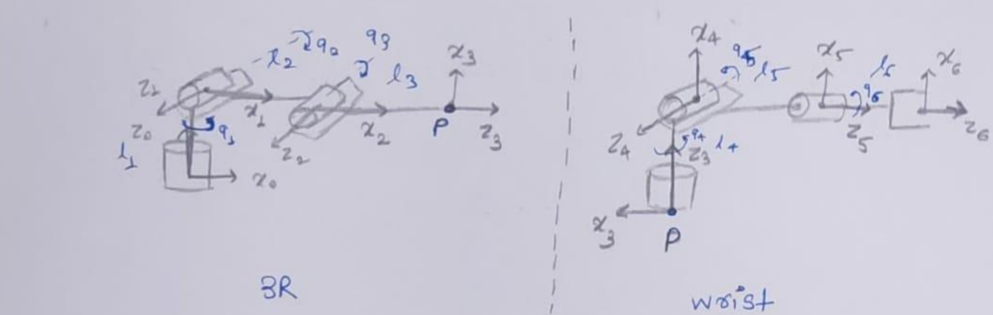
```
Manipulator Jacobian:
[[ 0.  0.  1.  0.]
 [ 0.  1.  0.  0.]
 [ 1.  0.  0. -1.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]]

End effector position:
[[ 3.7]
 [ 2.5]
 [-4.7]]
```

So manually calculated values are equal to values given by code.

Q6.
Ans: problem 3-8 in the textbook

Q.6.
→ let's split 3R & wrist.



DH :-

link	d	θ	a	α
1	l_1	q_1	0	$\pi/2$
2	0	q_2	l_2	0
3	0	q_3	l_3	$\pi/2$
4	l_4	q_4	0	$-\pi/2$
5	0	q_5	l_5	$\pi/2$
6	l_6	q_6	a_6	0

$$H_{i-1}^{i} = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} C_3 & 0 & S_3 & l_3C_3 \\ S_3 & 0 & -C_3 & l_3S_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_3^4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & +C_4 & 0 \\ 0 & -1 & 0 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^5 = \begin{bmatrix} C_5 & 0 & S_5 & l_5C_5 \\ S_6 & 0 & -C_5 & l_5S_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_5^6 = \begin{bmatrix} C_6 & -S_6 & 0 & \cancel{l_6C_6} & 0 \\ S_6 & C_6 & 0 & \cancel{l_6S_6} & 0 \\ 0 & 0 & 1 & \cancel{0} & l_6 \\ 0 & 0 & 0 & & 1 \end{bmatrix}$$

$\therefore H_0^6 = H_0^1 \cdot H_1^2 \cdot H_2^3 H_3^4 \cdot H_4^5 \cdot H_5^6$

If we take $\theta_i = 0$ & $l_i = 1$

Then manually end effectorposition = $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Running code from Task 3 (Q3) for above values gives:

```
For Link 1:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 90

For Link 2:
d: 0
θ (degrees): 0
a: 1
α (degrees) : 0

For Link 3:
d: 0
θ (degrees): 0
a: 1
α (degrees) : 90

For Link 4:
d: 1
θ (degrees): 0
a: 0
α (degrees) : -90

For Link 5:
d: 0
θ (degrees): 0
a: 1
α (degrees) : 90

For Link 6:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 0
```

```
Manipulator Jacobian:
[[-0.  2.  2.  0.  1.  0.]
 [ 3.  0.  0. -1.  0.  0.]
 [ 0.  3.  2.  0.  1.  0.]
 [ 0.  0.  0.  0.  0.  0.]
 [ 0. -1. -1.  0. -1.  0.]
 [ 1.  0.  0. -1.  0. -1.]]

End effector position:
[[ 3.]
 [ 0.]
 [-1.]]
```

So manually calculated values are equal to values given by code.

Q7.
Ans:

Configurations for a 2R manipulator:

1. Direct Drive:

Configuration:
In a direct drive setup, the actuator is directly connected to the joints of the manipulator without any intermediate mechanisms.

Advantages:
Direct drive systems often provide high precision and accuracy since there are fewer components introducing play or backlash.
With fewer components, the response time can be faster, making it suitable for applications that require quick and precise movements.
The direct drive is relatively simple in terms of design and construction.

2. Remotely-Driven:

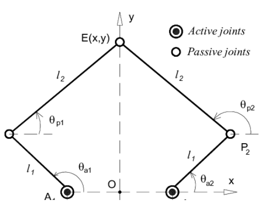
Configuration:
In this setup, the actuator is placed remotely and is connected to the manipulator joints through linkages or other transmission elements.

Advantages:
Since the actuator is not directly at the joints, it allows for a more compact design and can be beneficial in constrained spaces.
The mass of the actuators is not directly attached to the moving parts, which can reduce inertia and improve dynamic performance.
Actuators can be placed in locations that optimize the overall system's balance and stability.

3. 5-Bar Parallelogram:

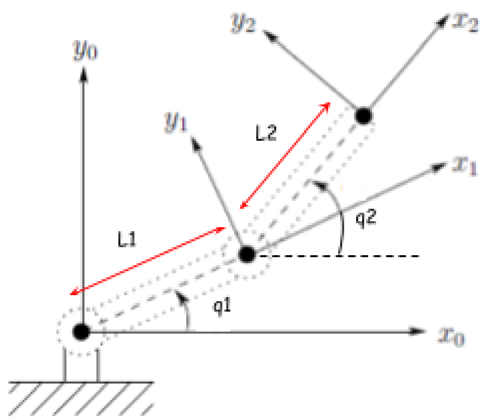
Configuration:
This configuration employs a 5-bar parallelogram linkage mechanism to drive the manipulator joints.

Advantages:
The parallelogram arrangement can provide increased stability during movement, especially in applications where stability is crucial.
Forces and torques are distributed more evenly, reducing the strain on individual joints and improving overall system longevity.
With a more balanced distribution of forces, wear and tear on individual components are often minimized.



Q8.
Ans:

2R- Manipulator (Planar)



$$w_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q_1} \end{bmatrix}, \quad w_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q_2} \end{bmatrix}$$

$$V_{c_1} = J_{c_1} \cdot \dot{q} = \begin{bmatrix} \frac{-l_1}{2} \cdot \sin(q_1) & 0 \\ \frac{l_1}{2} \cdot \cos(q_1) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix}$$

$$V_{c_2} = J_{c_2} \cdot \dot{q} = \begin{bmatrix} -l_1 \cdot \sin(q_1) & \frac{-l_2}{2} \cdot \sin(q_2) \\ l_1 \cdot \cos(q_1) & \frac{l_2}{2} \cdot \cos(q_2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix}$$

Inertia matrix D(q) is given as

$$D(q) = m_1 \cdot J_{c_1}^T \cdot J_{c_1} + m_2 \cdot J_{c_2}^T \cdot J_2 + \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$$

After simplification it becomes,

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Christoffels symbols are given as,

$$C_{ijk} = \frac{1}{2} \left(\frac{\partial q_{kj}}{\partial q_i} + \frac{\partial q_{ik}}{\partial q_j} - \frac{\partial q_{ij}}{\partial q_k} \right)$$

$$C_{111} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial q_1} + \frac{\partial d_{11}}{\partial q_1} - \frac{\partial d_{11}}{\partial q_1}\right) = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{21}}{\partial q_1} - \frac{\partial d_{21}}{\partial q_1}\right) = 0$$

$$C_{221} = \frac{1}{2}\left(\frac{\partial d_{12}}{\partial q_2} + \frac{\partial d_{21}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1}\right) = \frac{-m_2 l_1 l_2}{2}\sin(q_2 - q_1)$$

$$C_{112} = \frac{1}{2}\left(\frac{\partial d_{21}}{\partial q_1} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{11}}{\partial q_2}\right) = \frac{m_2 l_1 l_2}{2}\sin(q_2 - q_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial q_1} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial d_{12}}{\partial q_2}\right) = 0$$

$$C_{222} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial q_2} + \frac{\partial d_{22}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_2}\right) = 0$$

The potential energy is given as

$$V = \frac{m_1 g l_1}{2}\sin(q_1) + m_2 g \cdot \left(l_1 \sin(q_1) + \frac{l_2}{2}\sin(q_2)\right)$$

So potential terms are given as

$$\phi_1 = \frac{\partial V}{\partial q_1} = \left(\frac{m_1 g l_1}{2} + m_2 g l_1\right)\cos(q_1)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = \frac{m_2 g l_2}{2}\cos(q_1)$$

So torque is given by,

$$\boxed{.\quad \tau_k = \sum_{i}^{n} d_{ik} \cdot \ddot{q_i} + \sum_{i,j}^{n} C_{ijk} \cdot \ddot{q_i}\ddot{q_j} + \phi_k \quad .}$$

$$\tau_1 = \sum_{i}^{2} d_{i1} \cdot \ddot{q_i} + \sum_{i,j}^{2} C_{ij1} \cdot \ddot{q_i}\ddot{q_j} + \phi_1$$

$$= d_{11}\ddot{q_1} + d_{21}\ddot{q_2} + C_{111}\dot{q_1}^2 + C_{211}\dot{q_2}\dot{q_1} + C_{121}\dot{q_1}\dot{q_2} + C_{221}\dot{q_2}^2 + \phi_1$$

$$= d_{11}\ddot{q_1} + d_{21}\ddot{q_2} + C_{221}\dot{q_2}^2 + \phi_1$$

$$\tau_2 = \sum_{i}^{2} d_{i2} \cdot \ddot{q_i} + \sum_{i,j}^{2} C_{ij2} \cdot \ddot{q_i}\ddot{q_j} + \phi_2$$

$$= d_{12}\ddot{q_1} + d_{22}\ddot{q_2} + C_{112}\dot{q_1}^2 + C_{212}\dot{q_2}\dot{q_1} + C_{122}\dot{q_1}\dot{q_2} + C_{222}\dot{q_2}^2 + \phi_2$$

$$= d_{12}\ddot{q_1} + d_{22}\ddot{q_2} + C_{112}\dot{q_1}^2 + \phi_2$$

As there were hardware issues for the OSAKA 2R manipulator, hence we are not able to compare results.

Q10.
Ans:

10]

$\longrightarrow$ Given $D(q)$ and $V(q)$

Lagrangian eq^n is $\mathcal{L} = K - V$

where $K = \frac{1}{2}\dot{q}^T . D(q) . \dot{q}$

$V = V(q)$

$$\tau = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q}$$

where $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \sum_i d_{ik}\ddot{q}_i + \sum_{i,j} \frac{\partial d_{ik}}{\partial q_j}\dot{q}_i\dot{q}_j$

$\frac{\partial \mathcal{L}}{\partial q} = \frac{1}{2}\sum_{i,j}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial V(q)}{\partial q_k}$

$$\therefore \tau_k = \sum_i d_{ik}\ddot{q}_i + \frac{1}{2}\sum_{i,j}\left[\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + c_{ijk}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_i d_{ik}\ddot{q}_i + c_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q)$$

$= \cancel{D(q)} \cdot \ddot{q} +$

More commonly

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

where $D(q) = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & & & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix}$

So when we given $D(q)$ we will find $C(q,\dot{q})$

using $\sum_{i,j} c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{ik}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right)$

$g(q) = \frac{\partial V}{\partial q_k}$

Q12.
Ans: STANFORD TYPE MANIPULATOR

Forward kinematics

```
Provide DH parameters

For Link 1:
d: 1
θ (degrees): -45
a: 0
α (degrees) : -90

For Link 2:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 90

For Link 3:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 0
```

```
End effector position:
[[0.71]
 [0.71]
 [2.  ]]
```

Inverse kinematics

```
x = 0.707
y = 0.707
z = 2
length of link 1 (l1) = 1
length of link 2 (l2) = 1
height of link 3 (d)  = 1
q1 =  -44.99
q2 =  0.0
```

Hence results are matching

Q13.
Ans: SCARA MANIPULATOR

Forward kinematics

```
Provide DH parameters

For Link 1:
d: 1
θ (degrees): 30
a: 1
α (degrees) : 0

For Link 2:
d: 0
θ (degrees): 60
a: 1
α (degrees) : 180

For Link 3:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 0
```

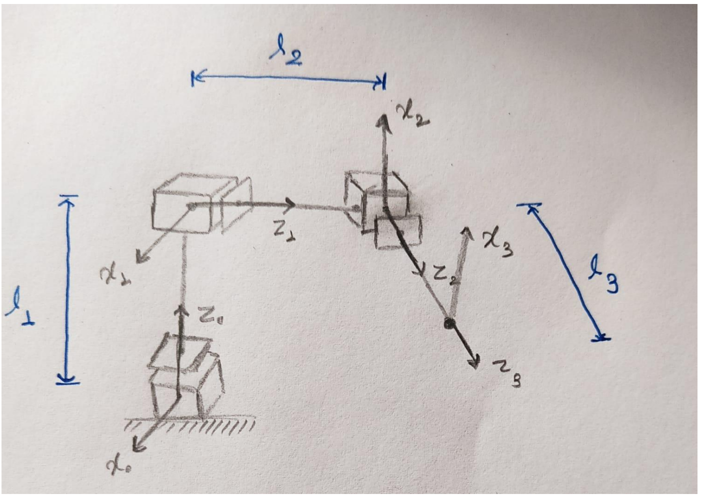
```
End effector position:
[[0.87]
 [1.51]
 [0.  ]]
```

Inverse kinematics

```
x = 0.87
y = 1.51
z = 0
height of link 1 (d1)  = 1
length of link 1 (l1) = 1
length of link 2 (l2) = 1
q1 =  30.67
q2 =  58.77
```

Q18.

Ans: 3D printer (PPP)



$DH =$

Link	d	θ	a	α
1	l_1	0	0	-90
2	l_2	-90	0	-90
3	l_3	0	0	0

Example1: Let, L1=L2=L3=1 then end effector position should be [1,1,1]

$DH =$

Link	d	θ	a	α
1	1	0	0	-90
2	1	-90	0	-90
3	1	0	0	0

Let’s use this value in code from task3

```
Type of joint 1: P
Type of joint 2: P
Type of joint 3: P

Provide DH parameters

For Link 1:
d: 1
θ (degrees): 0
a: 0
α (degrees) : -90

For Link 2:
d: 1
θ (degrees): -90
a: 0
α (degrees) : -90

For Link 3:
d: 1
θ (degrees): 0
a: 0
α (degrees) : 0
```

```
Manipulator Jacobian:
[[0. 0. 1.]
 [0. 1. 0.]
 [1. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]

End effector position:
[[1.]
 [1.]
 [1.]]
```

Hence results are the same.

```
Give input for q_dot for joints
Enter Linear velocity for Prismatic joint 1: 2
Enter Linear velocity for Prismatic joint 2: 2
Enter Linear velocity for Prismatic joint 3: 2

Velocity of end effector:
[[2.]
 [2.]
 [2.]
 [0.]
 [0.]
 [0.]]
```

Example2: Let, L1= 1, L2=-2.7, L3=1.5 then end effector position should be [1.5,-2.7,1]

$DH =$

Link	d	θ	a	α
1	1	0	0	-90
2	-2.7	-90	0	-90
3	1.5	0	0	0

Let's use this value in code from task3

```
Type of joint 1: P
Type of joint 2: P
Type of joint 3: P

Provide DH parameters

For Link 1:
d: 1
θ (degrees): 0
a: 0
α (degrees) : -90

For Link 2:
d: -2.7
θ (degrees): -90
a: 0
α (degrees) : -90

For Link 3:
d: 1.5
θ (degrees): 0
a: 0
α (degrees) : 0
```

```
Manipulator Jacobian:
[[0. 0. 1.]
 [0. 1. 0.]
 [1. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]
 [0. 0. 0.]]

End effector position:
[[ 1.5]
 [-2.7]
 [ 1. ]]
```

Hence results are the same.

```
Give input for q_dot for joints
Enter Linear velocity for Prismatic joint 1: 1.7
Enter Linear velocity for Prismatic joint 2: 0.2
Enter Linear velocity for Prismatic joint 3: 1.6

Velocity of end effector:
[[1.6]
 [0.2]
 [1.7]
 [0. ]
 [0. ]
 [0. ]]
```