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1)

To prove $R\,S(a)\,R^T = S(Ra)$

here R is a rotation matrix $S(a)$ is a skew symmetric matrix

and a is a column vector

Let b be another column vector

Then $R\,S(a)\,R^T b = R(a \times R^T b)$ [using $S(a)b = a \times b$ vector cross product]

$= Ra \times RR^T b$ [Distributivity of orthogonal matrix R]

$= Ra \times b$

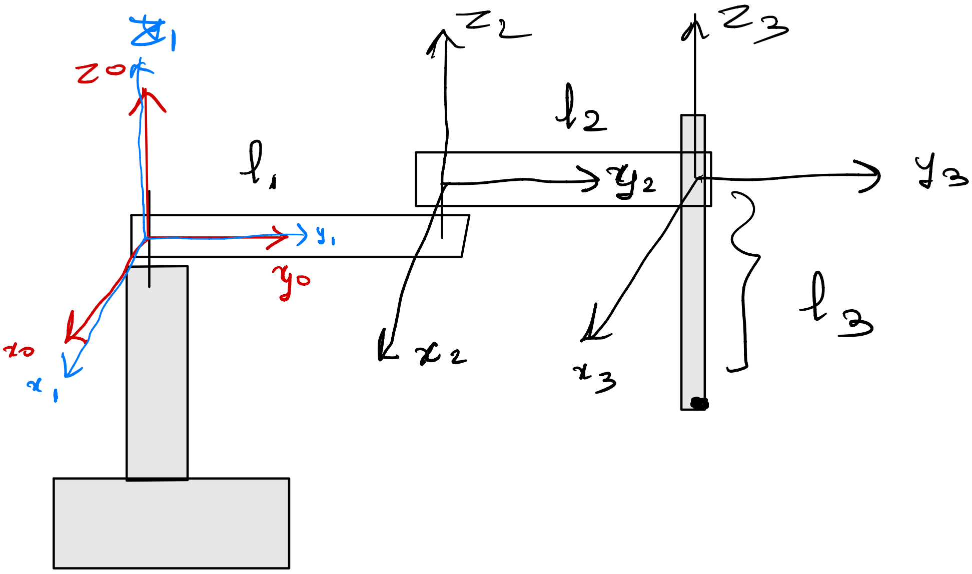
$= S(Ra)\,b$

$\therefore$ $R(S(a))R^T b = S(Ra)\,b$

since b is an arbitrary column vector & $b \neq 0$

$R\,S(a)\,R^T = S(Ra)$

2)



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^3 = H_0^1 \, H_1^2 \, H_2^3$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

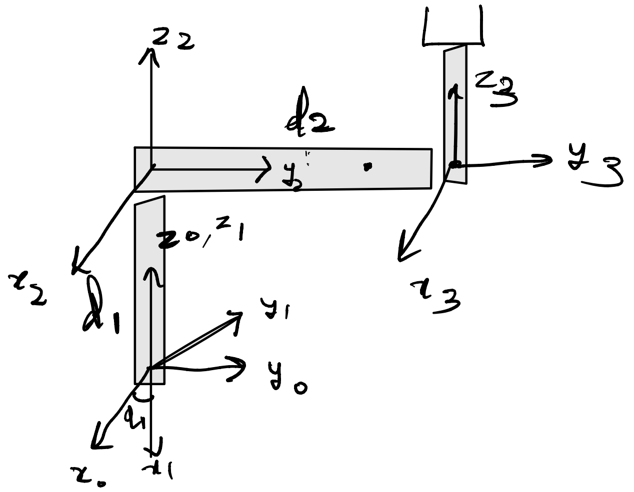
$$d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & d_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

4)



Here also

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^3 = H_0^1 H_1^2 H_2^3$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$

$$H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

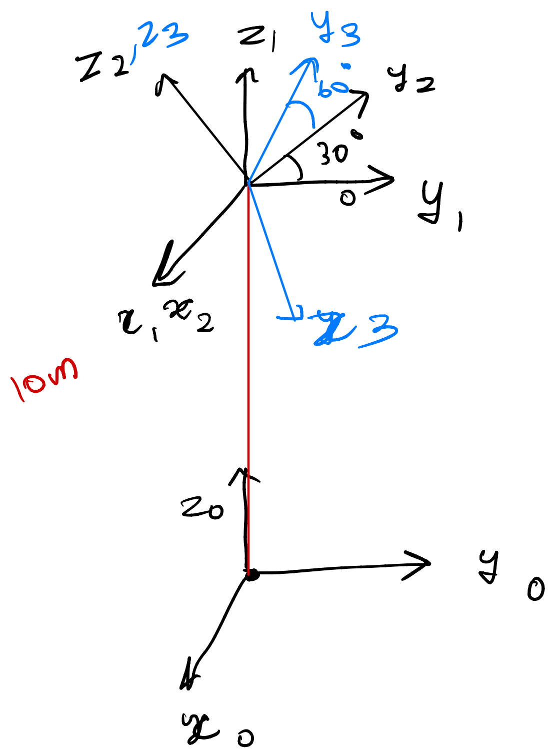
Illy

$$H_1^2 = \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin q_1 & 0 & \cos q_1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin q_1 & 0 & \cos q_1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

5) obstacle • 3m along z_3



Here

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

where $P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ ie position of obstacle along z axis of frame - 3

$$H_0^3 = H_0^1 \, H_1^2 \, H_2^3$$

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

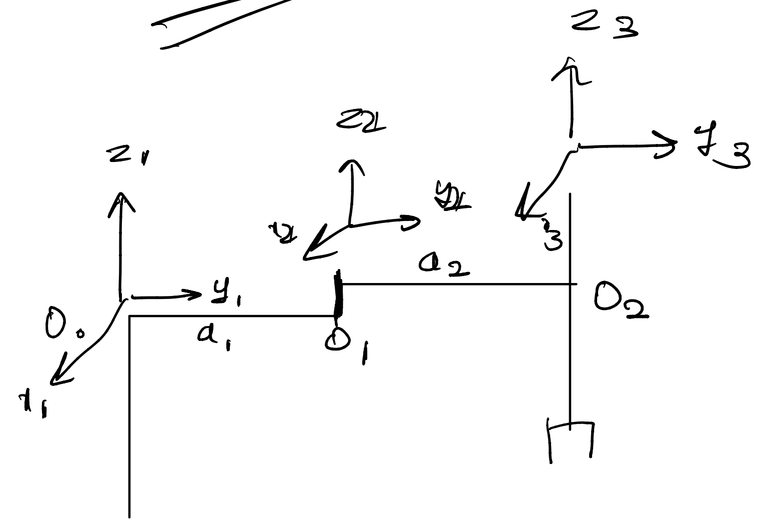
$$H_2^3 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.75 & 0.43 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P^0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \\ 1 \end{bmatrix}$$

$$P^0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \end{bmatrix}$$

7)



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = d_0^1 = \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix}$$

$$O_2 = R_0^1 d_1^2 + d_0^1$$

$$= \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 C_2 \\ a_2 S_2 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 C_{12} + a_1 C_1 \\ a_2 S_{12} + a_1 S_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^1 R_1^2 d_2^3 + R_0^1 d_1^2 + d_0^1$$

$$= \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ -a_3 \end{matrix} + O_2$$

$$= \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ -a_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (o_3 - o_0) \\ z_0 \end{bmatrix}$$ revolute joint

here $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$o_3 - o_0 = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ -a_3 \end{bmatrix}$$

$$z_0 \times (o_3 - o_0) = \begin{bmatrix} -(a_2 s_{12} + a_1 s_1) \\ a_2 c_{12} + a_1 c_1 \\ 0 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} -(a_2 s_{12} + a_1 s_1) \\ a_2 c_{12} + a_1 c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (o_3 - o_1) \\ z_1 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad o_3 - o_1 = \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ -a_3 \end{bmatrix}$$

$$z_1 \times (o_3 - o_1) = \begin{bmatrix} -a_2 s_{12} \\ a_2 c_{12} \\ 0 \end{bmatrix}$$

$$\therefore \quad J_2 = \begin{bmatrix} -a_2 s_{12} \\ a_2 c_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix} \rightarrow \text{prismatic joint}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Here $c_1 = \cos\theta_1$ $\quad s_1 = \sin\theta_1$

$c_2 = \cos\theta_2$ $\quad s_2 = \cos\theta_2$

$c_{12} = \cos(\theta_1 + \theta_2)$ $\quad s_{12} = \sin(\theta_1 + \theta_2)$

Task - 6)

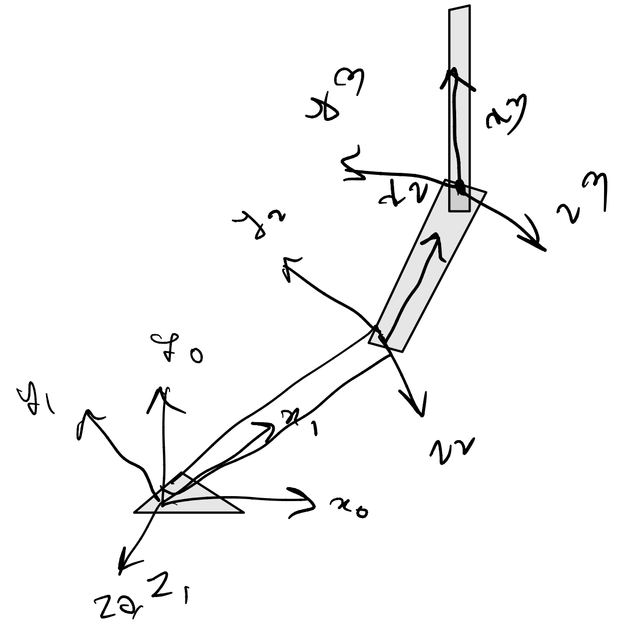
- Spur Gearbox:
    - Pros: Simple design, cost-effective, high efficiency for parallel shafts, reliable torque transmission.
    - Cons: Prone to noise and vibrations due to direct meshing teeth, limited torque capacity, less compact.
    - Applications: Industrial robots, conveyor systems, manufacturing equipment.
- Planetary Gearbox:
    - Pros: High torque density, efficient due to load sharing, compact design, good backlash control.
    - Cons: More complex and expensive to manufacture, slight efficiency loss in some designs.
    - Applications: Robotics arms, precision machinery, aerospace systems.
- Worm Gearbox:
    - Pros: High reduction ratios, self-locking ability, smooth motion transmission, compact design.
    - Cons: Lower efficiency due to sliding contact, potential heat generation, limited speed capabilities.
    - Applications: Robotic joints, conveyor systems, lifting mechanisms.
- Cycloidal Gearbox:
    - Pros: High shock load capacity, compact design, relatively low backlash, smooth operation.
    - Cons: Complex design, lower efficiency compared to some other types, challenging to manufacture.
    - Applications: Industrial robots, packaging machinery, material handling equipment.
- Harmonic Drive:
    - Pros: Zero backlash, high precision, compact size, excellent positional accuracy.
    - Cons: Expensive to produce, limited torque capacity, potential for wear over time.
    - Applications: Robotic arms, aerospace mechanisms, medical devices, precision optics.
- Bevel Gearbox:
    - Pros: Compact design, efficient torque transmission between non-parallel shafts, high precision.
    - Cons: Limited load capacity compared to some other types, potentially higher cost.
    - Applications: Robotic manipulators, automotive steering systems, printing presses.
- Helical Gearbox:
    - Pros: High load-bearing capacity, quieter operation compared to spur gears, smooth torque transmission.
    - Cons: More complex design, potential for axial thrust, slightly lower efficiency due to helix angle.
    - Applications: Robotics, machinery, automotive applications.
- Parallel Shaft Gearbox:

- Pros: Simple design, suitable for high torque applications, cost-effective.
- Cons: Limited reduction ratios compared to other types, larger size for higher ratios.
- Applications: Conveyors, heavy machinery, robotics.

b)

Regarding drones, as mentioned earlier, gearboxes are typically not used in their propulsion systems due to their drawbacks. Gearboxes add weight complexity and can reduce efficiency, which are detrimental to drone performance. Drones rely on direct drive systems where the motor is directly connected to the propeller shaft, allowing for lighter weight, simpler construction, and better agility. The goal of drone design is to maximize flight time, maneuverability, and overall efficiency, making gearboxes less suitable for this application.

9)



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad o_1 = d_0^1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = R_0^1 d_1^2 + d_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 c_2 \\ a_2 s_2 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^1 R_1^2 d_2^3 + O_2$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3 c_3 \\ a_3 s_3 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_3 c_{123} \\ a_3 s_{123} \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_3 c_{123} + a_2 c_{12} + a_1 c_1 \\ a_3 s_{123} + a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J_1 = \begin{bmatrix} z_0 \times (O_3 - O_0) \\ z_0 \end{bmatrix} = \begin{bmatrix} -(a_3 s_{123} + a_2 s_{12} + a_1 s_1) \\ a_3 c_{123} + a_2 c_{12} + a_1 c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (O_3 - O_1) \\ z_1 \end{bmatrix} = \begin{bmatrix} -(a_3 s_{123} + a_2 s_{12}) \\ a_3 c_{123} + a_2 c_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \times (O_3 - O_2) \\ z_2 \end{bmatrix} = \begin{bmatrix} -a_3 s_{123} \\ a_3 c_{123} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Here $S_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$ $C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$

$S_{12} = \sin(\theta_1 + \theta_2)$ $C_{12} = \cos(\theta_1 + \theta_2)$

$S_1 = \sin(\theta_1)$ $C_1 = \cos(\theta_1)$