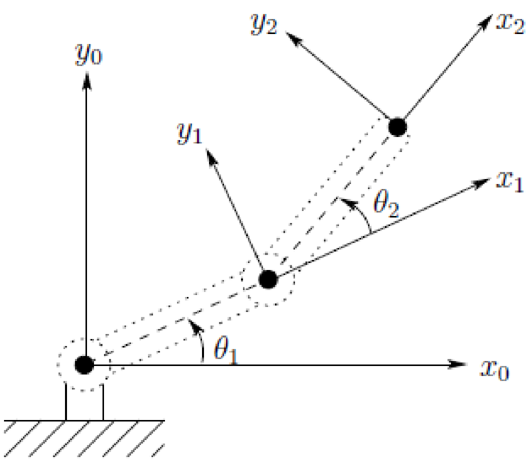
2R- Manipulator (Planar)



DH Parameter:

Link	d	θ	a	α
1	0	θ_1	l_1	0
2	0	θ_2	l_2	0

d = depth along the previous joint's Z-axis
θ = Rotation about the Z-axis to align the X-axis
a = length of common normal for both Z-axis
α = Rotation about the new X-axis to align the previous Z-axis.

$l_1 = 91.88\,mm,\ l_2 = 104.54mm$

Link	d	θ	a	α
1	0	θ_1	91.88	0
2	0	θ_2	104.54	0

Let $P_0 = (x_p\,,\,y_p)$ be end effector position w.r.t. Base frame.
Let's first calculate homogeneous transformation using DH parameters,

$$H_{i-1}^{i} = \begin{bmatrix} \cos\theta & -\sin\theta\cdot\cos\alpha & \sin\theta\cdot\sin\alpha & a.\cos\theta \\ \sin\theta & \cos\theta\cdot\cos\alpha & -\cos\theta\cdot\sin\alpha & a\cdot\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_1\cdot\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_1.\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ -------------- (1)

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2\cdot\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & l_2\cdot\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 \ = \ H_0^1 \cdot H_1^2$$

$$H_0^2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -sin(\theta_1+\theta_2) & 0 & l_1.\cos\theta_1 + l_2\cdot\cos(\theta_1+\theta_2) \\ sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & l_1.sin\theta_1 + l_2\cdot sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ -------------- (2)

P_2 is a position of end effector w.r.t. frame 2

$$P_2 \ = \ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} \ = \ H_0^2 \cdot \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

Using the above values gives

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} \ = \ \begin{bmatrix} l_1\cdot\cos\theta_1 + l_2\cdot\cos(\theta_1+\theta_2) \\ l_1\cdot sin\theta_1 + l_2\cdot sin(\theta_1+\theta_2) \\ 0 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} l_1\cdot\cos\theta_1 + l_2\cdot\cos(\theta_1+\theta_2) \\ l_1\cdot sin\theta_1 + l_2\cdot sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$x_p \ = \ l_1\cdot\cos\theta_1 \ + \ l_2\cdot\cos(\theta_1+\theta_2)$$
$$y_p \ = \ l_1\cdot\sin\theta_1 \ + \ l_2\cdot\sin(\theta_1+\theta_2)$$

The velocity of the end-effector can be calculated using the Jacobian matrix.

$$\begin{bmatrix} v_0^n \\ w_0^n \end{bmatrix} = J \cdot \dot{q} = [J_1 \quad J_2] \cdot \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix}$$

Both joints are revolute joints, hence

$$J = \begin{bmatrix} Z_0 \times (o_2 - o_0) & Z_1 \times (o_2 - o_1) \\ Z_0 & Z_1 \end{bmatrix}$$

Where, $Z_i = R_0^i . k = R_0^i . \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$Z_0 = Z_1 = Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

From (1) and (2),

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_1 \cdot \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_1 . \sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_0^1 $\qquad$ o_1

$$H_0^2 = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & l_1 . \cos\theta_1 + l_2 \cdot \cos(\theta_1 + \theta_2) \\ sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 & l_1 . sin\theta_1 + l_2 \cdot sin(\theta_1 + \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_0^2 $\qquad$ o_2

So, the Jacobian matrix becomes,

$$J = \begin{bmatrix} -l_1 \cdot \sin\theta_1 - l_2 \cdot \sin(\theta_1 + \theta_2) & -l_2 \cdot \sin(\theta_1 + \theta_2) \\ l_1 \cdot cos\theta_1 + l_2 \cdot cos(\theta_1 + \theta_2) & l_2 \cdot cos(\theta_1 + \theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

End effector velocity,

$$\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} -l_1 \cdot \sin\theta_1 - l_2 \cdot \sin(\theta_1 + \theta_2) & -l_2 \cdot \sin(\theta_1 + \theta_2) \\ l_1 \cdot cos\theta_1 + l_2 \cdot cos(\theta_1 + \theta_2) & l_2 \cdot cos(\theta_1 + \theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix}$$