Assignment 3-4

A-I Singular Configuration is a configuration where certain directions of motion ~~of~~ at the end-effectors becomes unattainable or problematic

OR

At ~~sign~~ singularitys, bounded end-effectos velocites may correspond to unbounded joint velocitys

OR

Near Singularities there will be not exist a unique solution to the inverse kinematics problem. In such cases there may be no sol^{n} or there may be infinitely many sol.

A singularity configuration can be detect by examining Jacobian. ~~...~~ When determinant of Jacobian approaches zero, the manipulator is nearby a singularity, which can be ~~avoided~~ imp for avoiding potential issues & such as unbounded joint velocitys, unattainable end-effector motion or difficulties in solving Inverse kinematic Problems.

A-7

	a) Drive Direct (2R)	Remotely Driven (2R)	5-Bar ||^{r} Arrangement (2R)
①	Both joints are directly driven by motors	one joint is mechanically connected with another	Both links are connected are to common base using ||^{r} linkage.
②	Controlled independently more precision.	Simplied wiring & control compared to Direct Drive.	Ensure End effector remai[n] parallel to base, Thus stab[le]
③	Increase Complexity	Reduced precision of movement.	Complex Machenism & High cost.

Ans 2 Denavit Hartenberg Representation (DH)

↳ Standardize } Reference frame
↳ Automate coding

0 to n links (0 - base/ground)
i^{th} - coordinate frame rigidly attached to i^{th} link

1 to n joints
i^{th} joint connects link i-1 and link i.

i^{th} joint variable is q_i
z_i are along joint axis (axis of rotation for R, axis of linear motion for P.

O_i need not be at the joint

Instead of descriptive representation, looking for a tabular representat

For each pair of coordinate frame (eg $i-1^{th}$ & i^{th}), ideally 6 parameters, (can reduce to four)

$$H_{i-1}^{i} = T_{trans\, z, d} \; Rot_{z,\theta} \; Trans_{x,a} \; Rot_{x,\alpha}$$

$$H = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} \\ 0 \\ \vdots \\ n \end{array}\begin{array}{cccc} d & \theta & a & \alpha \\ \hline \vdots & \vdots & \vdots & q_0 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & & & \end{array} \qquad \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_n^0 \begin{bmatrix} P_n \\ 1 \end{bmatrix}$$

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* DH (d, θ, a, α)

0 to n links

$\therefore H_{i-1}^{i} = Trans_{z,d}\, Rot_{z,\theta}\, Trans_{x,a}\, Rot_{x,\alpha}$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & -s\theta & 0 & 0 \\ s\theta & c\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

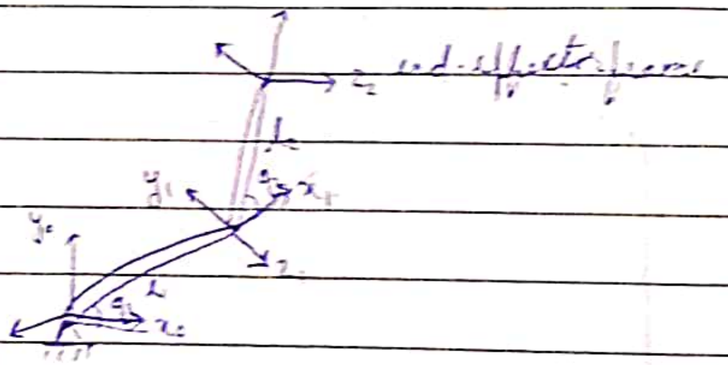
$$\therefore H_{i-1}^{i} = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & a\cos\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & a s\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore H_{i-1}^{i} = \underline{Rot_{z,\theta}\, Trans_{z,d}}\, Trans_{x,a}\, Rot_{x,\alpha}$

↳ Both this commute $(a.b = b.a)$

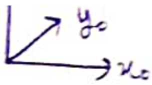
$R \in SO(3)$ → 3 diff. Means of Transformation Matrix H A T

* z_i are axis of joint $i+1$



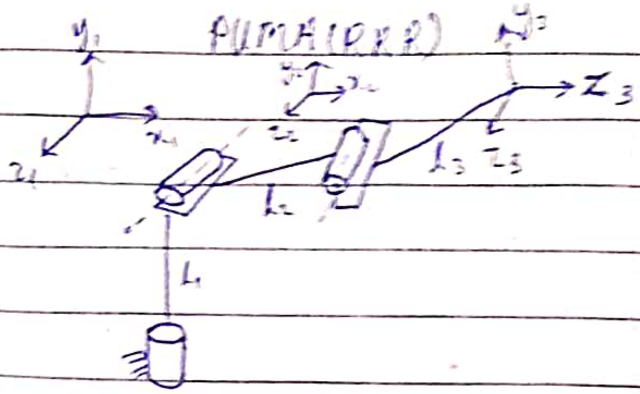
link	d	θ	a	α
1	0	q_1^*	l_1	0
2	0	q_2^*	l_2	0

q_1^*, q_2^* are variable/changing joint angle.

$q_1 \rightarrow$  $q_1 + \pi/2 \rightarrow$ 

x_i is along common normal of $i+1$ & i^{th} frame

PUMA (RRR)



Link	d	θ	a	α
1	l_1	q_1^*	0	$\pi/2$
2	0	q_2^*	l_2	0
3	0	q_3^*	l_3	0

(DH1) The axis x_{i+1} is perpendicular to z_i

(DH2) The axis x_{i+1} intersects the axis z_i

Ans 10

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{L_1} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

$$c_{111} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_1} + \cancel{\frac{\partial d_{11}}{\partial q_1}} - \cancel{\frac{\partial d_{11}}{\partial q_1}}\right] = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \cancel{\frac{\partial d_{21}}{\partial q_1}} - \cancel{\frac{\partial d_{12}}{\partial q_1}}\right] = \frac{1}{2}\cdot\frac{\partial d_{11}}{\partial q_2} =$$

$$\underset{ijk}{c_{221}} = \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_2} + \frac{\partial d_{21}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1}\right] = \frac{\partial d_{21}}{\partial q_2} - \cancel{\frac{1}{2}\frac{\partial d_{22}}{\partial q_1}}^{\,0} = \frac{\partial d_{21}}{\partial q_2} = -m_2 \frac{l_1 l_2}{2}\sin(q_2 - q_1)$$

$(-)\ (-q_1) \Rightarrow +$

$$c_{112} = \frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_1} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{11}}{\partial d_2}\right] = \frac{\partial d_{12}}{\partial d_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2 \frac{l_1 l_2}{2}\sin(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\left[\frac{\partial d_{22}}{\partial q_1} + \cancel{\frac{\partial d_{12}}{\partial q_2}} - \cancel{\frac{\partial d_{21}}{\partial q_2}}\right] = \frac{1}{2}\left\{\frac{\partial d_{22}}{\partial q_{22}}\right\}^{0} = 0$$

$$\phi_1 = \frac{\partial V}{\partial q_1} \qquad \phi_2 = \frac{\partial V}{\partial q_2}$$

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$$C_{222} = \frac{1}{2}\left[\frac{\partial d_{22}}{\partial q_2} + \cancel{\frac{\partial d_{22}}{\partial q_2}} - \cancel{\frac{\partial d_{22}}{\partial q_2}}\right] = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential Energy:-

$$V = m_1 g \frac{l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$\Rightarrow \phi_1 = \frac{\partial V}{\partial q_1} = \left(m_1\frac{l_1}{2} + m_2 l_1\right) g \cos q_1$$

$$\Rightarrow \phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2}\cos q_2$$

$\Rightarrow$ Final Equations: **Ans 8**

$$\left.\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 &= \tau_2 \end{aligned}\right\} \rightarrow$$

Should be same as eq^n ⑥ in miniproject-I.