Assignment 2

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Q.1] ~~Given~~ To prove : ~~RCS~~ $R\,S(a)\,R^T = S(Ra)$

Now, ~~LHS = $R S(a) R^T$~~

~~$= R(a \times R^T)$ .... [∵ $S(a) R^T = a \times R^T$]~~

Lets us multiply LHS and RHS by a vector b,

$\Rightarrow$ ~~$R S(a)$~~ $R S(a) R^T b = S(Ra)\, b.$

Now, LHS $= R S(a) R^T b$

$= R(a \times R^T b)$ .... [∵ $S(a) p = a \times p$]

$= (Ra) \times R R^T b$ .... [∵ $R(a \times b) = Ra \times Rb$]

$= (Ra) \times b$

LHS $= S(Ra)\, b$ ..... [∵ $S(a) p = a \times p$]

$\boxed{\text{LHS} = \text{RHS}}$

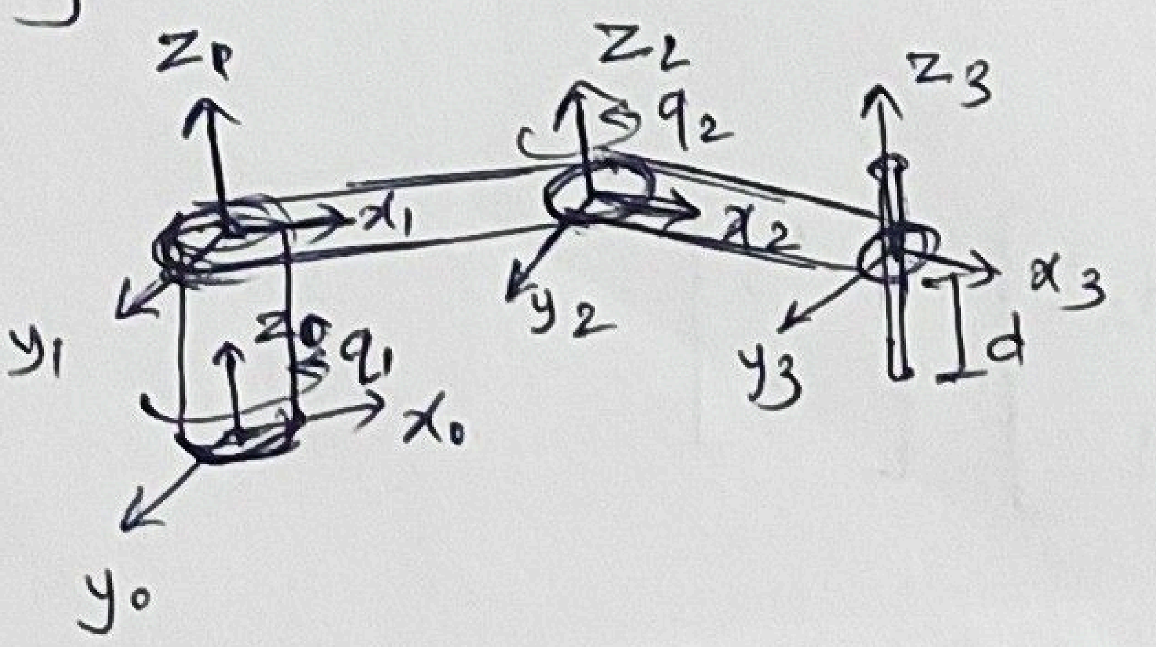
$\Rightarrow R(S(a)) R^T b = S(Ra)\, b$

Cancelling out b gives,

$R(S(a)) R^T = S(Ra)$

Hence Proved.

Q.2]



z_1 and z_2 are the axis of rotation of two rotary joint and z_3 is the axis of translation of the prismatic joint.

Let, us assume the clearance of the prismatic joint to be d.

The axis z_0 is rotated by q_1 angle to transform into z_1 and z_1 is ~~transfo~~ rotated by q_2 angle to transform into z_2.

We are aligning all x axes along the link length.
Now, we know, Homogeneous transformation of the axis will be,

~~$P_0 = R_0^1 R_1^2 R_1^3 P_0$~~

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Now for H_0^1, $R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .. (∵ no translation from 0 to 1 frame)

$\rightarrow$ $H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$

Now, for H_1^2, $R_1^2 = R_{z,q_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$ ... [where l_1 is the length of link 1]

$\Rightarrow$ $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$

Now, for H_2^3, $R_2^3 = R_{z,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .. (∵ no rotation happen ~~during~~ between 2 & 3 frames)

$= I$

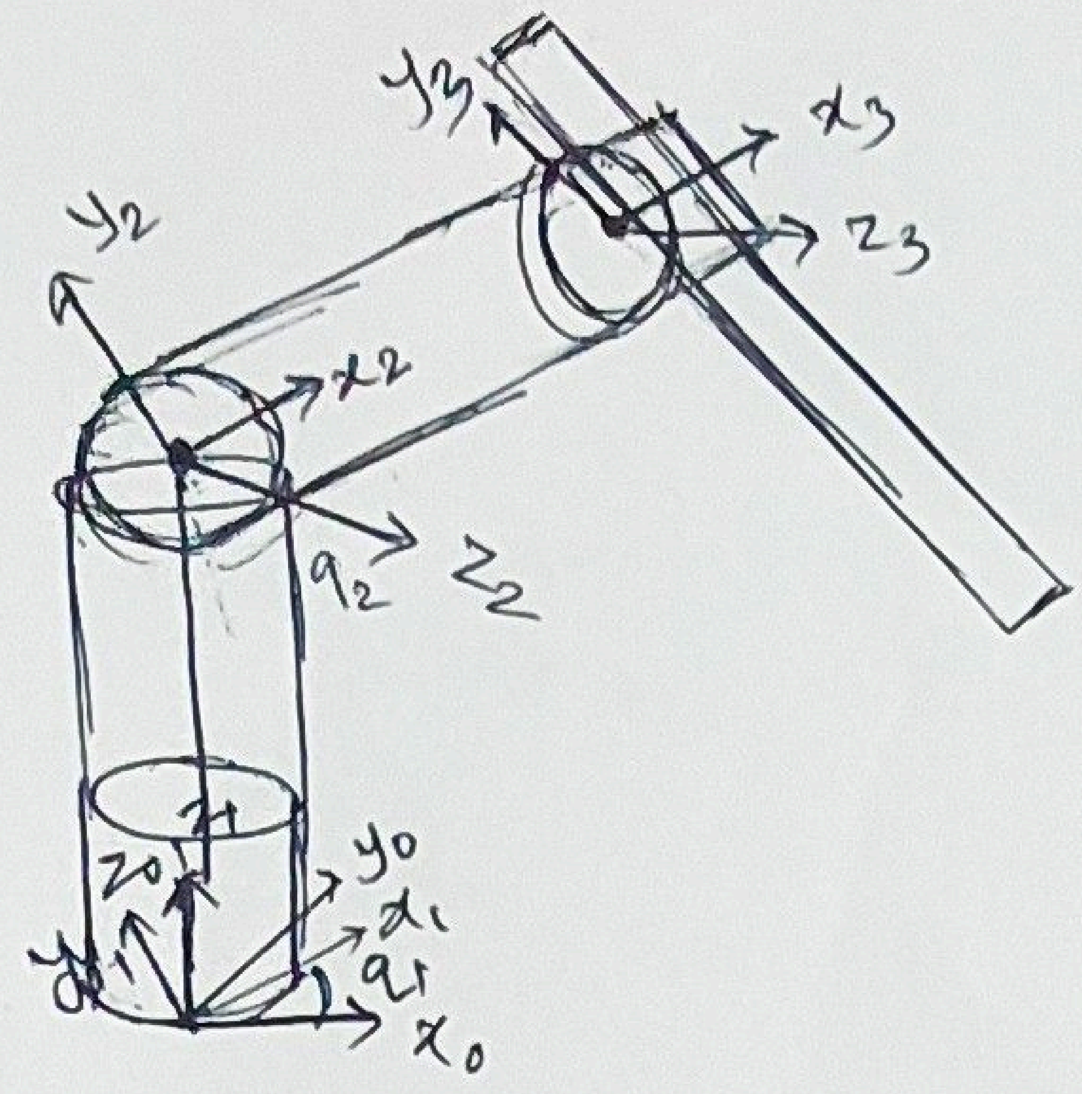
$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ d \end{bmatrix}$ ... [where l_2 is the length of link 2]
[and d is the length of prismatic link when no translation has occured]

$\Rightarrow$ $H_2^3 = \begin{bmatrix} I & d_2^3 \\ 0 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$$

where $P_3 = \begin{bmatrix} 0 \\ 0 \\ -l_3 \end{bmatrix}$

Q.4]



~~Here~~, We have alligned all the rotating axis about z axis and traslatation about x axis. ~~Hence the axis~~ First rotation is occuring ~~to~~ about z_0 axis by angle q_1 and then second rotation occurs ~~at~~ about x_1 axis by $\pi/2$ and z_2 axis by q_2 angle. l_1, ~~and~~ l_2 and l_3 are lengths of link 1, 2 and 3 respectively. Now, by homogeneous transfoomation, we can write,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$$

where P_0 is position vector of end effect in base frame and P_3 is position vector of end effector in frame x_3, y_3, z_3.

Now, for H_0^1, $R_0^1 = R_{z,q_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .. [$\because$ no translation occur btn 0 & 1 frame]

for H_1^2, $R_1^2 = R_{x_1 \frac{\pi}{2}} R_{z,q_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

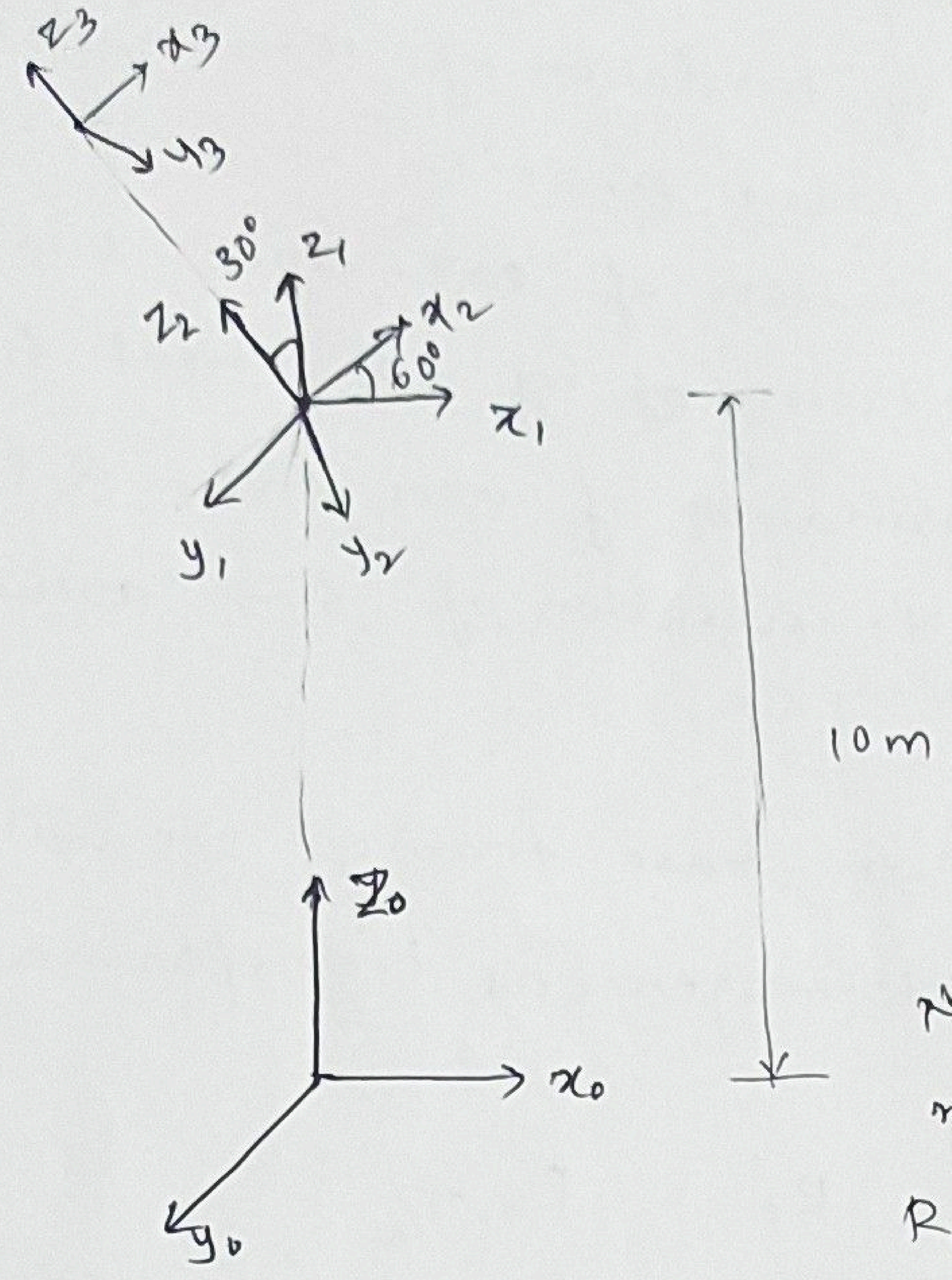
for H_2^3, $R_2^3 = R_{z\,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .. ($\because$ no rotation occurs)

$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

and $P_3 = \begin{bmatrix} 0 \\ -l_3 \\ 0 \end{bmatrix}$ ($\because$ y_3 axis is aligned along the prismatic joint translation)

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 0 \end{bmatrix}$$

Q.5]



Here, transformation of ~~matrix~~ axes from 0-1 frame is by translation of 10m along z_0 axis

Then, transformation of axes from 1-2 frame is by rotation of axes about x_1 by 30° followed by rotation of axes about z_2 by 60°.

Then, finally, transformation of axes from 2-3 takes place by translation of 3m along z_2 axis.

Now, defining the homogeneous transformation matrices,

$$R_0^1 = [R_{z,0}] \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\Rightarrow \quad H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{x,\pi/6} \cdot R_{z,\pi/3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} R_1^2 &= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix} \\ d_1^2 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}\right\} \Rightarrow H_1^2 = \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$\left.\begin{aligned} R_2^3 &= R_{z,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ d_2^3 &= \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \end{aligned}\right\} \Rightarrow H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, by applying homogeneous transformation,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ \phi \end{bmatrix}.$$

$$= \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

$$\rightarrow \boxed{p_0 = \begin{bmatrix} 0 & \frac{-3}{2} & \frac{3\sqrt{3}}{2} + 10 \end{bmatrix}^T}$$ Ans.

$$p_0 = \begin{bmatrix} 0 & -1.5 & 12.598 \end{bmatrix}^T$$

Q.7] From question 2 we have the values.

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} ; \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ d \end{bmatrix} ; \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ d - l_3 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} R_0^1 R_1^2 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} cq_1 cq_2 - sq_1 sq_2 & -cq_1 sq_2 - sq_1 cq_2 & 0 \\ sq_1 cq_2 + sq_2 cq_1 & -sq_1 sq_2 + cq_1 cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

need to ... calculat...

$$R_0^3 = R_0^2 R_2^3 = \begin{bmatrix} cq_1 cq_2 - sq_1 sq_2 & -cq_1 sq_2 - sq_1 cq_2 & 0 \\ sq_1 cq_2 + sq_2 cq_1 & -sq_1 sq_2 + cq_1 cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^3 = \begin{bmatrix} cq_1 cq_2 - sq_1 sq_2 & -cq_1 sq_2 - sq_1 cq_2 & 0 \\ sq_1 cq_2 + sq_2 cq_1 & -sq_1 sq_2 + cq_1 cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{d_0^2 = d_0^1 + R_0^1 d_1^2}$$

$$H_0^2 = H_0^1 H_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_1 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & cq_1 l_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & sq_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad d_0^2 = \begin{bmatrix} cq_1 l_1 \\ sq_1 l_1 \\ 0 \end{bmatrix}$$

$$H_0^3 = H_0^2 H_2^3 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & cq_1 l_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & sq_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & cq_1 l_1 + c(q_1+q_2) l_2 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & sq_1 l_1 + s(q_1+q_2) l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} cq_1 l_1 + c(q_1+q_2) l_2 \\ sq_1 l_1 + s(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

For calculating the Jacobian matrix, we need to use below relations.

	Prismatic	Revolute
Linear	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_n - d^0_{i-1})$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R^0_{i-1}\begin{bmatrix}0\\0\\1\end{bmatrix}$

~~Usa~~ where $n = 3$ in our case,

$$\therefore\ J = \begin{bmatrix} R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_3-d^0_0) & R^0_1\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_3-d^0_1) & R^0_2\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_3-d^0_2) \\ & & \begin{bmatrix}0\\0\\0\end{bmatrix} \\ R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix} & R^0_1\begin{bmatrix}0\\0\\1\end{bmatrix} & \end{bmatrix}$$

$$\Rightarrow\ R^0_0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_3-d^0_0) = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\begin{bmatrix}cq_1 l_1 + c(q_1+q_2)l_2\\ sq_1 l_1 + s(q_1+q_2)l_2\\ 0\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix}\times\begin{bmatrix}cq_1 l_1 + c(q_1+q_2)l_2\\ sq_1 l_1 + s(q_1+q_2)l_2\\ 0\end{bmatrix}$$

$$= \begin{bmatrix}-sq_1 l_1 - s(q_1+q_2)l_2\\ cq_1 l_1 + c(q_1+q_2)l_2\\ 0\end{bmatrix}$$

$$\text{Now, } R^0_1\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d^0_3-d^0_1) = \begin{bmatrix}cq_1&-sq_1&0\\sq_1&cq_1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\begin{bmatrix}cq_1 l_1 + c(q_1+q_2)l_2\\ sq_1 l_1 + s(q_1+q_2)l_2\\ 0\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\1\end{bmatrix}\times\begin{bmatrix}cq_1 l_1 + c(q_1+q_2)l_2\\ sq_1 l_1 + s(q_1+q_2)l_2\\ 0\end{bmatrix} = \begin{bmatrix}-sq_1 l_1 - s(q_1+q_2)l_2\\ cq_1 l_1 + c(q_1+q_2)l_2\\ 0\end{bmatrix}$$

Now, $R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \cancel{J = \begin{bmatrix} -sq_1 l_1 - s(q_1+q_2) l_2 & -sq_1 l_1 - s(q_1+q_2) l_2 & 0 \\ cq_1 l_1 + c(q_1+q_2) l_2 & cq_1 l_1 + c(q_1+q_2) l_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}}$$

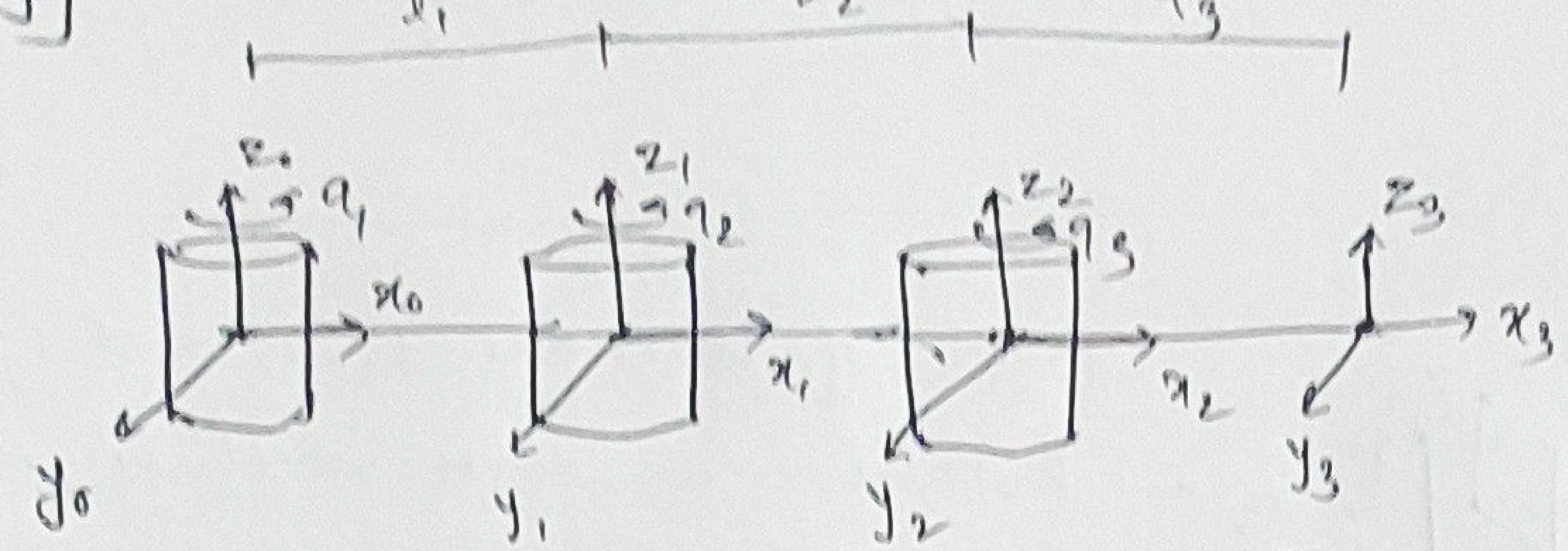
$$R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_2^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} c(q_1+q_2) l_2 \\ s(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -s(q_1+q_2) l_2 \\ c(q_1+q_2) l_2 \\ 0 \end{bmatrix}$$

Now, $R_3^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\Rightarrow J = \begin{bmatrix} -sq_1 l_1 - s(q_1+q_2) l_2 & -s(q_1+q_2) l_2 & 0 \\ cq_1 l_1 + c(q_1+q_2) l_2 & c(q_1+q_2) l_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q.9]



Now, $H_0^1 = \begin{bmatrix} R_{z,q_1} & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & l_1 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_1^2 = \begin{bmatrix} R_{z,q_2} & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_2^3 = \begin{bmatrix} R_{z,q_3} & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_3 & -sq_3 & 0 & l_3 \\ sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now, $H_0^2 = H_0^1 * H_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 & l_1 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 \\ sq_2 & cq_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

$\therefore H_0^2 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & l_2 cq_1 + l_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_2 sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_0^3 = H_0^2 H_2^3 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & l_2 cq_1 + l_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_2 sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_3 & -sq_3 & 0 & l_3 \\ sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 & c(q_1+q_2)l_3 + l_2 cq_1 + l_1 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 & s(q_1+q_2)l_3 + l_2 sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

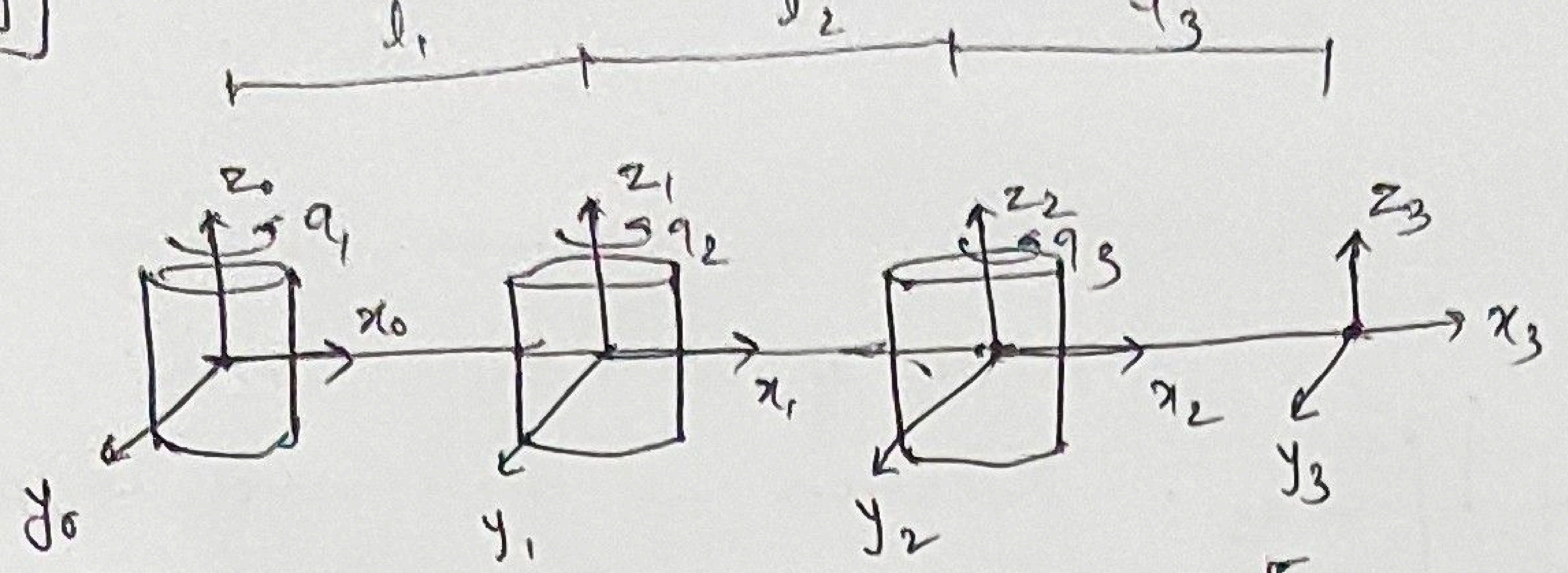
$$\therefore \quad J = \begin{bmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (d_3^0 - d_2^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} c(q_1+q_2) l_3 + l_2 c q_1 + l_1 \\ s(q_1+q_2) l_3 + l_2 s q_2 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} c(q_1+q_2) l_3 + l_2 c q_1 \\ s(q_1+q_2) l_3 + l_2 s q_2 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} c(q_1+q_2) l_3 \\ s(q_1+q_2) l_3 \\ 0 \end{bmatrix} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Ans

$$J = \begin{bmatrix} -(s(q_1+q_2) l_3 + l_2 s q_2) & -(s(q_1+q_2) l_3 + l_2 s q_2) & -s(q_1+q_2) l_3 \\ c(q_1+q_2) l_3 + l_2 c q_1 + l_1 & c(q_1+q_2) l_3 + l_2 c q_1 & c(q_1+q_2) l_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q.9]



Now, $$H_0^1 = \begin{bmatrix} R_{z,q_1} & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_1 & -sq_1 & 0 & l_1cq_1 \\ sq_1 & cq_1 & 0 & l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_{z,q_2} & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2cq_2 \\ sq_2 & cq_2 & 0 & l_2sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_{z,q_3} & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_3 & -sq_3 & 0 & l_3cq_3 \\ sq_3 & cq_3 & 0 & l_3sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, $$H_0^2 = H_0^1 * H_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 & l_1cq_1 \\ sq_1 & cq_1 & 0 & l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2cq_2 \\ sq_2 & cq_2 & 0 & l_2sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\therefore H_0^2 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & l_2c(q_1+q_2) + l_1cq_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_2s(q_1+q_2) + l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = H_0^2 H_2^3 = \begin{bmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 & l_2c(q_1+q_2)+l_1cq_1 \\ s(q_1+q_2) & c(q_1+q_2) & 0 & l_2s(q_1+q_2)+l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cq_3 & -sq_3 & 0 & l_3cq_3 \\ sq_3 & cq_3 & 0 & l_3sq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(q_1+q_2+q_3) & -s(q_1+q_2+q_3) & 0 & l_3c(q_1+q_2+q_3) + l_2c(q_1+q_2) + l_1cq_1 \\ s(q_1+q_2+q_3) & c(q_1+q_2+q_3) & 0 & l_3s(q_1+q_2+q_3) + l_2s(q_1+q_2) + l_1sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore\; J = \begin{bmatrix} R_0^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} (d_3^0 - d_2^0) \\ R_0^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 c q_1 \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 s q_1 \\ 0 \end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) \\ l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) \\ 0 \end{bmatrix} & \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} l_3 c(q_1+q_2+q_3) \\ l_3 s(q_1+q_2+q_3) \\ 0 \end{bmatrix} \\ R_0^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_1^0 \begin{bmatrix}0\\0\\1\end{bmatrix} & R_2^0 \begin{bmatrix}0\\0\\1\end{bmatrix} \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_3 s(q_1+q_2+q_3) + l_2 s(q_1+q_2) + l_1 s q_1) & -(l_3 s(q_1+q_2+q_2) + l_2 s(q_1+q_2)) & -l_3 s(q_1+q_2+q_3) \\ l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) + l_1 s q_1 & l_3 c(q_1+q_2+q_3) + l_2 c(q_1+q_2) & l_3 c(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q 06

There are many types of gearboxes used in robotics applications. Some main types are:

1. **Planetary Gearbox:**
   This type of gearbox offers high torque output and efficiency due to their multiple gear stages. They have a compact size and good shock load resistance. But they can be relatively expensive and have complex design and assembly. These type of gearboxes are used in industrial robotic applications where high precision is required.
2. **Worm Gearbox:**
   It provides high gear reduction ratios, excellent self locking capability. It also has a compact design. But they are less efficient and generate significant heat during application. These are used in robotic arm joints where high reduction ratios are required.
3. **Spur Gearbox:**
   These types of gearboxes are simple, cost-effective, and provide efficient power transmission. But they can be noisy and have limited torque capacity. It is used in lightweight robot applications.
4. **Harmonic Drive Gearbox:**
   These types of gearbox have high precision, zero backlash, and excellent torque-to-weight ratios. But they can be expensive and have limited torque capacity in larger sizes. It is used in robotic arms.

In drones, it is common to see gearboxes used in conjunction with the motors, especially in high-performance drones and UAVs. It helps in increasing the efficiency of the motor by matching the RPM to the optimal speed for propellers. It also helps in reducing the noise created by high RPM motors and also improves torque.