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Intro to Robotics
ME 639
Assignment - 3

Q1. The rank of a matrix is said to be the number of linearly independant columns.

i.e rank $J \leq \min(6, n)$ for a n-link arm

The rank of a manipulator jacobian depends on configuration q, and that configurations for which rank $J(q)$ is less than its maximum value are called singularities or singular configuration.

A particular method to determine the singularity of a Jacobian, where we consider the fact that square matrix is singular when its determinant is equal to zero.

i.e $\det J(q) = 0$

We resolve it into 2 singularities to solve it easily :- Arm singularities and wrist singularities

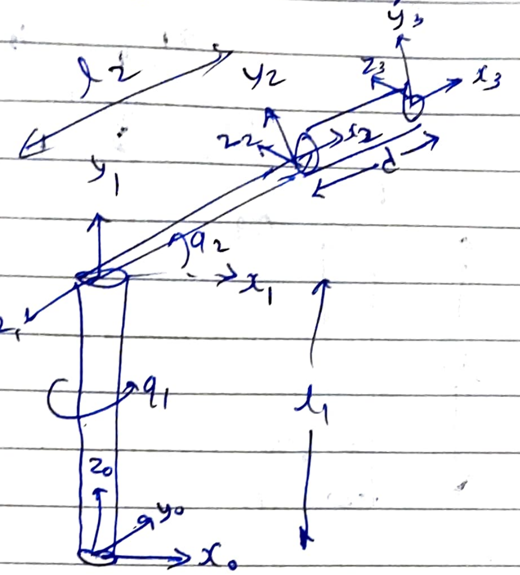
$\det J(q) = 0$

where $J = \{J_p \mid J_o\}$

For a random $m \times n$ jacobian matrix, if $\det(J)$ is near zero, it is near singular configuration.

=> Also, we need to check if columns are linearly independant or not.

Q4. For RRP - Stanford manipulator



D-H Parameters table

Link	θ_i	d_i	a_i	α_i
	Link parameters			
1	q_1^*	l_1	0	$\pi/2$
2	q_2^*	0	l_2	0
3	0	0	d^*	0

putting same values in D-H parameter and the manually solved code for last submission.

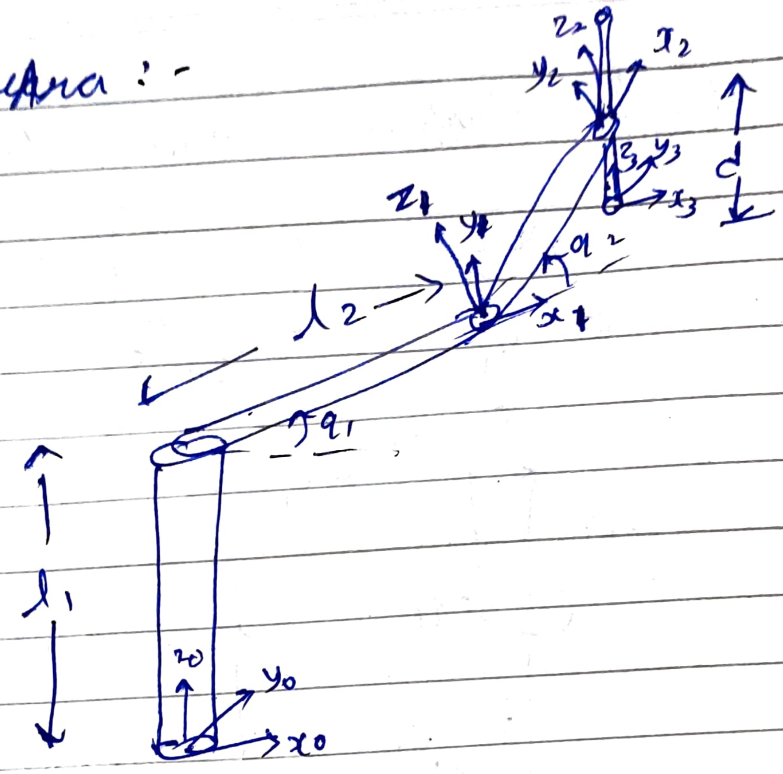
We get values close to each other for eg.

for	θ_i	d_i	a_i	α_i
1	30	4	0	$\pi/2$
2	60	0	7	0
3	0	0	5	0

manually solved = [5.196 3 14.3]

DH param = [4.7 2.1 15.7]

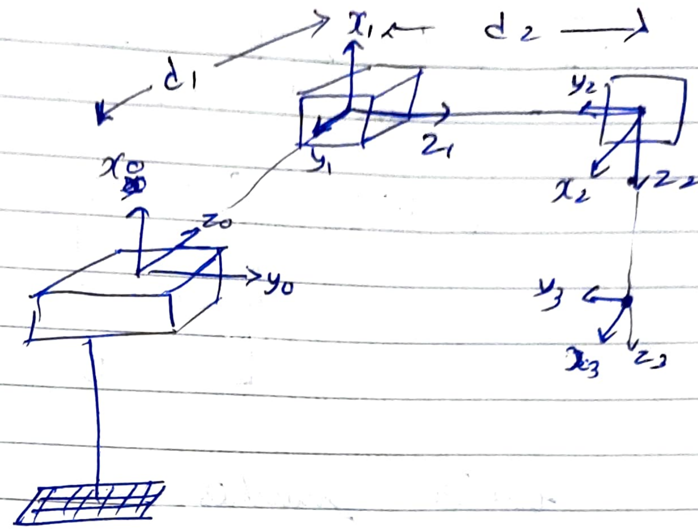
RRP Scara :-



D-H Param table

Link	a_i	d_i	a_i	L_i
1	q_1^*	l_1	l_2	0
2	q_2^*	0	l_3	0
3	0	d	0	0

Q5.



D-H parameter table:-

Link	θ_i	d_i	a_i	α_i
1	0	d_1	0	$-\pi/2$
2	$\pi/2$	d_2	0	$-\pi/2$
3	0	d_3	0	0

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We put values in code 03

$d_1 = 1 \quad d_2 = 2 \quad d_3 = 3$

$$T_0(1,2,3) = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which matches.

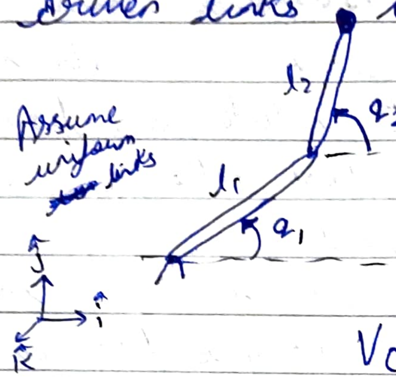
Q7. <u>Direct-drive Configuration</u>: There are no gears, so there is no backlash. Response time will be quicker as there are no gears in between.
→ as direct drive doesn't include gear box, it will have low cost. It is just like a motor attached directly to the joint.
→ To use direct-drive, we need a motor that can give high torque for lower RPM.

<u>Remotely-Driven Joint</u>: Uses some sort of gears or other mechanism to drive the joints while the motor is in the base frame.
→ Easier to implement as arms will not be having an additional weight of motor to lift.
→ No additional Torque is applied by the motor to the previous joint like directly-driven joints.

<u>5-bar ~~paralle~~ parallelogram arrangement</u>: both motors can be on ground and links are arranged such that it can achieve workspace similar to 2R manipulator

→ This can be thought as an extension to remotely driven joint or just another way to drive the joint. So, it ~~has~~ is almost same as remotely driven.
→ It has workspace similar to the serial manipulator but it is smaller in size due to constraint on other bars.

Q8. The elbow manipulator with remotely driven links using absolute angles.



$$V_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -l_2/2 \sin q_2 \\ l_1 \cos q_1 & l_2/2 \cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \quad , \quad \omega_2 = \dot{q}_2 \hat{k}$$

Now the Kinetic energy, K is given by:

$$K = \frac{1}{2}\sum_{i=1}^{n} m_i V_{ci}^T V_{ci} + \frac{1}{2}\sum_{i=1}^{n} w_i^T I_i w_i$$

where $V_{ci} = J_{V_{ci}}(q)\dot{q}$ $\quad w_i = R_i^T J_{w_i}(q)\dot{q}$

putting these values in K

$$K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{n}\left[m_i J_{v_{ci}}(q)^T J_{v_{ci}}(q) + J_{w_i}(q)^T R_i(q) I_i R_i(q)^T J_{w_i}(q)\right]\dot{q}$$

$$\therefore \quad K = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

~~D(q) for~~ In this case,

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2\frac{l_1 l_2}{2}\cos(q_2 - q_1) \\ m_2\frac{l_1 l_2}{2}\cos(q_2 - q_1) & m_2\frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Calculating the Christoffel's Symbols

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$C_{jik} = C_{ijk}$

$$C_{121} = C_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \cancel{\frac{\partial d_{12}}{\partial q_1}} - \cancel{\frac{\partial d_{12}}{\partial q_1}}\right]$$

$$= \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -m_2\frac{l_1 l_2}{2}\sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2\frac{l_1 l_2}{2}\sin(q_2 - q_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy is given by:

$$V = m_1 g\frac{l_1}{2}\sin q_1 + m_2 g\left(l_1\sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$\therefore \quad \phi_1 = \frac{\partial V}{\partial q_1} = \frac{m_1 g l_1}{2}\cos(q_1) + m_2 g l_1\cos(q_1)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g\frac{l_2}{2}\cos(q_2)$$

The final equations are

link ①
$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_1^2 + \phi_1 = \tau_1$$

link ②
$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

①
$$\boxed{\tau_1 = \left(m_1\frac{l_1^2}{4} + m_2 l_2^2 + \frac{1}{2} m l_1^2\right)\ddot{q}_1 + m_2\frac{l_1 l_2}{2}\cos(q_2-q_1)\ddot{q}_2 - m_2\frac{l_1 l_2}{2}\sin(q_2-q_1)\dot{q}_1^2 + m_1 g l_1\cos q_1 + m_2 g l_1\cos(q_1)}$$

$$\Rightarrow d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

~~$\tau_2 = m_2 l_1 l_2$~~

$$\boxed{\tau_2 = m_2\frac{l_1 l_2}{2}\cos(q_2-q_1)\ddot{q}_1 + \left(\frac{m_2 l_2^2}{4} + \frac{1}{12} m_2 l_2^2\right)\ddot{q}_2 + \frac{1}{2} m_2 l_1 l_2 \sin(q_2-q_1)\dot{q}_1^2 + m_2 g\frac{l_2}{2}\cos(q_2)}$$

②

① & ② are same as that of the equations derived in mini-project.

Q10. It is given that we have matrix $D(q)$ and $V(q)$.

The Kinetic energy (quadratic function of $\dot{q}$) is of form.

$$K = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q) \dot{q}_i \dot{q}_j = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

~~Here~~ Now, assuming Potential energy $P = P(q)$ is independent of $\dot{q}$.

The Euler-Lagrange equations for system can be derived as:

$$L = K - P = \frac{1}{2} \sum_{i,j} d_{ij}(q) \dot{q}_i \dot{q}_j - P(q)$$

$$\rightarrow \quad \frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q) \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q) \ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}(q) \dot{q}_j$$

$$= \sum_j d_{kj}(q) \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}(q)}{\partial q_i} \dot{q}_i \dot{q}_j$$

Also,

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}(q)}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

Thus, the Euler-Lagrange equations can be written as:

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\left(\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \quad -(1)$$

By symmetry,

$$\sum_{i,j}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i\dot{q}_j$$
$$= \frac{1}{2}\sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right\}\dot{q}_i\dot{q}_j$$

By using symmetry we can simplify the eq^n (1) to the following,

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

taking

$$C_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\} \;\&\; \phi_k(q) = \frac{\partial V}{\partial q_k}$$

$$\boxed{\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} C_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k}$$

in matrix form,

$$\boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau}$$