20110029 — Ashutosh-Goyal — ME639

Assignment - 2

(1) $\boxed{RS(a)R^T = S(Ra)}$

$\forall$ R is a Rotation matrix

We know that, In case of Rotation matrix

$$\boxed{R(a\times b) = (Ra)\times(Rb)} \quad -(1)$$

Also,

$$\boxed{S(a).p = a\times p} \quad -(2)$$

Also,

$$\boxed{S(\alpha a+\beta b) = \alpha S(a) + \beta S(b)}$$

Therefore, From (1) & (2) we can write

$$(Ra)\times(RR^Tb) = R(\underbrace{a\times R^Tb}_{S(a).R^Tb})$$

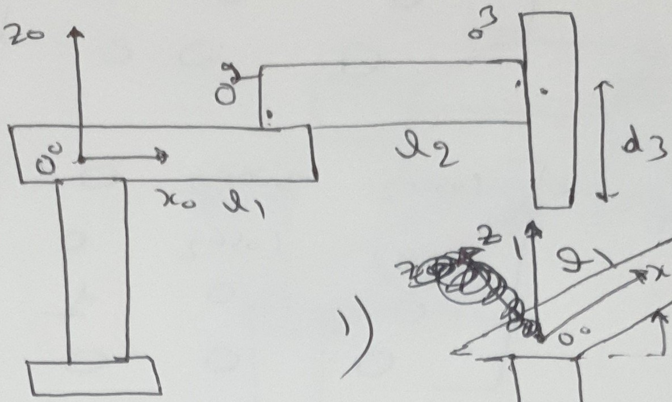
Hence, $RS(a).R^Tb = (Ra)\times(\underbrace{RR^T}_{I}b)$

$$= (Ra)\times b$$

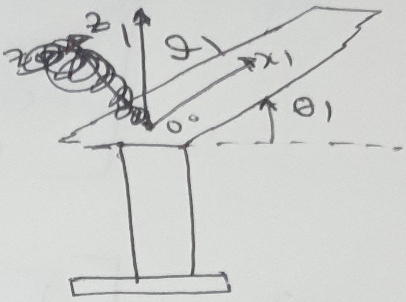
$$\boxed{RS(a).R^T.b = S(Ra)b}$$

Hence, we get $\boxed{RS(a)R^T = S(Ra)}$ → Hence proved

② SCARA



1)



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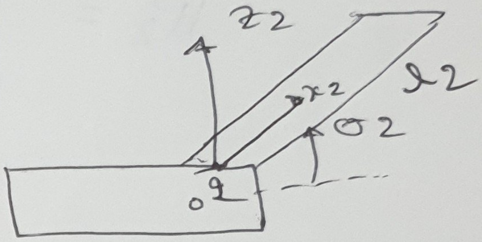

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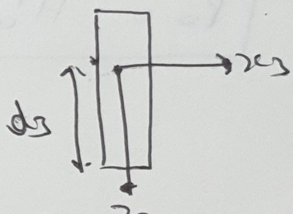


Now, let's suppose, we know the position of end-effector w.r.t 3rd frame and want to find out the w.r.t 0th frame.

$$\boxed{P_0 = H_0^1 \, H_1^2 \, H_2^3 \, P_3}$$

H_0^1 → Rotation about Z-axis by angle θ_1

H_1^2 → Rotation about z-axis by angle θ_2

H_2^3 → translation about z-axis by distance d_3

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

Hence, $H^1_0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

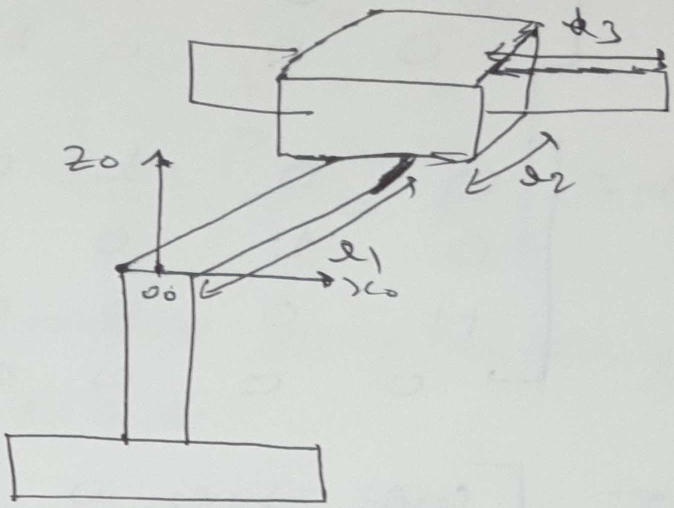
Similarly, $H^2_1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ As $d^2_1 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

$H^3_2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ As $d^3_2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

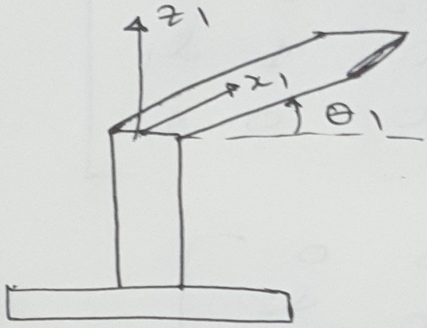
Hence, $\boxed{P_0 = H^1_0 H^2_1 H^3_2 P_3}$ — Ans $P_3 = \begin{bmatrix} 0 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}$

2

(4) Standford-type



1)



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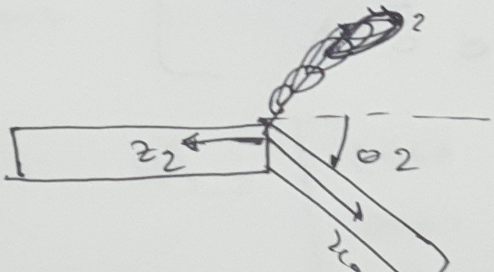

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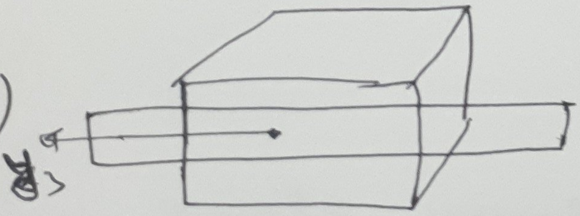


(moment is occuring inside the plane)

Hence, $\boxed{P_0 = \underbrace{H^1_0}\, H^2_1\, H^3_2\, P_3}$ $\Rightarrow$ $\boxed{P_0 = H^1_0\, R_{y,90°}\, H^2_1\, H^3_2\, P_3}$

There will also a Rotation about y-axis, if we want to align both the z-axis in same direction.

$\Rightarrow$

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{y,-90} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ d_3 \\ 0 \\ 1 \end{bmatrix}$$

(5) Initially, drone moves 10 m in z-direction

So,
$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, there is rotation about x-axis by an angle of $\theta = 30^\circ$

Hence,
$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, there is follow-up rotation of $\theta = 60^\circ$ about z-axis

$$\Rightarrow \quad H_2^3 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, Finally obstacle is at 3m above the drone. Hence

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

location of obstacle w.r.t current/last frame

Drone's Frame

Hence, $P_0 = H^1_0 H^2_1 H^3_2 P_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60 & -\sin 60 & 0 & 0 \\ \sin 60 & \cos 60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Hence, By solving this, we get

$$P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.98 \\ 1 \end{bmatrix}$$

Hence, the position vector of the obstace w.r.t base frame is $\boxed{(0, -1.5, 12.98)}$

Ans

(6) There are various types of gearbox which are generally used in motors to provides torque or to alter the velocity.

1) <u>Helical Gears</u> :- Helical gears have a diagonal tooth profile which allows them to be quiter and smoother then other gears. They have the ability to be mounted in parallel or crossed but they have disadvantage as they are less efficient.

2) <u>Spur gears</u> :- They have straight teeth that are easy to align and they are very efficient and there is minimal power lost due to slippage. But, we can only use them in parallel only. They also becomes noisy at high speeds.

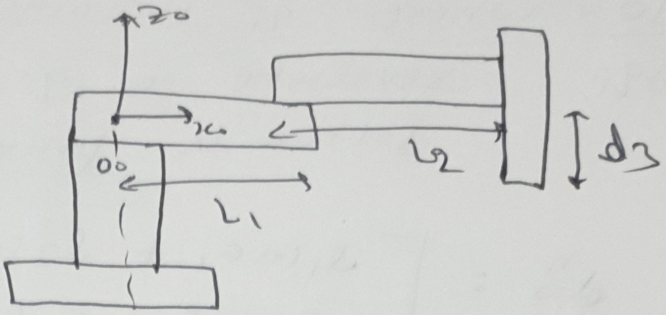
3) <u>Bevel gears</u> :- These are used for intersecting shafts and have a changeable operating angle due to their shape. They have a disadvantage like there are very difficult to assemble due to changeable operating angle.

4) <u>Rack gears</u> :- They have the advantage of like when paired with a spur gear or pinion; they can transfer rotary motion into linear motion. But they cannot run continuosly, since the rack will eventually end.

5) worm gears:- worm gears are self-locking and quiet, but suffer from high power loss and high thrust load on the worm.

The main work of the gearbox is providing the high torque which is generally done by reducing the speed. But, In drones we need very high speed. Therefore, we generally refuses to use the gearbox in drone. We generally use BLDC motor in drones which is gearless.

(7)



We know that, $\begin{bmatrix} V \\ \omega \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \dot{q}$

Here, 2 joints are revolute & 1 is prismatic

As, $J_\omega = \begin{bmatrix} \rho_1 z_0 & --- & \rho_n z_{n-1} \end{bmatrix}$

Therefore, $\boxed{\rho_1 = \rho_2 = 1}$ & $\boxed{\rho_3 = 0}$

Also, $\boxed{z_i = R_0^{i-1} \hat{k}}$

For First rotation:-

$$z_0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

similarly, $z_1 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Hence, $z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Hence, $J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Now, we know that i^{th} column of J_v is

$\frac{\partial d_0^n}{\partial q_i}$ where q_i is angle if i^{th} joint is revolute & q_i is distance if i^{th} joint is prismatic.

Finally, $d_0^n = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ -d_3 \end{bmatrix}$

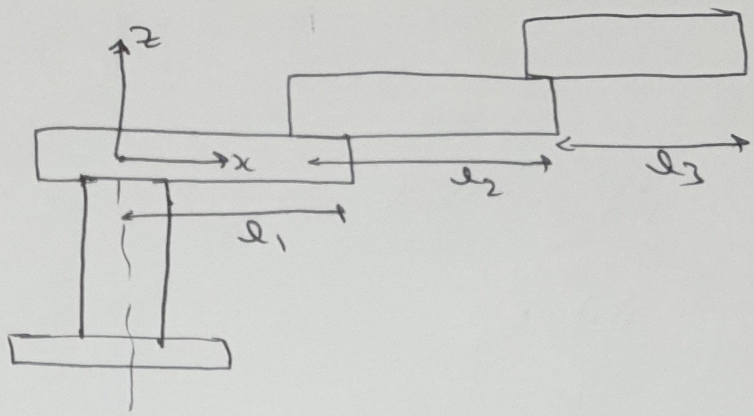
Hence, $J_v = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} & 0 \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Therefore $\boxed{J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ -(l_1S_1 + l_2S_{12}) & -l_2S_{12} & 0 \\ l_1C_1 + l_2C_{12} & l_2C_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}}$

Ans

Jacobian manipulator for SCARA

9



we know that, $\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \dot{q}$

Here all joints are revolute.

$$J_w = \begin{bmatrix} \rho_1 z_0 & \cdots\cdots & \rho_n z_{n-1} \end{bmatrix}$$

As all joints are revolute $\Rightarrow$ $\boxed{\rho_1 = \rho_2 = \rho_3 = 1}$

$$\boxed{z_i = R_0^{i-1} \hat{k}}$$

For first rotation:-

$$z_0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly, $z_1 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Similarly $z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Hence, $J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Now, we know that ~~a~~ i^{th} column of J_v is

$$\frac{\partial d_0^n}{\partial q_i}$$

Firstly, $d_0^n = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos(\theta_1+\theta_2+\theta_3) \\ l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin(\theta_1+\theta_2+\theta_3) \\ 0 \end{bmatrix}$

Now $J_v = \begin{bmatrix} -l_1S_1 - l_2S_{12} - l_3S_{123} & -l_2S_{12} - l_3S_{123} & -l_3S_{123} \\ +l_1C_1 + l_2C_{12} + l_3C_{123} & l_2C_{12} + l_3C_{123} & l_3C_{123} \\ 0 & 0 & 0 \end{bmatrix}$

Hence, $J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -(l_1S_1 + l_2S_{12} + l_3S_{123}) & -(l_2S_{12} + l_3S_{123}) & -l_3S_{123} \\ l_1C_1 + l_2C_{12} + l_3C_{123} & l_2C_{12} + l_3C_{123} & l_3C_{123} \\ 0 & 0 & 0 \end{bmatrix}$

Jacobian manipulator

Ans