

$R_2, q_1, R_2, q_2, R_2, q_3$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ l_2 \\ 0 \end{bmatrix}$$

$$d_3 = \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

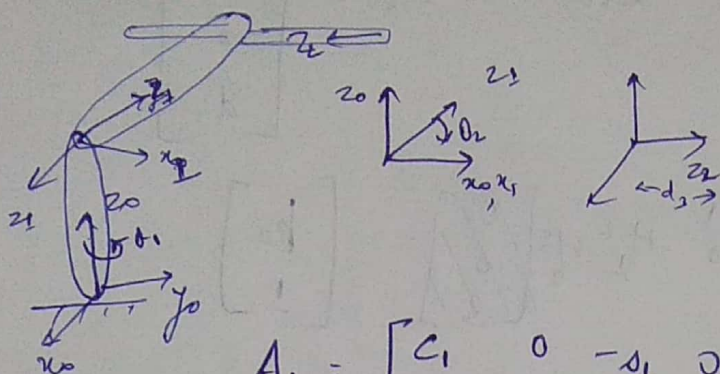
$$H_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & 0 \\ s_{q_2} & c_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$$

4. Stanford RRP manipulator



Link	d_i	a_i	α_i	θ_i
1	0	0	-90°	θ_1
2	d_2 (constant)	0	$+90^\circ$	θ_2
3	d_3	0	0	0

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

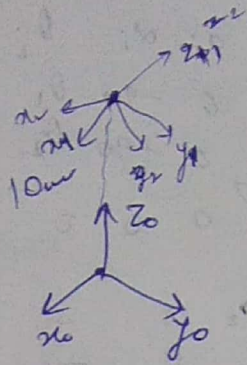
$$A_2 = \begin{bmatrix} c_2 & 1 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3$$

$$= \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & c_1 s_2 d_3 - s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 s_2 d_3 + c_1 d_2 \\ -s_2 & 0 & c_2 & c_2 d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$p = R_{x,30^\circ} \cdot R_{y,60^\circ} \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix} = H_0^{-1} H_1^{-1} H_2^{-1} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix} \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$$

$$H_0^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & \sin 30^\circ & 0 \\ 0 & -\sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^{-1} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} p_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^{-1} H_1^{-1} H_2^{-1} \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$$

$$[H_2^3] [p_3] = [p_2] = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$[H_1^2] [p_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix} = \begin{bmatrix} p_1 \\ 1 \end{bmatrix}$$

$$[H_0^1] [p_1] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

Position vector: $-\frac{3}{2}\hat{j} + \left(\frac{3\sqrt{3}}{2} + 10\right)\hat{k}$

i) Concentric gear box

Shaft arrangement: High & low-speed shafts are on the same horizontal and vertical plane.

Mounting - Foot-mounted

Industries - Agriculture, automotive, cement, energy, forest, mining

Applications -

- i) Heavy duty bulk material handling
- ii) Heavy duty process specific.

ii) Parallel gear box: In this, high and low speed shaft are on the same horizontal plane and parallel to each other.

Mounting: Foot mounted.

Industries: Agriculture, automotive, energy, mining.

Applications: Conveying, mill, crushers etc.

iii) Right angle gear box:

High and low-speed shafts are on the same horizontal and vertical plane.

Mounting: Foot mounted.

Industries: Automotive, food & beverage, cement

Applications: Conveying, elevators, agitators etc.

iv) Shaft mounted gear box:

The gearbox is mounted directly onto and supported by the driven shaft.

Applications: Belt conveyors, classifiers, separators

For RRP & SCARA manipulator, DH convention,

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

$$= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joints 1, 2 and 4 are revolute and joint 3 is prismatic, and $(O_4 - O_3)$ is parallel to z_3 (thus, $z_3 \times (O_4 - O_3) = 0$),

$$J = \begin{bmatrix} z_0 \times (O_4 - O_0) & z_1 \times (O_4 - O_1) & z_2 & 0 \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$

The origins of the DH frames are given by

$$O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

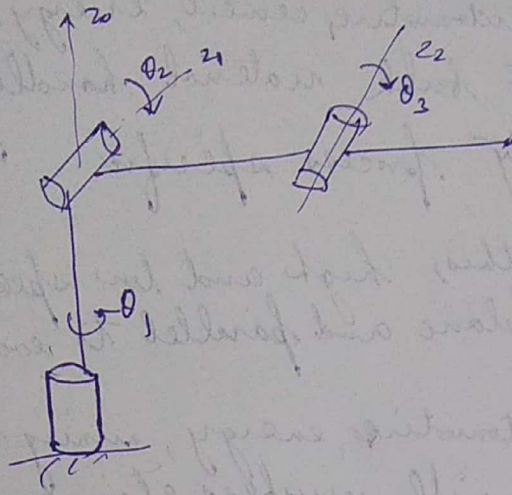
$$O_4 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

$$z_0 = z_1 = k,$$

$$z_2 = z_3 = -k$$

$$J = \begin{bmatrix} -a_1 s_1 & -a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

9.



$$z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1^0 = \begin{bmatrix} \sin \theta_1 \\ \cos \theta_1 \\ 0 \end{bmatrix}$$

$$z_2^0 = \begin{bmatrix} \cos \theta_1 \sin \theta_2 + \cos \theta_2 \sin \theta_1 \\ \sin \theta_1 \sin \theta_2 - \cos \theta_1 \cos \theta_2 \\ 0 \end{bmatrix}$$

$$O_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1^0 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$$

$$O_2^0 = \begin{bmatrix} d_2 \cos \theta_1 \\ d_2 \sin \theta_1 \\ d_1 \end{bmatrix}$$

$$O_3^0 = \begin{bmatrix} d_2 \cos \theta_1 + d_2 \cos \theta_2 - d_3 - d_1 \sin \theta_2 \sin \theta_1 \\ d_2 \sin \theta_1 + d_2 \sin \theta_2 + d_1 \cos \theta_2 \sin \theta_1 \\ d_1 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0^0 \times (O_3^0 - O_0^0) & z_1^0 \times (O_3^0 - O_1^0) & z_2^0 \times (O_3^0 - O_2^0) \\ z_0^0 & z_1^0 & z_2^0 \end{bmatrix}$$