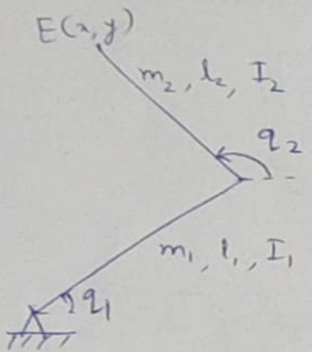
# ME - 639

## Mini project

### Task - 0)




Consider the 2R manipulater with $m_i$, $l_i$ and $I_i$ as the mass, length and moment of inertia of the respective links. Also, Let the co-ordinates of end effector be $(x, y)$.

$$\therefore \left.\begin{aligned} x &= l_1 \cos(q_1) + l_2 \cos(q_2) \\ y &= l_2 \sin(q_1) + l_2 \sin(q_2) \end{aligned}\right\} \quad \text{— (1)}$$

Differentiate w.r.t. time, we get

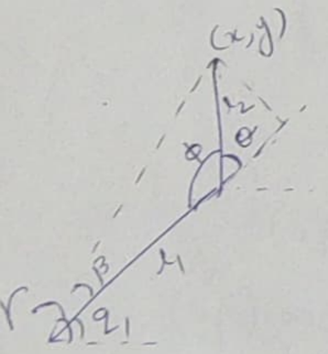
$$\dot{x} = -l_1 \sin(q_1)\dot{q}_1 - l_2 \sin(q_2)\dot{q}_2$$

$$\dot{y} = l_1 \cos(q_1)\dot{q}_1 + l_2 \cos(q_2)\dot{q}_2$$

~~Lineo~~ Above equations can be represented in matrix form as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin(q_1) & -l_2 \sin(q_2) \\ l_1 \cos(q_1) & l_2 \cos(q_2) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— (2)}$$

Also for the relation between $(x, y)$ and $q_1, q_2$ we can use inverse kinematics as,




Here, $q_2 = q_1 + \theta$

Using cosine rule we get

$$\cos(\phi) = \frac{l_1^2 + l_2^2 - x^2 - y^2}{2 l_1 l_2} = \cos(\pi - \theta)$$

$$\Rightarrow -\cos(\theta) = \frac{l_1^2 + l_2^2 \mp x^2 - y^2}{2 l_1 l_2}$$

$$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

Also, $q_1 = \gamma - \beta$

$\therefore$ ~~$\gamma = $ ta~~ $\tan\gamma = \dfrac{y}{x} \Rightarrow \gamma = \tan^{-1}\left(\dfrac{y}{x}\right)$

and, $\tan\beta = \dfrac{l_2 \sin\theta}{l_1 + l_2\cos\theta} \Rightarrow \beta = \tan^{-1}\left(\dfrac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right)$

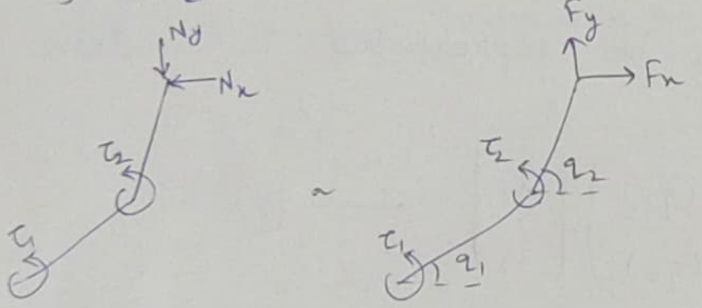
$$\therefore \boxed{q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right)}$$

And, $q_2 = q_1 + \theta$

$$\boxed{q_2 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right) + \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

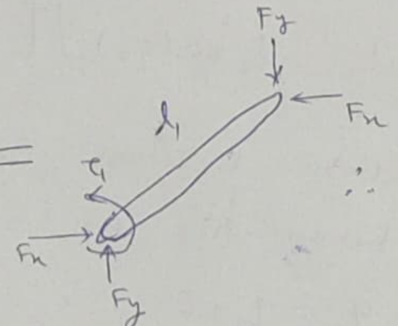
Combining all the bones we get equation (3).

For Finding forces we need a relation between torques provided by motors and force applied by end effector.



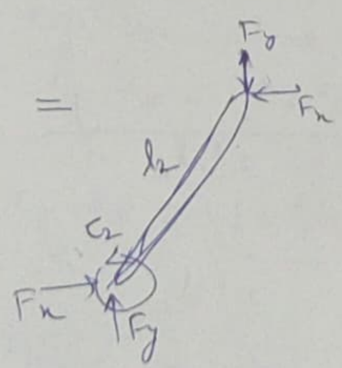

Under static equilibrium, Assuming no gravity. we ham

FBD of link -1 =




$$\therefore \tau_1 = -F_x l_1 \sin(q_1) + F_y l_1 \cos(q_1)$$

FBD of link -2 =




$$\tau_2 = -F_x l_2 \sin(q_2) + F_y l_2 \cos(q_2)$$

Representing in matrix form, we get

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin(q_1) & l_1 \cos(q_1) \\ -l_2 \sin(q_2) & l_2 \cos(q_2) \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \text{—— (4)}$$

Now for the robotic arm to show a virtual spring effect about ~~an~~ given point, we have

Let the given point be $(x_0, y_0)$.

$$\therefore \left. \begin{aligned} F_x &= k(x - x_0) \\ F_y &= k(y - y_0) \end{aligned} \right\} \text{— (5)}$$

Here $k$ is the user defined spring stiffness

The equations derived above did not take into ~~account~~ account any dynamics of robotic arm. For considering dynamics we need to use langrangian equation, where

$$\mathcal{L} = K - V \quad \text{and} \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'$$

where $i = 1, 2, \ldots$

$Q_i' =$ generalised force

The kinetic energy of links will be,

$$K = \frac{1}{2}\left\{ \left(\frac{1}{3} m_1 l_1^2 + m_2 l_1^2\right)\dot{q}_1^2 + \left(\frac{1}{3} m_2 l_2^2\right)\dot{q}_2^2 + \left(m_2 l_1 l_2 \cos(q_2 - q_1)\right)\dot{q}_1 \dot{q}_2 \right\}$$

and Potential Energy, $V = m_1 g \frac{l_1}{2} \sin(q_1) + m_2 g \left(l_1 \sin(q_1) + \frac{l_2}{2}\sin(q_2)\right)$

$$\therefore \text{(6)} \begin{cases} \tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 l_1 l_2 \ddot{q}_2 \cos(q_2 - q_1)/2 - m_1 l_1 l_2 \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)/2 \\ \quad - m_1 g l_1 \cos(q_1)/2 + m_2 g l_1 \cos(q_1) \\ \tau_2 = \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 l_2^2 \ddot{q}_2/4 + m_2 l_1 l_2 \ddot{q}_1 \cos(q_2 - q_1)/2 + m_2 g l_2 \sin(q_2)/2 \\ \quad - m_2 l_1 l_2 \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)/2 \end{cases}$$