Task-1

1) Show that $RS(a)R^T = S(Ra)$ where R is a rotation matrix.

Ans

$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}_{3\times 1} \qquad S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}_{3\times 3}$$

let $R \rightarrow R_{y,\theta}$

$$R = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}_{3\times 3}, \quad R^T = \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}$$

$$R(S(a))R^T = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}$$

$$= \begin{bmatrix} -a_y S\theta & -a_z C\theta + a_x S\theta & a_y C\theta \\ a_z & 0 & -a_x \\ -a_y C\theta & +a_z S\theta + a_x C\theta & -a_y S\theta \end{bmatrix} \begin{bmatrix} C\theta & 0 & -S\theta \\ 0 & 1 & 0 \\ S\theta & 0 & C\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_z C\theta + a_x S\theta & a_y \\ a_z C\theta - a_x S\theta & 0 & -a_z S\theta - a_x C\theta \\ -a_y & a_z S\theta + a_x C\theta & 0 \end{bmatrix}$$

this matrix is skew Symmetric for sure.

$= S(\omega)$

where $\omega = \begin{bmatrix} a_z S\theta + a_x C\theta \\ a_y \\ a_z C\theta - a_x S\theta \end{bmatrix}$

$$Ra = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$(Ra) = \begin{bmatrix} a_x C\theta + a_z S\theta \\ a_y \\ -a_x S\theta + a_z C\theta \end{bmatrix}$$

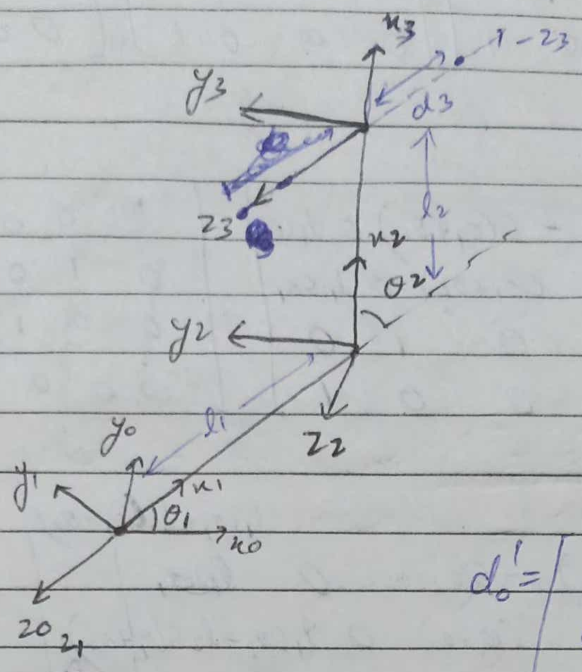
$\because \quad \omega = Ra$

So, $\quad R(S(a))R^T = S(\omega)$

$$\boxed{R\,S(a)\,R^T = S(Ra)}$$

hence proved.

(2)



$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ -d_3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C\theta_1 C\theta_2 - S\theta_1 S\theta_2$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & l_1 C\theta_1 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & l_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}$$

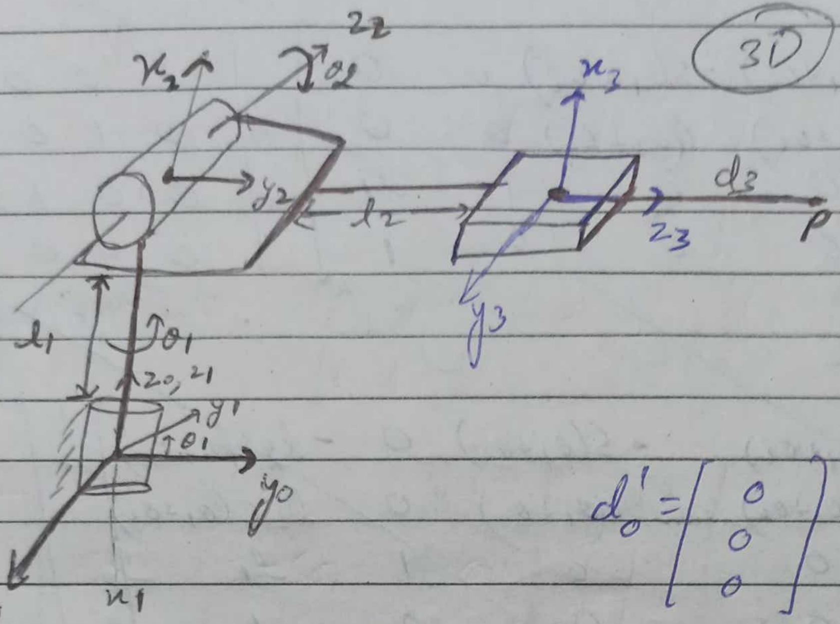
$$= \underset{4\times4}{\begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 & l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 & l_1 S\theta_1 + l_2 S(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 0 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) \\ l_1 S\theta_1 + l_2 S(\theta_1+\theta_2) \\ -d_3 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} P_{0x} &= l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ P_{0y} &= l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ P_{0z} &= -d_3 \end{aligned}}$$

(4)

3D



$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation of z_1 about y_1 such that z_1 [illegible]

$$H_1^2 = \begin{bmatrix} C(-90) & 0 & S(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -S(-90) & 0 & C(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \cancel{\begin{bmatrix} -S\theta_2 & -C\theta_2 & 0 & 0 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \begin{bmatrix} 0 & 0 & -1 & -l_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -l_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

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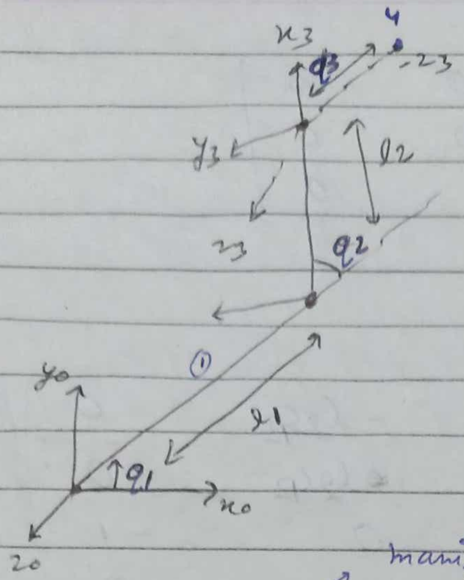

manipulator Jacobian

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J \dot{q}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$d_0^4 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ -q_3 \end{bmatrix}$$

$$J_v = \begin{bmatrix} \frac{\partial d_0^4}{\partial q_1} & \frac{\partial d_0^4}{\partial q_2} & \frac{\partial d_0^4}{d q_3} \end{bmatrix}$$

$$J_v = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$J\omega = \begin{bmatrix} 1 \times z_0 & 1 \times z_1 & 0 \times z_2 \\ & & \end{bmatrix}$$

$$= \begin{bmatrix} z_0 & z_1 & 0_{3\times3} \end{bmatrix}$$

$$z_0 = \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1 = R_0^1 \hat{k} = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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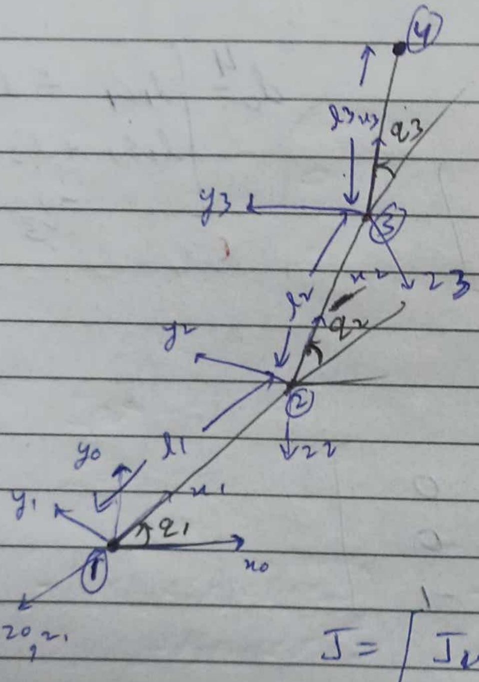

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J \dot{q}$$

$$J = \begin{bmatrix} J_v \\ J\omega \end{bmatrix}$$

$$\begin{bmatrix} d_0^4 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} d_3^4 \\ 1 \end{bmatrix} \qquad d_3^4 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \; 1$$

$$H_0^1 H_1^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 \\ s_{12} & c_{12} & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 \\ s_{12} & c_{12} & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & l_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_2 s_{12} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} d_0^4 \\ 1 \end{bmatrix} = \begin{bmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_2 s_{12} + l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} d_0^4 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3 c_{123} + l_1 c_1 + l_2 c_{12} \\ l_3 s_{123} + l_2 s_{12} + l_1 s_1 \\ 0 \\ 1 \end{bmatrix}$$

$$d_0^4 = \begin{bmatrix} l_3 c_{123} + l_1 c_1 + l_2 c_{12} \\ l_3 s_{123} + l_2 s_{12} + l_1 s_1 \\ 0 \end{bmatrix}$$

$$J_v = \begin{bmatrix} \frac{\partial d_0^4}{\partial q_1} & \frac{\partial d_0^4}{\partial q_2} & \frac{\partial d_0^4}{\partial q_3} \end{bmatrix}$$

$$J_v = \begin{bmatrix} -l_3 s_{123} - l_1 s_1 - l_2 s_{12} & -l_3 s_{123} - l_2 s_{12} & -l_3 s_{123} \\ l_3 c_{123} + l_2 c_{12} + l_1 c_1 & l_3 c_{123} + l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_\omega = \begin{bmatrix} 1 \times z_0 & 1 \times z_1 & 1 \times z_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_3 s_{123} + l_1 s_1 + l_2 s_{12}) & -(l_3 s_{123} + l_2 s_{12}) & -l_3 s_{123} \\ l_3 c_{123} + l_2 c_{12} + l_1 c_1 & l_3 c_{123} + l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$