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ME 631: Assignment 1

Question 2:

Types of Robots:

Mobile Robots (ground): identify their surroundings and move around in their environment
Examples:
Unmanned rovers: can carry out missions and studies on extraterrestrial bodies - for example, the curiosity rover
https://www.youtube.com/watch?v=3-MNAX1jgbA
Roomba: a consumer robot used for cleaning spaces - it uses sensors to avoid bumping into furniture and walls, and can be programmed by the user to clean at specific times of the day
https://www.youtube.com/watch?v=XIPzSmwClJ8

Aerial Robots (UAV): fly through air. Can be used to collect data for weather forecasting, for general surveillance, and so on.
Examples:
Quadcopter: multicopter propelled by four rotors
https://www.youtube.com/watch?v=w2itwFJCgFQ
Wing flapping robot: flies by flapping wings
https://www.youtube.com/watch?v=IoHzoeFP9Io

Underwater Robots (AUV): direct themselves and travel through water
Example: ROVs used in shipwreck surveys
https://www.youtube.com/watch?v=p1HmgP9l4VY

Soft Robots: robots made out of compliant and flexible materials that are often made to imitate physical characteristics of living organisms. They can be an example of biomimicry in engineering.
Example: An octopus-inspired robot that can twist and squeeze its way around
https://www.youtube.com/watch?v=A7AFsk40NGE

Micro Robots: mobile robots with dimensions less than 1mm
Example: Robotic crab designed to work in confined spaces (possibly inside the human body one day): https://www.youtube.com/watch?v=1IP7jptXjgQ

RRP (Stanford type): Two rotary and one prismatic joint. Axis of rotation of the first 2 R are perpendicular.
Example: the original stanford arm created by Scheinman in a hand-eye experiment (1971):
https://www.youtube.com/watch?v=laWnTCg5I9w

RRR (Puma type): Three revolutionary joints. Axis of rotation of the first 2 R are perpendicular, and the axis of rotation of the last 2 R are parallel.
Example: this robot can draw pre-programmed images with a pen it holds:
https://www.youtube.com/watch?v=aHV5oY7viBM

RRP (Scara type): Two rotary and one prismatic joint. Axis of rotation of the first 2 R are parallel.
Example: robot used for rapid pick and place operations
https://www.youtube.com/watch?v=-m1oKuFkSTE

Question 3:
Types of Motors:

Motors are broadly classified into two types - AC and DC. DC motors are classified into brushed and brushless motors. Brushless motors can be stepped (or just a regular brushless motor). AC motors are classified into synchronous and asynchronous (induction) motors. Servo motors are special cases - they can be powered using either AC or DC.

Brushed DC motors: made of stator and rotor. Stator stays in place, the rotor moves. Stator has copper/carbon brushes attached, which make contact with the copper pads on the rotor and are connected to the input and output windings. When they change the pad they touch, they change their magnetic polarity. The other side of the stator has 2 permanent magnets - one north pole, other south pole. The change in magnetic polarity of the pads is what keeps the motor moving. Higher DC power means higher speed. These motors also have a core to help the magnetic field.

AC induction motors: have windings to induce a magnetic field. The AC field provides an oscillating magnetic field. 1 phase means the motor has one core, 3 phases means it has 3 cores, each aligned at 120 degrees with each other. The rotor has closed loops of conducting material. When current is provided, it induces a magnetic field in the core. As a consequence of Lorenz law, a changing magnetic flux gives rise to an electric field. The electric field generated in the coils gives rise to electromagnetic force, which allows the coil to spin. No brushes means these last longer. Speed increases with increase in frequency of the alternating current.

AC synchronous motor: rotor speed and stator magnetic field speed are equal. Rotor has permanent magnets or electromagnets, not cores. AC voltage is provided to the wirings in the stator, and the magnets in the rotors align themselves with the external magnetic field. This allows the rotor and the stator magnetic field to rotate at the same speed.

Brushless DC motor: have a triple phase input. Stator has coils, the rotor has magnets. Alternate coils to make poles. Pole to magnet ratio is pre-defined. Inrunners - rotor is inside, stator is around it. Outrunners - stator is inside, rotor is around it. The rotor can also be front supported or back supported. 3 phases - A,B,C. Initially, phase A is given a positive voltage (south), B is grounded(north), C is left floating. Magnets inside align themselves accordingly. Floating phase - receives back emf, voltage drop. When this reaches its peak, we switch inputs -

A is still positive, but B is floating and C is grounding. This changes the direction of the magnetic field, forcing the magnets inside to align, and the motor to keep moving. Control speed using an electronic speed controller, which inputs a DC pulse signal.

Stepper: these are also brushless. The stator surrounds the rotor. The rotor is made of a simple magnet with north and south poles. The stator has 2 coils with multiple windings at equal angles from each other. These have teeth shaped magnetic conductors, which help create steps. These are also present on the rotor. Initially, coil A is positive, B is ground. Magnet moves by one step and aligns. Then we switch poles, and the magnet moves by another step. We can make this motion simpler by increasing the number of steps. This can be done by adding more coil windings, more magnets, or switching both coils on at the same time - creating half steps. Stepper drivers are used to control speed.

Servo: can be operated with either AC or DC. Main component is the feedback, which can be done through an encoder or potentiometer. More gears in the motor means more torque (but less speed). The feedback device is located below the main shaft of the motor. This helps us find the position of the shaft. More feedback means higher precision. A comparator can be used between the potentiometer and shaft, and can be used to control both the direction of rotation and position of the shaft.

⑥

$$R_0^1 = \begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 & \hat{j}_1\cdot\hat{i}_0 & \hat{k}_1\cdot\hat{i}_0 \\ \hat{i}_1\cdot\hat{j}_0 & \hat{j}_1\cdot\hat{j}_0 & \hat{k}_1\cdot\hat{j}_0 \\ \hat{i}_1\cdot\hat{k}_0 & \hat{j}_1\cdot\hat{k}_0 & \hat{k}_1\cdot\hat{k}_0 \end{bmatrix} \quad \text{columns: } \underset{①}{\begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 \\ \hat{i}_1\cdot\hat{j}_0 \\ \hat{i}_1\cdot\hat{k}_0 \end{bmatrix}} \; \underset{②}{\begin{bmatrix} \hat{j}_1\cdot\hat{i}_0 \\ \hat{j}_1\cdot\hat{j}_0 \\ \hat{j}_1\cdot\hat{k}_0 \end{bmatrix}} \; \underset{③}{\begin{bmatrix} \hat{k}_1\cdot\hat{i}_0 \\ \hat{k}_1\cdot\hat{j}_0 \\ \hat{k}_1\cdot\hat{k}_0 \end{bmatrix}}$$

If a matrix A is orthogonal $\Rightarrow A.A^T = 1$

$\Rightarrow$ If columns are orthogonal. $①①^T = I_{3\times3}$ $②②^T = I_{3\times3}$ $③③^T = I_{3\times3}$

$①①^T$:

$$\begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 \\ \hat{i}_1\cdot\hat{j}_0 \\ \hat{i}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 & \hat{i}_1\cdot\hat{j}_0 & \hat{i}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} (\hat{i}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{i}_0) & \overbrace{(\hat{i}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{j}_0)}^{0} & \overbrace{(\hat{i}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{k}_0)}^{0} \\ \underbrace{(\hat{i}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{i}_0)}_{0} & (\hat{i}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{j}_0) & \underbrace{(\hat{i}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{k}_0)}_{0} \\ \underbrace{(\hat{i}_1\cdot\hat{k}_0)(\hat{i}_1\cdot\hat{i}_0)}_{0} & \underbrace{(\hat{i}_1\cdot\hat{k}_0)(\hat{i}_1\cdot\hat{j}_0)}_{0} & (\hat{i}_1\cdot\hat{k}_0)(\hat{i}_1\cdot\hat{k}_0) \end{bmatrix}$$

$\hat{i}_0, \hat{j}_0, k_0 \rightarrow$ orthogonal $\quad \hat{i}_1, \hat{j}_1, \hat{k}_1 \rightarrow$ orthogonal

$\Rightarrow$ dot product of any $2 = 0$ $\quad$ dot product with itself $= 1$ (unit vectors)

$$\Rightarrow \begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 \\ \hat{i}_1\cdot\hat{j}_0 \\ \hat{i}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{i}_1\hat{i}_0 & \hat{i}_1\cdot\hat{j}_0 & \hat{i}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow ① \text{ is orthogonal}$$

$②②^T$:

$$\begin{bmatrix} \hat{j}_1\cdot\hat{i}_0 \\ \hat{j}_1\cdot\hat{j}_0 \\ \hat{j}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{j}_1\cdot\hat{i}_0 & \hat{j}_1\cdot\hat{j}_0 & \hat{j}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} (\hat{j}_1\cdot\hat{i}_0)(\hat{j}_1\cdot\hat{i}_0) & \overbrace{(\hat{j}_1\cdot\hat{i}_0)(\hat{j}_1\cdot\hat{j}_0)}^{0} & \overbrace{(\hat{j}_1\cdot\hat{i}_0)(\hat{j}_1\cdot\hat{k}_0)}^{0} \\ \underbrace{(\hat{j}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{i}_0)}_{0} & (\hat{j}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{j}_0) & \underbrace{(\hat{j}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{k}_0)}_{0} \\ \underbrace{(\hat{j}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{i}_0)}_{0} & \underbrace{(\hat{j}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{j}_0)}_{0} & (\hat{j}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{k}_0) \end{bmatrix}$$

$$\begin{bmatrix} \hat{j}_1\cdot\hat{i}_0 \\ \hat{j}_1\cdot\hat{j}_0 \\ \hat{j}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{j}_1\hat{i}_0 & \hat{j}_1\cdot\hat{j}_0 & \hat{j}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow I \Rightarrow ② \text{ is orthogonal}$$

$③③^T$:

$$\begin{bmatrix} \hat{k}_1\cdot\hat{i}_0 \\ \hat{k}_1\cdot\hat{j}_0 \\ \hat{k}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{k}_1\cdot\hat{i}_0 & \hat{k}_1\hat{j}_0 & \hat{k}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} (\hat{k}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{i}_0) & \overbrace{(\hat{k}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{j}_0)}^{0} & \overbrace{(\hat{k}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{k}_0)}^{0} \\ \underbrace{(\hat{k}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{i}_0)}_{0} & (\hat{k}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{j}_0) & \underbrace{(\hat{k}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{k}_0)}_{0} \\ \underbrace{(\hat{k}_1\cdot\hat{k}_0)(\hat{k}_1\cdot\hat{i}_0)}_{0} & \underbrace{(\hat{k}_1\cdot\hat{k}_0)(\hat{k}_1\cdot\hat{j}_0)}_{0} & (\hat{k}_1\cdot\hat{k}_0)(\hat{k}_1\cdot\hat{k}_0) \end{bmatrix}$$

$$\begin{bmatrix} \hat{k}_1\cdot\hat{i}_0 \\ \hat{k}_1\cdot\hat{j}_0 \\ \hat{k}_1\cdot\hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{k}_1\hat{i}_0 & \hat{k}_1\cdot\hat{j}_0 & \hat{k}_1\cdot\hat{k}_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow ③ \text{ is orthogonal}$$

$\Rightarrow$ columns of R_0^1 are orthogonal

⑦ $R_0^1 = R_{z,\theta_1} . R_{y,\theta_2} . R_{x,\theta_3}$

any rotation matrix can be decomposed into rotation matrices for rotation about the xyz axes

$\det(R_0^1) = \det(R_{z,\theta_1} . R_{y,\theta_2} . R_{x,\theta_3}) = \det(R_{z,\theta_1}) . \det(R_{y,\theta_2}) . \det(R_{x,\theta_3})$

$$R_{z,\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{y,\theta_2} = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

$$R_{x,\theta_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & +\sin\theta_3 & \cos\theta_3 \end{bmatrix}$$

$\det(R_{z,\theta_1}) = \cos\theta_1(\cos\theta_1) - (-\sin\theta_1)(\sin\theta_1) = \cos^2\theta_1 + \sin^2\theta_1 = 1$

$\det(R_{y,\theta_2}) = \cos\theta_2(\cos\theta_2) + \sin\theta_2(0 - (-\sin\theta_2)) = \cos^2\theta_2 + \sin^2\theta_2 = 1$

$\det(R_{x,\theta_3}) = 1.(\cos^2\theta_3 - (-\sin^2\theta_3)) = \cos^2\theta_3 + \sin^2\theta_3 = 1$

$\Rightarrow \det(R_{z,\theta_1}) . \det(R_{y,\theta_2}) . \det(R_{x,\theta_3}) = 1.1.1 = 1 = \det(R_0^1)$

$\rightarrow \det(R_0^1) = 1$