ME6~~8~~39: Assigment 2

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Q1) To prove $R S(a) R^T = S(Ra)$

We are give R as the rotational matrix.

Now let us take any $\vec{b} \in R^3$

Taking

$$R S(\vec{a}) R^T \vec{b}$$

$$= R(\vec{a} \times R^T \vec{b}) \qquad (\because S(\vec{a})\vec{p} = \vec{a} \times \vec{p})$$

$$= (R\vec{a}) \times (R R^T \vec{b})$$

$$(\because R R^T = I)$$

$$\therefore = (R\vec{a}) \times \vec{b}$$

$$= \cancel{S(\vec{a})} S(R\vec{a})\vec{b}$$

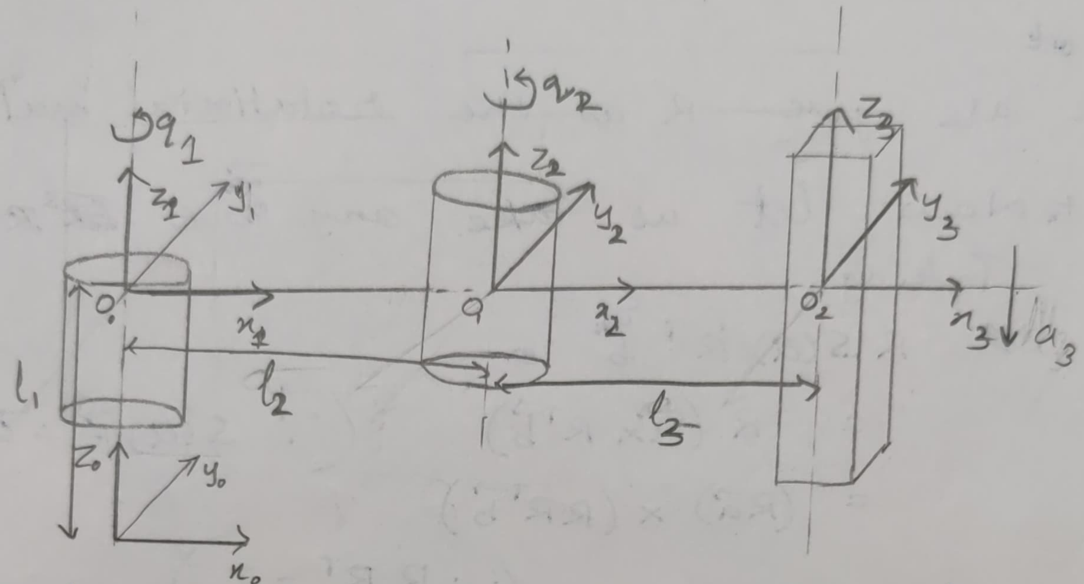
Hence on comparision we get

$$R S(\vec{a}) R^T = S(R\vec{a})$$

Hence proved

Q2)

SCARA configuration



We can write the rotation matrices as follows

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 1 \end{bmatrix}$$

here a_3, q_1 and q_2 are variables

Now P_0 is the end effector coordinates in the zeroth ~~fr~~ frame

Now we will get the relation ~~from~~ of P_0 and P_3 as following

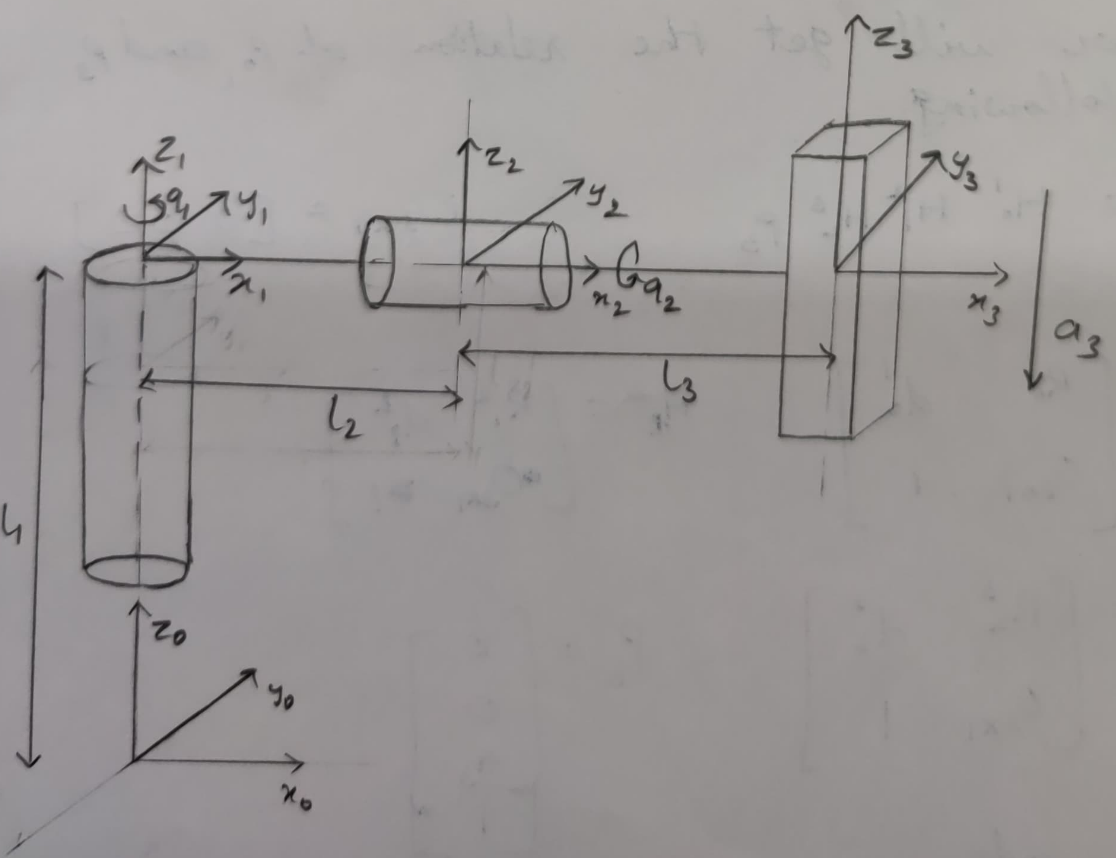
$$P_0 = H_0^1 H_1^2 H_2^3 P_3 \qquad O_{3\times1} = [0\ 0\ 0]$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ O_{3\times1} & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ O_{3\times1} & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ O_{3\times1} & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 1 \end{bmatrix}$$

Q3) Python code

Q4) RRP - Stanford type



We can write down the rotation matrices as following

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & -sq_2 \\ 0 & sq_2 & cq_2 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ q_3 \\ 1 \end{bmatrix}$$

here a_3, q_1 and a_2 are variables

P_0 is the end effector coordinates in the 0th frame

Now we will get the relation of P_0 and P_3 as following

$$P_0 = H_0^1 H_1^2 H_2^3 P_3 \qquad O_{3\times 1} = [0\ 0\ 0]$$

where

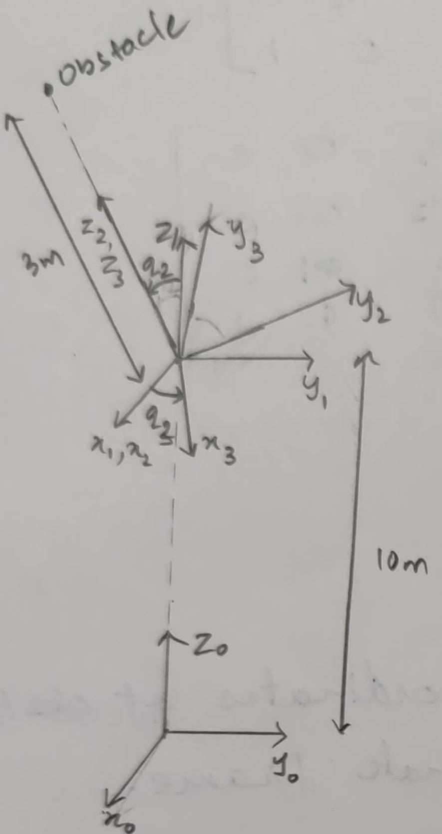
$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ O_{3\times 1} & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ O_{3\times 1} & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ O_{3\times 1} & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 1 \end{bmatrix}$$

Python code

Q5>

Let us first show how the axes are oriented with each other



The rotation matrices between different coordinate frames can be given ~~as~~ as following -

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & -sq_2 \\ 0 & sq_2 & cq_2 \end{bmatrix} \qquad q_2 = 30^\circ$$

$$R_2^3 = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad q_3 = 60^\circ$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Now lets calculate individual ~~homg~~ homogenous transformations.

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_2 & -sq_2 & 0 \\ 0 & sq_2 & cq_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_3 & -sq_3 & 0 & 0 \\ sq_3 & cq_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Let P_0 be the ~~end~~ coordinates of obstacle in ~~inertial~~ base coordinate frame.

$$P_0 = H_0^1 \, H_1^2 \, H_2^3 \, P_3$$

Substituting values

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 & 0 & 0 \\ \frac{\sqrt{3}}{2} & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -3/2 \\ \frac{3\sqrt{3}}{2} + 10 \\ 1 \end{bmatrix}$$

$\therefore$ The coordinates of obstacle in the base coordinate frame are -

$$(x, y, z) = \left(0, -3/2, \frac{3\sqrt{3}}{2} + 10\right)$$

$$\boxed{(x, y, z) = (0, -1.5, 12.5981)}$$

Q6)

There are generally 5 types of gearboxes -

i) Helical Gearbox

→ The helical gearbox is compact and consumes less power. This is generally used in ~~the~~ heavy duty operations. These are used in crushers, extruders, etc & in low power applications.

ii) Coaxial helical inline gearbox

→ They are used for heavy duty applications. They have very good quality and efficiency. They are manufactured with a high degree of specification, which allows one to maximize load and transmission rations. These are used in quarries, mining industry, etc.

iii) Skew Bevel helical Gearbox

→ They are noticed for ~~theidr~~ their rigid and monolithic structure, which makes them useful in heavy loads and other applications. These offer more ~~mechan~~ mechanical advantages once connected to the motors. They are used to move heavy loads.

iv) Worm reduction gearboxes

→ These are also used to do heavy duty operations. These are used when we need increased speed reduction between non-intersecting ~~or~~ crossed axis shafts. This uses ~~worm~~ worm wheel which has large diameter. They are used in conveyor belts, tuning instruments, etc.

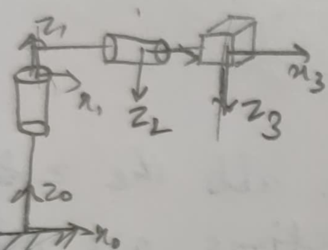
v) Planetary Gearbox

→ These have a central gear known as the sun gear and 3 to 4 planet gears revolving around it. This system provides equal power to the gears and achieves a higher torque in a small space. This type of gearbox is used in robotics and 3D printing.

7) For the SCARA manipulator (RRP) type ~~that we~~ the manipulator Jacobian can be given as following:-

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 (O_3 - O_1) & z_2 & 0 \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$

where $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}$ $\quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$



$$O_4 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

where d_4 is any variable length and d_3 is the z-distance between the 2nd and 3rd frame

$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

On substituting the above values we get

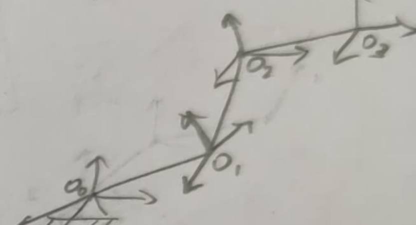
$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Q8) Python Code

Q9) ~~R~~ For RRR configuration we need to derive the manipulator Jacobian. (Planar robot)

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \quad O_2 \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 \end{bmatrix}$$



$$Z_0 = Z_1 = Z_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence all the z directions are parallel to each other.

The Jacobian is given as

$$J = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

On substituting values we get the Jacobian as

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q10) Python code