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Assignment 2

Task 1

To show

$R\,S(a)\,R^T = S(Ra)$ $\quad$ $R \to$ Rotation matrix

$S(a) \to$ skew symmetric matrix

1) $S(a)b = a \times b$ $\quad$ $a, b \in R^3$

2) $R(a \times b) = Ra \times Rb$ $\quad$ $a, b \in R^3$

& R be orthogonal.

let $b \in R^3$ arbitrary matrix

So $R\,S(a)\,R^T b = R(a \times R^T b)$

as R is orthogonal

$= Ra \times RR^T b = Ra \times b$

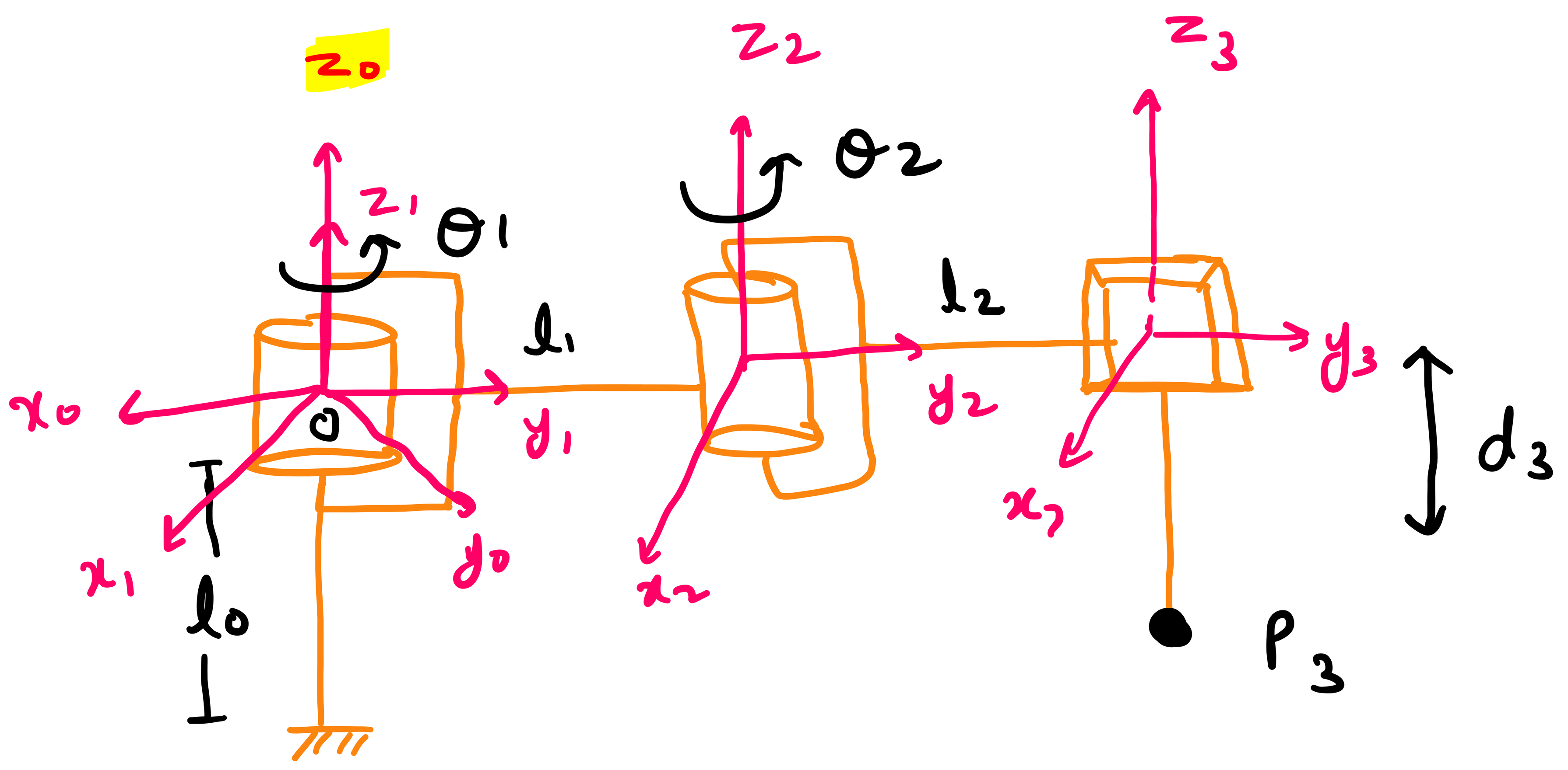
So $R\,S(a)\,R^T b = S(Ra)\,b$

from here we can see that

$R\,S(a)\,R^T = S(Ra)$

Task -2

Assuming box frame at O and not at ground.



So Rotations are $R_{z\theta_1}$, $R_{z\theta_2}$, $R_{z,0}$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ -d_3 \end{bmatrix}$$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

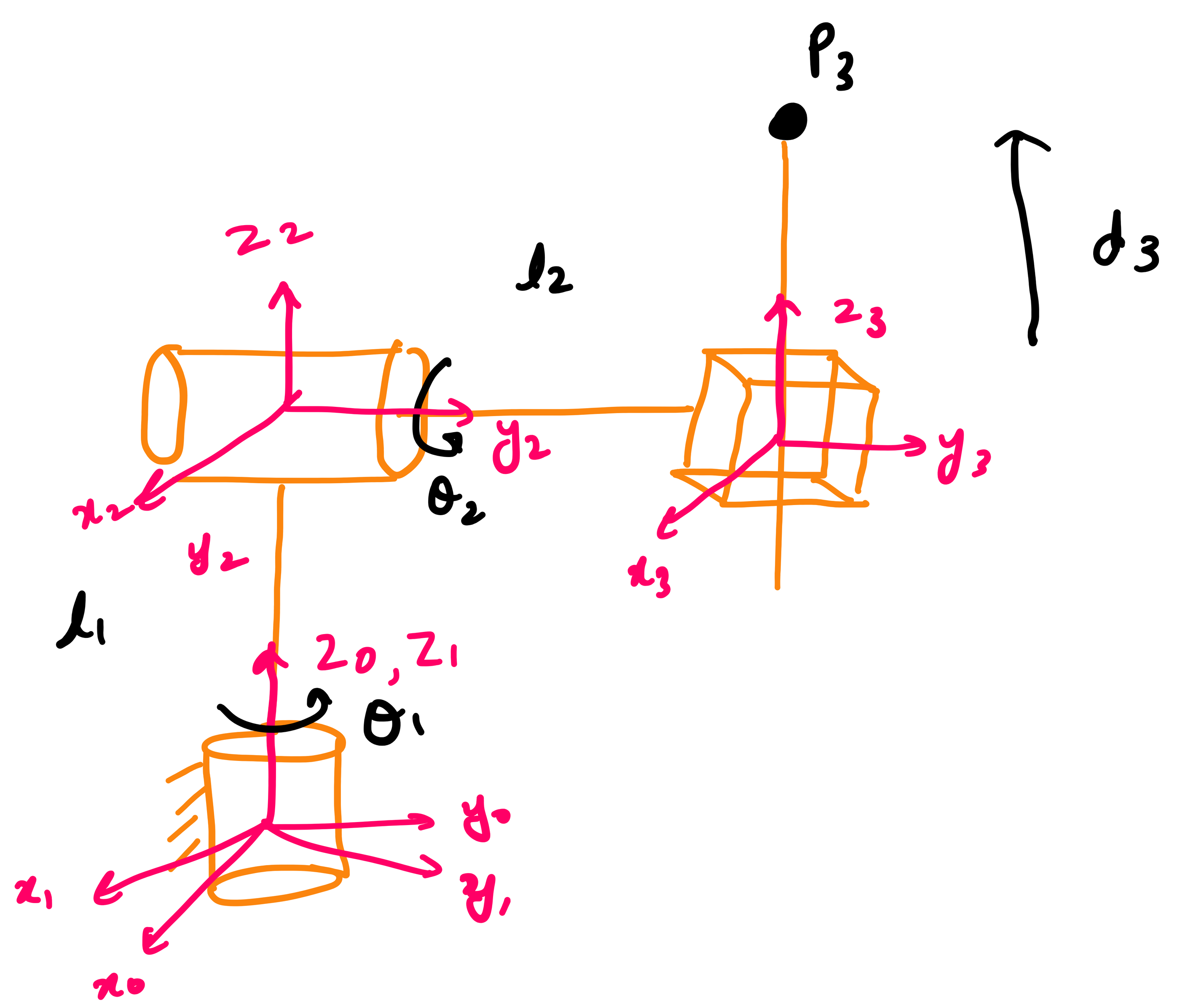
$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_{z,\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{z,\theta_2} & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{z,0} & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Solving.

$$P_0 = \begin{bmatrix} -l_2 S_{12} & -l_1 S_1 \\ l_2 C_{12} & +l_1 C_1 \\ & -d_3 \end{bmatrix}$$

Task - 4



Rotations → R_{z,θ_1} R_{z,θ_2} $R_{z_3,0}$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix}$$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_{z,\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{y,\theta_2} & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{z,0} & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -l_2 s_1 + d_3 c_1 s_1 \\ l_2 c_1 + d_3 s_1 s_2 \\ l_1 + d_3 c_2 \end{bmatrix}$$

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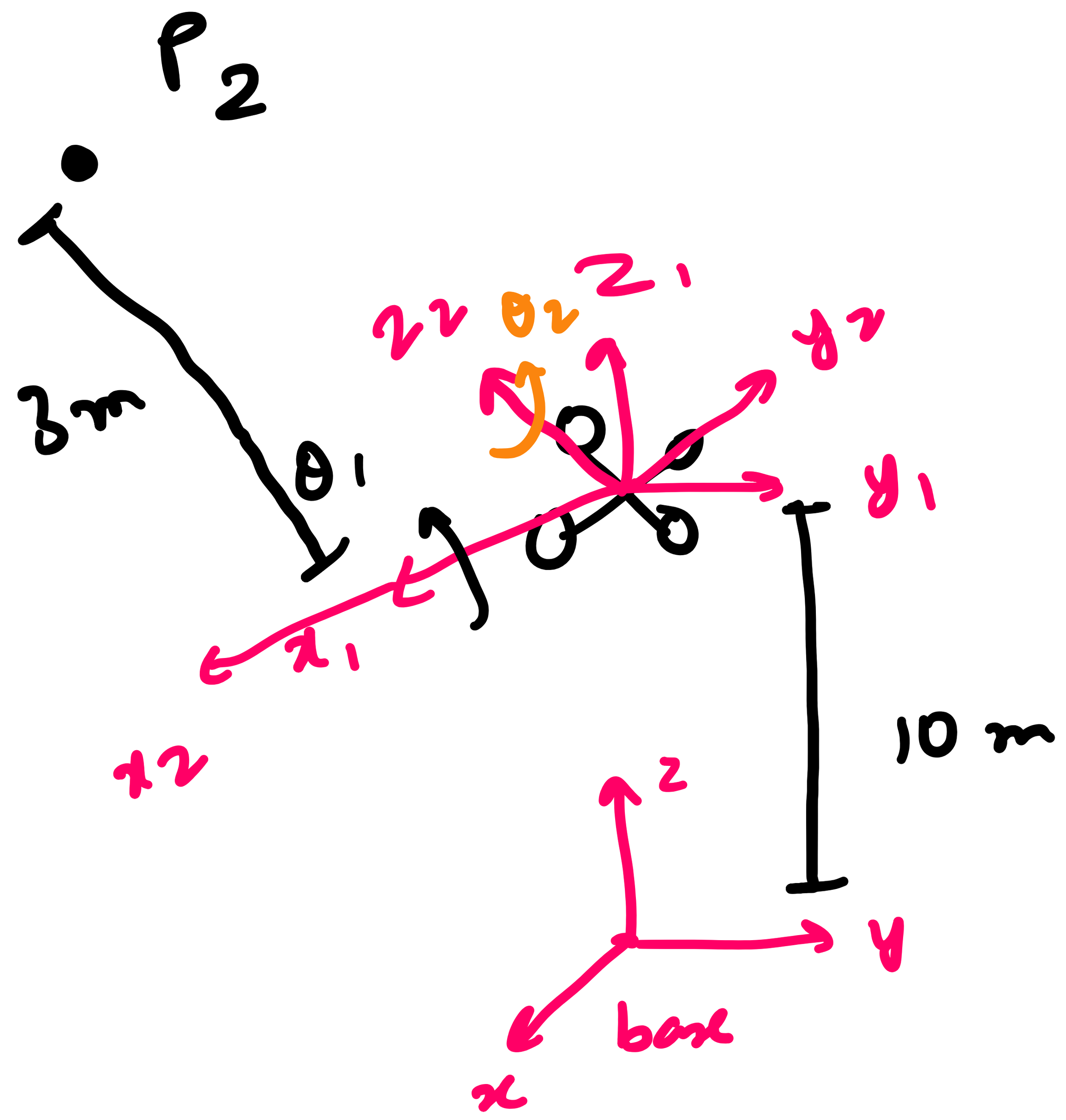
Task - 5



$\theta_1 = 30^\circ \quad \theta_2 = 60^\circ$

$R_{x_1, 30^\circ} \, R_{z_2, 60^\circ}$

obstacle 3m above (in drone frame) $\rightarrow z_2$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$R_1^2 = R_{x,30} \, R_{z\,60}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_{z,0} & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

solving

$$P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.5981 \end{bmatrix} m$$

Task 6

Gearbox in motors:

1) Helical (Parallel shaped geared)

These are used in application requiring greater strength, silent operation like carriage drivers, agitators.
They are small in size & used to achieve speed reduction.
Provide high torque & less noise than spur.

2) Planetary -

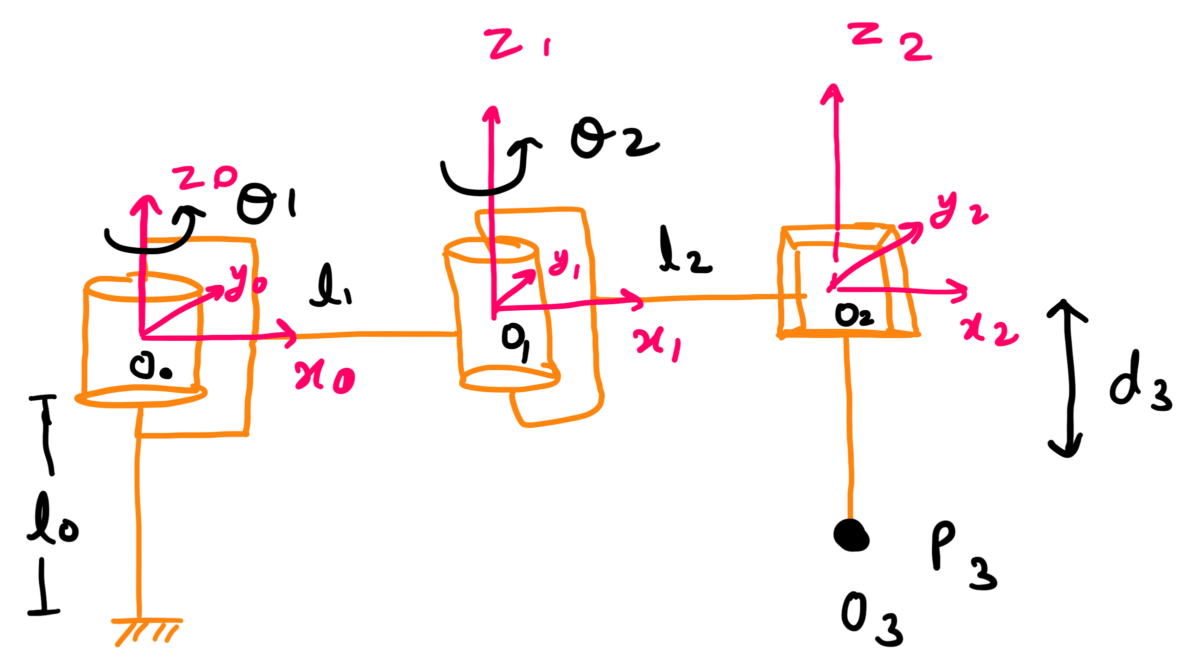
Low power loss and precise. It takes less space with more speed ratio options. They provide higher torque.
Mainly used in automatic automobiles. Due to complex structure they are expensive.

3) Right angle geared (worm, bevel)

Output shaft is rotated 90 degrees to motor shaft.
Generally used in cranes.

Gearbox are used to increase torque which in turn reduces speed. Drones require very high rpm to make it fly just by aerodynamics. So brushless motors are used in drones. Also if one thinks that we can get gearbox which increase speed but that will require high torque motors & will increase drone weight which is very important factor as it affects flight time & payload capacity adversely.

Task-7



Jacobian: $J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix} \rightarrow$ Revolute.

$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \rightarrow$ Prismatic

SCARA RRP 3 DOF

$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = z_1 = z_2$ as all are parallel

$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} l_1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $H_0^1 = \begin{bmatrix} R_{z,\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $H_1^2 = \begin{bmatrix} R_{z\theta_2} & d_1^2 \\ 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

$H_2^3 = \begin{bmatrix} R_{z,0} & d_2^3 \\ 0 & 1 \end{bmatrix}$ $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}$

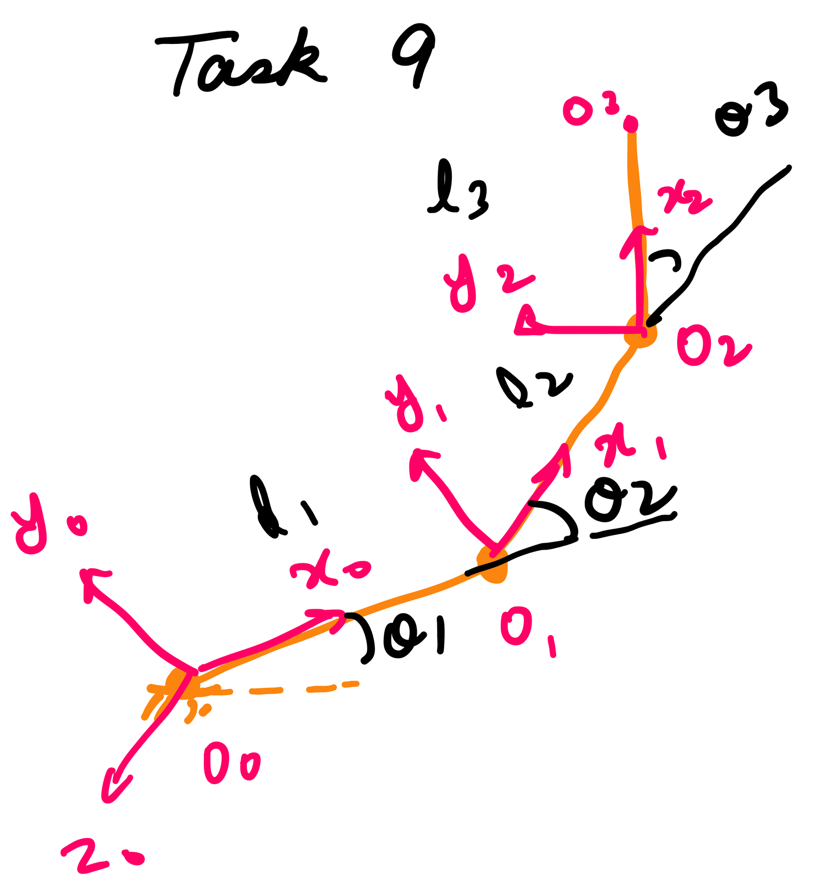
$O_1 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \end{bmatrix}$ $O_2 = \begin{bmatrix} l_2 C_{12} + l_1 C_1 \\ l_2 S_{12} + l_1 S_1 \\ 0 \end{bmatrix}$

$O_3 = \begin{bmatrix} l_2 C_{12} + l_1 C_1 \\ l_2 S_{12} + l_1 S_1 \\ -d_3 \end{bmatrix}$

$$J = \begin{bmatrix} z_0 \times O_3 & z_1 \times \begin{bmatrix} l_2 C_{12} \\ l_2 S_{12} \\ -d_3 \end{bmatrix} & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} & 0 \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Task 9



all z are parallel

so $z_0 = z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$J = \begin{bmatrix} z_0 (o_3 - o_0) & z_1 (o_3 - o_1) & z_2 (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} o_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \qquad H_0^1 = \begin{bmatrix} R_{z\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} o_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} o_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

all rotation about z

(In MATLAB)

so we have

$$o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \qquad o_2 = \begin{bmatrix} l_2 c_{12} + l_1 c_1 \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$$

$$o_3 = \begin{bmatrix} l_3 c_{123} + l_2 c_{12} + l_1 c_1 \\ l_3 s_{123} + l_2 s_{12} + l_1 s_1 \\ 0 \end{bmatrix}$$

so we have (In MATLAB)

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} - l_2 s_{12} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_3 c_{123} + l_2 c_{12} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$