Assignment - 3

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Ans-1 :-

(a) Singular configurations are the configurations for which the rank of the corresponding Jacobian matrix decreases. In singular configurations, the manipulator cannot attain certain direction of motion.

To find the singular configuration, we divide the problem into two parts.
(i) Finding wrist singularities
(ii) Finding arm singularities.

For arm singularities we mostly calculate the determinant of the part of the Jacobian corresponding to arm joints and equate it to zero.

For wrist singularities, if the wrist is simple as spherical wrist, we just check the linear dependence of the Jacobian matrix corresponding to the wrist part. If the wrist is complex, we use the same procedure as done for arm singularities.

(b) Yes, we can detect if a particular configuration is close to a singular configuration. We need to find the determinant of the corresponding manipulator Jacobian. If the determinant tends to zero or is zero, the configuration is close to a singular configuration.

Ans-4 :-

- RRP Standford

Analytical expression

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

$$\Rightarrow z_0 \times (o_3 - o_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix}$$

$$= \begin{bmatrix} -(s_1 s_2 d_3 + c_1 d_2) \\ c_1 s_2 d_3 - s_1 d_2 \\ 0 \end{bmatrix}$$

$$z_1 \times (o_3 - o_1) = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} c_1 s_2 d_3 - s_1 d_2 \\ s_1 s_2 d_3 + c_1 d_2 \\ c_2 d_3 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 c_2 d_3 \\ s_1 c_2 d_3 \\ -s_2 d_3 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} -(s_1 s_2 d_3 + c_1 d_2) & c_1 c_2 d_3 & c_1 s_2 \\ c_1 s_2 d_3 - s_1 d_2 & s_1 c_2 d_3 & s_1 s_2 \\ 0 & -s_2 d_3 & c_2 \\ 0 & -s_1 & 0 \\ 0 & c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

We will Check for 2 parameters :-

(i) $d_2 = 3$

$d_3 = 4$

$\theta_1 = 30^\circ$

$\theta_2 = 60^\circ$

corresponding DH - parameters are :-

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	30°
2	0	90	3	60°
3	0	0	4	0

According to Analytical expression,

$$J = \begin{bmatrix} -4.3301 & 1.732 & 0.75 \\ 1.5 & 1 & 0.433 \\ 0 & -3.464 & 0.5 \\ 0 & -0.5 & 0 \\ 0 & 0.866 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

According to the code,

$$J = \begin{bmatrix} -4.3301 & 1.7321 & 0.75 \\ 1.5 & 1 & 0.433 \\ 0 & -3.4641 & 0.5 \\ 0 & -0.5 & 0 \\ 0 & 0.866 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Hence both Jacobians match.

(ii) $d_2 = 2$
$d_3 = 3$
$\theta_1 = 0$
$\theta_2 = 90$

Corresponding DH-parameters are :-

Link	a_i	α_i	d_i	θ_i
1	0	-90	0	0
2	0	90	2	90
3	0	0	3	0

$$J_{analytical} = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_{code} = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Hence both Jacobians match.

- **RRP - SCARA**

To check that the wrist kinematics is also covered, we will consider RRP SCARA with 1 R wrist configuration.

$J_{analytical}$ as given in text is :

$$J_{analytical} = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Considering two cases :

(i) $a_1 = 2$ $\quad d_3 = 3$

$a_2 = 4$ $\quad d_4 = 5$

$\theta_1 = 30°$

$\theta_2 = 60°$, $\theta_4 = 45°$

Corresponding DH-parameters are :-

Joint	a_i	α_i	d_i	θ_i
1	2	0	0	30°
2	4	180	0	60°
3	0	0	3	0
4	0	0	5	45°

$$\therefore J_{analytical} = \begin{bmatrix} -5 & -4 & 0 & 0 \\ 1.732 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

$$J_{code} = \begin{bmatrix} -5 & -4 & 0 & 0 \\ 1.7321 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

(ii) $a_1 = 6$ $\quad d_3 = 10$

$a_2 = 4$ $\quad d_4 = 10$

$\theta_1 = 45°$

$\theta_2 = 45°$, $\theta_4 = 0$

Corresponding DH-parameters are :-

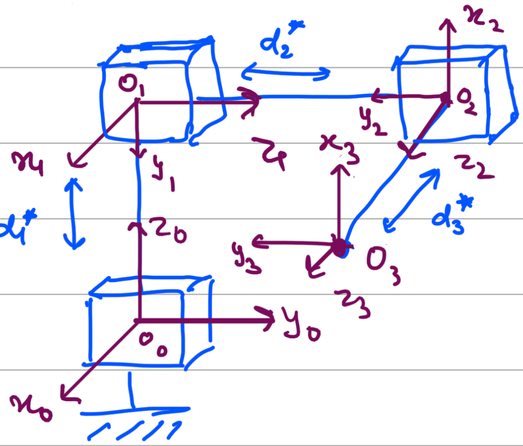
Joint	a_i	α_i	d_i	θ_i
1	6	0	0	45°
2	4	180	0	45°
3	0	0	10	0
4	0	0	10	0

$$
J_{analytical} = \begin{bmatrix} -8.242 & -4 & 0 & 0 \\ -4.242 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}
$$

$$
J_{code} = \begin{bmatrix} -8.2426 & -4 & 0 & 0 \\ 4.2426 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}
$$

Hence J_{code} is matching with $J_{analytical}$ in both the cases.

Ans-5   Given 3 link cartesian manipulator.



DH parameters:

Link	a_i	α_i	d_i	θ_i
1	0	−90	d_1^*	0
2	0	−90	d_2^*	−90
3	0	0	d_3^*	0

$$\therefore \; T_0^n = T_0^3 = A_1 A_2 A_3$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore\ T_0^3 = \begin{bmatrix} 0 & 0 & 1 & d_3^* \\ 0 & -1 & 0 & d_2^* \\ 1 & 0 & 0 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Position of the end effector in ground frame, given joint variables $[d_1^*, d_2^*, d_3^*]$,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = T_0^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_3^* \\ d_2^* \\ d_1^* \\ 1 \end{bmatrix}$$

$$\therefore\ P_0 = \begin{bmatrix} d_3^* \\ d_2^* \\ d_1^* \end{bmatrix}$$

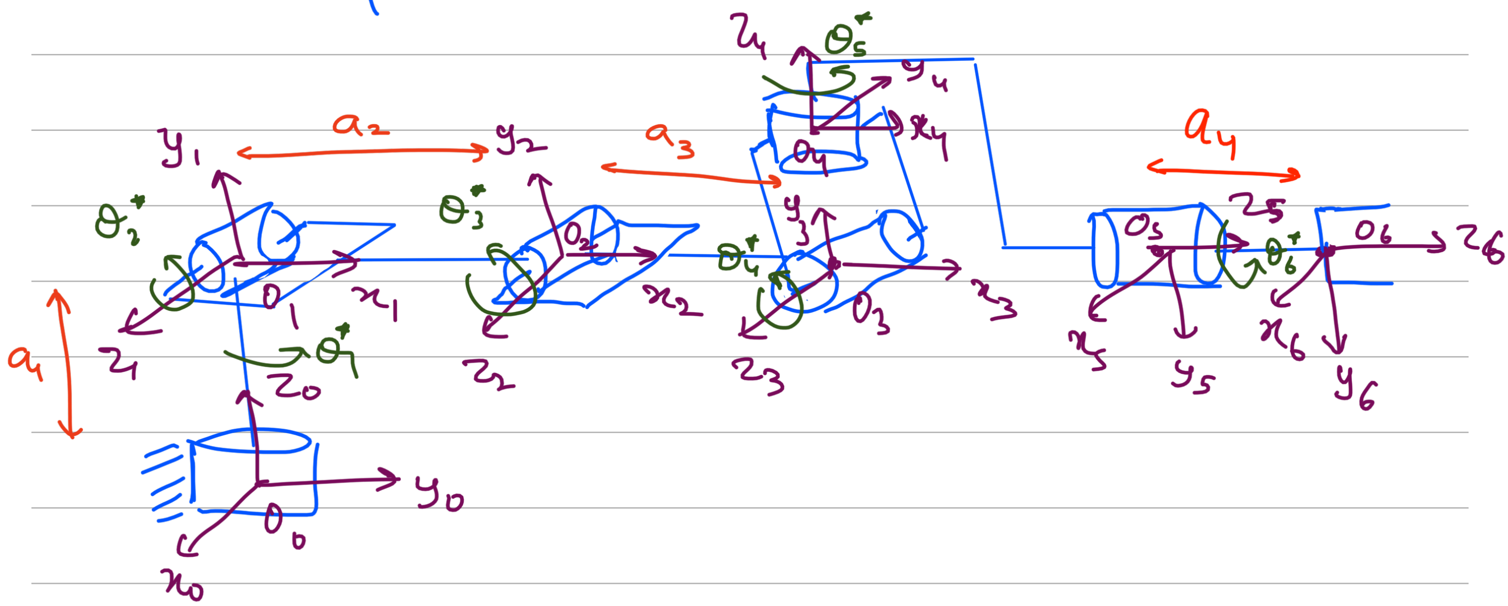
<u>Verification</u> <u>with</u> <u>code</u> :-

input joint variables :- $[3, 4, 5]$

Output according the analytical expression derived above $= \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$

Output of code $= \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$ $\Rightarrow$ Hence verified.

Ans-6 Given three link articulated manipulator with spherical wrist.



Here O_3, O_4, O_5 overlap on each other.

DH parameters :-

Link	a_i	α_i	d_i	θ_i
1	0	90	a_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*
4	0	-90	0	θ_4^*
5	0	-90	0	θ_5^*
6	0	0	a_4	θ_6^*

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

$$A_1 = \begin{bmatrix} c\theta_1^* & 0 & s\theta_1^* & 0 \\ s\theta_1^* & 0 & -c\theta_1^* & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c\theta_2^* & -s\theta_2^* & 0 & a_2 c\theta_2^* \\ s\theta_2^* & c\theta_2^* & 0 & a_2 s\theta_2^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c\theta_3^* & -s\theta_3^* & 0 & a_3 c\theta_3^* \\ s\theta_3^* & c\theta_3^* & 0 & a_3 s\theta_3^* \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c\theta_4^* & 0 & -s\theta_4^* & 0 \\ s\theta_4^* & 0 & c\theta_4^* & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c\theta_5^* & 0 & -s\theta_5^* & 0 \\ s\theta_5^* & 0 & c\theta_5^* & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c\theta_6^* & -s\theta_6^* & 0 & 0 \\ s\theta_6^* & c\theta_6^* & 0 & 0 \\ 0 & 0 & 1 & a_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For verification,

taking, $a_1 = 3$ & $\theta_1^* = 30°$ $\theta_5^* = 45°$

$a_2 = 4$ $\theta_2^* = 60°$ $\theta_6^* = 0°$

$a_3 = 5$ $\theta_3^* = 45°$

$a_4 = 6$ $\theta_4^* = 0°$

Analytical result,

$$T_0^6 = \begin{bmatrix} -0.512 & 0.836 & -0.195 & -0.559 \\ 0.521 & 0.483 & 0.704 & 4.576 \\ 0.683 & 0.259 & -0.683 & 7.196 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Position of End effector tip in $O_0X_0Y_0Z_0$ frame

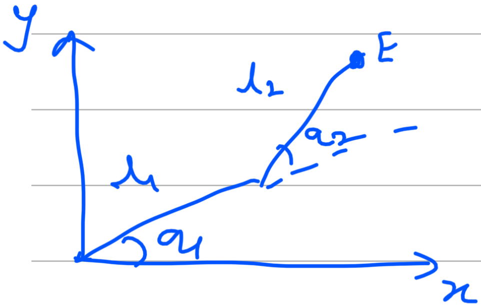
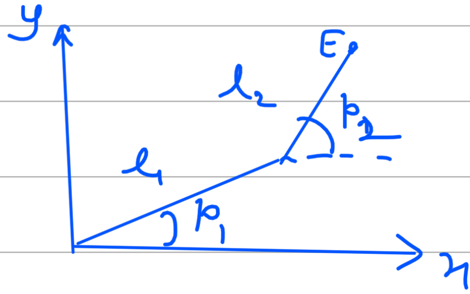
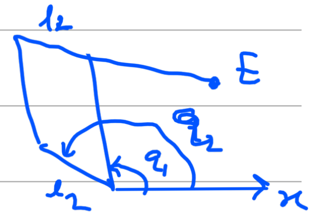
$$= \begin{bmatrix} -0.559 \\ 4.576 \\ 7.196 \end{bmatrix}$$

code result,

Position of End effector tip in $O_0X_0Y_0Z_0$ frame

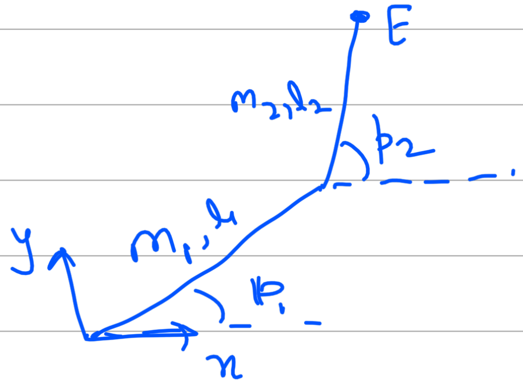
$$= \begin{bmatrix} -0.559 \\ 4.576 \\ 7.196 \end{bmatrix} \implies \boxed{\text{Hence verified}}$$

Ans-7

Directly Driven	Remotely Driven	5-bar parallelogram
		
• The angles are relative to each other. q_1 w.r.t x-axis. q_2 w.r.t to axis passing through l_1.	• The angles are absolute. p_1 & p_2 both are measured w.r.t to x-axis of ground.	• The angles are absolute. q_1 & q_2 both are measured w.r.t to x-axis of ground.
• The inclination of the links with x-axis is not independent.	The inclination of the links with x-axis is independent. Each link's inclination can be controlled from the ground independently.	Same as in case of remotely driven.
• Basic manipulator & easy to code. Forward kinematics can be derived using DH-Parameters.	Motors can be fixed at ground and can control the link angles from ground. DH parameters cannot be used.	Same as remotely driven and added stability and strength due to 4 bars is another advantage.

Ans - 8.

Given remotely driven planar elbow manipulator,



Given $D(p) = \begin{bmatrix} m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 & m_2 l_1 l_{c_2} \cos(p_2 - p_1) \\ m_2 l_1 l_2 \cos(p_2 - p_1) & m_2 l_{c_2}^2 + I_2 \end{bmatrix}$

$\therefore\ d_{11} = m_1 l_{c_1}^2 + m_2 l_1^2 + I_1$

$d_{12} = m_2 l_1 l_{c_2} \cos(p_2 - p_1) = d_{21}$

$d_{22} = m_2 l_{c_2}^2 + I_2$

$$c_{111} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial p_1} + \frac{\partial d_{11}}{\partial p_1} - \frac{\partial d_{11}}{\partial p_1}\right) = 0$$

$$c_{121} = \frac{1}{2}\left(\frac{\partial d_{21}}{\partial p_{\cancel{1}}} + \frac{\partial d_{11}}{\partial p_2} - \frac{\partial d_{12}}{\partial p_1}\right) = \frac{1}{2}\frac{\partial d_{11}}{\partial p_2} = 0$$

$$c_{211} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial p_2} + \frac{\partial d_{12}}{\partial p_1} - \frac{\partial d_{21}}{\partial p_1}\right) = \frac{1}{2}\frac{\partial d_{11}}{\partial p_2} = 0$$

$$c_{221} = \frac{1}{2}\left(\frac{\partial d_{12}}{\partial p_2} + \frac{\partial d_{12}}{\partial p_2} - \frac{\partial d_{11}}{\partial p_2}\right)$$

$$= \frac{\partial d_{12}}{\partial p_2} = -m_2 l_1 l_{c_2} \sin(p_2 - p_1)$$

$$c_{112} = \frac{1}{2}\left(\frac{\partial d_{21}}{\partial p_1} + \frac{\partial d_{21}}{\partial p_1} - \frac{\partial d_{11}}{\partial p_2}\right)$$

$$= \frac{\partial d_{21}}{\partial p_1} = m_2 l_1 l_{c_2} \sin(p_2 - p_1)$$

$$c_{122} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial p_1} + \frac{\partial d_{21}}{\partial p_2} - \frac{\partial d_{12}}{\partial p_2}\right)$$

$$= \frac{1}{2}\frac{\partial d_{22}}{\partial p_1} = 0$$

$$c_{212} = \frac{1}{2}\left(\frac{\partial d_{21}}{\partial p_2} + \frac{\partial d_{22}}{\partial p_1} - \frac{\partial d_{21}}{\partial p_2}\right)$$

$$= \frac{1}{2}\frac{\partial d_{22}}{\partial p_1} = 0$$

$$c_{222} = \frac{1}{2}\left(\frac{\partial d_{22}}{\partial p_2} + \frac{\partial d_{22}}{\partial p_2} - \frac{\partial d_{22}}{\partial p_2}\right) = \frac{1}{2}\frac{\partial d_{22}}{\partial p_2} = 0$$

Given,

$$V = m_1 g l_{c_1} \sin p_1 + m_2 g (l_1 \sin p_1 + l_{c_2} \sin p_2)$$

$$\phi_1 = \frac{\partial V}{\partial p_1} = m_1 g l_{c_1} \cos p_1 + m_2 g l_1 \cos p_1$$

$$= (m_1 l_{c_1} + m_2 l_1) g \cos p_1$$

$$\phi_2 = \frac{\partial V}{\partial p_2} = m_2 g l_{c_2} \cos p_2$$

$\therefore$ Dynamics eq^n:

$$\tau_1 = d_{11} \ddot{p}_1 + d_{12} \ddot{p}_2 + c_{221} \dot{p}_2^2 + \phi_1$$

$$= (m_1 l_{c_1}^2 + m_2 l_1^2 + I_1) \ddot{p}_1 + m_2 l_1 l_{c_2} \cos(p_2 - p_1) \ddot{p}_2$$
$$- m_2 l_1 l_{c_2} \sin(p_2 - p_1) \dot{p}_2^2 + (m_1 l_{c_1} + m_2 l_1) g \cos p_1$$

$$\tau_2 = d_{21} \ddot{p}_1 + d_{22} \ddot{p}_2 + c_{112} \dot{p}_1^2 + \phi_2$$

$$= m_2 l_1 l_{c_2} \cos(p_2 - p_1) \ddot{p}_1 + (m_2 l_{c_2}^2 + I_2) \ddot{p}_2 + m_2 l_1 l_{c_2} \sin(p_2 - p_1) \dot{p}_1^2$$
$$+ m_2 g l_{c_2} \cos p_2$$

Substituting, $l_{c_1} = \frac{l_1}{2}$, $l_{c_2} = \frac{l_2}{2}$, $I_1 = \frac{1}{12} m_1 l_1^2$, $I_2 = \frac{1}{12} m_2 l_2^2$

we get,

$$\left(m_1\frac{l_1^2}{4}+m_2l_1^2+\frac{1}{12}m_1l_1^2\right)\ddot{\phi}_1+\frac{m_2}{2}l_1l_2\cos(\phi_2-\phi_1)\ddot{\phi}_2$$

$$-\frac{m_2}{2}l_1l_2\sin(\phi_2-\phi_1)\dot{\phi}_2^2+\left(m_1\frac{l_1}{2}+m_2l_1\right)g\cos\phi_1=\tau_1$$

$\hookrightarrow$ (1)

$$\frac{m_2}{2}l_1l_2\cos(\phi_2-\phi_1)\ddot{\phi}_1+\left(m_2\frac{l_2^2}{4}+\frac{m_2l_2^2}{12}\right)\ddot{\phi}_2+\frac{m_2}{2}l_1l_2\sin(\phi_2-\phi_1)\dot{\phi}_1^2$$

$$+\frac{m_2}{2}gl_2\cos\phi_2=\tau_2$$

$\hookrightarrow$ (2)

$\hookrightarrow$ The above two equations were used in the miniproject.

Hence the derivation is verified.

Ans-10

When provided with $D(q)$ and $V(q)$ matrix, perform the following steps to derive the equations of motion.

The equation of motion in matrix form is

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \quad \text{—} \; (1)$$

The above form can be resolved into element summation form as follows:

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k,$$
$$k = 1, 2, \ldots, n \quad \text{—} \; (2)$$

here d_{kj} are the elements of matrix $D(q)$. They can be directly taken from $D(q)$ matrix.

Step-1 Calculate c_{ijk} coefficients.

$$c_{ijk} = \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right)$$

d_{kj}, d_{ki} & d_{ij} can be taken from $D(q)$ matrix as stated above.

Step-2 calculate $\phi_k(q)$.

$$\phi_k(q) = \frac{\partial V(q)}{\partial q_k}$$

here, $V(q)$ will be a function of q where q is the vector $[q_1 q_2 q_3 \dots q_n]$.

$V(q)$ is already provided.

Step-3 Substitute C_{ijk}, $\phi_k(q)$ & d_{jk} in eqⁿ (2).

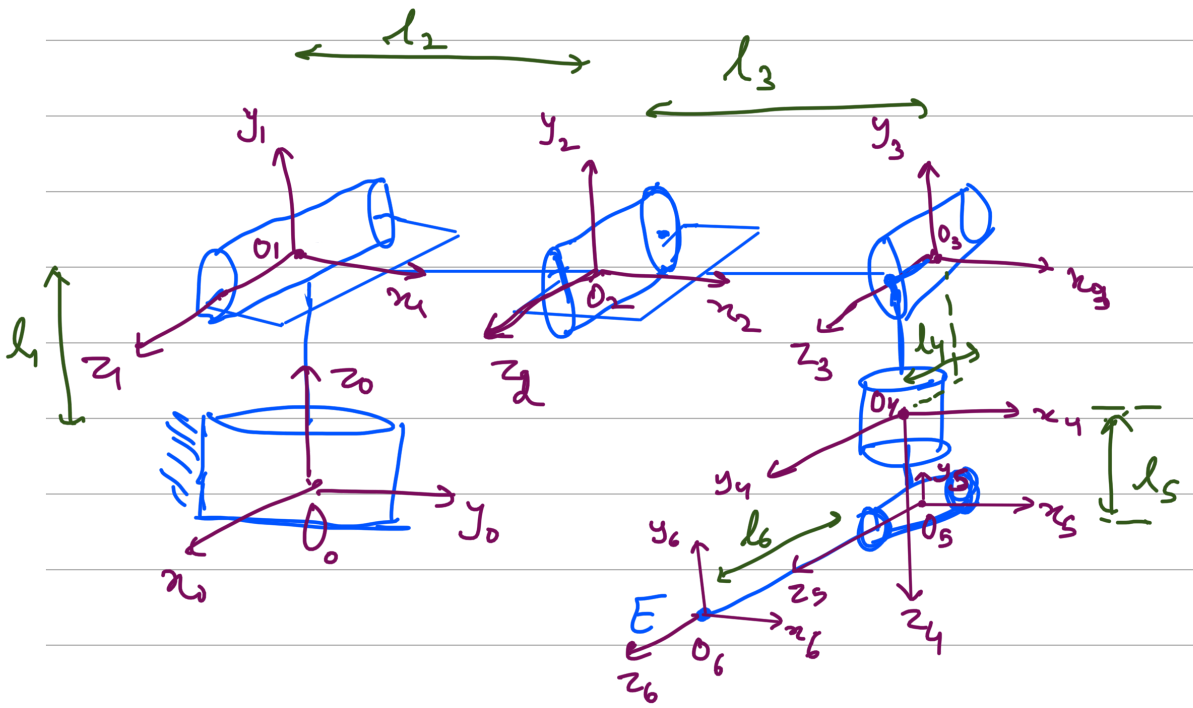
On substitution, the equation of motion will be desired.

Note :- q, $\dot{q}$ & $\ddot{q}$ are vectors. They are as follows,

$$q = [q_1\ q_2\ q_3 \dots\dots q_n]$$
$$\dot{q} = [\dot{q}_1\ \dot{q}_2\ \dot{q}_3 \dots\dots \dot{q}_n]$$
$$\ddot{q} = [\ddot{q}_1\ \ddot{q}_2\ \ddot{q}_3 \dots\dots \ddot{q}_n]$$

Ans-12

UR 5 robot :-



The are total 6 links and 6 joints in the UR5 robot. All joints are 'R' or revolute joints.

DH parameters :-

Link	a_i	α_i	d_i	θ_i
1	0	90°	l_1	θ_1^*
2	l_2	0	0	θ_2^*
3	l_3	0	0	θ_3^*
4	0	90°	l_4	θ_4^*
5	0	-90°	l_5	θ_5^*
6	0	0°	l_6	θ_6^*