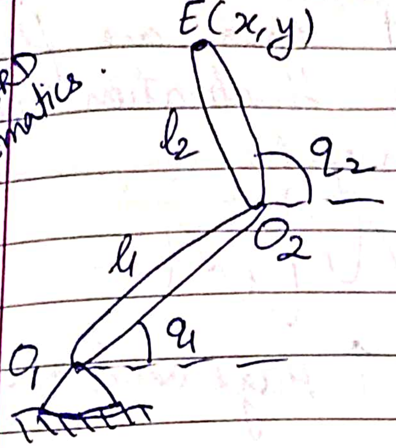
FORWARD Kynematics.



$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\} \longrightarrow (1)$$

Now velocity would be :-

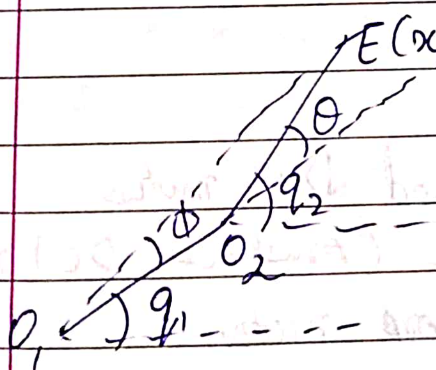
$$\dot{x} = -l_1 \dot{q}_1 \sin q_1 - l_2 \dot{q}_2 \sin q_2$$
$$\dot{y} = l_1 \dot{q}_1 \cos q_1 + l_2 \dot{q}_2 \cos q_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \longrightarrow (2)$$

cartesian space or task space for End Effector

Joint space.

INWARD KINEMATICS



$\theta = q_2 - q_1$

using cosine rule :-

$$\theta = \cos^{-1}\left(\frac{(x^2+y^2) - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

and, $\tan(q_1 + \phi) = y/x$.

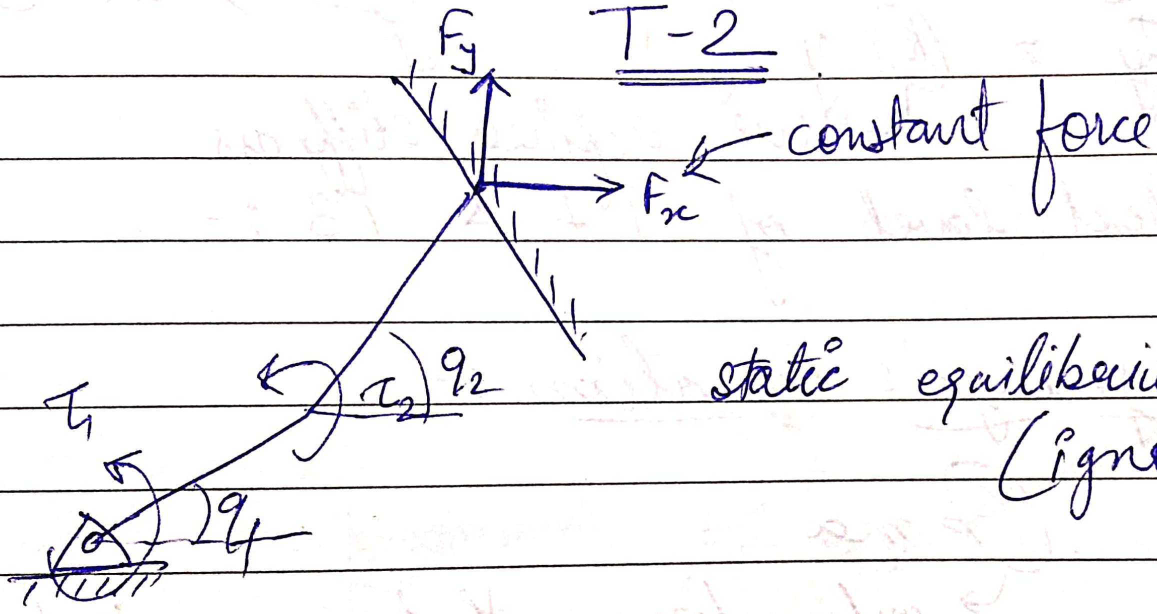
where, $\tan\phi = \left(\dfrac{l_2 \sin\theta}{(l_1 + l_2 \cos\theta)}\right)$

so,

$$\left.\begin{aligned} q_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right) \\ q_2 &= \theta + q_1 \end{aligned}\right\} (3)$$

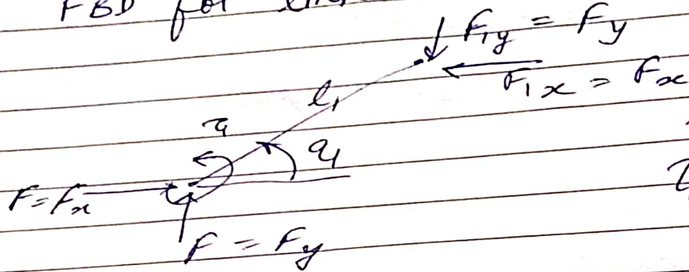
$q_2 = q_1 + \theta$

T-2



static equilibrium.
(ignoring gravity)

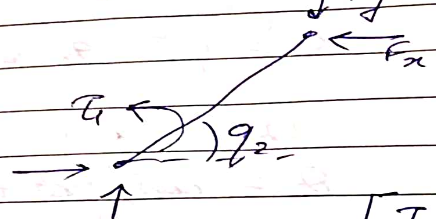
FBD for link 1 :-



now,

$$\tau_1 = -F_x l_1 sq_1 + F_y l_1 cq_1$$

FBD for link 2 :-



$$\tau_2 = -F_x l_2 sq_2 + F_y l_2 cq_2$$ (3)

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & l_1 cq_1 \\ -l_2 sq_2 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$ ⟶ (4)

(2) along with (4) can solve T2.

* For T3 and next level of T1 :-
↳ * for T3 (simple version).

$F_x = k(x - x_0)$
$F_y = k(y - y_0)$ } → use (4) after this.
k ↳ user defined stiffness

↳ Next level of T1 & T3 :-

<u>Lagrange's</u> <u>Equation</u> :-

$F = ma$
F ↳ only forces that account for work done.

<u>Lagrangian</u> :- $\mathcal{L} = K - V$

K ↳ kinetic energy, V ↳ potential energy

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i} \rightarrow \text{Torques.}$$

$i = 1, 2$

In this case :-

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of link 1}} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{link2 abt its CM}} + \underbrace{\frac{1}{2} m_2 v_{c_2}^2}_{\text{translation of C.M of link2}}$$

where,

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2 \dot{q}_2}{2}\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2 \ddot{q}_2}{2} \cos(q_2 - q_1) - \frac{m l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_1 + m_2 g l_1 c q_1$$

$$\tau_2 = \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_2 l_1}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s(q_2)$$

(6)

$$\left.\begin{aligned} F_x &= k(x - x_0) = k(l_1 c q_1 + l_2 c q_2 - x_0) \\ F_y &= k(y - y_0) = k(l_1 s q_1 + l_2 s q_2 - y_0). \end{aligned}\right] \rightarrow (7)$$