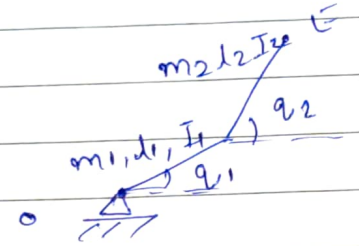
19110179 **task0** ITR miniproj

task0



let E(x, y) and O be (0, 0)

$$\therefore \quad \begin{aligned} x &= d_1 \cos q_1 + d_2 \cos q_2 \\ y &= d_1 \sin q_1 + d_2 \sin q_2 \end{aligned} \quad \Big] \quad -(1)$$

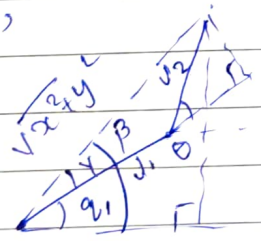
Now for velocity of end effector,

$$\begin{aligned} \dot{x} &= -d_1 \dot{q}_1 \sin q_1 - d_2 \dot{q}_2 \sin q_2 \\ \dot{y} &= d_1 \dot{q}_1 \cos q_1 + d_2 \dot{q}_2 \cos q_2 \end{aligned}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -d_1 \sin q_1 & -d_2 \sin q_2 \\ d_1 \cos q_1 & d_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad -(2)$$

$\dot{x}, \dot{y}$: task space; $\dot{q}_1, \dot{q}_2$: joint space

Also,



$$q_2 = q_1 + \theta$$

$$\cos(\pi - \theta) = \frac{d_1^2 + d_2^2 - (x^2+y^2)}{2 d_1 d_2}$$

$$+\cos\theta = \frac{x^2+y^2-d_1^2-d_2^2}{2 d_1 d_2}$$

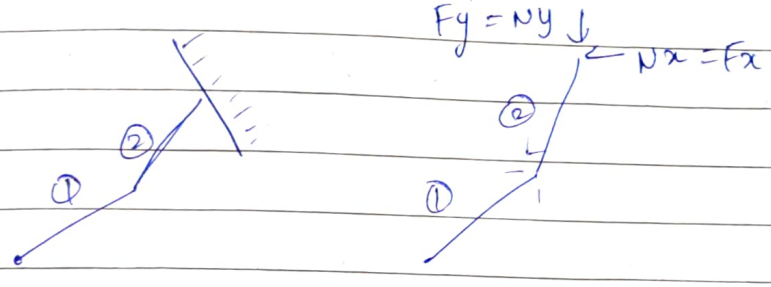
$$\theta = q_2 - q_1 = \cos^{-1}\left[\frac{x^2+y^2-d_1^2-d_2^2}{2 d_1 d_2}\right] \quad -(3)$$

$$\tan\beta = \frac{y}{x} \qquad \tan\gamma = \frac{d_2\sin\theta}{d_1 + d_2\cos\theta}$$

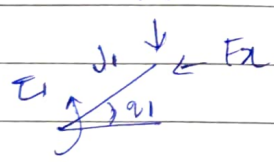
$$\beta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$q_1 = \beta - \gamma$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left[\frac{d_2\sin\theta}{d_1 + d_2\cos\theta}\right] \quad -③$$

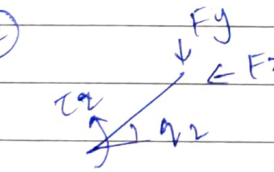


① neglect g



$$\tau_1 + F_x d_1 \sin q_1 - F_y d_1 \cos q_1 = 0$$

②



$$\tau_2 + F_x d_2 \sin q_2 - F_y d_2 \cos q_2 = 0$$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}-d_1\sin q_1 & d_1\cos q_1\\ -d_2\sin q_2 & d_2\cos q_2\end{bmatrix}\begin{bmatrix}F_x\\ F_y\end{bmatrix} \quad -④$$

for spring

$$\left.\begin{aligned} F_x &= k(x - x_0)\\ F_y &= k(y - y_0)\end{aligned}\right] -⑤$$

$L = K - V$.

$$Q_i = \frac{d}{dt}\frac{dL}{d\dot{q}_i} - \frac{dL}{dq_i}$$

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c2}^2$$

$$v_{c2}^2 = l_1^2\dot{q}_1^2 + \frac{l_2^2}{2}\dot{q}_2^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$\tau_1 = \frac{d}{dt}\frac{dL}{d\dot{q}_1} - \frac{dL}{dq_1}$$

$$= \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2}\ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2}\dot{q}_2(\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1) + m_1 g \frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1$$

(6)

$$\tau_2 = \frac{d}{dt}\frac{dL}{d\dot{q}_2} - \frac{dL}{dq_2}$$

$$= \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_1 \frac{l_2^2}{4}\ddot{q}_2 + m_2 \frac{l_1 l_2}{2}\ddot{q}_1 \cos(q_1 - q_2) - m_2 \frac{l_1 l_2}{2}\dot{q}_1 \sin(q_2 - q_1) + m_2 g \frac{l_2}{2}\sin q_2$$