

ITR Assignment 3

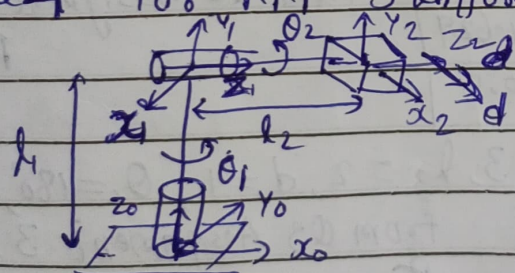
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Q1 There are certain configurations for a manipulator at which the velocity components are linearly dependent to each other. In these configurations we cannot solve for joint variables and \dot{q}_s since we cannot set the velocities independently. ~~Here~~ More formally, the rank of Jacobian matrix $J(q)$ decreases. These configurations are known as singular configurations.

We can check ~~the~~ if the columns of Jacobian are linearly dependent. This means its rank is less than its maximum value. This condition will be a singular configuration.

For a Jacobian, if its columns are close to becoming linearly dependent then it is close to singular configuration. ~~If~~ For a square Jacobian matrix, if its determinant is close to zero, ~~it is~~ the configuration is close to a singular one.

Q2 For RRP Stanford manipulator:



D-H Parameters table:

	θ_i	d_i	a_i	α_i
1	$\theta_1 - \frac{\pi}{2}$	l_1	0	$-\pi/2$
2	θ_2	l_2	0	$\pi/2$
3	0	0	d	0

Using Q4 from Assignment 2, for the following values: $l_1 = 1$, $l_2 = 2$, $d = 0.5$, $\theta_1 = 60^\circ$, $\theta_2 = 45^\circ$

$$P_0 = \begin{bmatrix} 1.306 \\ 1.555 \\ 0.646 \end{bmatrix}$$

From Q3 of Assignment 3 using the same above values

$$P_0 = \begin{bmatrix} 1.306 \\ 1.555 \\ 0.646 \end{bmatrix}$$

Second example:

Using Q4 from Assignment 2 for:

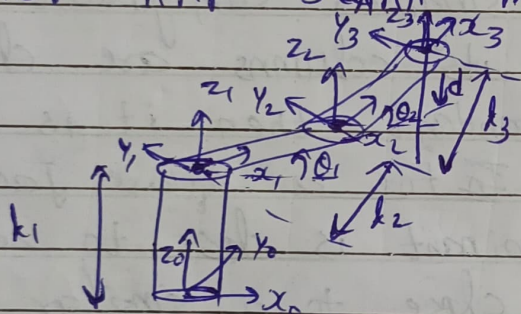
$$k_1 = 3, k_2 = 2, d = 1, \theta_1 = 180^\circ, \theta_2 = 30^\circ$$

$$P_0 = \begin{bmatrix} -2 \\ 0.866 \\ 2.5 \end{bmatrix}$$

From Q3 of Assignment 3 using the same values

$$P_0 = \begin{bmatrix} -2 \\ 0.866 \\ 2.5 \end{bmatrix}$$

For RRP SCARA manipulator



D-H Table

	θ_i	d_i	α_i	α_i
1	θ_1	k_1	θ_2	0
2	θ_2	0	k_3	0
3	0	-d	θ_3	0

① For values: $k_1 = 1, k_2 = 2, k_3 = 3, d = 0.5, \theta_1 = 60^\circ, \theta_2 = 45^\circ$

From Q3 Assignment 2:

$$P_0 = \begin{bmatrix} 0.229 \\ 4.629 \\ 0.5 \end{bmatrix}$$

From Q3 Assignment 3:

$$P_0 = \begin{bmatrix} 0.224 \\ 4.629 \\ 0.5 \end{bmatrix}$$

② For values: $k_1 = 4, k_2 = 3, k_3 = 2, d = 1, \theta_1 = 180^\circ, \theta_2 = 30^\circ$

From Q3 Assignment 2:

$$P_0 = \begin{bmatrix} -4.732 \\ -1 \\ 3 \end{bmatrix}$$

From Q3 Assignment 3:

$$P_0 = \begin{bmatrix} -4.732 \\ -1 \\ 3 \end{bmatrix}$$

for these values from Q8 assignment 2:

$$J = \begin{bmatrix} 1 & 1 & 0 \\ -4.732 & -1.732 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

from Q3:

$$J = \begin{bmatrix} 1 & 1 & 0 \\ -4.732 & -1.732 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

For values $k_1=1, k_2=2, k_3=3, d=0.5, \alpha_1=60^\circ, \alpha_2=45^\circ$

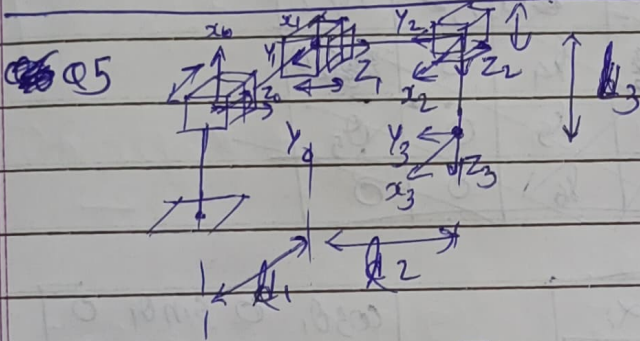
From Q8, Assignment 2:

$$J = \begin{bmatrix} -4.629 & -2.898 & 0 \\ 0.224 & -0.776 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

From Q3, Assignment 3:

$$J = \begin{bmatrix} -4.629 & -2.898 & 0 \\ 0.224 & -0.776 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

All the results match from what we calculated before.



DH table:

	α_i	d_i	a_i	α_i
1	0	l_1	0	$-\pi/2$
2	$-\pi/2$	l_2	0	$-\pi/2$
3	0	l_3	0	0

$$\therefore A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & 1 & -d_3 \\ 0 & 0 & 1 & -d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & 1 & -d_3 \\ 0 & 0 & 1 & -d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

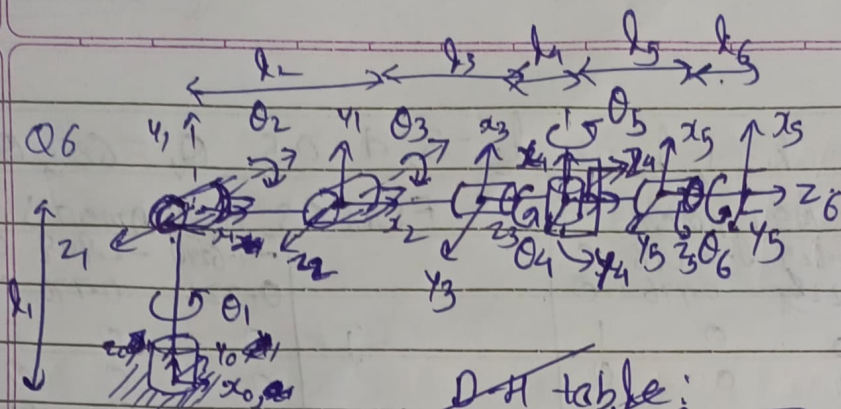
Putting values

$$\therefore T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & -1 & -l_3 \\ 0 & -1 & 0 & l_2 \\ -1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting values $l_1=3, l_2=2, l_3=1$ in Q3

$$\text{we get } T = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We get the same results.



DH table:

	Q_i	d_i	a_i	α_i
1	θ_1	l_1	0	$\pi/2$
2	θ_2	0	l_2	0
3	$\theta_3 + \frac{\pi}{2}$	0	l_3	$\pi/2$
4	θ_4	l_4	0	0
5	0	l_5	0	θ_5
6	θ_6	l_6	0	0

DH Table:

	Q_i	d_i	a_i	α_i
1	θ_1	0	0	$\pi/2$
2	θ_2	0	l_2	0
3	$\theta_3 + \frac{\pi}{2}$	0	l_3	$\pi/2$
4	$\theta_4 + \frac{\pi}{2}$	0	0	$\pi/2$
5	$\theta_5 + \frac{\pi}{2}$	0	0	$\pi/2$
6	θ_6	l_6	0	0

$$\therefore A_1 = \begin{bmatrix} \cos \theta_1 & 0 & \sin \theta_1 & 0 \\ \sin \theta_1 & 0 & -\cos \theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & l_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & l_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & l_3 \cos \theta_3 \\ \sin \theta_3 & \cos \theta_3 & 0 & l_3 \sin \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos \theta_4 & 0 & \sin \theta_4 & 0 \\ \sin \theta_4 & 0 & \cos \theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos \theta_5 & 0 & \sin \theta_5 & 0 \\ \sin \theta_5 & 0 & -\cos \theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & 0 \\ \sin \theta_6 & \cos \theta_6 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $Q_i = 0$ for all i , by multiplication, we get:

$$T_0^6 = \begin{bmatrix} 0 & 0 & 1 & l_2 + l_3 + l_6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Putting $l_2 = 1$, $l_3 = 2$ and $l_4 = 6$ in Q3
we get

$$T = \begin{bmatrix} 0 & 0 & 1 & 6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is the expected result.

Q7

Direct Drive:

The transmission of torque happens without gears. This gives a quicker response time and no backlash. In this case, the motor is directly attached to the joint. Hence this has the least cost. But, for this drive we need a high torque, low rpm motor.

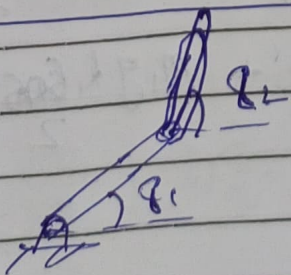
Remotely Driven Joint:

In this arrangement, Motor is in the base frame and the joint is driven by using a mechanism to transmit the torque for eg. gears. This is easy to implement. The weight of the motor will be not a problem since motor is at the base.

5-bar parallelogram:

In this arrangement, two motors are at the base frame and a 5-bar mechanism is arranged in which 4-bars form parallelogram and the remaining 1 bar and one of the 4-bars act as 2R mechanism. It is smaller in size than the other two. It has similar advantages as Remotely Driven Joints.

Q8



$$V_1 = \begin{bmatrix} -l_2 \sin(\theta_1/2) \\ l_2 \cos(\theta_1/2) \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$V_{c2} = \begin{bmatrix} -l_1 \sin \theta_1 & -l_2 \sin(\theta_2)/2 \\ l_1 \cos \theta_1 & l_2 \cos(\theta_2)/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad \begin{matrix} \omega_1 = \dot{\theta}_1 \hat{k} \\ \omega_2 = \dot{\theta}_2 \hat{k} \end{matrix}$$

$$K.E : K = \frac{1}{2} \sum_{i=1}^2 (m_i v_{c_i}^T v_{c_i} + \omega_i^T I_i \omega_i) = \sum_{i=1}^2 \frac{1}{2} \dot{\theta}^T D(\theta) \dot{\theta}$$

$$\text{Here, } D(\theta) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 l_2 \cos(\theta_2 - \theta_1) \\ m_2 l_1 l_2 \cos(\theta_2 - \theta_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

$$P.E : V = \frac{m_1 g l_1 \sin \theta_1}{2} + m_2 g l_1 \sin \theta_1 + \frac{m_2 g l_2 \sin \theta_2}{2}$$

we have equations:

$$\sum_j d_{kj}(\theta) \ddot{\theta}_j + \sum_{i,j} c_{ijk}(\theta) \dot{\theta}_i \dot{\theta}_j + \phi_k(\theta) = T_k$$

$$c_{ijk} = \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial \theta_i} + \frac{\partial d_{ki}}{\partial \theta_j} - \frac{\partial d_{ij}}{\partial \theta_k} \right]$$

$$\therefore c_{111} = c_{121} = c_{211} = c_{212} = c_{122} = c_{222} = 0$$

$$c_{112} = \frac{m_2 l_1 l_2 \sin(\theta_2 - \theta_1)}{2}, \quad c_{221} = -\frac{m_2 l_1 l_2 \sin(\theta_2 - \theta_1)}{2}$$

$$\phi_1 = \frac{\partial V}{\partial \theta_1} = \frac{m_1 g l_1 \cos \theta_1}{2} + m_2 g l_1 \cos \theta_1$$

$$\phi_2 = \frac{\partial V}{\partial \theta_2} = \frac{m_2 g l_2 \cos \theta_2}{2}$$

Hence the equations are:

$$T_1 = \left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + \frac{m_2 l_2^2}{12} \right) \ddot{\theta}_1 + \frac{m_2 l_1 l_2 \cos(\theta_2 - \theta_1)}{2} \ddot{\theta}_2 - \frac{m_2 l_1 l_2 \sin(\theta_2 - \theta_1)}{2} \dot{\theta}_1^2 + \frac{m_1 g l_1 \cos \theta_1}{2} + m_2 g l_1 \cos \theta_1$$

$$T_2 = \frac{m_2 l_1 l_2 \cos(\theta_2 - \theta_1) \ddot{q}_1}{2} + \frac{m_2 l_2^2 \ddot{q}_2}{3} + \frac{m_2 l_1 l_2 \sin(\theta_2 - \theta_1) \dot{q}_1^2}{2} + m_2 g l_2 \cos \theta_2 / 2$$

These are same equations as derived in Miniproject-1

Q 11 from

We know Euler Lagrange's Equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = T_k, \quad k=1, 2, \dots, n \quad \text{--- (1)}$$

Here, $L = K - V$

$$K.E. : K = \frac{1}{2} \sum_{i=1}^n (m_i v_{ci}^T v_{ci} + \omega_i^T I_i \omega_i) = \sum_{i=1}^n \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

P.E $V = V(q)$

$$\therefore L = \frac{1}{2} \sum_{i,j} d_{ij}(q) \dot{q}_i \dot{q}_j - V(q)$$

$$\therefore \frac{\partial L}{\partial \dot{q}_k} = \sum_{i=1}^n d_{ki}(q) \dot{q}_i \quad \left(\frac{\partial V(q)}{\partial \dot{q}_k} = 0 \text{ since } V(q) \text{ is not dependent on } \dot{q} \right)$$

$$\begin{aligned} \therefore \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) &= \sum_j \frac{d}{dt} (d_{kj}(q) \dot{q}_j) + \sum_j \frac{d}{dt} (d_{kj}(q) \dot{q}_j) \\ &= \sum_j d_{kj}(q) \ddot{q}_j + \sum_{ij} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j \end{aligned} \quad \text{--- (2)}$$

$$\text{And } \frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{ij} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k} \quad \text{--- (3)}$$

Substituting (2) and (3) in (1)

$$\therefore \sum_j d_{kj} \ddot{q}_j + \sum_{ij} \left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k} \right] \dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k} = T_k \quad \text{--- (4)}$$

Since $D(q)$ is symmetric,

$$\sum_{ij} \left(\frac{\partial d_{kj}}{\partial q_i} \right) \dot{q}_i \dot{q}_j = \frac{1}{2} \sum_{ij} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} \right] \dot{q}_i \dot{q}_j$$

Hence equation (4) becomes:

$$\sum_j^n d_{kj} \ddot{q}_j + \sum_{i,j} \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right] \dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k} = T_k$$

Christoffel symbols:

Putting $C_{ijk} = \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right] \dot{q}_i \dot{q}_j$

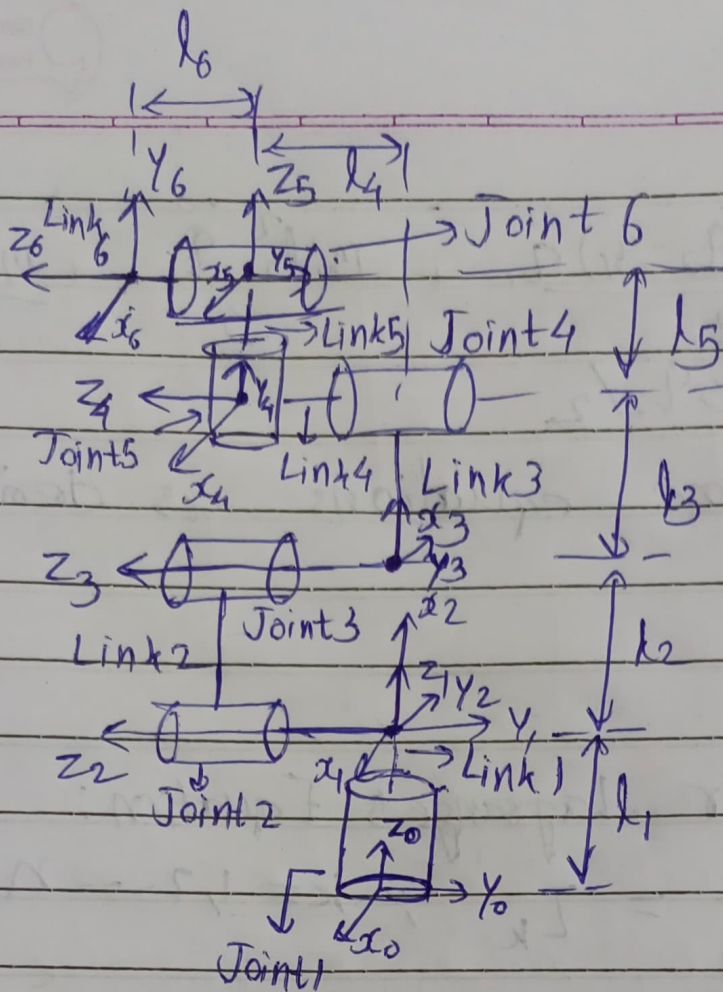
$$\therefore \sum_j^n d_{kj}(q) \ddot{q}_j + \sum_{i,j} C_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = T_k$$

Here, $\phi_k(q) = \frac{\partial V}{\partial q_k}$

We write the eqⁿ in matrix form:

$$D(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau$$

Q12



Number of Links
= 6

Number of joints = 6

All joints are R type

DH table:

	θ_i	d_i	a_i	α_i
1	θ_1	l_1	0	0
2	θ_2	0	l_2	$\pi/2$
3	θ_3	0	l_3	0
4	θ_4	l_4	0	0
5	θ_5	l_5	0	$\pi/2$
6	θ_6	l_6	0	$-\pi/2$