19110179 Aryan Shah Assignment 02

Q1 To show $R\,S(a)\,R^T = S(Ra)$

since R is a general rotational matrix we can assume it to be for z axis therefore

let $R_{z,\theta} = R = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$

LHS

$R\,S(a)\,R^T = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -s\theta a_z & -c\theta a_z & c\theta a_y + s\theta a_x \\ c\theta a_z & -s\theta a_z & s\theta a_y - c\theta a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -a_z & c\theta a_y + s\theta a_x \\ a_z & 0 & s\theta a_y - c\theta a_x \\ -a_y c\theta - a_x s\theta & -a_y s\theta + a_x c\theta & 0 \end{bmatrix}$

RHS

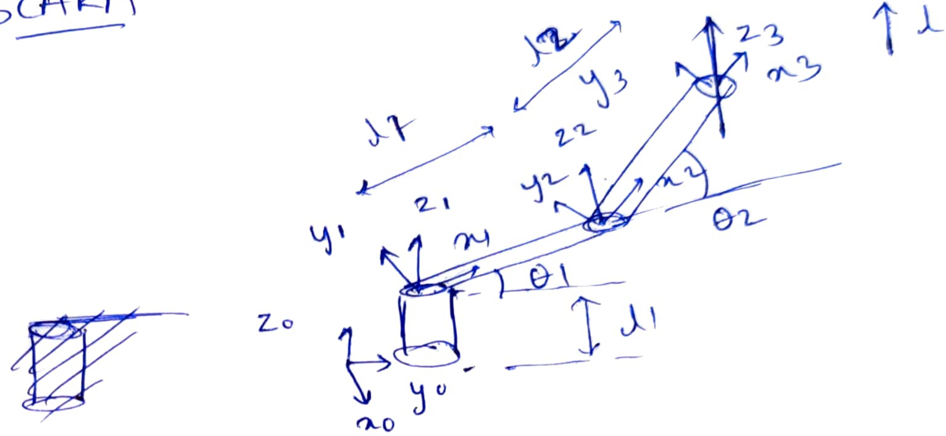
$S(Ra) \Rightarrow \quad Ra = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$

$= \begin{bmatrix} a_x c\theta - a_y s\theta \\ a_x s\theta + a_y c\theta \\ a_z \end{bmatrix}$

$$S(Ra) = \begin{bmatrix} 0 & -a_z & a_x s\theta + a_y c\theta \\ a_z & 0 & a_y s\theta - a_x c\theta \\ -a_y c\theta - a_x s\theta & a_x c\theta - a_y s\theta & 0 \end{bmatrix}$$

LHS = RHS   Hence proved.

Q2 SCARA



going from 0 to 1

$$R_0^1 = R_{z,\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{z,\theta_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \text{No Rotation} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Also,

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} d_7 \\ 0 \\ 0 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} d_8 \\ 0 \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & d_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = H_0^1 H_1^2 H_2^3 P_3$$

let $P_3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix}$

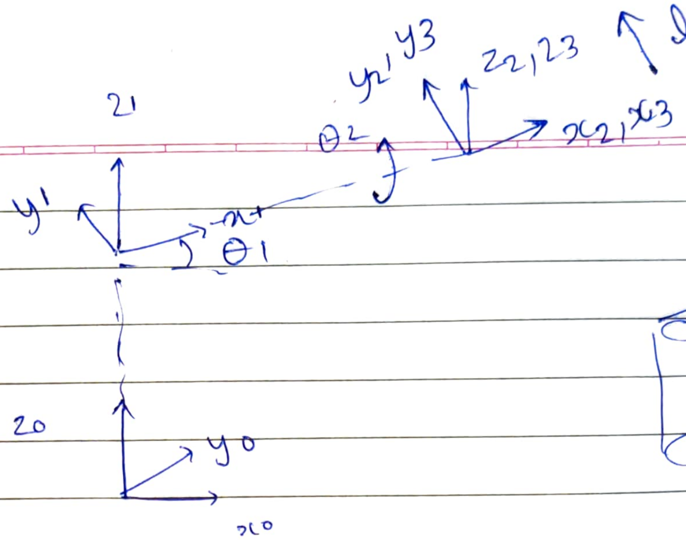
$$P_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

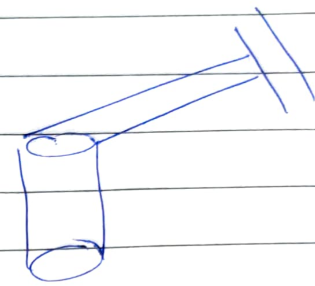
$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & d_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \cancel{d_3 \cos(\theta_1 + \theta_2} = \begin{bmatrix} d_2 c(\theta_1 + \theta_2) + d_1 c\theta_1 \\ d_2 s(\theta_1 + \theta_2) + d_1 s\theta_1 \\ d_1 + d \end{bmatrix}$$

Q4





$$R_0^1 = R_{z,\theta_1} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = R_{x,\theta_2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_2 & -s\theta_2 \\ 0 & s\theta_2 & c\theta_2 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} d_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & c\theta_2 & -s\theta_2 & 0 \\ 0 & s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Page No.
Date

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

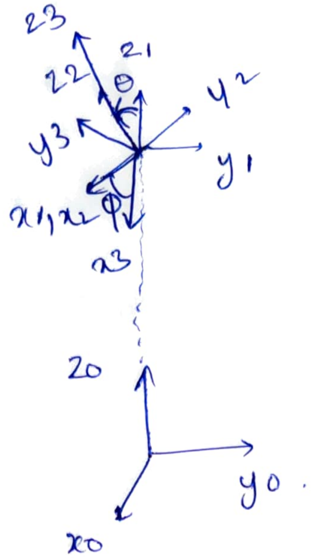
$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} d_2 \cos\theta_1 - d \cos\theta_1 \sin\theta_1 \\ d \cos^2\theta_1 + d_2 \sin\theta_1 \\ d_1 + d \sin\theta_2 \end{bmatrix}$$

Q5



$\theta = 30^\circ$

$\phi = 60^\circ$

$R_0^1 = \text{no rotation} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$R_1^2 = R_{x,30^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c30 & -s30 \\ 0 & s30 & c30 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2^3 = R_{z,60^\circ} = \cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c60 & -s60 \\ 0 & s60 & c60 \end{bmatrix}} \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \qquad H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cancel{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} 0 \\ -1.5 \\ 15.6 \\ 0 \end{bmatrix}} \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \\ 1 \end{bmatrix} \Rightarrow P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \end{bmatrix}$

Q6

There are basic three types of geared motors

① Right angle geared motor use (bevel, worm or hypoid gears)

→ These are angular geared motors with output shaft rotated 90 degrees to motor shaft. Used in cranes, packaging, conveyors

② Parallel shafted geared motor (helical)
↳ It uses gears to achieve speed reduction. Motor shaft and speed reducer shaft are parallel to eachother. Greater power, are silent, enduring. Used in agitators, carriage drivers

③ Inline geared motor (spur or helical or planetary sets)

↳ The gear output shaft is in line with motor shaft. Used for low speed high torque applications.

Each of these types can use any gear sets like Helical set has more torque capacity than spur set and usually emit less noise. Worm set perform well in cases of low torque angles and are suitable for speed reduction scenarios

Drones have brushless motors which are meant for high speed cases ~~and are~~. ~~These~~ Drones also doesnot require torques. On the other hand geared motors are used for high torques ~~cases~~ and low speed cases. Thus drone motors doesnot require geared motors.

Q7 SCARA

Jacobian

(using matrices from q3 as well assuming same fig as q3)

we know $J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$

$$J_{v_i} = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) & \text{revolute } i \\ z_{i-1} & \text{prismatic } i \end{bmatrix}$$

$$J_{w_i} = \begin{bmatrix} z_{i-1} & \text{revolute } i \\ 0 & \text{prismatic } i \end{bmatrix}$$

$i = 1, 2, 3$ here

$$z_{i-1} = R_0^{i-1} k_{i-1}$$

$$\begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} l_1 c\theta_1 \\ l_1 s\theta_1 \\ 0 \end{bmatrix}$$

so,

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} \cancel{0} \\ \cancel{0} \\ \cancel{d_1} \\ 1 \end{bmatrix} = \begin{bmatrix} \cancel{d_2 c\theta_1} & 0 \\ \cancel{d_2 s\theta_1} & 0 \\ \cancel{d_1} & \\ \cancel{1} & \end{bmatrix}$$

$$\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 \cancel{H_1^2} \begin{bmatrix} d_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cancel{d_3 c(\theta_1+\theta_2) + d_2 c\theta_1} \\ \cancel{d_3 s(\theta_1+\theta_2) + d_2 s\theta_1} \\ \cancel{d_1} \\ 1 \end{bmatrix} \begin{bmatrix} d_2 \cos\theta_1 \\ d_2 \sin\theta_1 \\ d_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = \text{derived} = \begin{bmatrix} d_3 c(\theta_1+\theta_2) + d_2 c\theta_1 \\ d_3 s(\theta_1+\theta_2) + d_2 s\theta_1 \\ d_1 \\ 1 \end{bmatrix}$$

$i = 1, 2$ revolute $\quad 3 =$ prismatic

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$z_{i-1} = R_0^{i-1} k_{i-1}$$

$$z_0 = R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1 = R_0^1 k_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

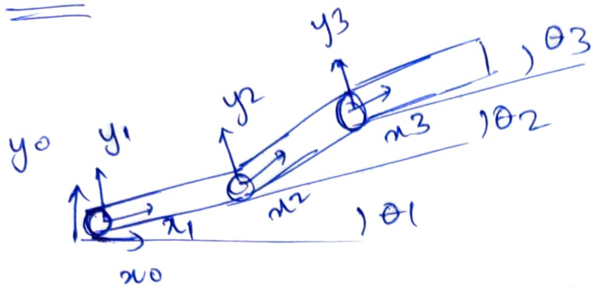
$$z_2 = R_0^2 k_2 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_2 s(\theta_1+\theta_2) - l_1 s\theta_1 & -l_2 s(\theta_1+\theta_2) & 0 \\ l_2 c(\theta_1+\theta_2) + l_1 c\theta_1 & l_2 c(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q9 RRR

All z axis out of plane



$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \qquad J_{vi} = \begin{cases} z_{i-1} \times (O_n - O_{i-1}) & \text{revolute } i \\ z_{i-1} & \text{pris } i \end{cases}$$

$$z_{i-1} = R_0^{i-1} k_{i-1} \qquad J_{\omega i} = \begin{cases} z_{i-1} & \text{revolute } i \\ 0 & \text{pris } i \end{cases}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_1 c\theta_1 \\ d_1 s\theta_1 \\ 0 \\ 1 \end{bmatrix}$$

$$\cancel{O_2 = R_0^1 R_1^2 \begin{bmatrix} d_2 \\ 0 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} d_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & d_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_2 c(\theta_1+\theta_2) + d_1 c\theta_1 \\ d_2 s(\theta_1+\theta_2) + d_2 s\theta_1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} O_3 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} d_3 \\ 0 \\ 0 \end{bmatrix}$$

following symmetry and also I/y as above

$$= \begin{bmatrix} d_3 c(\theta_1+\theta_2+\theta_3) + d_2 c(\theta_1+\theta_2) + d_1 c\theta_1 \\ d_3 s(\theta_1+\theta_2+\theta_3) + d_2 s(\theta_1+\theta_2) + d_1 s\theta_1 \\ 0 \\ 1 \end{bmatrix}$$

$i = 1, 2, 3$ revolute

$\therefore$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 = R_0^0 k_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

IIly all $z_1, z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Note let $\cancel{O_3 - O_0}$ $O_n - O_{i-1} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\quad z_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\cancel{z_i \times (O_3 - O_0)}$ $z_i \times (O_n - O_{i-1}) = \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$

$\therefore$

$$J = \begin{bmatrix} -l_3 s(\theta_1+\theta_2+\theta_3) - l_2 s(\theta_1+\theta_2) - l_1 s(\theta_1) & -l_3 s(\theta_1+\theta_2+\theta_3) - l_2 s(\theta_1+\theta_2) & -l_3 \sin(\theta_1+\theta_2+\theta_3) \\ l_3 c(\theta_1+\theta_2+\theta_3) + l_2 c(\theta_1+\theta_2) + l_2 c(\theta_1) & l_3 c(\theta_1+\theta_2+\theta_3) + l_2 c(\theta_1+\theta_2) & l_3 c(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$