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1)Singularities: The configurations for which the rank of J (the jacobian, which maps dq to dx) reduces are called singularities. The loss of rank represents the loss of a degree of freedom/ ability to move in a certain direction. Bounded gripper velocities, forces, torques, correspond to unbounded gripper velocities, forces, and torques.

Singular configurations are usually (but not always) found at the boundary of the workspace or where the end effector cannot reach with small disturbances.

When the determinant of a manipulator jacobian is zero, then the configuration is a singular one. So, if the determinant of a configuration is close to zero, the robot is close to a singular configuration. Also, the inverse kinematics problem will not have a unique solution - it may have more than one or infinitely many solutions.

2)DH parameters:

End effector frame:
Default assumption of a rotating wrist being attached to the end effector. In which case, their DH parameters would be:

Transformation	d	θ (rad)	r	α (rad)
0 → 1	d1	q1	0	0

Value of d1 would be dependent on the previous joint - whether it's prismatic or revolute. Prismatic joints would have d1 ≠ 0, whereas a revolute joint would have d1 = 0.

Spherical Wrist:
Treat it as 3 revolute joints attached to the robot (with the last revolute joint having a rotating wrist type end effector of its own)

Transformation	d	θ (rad)	r	α (rad)
3 → 4	d1	q3	0	-π/2
4 → 5	0	q4	0	π/2
5 → 6	0	q5	0	0

Again, d1 depends on whether the previous joint was prismatic or not.

4)Stanford Manipulator:
DH values

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0	q1	l1	0
1 → 2	0	q2	l2	π/2
2 → 3	q3	q4	0	0

Jacobian:

$$\begin{bmatrix} q_3c_{12} - l_2s_{12} - l_1s_1 & q_3c_{12} - l_2s_{12} & s_{12} & 0 \\ q_3s_{12} + l_2c_{12} + l_1c_1 & q_3s_{12} + l_2c_{12} & -c_{12} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{12} \\ 0 & 0 & 0 & -c_{12} \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Different test cases:

Case 1:
DH parameters

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0	π/4	1	0
1 → 2	0	π/4	1	π/2
2 → 3	0.2	0	0	0

Calculated:

$$\begin{bmatrix} -1.7071 & -1 & 1 & 0 \\ 0.9071 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

From Code:

```
[[-1.70710605e+00 -9.99999735e-01  1.00000000e+00 -0.00000000e+00]
 [ 9.07108577e-01  2.00001327e-01 -1.32679490e-06  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.32679490e-06  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.32679490e-06]
 [ 1.00000000e+00  1.00000000e+00  0.00000000e+00  1.32679490e-06]]
```

Case 2:
DH parameters

Transformation	d	θ (rad)	r	α (rad)
$0 \rightarrow 1$	0	π/6	1	0
$1 \rightarrow 2$	0	π/2	1	π/2
$2 \rightarrow 3$	1	0	0	0

Calculated:

$$\begin{bmatrix} -1.866 & -1.366 & 0.866 & 0 \\ 1.232 & 0.366 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.866 \\ 0 & 0 & 0 & 0.5 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

From Code:

```
[[-1.86602437e+00 -1.36602476e+00  8.66026288e-01  0.00000000e+00]
 [ 1.23205345e+00  3.66027820e-01  4.99998468e-01  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.32679490e-06  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  8.66026288e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  4.99998468e-01]
 [ 1.00000000e+00  1.00000000e+00  0.00000000e+00  1.32679490e-06]]
```

SCARA Manipulator:
DH values

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0	q1	l1	0
1 → 2	0	q2	l2	π
2 → 3	q3	q4	0	0

3rd frame: end effector

Jacobian:

$$\begin{bmatrix} -l_2 s_{12} - l_1 s_1 & -l_2 s_{12} & 0 & 0 \\ l_2 c_{12} + l_1 c_1 & l_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Different test cases:

Case 1:

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0	π/6	1	0
1 → 2	0	π/6	1	π
2 → 3	1	0	0	0

Calculated:

$$\begin{bmatrix} -1.366 & -0.866 & 0 & 0 \\ 1.366 & 0.5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

From Code:

```
[[-1.36602325e+00 -8.66023635e-01  2.29807500e-06  0.00000000e+00]
 [ 1.36602869e+00  5.00003064e-01 -1.32679693e-06 -0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00 -1.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.29807500e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.32679693e-06]
 [ 1.00000000e+00  1.00000000e+00  0.00000000e+00 -1.00000000e+00]]
```

Case 2:

Transformation	d	θ (rad)	r	α (rad)
$0 \rightarrow 1$	0	π/4	1	0
$1 \rightarrow 2$	0	π/3	1	π
$2 \rightarrow 3$	0.2	0	0	0

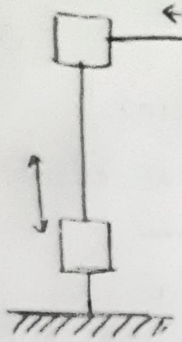
Calculated:

$$\begin{bmatrix} -1.673 & -0.9659 & 0 & 0 \\ 0.448 & -0.2588 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

From Code:

```
[[-1.67303268e+00 -9.65926364e-01  2.56317198e-06  0.00000000e+00]
 [ 4.48290213e-01 -2.58817037e-01  6.86795609e-07 -0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00 -1.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.56317198e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  6.86795609e-07]
 [ 1.00000000e+00  1.00000000e+00  0.00000000e+00 -1.00000000e+00]]
```

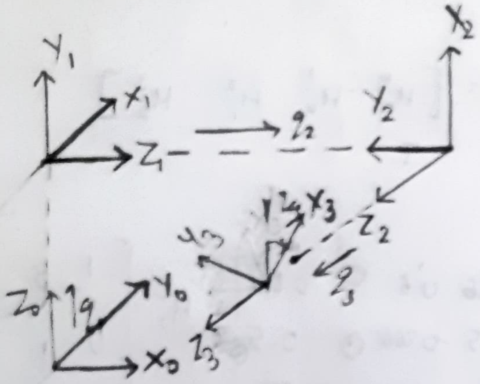
5



$$H_0^3 = H_0^1 H_1^2 H_2^3 = [Z_1][X_1][Z_2][X_2][Z_3][X_3]$$

$$Z_1 = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$X_1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$



$$H_0^1 = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

d-h parameters

	d	θ	r	α
$0 \to 1$	q_1	$\pi/2$	0	$\pi/2$
$1 \to 2$	q_2	$\pi/2$	0	$\pi/2$
$2 \to 3$	q_3	$+q_4$	0	0

$$Z_2 = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad X_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad H_1^2 = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$Z_3 = \left[\begin{array}{ccc|c} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad X_3 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad H_2^3 = \left[\begin{array}{ccc|c} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$J = [\, J_1 \quad J_2 \quad J_3 \quad J_4 \,]$

$$J_1 = \begin{bmatrix} Z_0 \\ 0 \end{bmatrix} \quad J_2 = \begin{bmatrix} Z_1 \\ 0 \end{bmatrix} \quad J_3 = \begin{bmatrix} Z_2 \\ 0 \end{bmatrix} \quad J_4 = \begin{bmatrix} Z_3 \times (O_3 - O_3) \\ Z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ Z_3 \end{bmatrix}$$

$$Z_0 = R_0^0 \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad Z_1 = R_0^1 \hat{k} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad Z_2 = R_0^2 \hat{k} \quad Z_3 = R_0^3 \hat{k}$$

$$H_0^2 = H_0^1 H_1^2 = \left[\begin{array}{ccc|c} 0 & 1 & 0 & q_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & q_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad H_0^3 = H_0^2 H_2^3 = \left[\begin{array}{ccc|c} S_4 & C_4 & 0 & q_2 \\ 0 & 0 & 1 & q_3 \\ C_4 & -S_4 & 0 & q_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

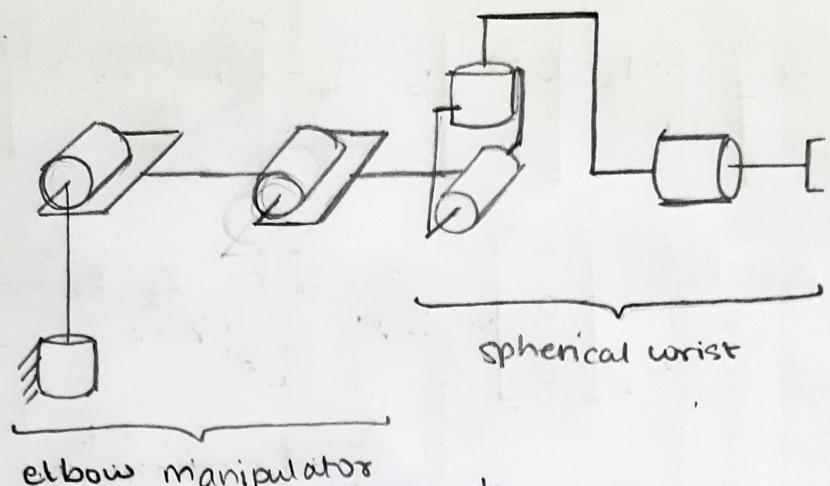
$$Z_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad Z_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

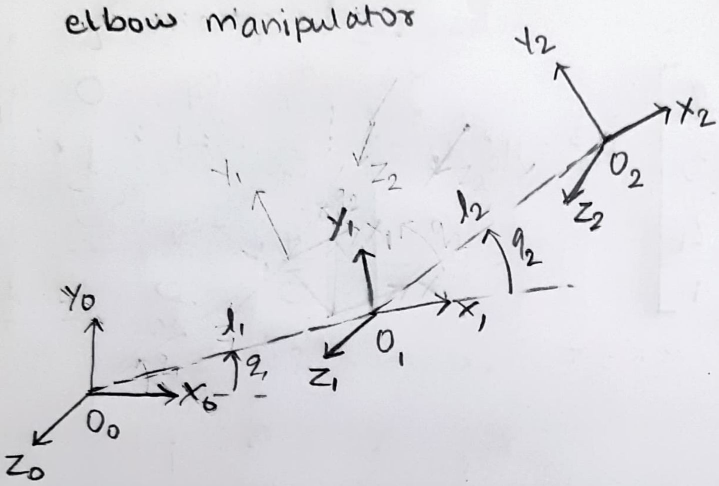
$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

end-effector position: $O_3 - O_0 = \begin{bmatrix} q_2 \\ q_3 \\ q_4 \end{bmatrix}$

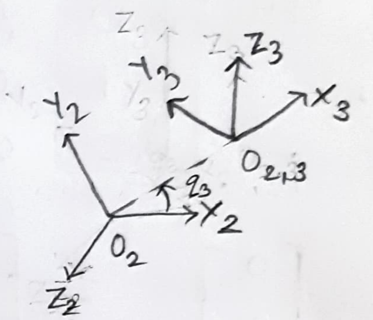
$$\dot{X} = J\dot{q}$$

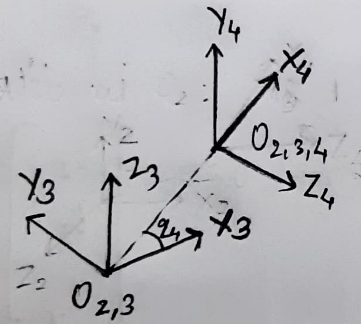
⑥

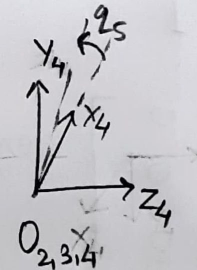


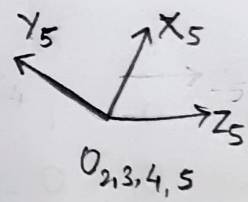


at O_2:









also at O_2

d-h parameters

	d	θ	r	α
$0\to1$	0	q_1	l_1	0
$1\to2$	0	q_2	l_2	0
$2\to3$	0	q_3	0	$-\pi/2$
$3\to4$	0	q_4	0	$\pi/2$
$4\to5$	0	q_5	0	0

(Just assuming d_5 to be 0)

$J = [J_1 \quad J_2 \quad J_3 \quad J_4 \quad J_5 \quad J_6]$

$$J_1 = \begin{bmatrix} z_0 \times (O_5 - O_0) \\ z_0 \end{bmatrix} \quad J_2 = \begin{bmatrix} z_1 \times (O_5 - O_1) \\ z_1 \end{bmatrix} \quad J_3 = \begin{bmatrix} z_2 \times (O_5 - O_2) \\ z_2 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} z_3 \times (O_5 - O_3) \\ z_3 \end{bmatrix} \quad J_5 = \begin{bmatrix} z_4 \times (O_5 - O_4) \\ z_4 \end{bmatrix} \quad J_6 = \begin{bmatrix} z_5 \times (O_5 - O_5) \\ z_5 \end{bmatrix}$$

$O_5, O_4, O_3, O_2 \rightarrow$ same

$z_0 = R_0^0 \hat{k} \quad z_1 = R_0^1 \hat{k} \quad z_2 = R_0^2 \hat{k} \quad z_3 = R_0^3 \hat{k} \quad z_4 = R_0^4 \hat{k}$

$z_5 = R_0^5 \hat{k}$

$$J_1 = \begin{bmatrix} z_0 \times (O_5 - O_0) \\ z_0 \end{bmatrix} \quad J_2 = \begin{bmatrix} z_1 \times (O_5 - O_1) \\ z_1 \end{bmatrix} \quad J_3 = \begin{bmatrix} 0 \\ z_2 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} 0 \\ z_3 \end{bmatrix} \quad J_5 = \begin{bmatrix} 0 \\ z_4 \end{bmatrix} \quad J_6 = \begin{bmatrix} 0 \\ z_5 \end{bmatrix} \quad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$H_0^5 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 = [Z_1][X_1][Z_2][X_2][Z_3][X_3][Z_4][X_4][Z_5][X_5]$

$$Z_1 = \left[\begin{array}{ccc|c} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad X_1 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad H_0^1 = \left[\begin{array}{ccc|c} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$Z_2 = \left[\begin{array}{ccc|c} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad X_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad H_1^2 = \left[\begin{array}{ccc|c} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$Z_3=\left[\begin{array}{ccc|c} C_3 & -S_3 & 0 & 0\\ S_3 & C_3 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad X_3=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0\\ 0 & 0 & +1 & 0\\ 0 & -1 & 0 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad H_2^3=\left[\begin{array}{ccc|c} C_3 & 0 & -S_3 & 0\\ S_3 & 0 & C_3 & 0\\ 0 & -1 & 0 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$Z_4=\left[\begin{array}{ccc|c} C_4 & -S_4 & 0 & 0\\ S_4 & C_4 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad X_4=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad H_3^4=\left[\begin{array}{ccc|c} C_4 & 0 & S_4 & 0\\ S_4 & 0 & -C_4 & 0\\ 0 & 1 & 0 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$Z_5=\left[\begin{array}{ccc|c} C_5 & -S_5 & 0 & 0\\ S_5 & C_5 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad X_5=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad H_4^5=\left[\begin{array}{ccc|c} C_5 & -S_5 & 0 & 0\\ S_5 & C_5 & 0 & 0\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$H_0^2=H_0^1H_1^2=\left[\begin{array}{ccc|c} C_{12} & -S_{12} & 0 & l_2C_{12}+l_1C_1\\ S_{12} & C_{12} & 0 & l_2S_{12}+l_1S_1\\ 0 & 0 & 1 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right] \quad H_0^3=H_0^2H_2^3=\left[\begin{array}{ccc|c} C_{123} & 0 & -S_{123} & l_2C_{12}+l_1C_1\\ S_{123} & 0 & C_{123} & l_2S_{12}+l_1S_1\\ 0 & -1 & 0 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$H_0^4=H_0^3H_3^4=\left[\begin{array}{ccc|c} C_4C_{123} & -S_{123} & S_4C_{123} & l_2C_{12}+l_1C_1\\ C_4S_{123} & C_{123} & S_4S_{123} & l_2S_{12}+l_1S_1\\ -S_4 & 0 & C_4 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$H_0^5=H_0^4H_4^5=\left[\begin{array}{ccc|c} C_4C_{123}C_5-S_{123}S_5 & -S_5C_4C_{123}-S_{123}C_5 & S_4C_{123} & l_2C_{12}+l_1C_1\\ C_4C_5S_{123}+C_{123}S_5 & -S_5C_4S_{123}+C_5C_{123} & S_4S_{123} & l_2S_{12}+l_1S_1\\ -S_4C_5 & S_4S_5 & C_4 & 0\\ \hline 0 & 0 & 0 & 1\end{array}\right]$$

$$Z_1=\begin{bmatrix}0\\0\\1\end{bmatrix} \quad Z_2=\begin{bmatrix}0\\0\\1\end{bmatrix} \quad Z_3=\begin{bmatrix}-S_{123}\\C_{123}\\0\end{bmatrix} \quad Z_4=\begin{bmatrix}S_4C_{123}\\S_4S_{123}\\C_4\end{bmatrix} \quad Z_5=\begin{bmatrix}S_4C_{123}\\S_4S_{123}\\C_4\end{bmatrix}$$

$$O_5-O_0=\begin{bmatrix}l_2C_{12}+l_1C_1\\l_2S_{12}+l_1S_1\\0\end{bmatrix} \quad O_1-O_0=\begin{bmatrix}l_1C_1\\l_1S_1\\0\end{bmatrix} \quad O_5-O_1=\begin{bmatrix}l_2C_{12}\\l_2S_{12}\\0\end{bmatrix}$$

$$z_0 \times (O_5 - O_0) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ O_x & O_y & O_z \end{bmatrix} = -O_y\hat{i} + O_x\hat{j}$$

$$= -(l_2S_{12} + l_1S_1)\hat{i} + (l_2C_{12} + l_1C_1)\hat{j}$$

$$z_1 \times (O_5 - O_1) = -O_y\hat{i} + O_x\hat{j} = -l_2S_{12}\hat{i} + l_2C_{12}\hat{j}$$

$$J = \begin{bmatrix} -l_2S_{12} - l_1S_1 & -l_2S_{12} & 0 & 0 & 0 & 0 \\ l_2C_{12} + l_1C_1 & l_2C_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -S_{123} & S_4C_{123} & S_4C_{123} \\ 0 & 0 & 0 & C_{123} & S_4S_{123} & S_4S_{123} \\ 1 & 1 & 1 & 0 & C_4 & C_4 \end{bmatrix}$$

5) End effector position: $O_3 - O_0 = (q2, q3, q1)$
Different test cases:

Case 1:
DH parameters:

Transformation	d	θ (rad)	r	α (rad)
0 → 1	1	π/2	0	π/2
1 → 2	0.5	π/2	0	π/2
2 → 3	0.75	0	0	0

Calculated Jacobian:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From Code:

```
[[ 0.00000000e+00  1.00000000e+00  2.65359155e-06  0.00000000e+00]
 [ 0.00000000e+00 -1.32679490e-06  1.00000000e+00 -0.00000000e+00]
 [ 1.00000000e+00  1.32679490e-06 -1.32679314e-06  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.65359155e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.32679314e-06]]
```

Case 2:
DH parameters:

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0.25	π/2	0	π/2
1 → 2	0.3	π/2	0	π/2
2 → 3	1	0	0	0

Calculated Jacobian:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From Code:

```
[[ 0.00000000e+00  1.00000000e+00  2.65359155e-06  0.00000000e+00]
 [ 0.00000000e+00 -1.32679490e-06  1.00000000e+00 -0.00000000e+00]
 [ 1.00000000e+00  1.32679490e-06 -1.32679314e-06  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.65359155e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.32679314e-06]]
```

6) End effector position = O_5 - O_0
Different test cases:

Case 1:
DH parameters:

Transformation	d	θ (rad)	r	α (rad)
0 → 1	0	π/2	1	0
1 → 2	0	π/3	1	0
2 → 3	0	π/4	0	- π/2
3 → 4	0	π/6	0	π/2
4 → 5	0	π/8	0	0

Calculated:

$$\begin{bmatrix} -1.5 & -0.5 & 0 & 0 & 0 & 0 \\ -0.866 & -0.866 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2588 & -0.4829 & -0.4829 \\ 0 & 0 & 0 & -0.9659 & -0.1294 & -0.1294 \\ 1 & 1 & 1 & 0 & 0.866 & 0.866 \end{bmatrix}$$

From Code:

```
[[-1.50000192e+00 -5.00001915e-01  0.00000000e+00 -0.00000000e+00
  -0.00000000e+00 -0.00000000e+00]
 [-8.66022971e-01 -8.66024298e-01  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.58816268e-01
  -4.82962869e-01 -4.82962869e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -9.65926570e-01
  -1.29408207e-01 -1.29408207e-01]
 [ 1.00000000e+00  1.00000000e+00  1.00000000e+00  1.32679490e-06
   8.66025625e-01  8.66025625e-01]]
```

Case 2:
DH parameters:

Transformation	d	θ (rad)	r	α (rad)
$0 \rightarrow 1$	0	π/8	1	0
$1 \rightarrow 2$	0	π/4	1	0
$2 \rightarrow 3$	0	π	0	- π/2
$3 \rightarrow 4$	0	π/2	0	π/2
$4 \rightarrow 5$	0	π/6	0	0

Calculated:

$$\begin{bmatrix} -1.30656 & -0.92387 & 0 & 0 & 0 & 0 \\ 1.30656 & 0.38268 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.923879 & -0.382683 & -0.382683 \\ 0 & 0 & 0 & -0.382683 & 0.923879 & 0.923879 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

From Code:

```
[[-1.30656228e+00 -9.23879152e-01  0.00000000e+00 -0.00000000e+00
  -0.00000000e+00 -0.00000000e+00]
 [ 1.30656401e+00  3.82684352e-01  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  9.23878136e-01
  -3.82685578e-01 -3.82685578e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -3.82686803e-01
  -9.23878644e-01 -9.23878644e-01]
 [ 1.00000000e+00  1.00000000e+00  1.00000000e+00  1.32679490e-06
   1.32679666e-06  1.32679666e-06]]
```

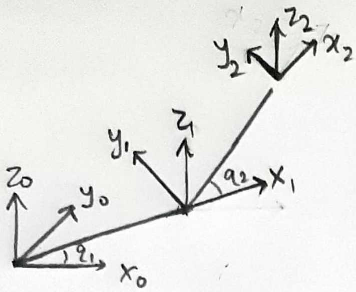
7) 2R manipulator: all of the below have 2 degrees of freedom

Direct drive: Have motors directly attached to joints of a 2R manipulator. Does not involve transmission elements between actuators and joints. Advantage: behavior of system more predictable,

Remotely driven: Have motors attached to base, rotation of links controlled from there (using belts and other such methods). Advantages: more compact (than direct drive), reduction in weight.

5-bar parallelogram arrangement: made from 5 links connected together in a closed chain (one of them being the base). Advantage: cheaper and simpler to construct. Disadvantage: non-linearity

8



d-h parameters

	d	θ	r	α
$0 \to 1$	0	q_1	l_1	0
$1 \to 2$	0	q_2	l_2	0

$$J = [J_1 \;\; J_2]$$

$$J_1 = \begin{bmatrix} z_0 \times (O_2 - O_0) \\ z_0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} z_1 \times (O_2 - O_1) \\ z_1 \end{bmatrix}$$

$$z_0 = R_0^0 \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad H_0^2 = H_0^1 H_1^2 = [Z_1][X_1][Z_2][X_2]$$

$$[Z_1] = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [X_1] = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_0^1 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Z_2] = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_1 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [X_2] = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^2 = H_0^1 H_1^2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_2C_{12}+l_1C_1 \\ S_{12} & C_{12} & 0 & l_2S_{12}+l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad O_2 - O_0 = \begin{bmatrix} l_2C_{12}+l_1C_1 \\ l_2S_{12}+l_1S_1 \\ 0 \end{bmatrix} \quad O_2 - O_1 = \begin{bmatrix} l_2C_{12} \\ l_2S_{12} \\ 0 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad J = \begin{bmatrix} -l_2S_{12}-l_1S_1 & -l_2S_{12} \\ +l_2C_{12}+l_1C_1 & l_2C_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

end effector location = $(O_2 - O_0)$

$$\Rightarrow \quad x = l_2 c_{12} + l_1 c_1$$
$$y = l_2 s_{12} + l_1 s_1$$

velocities: $\dot{x} = J\dot{q}$

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} -l_2 s_{12} - l_1 s_1 & -l_2 s_{12} \\ l_2 c_{12} + l_1 c_1 & l_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} (-l_2 s_{12} - l_1 s_1)\dot{q}_1 - l_2 s_{12}\dot{q}_2 \\ (l_2 c_{12} + l_1 c_1)\dot{q}_1 + l_2 c_{12}\dot{q}_2 \\ 0 \\ 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 \end{bmatrix}$$

$$\dot{x} = (-l_2 s_{12} - l_1 s_1)\dot{q}_1 - l_2 s_{12}\dot{q}_2 \qquad w_x = 0$$
$$\dot{y} = (l_2 c_{12} + l_1 c_1)\dot{q}_1 + l_2 c_{12}\dot{q}_2 \qquad w_y = 0$$
$$\dot{z} = 0 \qquad w_z = \dot{q}_1 + \dot{q}_2$$

force - Torque Relationship: $\tau = J^T F$

$$\tau = \begin{bmatrix} -l_2 s_{12} - l_1 s_1 & l_2 c_{12} + l_1 c_1 & 0 & 0 & 0 & 1 \\ -l_2 s_{12} & l_2 c_{12} & 0 & 0 & 0 & 1 \end{bmatrix}_{2\times 6} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}_{6\times 1}$$

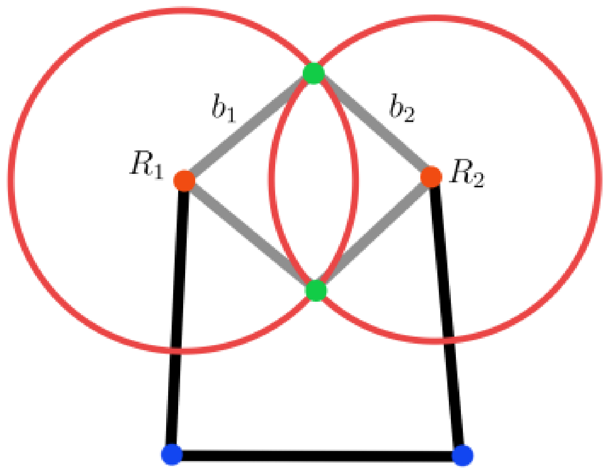
$$\tau = \begin{bmatrix} (-l_2 s_{12} - l_1 s_1) f_x + (l_2 c_{12} + l_1 c_1) F_y + M_z \\ (-l_2 s_{12}) F_x + (l_2 c_{12}) F_y + M_z \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

eq^n are same, but torque eq^n has moments accounted for now

9)Dynamic Equations:

Remotely driven: derived using lagrangian method (as in Q10,Q11)

5-bar parallelogram arrangement:



R1 and R2 are found using usual trigonometric methods used. b1 and b2 are radii of circles centered at R1 and R2. The intersection of the circles gives us the end effector position. Differentiating the equations gives us the velocities.

References:

https://www.universal-robots.com/articles/ur/application-installation/dh-parameters-for-calculations-of-kinematics-and-dynamics/

https://www.researchgate.net/figure/UR5-robot-parameters-by-Denavit-Hartenberg-method_fig2_347021253

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https://scholarworks.uvm.edu/cgi/viewcontent.cgi?article=1416&context=hcoltheses

(10) $\mathcal{L} = K - V$

$$K = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i \dot{q}_j = \frac{1}{2}\dot{q}^T D \dot{q} \qquad V = V(q)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_k} = \sum_j d_{kj}\dot{q}_j = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_k}\right) = \sum_j d_{kj}\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}\dot{q}_j = \sum_j d_{kj}\ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i \dot{q}_j$$

$$\frac{\partial \mathcal{L}}{\partial q_k} = \frac{1}{2}\sum_{ij}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial V}{\partial q_k}$$

$$\sum d_{kj}\ddot{q}_j + \sum_{i,j}\frac{\partial d_{ki}}{\partial q_i}\dot{q}_i\dot{q}_j - \frac{1}{2}\sum_{ij}\frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \text{, where } k = 1, 2, \ldots, n \quad -①$$

$n \downarrow$ no. of links

$$\sum d_{kj}\ddot{q}_j + \sum\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

$$\sum_{ij}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i\dot{q}_j = \frac{1}{2}\sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_i\dot{q}_j$$

$$\sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j = \frac{1}{2}\sum_{ij}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j = \sum_{ij} c_{ijk}\dot{q}_i\dot{q}_j$$

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right] \rightarrow \text{substitute in } ①$$

$$\sum d_{kj}\ddot{q}_j + \sum_{ij} c_{ijk}\dot{q}_i\dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \Rightarrow D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_k$$

substituting i, j, k values $\rightarrow$ eqⁿ of motion

n dof $\Rightarrow$ n eqⁿ

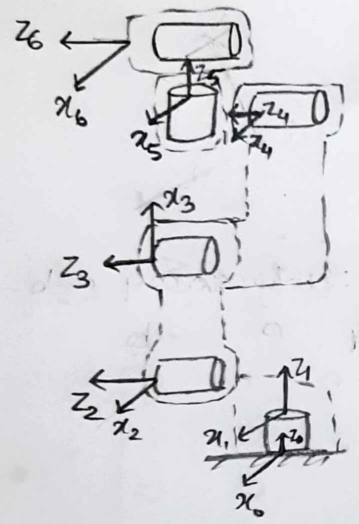
(12) UR5 robot: "universal robot", is used in industrial applications. It has 6 degrees of freedom

No. of links: 6

No. of joints: 6
Base, Shoulder, Elbow, 3 wrists

Nature of Joints: Revolute (all of them)

Link geometry:



d-h parameters

	d	θ	r	α
1	0.1519	0	0	$\pi/2$
2	0	0	-0.24365	0
3	0	0	-0.21325	0
4	0.11235	0	0	$\pi/2$
5	0.08535	0	0	$-\pi/2$
6	0.0819	0	0	0

Values of d, r → based on construction