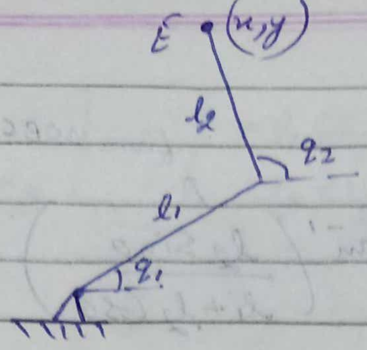
Task 0



$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2 \qquad — ①$$

Kinematics

Differenciate w.r.t to time

$$\dot{x} = -l_1 \sin q_1 \dot{q}_1 - l_2 \sin q_2 \dot{q}_2$$

$$\dot{y} = l_1 \cos q_1 \dot{q}_1 + l_2 \cos q_2 \dot{q}_2$$

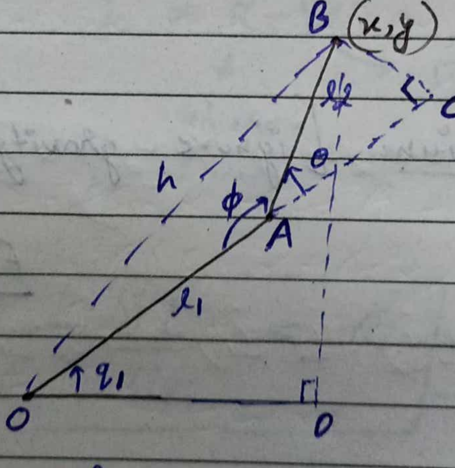
velocity of endeffector

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad — ②$$

Velocity Kinematics

Inverse Kinematics

$q_2 = q_1 + \theta$, $\theta = \pi - \phi$



From Cosine rule

$$h^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\phi$$

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2} \right)$$

In Right triangle OBC

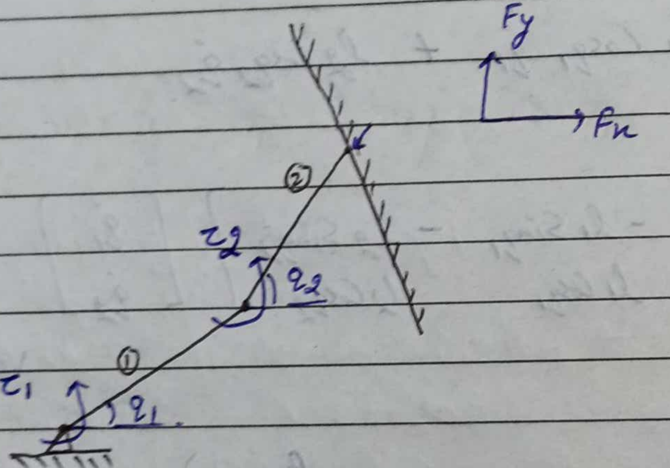
$$q_1 = \underbrace{\tan^{-1}\left(\frac{y}{x}\right)}_{\text{from } \Delta OBD} - \underbrace{\tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)}_{\text{from } \Delta OBC}$$

eq ③ ←

$$q_2 = q_1 + \theta$$

$$q_2 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right) + \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

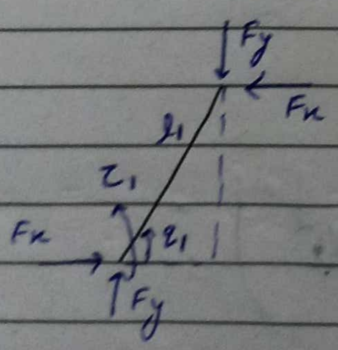
T-2

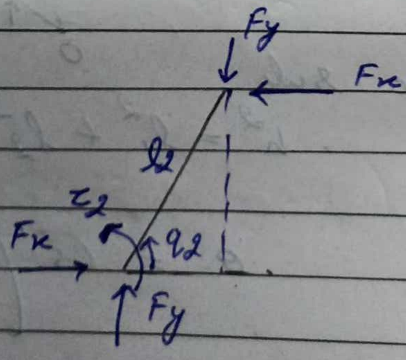


Static Equillibrium [ignore gravity]

FBD of link 1

FBD of link 2.





Balancing torque

$$\tau_1 = F_y l_1 \cos q_1 - F_x l_1 \sin q_1$$

$$\tau_2 = F_y l_2 \cos q_2 - F_x l_2 \sin q_2$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & +l_1 \cos q_1 \\ -l_2 \sin q_2 & +l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad — \text{④}$$

eq-③ along with eq-④ is the answer for T2

<u>T3</u> (virtual spring)

[assuming no damping]

$F_x = K(x - x_0)$

$F_y = K(y - y_0)$

$(x_0, y_0) \rightarrow$ mean position

Need to account for dynamics

<u>Lagrange's Equation</u>

take forces that perform work

$F = ma$

Lagrangian : $\mathcal{L} = K - V$

$K \rightarrow$ Kinetic Energy

$V \rightarrow$ Potential Energy

$$\boxed{\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'} \quad i = 1, 2, 3, 4 \text{ ---- } n$$

$\rightarrow$ eq ⑤

$n \rightarrow$ no. of joints

Q_i' – generalised forces

pure rotation of link-1 ⟷ rotation of link 2 about its Centre of mass [C.O.M]

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 (V_{c_2})^2$$

Kinetic energy

translation of C.O.M of link 2

$$(V_{c_2})^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_2)$$

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$$

potential energy

Now from eq ⑤

$$\boxed{\begin{aligned}
\tau_1 &= \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2 \ddot{q}_2}{2} \cos(q_2 - q_1) \\
&\quad - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 \\
\tau_2 &= \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2 \ddot{q}_2}{4} + \frac{m_2 l_1 l_2 \ddot{q}_1}{2} \cos(q_2 - q_1) \\
&\quad - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} \sin q_2
\end{aligned}}$$

eq. ⑥

dynamic

$$\left.\begin{array}{l} F_x = k(x - x_0) \quad = k(l_1 \cos q_1 + l_2 \cos q_2 - x_0) \\ F_y = k(y - y_0) \quad = k(l_1 \sin q_1 + l_2 \sin q_2 - y_0) \end{array}\right] - \text{eq. } 7$$

$$\tau_1 \text{ apply} = \tau_1 + \tau_{1s}$$

$$\tau_2 \text{ apply} = \tau_2 + \tau_{1s}$$

τ_1, τ_2: eq 6

τ_{1s}: from eq 7 → 4