Introduction to Robotics
Assignment I
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2. Examples for 7 categories of robots

1) Serial Manipulator - IRB 910SC (SCARA) - link
   It is a SCARA robot mainly used for the assembly of small parts. It can also be used for material handling and packaging. It also finds application in inspection and comes in 3 different variants.
2) Parallel Manipulator - Stewart Platform - link
   It is a 6D.O.F parallel manipulator used for research and testing purposes. One common application is flight simulations.
3) Mobile Robots - MiR250 - link
   It is an automated material retrieval robot used for small payloads. It has proximity and other sensors to avoid collisions.
4) Aerial Robots - Lancaster 5 - link
   It is an AUV used in the agricultural field for inspection. It comes with thermal sensors and has a load capacity of 2.2lbs.
5) Underwater Robots - NemoSens - link
   This AUV is used to monitor chemical and physical properties of the ocean/any water body. It is small and handy.
6) Soft Robots - Amoeba Energy's soft Robot - link
   It is a climbing robot with additional grips (essentially made of soft material) that helps it climb irregular terrains.
7) Micro Robots - Capsule Robots - link
   These robots find application in Medical drug delivery majorly. Device is usually of size within a few millimeters.

3. Common Types of Motors

1) AC Motors (Synchronous and Asynchronous) - These are driven by AC current, and have high torque capacities. They can be synchronous(rotation of motor is synchronized with power supply frequency) and asynchronous.
2) Brushed DC Motor - The brushed DC motors use the brushes to conduct current between the source and the armature.The Brush DC motors consist of six different components: axle, commutator, armature, stator, magnets, and brushes. Its stator remains stationary, while the rotor rotates with respect to the stator.Its stator remains stationary, while the rotor rotates with respect to the stator.
3) Brushless DC motor - These motors have permanent magnets which rotate a fixed armature. Unlike brushed DC motors, these motors eliminate the physical contact between the commutator and brushes, hence increasing the life of the motor.
4) Geared DC motor - are an upgrade to brushed DC motor, that have a gear box attached in addition which helps in increasing the torques produced.
5) Servo motor - Servo motors are essentially DC motors with in-built feedback mechanism, that is the position of the shaft after each rotation can be known. This helps to get required radians of rotation.
6) Stepper motor - It can be a brushless DC motor, such that the rotor has multiple permanent magnets placed in alternating poles fashion. This enables us to divide the rotation into smaller steps.

Assignment I

6.) Show that the columns of the rotation matrix R_0^1 are orthogonal.

Consider $R_0^1 = \begin{bmatrix} i_0 \cdot i_1 & j_0 \cdot i_1 & k_0 \cdot i_1 \\ i_0 \cdot j_1 & j_0 \cdot j_1 & k_0 \cdot j_1 \\ i_0 \cdot k_1 & j_0 \cdot k_1 & k_0 \cdot k_1 \end{bmatrix} \Rightarrow C_1 = \begin{bmatrix} i_0 \cdot i_1 \\ i_0 \cdot j_1 \\ i_0 \cdot k_1 \end{bmatrix} \quad C_2 = \begin{bmatrix} j_0 \cdot i_1 \\ j_0 \cdot j_1 \\ j_0 \cdot k_1 \end{bmatrix} \quad C_3 = \begin{bmatrix} k_0 \cdot i_1 \\ k_0 \cdot j_1 \\ k_0 \cdot k_1 \end{bmatrix}$

To show $C_1 / C_2 / C_3$ are orthogonal, we need to show

$$C_1 C_1^T = C_2 C_2^T = C_3 C_3^T = I$$

Now, Consider $C_1 C_1^T$,

$$C_1 C_1^T = \begin{bmatrix} i_0 i_1 \\ i_0 j_1 \\ i_0 k_1 \end{bmatrix} \begin{bmatrix} i_0 i_1 & i_0 j_1 & i_0 k_1 \end{bmatrix} = \begin{bmatrix} i_0 i_1 \, i_0 i_1 & i_0 i_1 \, i_0 j_1 & i_0 i_1 \, i_0 k_1 \\ i_0 j_1 \, i_0 i_1 & i_0 j_1 \, i_0 j_1 & i_0 j_1 \, i_0 k_1 \\ i_0 k_1 \, i_0 i_1 & i_0 k_1 \, i_0 j_1 & i_0 k_1 \, i_0 k_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \because \left(i_0 \cdot i_0 = i_1 \cdot i_1 = j_0 j_0 = j_1 j_1 = k_0 k_0 = k_1 k_1 = 1 \text{ and } i_0 j_0 = j_0 k_0 = k_0 i_0 = 0 \right)$$

$|||^{arly}$

$$C_2 C_2^T = \begin{bmatrix} j_0 i_1 \\ j_0 j_1 \\ j_0 k_1 \end{bmatrix} \begin{bmatrix} j_0 i_1 & j_0 j_1 & j_0 k_1 \end{bmatrix} = \begin{bmatrix} j_0 i_1 j_0 i_1 & j_0 i_1 j_0 j_1 & j_0 i_1 j_0 k_1 \\ j_0 j_1 j_0 i_1 & j_0 j_1 j_0 j_1 & j_0 j_1 j_0 k_1 \\ j_0 k_1 j_0 i_1 & j_0 k_1 j_0 j_1 & j_0 k_1 j_0 k_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 C^T = \begin{bmatrix} k_0 i_1 \\ k_0 j_1 \\ k_0 k_1 \end{bmatrix} \begin{bmatrix} k_0 i_1 & k_0 j_1 & k_0 k_1 \end{bmatrix} = \begin{bmatrix} k_0 i_1 k_0 i_1 & k_0 i_1 k_0 j_1 & k_0 i_1 k_0 k_1 \\ k_0 j_1 k_0 i_1 & k_0 j_1 k_0 j_1 & k_0 j_1 k_0 k_1 \\ k_0 k_1 k_0 i_1 & k_0 k_1 k_0 j_1 & k_0 k_1 k_0 k_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7) Show that $\det(R_0') = 1$

We know $R_0' = \begin{bmatrix} i_0 i_1 & j_0 i_1 & k_0 i_1 \\ i_0 j_1 & j_0 j_1 & k_0 j_1 \\ i_0 k_1 & j_0 k_1 & k_0 k_1 \end{bmatrix}$

Consider, 'R_0'' to be the rotation matrix obtained upon rotation of 'θ' about z,

then, $R_{\theta,z} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now $\det(R_{\theta z}) = \cos\theta[\cos\theta] - (-\sin\theta)[\sin\theta]$

$= \cos^2\theta + \sin^2\theta = 1$

$\therefore \boxed{\det(R_{\theta,z}) = 1}$

$|||^{arly}$, Consider $R_{\theta x}$. $R_{\theta,x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

$\det(R_{\theta,x}) = 1[(\cos\theta)(\cos\theta) - (\sin\theta)(-\sin\theta)]$

$= \cos^2\theta + \sin^2\theta = 1$

$\therefore \boxed{\det(R_{\theta,x}) = 1}$

$R_{\theta,y} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

$\det(R_{\theta,y}) = \cos\theta(\cos\theta) + \sin\theta(0 - (-\sin\theta))$

$= \cos^2\theta + \sin^2\theta = 1$

$\therefore \boxed{\det(R_{\theta,y}) = 1}$