ME-639

Assignment-2

Harshil Sati
20110073

1.) To show that, $R\,S(a)\,R^T = S(Ra)$

~~Let~~ Let $b \in \mathbb{R}^3$ then we have,

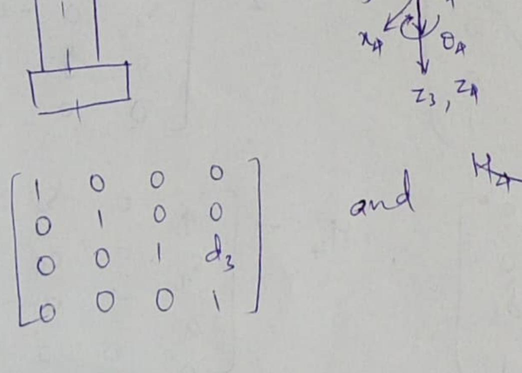
$$R\,S(a)R^Tb = R\,(a \times R^Tb)$$
$$= Ra \times RR^Tb \qquad [\because R \text{ is orthogonal}]$$
$$= (Ra)\times b$$
$$= S(Ra)b$$
$$\Rightarrow \boxed{R\,S(a)\,R^T = S(Ra)}$$

2.) Using DH Parameters for the SCARA (RRP config.) we have

Joint	a_i	d_i	α_i	θ_i
1	l_1	0	0	*
2	l_2	0	180	*
3	0	*	0	0
~~4~~	~~0~~	~~d_4~~	~~0~~	~~*~~

where * - joint variables

$$\therefore\ H_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & l_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$H_2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & l_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & l_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ and $$\cancel{H_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$P_0 = H\,P_3 = H_0^1 H_1^2 H_2^3 P_3$$

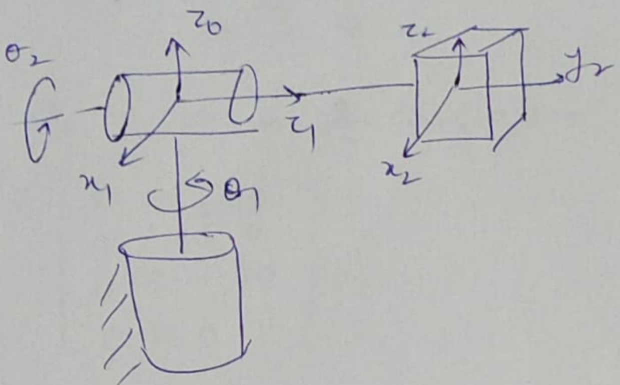
$$H = H_0^1 H_1^2 H_2^3$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & l_1c_1 \\ s_1 & c_1 & 0 & l_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & l_2c_2 \\ s_2 & c_2 & 0 & l_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1 + l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} c_{12} & -s_{12} & 0 & l_1c_1 + l_2c_{12} \\ s_{12} & c_{12} & 0 & l_1s_1 + l_2s_{12} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.) For Stanford type RRP configuration



$$\therefore H_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore H = H_1 H_2 H_3$$

$$= \begin{bmatrix} C_1 C_2 & -S_1 & C_1 S_2 & -d_2 S_1 \\ S_1 C_2 & C_1 & S_1 S_2 & d_2 C_1 \\ -S_2 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} C_1 C_2 & -S_1 & C_1 S_2 & d_3 C_1 S_2 - d_2 S_1 \\ S_1 C_2 & C_1 & S_1 S_2 & d_3 S_1 S_2 + d_2 C_1 \\ -S_2 & 0 & C_2 & d_3 C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5) We have to find p_0,

$\therefore p_0 = H_0^1 H_1^2 H_2^3 \, P_3$

where H_0^1 – 10 m translation ~~about~~ about z-axis

$$\therefore H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H_1^2 = 30° rotation about x-axis

$$\therefore H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 0 \\ 0 & 0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

H_2^3 – 60° rotation about z-axis

$$H_2^3 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.866 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore P_0 = H_0^1 H_1^2 H_2^3 P_3$$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ \cancel{0.749}\ 0.75 & 0.433 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore p_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \\ 1 \end{bmatrix}$$

$\Rightarrow$ Co-ordinates of obstacle in base frame is $(0, -1.5, 12.598)$

6.) Some types of gearbones are :

- spur gears - The most common and simple gears which are used to transmit rotational force. ~~to~~

- Planetary gears - It contains ~~an outer~~ a ring gear with one or more outer gears revolving around a central gear.

- Bevel gears - These are similar to spur gears except they are intended to transfer rotation ~~trow~~ through ~~90~~ a 90 degree translation

- Worm gears - Similar to bevel gear but allows high ratio between ~~ip~~ input and output.

7.) For a ~~scara~~ SCARA type robot, manipulator jacobian will be as follows

$$J = \begin{bmatrix} z_0 \times (o_4 - o_1) & z_1 \times (o_4 - o_1) & z_2 & 0 \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$

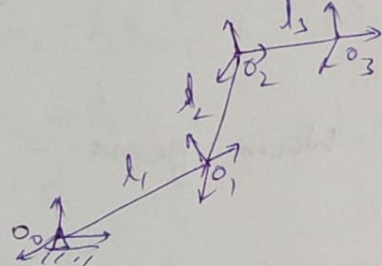
where $o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}$ $\quad o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$

$$o_4 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

9.) For a RRR configuration (planar)

Here, $o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}$



$$o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$$

$$o_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 \end{bmatrix} = \begin{bmatrix} a_{31} \\ a_{32} \\ 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \times (o_3 - o_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

As all 3 R joints, $z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\therefore z_0 \times (o_3 - o_0) = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ a_{31} & a_{32} & 0 \end{vmatrix} = \begin{bmatrix} -a_{32} \\ a_{31} \\ 0 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ l_2 c_{12} + l_3 c_{123} & l_2 s_{12} + l_3 s_{123} & 0 \end{vmatrix} = \begin{bmatrix} -l_2 s_{12} - l_3 s_{123} \\ l_2 c_2 + l_3 c_{123} \\ 0 \end{bmatrix}$$

$$z_2 \times (O_3 - O_2) = \begin{bmatrix} -l_3 s_{123} \\ l_3 c_{123} \\ 0 \end{bmatrix}$$

$$\therefore \quad J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_2 + l_3 c_{123} & + l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$