Assignment 2 18110066

Prasanna Venkatesh [ME] 2020-09-07

SOLUTION CHART

1. Let there be two orthonormal basis set (x_1, y_1, z_1) and (x_2, y_2, z_2).

Define the rotation matrix

$$R_1^2 = \begin{bmatrix} \hat{x}_1 \cdot \hat{x}_2 & \hat{x}_1 \cdot \hat{y}_2 & \hat{x}_1 \cdot \hat{z}_2 \\ \hat{y}_1 \cdot \hat{x}_2 & \hat{y}_1 \cdot \hat{y}_2 & \hat{y}_1 \cdot \hat{z}_2 \\ \hat{z}_1 \cdot \hat{x}_2 & \hat{z}_1 \cdot \hat{y}_2 & \hat{z}_1 \cdot \hat{z}_2 \end{bmatrix}$$

R_1^2 converts a vector p described in (x_1, y_1, z_1) basis to (x_2, y_2, z_2) basis.

$p^2 = R_1^2 \cdot p^1$

Let a, b, c be 3 vectors such that $c = a \times b$

When expressed in either basis - $c^1 = a^1 \times b^1$ and $c^2 = a^2 \times b^2$

Let $R(p) = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$

Using the rotation matrix,

$c^2 = a^2 \times b^2$

$\Rightarrow R_1^2 \cdot c^1 = (R_1^2 \cdot a) \times (R_1^2 \cdot b)$

$\Rightarrow R_1^2 \cdot (a \times b^1) = (R_1^2 \cdot a) \times R_1^2 \cdot b^1$

$\Rightarrow R_1^2 \cdot [R(a) \cdot b^1]^\vee = (R_1^2 \cdot a) \times R_1^2 \cdot b^1$

$\Rightarrow R_1^2 \cdot R(a) \cdot (R_1^2)^T \cdot R_1^2 \cdot b^1 = (R_1^2 \cdot a) \times R_1^2 \cdot b^1$

$\Rightarrow R_1^2 \cdot R(a) \cdot (R_1^2)^T \cdot b^2 = (R_1^2 \cdot a) \times b^2$

$\therefore b^2$ is an arbitrary vector,

$R_1^2 \cdot R(a) \cdot (R_1^2)^T = R(R_1^2 \cdot a)$

---

2. Given : RRP

DH parameters (θ, d, r, α)

J1 : $(\theta_1, 0, 0, 0)$

Homogeneous transform : $H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

J2 : $(\theta_2, 0, 0, 0)$

Homogeneous transform : $H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

J3 : $(0, \theta_3, 0, 0)$

Homogeneous transform : $H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\therefore P^0 = \begin{bmatrix} P^3 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot P^3 = H_0^3 \cdot \begin{bmatrix} P^3 \\ 1 \end{bmatrix}$

---

4. Stanford : RRP

DH parameters : (θ, d, r, α)

J1 : $(\theta_1, 0, 0, -\frac{\pi}{2})$

Homogeneous transform : $H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & 0 \\ 0 & \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

J2 : $(\theta_2, 0, 0, -\frac{\pi}{2})$

Homogeneous transform : $H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & 0 \\ 0 & \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

J3 : $(0, \theta_3, 0, 0)$

Homogeneous transform : $H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\therefore P^0 = \begin{bmatrix} P^3 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot P^3 = H_0^3 \cdot \begin{bmatrix} P^3 \\ 1 \end{bmatrix}$

5.

$P^3 = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$H_2^3 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} P^0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot \begin{bmatrix} P^3 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.75 & 0.433 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2.5 \\ 12.5 \\ 1 \end{bmatrix}$$

$\Rightarrow P^0 = \begin{bmatrix} 0 \\ -2.5 \\ 12.5 \end{bmatrix}$

---

6. (a) Planetary Gearbox

- High torque ratios in compact spaces
- Modular design - stages may be added to get desired torque.
- Multiple contacts using planets delivers higher efficiency.
- High input RPM

Cons -

- Heavy and bulky
- Backlash.

Applications : Central power delivery from motors

(b) Strain Wave Gearbox

- High torque to weight ratio
- Large torque transference.

Cons -

- Low efficiencies when load on effector changes frequently

Application : Joints close to end-effector.

(c) Cycloid Drives

- Less maintenance
- Withstand extremely high load / impact load

Cons -

- Low input RPM
- Backlash occurs

Applications : Robotic individual manipulators like cranes, CNC etc.

Ref: Gramisci P.L., Cycloid E., Gearless E., Vocabulator. T., Lapidus D (2008)

"Compact Gearbox for Modern Robotics : A Review"

Proceed. Robot. 81 3:105

Drives require extremely high speed motors (low torque) to yield large amounts of work. Pushing out required little torque, but the efficiency is positively related to the amount and speed of work. Electric motors typically have high speed and low torque, and gearboxes are attached to achieve high torque. Much more generally direct require gearboxes.

---

9. SCARA : RRP

$J = \begin{bmatrix} R_1 \times (O_3 - O_1) & R_2 \times (O_3 - O_2) & R_3 \\ R_1 & R_2 & 0 \end{bmatrix}$

$R_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad R_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$R_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \end{bmatrix}$

$R_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

To get O_3 : $H_0^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \cdot H_2^3 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = H_0^2 \cdot H_2^3 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

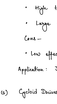
$$= H_0^2 \cdot \begin{bmatrix} l_2 \cos\theta_2 \\ l_2 \sin\theta_2 \\ q_3 \\ 1 \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ q_3 \\ 1 \end{bmatrix}$$

$\therefore O_3 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ q_3 \end{bmatrix}$

$S(R) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore J = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) & 0 \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

8.



$R_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \end{bmatrix}$

$R_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$O_2 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$

$R_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$O_3 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3) \\ 0 \end{bmatrix}$

$R_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$O_3 = O_3 = O_3$

$S(R_0) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$J = \begin{bmatrix} R_0 \times (O_3 - O_0) & R_1 \times (O_3 - O_1) & R_2 \times (O_3 - O_2) \\ R_0 & R_1 & R_2 \end{bmatrix}$

$$= \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) & -l_3 \sin(\theta_1+\theta_2+\theta_3) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) & l_3 \cos(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$