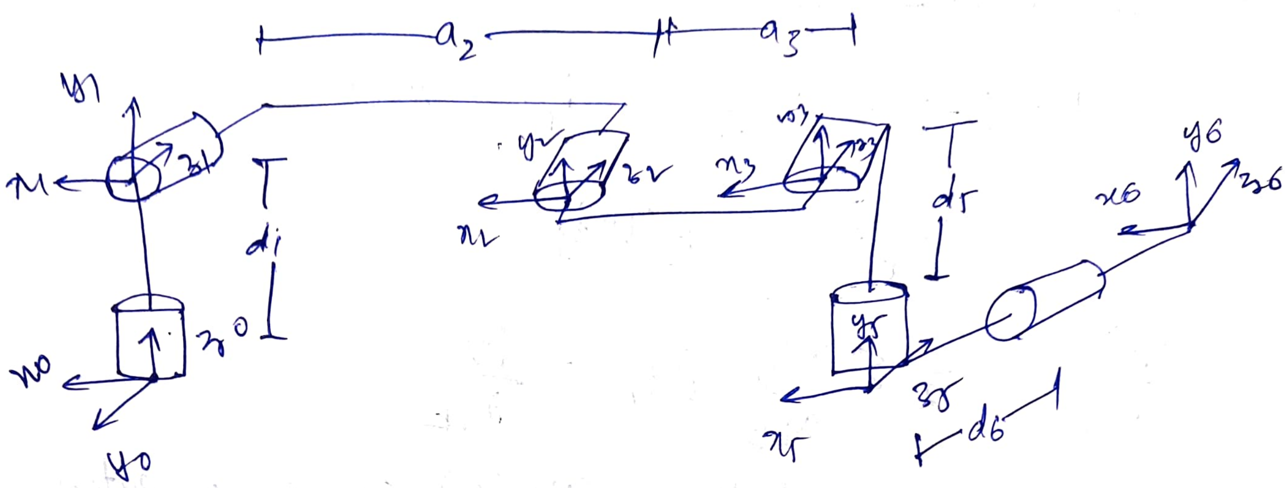
(12) For, a UR-5 robot which is also called as Universal robot.



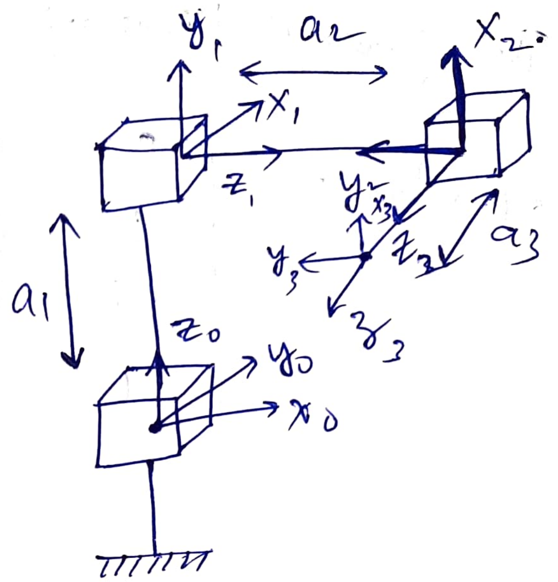
From the figure we can see,
6 links, 6 joints and all of them are revolute joints.

DH parameters for UR-5 Robot are.

Joint	θ	α	a	d
1	θ_1	$\pi/2$	0	0.089
2	θ_2	0	-425	0
3	θ_3	0	-0392	0
4	θ_4	$\pi/2$	0	0.1091
5	θ_5	$-\pi/2$	0	0.0945
6	θ_6	0	0	0.082

3.7 5

D-H parameters for the following Manipulator is.



θ	α	r	d
$\cancel{\theta_1}+90$	-90	0	a_1
$\cancel{\theta_2}+90$	-90	0	a_2
0	0	a_3	0

$$A = \begin{bmatrix} C\theta & -S\theta\, C\alpha & S\theta\, S\alpha & rC\theta \\ S\theta & C\theta\, C\alpha & -C\theta\, S\alpha & rS\theta \\ 0 & S\alpha & C\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematic equation is.

$$= A_1 A_2 A_3 = \begin{bmatrix} 0 & 1 & 0 & -a_2 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & +1 & -a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(7) Different types of configurations for 2R manipulator are :-

(1) Direct drive

→ Both the joints are driven by motors mounted at joints. for control we need to take care of the masses of the motor.

→ Some advantages of this are low friction, no backlash of motors and low compliance.

(2) Remotely driven

→ In this both the motors are driven by base motors.

→ advantages of this configuration is that the Coriolis forces are eliminated.

→ In this there is no need to consider the masses of motors while designing control.

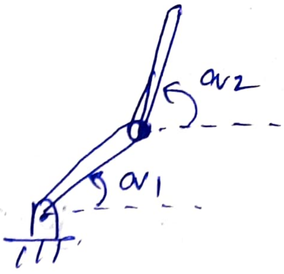
(3) 5-bar parallelogram

→ 5-bar parallelogram is a closed type of kinematic chain.

→ The equation of the manipulator are decayed so the q_1, q_2 are independently controlled.

→ only 2-DOF is possible.

(8) 2R - elbow manipulator :



q_1, q_2 are the joint varibles.

m_1, m_2 are the ~~mo~~ link mases

l_1, l_2 are the link lengths

$$V_{c_1} = \begin{bmatrix} -l_{c_1} \sin q_1 \\ l_{c_2} \cos q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -l_{c_2} \sin q_2 \\ l_1 \cos q_1 & + \; l_{c_2} \cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$w_1 = \dot{q}_1 k \;\; ; \;\; w_2 = \dot{q}_2 k$

the kinetic energy of the manipulator can be written as $K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$

$$D(q) = \begin{bmatrix} m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 & m_2 l_1 \cos(q_2 - q_1) \\ m_2 l_1 l_{c_2} \cos(q_2 - q_1) & m_2 l_{2c}^2 + I_2 \end{bmatrix}$$

the cristoff symbols are :-

$$C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1}$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2}$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

from there,

$$\phi_1 = (m_1 l_{c1} + m_2 l_1)\, g \cos q_1$$

$$\phi_2 = m_2 l_{c2}\, g \cos q_2$$

the final dynamic eq are :-

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

① · Let, Jacobian $J(q)$ matrix defines as.

$$x' = J(q)\,\dot{q}$$

$\dot{q}$ are the joint velocities.

$$\dot{x} = (v, \omega^T)$$

Now, we can write.

$$dx = J(q)\,dq$$

As jacobian is a function of q, the configurations for which the rank of J decreases. Such configurations are called singularities. or singular configurations.

$$\det J(q) = 0$$

$$J = [J_P \mid J_O] = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

$$J_o = \begin{bmatrix} z_3 \times (O_6 - O_3) & z_4 \times (O_6 - O_4) & z_4 \times (O_6 - O_5) \\ z_3 & z_4 & z_5 \end{bmatrix}$$

Since the wrist axes intersect at common point O

$O_3 = O_4 = O_5 = O$

$$J_o = \begin{bmatrix} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \times (O - O_{i-1}) \\ z_{i-1} \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

$$J = \begin{pmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{pmatrix}$$

$\det J = \det J_{11} \det J_{22}$

$J_{22} = [z_3 \; z_4 \; z_5]$

We can also check that if in a manipulator Jacobian, there are any rows with zero entries then it can be considered closed to singular configuration.

(10) From the Question we were provided with $D(q)$ and $V(q)$.

→ The euler-lagrangian eq. can be written as

$$L = L - V = \frac{1}{2} \sum_{i,j} d_{ij}(q)\dot{q}_i\dot{q}_j - V(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_k - \frac{\partial V}{\partial q_k}$$

Using Cristoffon symbols, $C_{ijk}(q)$ can be computed.

$$C_{kj} = \sum_{i=1}^{n} C_{ijk}(q)\dot{q}_i$$

$$= \sum_{r=1}^{n} \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot{q}_i$$

By using these d_i and C's we can get equations of motions for each link.