Assignment-3

Task-1: → Singularity is a point in robot workspace, where jacobian matrix looses its rank which means that certain controlled variables are unable to change the end-effector states.

→ At these points, certain directions of motion of end-effector may be unattainable. We can check singular configuration by checking rank of manipulator jacobian at a particular configuration. Set of singular configurations is the union of arm configurations satisfying $\det J_{11} = 0$ and $\det J_{22} = 0$.

→ Yes, we can detect if a particular configuration is close to a singular configuration using Manipulator Jacobian.

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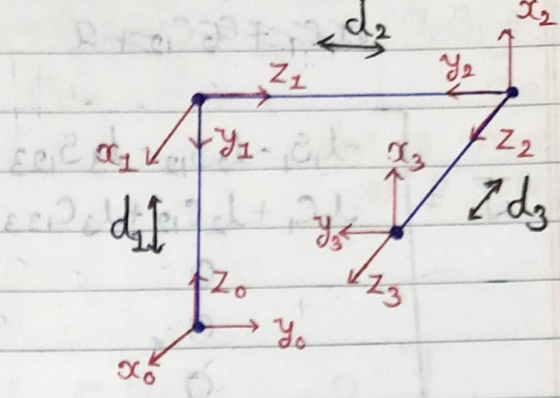
Task-5 3-link cartesian manipulator

link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	d_1	0
2	0	$-\pi/2$	d_2	$-\pi/2$
3	0	0	d_3	0



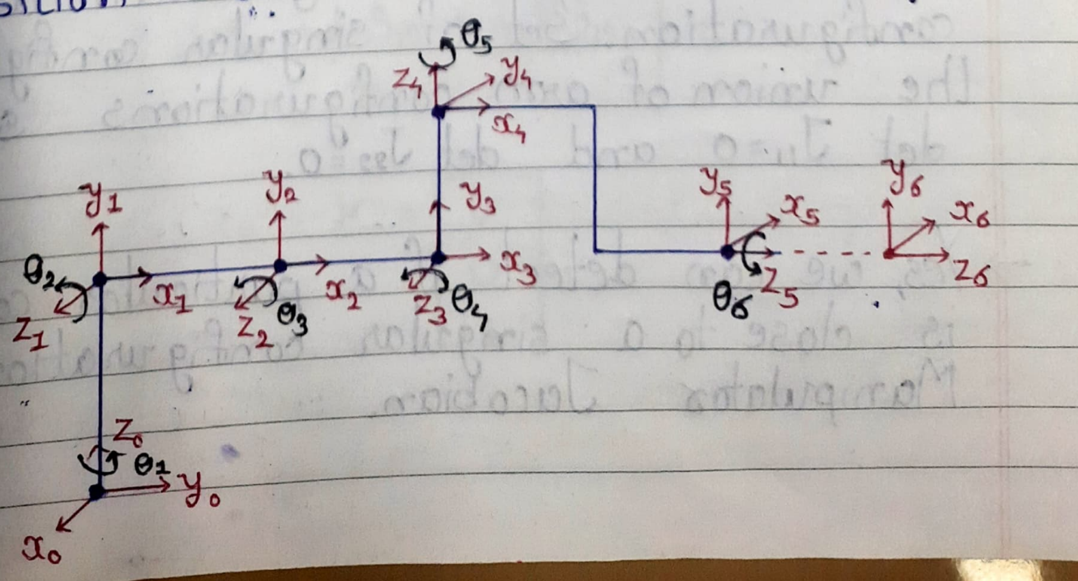
$$A = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_0^1 A_1^2 A_2^3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ position of end-effector ⇒ (d_3, d_2, d_1). in inertial frame

Task-6



link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$-\pi/2$	d_4	θ_4
5	a_5	$\pi/2$	d_5	θ_5
6	a_6	0	0	θ_6

$$A = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_1^2 = \begin{bmatrix} C_{\theta_2} & S_{\theta_2} & 0 & a_2C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & a_2S_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^2 = \begin{bmatrix} C_{\theta_1} & C_{\theta_1}S_{\theta_2} & S_{\theta_1} & a_2C_{\theta_1}C_{\theta_2} \\ S_{\theta_1}C_{\theta_2} & S_{\theta_1}S_{\theta_2} & C_{\theta_1} & a_2S_{\theta_1}C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^3 = \begin{bmatrix} C_{\theta_3} & S_{\theta_3} & 0 & a_3C_{\theta_3} \\ S_{\theta_3} & C_{\theta_3} & 0 & a_3S_{\theta_3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^4 = \begin{bmatrix} C_{\theta_4} & 0 & -S_{\theta_4} & 0 \\ S_{\theta_4} & 0 & -C_{\theta_4} & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^4 = \begin{bmatrix} C_{\theta_3}C_{\theta_4}+S_{\theta_3}S_{\theta_4} & 0 & -C_{\theta_3}S_{\theta_4}-S_{\theta_3}C_{\theta_4} & a_3C_{\theta_3} \\ S_{\theta_3}C_{\theta_4}+C_{\theta_3}S_{\theta_4} & 0 & -S_{\theta_3}S_{\theta_4}-C_{\theta_3}C_{\theta_4} & a_3S_{\theta_3} \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^5 = \begin{bmatrix} C_{\theta_5} & 0 & S_{\theta_5} & a_5C_{\theta_5} \\ S_{\theta_5} & 0 & C_{\theta_5} & a_5S_{\theta_5} \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5^6 = \begin{bmatrix} C_{\theta_6} & 0 & 0 & a_6C_{\theta_6} \\ S_{\theta_6} & 0 & 0 & a_6S_{\theta_6} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^6 = \begin{bmatrix} C_{\theta_5}C_{\theta_6} & 0 & 0 & a_6C_{\theta_5}C_{\theta_6} + a_5C_{\theta_5} \\ S_{\theta_5}C_{\theta_6} & 0 & 0 & a_6S_{\theta_5}C_{\theta_6} + a_5S_{\theta_5} \\ S_{\theta_6} & 0 & 0 & a_6S_{\theta_6} + d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_0^6 = A_0^2 \cdot A_2^4 \cdot A_4^6}$$

Task-7 ⇒ 3 different configuration of 2R- manipulator.

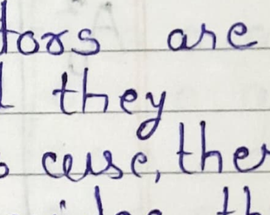
(i) Direct Drive 2R Manipulator:-

In this configuration, actuators are mounted directly at joint. In this robot, problems of backlash, friction & compilance due to gears are eliminated. This configuration has controls terms in equation of motion which is little complex. Dynamics of motor is also tricky. They are also coupled equations.

(ii) Remotly Driven 2R Manipulator :-

In this configuration, actuators are not directly mounted on links and they won't be moving with links. In this case, there is amplification of inertia. But it provides the easier dynamics of motor. The configuration doesn't have coriolis term in EOM but they are coupled. The angle produced at each joint can be easily calculated but calculation of trajectory gets sligtly complicated.

(iii) 5 bar parallelogram arrangement:



In this arrangement, there are 5 joints and 2 grounded joints that are attached to motor. 5 bar linkage provides coriolis ~~force~~ free & decoupled equation of motion. Both joints can be calculate (control) independently.

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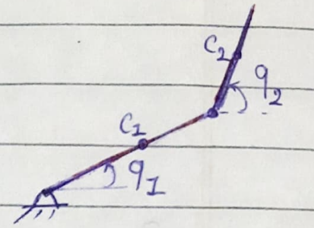
controlling these 2 motor would help in moving the tip through entire workspace. This arrangement sustain higher load input.

Task-8



$$V_{C_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix}, \quad V_{C_2} = \begin{bmatrix} -l_1 S q_1 & -\frac{l_2}{2} S q_2 \\ l_1 C q_1 & l_2/2\, C q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}$$

$$\omega_1 = \dot q_1 \hat k \qquad \omega_2 = \dot q_2 \hat k$$

$$K = \frac{1}{2}\sum_{i=1}^{n} m_i V_{C_i}^T V_{C_i} + \frac{1}{2}\sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

where $V_{C_i} = J_{V_{C_i}}(q)\dot q \qquad \omega_i = R_i^T J_{\omega_i}(q)\dot q$

$$\therefore K = \frac{1}{2}\dot q^T \sum_{i=1}^{n}\left[m_i J_{V_{C_i}}(q)^T J_{V_{C_i}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q)\right]\dot q$$

$$K = \frac{1}{2}\dot q^T D(q)\dot q$$

→ For 2R Manipulator

$$D(q) = \begin{bmatrix} m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & m_2\frac{l_2^2}{4} + I_2 \end{bmatrix}$$

→ Computing christoffel symboles.

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$\hookrightarrow C_{111} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_1} + \frac{\partial d_{11}}{\partial q_1} - \frac{\partial d_{11}}{\partial q_1}\right] = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial q_1}\right)$$

$$= \frac{1}{2}\frac{\partial}{\partial q_1}\left(m_1\frac{l_1^2}{4} + m_2 l_1^2 + I_1\right)$$

$$C_{111} = 0$$

$$\cancel{C_{211}} = \cancel{C_{121}} \quad C_{221} = \frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1}\right] = \cancel{\tfrac{1}{2}}\,\partial$$

$$= \frac{\partial}{\partial q_2}\left(m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1)\right) - \frac{1}{2}\frac{\partial}{\partial q_1}\left(\frac{m_2 l_2^2}{4} + I_2\right)$$

$$= -m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$\cancel{C_{221}} = \cancel{\frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1}\right]}$$

$$= \cancel{\frac{\partial}{\partial q_2}\left(m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1)\right) - \frac{1}{2}\frac{\partial}{\partial q_1}\frac{m_2 l_2^2}{4}}$$

$$C_{211} = C_{121} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_1}\right] = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$= \frac{1}{2}\frac{\partial}{\partial q_2}\left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)$$

$$= 0$$

$$C_{112} = \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_1} + \frac{\partial d_{21}}{\partial q_1} - \frac{\partial d_{11}}{\partial q_2}\right]$$

$$= \frac{\partial}{\partial q_1}\left(m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1)\right) - \frac{1}{2}\frac{\partial}{\partial q_2}\left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)$$

$$= m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{122} = C_{212} = \frac{1}{2}\left[\frac{\partial d_{22}}{\partial q_1} + \frac{\partial d_{21}}{\partial q_2} - \frac{\partial d_{12}}{\partial q_2}\right]$$

$$= \frac{1}{2}\frac{\partial}{\partial q_1}\left(\frac{m_2 l_2^2}{4} + I_2\right)$$

$$= 0$$

$$C_{222} = \frac{1}{2}\left[\frac{\partial d_{22}}{\partial q_2} + \frac{\partial d_{22}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_2}\right] = 0$$

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→ We have potential energy as

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = \frac{m_1 g l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2} \cos q_2$$

→ We have $\sum_{j=1}^{2} d_{kj}(q) \ddot{q}_j + \sum_{ij} C_{ijk}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k$ $\quad K = 1,2$

$$\tau_1 = d_{11}(q) \ddot{q}_1 + d_{12}(q) \ddot{q}_2 + \cancelto{0}{C_{111}}(q) \dot{q}_1 \dot{q}_1 + \cancelto{0}{C_{121}}(q) \dot{q}_1 \dot{q}_2 + \cancelto{0}{C_{211}}(q) \dot{q}_2 \dot{q}_1 + C_{221}(q) \dot{q}_2 \dot{q}_2 + \phi_1(q)$$

$$\boxed{\tau_1 = d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 + C_{221} \dot{q}_2^2 + \phi_1(q)}$$

$$\rightarrow \tau_2 = d_{21}(q) \ddot{q}_1 + d_{22}(q) \ddot{q}_2 + C_{112}(q) \dot{q}_1 \dot{q}_1 + \cancelto{0}{C_{122}}(q) \dot{q}_1 \dot{q}_2 + \cancelto{0}{C_{212}}(q) \dot{q}_2 \dot{q}_1 + \cancelto{0}{C_{222}}(q) \dot{q}_2 \dot{q}_2 + \phi_2(q)$$

$$\boxed{\tau_2 = d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + C_{112} \dot{q}_1^2 + \phi_2(q)}$$

Task-10 → given $D(q)$ and $V(q)$

① We know $D(q)$ matrix and its elements as d_{ij} hence, we can compute christoffel symbols.

$$C_{ijk} = \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right]$$

② We have $V(q)$ equation and computing variable $\phi_k(q)$

$$\left[\phi_k(q) = \frac{\partial V}{\partial q_k} \right]$$

③ Putting above values in euler-Lagrange's eq^n below,

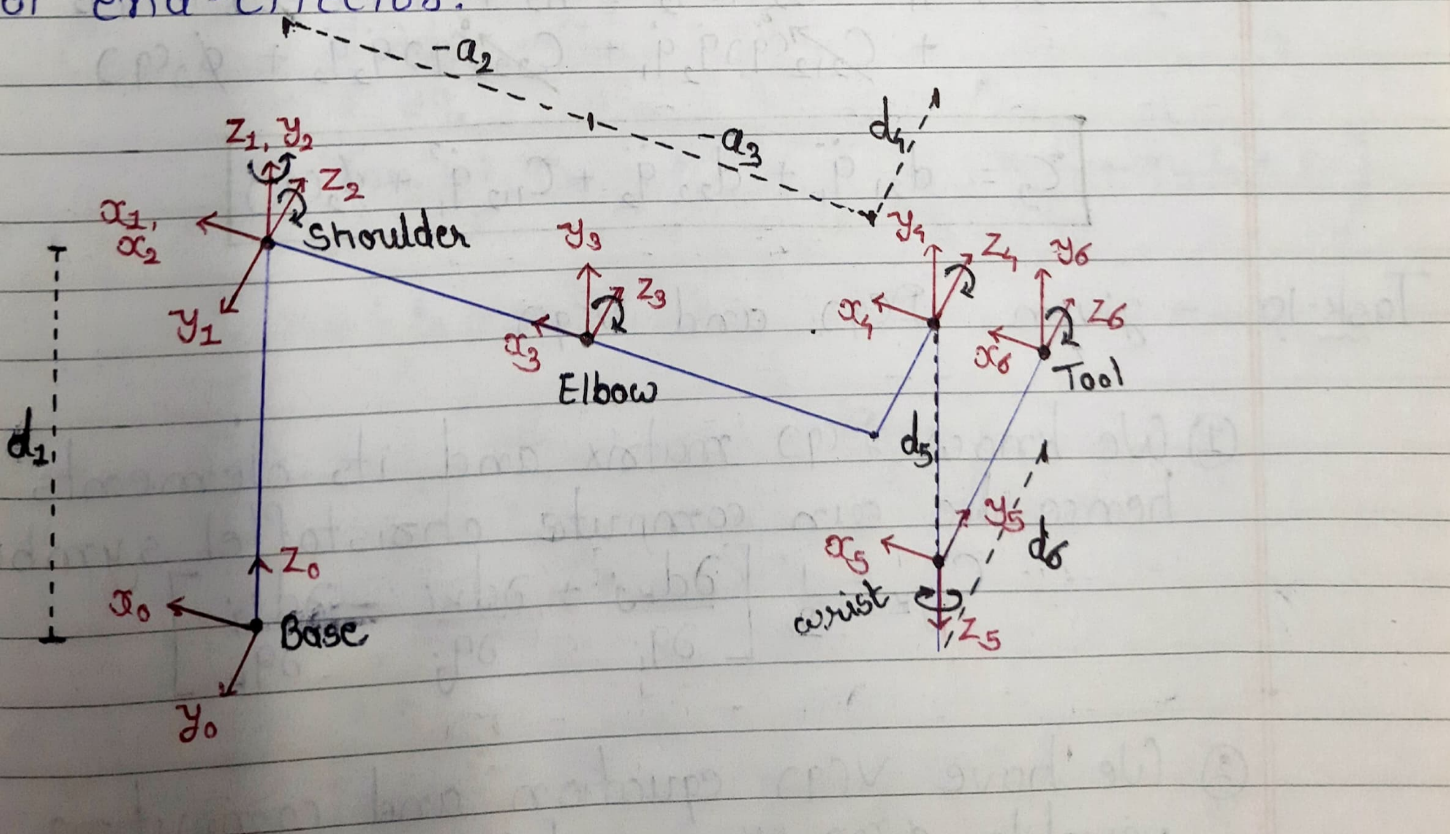
$$\sum_{j=1}^{2} d_{kj}(q)\ddot{q}_j + \sum_{ij} C_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

④ More comman to write as

$$\left[D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau\right]$$ ... all n equation together in matrix Form.

Task-12 → The UR5 robot has 6 DoFs, and it is composed of 6 links connected by revolute(R) joints.

→ Its kinematics is similar to an anthropomorphic arm, with the noticeable difference that the last 3 R joints are not arranged in a spherical wrist fashion, so that all 6 joints contribute to both the translational and rotational motion of end-effector.



i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	0	$\pi/2$	0	θ_2
3	a_2	0	0	θ_3
4	a_3	0	d_4	θ_4
5	0	$\pi/2$	d_5	θ_5
6	0	$-\pi/2$	d_6	θ_6

$a_i \Rightarrow$ distance from Z_i to Z_{i+1} measured along X_i

$\alpha_i \Rightarrow$ angle from Z_i to Z_{i+1} measured about X_i

$d_i \Rightarrow$ distance from X_{i-1} to X_i measured along Z_i

$\theta_i \Rightarrow$ angle from X_{i-1} to X_i measured about Z_i