Ans 1 :-

1. Jacobian is function of configuration q.
when jacobian matrix is rank-deficient
we say configuration is singular.

2. Singular configuration can be identified
when →

a) Bounded gripper velocities correspond to unbounded joint velocities ≈ (gripper F vs τ)

b) certain direction of motion or force application not possible from a given configuration.

c) Points on the boundaries of workspace usually correspond to singular config.

d) Near singularities inverse kinematic solution fails. (no solution or ∞ly many solutions.)

3. Rank (Jacobian Matrix) < max (6, n)
n is number of joints.
is condition for singularity.

Q 2 Done.

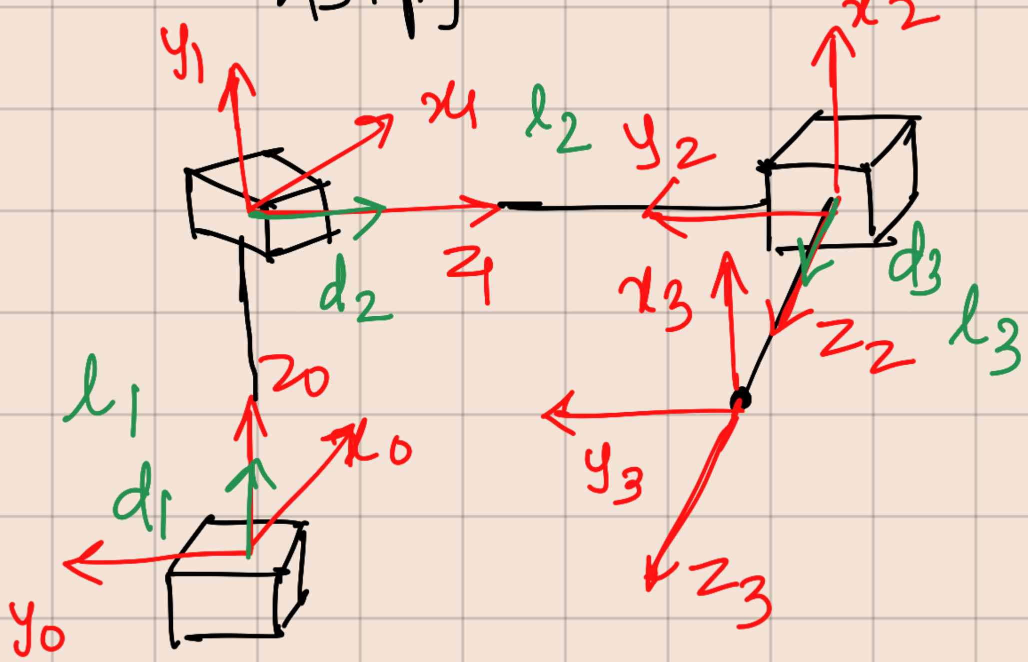
Q3 a) ✓ A3.py

b) ✓

c) ✓

Q4 ✓ A3.py

Q5.



i	d	a	α	θ
1	$l_1 + d_1^*$	0	$+90^\circ$	0
2	$l_2 + d_2^*$	0	-90°	90°
3	$l_3 + d_3^*$	0	0	0

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Using eq (1)

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 + d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_2 + d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_3 + d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = A_1 A_2 A_3$$

$$A = \begin{bmatrix} 0 & 0 & -1 & -(l_3 + d_3^*) \\ 0 & 1 & 0 & -(l_2 + d_2^*) \\ 1 & 0 & 0 & l_1 + l_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q6.

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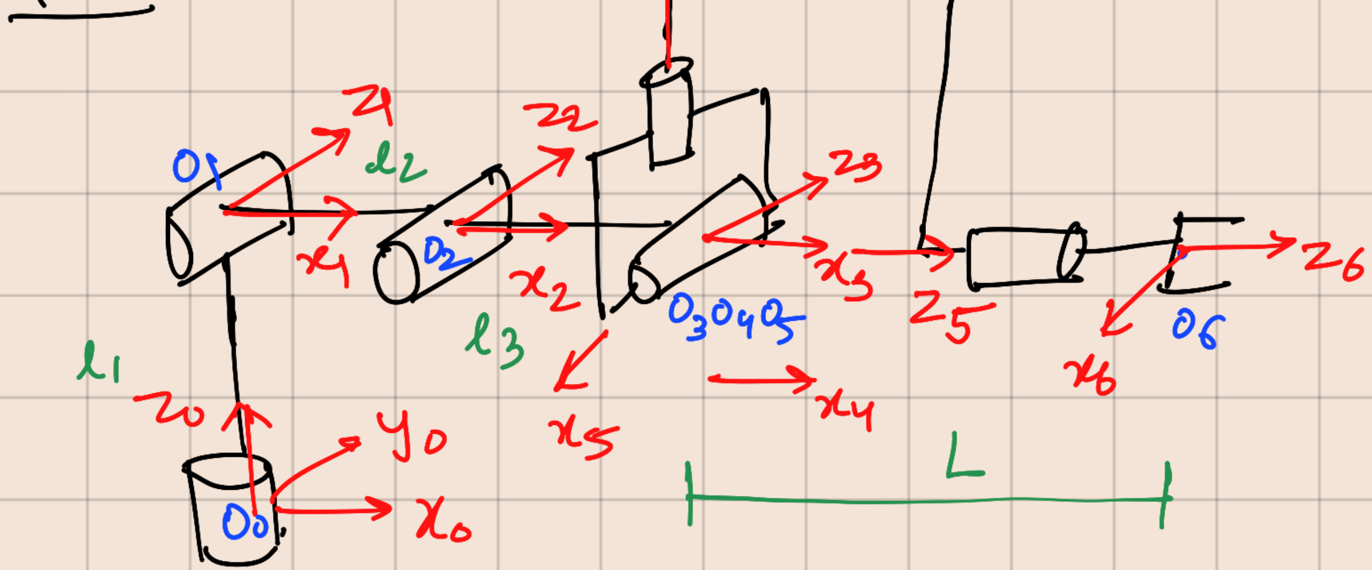


i	d	a	α	θ
1	l_1	0	$-90°$	θ_1^*
2	0	l_2	0	θ_2^*
3	0	l_3	0	θ_3^*
4	0	0	$90°$	θ_4^*
5	0	0	$90°$	$-\theta_5^*$
6	L	0	0	θ_6^*

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & l_2 c\theta_2 \\ c\theta_2 & c\theta_2 & 0 & l_2 s\theta_2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & l_3 \, c\theta_3 \\ c\theta_3 & c\theta_3 & 0 & l_3 \, s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c\theta_5 & 0 & -s\theta_5 & 0 \\ -s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = A_1 A_2 A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} x \end{bmatrix}$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = A(1:3,4)$$

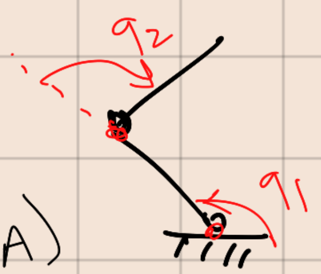
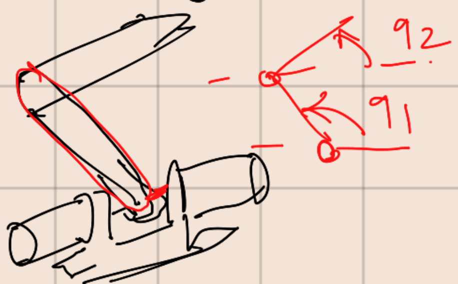
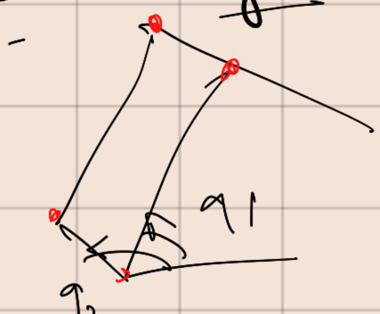
$$x = l_2 c\theta_2 - L(c\theta_5\, c\theta_2\, s\theta_3 + c\theta_3\, s\theta_2 + c\theta_4\, s\theta_5 (c\theta_2 c\theta_3 - s\theta_2 s\theta_3 + l_3 c\theta_2 c\theta_3 - l_3 s\theta_2 s\theta_3)$$

$$= l_2 c\theta_2 - L(c\theta_5 c\theta_2 c\theta_3 + c(\theta_3+\theta_2) + l_3 c(\theta_2+\theta_3) + c\theta_3 c\theta_2 + c\theta_4 s\theta_5 c\theta_2 c\theta_3.)$$

$$y = -L s\theta_4 s\theta_5$$

$$z = l_1 - L(c\theta_5\, c\theta_2 c\theta_3 - s\theta_2 s\theta_3 - c\theta_4 s\theta_5 \, s\theta_3\, c\theta_2 + c\theta_3 s\theta_2) - l s\theta_2 - l_3 c\theta_2\, s\theta_3 - l_3 c\theta_3\, s\theta_2$$

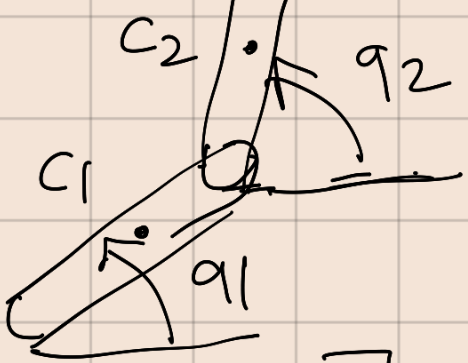
7.)

Direct drive	Remotely Driven	5 bar linkage.
		
A) 1. motors directly attached	A) 1. remotely driven link motor also	A) motors mounted at base same

to joints. 2. Motors near base have to carry additional weight of motors near endeffector. 3. Costlier high torque motors or gearbox required.	at base. hence lower torque requirement. Cheaper. Remote link is driven by belt pulley.	as remotely driven.
B) Coriolis component appear in dynamics	No coriolis components in dynamics	No coriolis components.
c) relative angle measured.	absolute angle	absolute angle.
D) coupled dynamics	coupled dynamics	decoupled dynamics

Q8

Q8



$$J_{vc_1} = \begin{bmatrix} -\frac{\ell_1}{2}\sin q_1 \\ \frac{\ell_1}{2}\cos q_1 \\ 0 \end{bmatrix} \qquad J_{vc_2} = \begin{bmatrix} -\ell_1 \sin q_1 & -\frac{\ell_2}{2} s q_2 \\ \ell_1 c q_1 & \frac{\ell_2}{2} c q_2 \\ 0 & 0 \end{bmatrix}$$

$$v_{c_1} = J_{vc_1}\dot{q} \qquad v_{c_2} = J_{vc_2}\dot{q}$$

$$v_{c_1} = \begin{bmatrix} -\frac{\ell_1}{2}\sin q_1 \\ \frac{\ell_1}{2}\cos q_1 \\ 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad \omega_1 = \dot{q}_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\omega_2 = \dot{q}_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v_{c_2} = \begin{bmatrix} -\ell_1 s q_1 & -\frac{\ell_2}{2} s q_2 \\ \ell_1 c q_1 & \frac{\ell_2}{2} c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$D(q) = \begin{bmatrix} m_1 \frac{\ell_1}{2}^2 + m_2 \ell_1^2 + I_1 & m_2 \ell_1 \frac{\ell_2}{2} c(q_2 - q_1) \\ m_2 \ell_1 \ell_1 \cos(q_2 - q_1) & m_2 \ell_2^2 + I_2 \end{bmatrix}$$

$$\left[m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \qquad m_2 \frac{l_2^2}{4} + I_2 \right]$$

$$c_{ijk} = \frac{1}{2}\left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\}$$

$$c_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 \frac{l_2}{2} s(q_2 - q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2 l_1 \frac{l_2}{2} s(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2 \right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = \left(m_1 \frac{l_1}{2} + m_2 l_1 \right) g \cos q_1$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 \frac{l_2}{2} g \cos q_2$$

$\partial q_2 \quad - \quad \frac{2}{2} g \cos q_2$

Dynamic equations :→

$$\left(m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 +$$

$$m_2 \left(l_1 \frac{l_2}{2}\right) \cos(q_2 - q_1)\, \ddot{q}_2 +$$

$$- m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)\, \dot{q}_2^2 +$$

$$\left(m_1 \frac{l_1}{2} + m_2 l_1\right) g \cos q_1 = \tau_1$$

$$m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1)\, \ddot{q}_1 + \left(m_2 \frac{l_2^2}{4} + I_2\right) \ddot{q}_2$$

$$+ m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)\, \dot{q}_1^2 +$$

$$m_2 \frac{l_2}{2} g \cos q_2 = \tau_2$$

9.) Done

10.) If D and V is given,

find

$$c_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$$

$$\phi_k = \frac{\partial V}{\partial q_k}$$

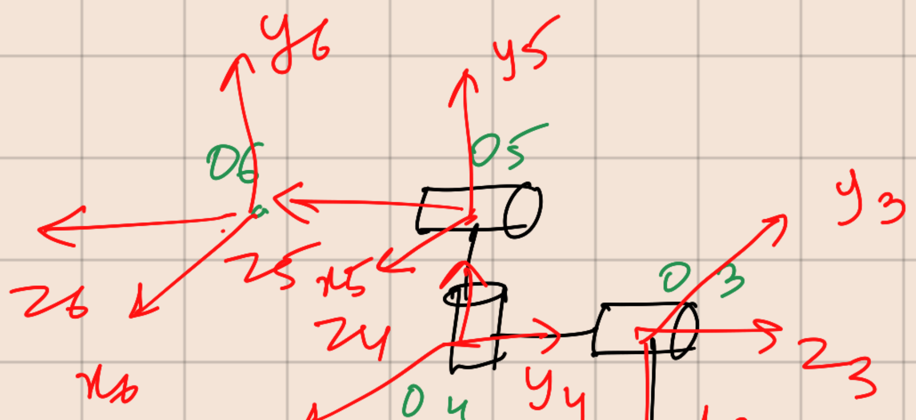
Equation of motion is then given by,

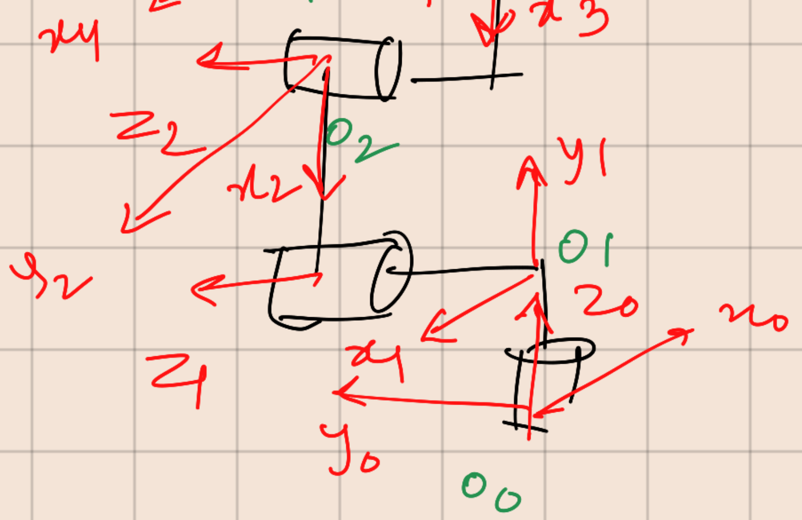
$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

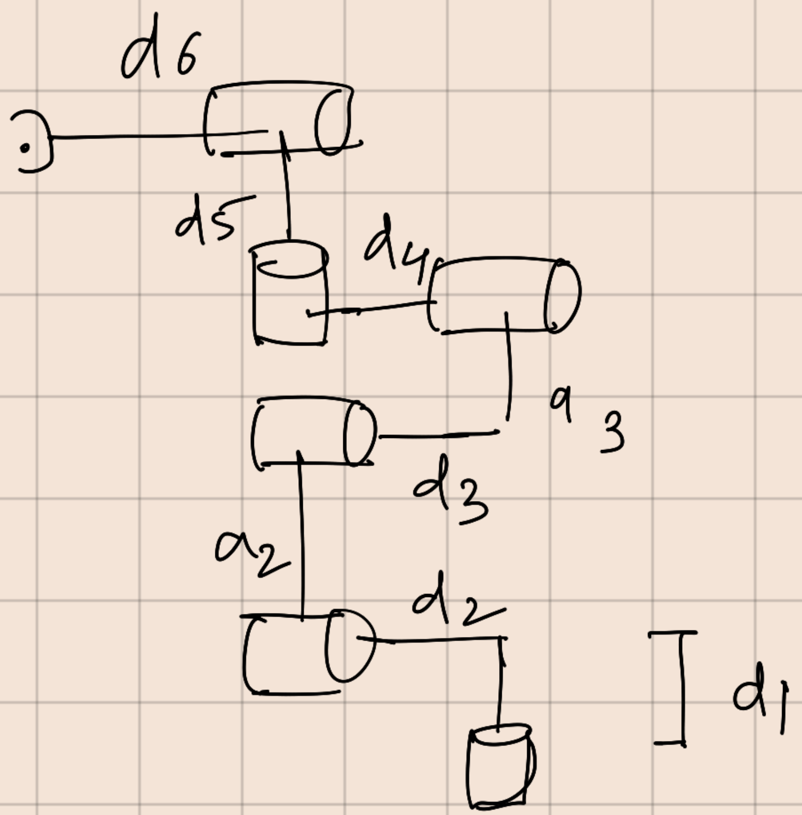
$$k = 1, 2 \cdots n.$$

11.) dynamics.py

12.) UR5 ROBOT







i	d	a	α	θ
1	d_1	0	90°	$180^\circ + \theta_1^*$
2	d_2	$-a_2$	0°	$-90^\circ + \theta_2^*$
3	$-d_3$	$-a_3$	180°	θ_3^*
4	d_4	0	90°	$-90^\circ + \theta_4^*$
5	d_5	0	90°	θ_5^*
6	d_6	0	0	θ_6^*