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Assignment 3 - ME639

References: 1. Classnotes of figures. 2. Robot Dynamics and Control textbook.

1.

let the Jacobian $J(q)$ mapping be defines as -

$$\dot{x} = J(q)\dot{q}$$

between the $\dot{q}$ and $\dot{x} = (v, \omega)^T$ of end effector velocities. on infinitesimally small scale this above equation will be a Linear transformation -

$$dx = J(q)dq$$

The Jacobian is a function of the configuration q, those configurations for which the rank of J decreases are of Special Significance.

let suppose that $n=6$, that is the manipulator consists of a 3 DOF arm with a 3 DOF Spherical wrist. In this case the Jacobian is a 6×6 matrix and a configu-ration q is singular if and only if

$$\det J(q) = 0$$

$$J = [J_P \mid J_O] = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

then, Since the final three joints are always revolute

$$J_o = \begin{bmatrix} z_3 \times (o_6 - o_3) & z_4 \times (o_6 - o_4) & z_4 \times (o_6 - o_5) \\ z_3 & z_4 & z_5 \end{bmatrix}$$

Since the wrist axes intersect at a common point o, if we choose the co-ordinate frames so that $o_3 = o_4 = o_5 = o_6 = o$, then J_o becomes

$$J_o = \begin{bmatrix} 0 & 0 & 0 \\ z_3 & z_4 & z_5 \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \times (o - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}$$

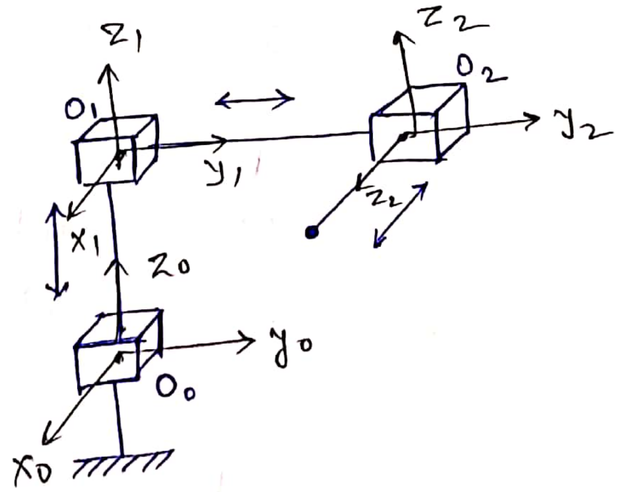
$$\det J = \det J_{11} \det J_{22}$$

Where, J_{11} and J_{22} are each 3x3 matrices. J_{11} has i-th column $z_{i-1} \times (o - o_{i-1})$ if joint i is revolute, and z_{i-1} if joint i is prismatic, while

$$J_{22} = [\, z_3 \; z_4 \; z_5 \,]$$

2. Reading assignment.

5.



Link	d_i	a_i	α_i	θ_i
1	d_1	0	$\pi/2$	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*
4				

$$A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1} \times 0 & S_{\theta_1} \cdot 1 & 0 \\ S_{\theta_1} & C_{\theta_1} \times 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & 0 \\ S_{\theta_2} & C_{\theta_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & 0 \\ S_{\theta_3} & C_{\theta_3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Cont...

$$T_0^3 = A_1 A_2 A_3$$

We can calculate the ~~foor~~ward kinematics equation using above equation.

6.

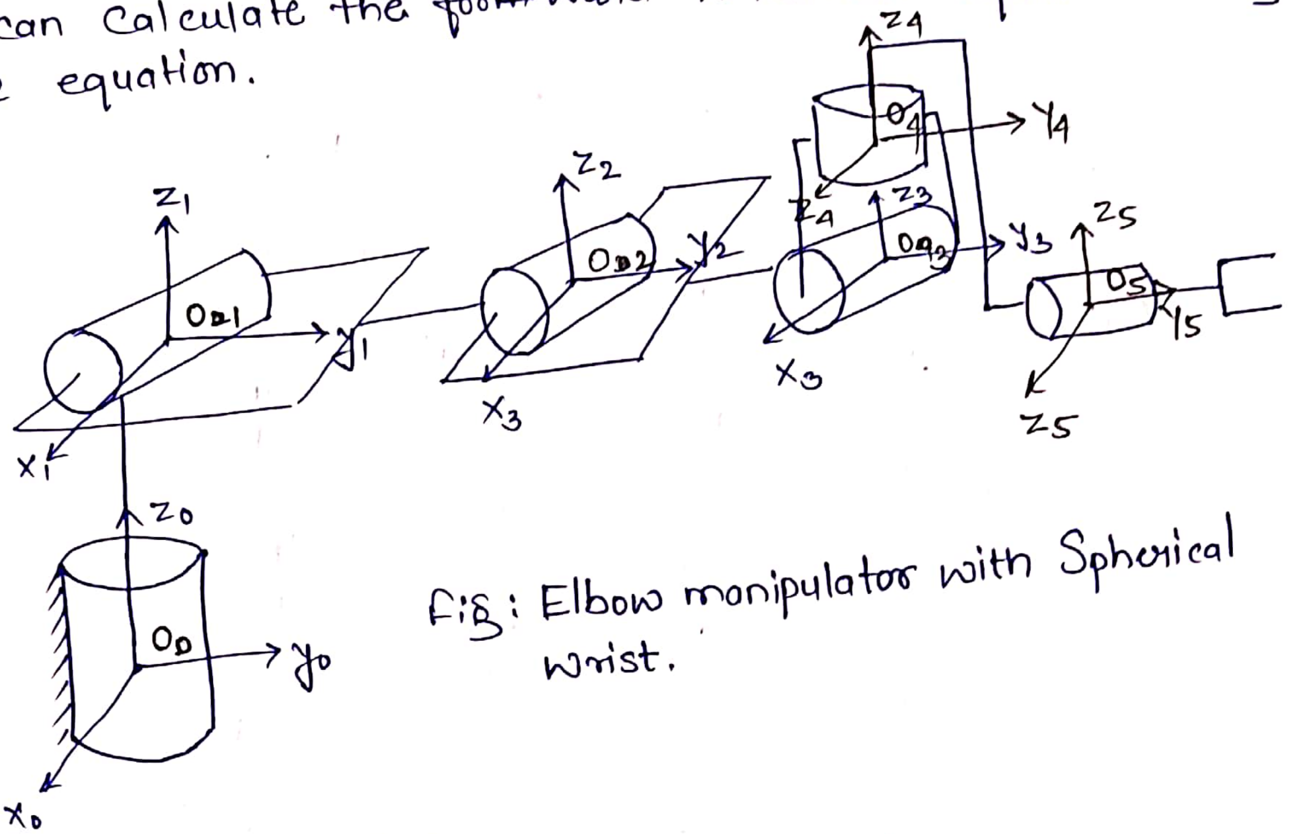


fig: Elbow manipulator with Spherical wrist.

D-H parameter Table:

Link	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	d_1	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$-\pi/2$	0	θ_4^*
5	0	$\pi/2$	0	θ_5^*
6	0	0	d_6	θ_6^*

Each homogenous transformation A_i is represented as a product of four "basic" transformations

$$A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1}(0) & S_{\theta_1} & (0)\cdot C_{\theta_1} \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & a_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & a_2 S_{\theta_2} \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_{\theta_3^*} & 0 & -S_{\theta_3^*} & 0 \\ S_{\theta_3^*} & 0 & -C_{\theta_3^*} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_{\theta_4^*} & 0 & -S_{\theta_4^*} & 0 \\ S_{\theta_4^*} & 0 & -C_{\theta_4^*} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

10.

The kinetic energy is a quadratic function of the vector $\dot{q}$ of the form -

$$K = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{q}^T D(q)\dot{q}$$

The Euler-Lagrange equations for such a system can be derived as follows.

$$L = K - V = \frac{1}{2}\sum_{i,j} d_{ij}(q)\dot{q}_i\dot{q}_j - V(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\dot{q}_j$$

and,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}(q)\dot{q}_j$$

$$= \sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

also,

$$\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum \frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_k - \frac{\partial V}{\partial q_k}$$

Thus the Euler-Lagrange equations can be written

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial d_k}\right\}\dot{q}_i\dot{q}_k - \frac{\partial V}{\partial q_k} = \tau_k$$

$$k = 1, \dots, n$$

$$c_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$$

$$\phi_k = \frac{\partial V}{\partial q_k}$$

$$\sum_j d_{kj}(q)(\ddot{q}_j) + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

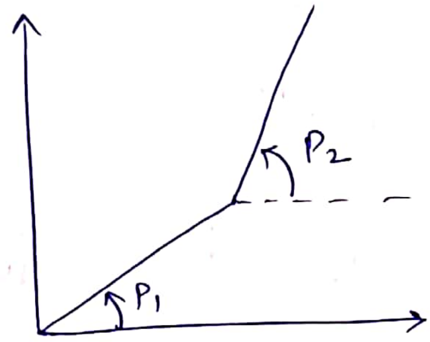
$$k = 1, 2, 3, \dots, n$$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$C_{kj} = \sum_{i=1}^{n} c_{ijk}(q)\dot{q}_i$$

8.

Planar Elbow manipulator with Remotely Driven Link.



P_1 and P_2 are not the joint angles used earlier, we cannot use the velocity Jacobians derived in Chapter.

$$V_{c1} = \begin{bmatrix} -l_{c1}\sin P_1 \\ l_{c2}\cos P_1 \\ 0 \end{bmatrix} \dot{P}_1$$

$$V_{c2} = \begin{bmatrix} -l_1\sin P_1 & -l_{c2}\sin P_2 \\ l_1\cos P_1 & l_{c2}\cos P_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \end{bmatrix}$$

$$\omega_1 = \dot{P}_1 k, \quad \omega_2 = \dot{P}_2 k$$

Hence, the kinetic energy of the manipulator equals

$$K = \dot{P}^T D(P) \dot{P}$$

where,
$$D(P) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_1^2 + I_1 & m_2 l_1 l_{c2}\cos(P_2 - P_1) \\ m_2 l_1 l_{c2}\cos(P_2 - P_1) & m_2 l_{2c}^2 + I_2 \end{bmatrix}$$

Computing the christoffel Symbols –

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial P_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial P_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial P_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial P_1}$$

$$= -m_2 l_1 l_{c2} \sin(P_2 - P_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial P_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial P_2}$$

$$= m_2 l_1 l_{c2} \sin(P_2 - P_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial P_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial P_2} = 0$$

The potential energy of the manipulator, in terms of P_1 and P_2 equals

$$V = m_1 g l_{c1} \sin P_1 + m_2 g (l_1 \sin P_1 + l_{c2} \sin P_2)$$

Hence, $$\phi_1 = (m_1 l_{c1} + m_2 l_1) g \cos P_1$$

$$\phi_2 = m_2 l_{c2} g \cos P_2$$

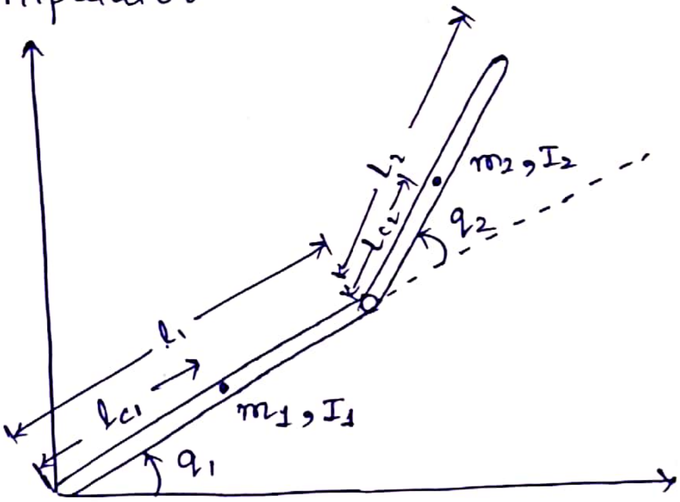
finally, the dynamic equations are –

$$d_{11}\ddot{P}_1 + d_{12}\ddot{P}_2 + C_{221}\dot{P}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{P}_1 + d_{22}\ddot{P}_2 + C_{112}\dot{P}_1^2 + \phi_2 = \tau_2$$

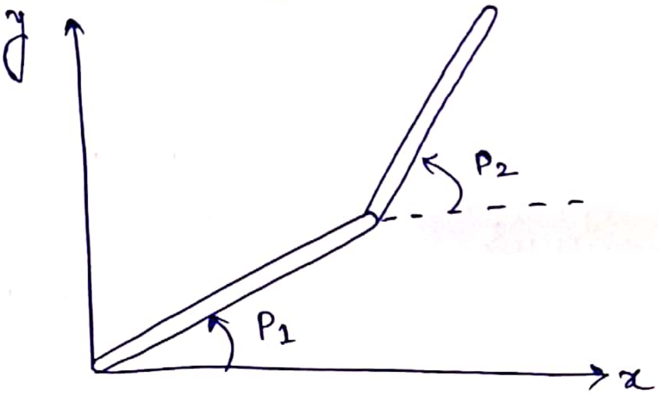
87.

(1) 2R Manipulator direct drive.



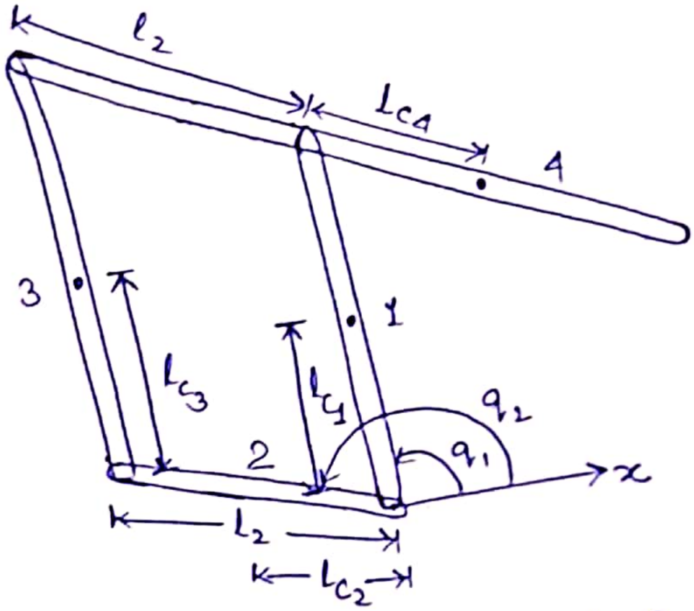
Planar Elbow manipulator both joints are driven by motors mounted at the joints. For Control we need to take care of the masses of the motors.

(2) Planar Elbow manipulator with remotely driven link.



Both joints are driven by motors mounted at the base. The first joint is turned directly by one of the motors, While other is turned via a gearing mechanism or a timing belt. No need to consider the masses of motors while designing Control.

(3) 5-bar parallelogram arrangement

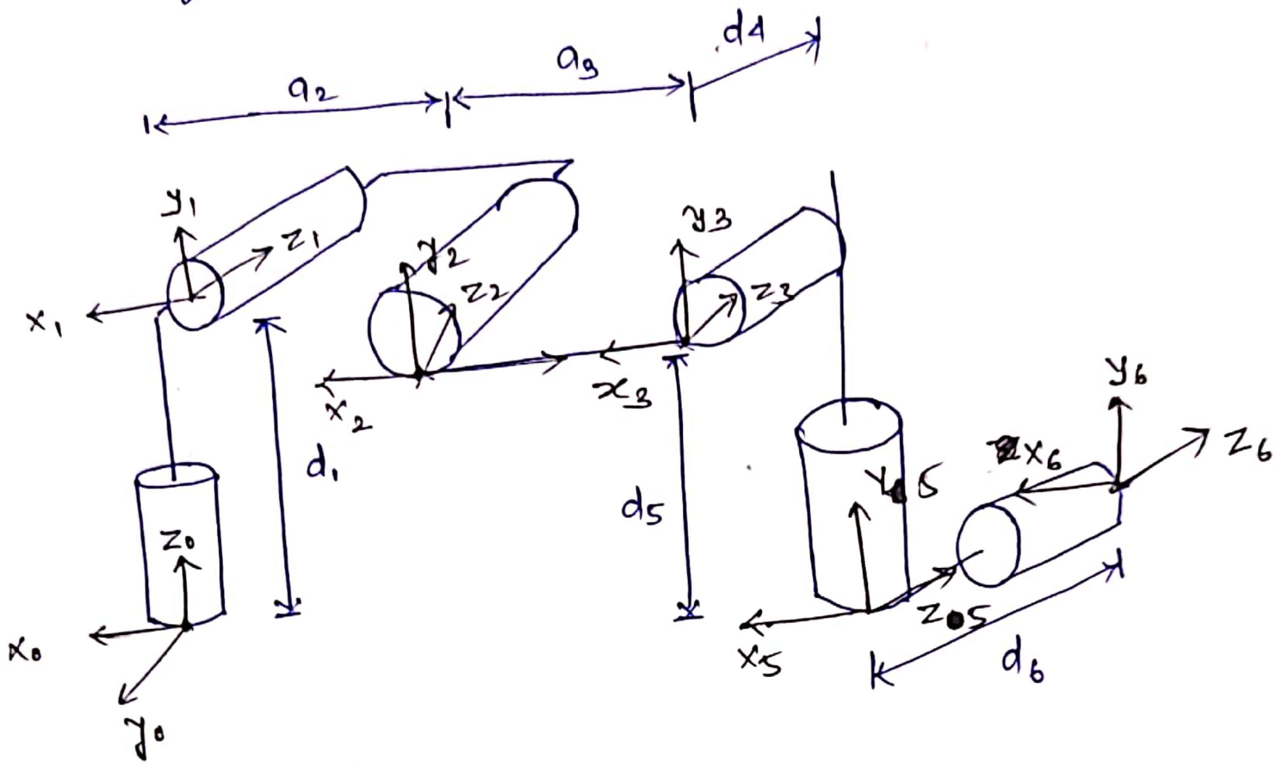


5-bar parallelogram arrangment is one of the closed kinematic chain (through of a particularly simple kind). So the equation of Jacobian matrices can not be used as of other direct driven or remotely driven 2R manupulator.

Only 2 Degree of freedom is possible instead of having ~~4~~ ~~4~~ 5 link arrangment.

12. UR5 Robot

Number of links : 6
Number of joints : 6
Nature of joints : Revolute joints



Joint	a	α	d	θ
1	0	$\pi/2$	0.089	θ_1
2	-0.42	0	0	θ_2
3	-0.39	0	0	θ_3
4	0	$\pi/2$	0.109	θ_4
5	0	$-\pi/2$	0.094	θ_5
6	0	0	0.082	θ_6

D-H parameters.

As with any 6-DOF robot, the homogenous transformation from the base frame to the gripper can be defined as follows:

$$T_6^0(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = T_1^0(\theta_1)\,T_2^1(\theta_2)\,T_3^2(\theta_3)\,T_4^3(\theta_4)\,T_5^4(\theta_5) \times T_6^5(\theta_6)$$

Also, remember that a homogenous transformation T_j^i has the following form:

$$T_j^i = \begin{bmatrix} R_j^i & \vec{P_j^i} \\ 0 & 1 \end{bmatrix}$$

$$T_j^i = \begin{bmatrix} x_x & y_x & z_x & (P_j^i)_x \\ x_y & y_y & z_y & (P_j^i)_y \\ x_z & y_z & z_z & (P_j^i)_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$