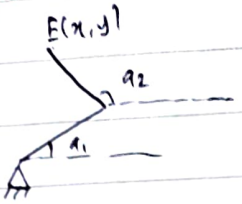
Miniproject derivation



$$x = l_1\cos q_1 + l_2\cos q_2 = l_1 c q_1 + l_2 c q_2$$
$$y = l_1\sin q_1 + l_2\sin q_2 = l_1 s q_1 + l_2 s q_2$$
$$\text{---------} \quad (1)$$

Differentiate

$$\dot{x} = -l_1 s q_1 \dot{q_1} - l_2 s q_2 \dot{q_2}$$
$$\dot{y} = l_1 c q_1 \dot{q_1} + l_2 c q_2 \dot{q_2}$$

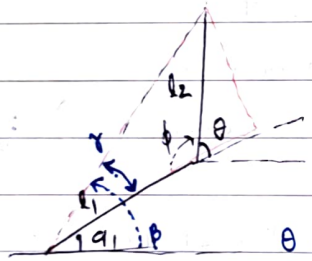
Cartesian space / Task space — Joint space

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \end{bmatrix} \quad \text{---------} \quad (2)$$

This much is not enough for T_1

Inrerse kinematics option1: numerically

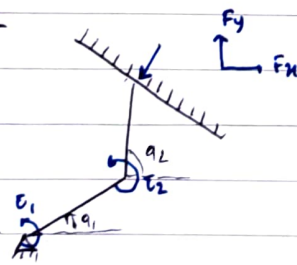
option2: Derive closed-form expressions



$$q_2 = q_1 + \theta$$
$$l_2^2 + l_1^2 - 2 l_1 l_2 \cos\phi = h^2$$
$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

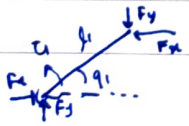
$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right) \quad \text{---------} \quad (3)$$

T2

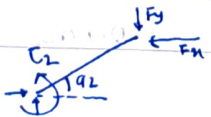


static equilibrium

FBD of link 1



FBD of link 2



$\tau_1 = -F_x l_1 s q_1 + F_y l_1 c q_1$

$\tau_2 = -F_x l_2 s q_2 + F_y l_2 c q_2$

Task space

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 - l_1 c q_1 \\ -l_2 s q_2 \quad + l_2 c q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \text{Joint space} \quad \text{---------} \quad (4)$$

TS

Feed $F_x = k(x - x_0) = k(l_1 c q_1 + l_2 c q_2 - x_0)$ ---- (5)

$F_y = k(y - y_0) = k(l_1 s q_1 + l_2 s q_2 - y_0)$

Next level of T_1 & T_3

Need to account for dynamics

Lagrange's Equations

Lagrangian : $\mathcal{L} = K - V$

kinetic energy, potential energy

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i$$

Q_i : generalized forces

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

pure rotation of link 1 — rotation of link 2 abt its CoM — translation of CoM of link 2

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - \frac{m_2 l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ \frac{m_1 g l_1}{2} c q_2 + m_2 g l_1 c q_2 \quad *$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$$

(6)

Tasks

$$\tau_{1s} + \tau_1 = \tau_{1\,apply}$$

$$\tau_{2s} + \tau_2 = \tau_{2\,apply}$$