Assignment - 3.

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1.) <u>Singularity</u> - may mean different things in the Context of inverse kinematics. A few of them are listed below -

(i) Configurations from which motion is not possible along a particular direction.

(ii) For a particular end effector velocities, there are no to infinitely many joint velocities possible.

(iii) Hence, there will not exist a unique solution to inverse kinematics at singular configurations.

<u>Finding singular Configurations</u> -

One of the methods is to decouple the wrist and manipulator singularities.

For instance, if we consider a 6 D.O.F freedom, its jacobian can be partitioned as below.-

$$J = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

where J_{11} will be 3×3 matrix describing the manipulator end point velocities & joint velocities relation, J_{12} will similarly describe those for wrist

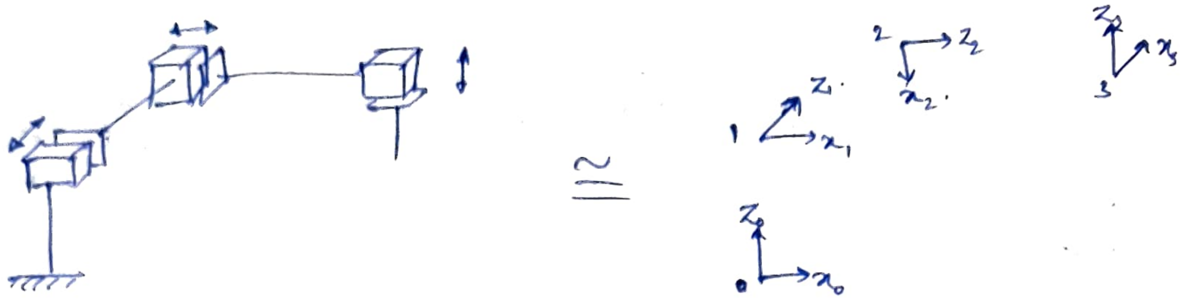
Typically, $J_{12} = [Z_3 (O_6 - O_3) \;\; Z_4 \times (O_6 - O_4) \;\; Z_5 \times (O_6 - O_5)]$

Since O_6, O_3, O_4 and O_5 coincide at single point (assumption),

$$J_{12} = 0$$

Hence, $|J| = \left|\begin{array}{c|c} J_{11} & 0 \\ \hline J_{21} & J_{22} \end{array}\right| = J_{11} \cdot J_{22} = 0$ would give the desired configurations.

(5) Consider the 3-link Cartesian manipulator. Derive forward kinematic equations using D-H Convention.



3R

The DH parameters for the above Configuration can be written as –

	α	a	d	θ
link 1	$-\pi/2$	l_1	d_1	0
link 2	$\pi/2$	l_2	d_2	$\pi/2$
link 3	$-\pi/2$	l_3	d_3	$\pi/2$

DH $\to$ T = $\begin{bmatrix} C\theta & -S\theta C\alpha & S\theta S\alpha & aC\theta \\ S\theta & C\theta C\alpha & -C\theta S\alpha & aS\theta \\ 0 & S\alpha & C\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$

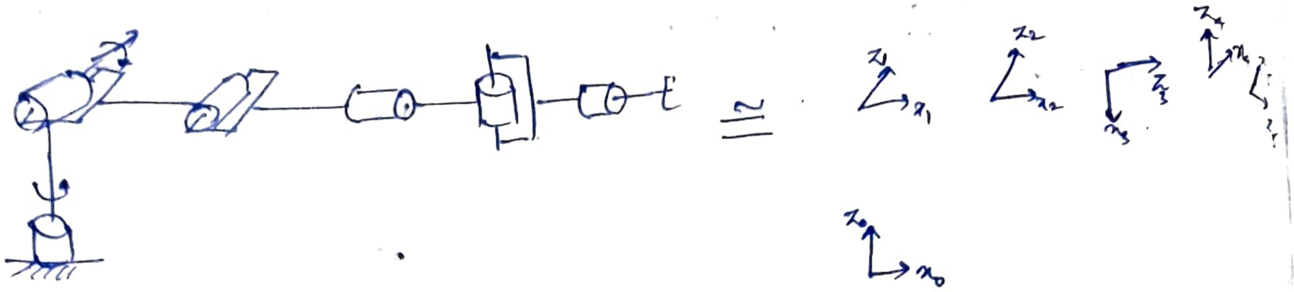
$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = T_0^1 T_1^2 T_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & l_1 \\ 0 & -1 & 0 & -d_2 \\ 1 & 0 & 0 & l_2+d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & d_3+l_1 \\ -1 & 0 & 0 & -(l_3+d_2) \\ 0 & 0 & 1 & l_2+d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(6.) Attached spherical wrist to a 3-link articulated manipulator. Derive the forward kinematics equations.



The D-H parameters for the above configuration can be written as:-

	α	a	d	θ
link 1	$-\pi/2$	l_1	0	θ_1
link 2	0	l_2	0	θ_2
link 3	$\pi/2$	l_3	0	$\theta_3 + \frac{\pi}{2}$
link 4	$-\pi/2$	l_4	0	$\theta_4 + \pi/2$
link 5	$\pi/2$	l_5	0	θ_5
link 6	0	l_6	0	θ_6

Considering the wrist centers to coincide,

$\Rightarrow l_4 = l_5 = l_6 = 0$.

$$\text{D-H } T_n = \begin{bmatrix} C\theta & -S\theta C\alpha & S\theta S\alpha & aC\theta \\ S\theta & C\theta C\alpha & -C\theta S\alpha & aS\theta \\ 0 & S\alpha & C\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} C_1 & 0 & S_1 & l_1C_1 \\ S_1 & 0 & -C_1 & l_1S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^3 = \begin{bmatrix} -S_3 & 0 & C_3 & -l_3S_3 \\ C_3 & 0 & +S_3 & l_3C_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} -S_4 & 0 & -C_4 & 0 \\ C_4 & 0 & -S_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4^5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5^6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4 \, T_4^5 \, T_5^6$$

$$= \underbrace{\begin{bmatrix} C_1 & 0 & S_1 & l_1C_1 \\ S_1 & 0 & -C_1 & l_1S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} -S_3 & 0 & C_3 & -l_3S_3 \\ C_3 & 0 & S_3 & l_3C_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -S_4 & 0 & -C_4 & 0 \\ C_4 & 0 & -S_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -S_2C_1 & S_1 & l_2C_2C_1 + l_1C_1 \\ S_1C_2 & -S_2S_1 & -C_1 & l_2C_2S_1 + l_1S_1 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} -C_1 S_{23} & S_1 & C_1 C_{23} & l_3 C_1 S_{23} + l_2 C_2 C_1 + l_1 C_1 \\ -S_1 S_{23} & -C_1 & S_1 S_{23} & l_3 S_1 S_{23} + l_2 C_2 S_1 + l_1 S_1 \\ C_{23} & 0 & S_{23} & l_3 C_{23} + l_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{end position without wrist}} \times \underbrace{\begin{bmatrix} S_4 C_5 C_6 - C_4 C_6 & S_6 S_4 C_5 - C_4 C_6 & -S_4 S_5 & 0 \\ C_4 C_6 C_5 - S_4 S_6 & -S_6 C_4 C_5 - S_4 C_6 & C_4 S_5 & 0 \\ S_5 C_6 & -S_6 S_5 & -C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{wrist}}$$

(7.) Compare the 3 different Configurations for 2R manipulator

1. Direct drive – Has high driving torque and accurate positioning abilities. However, the motion of robot is strongly affected by inertial forces and non-linear forces.

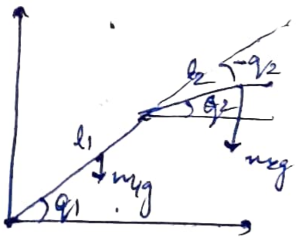
2. Remotely driven – though the inertial forces are dealt with there is lower accuracy and hence the need to ~~tak~~ incorporate sensors and feedback system.

3. 5-bar parallelogram arrangement –
Closed loop mechanism with motor mounted at base. The external load gets distributed among the links and hence has high load bearing capacity.

(8.) 2 link Planar manipulator.

Computing the jacobian for elbow manipulator, the linear velocity, V_{c1} of link 1 is given as

$$V_{c1} = J_{vc1} \dot{q}.$$



from geometry J_{vc1},

$$J_{vc1} = \begin{bmatrix} -l_{c1}\sin q_1 & 0 \\ l_{c1}\cos q_1 & 0 \\ 0 & 0 \end{bmatrix}$$ and

$$V_{c2} = J_{vc2} \dot{q}, \text{ where}$$

$$J_{vc2} = \begin{bmatrix} -l_1 \sin q_1 + -l_{c2}\sin(q_1+q_2) & -l_{c2}\sin(q_1+q_2) \\ l_1 \cos q_1 + l_{c2}\cos(q_1+q_2) & l_{c2}\cos(q_1+q_2) \\ 0 & 0 \end{bmatrix}$$

Translation part of the kinetic energy,

$$\frac{1}{2}m_1 V_{c1}^T V_{c1} + \frac{1}{2}m_2 V_{c2}^T V_{c2} = \frac{1}{2}\dot{q}\left\{ m_1 J_{vc1}^T J_{vc1} + m_2 J_{vc2}^T J_{vc2} \right\}\dot{q}$$

Considering the rotation of links & angular velocity,

$$\omega_1 = \dot{q}_1 k \qquad \omega_2 = (\dot{q}_1 + \dot{q}_2)k.$$

Total rotational K.E,

$$\frac{1}{2}\dot{q}^T \left\{ I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\} \dot{q}.$$

Calculating $D(q)$,

$$D(q) = m_1 J_{vc1}^T J_{vc1} + m_2 J_{vc2}^T J_{vc2} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

upon substituting and simplifying,

$$d_{11} = m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2}^2 + 2l_1 l_{c2} \cos q_2) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_2) + I_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2$$

Now calculating Christoffel symbols,

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2} \sin q_2 = h$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h.$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$$

$$C_{122} = C_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

The potential energy of links,

$$P_1 = m_1 g l_{c1} \sin q_1$$

$$P_2 = m_2 g (l_1 \sin q_1 + l_{c2} \sin(q_1 + q_2))$$

$$P = P_1 + P_2$$

&

$$\phi_1 = \frac{\partial P}{\partial q_1} = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2)$$

$$\phi_2 = \frac{\partial P}{\partial q_2} = m_2 l_{c2} \cos(q_1 + q_2)$$

Substituting in

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{121}\dot{q}_1\dot{q}_2 + C_{211}\dot{q}_2\dot{q}_1 + C_{221}\dot{q}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

(10) Steps to derive equations of motion when $D(q)$ and $V(q)$ are given.

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1. From $D(q)$, derive all possible christoffel symbols using the below mentioned -

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right].$$

2. From $V(q)$, derive all ϕs, with respect to each joint angle. The below mentioned formula can be used -

$$\phi_k(q_k) = \frac{\partial V}{\partial q_k}.$$

3. These values can be substituted in the below mentioned to obtain the dynamic equations of motion.

$$\sum_{j=1}^{n} d_{kj}\ddot{q}_j + \sum_{i,j} C_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k.$$

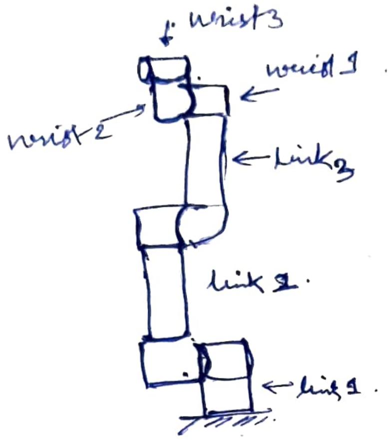
(k equations of k D.O.F system).

Assignment 3.

12

0) UR5 robot — is a 6R manipulator, including the 3 revolute joints in the wrist of the manipulator.
If we ignore the 3Rs which describe the orientation, UR5 is a 3R robot

1)



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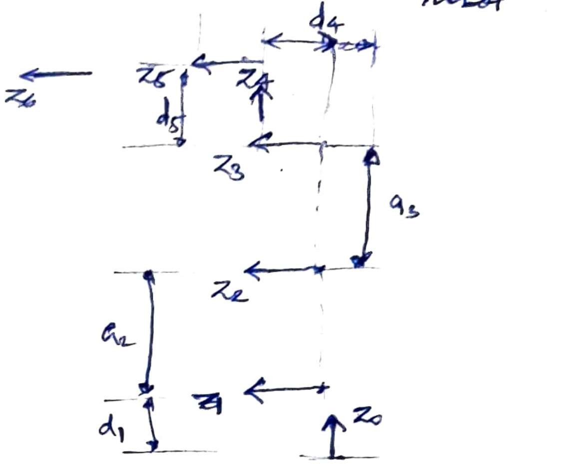

1) Arbitrary Config of UR5

2) Selection of Z-axes

3) D-H parameters

link	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	d_1	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
Wrist 1	$\pi/2$	0	d_4	θ_4
Wrist 2	$-\pi/2$	0	d_5	θ_5
Wrist 3	0	0	ϕ	θ_6

4) Joint Transformation matrices

we know D-H is given as below,

$$H = \begin{bmatrix} C_i & -S_i C_{\alpha i} & S_i S_{\alpha i} & C_i a_i \\ S_i & C_i C_{\alpha i} & -C_i S_{\alpha i} & S_i a_i \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Substituting the D-H parameters for every link, we get

For link 1,

$$H_0^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 2,

$$H_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 3,

$$H_2^3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 4 / wrist 1,

$$H_3^4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 5 / wrist 2,

$$H_4^5 = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For link 6 / wrist 3,

$$H_5^6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.) Total Transformation matrix from base frame to end effector frame.

$$H_0^5 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6$$

$$= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times$$

$$\begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & a_2c_2c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & a_2c_2s_1 \\ s_2 & c_2 & 0 & a_2s_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & a_3C_1(C_{23})+a_2C_2C_1 \\ S_1C_{23} & -S_1S_{23} & -C_1 & a_3S_1C_{23}+a_2C_2S_1 \\ S_{23} & C_{23} & 0 & a_3S_{23}+a_2S_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_{234} & S_1 & C_1S_{234} & S_1d_4+a_3C_1C_{23}+a_2C_2C_1 \\ S_1C_{234} & -C_1 & S_1S_{234} & -C_1d_4+a_3S_1C_{23}+a_2C_2S_1 \\ S_{234} & 0 & -C_{234} & a_3S_{23}+a_2S_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_5C_{234}+S_1S_5 & C_1S_{234} & C_1C_{234}S_5+S_1C_5 & C_1S_{234}d_5+S_1d_4+a_3C_1C_{23}+a_2C_2C_1 \\ S_1C_{234}C_5-C_1S_5 & S_1S_{234} & S_1S_5C_{234}-C_1C_5 & S_1S_{234}d_5-C_1d_4+a_3S_1C_{23}+a_2C_2S_1 \\ C_5S_{234} & -C_{234} & S_5S_{234} & -C_{234}d_5+a_3S_{23}+a_2S_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^6 = \begin{bmatrix} (C_1C_5C_{234}+S_1S_5)C_6 + C_1S_{234}S_6 & -(C_1C_5C_{234}+S_1S_5)S_6 + C_1S_{234}C_6 & C_1C_{234}S_5 + S_1C_5 & C_1S_{234}d_5+S_1d_4+a_3C_1C_{23}+a_2C_2C_1 \\ (S_1C_{234}C_5-C_1S_5)C_6 + S_1S_{234}S_6 & -(S_1C_{234}C_5-C_1S_5)S_6 + S_1S_{234}C_6 & S_1S_5C_{234}-C_1C_5 & S_1S_{234}d_5 - C_1d_4 + a_3S_1C_{23}+a_2C_2S_1 \\ C_5S_{234}C_6 - C_{234}S_6 & -C_5S_{234}S_6 - C_{234}C_6 & S_5S_{234} & -C_{234}d_5+a_3S_{23}+a_2S_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is the required transformation matrix.