ITR - Assignment - 2

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Ans-1 To derive this, we will use two important properties of a Skew-symmetric matrix.

① $S(a) \times b = a \times b$ (where a & b are vectors)

② $R(a \times b) = Ra \times Rb$ (where R is an orthogonal matrix (in this case, a Rotationtional matrix), a & b are vectors)

Proof of ①:

Let $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$$S(a) \times b = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$= \begin{bmatrix} b_3a_2 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \underline{a \times b}$$

Proof of 2 is not given. It is directly used from the textbook.

Now consider a term,

$\Rightarrow \quad R\,S(a)\,R^T\,b$: a & b are vectors
S is skew symmetric matrix
R is Rotational matrix

$= R\left(a \times (R^T b)\right)$ → using eq^n 1

$= \left(Ra \times R(R^T b)\right)$ → using eq^n 2

$= Ra \times b$ → $(RR^T = I$, orthogonality of $R)$

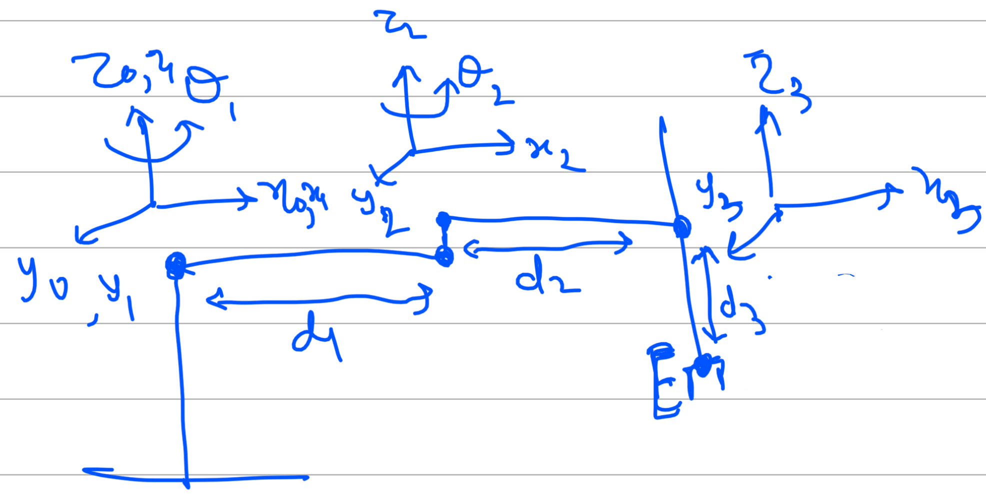
$= S(Ra)\,b$ → using eq^n 1 again

$\therefore \quad R\,S(a)\,R^T\,b = S(Ra)\,b$

$$\therefore \quad \boxed{R\,S(a)\,R^T = S(Ra)}$$

hence proved.

Ans - 2   SCARA RRR :-



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} d_1 \\ 0 \\ 0 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} d_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = R_{z,\theta_1}$$

$$R_1^2 = R_{z,\theta_2}$$

$$R_2^3 = I \quad (\text{Identity matrix})$$

$$H_0^1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

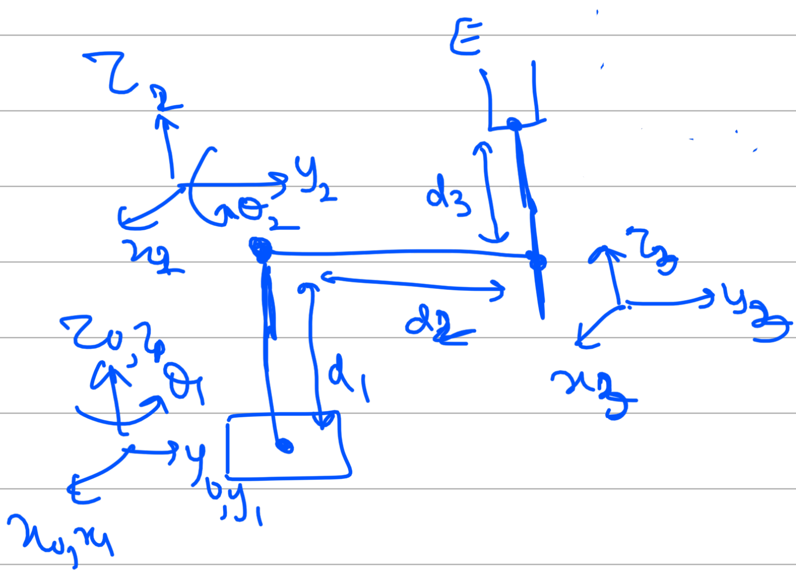
$$H_1^2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & d_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & d_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Ans-4 Standford RRP:



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} 0 \\ d_2 \\ 0 \end{bmatrix}$$

$$R_0^1 = R_{z, \theta_1}$$

$$R_1^2 = R_{y, \theta_2}$$

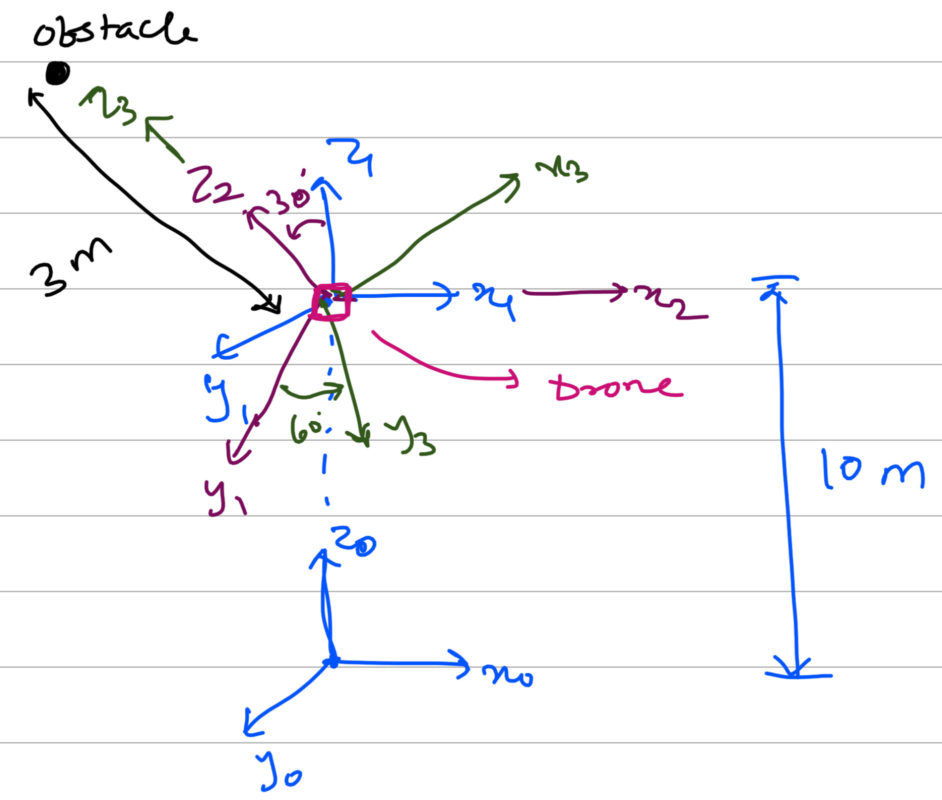
$$R_2^3 = I \quad (\text{Identity matrix})$$

$$H_0^1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_2 & 0 & c\theta_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_2 & 0 & c\theta_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

Ans-5



Given the location of obstacle in drone from is 3 m along the z-axis.

$$\therefore P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$R_0^1 = I$ (Identity matrix)

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = R_{x,30^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = R_{z,60^\circ} = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$H_2^3 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 & 0 \\ 0 & \sin 30 & \cos 30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.75 & 0.433 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \\ 1 \end{bmatrix}$$

$$\boxed{\therefore P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \end{bmatrix}}$$

→ position vector of the obstacle in base co-ordinate frame.

Ans 6

a) Some types of GearBoxes are:

- **Spur GearBox**
  - Pros
    - They are very cheap and easy to handle.
    - They work very well at low speeds.
  - Cons
    - Reductions are too low. For large reductions, many gears are needed.
    - Does not work well at high speeds and large torques.

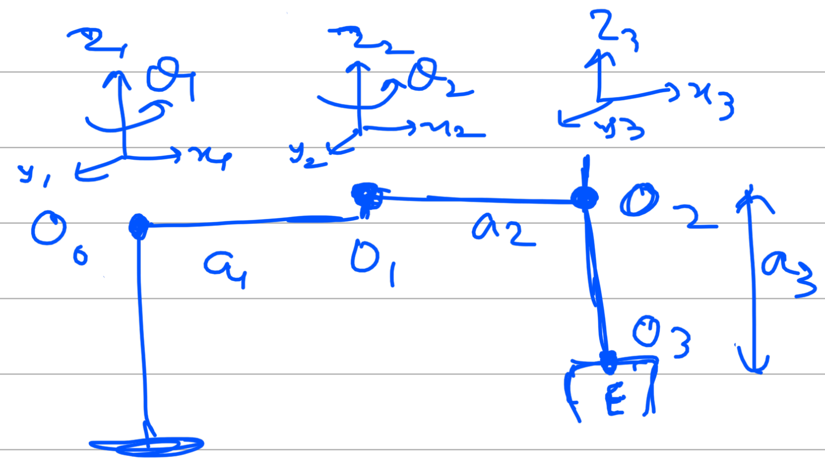
- **Worm GearBox**
  - Pros
    - Maintainance is easy.
    - Large reductions are possible.
    - Has a high thermal capacity and large shock load capability.
  - Cons
    - Is costly.
    - The packaging size of the design is large.
    - Has a moderate efficiency.

- **6 Speed GearBox**
  - Pros
    - 6 choices of gear ratios can be made according to the need.
  - Cons
    - Moderately costly.

- **Planetary GearBox**
  - Pros
    - Have a small size and high efficiency.
    - Very large reductions are possible.
    - Large torque can be provided.
  - Cons
    - They are more complex as compared to other gearboxes.
    - There a some thermal limitations for smaller sizes.

b) Most Drones donot use GearBoxes. GearBoxes are used when the highspeed motor needs to be converted to high torque providing motor. Mostly drones need high speed and not high torque. But some drones indeed need gearboxes. Some large sized drones have very large propellers which need high torque for rotation. In those cases, gearboxes are used in drones. One such example of drone that use extra gears is Syma X5 drone.

Ans - 7   SCARA RRP:



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = d_0^1$$

$$= \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = R_0^1 d_1^2 + d_0^1$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 c_2 \\ a_2 s_2 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^1 R_1^2 d_2^3 + R_0^1 d_1^2 + d_0^1$$

$$R_1^2 = I$$

$$d_2^3 = \begin{bmatrix} 0 \\ 0 \\ -a_3 \end{bmatrix}$$

$$\therefore O_3 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -a_3 \end{bmatrix} + O_2$$

$$= \begin{bmatrix} 0 \\ 0 \\ -a_3 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ -a_3 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} z_0 \times (O_3 - O_0) \\ z_0 \end{bmatrix} \rightarrow \text{Revolute joint}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_3 - O_0 = \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ -a_3 \end{bmatrix}$$

$$Z_0 \times (O_3 - O_0) = \begin{bmatrix} -(a_2 s_{12} + a_1 s_1) \\ a_2 c_{12} + a_1 c_1 \\ 0 \end{bmatrix}$$

$$\therefore \quad J_1 = \begin{bmatrix} -(a_2 s_{12} + a_1 s_1) \\ a_2 c_{12} + a_1 c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (O_3 - O_1) \\ z_1 \end{bmatrix} \rightarrow \text{Revolute joint}$$

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad O_3 - O_1 = \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ -a_3 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -a_2 s_{12} \\ a_2 c_{12} \\ 0 \end{bmatrix}$$

$$\therefore J_2 = \begin{bmatrix} -a_2 s_{12} \\ a_2 c_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix} \longrightarrow \text{Prismatic joint}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore J_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

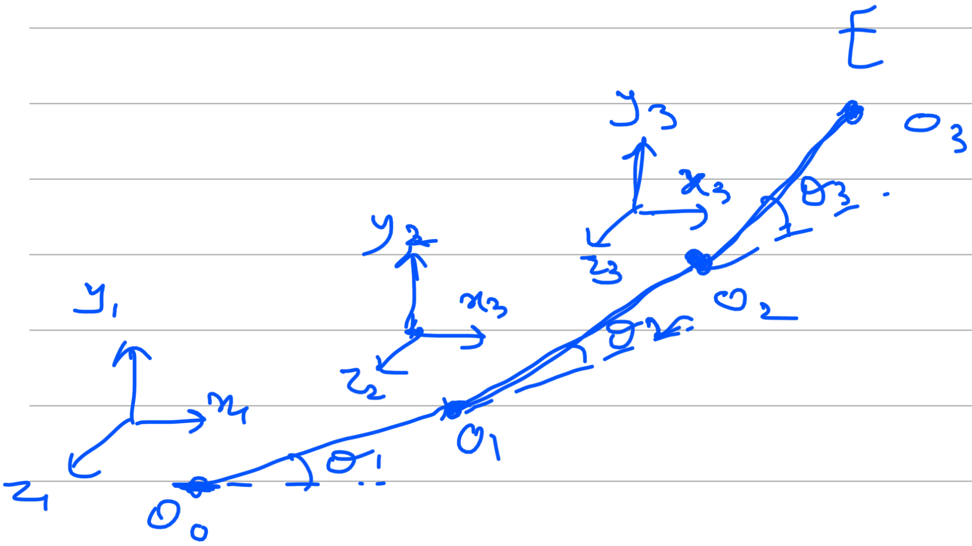
$$\therefore \; J = \begin{bmatrix} -(a_2 s_{12} + a_1 s_1) & -a_2 s_{12} & 0 \\ a_2 c_{12} + a_1 c_1 & a_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Note :- Here $c_1 = \cos\theta_1$, $c_2 = \cos\theta_2$

$s_1 = \sin\theta_1$, $s_2 = \sin\theta_2$

$c_{12} = \cos(\theta_1 + \theta_2)$, $s_{12} = \sin(\theta_1 + \theta_2)$

Ans-9 Planar RRR:



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = d_0^1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = R_0^1 \times d_1^2 + d_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 c_2 \\ a_2 s_2 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = R_0^2 \times d_2^3 + R_0^1 \times d_1^2 + d_0^1$$

$$= R_0^1 \times R_1^2 \times d_2^3 + O_2$$

$$= \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3 c_3 \\ a_3 s_3 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3 c_3 \\ a_3 s_3 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_3 c_{123} \\ a_3 s_{123} \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_3 c_{123} + a_2 c_{12} + a_1 c_1 \\ a_2 s_{123} + a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \; J_1 = \begin{bmatrix} z_0 \times (O_3 - O_0) \\ z_0 \end{bmatrix}$$

Revolute joint

$$= \begin{bmatrix} -(a_3 s_{123} + a_2 s_{12} + a_1 s_1) \\ a_3 c_{123} + a_2 c_{12} + a_1 c_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (o_3 - o_1) \\ z_1 \end{bmatrix}$$

↳ Revolute joint

$$= \begin{bmatrix} -(a_3 s_{123} + a_2 s_{12}) \\ a_3 c_{123} + a_2 c_{12} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} z_2 \times (o_3 - o_2) \\ z_2 \end{bmatrix} = \begin{bmatrix} -a_3 s_{123} \\ a_3 c_{123} \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

↳ Revolute Joint

$$\therefore \quad J = \begin{bmatrix} -(a_3 s_{123} + a_2 s_{12} + a_1 s_1) & -(a_3 s_{123} + a_2 s_{12}) & -a_3 s_{123} \\ a_3 c_{123} + a_2 c_{12} + a_1 c_1 & a_3 c_{123} + a_2 c_{12} & a_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Note: Here,

$$c_1 = \cos\theta_1 \ , \ s_1 = \sin\theta_1$$
$$c_2 = \cos\theta_2 \ , \ s_2 = \sin\theta_2$$
$$c_3 = \cos\theta_3 \ , \ s_3 = \sin\theta_3$$
$$c_{12} = \cos(\theta_1+\theta_2) \ , \ s_{12} = \sin(\theta_1+\theta_2)$$
$$c_{123} = \cos(\theta_1+\theta_2+\theta_3) \ , \ s_{123} = \sin(\theta_1+\theta_2+\theta_3)$$