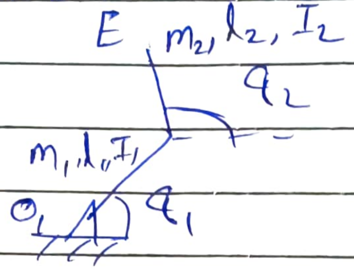
## Task 0 :




$E(x,y)$ End effector

O is origin $(0,0)$

$\therefore x = l_1 \cos q_1 + l_2 \cos q_2$

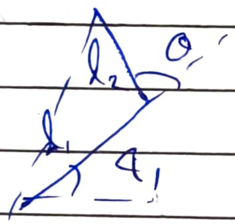
$y = l_1 \sin q_1 + l_2 \sin q_2$

$\therefore \dot{x} = -l_1 \sin q_1 \, \dot{q}_1 - l_2 \sin q_2 \, \dot{q}_2$

$\dot{y} = l_1 \cos q_1 \, \dot{q}_1 + l_2 \cos q_2 \, \dot{q}_2$

$$\therefore \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Here $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$ is task space and $\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$ is joint space



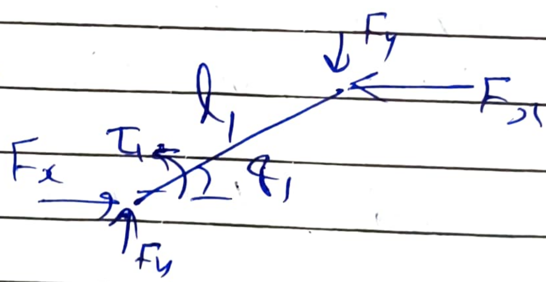

Here, $q_2 = q_1 + \theta$

Using cosine rule

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

and $q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \sin\theta}\right)$, $q_2 = q_1 + \theta$

FBD of link 1 :




$\therefore \tau_1 + F_x l_1 \sin q_1 - F_y l_1 \cos q_1 = 0$

$\tau_2 + F_x l_2 \sin q_2 - F_y l_2 \cos q_2 = 0$

$$\therefore \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & l_1 \cos q_1 \\ -l_2 \sin q_2 & l_2 \cos q_2 \end{bmatrix}$$

Now, Writing for task 3, $F_x = k(x - x_0)$

and $F_y = k(y - y_0)$

We know Lagrange's eq$^n$ : $\mathcal{L} = K - V$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i' \rightarrow \text{Generalised forces}$$

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2 \dot{q}_2}{2}\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} sq_1 + m_2 g \left(l_1 sq_1 + \frac{l_2 \, sq_2}{2}\right)$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2 \ddot{q}_2 \cos(q_2 - q_1)}{2} - \frac{m_2 l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} cq_2 + m_2 g l_1 cq_2 = \tau_1$$ → Add this to torque from $F_x = k(x - x_0)$ etc.

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2 \ddot{q}_2}{4} + \frac{m_2 l_1 l_2 \ddot{q}_1 \cos(q_2 - q_1)}{2} - \frac{m_2 l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ \frac{m_2 g l_2 \sin q_2}{2} = \tau_2$$

$$F_x = k(x - x_0) = k(l_1 cq_1 + l_2 cq_2 - x_0)$$
$$F_y = k(y - y_0) = k(l_1 sq_1 + l_2 sq_2 - y_0)$$