ME639

MINI PROJECT - 1

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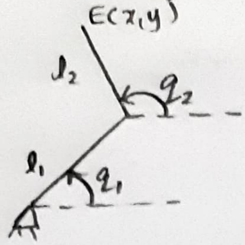
**Task 0:** the 6 equations




Now,

$$x = l_1\cos q_1 + l_2\cos q_2$$
$$y = l_1\sin q_1 + l_2\sin q_2 \quad - (1) - \quad x = l_1 cq_1 + l_2 cq_2, \quad y = l_1 sq_1 + l_2 sq_2$$

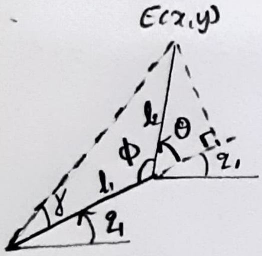
Differentiating on both sides wrt time:

$$\dot{x} = -l_1 sq_1(\dot{q}_1) - l_2 sq_2(\dot{q}_2) \quad \text{(forward kinematics)}$$
$$\dot{y} = l_1 cq_1(\dot{q}_1) + l_2 cq_2(\dot{q}_2)$$

In matrix form:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad - (2)$$

Need inverse kinematics:




$$\beta = \gamma + q_1 \qquad x^2+y^2 = h^2$$
$$q_2 = q_1 + \theta \qquad l_1^2 + l_2^2 - 2l_1l_2\cos\phi = h^2$$

(By cosine rule)

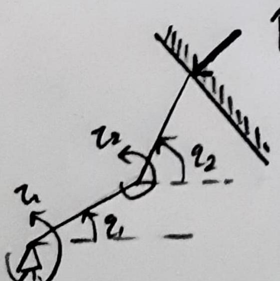
$$\phi = 180 - \theta \Rightarrow l_1^2 + l_2^2 + 2l_1l_2\cos\theta = h^2 = x^2+y^2$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right) \quad (3.1)$$

$$q_2 = q_1 + \theta \quad (3.2)$$

$$q_1 : \text{from } (---) \text{ triangle} = \beta - \gamma = \underbrace{\tan^{-1}(y/x)}_{\beta} - \underbrace{\tan^{-1}\left(\frac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right)}_{\gamma} \quad (3.3)$$

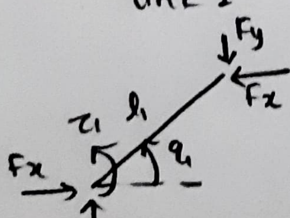
(3)



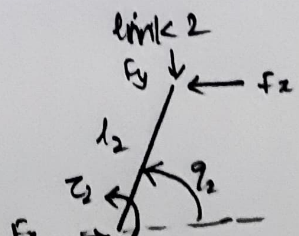

Static eq$^{lb}$, ignoring gravity

FBD:

link 1




link 2




$$\tau_1 = -F_x l_1\sin q_1 + F_y l_1\cos q_1 \qquad \tau_2 = -F_x l_2\sin q_2 + F_y l_2\cos q_2$$

In matrix form,

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1\sin q_1 & l_1\cos q_1 \\ -l_2\sin q_2 & l_2\cos q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad - (4)$$

$\mathcal{L} = K - V$ (Lagrange's Equations)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i \qquad i = 1, 2, \ldots n$$

↳ generalized forces

$$K = \underbrace{\frac{1}{2}\left(\frac{m_1 l_1^2}{3}\right)\dot{q}_1^2}_{\text{pure rotation, link 1}} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{rotation of link 2 abt C.O.M}} + \underbrace{\frac{m_2 v_{c_2}^2}{2}}_{\text{translation, C.O.M, link 2}} \text{ : Kinetic energy}$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2 \dot{q}_2}{2}\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

potential energy:

$$V = \frac{m_1 g l_1 s q_1}{2} + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right) \quad \text{(C.OM of link 1 + link 2)}$$

substituting K and V in $\mathcal{L} = K - V$, taking generalized forces $F_x, F_y$, & using eq^n 4 to replace them with $\tau_1$ & $\tau_2$,

$$\frac{d}{dt}\left(\frac{\partial (K-V)}{\partial \dot{q}_i}\right) - \frac{\partial (K-V)}{\partial q_i} = F$$

$$\Rightarrow \frac{m_1 l_1^2 \ddot{q}_1}{3} + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2 \dot{q}_2}{2} (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 c q_1 = \tau_1 \quad - \text{(5.1)}$$

$$\frac{m_2 l_2^2 \ddot{q}_2}{3} + \frac{m_2 l_2^2 \ddot{q}_2}{4} + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} \sin q_2$$

$$= \tau_2 \quad - \text{(5.2)}$$

Moment Balance:

$$\left.\begin{aligned} &\tau_{1\,spring} + \tau_1 \text{(ext drive)} = \tau_{1\,apply} \\ &\tau_{2\,spring} + \tau_2 \text{(ext drive)} = \tau_{2\,apply} \end{aligned}\right\} \text{(6)}$$