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Q3

The types of motors can be generaly divided into two: AC and DC motors

- AC motors can be divided into two synchronous and Asynchronous motors
  - In synchronous motors the speed remains constant even with varying loads as the supply current frequency is synchronised with rotor rotation. Since the speed is constant these are used in machines
  - Asynchronous motors works on the principle of electromagnetic induction. This induction is caused due to magnetic field on rotor. Broadly divided into two single and three phase motors. Single phase are used for smaller appliances eg. home appliance and three phase has industrial application for eg conveyor belts.

- DC motors are divided into brushed and brushless motors.
  - Brushed motors has permanant magnet inside its body and has an rotating armature inside. Magnets are stationary and are called stator whereas the rotating armature has an electromagnet called rotor. It has carbon brushes inside and thus are called brushed motors. These have high torques hence are good for Industrial applications. Its cheap too.
  - Brushless are inside out brushed motors and hence does not have brushes. It has permanant magnets on rotor and electromagnets on stator. It has a long life span, have better efficiency, are high speed DC motors and are quiet.

- Servo motors is a motor with position control inbuilt thus are used a lot in robotics. It is a rotatary or linear actuator which allows for precise control of angular, linear position as well as velocity and acceleration.
- Stepper motors have an internal rotor manipulated by magnets outside. Rotor is made with permanant magnets. Rotor teeth aligns with magnetic field once the windings are energised thus it rotates with fixed increments from position to position.

Q6 to prove the columns of R_0^1 are orthogonal.

$$R_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

let x_1 axis make α with x_0, β with y_0, γ with k_0.

So for $x_0-y_0-z_0$ x_1 is a vector making α, β, γ angles respectively

we know $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore$ taking column ①,

$$C \cdot C^T = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{i}_1 \cdot \hat{j}_0 & \hat{i}_1 \cdot \hat{k}_0 \end{bmatrix} = \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} \begin{bmatrix} \cos\alpha & \cos\beta & \cos\gamma \end{bmatrix}$$

$$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

lly for other two columns

Hence proved.

Q7

we know,

$$R_{Z\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{x\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \quad R_{y,\psi} = \begin{bmatrix} c\psi & 0 & s\psi \\ 0 & 1 & 0 \\ -s\psi & 0 & c\psi \end{bmatrix}$$

$\det R_{Z\theta} = \cos^2\theta + \sin^2\theta = 1$

lly $R_{x\phi} = \cos^2\phi + \sin^2\phi = 1$

lly $R_{y\psi} = \cos^2\psi + \sin^2\psi = 1$

since any rotation matrix is formed by the product of these.

by property $\det(A \cdot B \cdot C) = \det(A) \cdot \det(B) \cdot \det(C)$

$$\boxed{\det R = 1 \times 1 \times 1 = 1}$$

Hence proved.