ASSIGNMENT 2

1) We know that for any vector, $p = (p_x, p_y, p_z)^T$

$$S(a)p = a \times p.$$

and if $R \in SO(3)$ and a, b are vectors in $\mathbb{R}^3$, it can be shown that, $R(\vec{a} \times \vec{b}) = Ra \times Rb$.

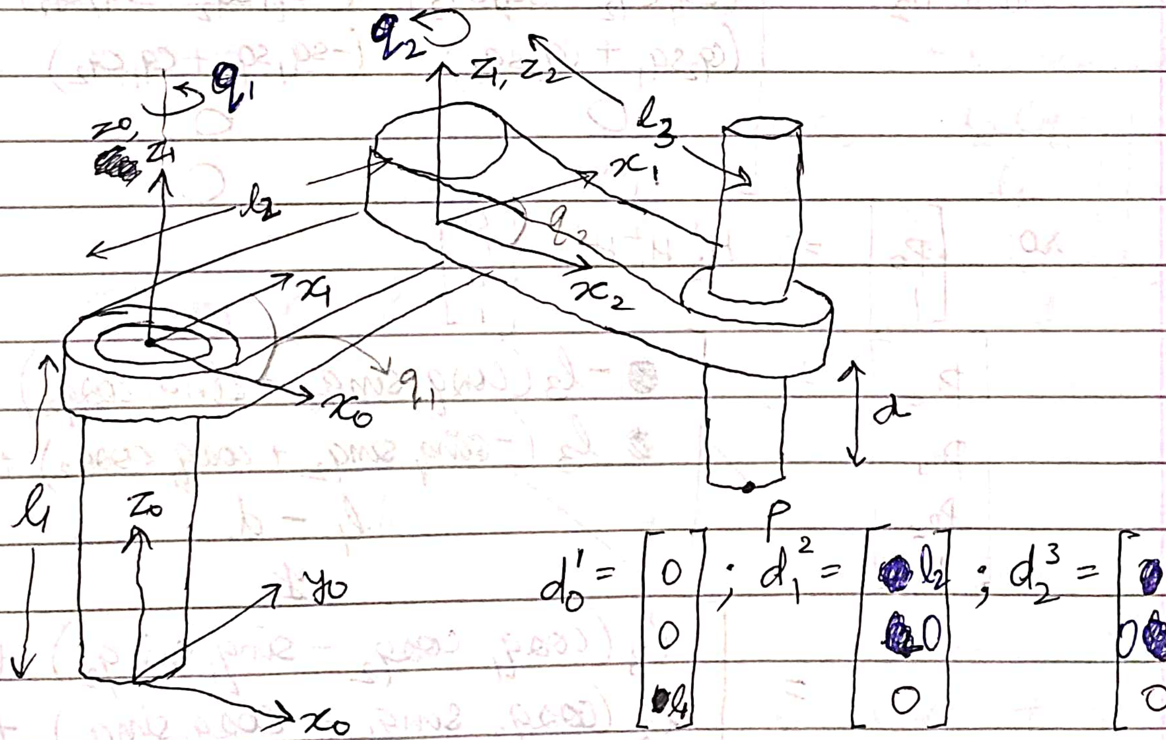
because R is orthogonal.

Using these and for any $R \in SO(3)$ and any $b \in \mathbb{R}^3$:-

$$\begin{aligned} R(S(a)R^T)b &= R(a \times R^T b) \\ &= (Ra) \times (RR^T b) \\ &= (Ra) \times b \\ &= S(Ra)b \end{aligned}$$

so, $R\,S(a)\,R^T = \underline{S(Ra)}$. Hence Proved

2)



$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}; \quad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}; \quad d_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}; \quad p = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

R_{z,q_1} $\qquad R_{z,q_2}$ $\qquad R_{z,0}$.

now, $\begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} p_3 \\ 1 \end{bmatrix}$

$$H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad H_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 H_1^2 = \begin{bmatrix} (\cos q_1 \cos q_2 - \sin q_1 \sin q_2) & (-cq_1 sq_2 - sq_1 cq_2) & 0 & + l_2 \cos q_1 \\ (cq_2 sq_1 + cq_1 sq_2) & (-sq_1 sq_2 + cq_1 cq_2) & 0 & l_2 \sin q_1 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

now, $$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$l_3(cq_1 cq_2 -$

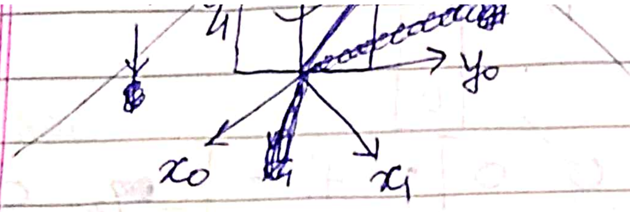
so, $$H_0^1 H_1^2 H_2^3 = \begin{bmatrix} (cq_1 cq_2 - sq_1 sq_2) & (-cq_1 sq_2 - sq_1 cq_2) & 0 & \cancel{l_3(cq_1 sq_2 \dots)} \\ (cq_2 sq_1 + cq_1 sq_2) & (-sq_1 sq_2 + cq_1 cq_2) & 0 & \cancel{l_3(cq_1 sq_2 + \dots)} \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$l_3(cq_2 sq_1 + cq_1 sq_2$

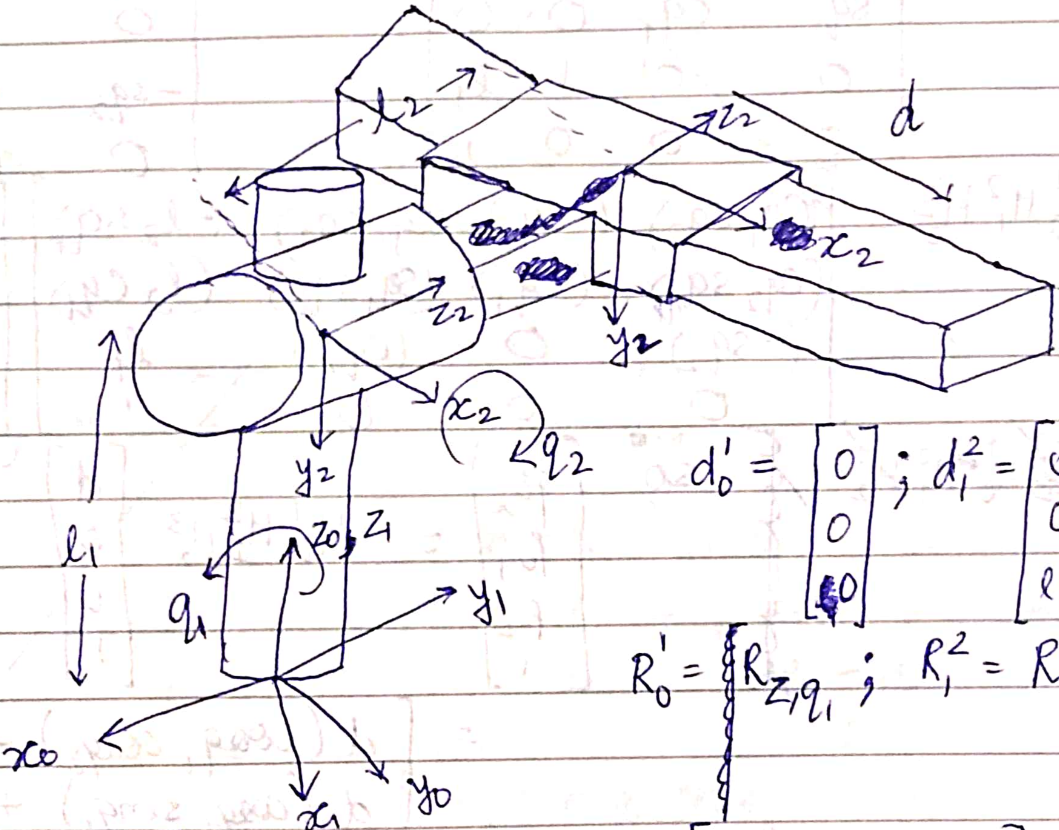
so, $$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} l_3(\cos q_1 \cos q_2 - \sin q_1 \sin q_2) + l_2 \cos q_1 \\ l_3(\cos q_2 \sin q_1 + \cos q_1 \sin q_2) + l_2 \sin q_1 \\ l_1 - d \\ 1 \end{bmatrix}$$



4)



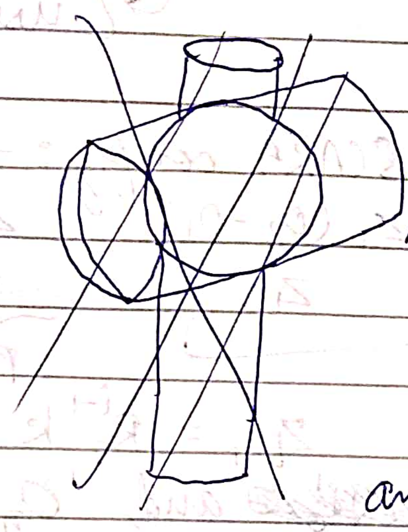
$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \; d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}; \; d_2^3 = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}; \; p = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = R_{z,q_1}; \quad R_1^2 = R_{z,0}; \; R_2^3 = R_{x,-90^\circ} \times R$$

$$H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; \cancel{H_2^3 =}$$

$$R_{x,-90} \times R_{z,q_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ \end{bmatrix} \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ \end{bmatrix} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ 0 & 0 & -1 \\ \end{bmatrix}$$

4) $$H_2^3 = \begin{bmatrix} cq_2 & -sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ sq_2 & cq_2 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



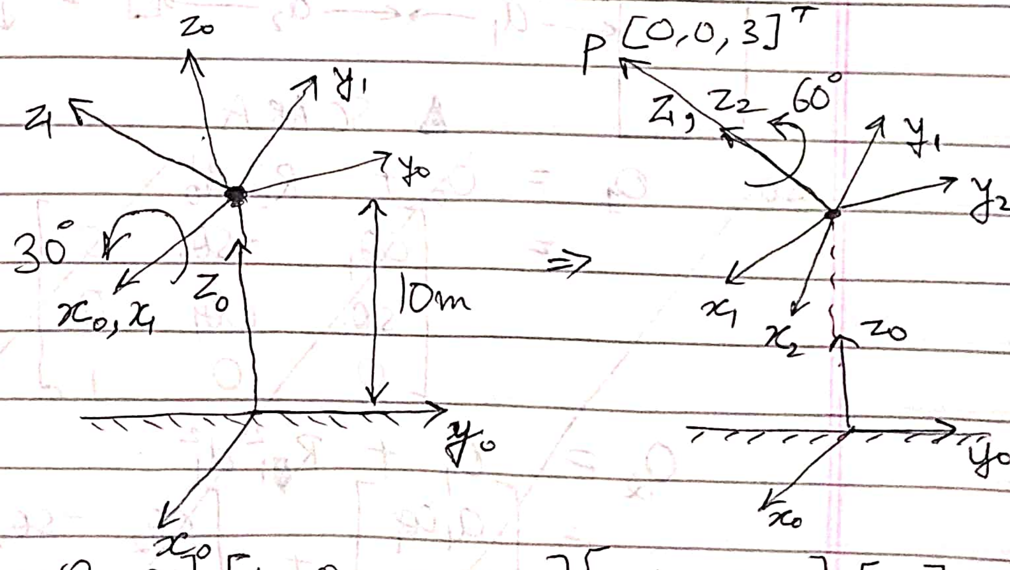
$$H_0^1 H_1^2 = \begin{bmatrix} cq_1 & -sq_2 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & l_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and, $H_0^1 H_1^2 H_2^3$:-

$$\begin{bmatrix} (cq_1 cq_2) & (-cq_1 sq_2) & sq_2 & 0 \\ (sq_1 cq_2) & (-sq_1 sq_2) & -cq_1 & 0 \\ (l_1 sq_2) & (l_1 cq_2) & 0 & l_1 l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow p_0 = \text{''} \times \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

so, $$\begin{bmatrix} p_{0x} \\ p_{0y} \\ p_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} d(\cos q_1 \cos q_2) \\ d(\sin q_1 \cos q_2) \\ +(d\,\cancel{\cos q_1}\, l_1 \sin q_2) + (l_1 l_2) \\ 1 \end{bmatrix}$$

5) Given :-



so,

$$p_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (c30°) & (-s30°) & 0 \\ 0 & (s30°) & (c30°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (c60°) & (-s60°) & 0 & 0 \\ (s60°) & (c60°) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = [0. \ , -1.5 \ , \ 12.5981 \ , \ 1.]^T$$ ↳ [using python]

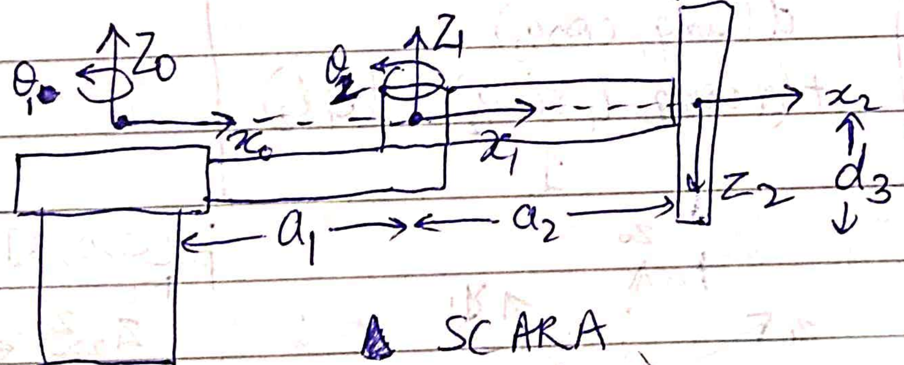
Note: Answer to Q6 is at the last of this pdf

7). Jacobian for RRP SCARA config:-

$$J = \begin{bmatrix} \underbrace{\begin{matrix} Z_0 (O_3 - O_0) & , Z_1 (O_3 - O_1) \\ Z_0 & Z_1 \end{matrix}}_{R} & \underbrace{\begin{matrix} Z_2 \\ 0 \end{matrix}}_{P} \end{bmatrix}$$

where $O_k = d_0^k$ and $Z_{i-1} = R_0^{i-1} k$.

now, $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and, $O_n = O_{i-1} + R_0^{i-1} d_{i-1}^n$



SCARA

$$\begin{bmatrix} s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O_1 = d_0^1 = R_{z,\theta_1} \times \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c\theta_1 & s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 \cos\theta_1 \\ a_1 \sin\theta_1 \\ 0 \end{bmatrix}$$ (1)

$$O_2 = d_0^2 = d_0^1 + R_{z,\theta_1} R_{z,\theta_2} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2(-s_1 s_2 + c_1 c_2) \\ a_2(s_1 c_2 + s_2 c_1) \\ 0 \end{bmatrix}$$ (2)

$$= \begin{bmatrix} a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) \\ a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}.$$

and, $$O_3 = d_0^3 = d_0^2 + R_{z,\theta_1} R_{z,\theta_2} R_{z,0} \begin{bmatrix} 0 \\ 0 \\ d_3 \end{bmatrix}$$ (3)

$$= \begin{bmatrix} a_1 \cos\theta + a_2 \cos(\theta_1+\theta_2) \\ a_1 \sin\theta + a_2 \sin(\theta_1+\theta_2) \\ d_3 \end{bmatrix}.$$

now, $z_0 = z_1 = k$ while $z_2 = -k$.

Since there are 3 ~~degrees of freedom~~ joint variables, we have :- Jacobian of 6×3 dimension :-

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

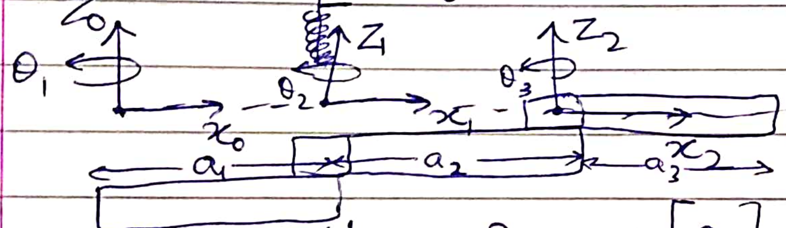
so, $$z_0 \times (O_3 - O_0) = \begin{bmatrix} -(a_1 \sin\theta + a_2 \sin(\theta_1+\theta_2)) \\ a_1 \cos\theta + a_2 \cos(\theta_1+\theta_2) \\ 0 \end{bmatrix}.$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -a_2 \sin(\theta_1+\theta_2) \\ a_3 \cos(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

so, $$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{1+2} & -a_2 s_{1+2} & 0 \\ a_1 c_1 + a_2 c_{1+2} & a_2 c_{1+2} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q.9) Jacobian for RRR config:-

$$J = \begin{bmatrix} z_0(O_3 - O_0) & z_1(O_3 - O_1) & z_2(O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = d_0^1 = R_{z,\theta_1} \times \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 \cos\theta_1 \\ a_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

$$O_2 = d_0^2 = d_0^1 + R_{z,\theta_1} R_{z,\theta_2} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} + \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_3 = d_0^3 = d_0^2 + R_{z,\theta_1} R_{z,\theta_2} R_{z,\theta_3} \begin{bmatrix} a_3 \\ 0 \\ 0 \end{bmatrix} = \text{''} + \begin{bmatrix} c_{123} & -s_{123} & 0 \\ s_{123} & c_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ 0 \\ 0 \end{bmatrix}$$

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$$O_3 = \begin{bmatrix} a_3 c_{123} + a_2 c_{12} + a_1 c_1 \\ a_3 s_{123} + a_3 s_{12} + a_2 s_1 \\ 0 \end{bmatrix}$$

and $z_0 = z_1 = z_2 = \hat{k}$

so, $J = \begin{bmatrix} (-a_1 s_1 - a_2 s_{12} - a_3 s_{123}) & (-a_2 s_{12} - a_1 s_1) & (-a_1 s_1) \\ (a_1 c_1 + a_2 c_{12} + a_3 c_{123}) & (a_1 c_1 + a_2 c_{12}) & (a_1 c_1) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Q6.

Type of Gearbox	Suitable Industries and Applications	Advantages	Disadvantages
Bevel	Print Press Power Plants Automobiles Steel Plants Hand Drills Differential Drives	Right angle configuration Durable	Axes must be able to support forces Poorly cut teeth may result in excessive vibration and noise during operation
Helical	Oil Industry Blowers Food and Labelling Cutters Elevators	Can be meshed in parallel or cross orientation Smooth and quiet operation Efficient High horsepower	Resistant thrust along axis of gear Additives to lubrication
Spur	Cut-to-Length Packaging Speed Control Construction Power Plants	Cost-effective High gear ratios Compact High torque output	Noisy Prone to wear
Worm	Mining Rolling Mills Presses Elevators/Escalator Drive Systems	High precision Right-angle configurations Low noise Maintenance-free	Non-reversible Low efficiency
Planetary	Slewing Drives Lifts Cranes Machine Tools Automotive	High power density Compact High efficiency in power transmission Greater stability Load distribution among planetary gears	High bearing loads Inaccessibility