Assignment 3 18110166

Wednesday, 14 September 2022 12:50 PM

1. A singular configuration occurs when the robot's end effector's velocity (linear + angular) space has dimension less than the number of joint variables.

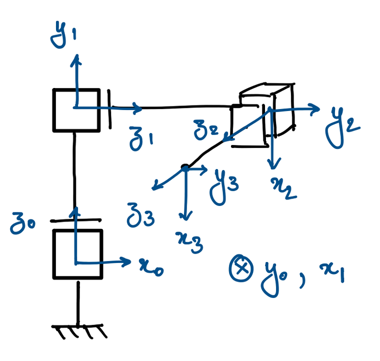
Mathematically, singularities occur when determinant of the manipulator jacobian is zero.

$\det(J) = 0$

We can find singular configurations by finding all vectors q for which the jacobian $J(q)$ is singular.

Conversely, for a given configuration represented by vector q if $\det(J(q))$ is zero or close to zero, we can say we are near a singular configuration.

5.



D-H parameters : $\theta_i, d_i, a_i, \alpha_i$

J1 : $(\frac{\pi}{2}, q_1, 0, \frac{\pi}{2})$

J2 : $(-\frac{\pi}{2}, q_2, 0, \frac{\pi}{2})$

J3 : $(0, q_3, 0, 0)$

$$T^0_1 = R_{z,\frac{\pi}{2}} T_{z,q_1} T_{x,0} R_{x,\frac{\pi}{2}} = \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^1_2 = R_{z,-\frac{\pi}{2}} T_{z,q_2} T_{x,0} R_{x,\frac{\pi}{2}} = \begin{bmatrix} \cos-\frac{\pi}{2} & -\sin-\frac{\pi}{2} & 0 & 0 \\ \sin-\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

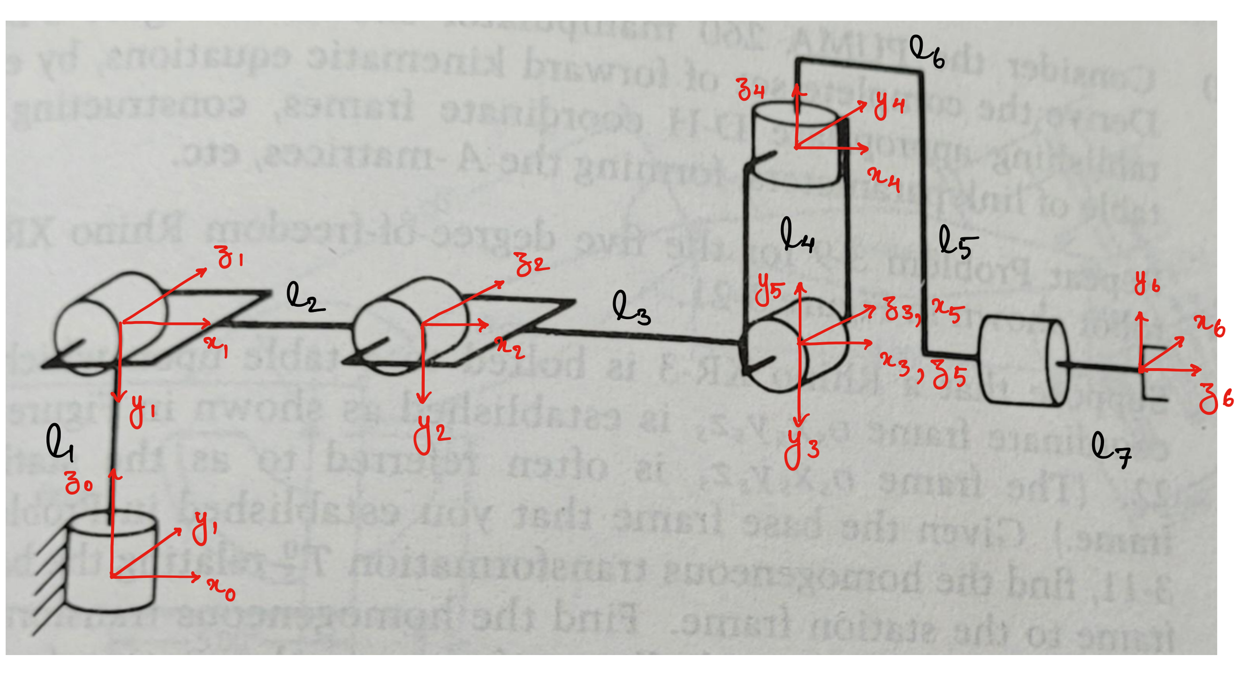
$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^2_3 = R_{z,0} T_{z,q_3} T_{x,0} R_{x,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore T^0_3 = T^0_1 T^1_2 T^2_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -q_3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 0 & 0 & -1 & -q_3 \\ -1 & 0 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.



D-H parameters : $\theta_i, d_i, a_i, \alpha_i$

J1 : $(q_1, l_1, 0, -\frac{\pi}{2})$

J2 : $(q_2, 0, l_2, 0)$

J3 : $(q_3, 0, l_3, 0)$

J4 : $(q_4, 0, 0, \frac{\pi}{2})$

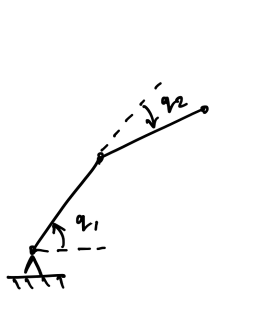
J5 : $(q_5, 0, 0, \frac{\pi}{2})$ (Currently $q_5 = \frac{\pi}{2}$)

J6 : $(q_6, 0, 0, 0)$

J4, J5, J6 are wrist joints

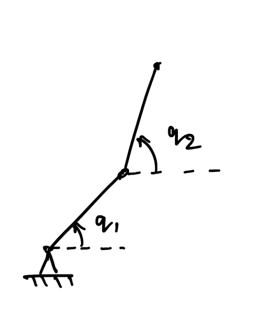
$\therefore l_4 = l_5 = l_6 = l_7 = 0$

7.



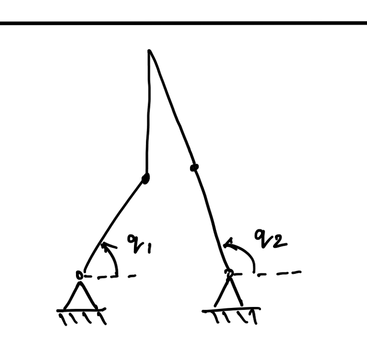
2R Direct Drive

- Relative angles as joint variables
- Standardized kinematics and manipulator jacobian using D-H parameters.



2R Remotely driven

- Absolute angles as joint variables
- Motors are stationary. Hence, dynamic analysis does not require to account for motor and gearbox mass, inertia and angular momenta.



5-bar parallelogram

- Doesnot form a kinematic chain
- Motors are stationary. Moreover, no belts/pulleys required to transfer torque to remote links as they are automatically constrained by geometry.

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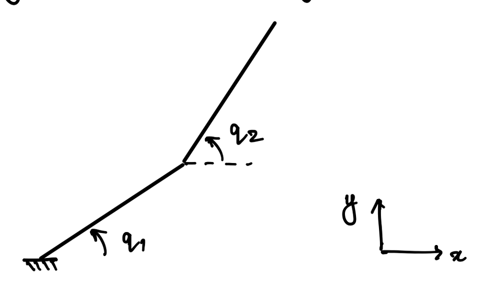
8. Elbow manipulator with remotely driven links using absolute angles



$$v_{c1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 & 0 \\ \frac{l_1}{2}\cos q_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad \omega_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$v_{c2} = \begin{bmatrix} -l_1\sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1\cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad \omega_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$T = \frac{1}{2}\sum_{i=1}^{2} m_i v_i^T v_i + \frac{1}{2}\sum_{i=1}^{2} \omega_i^T I_i \omega_i = \frac{1}{2}\dot{q}^T D \dot{q}$$

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$$V = m_1 g \frac{l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$$

$$c_{112} = -c_{221} = \frac{1}{2} m_2 l_1 l_2 \sin(q_2 - q_1)$$

$$c_{111} = c_{121} = c_{122} = c_{211} = c_{212} = c_{222} = 0$$

$$\phi_1 = m_1 g \frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1$$

$$\phi_2 = m_2 g \frac{l_2}{2}\cos q_2$$

$$\therefore \sum_j d_{kj}\ddot{q}_j + \sum_{ij} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

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10. Given $D(q) = [d_{ij}(q)]$ and $V(q)$

- Find $\frac{\partial d_{ij}}{\partial q_k}$ for each element d_{ij} in $D(q)$

  and for each joint variable q_k

- Find Christoffel's symbols of 1st kind –

  $$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

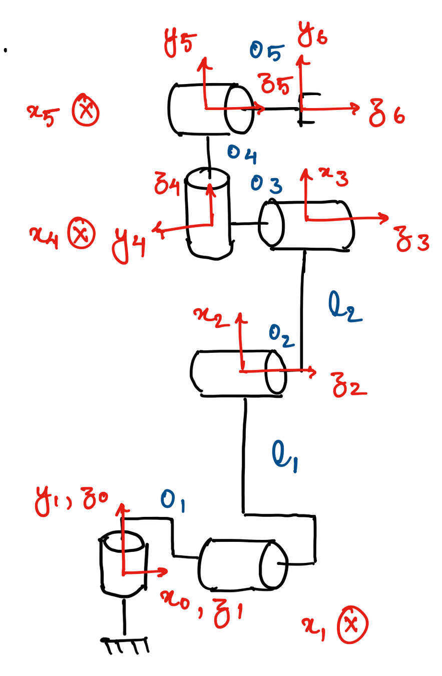
- Find forces from the potential field

  $$\phi_k = \frac{\partial V(q)}{\partial q_k}$$

After calculating all the terms, write the k^{th} equation as

$$\sum_j d_{kj}\ddot{q}_j + \sum_{ij} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

12.



Number of links : 2

Number of joints : 6

Nature of joints : All are Revolute

D-H parameters : $\theta_i, d_i, a_i, \alpha_i$

J1 : $(q_1, 0, 0, \frac{\pi}{2})$ currently $q_1 = \frac{\pi}{2}$

J2 : $(q_2, o_1, l_1, 0)$ currently $q_2 = \frac{\pi}{2}$

J3 : $(q_3, o_2, l_2, 0)$ currently $q_3 = 0$

J4 : $(q_4, o_3, 0, -\frac{\pi}{2})$ currently $q_4 = -\frac{\pi}{2}$

J5 : $(q_5, o_4, 0, \frac{\pi}{2})$ currently $q_5 = 0$

J6 : $(q_6, o_5, 0, 0)$ currently $q_6 = 0$

o_1, o_2, o_3, o_4, o_5 are offsets

l_1 and l_2 are link lengths.