ME - 639

Assignment - 3

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1.) When the dimension of the robot's end effector's velocity (linear + angular) space is fewer than the total number of joint variables, a singular configuration occurs.

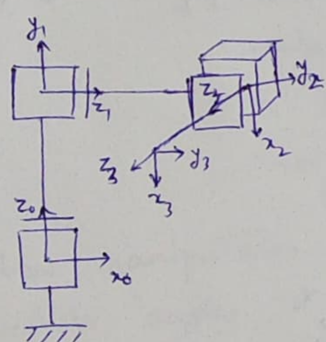
Mathematically, ~~a~~ singularities happen when the manipulator jacobian's determinant is zero.

$$\Rightarrow \quad \det(J(q)) = 0$$

By identifying all vectors q for which the jacobian $J(q)$ is singular, we can identify singular configurations.

On the other hand, if $\det(J(q)) = 0$ or close to zero for a particular configuration represented by vector q, we can argue that we are getting close to a unique configuration.

5.)



Let DH parameters be in order $(\theta_i, d_i, a_i, \alpha_i)$

$\therefore$ $J_1 : (\frac{\pi}{2}, q_1, 0, \frac{\pi}{2})$

$J_2 : (-\frac{\pi}{2}, q_2, 0, \frac{\pi}{2})$

$J_3 : (0, q_3, 0, 0)$

$$\therefore H_0^1 = H_{z,\frac{\pi}{2}} \cdot H_{z,q_1} \cdot H_{x,0} \cdot H_{x,\frac{\pi}{2}}$$

$$= \begin{bmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) & 0 & 0 \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) & 0 \\ 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = H_{z,-\frac{\pi}{2}} \; H_{z,q_2} \; H_{x,0} \; H_{x,\frac{\pi}{2}}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

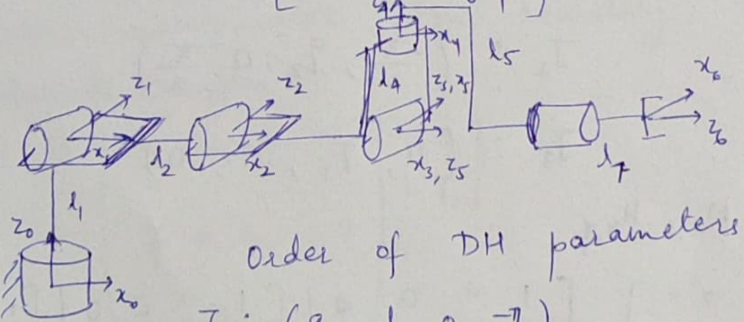
$$H_2^3 = H_{z,0} \; H_{z,q_3} \; H_{x,0} \; H_{x,0}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = H_0^1 H_1^2 H_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & q_2 \\ 0 & 0 & -1 & -q_3 \\ -1 & 0 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.)



Order of DH parameters : $(\theta_i, d_i, a_i, \alpha_i)$

$J_1 : (q_1, l_1, 0, -\frac{\pi}{2})$

$J_2 : (q_2, 0, l_2, 0)$, $J_3 : (q_3, 0, l_3, 0)$

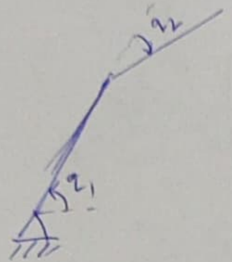
$J_4 : (q_4, 0, 0, \frac{\pi}{2})$, $J_5 : (q_5, 0, 0, \frac{\pi}{2})$

$J_6 : (q_6, 0, 0, 0)$

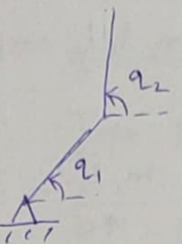
J_4, J_5, J_6 are wrist joints.

$\therefore l_4 = l_5 = l_6 = l_7 = 0$

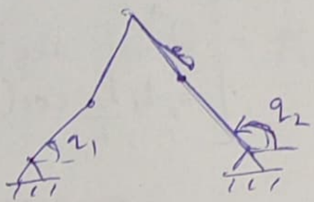
7.)



2R Direct Drive



2R Remotely driven



5-bar parallelogram

(i) Relative angles are used as joint variables

(ii) Standardized ~~kinema~~ kinematics and manipulator jacobian using D-H parameters.

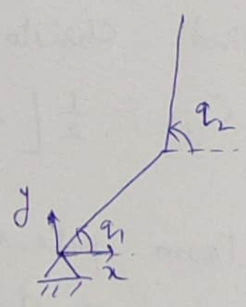
(i) Absolute angles are used as joint variables.

(ii) The motors are not moving. Thus, it is not necessary to take into consideration the mass, inertia and ~~the~~ angular momentum of the motor and gearbox in the dynamic analysis.

(i) This does not form a kinematic chain.

(ii) The motors are not moving. Furthermore, because remote links are automatically restricted by geometry, no belts / pulley are needed to deliver torque to them.

8.) Elbow manipulator with remotely driven links ~~via~~ using absolute angles.



$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin(q_1) & 0 \\ \frac{l_1}{2}\cos(q_1) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$v_{c_2} = \begin{bmatrix} -l_1\sin(q_1) & -\frac{l_2}{2}\sin(q_2) \\ l_1\cos(q_1) & \frac{l_2}{2}\cos(q_2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$K = \frac{1}{2}\sum_{i=1}^{2} m_i v_i^T v_i + \frac{1}{2}\sum_{i=1}^{2} \omega_i^T I_i \omega_i = \frac{1}{2}\dot{q}^T D \dot{q}$$

$$\therefore \quad D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2}\cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

$$V = m_1 g \frac{l_1}{2}\sin(q_1) + m_2 g\left(l_1 \sin(q_1) + \frac{l_2}{2}\sin(q_2)\right)$$

Here,

$$c_{112} = -c_{221} = \frac{1}{2} m_2 l_1 l_2 \sin(q_2 - q_1)$$

and, $c_{111} = c_{121} = c_{122} = c_{211} = c_{212} = c_{222} = 0$

Also, $\phi_1 = m_1 g \frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1$

$\phi_2 = m_2 g \frac{l_2}{2}\cos q_2$

$$\therefore \quad \sum_j d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

10.) Given, $D(q) = [d_{ij}(q)]$ and $V(q)$

(i) Find $\frac{\partial d_{ij}}{\partial q_k}$ for each element d_{ij} in $D(q)$ and for each joint variable q_k

(ii) Find Christoffel's symbols of 1st kind -

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

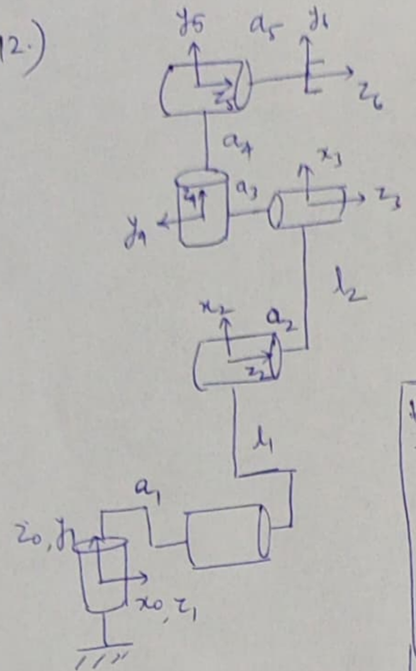
(ii) From potential field,

$$\phi_k = \frac{\partial V(q)}{\partial q_k}$$

After calculating all the terms, write the k^{th} eq^n^ as.

$$\sum_j d_{kj}\ddot{q}_j + \sum_{ij} c_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$$

12.)



Here,

Number of links = 2

Number of joints = 6

~~Type~~ Nature of all joints = R

DH parameters :

~~links~~ joints	θ_i	d_i	a_i	α_i
1	q_1	0	0	$\frac{\pi}{2}$
2	q_2	a_1	l_1	0
3	q_3	a_2	l_2	0
4	q_4	a_3	0	$-\pi/2$
5	q_5	a_4	0	$\pi/2$
6	q_6	a_5	0	0

currently, $q_1 = \frac{\pi}{2}$

$q_2 = \frac{\pi}{2}$

$q_3 = 0$

$q_4 = -\frac{\pi}{2}$

$q_5 = 0$

$q_6 = 0$

Also, a_1, a_2, a_3, a_4, a_5 - ~~offes~~ offsets

l_1, l_2 - link lengths.