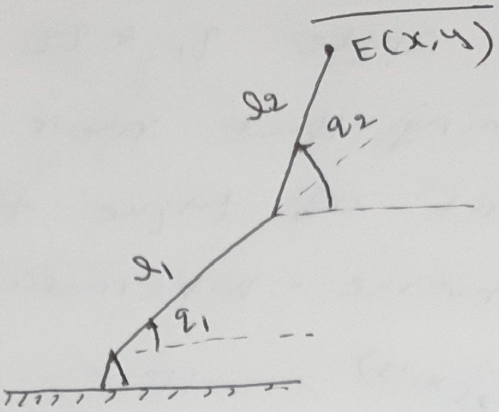
Ashutosh-Goyal 20110029 ME-639

miniproject

E:→ End-effector

Task-0:-



First of all, by using ~~Forw~~ Forward kinematics, we can write:→

$$\boxed{\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}} \quad \text{— (1)}$$

Therefore, to find the velocity of end-effector, Differentiate both side

$$\Rightarrow \boxed{\begin{aligned} \dot{x} &= -l_1 \sin q_1 \cdot \dot{q}_1 - l_2 \sin q_2 \, \dot{q}_2 \\ \dot{y} &= l_1 \cos q_1 \cdot \dot{q}_1 + l_2 \cos q_2 \cdot \dot{q}_2 \end{aligned}}$$

$$\boxed{\begin{aligned} \dot{x} &= \frac{dx}{dt} \\ \dot{q} &= \frac{dq}{dt} \end{aligned}}$$

$$\Rightarrow \boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & -l_2 \sin q_2 \\ l_1 \cos q_1 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}} \quad \text{— (2)}$$

Angular velocity provided by motor

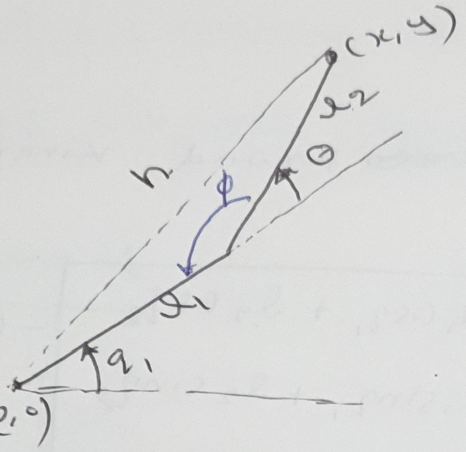
Velocity of end-effector

joint Variables or joint space

Inverse-Kinematics :-

Actually, we know the position of end-effector (x, y) and need to find angle q_1 & q_2 of the robotic arm by assuming that motor provided the sufficient amount of torque to robotic arm.

Hence, we required inverse-kinematics



Here,

$$q_2 = q_1 + \theta$$

Also,

$$\boxed{\pi - \phi = \theta}$$

$$\Rightarrow \boxed{\cos\phi = -\cos\theta}$$

$$\boxed{h^2 = x^2 + y^2}$$

Therefore, by applying cosine rule, we get,

$$\cancel{\theta = \cos^{-1}} \quad \cos\phi = \frac{l_1^2 + l_2^2 - h^2}{2 l_1 l_2}$$

$$\Rightarrow \cos\theta = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

Hence,

$$\boxed{q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)}$$

Therefore, q_2 will be

$$\boxed{q_2 = q_1 + \theta}$$

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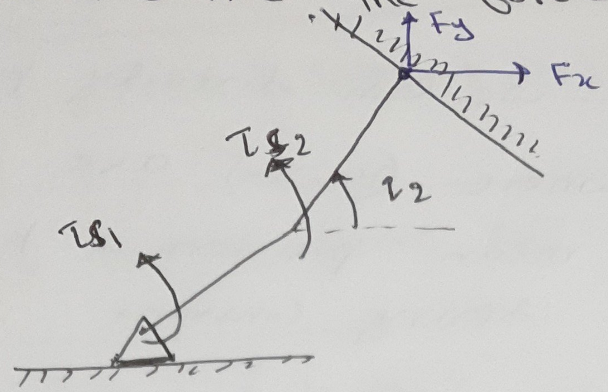
Now, Let's derive the forced eqn.

Task-2



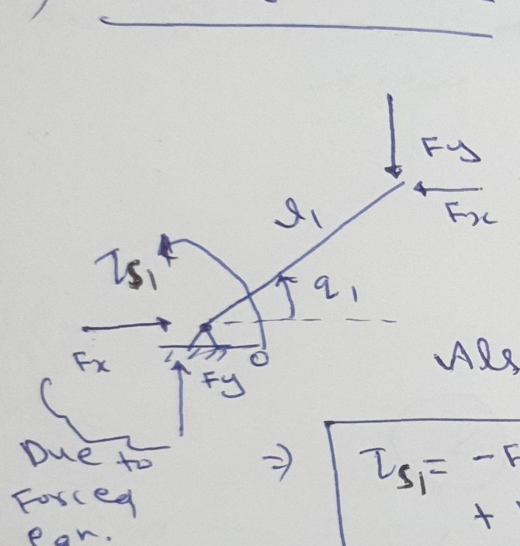
$\tau_1, \tau_2 \rightarrow$ Torque provided by motor

$F_x, F_y \rightarrow$ Force applied by the robotic arm of the wall

Assumption :- 1) Let's assume the eqm to be static eqm

2) we are ignoring the gravity.

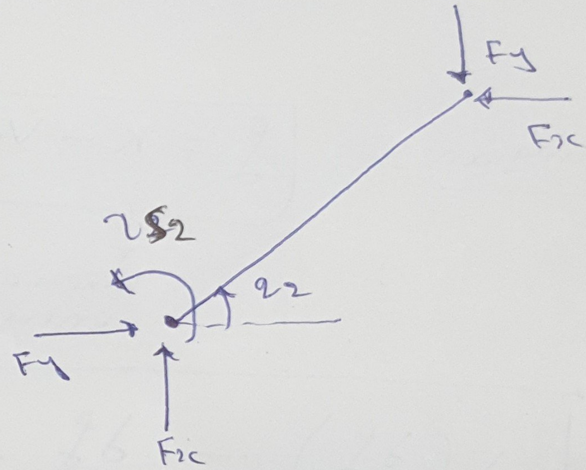
$\Rightarrow$ FBD of link-1



Also, $\sum \vec{m}_o = 0$

$\Rightarrow$ $\boxed{\tau_{s_1} = -F_x l_1 \sin q_1 + F_y l_1 \cos q_1}$

$\Rightarrow$ FBD of link-2



$\Rightarrow$ $\boxed{\tau_{s_2} = -F_x l_2 \sin q_2 + F_y l_2 \cos q_2}$

$$\Rightarrow \boxed{\begin{bmatrix} \tau_{s_1} \\ \tau_{s_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin q_1 & l_1 \cos q_1 \\ -l_2 \sin q_2 & l_2 \cos q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}} \quad (4)$$

⇒ Task-3 :-

i) Simple version :-

To make a virtual spring - we can directly put

$$\boxed{\begin{aligned} F_x &= k(x-x_0) \\ F_y &= k(y-y_0) \end{aligned}}$$

(where (x_0, y_0) are the mean position & k is spring constant)

Without taking care of dynamics

Dynamics :- By Lagrange's eq^n :- $\boxed{F = ma}$

Forces, which do account for some work

Lagrangian :- $\boxed{\mathcal{L} = K - V}$

V → potential energy

K → kinetic energy

⇒
$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i}$$

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$i = 1, 2, 3, \ldots n$

Q_i → Generalized forces

⇒ In our case :-

Kinetic energy =
$$\boxed{K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of link 1}} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{rotation of link 2 about it's C.O.M}} + \underbrace{\frac{1}{2} m_2 v_{c_2}^2}_{\text{translation of C.O.M of link 2}}}$$

where

$$\boxed{v_{c_2}^2 = (l_1\dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)}$$

Also, P.E =

$$\boxed{V = m_1 g \frac{l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)}$$

Now, By putting above eqn in Lagrangian, we get

$$\boxed{\begin{aligned}\tau_1 = {} & \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) \\ & - m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1) + m_1 g \frac{l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1\end{aligned}}$$

Also,

$$\boxed{\begin{aligned}\tau_2 = {} & \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + \frac{m_2 l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) \\ & + m_2 g \frac{l_2}{2}\sin q_2 - m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1)\sin(q_2 - q_1)\end{aligned}}$$

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Also, we can write.

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$$\boxed{\begin{aligned}F_x &= K(x - x_0) = K(l_1 \cos q_1 + l_2 \cos q_2 - x_0) \\ F_y &= K(y - y_0) = K(l_1 \sin q_1 + l_2 \sin q_2 - y_0)\end{aligned}}$$

Hence, In the place of τ_{s1} & τ_{s2}. If we are including dynamics, we will apply.

$\tau_{s1} + \tau_1$

& $\tau_{s2} + \tau_2$