# ME 351 : Mini Project

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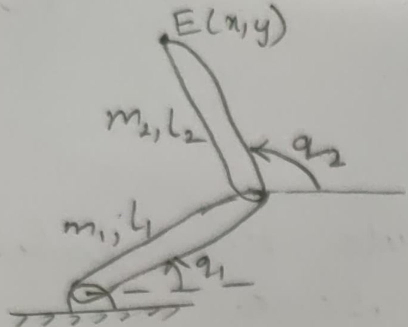
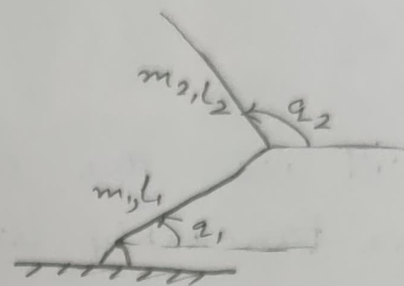



Now we can locate the coordinates of end effector by following -

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

The above equ can also be represented as

$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2 \end{aligned}\right] - (1)$$ Kinematics Equation

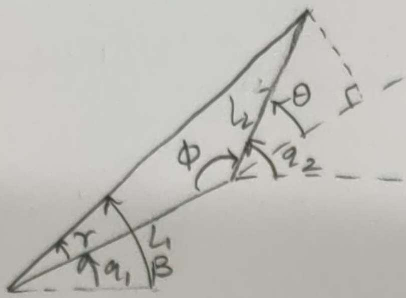
On differentiating the above eqn we get -

$$\dot{x} = -l_1 sq_1 \dot{q}_1 - l_2 sq_2 \dot{q}_2$$
$$\dot{y} = l_1 cq_1 \dot{q}_1 + l_2 cq_2 \dot{q}_2$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - (2)$$ Velocity Kinematics

→ Now since we require angles of the first and second joints which are difficult to calculate through the above eqn.

→ Therefore we need to use inverse Kinematics




$q_2 = q_1 + \theta$ (from ~~above~~ side figure)

$L_1^2 + L_2^2 = h^2$ (from right angle triangle)

→ We will use the cossine rule to calculate $\phi$ and simultaneously write $\phi = 180 - \theta$

$\therefore \; \theta = 180 - \phi$

→ On applying cosine rule, we get −

$$\boxed{\theta = \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)}$$

→ By using a property of right angled triangle

$$\boxed{q_1 = \underbrace{\tan^{-1}\left(\frac{y}{x}\right)}_{\beta} - \underbrace{\tan^{-1}\left(\frac{L_2\sin\theta}{L_1 + L_2\cos\theta}\right)}_{\gamma}}$$

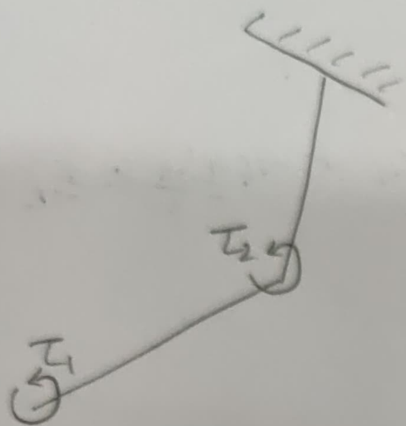
$q_1 = \beta - \gamma$

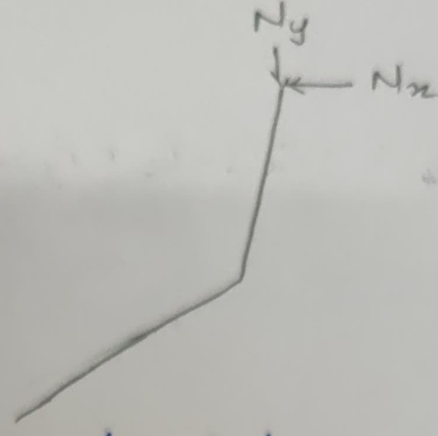
$$\boxed{q_2 = q_1 + \theta}$$

③ Inverse Kinematics

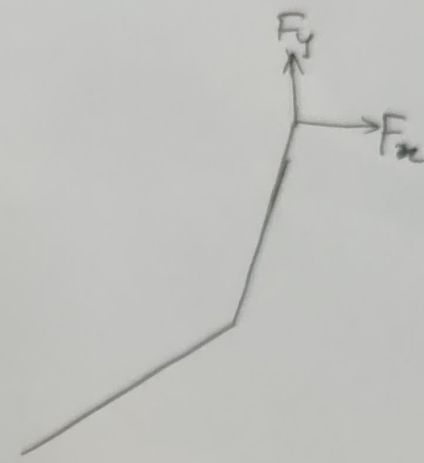
→ Now if we are applying a force on a wall of a particaular amount, the equations for the angle will be the same for a given location whereas for torque the equations can be given as following.

→ Torque calaulation

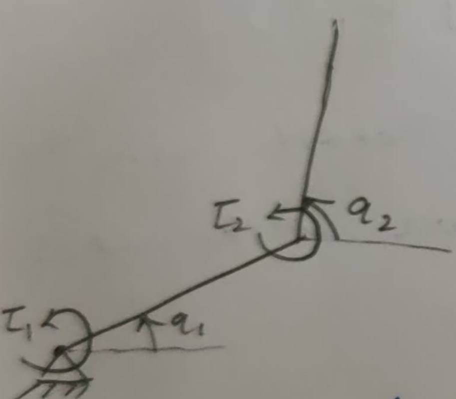



Normal applied by the wall on the manipulator



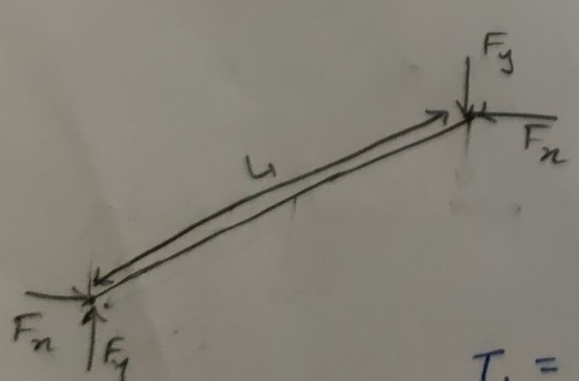

Force applied by the manipulator.




Under Static Equiliblium

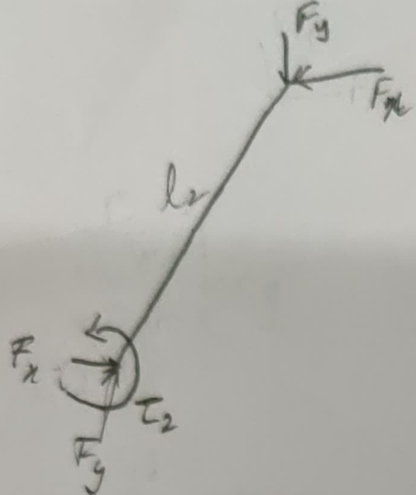
Assuming no gravity

FBD for link 1




$$\tau_1 = -F_x l_1 s q_1 + F_y l_1 c q_1$$

# FBD of Link 2




$$\tau_2 = -F_x l_2 s q_2 + F_y l_2 c q_2 \cancel{+ m_2 g \frac{l_2}{2} c q_{12}}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad (4)$$

→ Now if we want the ~~...~~ robotic arm to stay at a point from wherever ~~to~~ the user deflects it.

→ Now if we take a point $(x_0, y_0)$ where we ~~...~~ want the end effector to stay.

→ Now the force ~~...~~ required to do so is –

$$\left.\begin{aligned} F_x &= k(x - x_0) \\ F_y &= k(y - y_0) \end{aligned}\right] \text{ k - user defined stiffness}$$

How we need to account for dynamics

Lagrange's Equations

Lagrangian $L = K - U$

$K \downarrow$ Kinetic energy

$U \hookrightarrow$ Potential energy

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = Q_i'} \quad (5)$$

$i = 1, 2, 3, \cdots, n$

$Q_i'$ = generalized forces

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{Pure rotation of line 1}} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{Rotation of link 2 about its cm}}$$

$$+ \underbrace{\frac{1}{2} m_2 V_{c_2}^2}_{\text{Translation of CM of Link 2}}$$

$$V_{c_2}^2 = (l_1\dot{q}_1)^2 + \left(\frac{1}{2} l_2 \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

P.E.

$$V = m_1 g \frac{l_1}{2} sq_1 + m_2 g \left(l_1 sq_1 + \frac{l_2}{2} sq_2\right)$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} cq_2 + m_2 g l_1 cq_2 = \tau_1 \quad -(i)$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_1 - \dot{q}_2) \sin(q_1 - q_2)$$

$$+ m_2 g \frac{l_2}{2} s q_2 = \tau_2 \quad \text{— (ii)}$$

(i) and (ii) both combine to give eqn 6