# Assignment - 2

Q.1. $R \rightarrow$ Rotation matrix

we know that $RR^T = I$ & $a \times b = S(a)b$

$$a \times b = S(a)b$$
$$R(a \times b) = RS(a)b$$
$$Ra \times Rb = RS(a)b$$
$$S(Ra)Rb = RS(a)b$$
$$R^T S(Ra)Rb = R^T R S(a)b$$
$$R^T S(Ra)Rb = I \cdot S(a)b$$
$$R^T S(Ra)R = S(a) \qquad \because b \rightarrow \text{an arbitrary vector}$$
$$RR^T S(Ra)R = RS(a)$$
$$S(Ra)RR^T = RS(a)R^T$$
$$\boxed{S(Ra) = RS(a)R^T}$$

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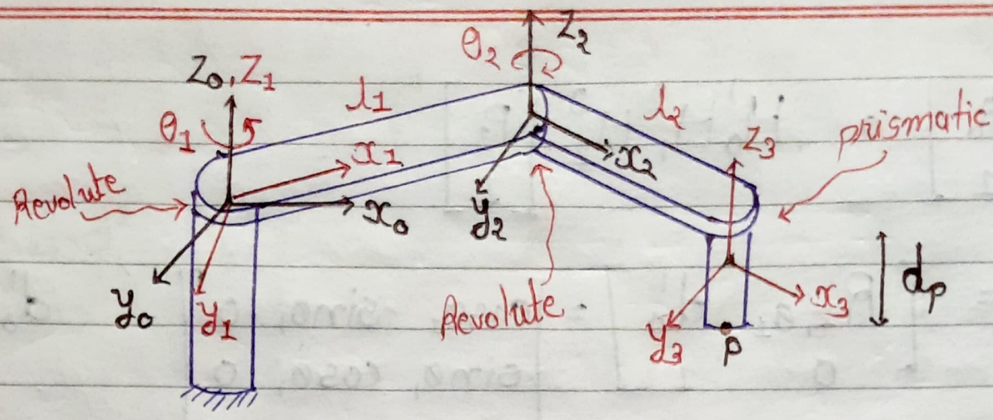
Q.2




(RRP SCARA Configuration)

we can define as

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

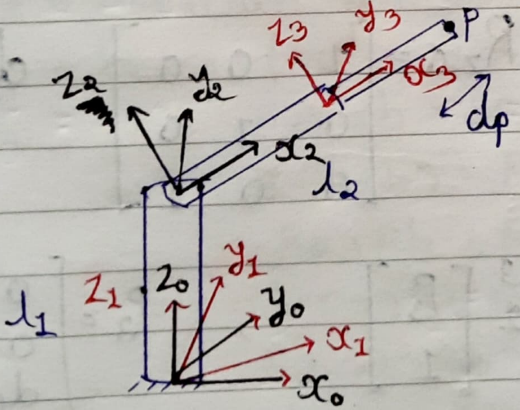
$\rightarrow$ $H_0^1 = \begin{bmatrix} R_{z,\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix}$ $R_{z,\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\rightarrow$ $H_1^2 = \begin{bmatrix} R_{z,\theta_2} & d_1^2 \\ 0 & 1 \end{bmatrix}$ $R_{z,\theta_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

$\rightarrow$ $H_2^3 = \begin{bmatrix} R_{z,0} & d_2^3 \\ 0 & 1 \end{bmatrix}$ $R_{z,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 \\ 0 \\ d_p \end{bmatrix}$$

Q.3




( RRP Stanford Configuration )

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_{z,\theta_1} & d_0^1 \\ 0 & 1 \end{bmatrix} \cdot R_{z,\theta_1} = \begin{bmatrix} \cos\theta, & -\sin\theta, & 0 \\ \sin\theta, & \cos\theta, & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} \qquad R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & 0 & -1 \\ \sin\theta_2 & \cos\theta_2 & 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}, \quad R_2^3 = I \quad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} d_p \\ 0 \\ 0 \end{bmatrix}$$

Q.6 There are many types of gearboxes typically used with motors in a robotic application, here are a few examples for the same.

(i) Cycloid Drives :-

A cycloid drives is a mechanism for reducing the speed of an input shaft by a certain ratio. It consists of a planet wheel which moves in a wobly cycloidal manner given input motion. Cycloidal speed reducers are capable of relatively high ratio in compact sizes. They are used when there is a heavy cam wheel. They are used in cranes, boats etc.

(ii) Planetary gearhead:

They are used to transfer the largest torque in the most compact form. They consist of planet gears. They are suited for rotating prime mover like electrical motors. They are used in helicopters, automobiles, etc.

→ Drones have propellers which require high speed. i.e. high RPM to help them hover and move. Gear boxes are generally used to increase the torque to the system. But in drons we only require high speed propellers. So a simple gear will suffice i.e. gear connecting the motor and the propellers.

Q.5 Here, there are total 4 transformation.

↳ drone base (z-direction) = 10 m
↳ drone rotation about x = 30°
↳ drone rotation about [current z] = 60°
↳ obstacle (z-direction) = 3 m [above drone]

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_4 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \quad R_0^1 = R_{z,0} = I_{3\times3} \quad \& \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \quad R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & \sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \quad \& \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \quad R_2^3 = \begin{bmatrix} \cos 60 & \sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \& \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & -\sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & -\sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.59 \\ 1 \end{bmatrix} \qquad \boxed{P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.59 \end{bmatrix}}$$

Q.7 RRP SCARA configuration

$$J = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \\ Z_0 & Z_1 & 0 \end{bmatrix}$$

Revolute joint — Revolute joint — prismatic joint

$$O_1 = \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ 0 \end{bmatrix} \quad O_3 = \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ d_3 \end{bmatrix}$$

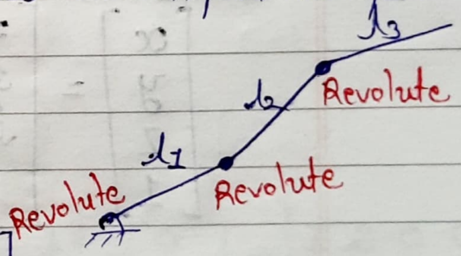
Prismatic

due to movement of 'P' joint

$$Z_0 = Z_1 = K = [0\ 0\ 1]^T$$
$$Z_2 = -K$$

$$J = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} & -a_2 S_{12} & 0 \\ a_1 C_1 + a_2 C_{12} & a_2 C_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

**Q.9** RRR configuration [Elbow manipulator (planner)]




$$J = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 C_1 \\ l_1 S_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} \\ l_1 S_1 + l_2 S_{12} + l_3 S_{123} \\ 0 \end{bmatrix}$$

$$Z_0 = Z_1 = Z_2 = k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\cancel{J} = \begin{matrix} \cancel{-a_1 s_1 - a_2 s_{12} - a_3 s_{123}} \\ \cancel{a_1 c_1 + a_2 c_{12} + a} \end{matrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$