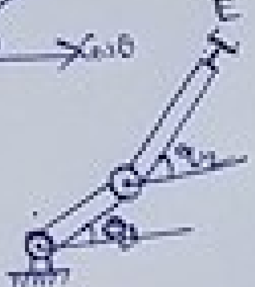
a) Forward Kinematics [2R manipulator]



l_1, l_2 lengths of links

m_1, m_2 mass of links

(x,y) E is the end effector coordinates.

For getting the location of E we need angles made by each joints.

$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\} \rightarrow (1) \text{ Position of E}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cq_1 & cq_2 \\ sq_1 & sq_2 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

for velocity coordinates/directions $\frac{dx}{dt} = v = \dot{x}$ $\left[\frac{d\theta}{dt} = \omega\right]$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -sq_1 & -sq_2 \\ cq_1 & cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \rightarrow (2)$$ $\left.\begin{matrix}\dot{q}_1 \\ \dot{q}_2\end{matrix}\right\} \rightarrow$ angular velocities

Inverse Kinematics :-

For getting exact location of end effect we need to get the angles q_1, q_2 these can be calculated through inverse kinematics.

from eq (1)
$$x = l_1 cq_1 + l_2 cq_2$$
$$y = l_1 sq_1 + l_2 sq_2$$

squaring on b.s and adding

$$x^2 + y^2 = l_1^2(c^2q_1 + s^2q_1) + l_2^2(c^2q_2 + s^2q_2) + 2l_1l_2(cq_1 \cdot cq_2 + sq_1 \cdot sq_2)$$

$\because \sin^2\theta + \cos^2\theta = 1$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

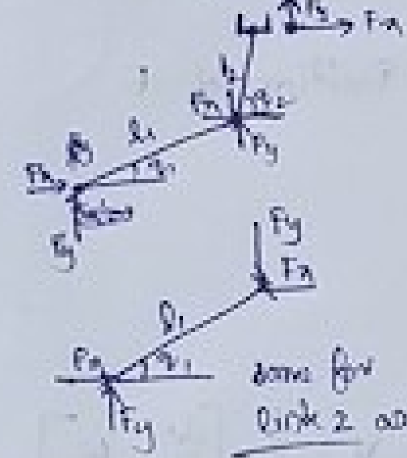
$$x^2+y^2 = l_1^2 + l_2^2 + 2l_1l_2\cos(q_2-q_1)$$

$$q_2-q_1 = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

as $q_2 - q_1 = \theta_2$

$$\theta_1 = \tan^{-1}(y/x) - \tan^{-1}\left(\frac{l_2\sin\theta_2}{l_1+l_2\cos\theta_2}\right)$$

$$q_2 = q_1 + \theta$$



for link 1 (assume gravity is negligible)

Torque $\tau_1 = -l_1F_x\sin q_1 + l_1F_y\cos q_1$

Torque $\tau_2 = -l_2F_x\sin q_2 + l_2F_y\cos q_2$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}-l_1\sin q_1 & l_1\cos q_1\\ -l_2\sin q_2 & l_2\cos q_2\end{bmatrix}\begin{bmatrix}F_x\\ F_y\end{bmatrix}$$

same for link 2 as well

- from lagrangian equation $L = K - V$

$L = K - V$ (kinetic energy - Potential energy)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad i = 1, 2, 3, \ldots n$$

$$K = \underbrace{\tfrac{1}{2}\left(\tfrac{1}{3}m_1l_1^2\right)\dot q_1^2}_{\text{rotation of link 1}} + \underbrace{\tfrac{1}{2}\left(\tfrac{1}{12}m_2l_2^2\right)\dot q_2^2}_{\text{rotation of link 2 about centre of mass}} + \underbrace{\tfrac{1}{2}m_2v_{c2}^2}_{\text{translation of centre of mass}}$$

$$v_{c_2}^2 = (l_1\dot q_1)^2 + \left(\frac{l_2}{2}\dot q_2\right)^2 + 2l_1\dot q_1\frac{l_2}{2}\dot q_2\cos(q_2-q_1)$$

$$V = m_1g\frac{l_1}{2}s q_1 + m_2g\left(l_1 s q_1 + \frac{l_2}{2}s q_2\right)$$

$$\tau_1 = \tfrac{1}{3}m_1l_1^2\ddot q_1 + m_2l_1^2\ddot q_1 + m_2\frac{l_1l_2}{2}\ddot q_2\cos(q_2-q_1) - \frac{m_2l_1l_2}{2}\dot q_2(\dot q_2-\dot q_1)\sin(q_2-q_1)$$
$$+ m_1g\frac{l_1}{2}\cos q_1 + m_2g\frac{l_2}{2}\cos q_2$$

$$\tau_2 = \tfrac{1}{3}m_2l_2^2\ddot q_2 + \frac{m_2l_2^2}{4}\ddot q_2 + \frac{m_2l_1l_2}{2}\ddot q_1\cos(q_2-q_1) - \frac{m_2l_1l_2}{2}\dot q_1(\dot q_2-\dot q_1)\sin(q_2-q_1)$$
$$+ \frac{m_2gl_2}{2}\sin q_2$$