30/08/22 ME639 | ASSIGNMENT - 2

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1.
$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & +a_x & 0 \end{bmatrix}$$

for simplicity, assume
$a_x = c$
$a_y = b$
$a_z = a$

L.H.S = $R S(a) R^T$

Let $R = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R^T = \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$R S(a) R^T = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -a & b \\ +a & 0 & -c \\ -b & c & 0 \end{bmatrix} \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -as_\theta & -ac_\theta & bc_\theta + cs_\theta \\ ac_\theta & -as_\theta & bs_\theta - cc_\theta \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a & bc_\theta + cs_\theta \\ a & 0 & bs_\theta - cc_\theta \\ -(bc_\theta + cs_\theta) & -bs_\theta + cc_\theta & 0 \end{bmatrix} \quad \text{— (1)}$$

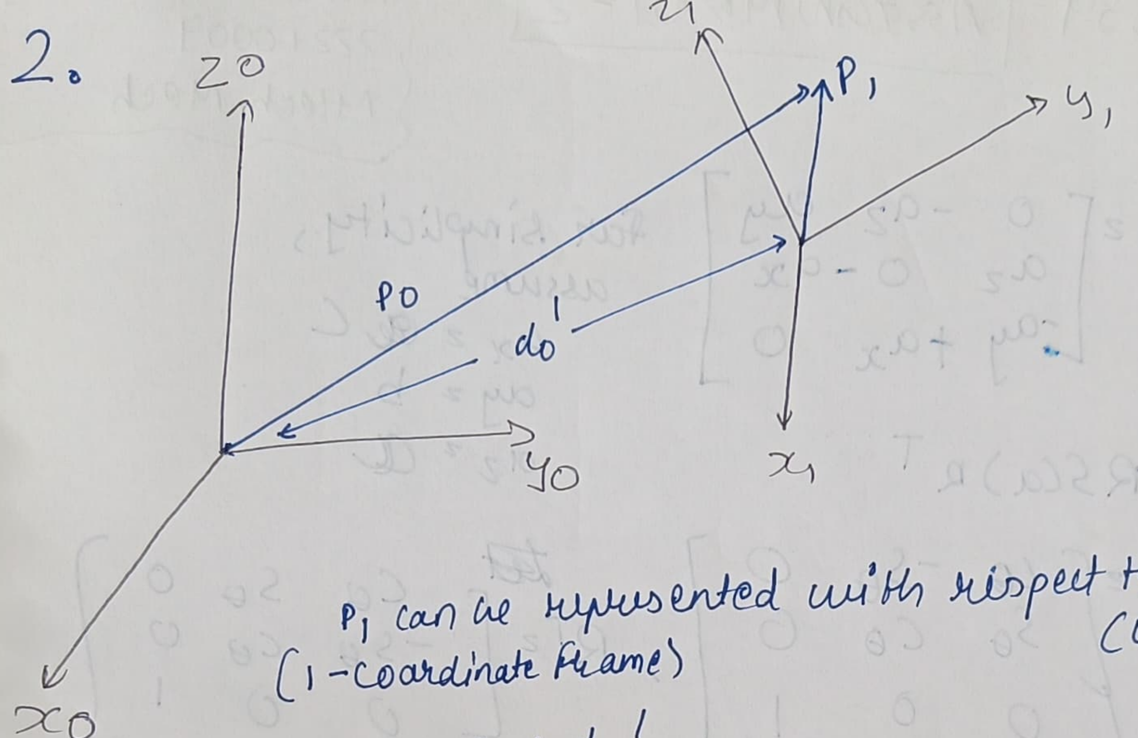
RHS = $S(Ra)$

$$Ra = \begin{bmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} cc_\theta - bs_\theta \\ cs_\theta + bc_\theta \\ a \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a & bc_\theta + cs_\theta \\ a & 0 & bs_\theta - cc_\theta \\ -(bc_\theta + cs_\theta) & -bs_\theta + cc_\theta & 0 \end{bmatrix} \quad \text{— (2)}$$

Hence, LHS = RHS

2.



P_1 can be represented with respect to P_0 (1-coordinate frame) (0-coordinate frame)

$$P_1 = P_0 + d_0^1$$

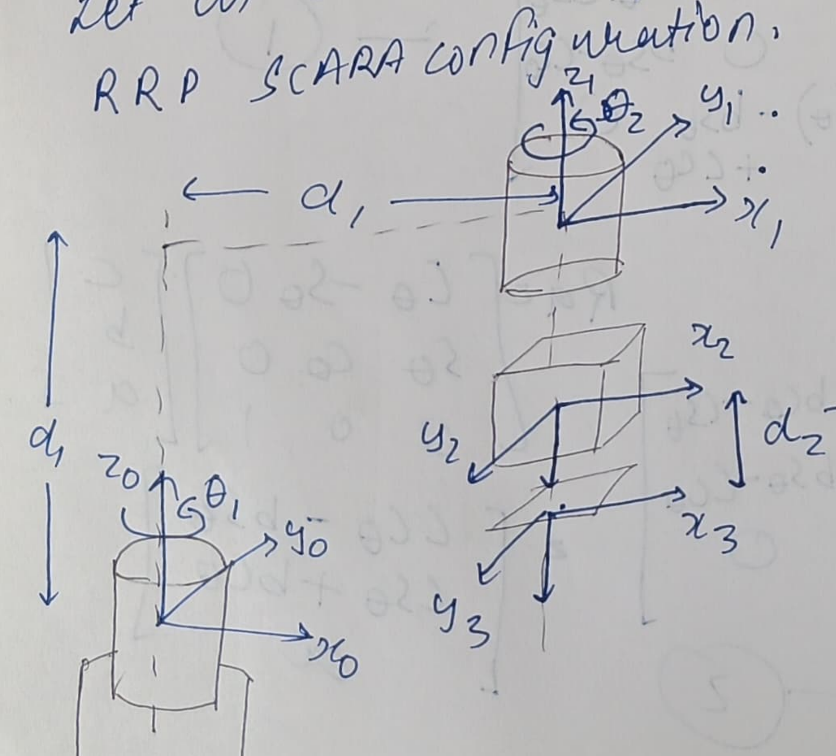
$$P_0 = R_0^1 P_1 + d_0^1$$

similarly

$$\therefore P_1 = R_1^2 P_2 + d_1^2$$

$$\therefore P_0 = R_0^1 R_1^2 P_2 + R_0^1 d_1^2 + d_0^1$$

Using same concept of coordinate ~~plan~~ frame let us derive Homogenous transformation for RRP SCARA configuration.



DH Parameter

Link	θ	α	a	d
1	θ_1	0	a_1	d_1
2	θ_2	180	0	0
3	0	0	0	d_2

$$H_n^{n-1} = \begin{bmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & r_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -s\alpha_n c\theta_n & r_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad d_1 = 0$$

$$H_2^1 = \begin{bmatrix} c\theta_2 & s\theta_2 & 0 & 0 \\ s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

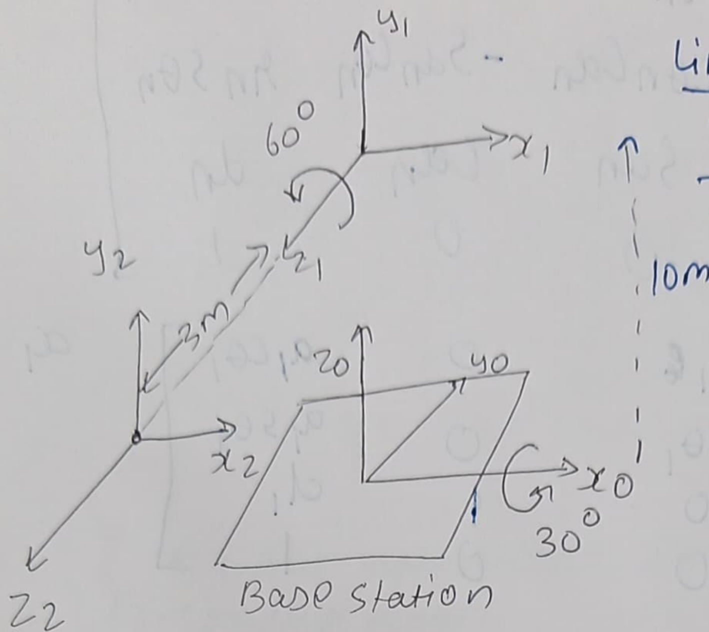
$$H_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^0 = H_1^0 H_2^1 H_3^2 = \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) & 0 & a_1 c\theta_1 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -d_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_3^0 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} a_1 c\theta_1 \\ a_1 s\theta_1 \\ d_1 \end{bmatrix} = P_0}$$

5.



Link	θ	α	a	d
1	0	30°	0	10
2	60°	0	3	0

$$H^0_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{30} & -S_{30} & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H^1_2 = \begin{bmatrix} 1 & -S_{60} & 0 & 0 \\ 0 & C_{60} & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H^0_2 = \begin{bmatrix} 1 & -S_{60} & 0 & 0 \\ 0 & C_{30}C_{60} & -S_{30} & 3C_{30} \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H^0_2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -3S_{30} \\ 13 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 13 \end{bmatrix}$$

6. Pros & cons of 6-speed Gearbox

Pros :-

1. Provides 6 diff gear ratios from 11.6:1 to 1300.9:1
2. Cam is also installed to change vertical to rectilinear movement.

Cons :-

1. Gearbox ~~is~~ has complex structure so requires more space to fit in.

(2) 4-speed GB

Pros :- 1. Higher speed setting is suitable for car model.
2. Lower speed setting is suitable for tracked vehicle model.

Cons :- ~~So~~.

(3) worm gearbox

Pros: 1. Can produce very high speed reduction.

(5) Planetary gearbox

Pros: 1. Conversion of high RPM to high torque low RPM
2. Used in precision instrument for reliability & accuracy

(one/two of them are actually seen during lectures)

~~(4) High~~

7. Jacobian · For RRP SCARA (DOF=3)

For revolute joint

$$J_i^0 = \begin{bmatrix} z_{i-1}^0 \times (O_n - O_{i-1}^0) \\ z_{i-1}^0 \end{bmatrix}$$

For prismatic joint

$$J_i^0 = \begin{bmatrix} z_{i-1}^0 \\ 0 \end{bmatrix}$$

"Assuming no losses in Power".

Considering 1, 2 joints to be revolute & 3 as prismatic.

$$J = [J_1 \ J_2 \ J_3] = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

For revolute, $d_0^n = d_0^{i-1} + R_0^{i-1} d_{i-1}^n$

$$d_0^n - d_0^{i-1} = R_0^{i-1} d_{i-1}^n$$

$$O_n - O_{i-1} = R_0^{i-1} d_{i-1}^n$$

$\therefore O_3 - O_0 = R_0^0 d_0^3 = I d_0^3$

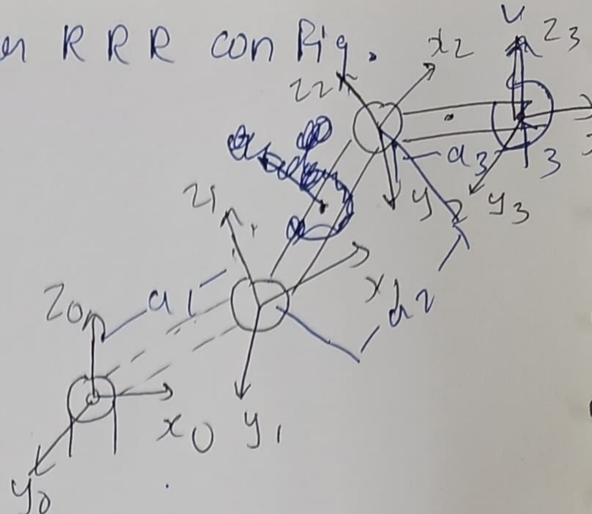
$$O_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{\theta_1} c_{\theta_2} \\ a_1 s_2 + a_2 s_{\theta_1} s_{\theta_2} \\ d_3 \end{bmatrix}, \quad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{\theta_1} c_{\theta_2} \\ a_1 s_1 + a_2 s_{\theta_1} s_{\theta_2} \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

Let take $z_0 = z_1 = k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $z_2 = -k = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{\theta_1} s_{\theta_2} & -a_2 s_{\theta_1} s_{\theta_2} & 0 \\ a_1 c_1 + a_2 c_{\theta_1} c_{\theta_2} & a_2 c_{\theta_1} c_{\theta_2} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

9. Manipulater Jacobian for RRR config.



$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J \dot{q}$$

$$J = [J_1 \;\; J_2 \;\; J_3]$$

$$J_1 = \begin{bmatrix} z_0 \times (O_3 - O_0) \\ z_0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} z_1 \times (O_3 - O_1) \\ z_1 \end{bmatrix} \qquad J_3 = \begin{bmatrix} z_2 \times (O_3 - O_2) \\ z_2 \end{bmatrix}$$

$$\cancel{J = [z_0} \qquad O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c\theta_1 c\theta_2 \\ a_1 s_1 + a_2 s\theta_1 s\theta_2 \\ 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad O_3 = \begin{bmatrix} a_1 c_1 + a_2 c\theta_1 c\theta_2 \\ a_1 s_2 + a_2 s\theta_1 s\theta_2 \\ a_3 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s\theta_1 s\theta_2 & -a_2 s\theta_1 s\theta_2 & 0 \\ a_1 c_1 + a_2 c\theta_1 c\theta_2 & a_2 c\theta_1 c\theta_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$