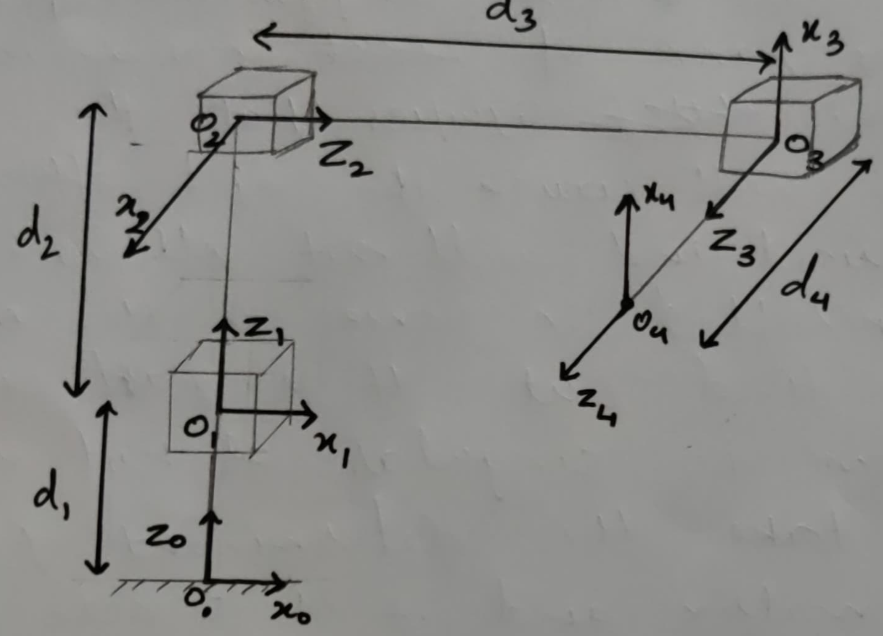
ME 639: Assignment 3

Name: Ravi Dhorajla
Roll No: 20110162

Q1) Singularities are ~~$~~ those configurations for which the rank of ~~mat~~ Jacobian matrix ~~de~~creases. This ~~thtf~~ signifies that the singularities will cause the end effector to be constrained and will not allow it to be controlled in some or the other way. To find out if a particular configuration is singular or not we have to take the determinant of the Jacobian matrix and if it is zero than it will be a singular configuration.

~~we can also take~~

Q 5)



D H Parameters

	a_i	α_i	d_i	θ_i
1	0	0	d_1^*	0
2	0	$-\pi/2$	d_2^*	$-\pi/2$
3	0	$-\pi/2$	d_3^*	$-\pi/2$
4	0	0	d_4^*	0

$*$ - variable

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

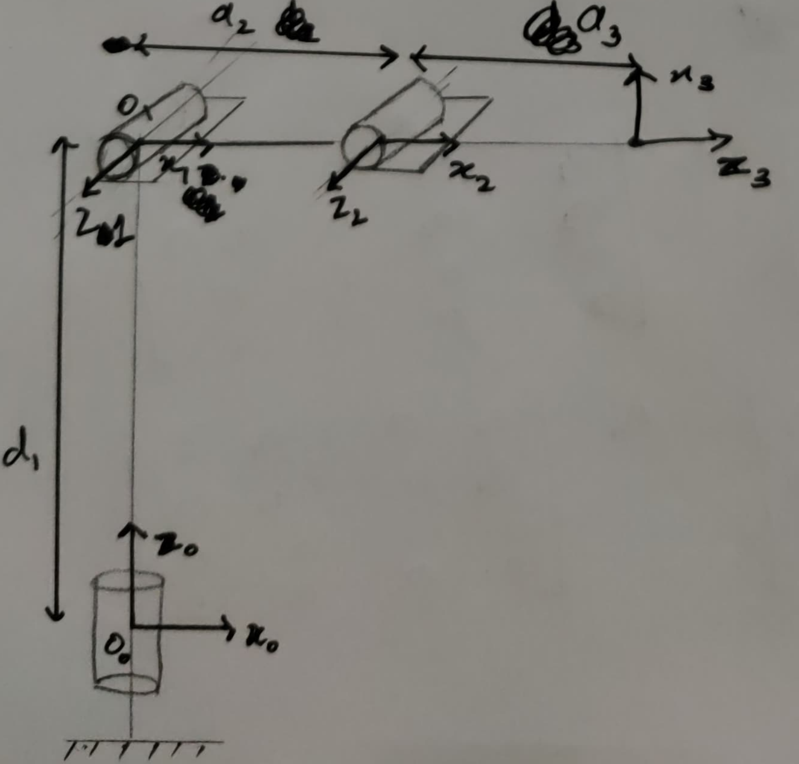
$$T_0^4 = A_1 A_2 A_3 A_4$$

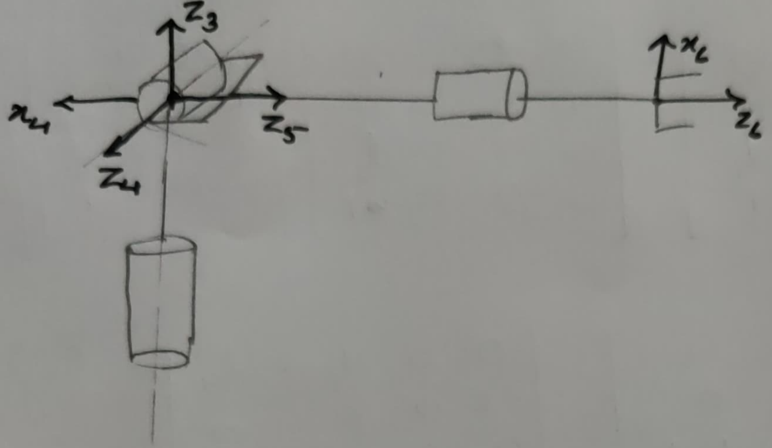
$$T_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & d_4 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_0^4 = \begin{bmatrix} 0 & -1 & 0 & d_3 \\ 0 & 0 & -1 & -d_4 \\ 1 & 0 & 0 & d_1+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Q6)





D H Parameters

	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	$\pi/2$	0	θ_3^*
4	0	$-\pi/2$	0	θ_4^*
5	0	$\pi/2$	0	θ_5^*
6	0	0	d_6	θ_6^*

$*$ - variable

$$A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c\theta_3 & 0 & s\theta_3 & a_3 c\theta_3 \\ s\theta_3 & 0 & -c\theta_3 & a_3 s\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now the ~~*~~foward ~~k~~kinematic equations are given as following

$$T_0^6 = A_1 A_2 A_3 A_4 A_5 A_6$$

~~a~~

Q7) The ~~thre~~ three different configurations are -

1) ~~R~~ Direct driven
2) Remotely driven
3) 5-bar parallelogram ~~at the~~ arrangement

Direct Driven

→ The angles are measured ~~with~~ relative to the previous one.

~~→ The motor link 2 stays~~

→ The motors need ~~to~~ to be constrained in such a way that we get ~~another~~ 2nd angle relative to the 1st one.

Remotely Driven

→ The angles are individually measured ~~fr~~ from the horizontal reference line and the are indipendent of each others.

→ The motors need to be ~~in a wa~~ attached at ~~at~~ the base and both of the are driven ~~s~~ seperately.

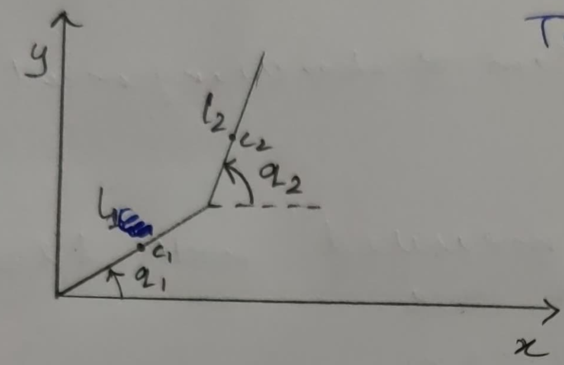
5 - bar Parallelogram arrangement

→ The angles are ~~at~~ measured individually wrt the links 1 and 2, ~~and~~ and they ~~do not di~~ are not relative in nature.

The link ~~on~~ 1 and 2 are attached to ~~The motors should be to~~ the same point.

→ The motors should be independently controlled but need to be at the base as well as at the same line.

Q8)



~~Taking~~ Taking I_1, I_2 as moment of inertia at centre of individual links

l_{c_1} and l_{c_2} are COM.

l_1 and l_2 are lengths

m_1 and m_2 are mass

$$V_{c_1} = \begin{bmatrix} -l_{c_1} s q_1 \\ l_{c_1} c q_1 \\ 0 \end{bmatrix} \dot{q}_1 \qquad V_{c_2} = \begin{bmatrix} -l_1 s q_1 & -l_{c_2} s q_2 \\ l_1 c q_1 & l_{c_2} c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

$$D(q) = \begin{bmatrix} m_1 l_{c_1}^2 + m_2 l_1^2 + I_1 & m_1 l_1 l_{c_2} c(q_2 - q_1) \\ m_2 l_1 l_{c_2} c(q_2 - q_1) & m_2 l_{c_2}^2 \; I_2 \end{bmatrix}$$

Christoffel coefficients

$$C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = -m_2 l_1 l_{c_2} \, s(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c_2} \sin(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

$$V = m_1 g l_{c_1} s q_1 + m_2 g (l_1 s q_1 + l_{c_2} s q_2)$$

$$\Phi_1 = \frac{\partial V}{\partial q_1} = (m_1 l_{c_1} + m_2 l_1) g \, c q_1$$

$$\Phi_2 = \frac{\partial V}{\partial q_2} = m_2 l_{c_2} g \cos q_2$$

$$\boxed{\begin{aligned} \tau_1 &= d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_2^2 + \phi_1 \\ \tau_2 &= d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \Phi_2 \end{aligned}}$$

where

$$d_{11} = m_1 l_{c_1}^2 + m_2 l_1^2 + I_1$$

$$d_{12} = d_{21} = m_1 l_1 l_{c_2} \, c(q_2 - q_1)$$

$$d_{22} = m_2 l_{c_2}^2 + I_2$$

We get the same result as that of the mini-project.

Q 10) Equations of Motion

Provided $D(q)$ and $V(q)$

$$L = K - V$$

where $K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$

$V = V(q)$

$$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k}\right] \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$$c_{ijk} = \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\}$$

Christoffel symbols

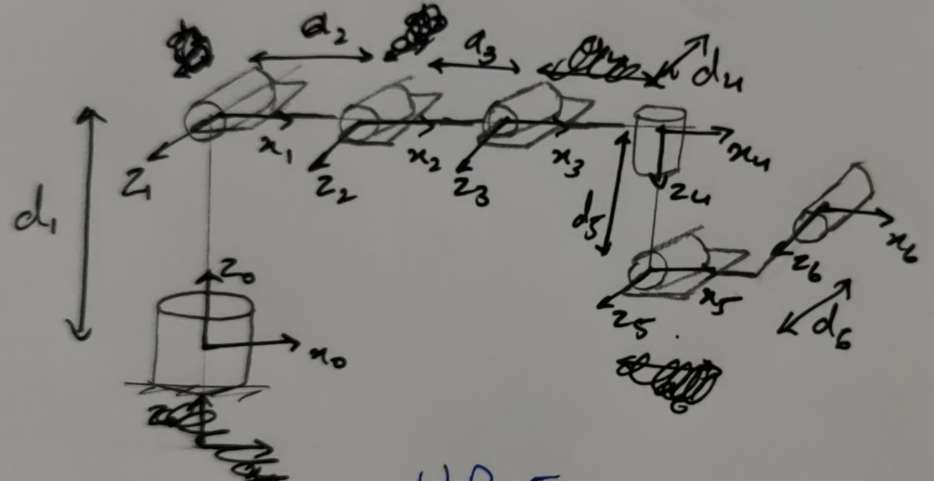
$$\phi_k = \frac{\partial V}{\partial q_k}$$

$$\tau_k = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} c_{ijk} \dot{q}_i \dot{q}_j + \phi_k(q)$$

$$\boxed{\tau = D(q)\, \ddot{q} + C(q, \dot{q})\, \dot{q} + g(q)}$$

Q12) ~~That~~ Universal Robot 5 (UR5)

The robot has 2 links in total and 6 joint in total. All of the joints are revolute joints.



UR5

DH Parameters

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*
4	0	d_4	$\pi/2$	θ_4^*
5	0	d_5	$-\pi/2$	θ_5^*
6	0	d_6	0	θ_6^*

* variable