Q1. To prove :- $R(S(a))R^T = S(Ra)$

consider an arbitrary vector b.

$$S(Ra)b = (Ra) \times b$$

$$R^T S(Ra) b = R^T Ra \times R^T b.$$

because R^T is orthogonal vector

$$R^T(a \times b) = R^T a \times R^T b.$$

$$R^T S(Ra) b = a \times (R^T b)$$

As $R^T R = I$ as R is orthogonal.

$$R^T S(Ra) b = S(a) R^T b.$$

multiplying both sides by R.

$$R R^T S(Ra) b = R S(a) R^T b.$$

$$S(Ra) b = R(S(a)) R^T b.$$

since b was any arbitrarily chosen vector the above expression satisfies for all b.

$$\therefore S(R(a)) = R(S(a)) R^T$$

Hence proved.

2.)

$$H_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rotate about z_0 by θ_1

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

translate along link to find O_1

$$H_3 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \ldots \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\pi & -s\pi & 0 \\ 0 & s\pi & c\pi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

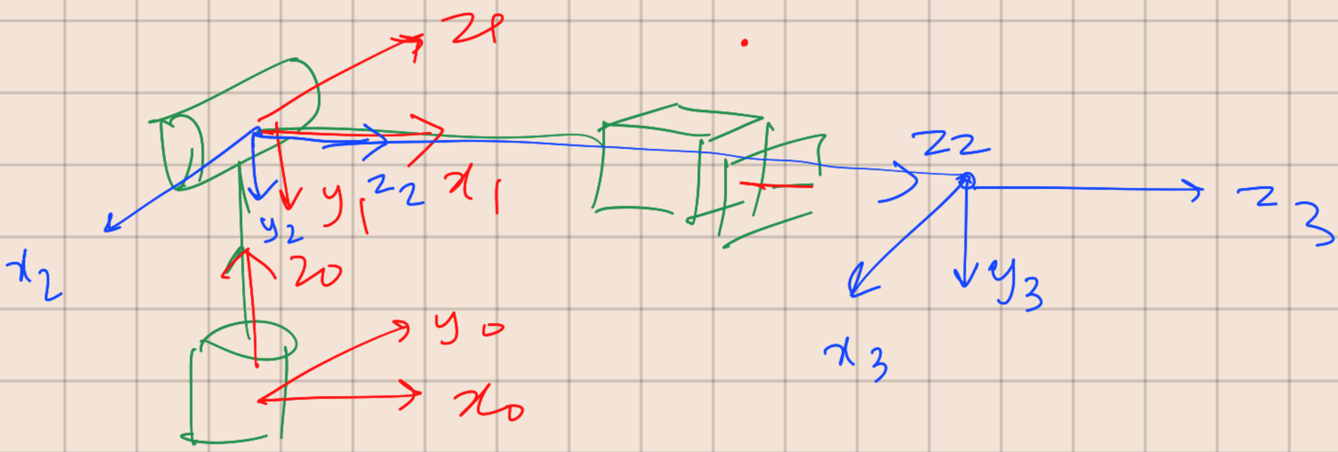
$$H_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6$$

$$= \begin{bmatrix} c(\theta_1+\theta_2) & s(\theta_1+\theta_2) & 0 & a_2 c(\theta_1+\theta_2) + a_1 c\theta_1 \\ s(\theta_1+\theta_2) & -c(\theta_1+\theta_2) & 0 & a_2 s(\theta_1+\theta_2) + a_1 s\theta_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} a_2 c(\theta_1+\theta_2) + a_1 c\theta_1 \\ a_2 s(\theta_1+\theta_2) + a_1 s\theta_1 \end{bmatrix}$$

4.)



$$H_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c(-\pi/2) & -s(-\pi/2) & 0 \\ 0 & s(\pi/2) & c\pi/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} c\pi/2 & 0 & -s\pi/2 & 0 & 0 \\ 0 & 1 & 0 & & \\ s\pi/2 & 0 & c\pi/2 & 0 & 0 \\ & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \end{bmatrix}$$

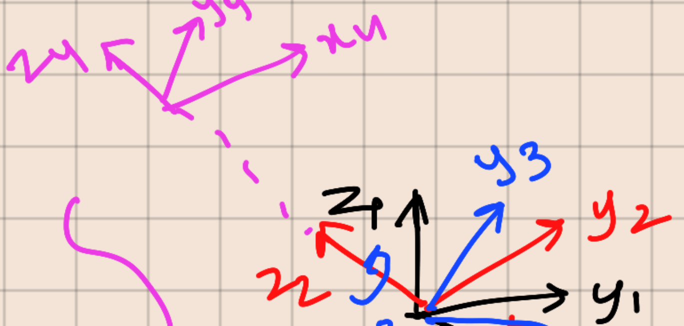
$$H_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_2+d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^5 = H_1 H_2 H_3 H_4 H_5$$

$$T_0^5 = \begin{bmatrix} -s\theta_1 & -c\theta_1 s\theta_2 & -c\theta_1 c\theta_2 & -c\theta_1 c\theta_2(a_2+d_3) \\ c\theta_1 & -s\theta_1 s\theta_2 & -c\theta_2 s\theta_1 & -c\theta_2 s\theta_1 (a_2+d_3) \\ 0 & -c\theta_2 & s\theta_2 & d_1 + s\theta_2 (a_2+d_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p_0 = \begin{bmatrix} -c\theta_1 c\theta_2 (a_2+d_3) \\ -c\theta_2 s\theta_1 (a_2+d_3) \\ d_1 + s\theta_2 (a_2+d_3) \end{bmatrix}$$

5.)



$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c30 & -s30 & 0 \\ 0 & s30 & c30 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} c60 & -s60 & 0 & 0 \\ s60 & c60 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^4 = H_0^1 H_1^2 H_2^3 H_3^4$$

$$H_0^4 = \begin{bmatrix} 0.5 & -8.66 & 0 & 0 \\ 0.75 & 0.433 & -0.5 & -1.5 \\ 0.433 & 0.25 & 0.866 & 12.5981 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0^4 = \begin{bmatrix} 0 \\ -1.5 \\ 12.5981 \end{bmatrix}$$

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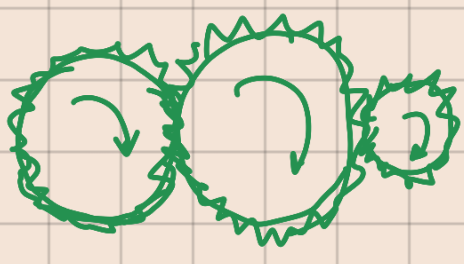
6.) Types of gearbox.

1. Spur Gear Box.



Spur gear engage to transmit power accross shafts.

Pros:-

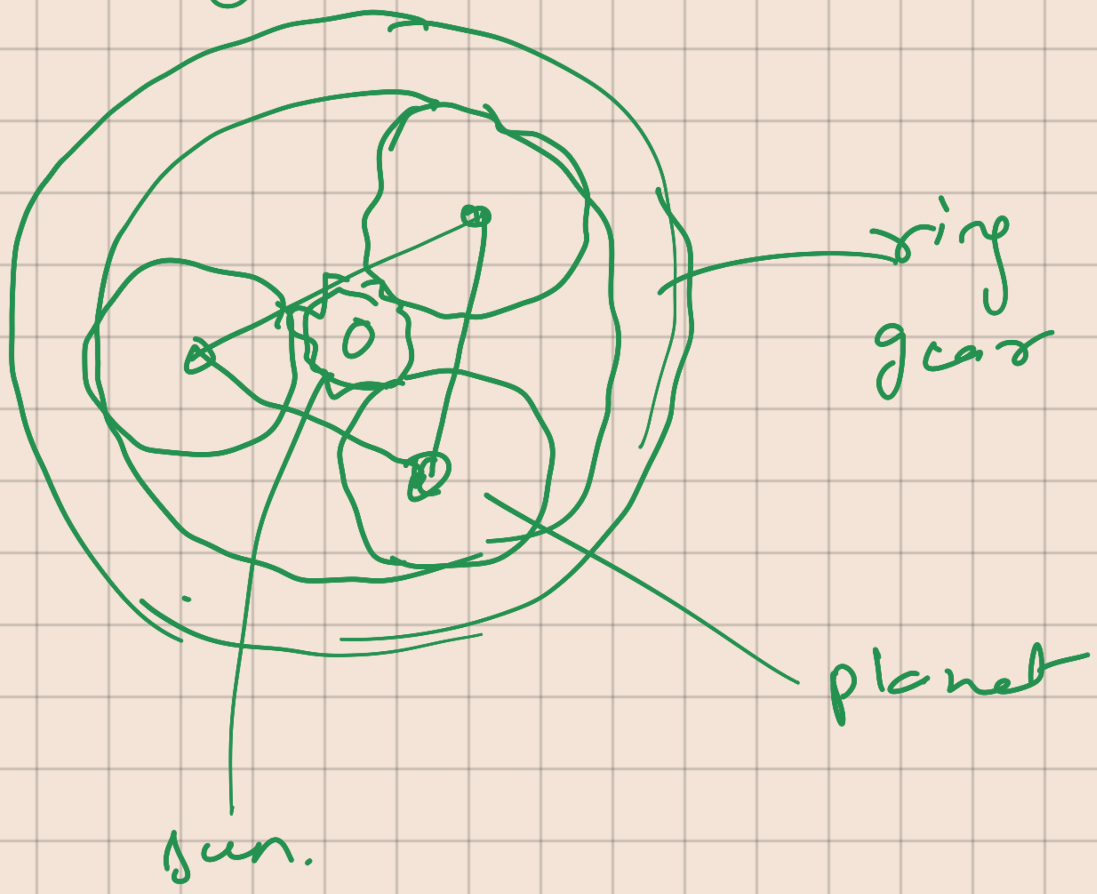
1. Simple mechanism required when shifting gears with spur gear box.

2. Ease of manufacturing.

Cons:-

1. Not compact
2. Not symmetric

2 Planetary Gearbox.



Ring gear is fixed. Planets connected through output shaft. Sun at input.

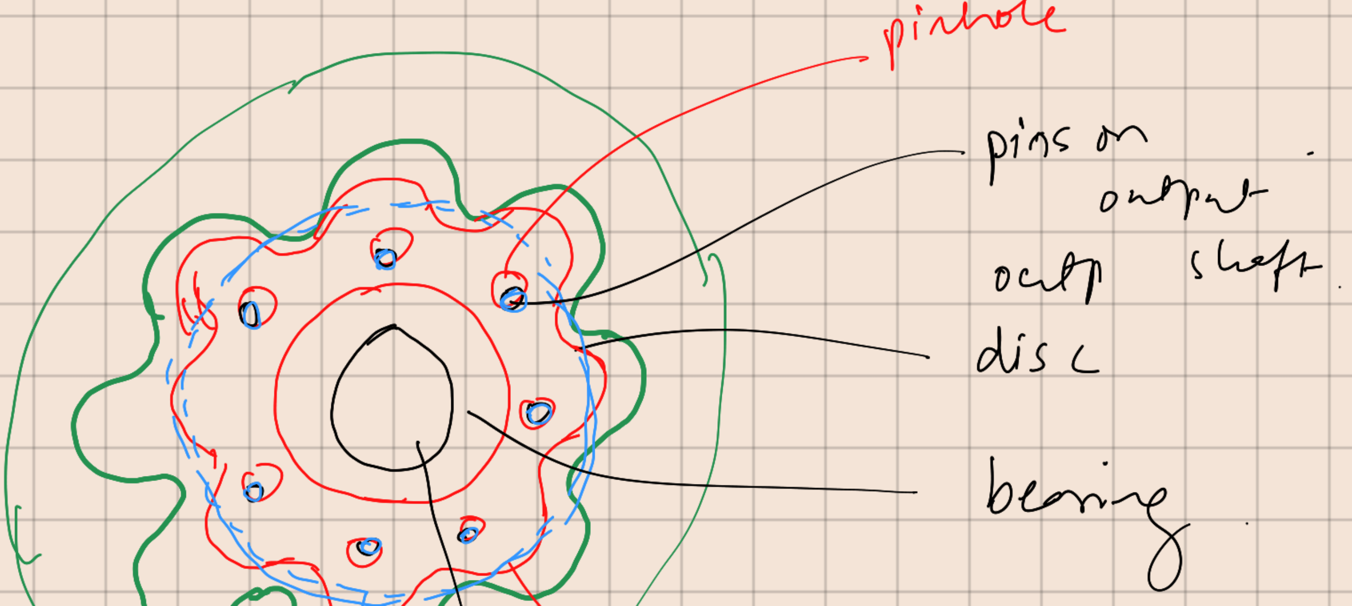
Pros :-

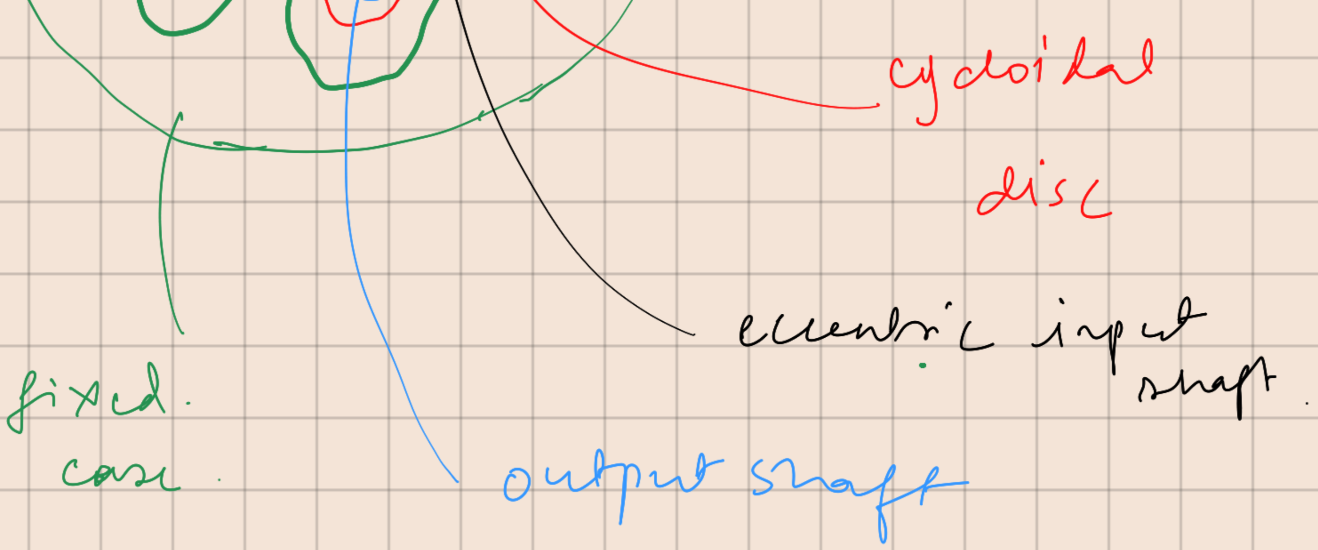
1. compact
2. high gear ratio using multistaging.
3. For a single stage friction is less hence backdrivable.
4. Symmetric. (less vibrations)

Cons :-

1. Gear ratio limited. compared to cycloidal & harmonic.

3. Cycloidal Gearbox.





Eccentric shaft rotates cycloidal disc. which rotates in case pushing the pins of output shaft.

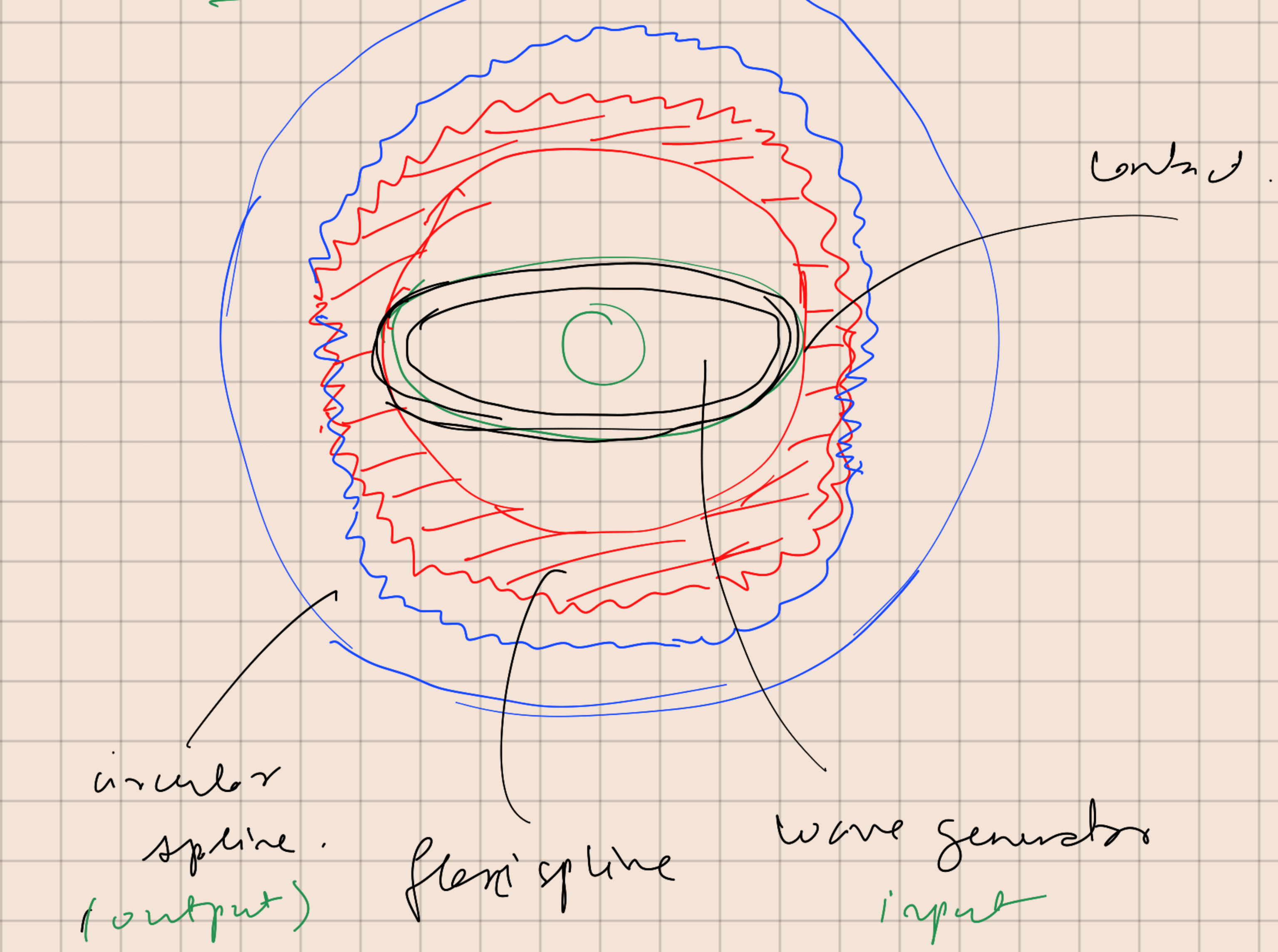
Pros

1. High gear ratio in compact size.
2. Backdrivable even at high gear ratio.
3. Gear ratio > planetary for same size.

Cons

1. Difficult construction
2. Cost
3. Vibrations due to eccentricity.
4. Not a constant torque output Small variations.

4. Harmonic Drive.



Flexispline is flexible material object. When pushed locally by wave generator (elliptical) flexispline teeth engage with circular spline output near the contact.

Pros :-

1. large gear ratio for size.

Harmonic > cycloidal > Planetary.

2. Silent operation.

Cons :- 1. Durable and flexible part costly.

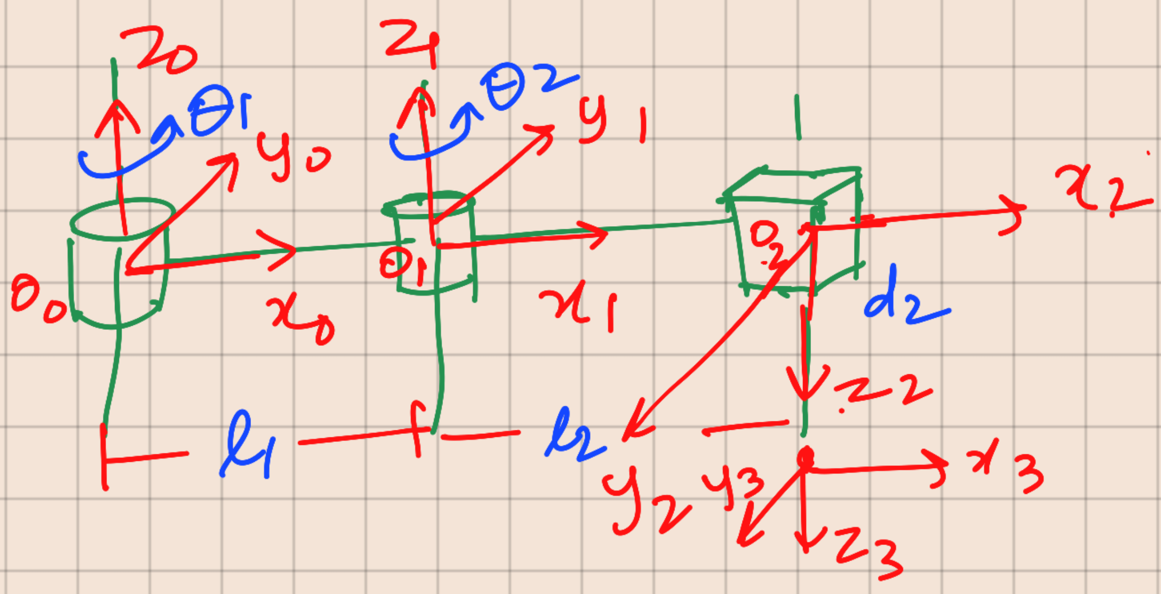
2. complex manufacturing.

Why drones typically don't have gearboxes ?

Drones require high speed motors for small propellors and high torque motors for large diameter propellors. Hence gearboxes are desirable but !! Every ounce you add to weight of drone reduces thrust to weight ratio. Hence appropriate motors are chosen to avoid weight of gearboxes.

Gearboxes contain friction losses and this reduces efficiency and hence flight time for same weight. "Hence it is desirable to choose right motor without gears.

7)



$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$J_v = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix} \quad o_1 = \begin{bmatrix} a_1 c_1 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} a_1 s_1 \\ 0 \end{bmatrix}$$

$$o_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$o_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ -d_3 \end{bmatrix}$$

$$o_3 - o_0 = o_3$$

$$o_3 - o_1 = \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ -d_3 \end{bmatrix}$$

$$z_0 \times (o_3 - o_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ -d_3 \end{bmatrix}$$

$$= \begin{pmatrix} -a_2 s_{12} \\ a_2 c_{12} \end{pmatrix}$$

$$0$$

$$z_0 \times (o_3) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c_{12} + a_1 c_1 \\ a_2 s_{12} + a_1 s_1 \\ -d_3 \end{bmatrix}$$

$$= \begin{bmatrix} -a_2 s_{12} - a_1 s_1 \\ +a_2 c_{12} + a_1 c_1 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_2 s_{12} - a_1 s_1 & -a_2 s_{12} & 0 \\ a_2 c_{12} + a_1 c_1 & a_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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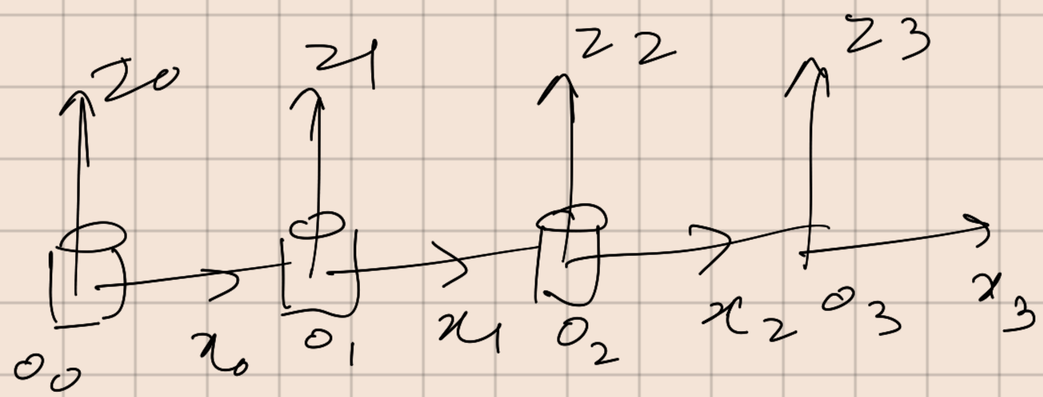
a)



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad O_1 = \begin{bmatrix} a_2 c_1 \\ a_2 s_1 \\ 0 \end{bmatrix} \quad O_2 = \begin{bmatrix} a_1 c_{12} \\ a_1 s_{12} \\ 0 \end{bmatrix} + O_1$$

$$O_3 = \begin{bmatrix} a_3 c_{123} \\ a_3 s_{123} \\ 0 \end{bmatrix} + O_2$$

$$O_3 - O_0 = \begin{bmatrix} a_3 c_{123} + a_1 c_{12} + a_1 c_1 \\ a_3 s_{123} + a_2 s_{12} + a_2 s_1 \\ 0 \end{bmatrix}$$

$$O_3 - O_1 = \begin{bmatrix} a_3 c_{123} + a_1 c_{12} \\ a_3 s_{123} + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 - O_2 = \begin{bmatrix} a_3 c_{123} \\ a_3 s_{123} \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_3 s_{123} - a_2 s_{12} - a_2 s_1 & -a_3 s_{123} - a_2 s_{12} & -a_3 s_{123} \\ a_3 c_{123} + a_1 c_{12} + a_1 c_1 & a_3 c_{123} - a_2 c_{12} & a_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$