ITR Assignment 2

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Q1 We know, $S(a)\,b = a \times b$

In the above eq^n, we replace b with $R^T b$

$\therefore$ ~~$S(a)b =$~~ $S(a)\,R^T b = a \times (R^T b)$

Pre-multiplying with R

$\therefore R(S(a)\,R^T b) = R(a \times (R^T b))$

$\therefore R\,S(a)\,R^T b = Ra \times RR^T b$ (Using identity: $R(a \times b) = Ra \times Rb$)

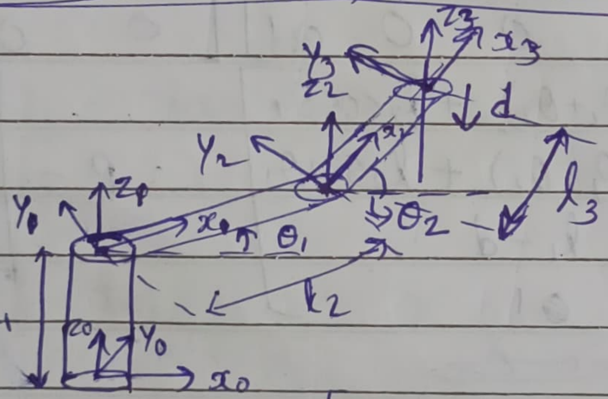
$\therefore R\,S(a)\,R^T b = Ra \times b$

Writing $Ra \times b = S(Ra)\,b$ (from property on the first line)

$\therefore R\,S(a)\,R^T b = S(Ra)\,b$

$\boxed{\therefore R\,S(a)\,R^T = S(Ra)}$ Hence proved

Q2



~~Here,~~

Here, $R_0^1 = R_{z,\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ \ell_1 \end{bmatrix}$.

$R_1^2 = R_{z,\theta_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_1^2 = \begin{bmatrix} \ell_2 \\ 0 \\ 0 \end{bmatrix}$

$R_2^3 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_2^3 = \begin{bmatrix} \ell_3 \\ 0 \\ 0 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$

$$\therefore H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

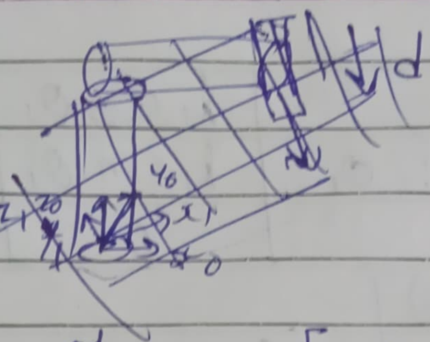
Also, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$

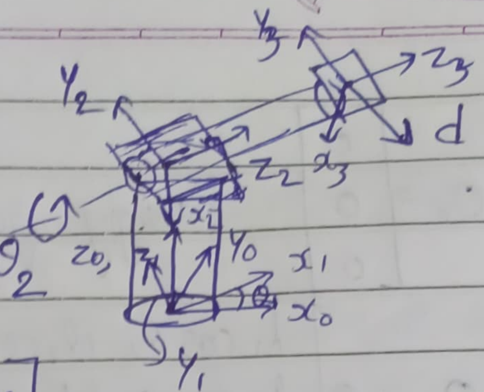
$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & l_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 \\ l_3\sin(\theta_1+\theta_2) + l_2\sin\theta_1 \\ l_1 - d \\ 1 \end{bmatrix} \quad \therefore \cancel{P_0}$$

$$\boxed{\therefore P_0 = \begin{bmatrix} l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 \\ l_3\sin(\theta_1+\theta_2) + l_2\sin\theta_1 \\ l_1 - d \end{bmatrix}}$$

Q4





~~Here, R_0^1~~

Here, $R_0^1 = R_{z,\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We now align z axis ~~wrt~~ with the axis of rotation

$$\therefore R_1^2 = R_{y,\frac{\pi}{2}} R_{z\theta_2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since no rotation from 2 to 3, $R_2^3 = I$

also, $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$, $d_2^3 = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}$, $\cancel{P_2} = \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix}$

$$\therefore H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ -\cos\theta_2 & \sin\theta_2 & 0 & l_1 \\ 0 & 0 & 0 & \phi \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & \phi \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ -\cos\theta_2 & \sin\theta_2 & 0 & l_1 \\ 0 & 0 & 0 & \phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & \phi \end{bmatrix} \begin{bmatrix} 0 \\ -d \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P_0 = \begin{bmatrix} l_2\cos(\theta_1) + d\cos(\theta_2)\sin(\theta_1) \\ l_2\sin(\theta_1) - d\cos(\theta_1)\cos(\theta_2) \\ l_1 - d\sin(\theta_2) \end{bmatrix}$$

Q5

First, drone is 10m from the ground

$\therefore R_0^1 = I, \; d_0^1 = [0 \;\; 0 \;\; 10]^T$

The next rotation is 30° about x-axis and 60° about z axis

$$\therefore R_1^2 = R_{x,30^\circ} R_{z,60^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R_1^2 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix}, \text{ since no disp., } d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, obstacle is 3m directly above drone in drone frame.

$$\therefore P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

From the above eq^n we get : $P_0 = \begin{bmatrix} 0 \\ -1.5 \\ 12.598 \end{bmatrix}$ m

Q6 Below are some types of gearboxes used with motors in robotic applications

Helical Gearbox:

It is small in size, ~~consumes low power~~ and has longer lifetime, silent operation and high strength because of which it is used in heavy industries. But due to friction between the teeths, the energy loss percentage is more.

Planetary Gearbox:

It has low power loss, more accurate and precise, and the options of speed ratio are more. Used in automobiles, etc where higher torque is needed. But due to complex mechanism, these are more expensive and need cooling for high speed cases.

Generally gearbox output higher torque but low angular speed. In drones, we need ~~high~~ very high speeds. In case we use overdrive gear, it will need high torque to operate which increases the weight of motor but we don't want that. Hence gearboxes are not used in drones.

Q7 Manipulator Jacobian for RRP SCARA

Refer to the figure in Q3

For $O - x_0 - y_0 - z_0$, $O_0 = [0 \quad 0 \quad 0]^T$

$$\begin{bmatrix} O_1 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ We know H_0^1 from Q3

$$\therefore \quad O_1 = [l_2 \cos q_1 \quad \cancel{l_2 \sin} \quad l_2 \sin q_1 \quad l_1]^T$$

$$\begin{bmatrix} O_2 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$ Using H_0^1 and H_1^2 from Q3

$$\therefore O_2 = \begin{bmatrix} l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 \\ l_3\sin(\theta_1+\theta_2) + l_2\sin\theta_1 \\ l_1 \end{bmatrix}, \quad O_3 = \begin{bmatrix} l_3\cos(\theta_1+\theta_2) + l_2\cos\theta_1 \\ l_3\sin(\theta_1+\theta_2) + l_2\sin\theta_1 \\ l_1 - d \end{bmatrix}$$

Since joints 1, 2 and 4 are revolute and joint 3 is prismatic and $O_4 - O_3$ is parallel to z_3,

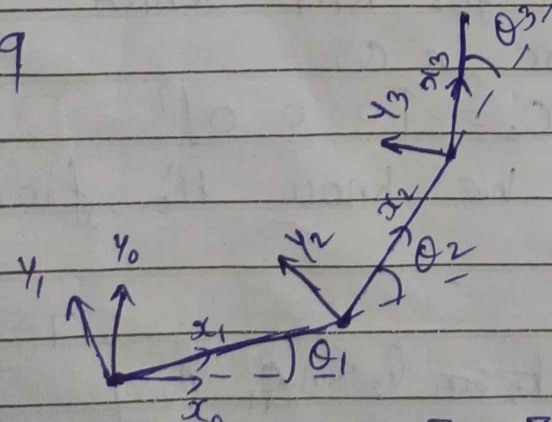
$$J = \begin{bmatrix} z_0 \times (O_2 - O_0) & z_1 \times (O_2 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$z_0 = R_0^0 \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = R_0^1 \hat{k} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore z_2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} -l_2\sin\theta_1 - l_3\sin(\theta_1+\theta_2) & -l_3\sin(\theta_1+\theta_2) & 0 \\ l_2\cos\theta_1 + l_3\cos(\theta_1+\theta_2) & l_3\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q9



In this case there is only rotation about z-axis

Hence z-axis will be the same

$$z_0 = z_1 = z_2 = z_3 = [0 \;\; 0 \;\; 1]^T$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{Using geometry: } O_1 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_2 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$ (From class notes)

$$O_3 = O_2 + R_0^3 \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} l_3 \cos(\theta_1+\theta_2+\theta_3) + l_2 \cos(\theta_1+\theta_2) + l_1 \cos\theta_1 \\ l_3 \sin(\theta_1+\theta_2+\theta_3) + l_2 \sin(\theta_1+\theta_2) + l_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

$$\therefore Z_0 \times (O_3 - O_0) = \begin{bmatrix} -l_3 \sin(\theta_1+\theta_2+\theta_3) - l_2 \sin(\theta_1+\theta_2) - l_1 \sin\theta_1 \\ l_3 \cos(\theta_1+\theta_2+\theta_3) + l_2 \cos(\theta_1+\theta_2) + l_1 \cos\theta_1 \\ 0 \end{bmatrix}$$

$$Z_1 \times (O_3 - O_1) = \begin{bmatrix} -l_3 \sin(\theta_1+\theta_2+\theta_3) - l_2 \sin(\theta_1+\theta_2) \\ l_3 \cos(\theta_1+\theta_2+\theta_3) + l_2 \cos(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$Z_2 \times (O_3 - O_2) = \begin{bmatrix} -l_3 \sin(\theta_1+\theta_2+\theta_3) \\ l_3 \cos(\theta_1+\theta_2\,\theta_3 \\ 0 \end{bmatrix}$$

Hence the Jacobian matrix is :

$$J = \begin{bmatrix} -l_3 \sin(\theta_1+\theta_2+\theta_3) - l_2 \sin(\theta_1+\theta_2) - l_1 \sin\theta_1 & -l_3 \sin(\theta_1+\theta_2+\theta_3) - l_2 \sin(\theta_1+\theta_2) & -l_3 \sin(\theta_1+\theta_2+\theta_3) \\ l_3 \cos(\theta_1+\theta_2+\theta_3) + l_2 \cos(\theta_1+\theta_2) + l_1 \sin\theta & l_3 \cos(\theta_1+\theta_2+\theta_3) + l_2 \sin(\theta_1+\theta_2) & l_3 \cos(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$