A Channel-Pruned and Weight-Binarized Convolutional Neural Network for Keyword Spotting

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Abstract. We study channel number reduction in combination with weight binarization (1-bit weight precision) to trim a convolutional neural network for a keyword spotting (classification) task. We adopt a group-wise splitting method based on the group Lasso penalty to achieve over 50 % channel sparsity while maintaining the network performance within 0.25 % accuracy loss. We show an effective three-stage procedure to balance accuracy and sparsity in network training.

Keywords: Convolutional Neural Network \cdot Channel Pruning \cdot Weight Binarization \cdot Classification.

1 Introduction

Reducing complexity of neural networks while maintaining their performance is both fundamental and practical for resource limited platforms such as mobile phones. In this paper, we integrate two methods, namely channel pruning and weight quantization, to trim down the number of parameters for a keyword spotting convolutional neural network (CNN, [4]).

Channel pruning aims to lower the number of convolutional channels, which is a group sparse optimization problem. Though group Lasso penalty [8] is known in statistics, and has been applied directly in gradient decent training of CNNs [7] earlier, we found that the direct approach is not effective to realize sparsity for the keyword CNN [4,6]. Instead, we adopt a group version of a recent relaxed variable splitting method [2]. This relaxed group-wise splitting method (RGSM, see [10] for the first study on deep image networks) accomplished over 50% sparsity while keeping accuracy loss at a moderate level. In the next stage (II), the original network accuracy is recovered with a retraining of float precision weights while leaving out the pruned channels in stage I. In the last stage (III), the network weights are binarized into 1-bit precision with a warm start training based on stage II. At the end of stage III, a channel pruned (over 50 %) and

weight binarized slim CNN is created with validation accuracy within 0.25 % of that of the original CNN.

The rest of the paper is organized as follows. In section 2, we review the network architecture of keyword spotting CNN [4,6]. In section 3, we introduce the proximal operator of group Lasso, RGSM, and its convergence theorem where an equilibrium condition is stated for the limit. We also outline binarization, the BinaryConnect (BC) training algorithm [1] and its blended version [11] to be used in our experiment. Through a comparison of BC and RGSM, we derive a hybrid algorithm (group sparse BC) which is of independent interest. In section 4, we describe our three stage training results, which indicate that RGSM is the most effective method and produces two slim CNN models for implementation. Concluding remarks are in sections 4.

2 Network Architecture

Let us briefly describe the architecture of keyword CNN [4,6] to classify a one second audio clip as either silence, an unknown word, 'yes', 'no', 'up', 'down', 'left', 'right', 'on', 'off', 'stop', or 'go'. After pre-processing by windowed Fourier transform, the input becomes a single-channel image (a spectrogram) of size $t \times f$, same as a vector $v \in \mathbb{R}^{t \times f}$, where t and f are the input feature dimension in time and frequency respectively. Next is a convolution layer that operates as follows. A weight tensor $W \in \mathbb{R}^{(m \times r) \times 1 \times n}$ is convolved with the input v. The weight tensor is a local time-frequency patch of size $m \times r$, where $m \leq t$ and $r \leq f$. The weight tensor has n hidden units (feature maps), and may down-sample (stride) by a factor s in time and u in frequency. The output of the convolution layer is n feature maps of size $(t-m+1)/s \times (f-r+1)/u$. Afterward, a max-pooling operation replaces each $p \times q$ feature patch in timefrequency domain by the maximum value, which helps remove feature variability due to speaking styles, distortions etc. After pooling, we have n feature maps of size $(t-m+1)/(sp) \times (f-r+1)/(uq)$. An illustration is in Fig. 1. The keyword CNN has two convolutional (conv) layers and a fully connected layer. There is 1 channel in the first conv. layer and there are 64 channels in the second. The weights in the second conv. layer form a 4-D tensor $W^{(2)} \in \mathbb{R}^{W \times H \times C \times N}$, where (W, H, C, N) are dimensions of spatial width, spatial height, channels and filters, C = 64.

3 Complexity Reduction and Training Algorithms

3.1 Group Sparsity and Channel Pruning

Our first step is to trim the 64 channels in the second conv. layer to a smaller number while maintaining network performance. Let weights in each channel form a group, then this becomes a group sparsity problem for which group Lasso (GL) has been a classical differentiable penalty [8]. Let vector

$$w = (w_1, \dots, w_q, \dots, w_G), \ w_q \in \mathbb{R}^d, \ w \in \mathbb{R}^{d \times G},$$

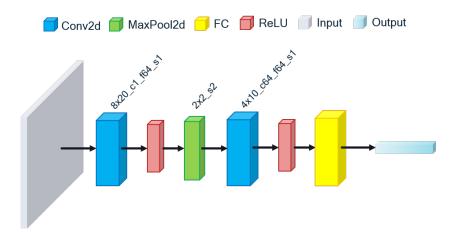


Fig. 1. A convolutional model with single-channel input. We use a simple notation to indicate conv. and max-pooling layers. For instance, $8x20_c1_f64_s1$ indicates a conv. layer with kernel size 8×20 , channel number 1, filter number 64 and stride 1.

where G is the number of groups. Let I_g be the indices of w in group g. The group-Lasso penalty is [8]:

$$||w||_{GL} := \sum_{g=1}^{G} ||w_g||_2. \tag{3.1}$$

It is easy to implement GL as an additive penalty term for deep neural network training [7] by minimizing a penalized objective function of the form:

$$f(w) := \ell(w) + \mu P(w), \quad \mu > 0,$$
 (3.2)

where $\ell(w)$ is a standard loss function on data such as cross entropy [12], and P(w) is a penalty function equal to sum of weight decay (ℓ_2 norm all network weights) and GL. For ease of notation, we merge the weight decay term with ℓ and take P as GL below.

In a case study of training CNN with un-structured weight sparsity [9], a direct minimization of ℓ_1 type penalty as an additive term in the training objective function provides less sparsity and accuracy (Table 4 of [9]) than the Relaxed Variable Splitting Method (RVSM [2]). In the group sparsity setting here, we shall see that the direct minimization of GL in (3.2) is also not efficient. Instead, we adopt a group version of RVSM [2], which minimizes the following Lagrangian function of (u, w) alternately:

$$\mathcal{L}_{\beta}(u, w) = \ell(w) + \mu P(u) + \frac{\beta}{2} \|w - u\|_{2}^{2}, \tag{3.3}$$

for a parameter $\beta > 0$.

The *u*-minimization is in closed form for GL. To see this, consider finding the GL proximal (projection) operator by solving:

$$y^* = \operatorname{argmin}_y \frac{1}{2} \|y - w\|^2 + \lambda \|y\|_{GL},$$
 (3.4)

for parameter $\lambda > 0$ or group-wise:

$$y_g^* = \operatorname{argmin}_{y_g} \lambda \|y_g\| + \frac{1}{2} \sum_{i \in I_g} \|y_{g,i} - w_{g,i}\|^2.$$
 (3.5)

If $y_g^* \neq 0$, the objective function of (3.5) is differentiable and setting gradient to zero gives:

$$y_{g,i} - w_{g,i} + \lambda y_{g,i} / ||y_g|| = 0,$$

or:

$$(1 + \lambda/||y_a||)y_{a,i} = w_{a,i}, \ \forall i \in I_a,$$

implying:

$$(1 + \lambda/||y_q||) ||y_q|| = ||w_q||,$$

or:

$$||y_a^*|| = ||w_a|| - \lambda, \text{ if } ||w_a|| > \lambda.$$
 (3.6)

Otherwise, the critical equation does not hold and $y_g^* = 0$. The minimal point formula is:

$$y_{q,i}^* = w_{q,i}(1 + \lambda/(\|w_g\| - \lambda))^{-1} = w_{q,i}(\|w_g\| - \lambda)/\|w_g\|, \text{ if } \|w_g\| > \lambda;$$

otherwise, $y_q^* = 0$. The result can be written as a soft-thresholding operation:

$$y_q^* = \text{Prox}_{GL,\lambda}(w_g) := w_g \max(\|w_g\| - \lambda, 0) / \|w_g\|$$
 (3.7)

The w minimization is by gradient descent, implemented in practice as stochastic gradient descent (SGD). Combining the u and w updates, we have the Relaxed Group-wise Splitting Method (RGSM):

$$u_g^t = \text{Prox}_{GL,\lambda}(w_g^t), \ g = 1, \dots, G,$$

 $w^{t+1} = w^t - \eta \nabla \ell(w^t) - \eta \beta (w^t - u^t),$ (3.8)

where η is the learning rate.

3.2 Theoretical Aspects

The main theorem of [2] guarantees the convergence of RVSM algorithm under some conditions on the parameters (λ, β, η) and initial weights in case of one convolution layer network and Gaussian input data. The latter conditions are used to prove that the loss function ℓ obeys Lipschitz gradient inequality on the iterations. Assuming that the Lipschitz gradient condition holds for ℓ , we adapt the main result of [2] into:

Theorem 1. Suppose that ℓ is bounded from below, and satisfies the Lipschitz gradient inequality: $\|\nabla \ell(x) - \nabla \ell(y)\| \leq L \|x - y\|$, $\forall (x, y)$, for some positive constant L. Then there exists a positive constant $\eta_0 = \eta_0(L, \beta) \in (0, 1)$ so that if $\eta < \eta_0$, the Lagrangian function $\mathcal{L}_{\beta}(u^t, w^t)$ is descending and converging in t, with (u^t, w^t) of RGSM algorithm satisfying $\|(u^{t+1}, w^{t+1}) - (u^t, w^t)\| \to 0$ as $t \to +\infty$, and subsequentially approaching a limit point (\bar{u}, \bar{w}) . The limit point (\bar{u}, \bar{w}) satisfies the equilibrium system of equations:

$$\bar{u}_g = \operatorname{Prox}_{GL,\lambda}(\bar{w}_g), \ g = 1, \cdots, G,$$

$$\nabla \ell(\bar{w}) = \beta \ (\bar{u} - \bar{w}). \tag{3.9}$$

Remark 1. The system (3.9) serves as a "critical point condition". The \bar{u} is the desired weight vector with group sparsity that network training aims to reach.

Remark 2. The group- ℓ_0 penalty is:

$$||w||_{GL0} := \sum_{g=1}^{G} 1_{(w_g:||w_g||_2 \neq 0)}$$
(3.10)

Then the GL proximal problem (3.5) is replaced by:

$$y_g^* = \operatorname{argmin}_{y_g} \lambda 1_{\|y_g\| \neq 0} + \frac{1}{2} \sum_{i \in I_g} \|y_{g,i} - w_{g,i}\|^2.$$
 (3.11)

If $y_g = 0$, the objective equals $||w_g||_2^2/2$. So if $\lambda \ge ||w_g||^2/2$, $y_g = 0$ is a minimal point. If $\lambda < ||w_g||^2/2$, $y_g = w_g$ gives minimal value λ . Hence the thresholding formula is:

$$y_g^* := \text{Prox}_{GL0,\lambda}(w_g) = w_g \, 1_{\|w_g\|_2 > \sqrt{2\lambda}}.$$
 (3.12)

Theorem 1 remains true with (3.9) modified where $\operatorname{Prox}_{GL,\lambda}$ is replaced by $\operatorname{Prox}_{GL0,\lambda}$.

3.3 Weight Binarization

The CNN computation can speed up a lot if the weights are in the binary vector form: float precision scalar times a sign vector $(\cdots, \pm 1, \pm 1, \cdots)$, see [3]. For the keyword CNN, such weight binarization alone doubles the speed of an Android app that runs on Samsung Galaxy J7 cellular phone [5] with standard tensorflow functions such as 'conv2d' and 'matmul'.

Weight binarized network training involves a projection operator or the solution of finding the closest binary vector to a given real vector w. The projection is written as $\operatorname{proj}_{\mathbb{Q}} w$, for $w \in \mathbb{R}^D$, $\mathbb{Q} = \mathbb{R}_+ \times \{\pm 1\}^D$. When the distance is Euclidean (in the sense of ℓ_2 norm $\|\cdot\|$), the problem:

$$\operatorname{proj}_{\mathbb{O},a}(w) := \operatorname{argmin}_{z \in \mathbb{O}} \|z - w\| \tag{3.13}$$

has exact solution [3]:

$$\operatorname{proj}_{\mathbb{Q},a}(w) = \frac{\sum_{j=1}^{D} |w_j|}{D} \operatorname{sgn}(w)$$
(3.14)

where $\operatorname{sgn}(w) = (q_1, \cdots, q_j, \cdots, q_D)$, and

$$q_j = \begin{cases} 1 & \text{if } w_j \ge 0 \\ -1 & \text{otherwise.} \end{cases}$$

The projection is simply the sgn function of w times the arithmetic average of the absolute values of the components of w.

The standard training algorithm for binarized weight network is Binary Connect [1]:

$$\mathbf{w}_f^{t+1} = \mathbf{w}_f^t - \eta \,\nabla \ell(\mathbf{w}^t), \ \mathbf{w}^{t+1} = \operatorname{proj}_{\mathbb{Q}, a}(\mathbf{w}_f^{t+1}), \tag{3.15}$$

where $\{\mathbf{w}^t\}$ denotes the sequence of binarized weights, and $\{\mathbf{w}_f^t\}$ is an auxiliary sequence of floating weights (32 bit). Here we use the blended version [11]:

$$\mathbf{w}_f^{t+1} = (1 - \rho) \mathbf{w}_f^t + \rho \mathbf{w}^t - \eta \nabla \ell(\mathbf{w}^t), \ \mathbf{w}^{t+1} = \operatorname{proj}_{\mathbb{Q}, a}(\mathbf{w}_f^{t+1}), \tag{3.16}$$

for $0 < \rho \ll 1$. The algorithm (3.16) becomes the classical projected gradient descent at $\rho = 1$, which suffers from weight stagnation due the discreteness of \mathbf{w}^t however. The blending in (3.16) leads to a better theoretical property [11] that the sufficient descent inequality holds if the loss function ℓ has Lipschitz gradient.

Remark 3. In view of (3.8) and (3.16), we see an interesting connection that both involve a projection step, as Prox is a projection in essence. The difference is that $\nabla \ell$ in BC is evaluated at the projected weight \mathbf{w}^t . If we mimic such a BC-gradient, and evaluate the gradient of Lagrangian in w at u^t instead of w^t , then (3.8) becomes:

$$u_g^t = \operatorname{Prox}_{GL,\lambda}(w_g^t), \quad g = 1, \cdots, G,$$

$$w^{t+1} = w^t - \eta \nabla \ell(u^t). \tag{3.17}$$

We shall call (3.17) a Group Sparsity BinaryConnect (GSBC) algorithm and compare it with RGSM in our experiment.

4 Experimental Results

In this section, we show training results of channel pruned and weight binarized audio CNN based on GL, RGSM, and GSBC. We assume that the objective function under gradient descent is $\ell(\cdot) + \mu \| \cdot \|_{GL}$, with a threshold parameter λ . For GL, $\mu > 0$, $\lambda = 0$, $\beta = 0$. For RGSM, $\mu = 0$, $\lambda > 0$, $\beta = 1$. For GSBC, $\mu = 0$, $\lambda > 0$, $\beta = 0$. The experiment was conducted in TensorFlow on a

single GPU machine with NVIDIA GeForce GTX 1080. The overall architecture [6] consists of two convolutional layers, one fully-connected layer followed by a softmax function to output class probabilities. The training loss $\ell(\cdot)$ is the standard cross entropy function. The learning rate begins at $\eta=0.001$, and is reduced by a factor of 10 in the late training phase. The training proceeds in 3 stages:

- Stage I: channel pruning with a suitable choice of μ or λ so that sparsity emerges at a moderate accuracy loss.
- Stage II: retrain float precision (32 bit) weights in the un-pruned channels at the fixed channel sparsity of Stage I, aiming to recover the lost accuracy in Stage I.
- Stage III: binarize the weights in each layer with warm start from the pruned network of Stage II, aiming to nearly maintain the accuracy in Stage II.

Stage I begins with random (cold) start and performs 18000 iterations (default, about 50 epochs). Fig. 2 shows the validation accuracy of RGSM at $(\lambda, \beta) = (0.05, 1)$ vs. epoch number. The accuracy climbs to a peak value above 80 % at epoch 20, then comes down and ends at 59.84 %. The accuracy slide agrees with channel sparsity gain beginning at epoch 20 and steadily increasing to nearly 56 % at the last epoch seen in Fig. 3. The bar graph in Fig. 4 shows the pruning pattern and the remaining channels (bars of unit height). At $(\lambda, \beta) = (0.04, 1)$, RGSM stage I training yields a higher validation accuracy 76.6 % with a slightly lower channel sparsity 51.6 %. At the same (λ, β) values, GSBC gives an even higher validation accuracy 80.9 % but much lower channel sparsity of 26.6 %. The GL method produces minimal channel sparsity in the range $\mu \in (0,1)$ covering the corresponding λ value where sparsity emerges in RGSM. The reason appears to be that the network has certain internal constraints that prevent the GL penalty from getting too small. Our experiments show that even with the cross-entropy loss $\ell(\cdot)$ removed from the training objective, the GL penalty cannot be minimized below some positive level. The Stage-I results are tabulated in Table 1 with a GL case at $\mu = 0.6$. It is clear that RGSM is the best method to go forward with to stage II.

Table 1. Validation Accuracy (%) and Channel (Ch.) Sparsity (%) after Stage I (Ch. pruning).

Model	$ oldsymbol{eta} $	λ	μ	Accuracy	Ch. Sparsity
Original Audio-CNN	0	0	0	88.5	0
GL Ch-pruning	0	0	0.6	66.8	0
RGSM Ch-pruning	1	4.e-2	0	76.6	51.6
RGSM Ch-pruning	1	5.e-2	0	59.8	56.3
GSBC Ch-pruning	0	4.e-2	0	80.9	26.6

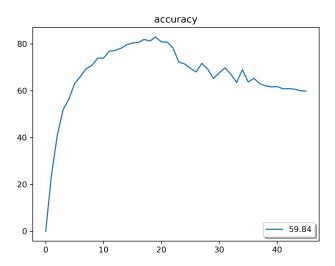


Fig. 2. Validation accuracy vs. number of epochs in Stage-1 training by RGSM at $(\lambda, \beta) = (0.05, 1)$. The accuracy at the last epoch is 59.84 %.

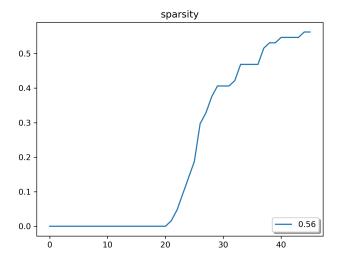


Fig. 3. Channel sparsity vs. number of epochs in Stage-1 training by RGSM at $(\lambda, \beta) = (0.05, 1)$. The sparsity at the last epoch is 56.3 %.

In Stage II, we mask out the pruned channels to keep sparsity invariant (Fig. 5), and retrain float precision weights in the complementary part of the network.

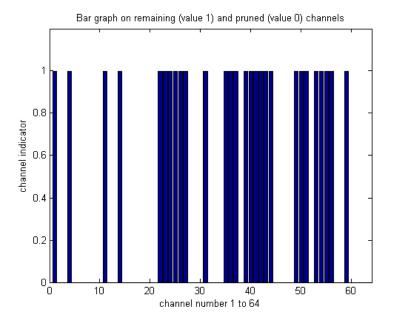


Fig. 4. Remaining channels illustrated by bars vs. channel number (1 to 64) after Stage-1 training by RGSM at $(\lambda, \beta) = (0.05, 1)$. The sparsity (% of 0's) is 56.3 %.

Fig. 7 shows that with a dozen epochs of retraining, the accuracy of the RGSM pruned model at $\lambda = 0.04$ (0.05) in Stage I reaches 89.2 % (87.9 %), at the level of the original audio CNN.

In Stage III, with blending parameter $\rho=1.e$ -5, the weights in the network modulo the masked channels are binarized with validation accuracy 88.3 % at channel sparsity 51.6 %, and 87 % at channel sparsity 56.3 %, see Fig. 6 and Table 3.

Table 2. Validation Accuracy (%) and Channel (Ch.) Sparsity (%) after Stage II (float precision weight retraining).

Model		λ	μ	Accuracy	$Ch. \ Sparsity$
Original Audio-CNN	0	0	0	88.5	0
RGSM Ch-pruning + Float Weight Retrain	1	4.e-2	0	89.2	51.6
RGSM Ch-pruning + Float Weight Retrain	1	5.e-2	0	87.9	56.3

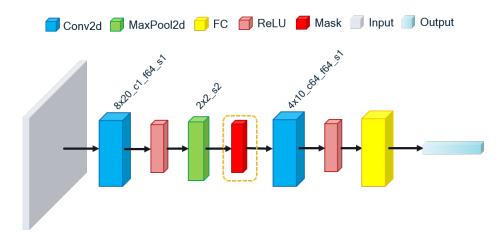


Fig. 5. The CNN model with pruned channels masked out (the masking layer in red).

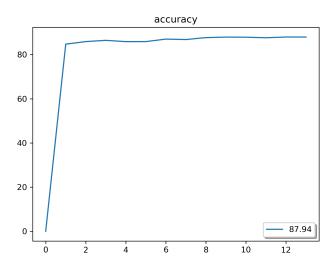


Fig. 6. Validation accuracy vs. number of epochs in Stage-2 float (32 bit) weight retraining. The accuracy at the last epoch is 87.94 %. Channel sparsity is 56.3 %.

5 Conclusion and Future Work

We successfully integrated a group-wise splitting method (RGSM) for channel pruning, float weight retraining and weight binarization to arrive at a slim yet almost equally performing CNN for keyword spotting. Since channel pruning involves architecture change, there is additional work to speed up a hardware

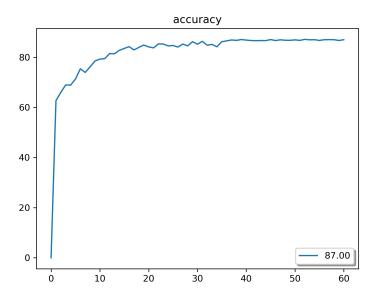


Fig. 7. Validation accuracy vs. number of epochs in Stage-3 binary (1-bit) weight training. The accuracy at the last epoch is 87%. Channel sparsity is 56.3%.

implementation. Preliminary test on a MacBook Air with a CPU version of Tensorflow shows as much as 28.87 % speed up by the network structure with float precision weight in Fig. 5. An efficient way to implement the masking layer without resorting to an element-wise tensor multiplication (especially on a mobile phone) is worthwhile for our future work.

We also plan to study other penalties [2] such as group- ℓ_0 (transformed- ℓ_1) in the RGSM framework as outlined in Remark 2, and extend the three stage process developed here to multi-level complexity reduction on larger CNNs and other applications in the future.

Table 3. Validation Accuracy (%) and Channel (Ch.) Sparsity (%) after Stage III (weight binarization training).

Model		λ	μ	Accuracy	Ch. Sparsity
Original Audio-CNN		0	0	88.5	0
RGSM Ch-pruning + Weight Binarization	1	4.e-2	0	88.3	51.6
RGSM Ch-pruning + Weight Binarization	1	5.e-2	0	87.0	56.3

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References

- M. Courbariaux, Y. Bengio and J. David, Binary Connect: Training Deep Neural Networks with Binary Weights during Propagations, Conference on Neural Information Processing Systems (NIPS), pp. 3123-3131, 2015.
- 2. T. Dinh, J. Xin, "Convergence of a relaxed variable splitting method for learning sparse neural networks via ℓ_1 , ℓ_0 , and transformed- ℓ_1 penalties", arXiv preprint arXiv:1812.05719.
- 3. M. Rastegari, V. Ordonez, J. Redmon and A. Farhadi, XNOR-Net: ImageNet Classification Using Binary Convolutional Neural Networks, European Conference on Computer Vision (ECCV), 2016.
- 4. T. Sainath and C. Parada, Convolutional Neural Networks for Small-footprint Keyword Spotting, Interspeech 2015, pp. 1478-1482, Dresden, Germany, Sept. 6-10.
- S. Sheen, J. Lyu, Median Binary-Connect Method and A Binary Weight Convolutional Neural Network for Word Recognition, arXiv:1811.02784; IEEE Data Compression Conference (DCC), 2019; DOI: 10.1109/DCC.2019.00116.
- 6. Simple audio recognition tutorial, tensorflow.org, last access Aug. 10, 2019.
- W. Wen, C. Wu, Y. Wang, Y. Chen, and H. Li, "Learning structured sparsity in deep neural networks," in NIPS, 2016.
- 8. M. Yuan and Y. Lin, "Model selection and estimation in regression with grouped variables," Journal of the Royal Statistical Society, Series B, 68(1):49-67, 2007.
- 9. F. Xue, J. Xin, "Learning Sparse Neural Networks via L0 and TL1 by a Relaxed Variable Splitting Method with Application to Multi-scale Curve Classification," arXiv preprint arXiv: 1902.07419; in Proc. World Congress Global Optimization, Metz, France, July, 2019. DOI:10.1007/978-3-030-21803-4_80.
- B. Yang, J. Lyu, S. Zhang, Y-Y Qi, J. Xin "Channel Pruning for Deep Neural Networks via a Relaxed Group-wise Splitting Method", In Proc. of 2nd International Conference on AI for Industries, Laguna Hills, CA, Sept. 25-27, 2019
- 11. P. Yin, S. Zhang, J. Lyu, S. Osher, Y-Y. Qi, J. Xin, "Blended coarse gradient descent for full quantization of deep neural networks". Research in the Mathematical Sciences 6(1), 14 (2019). DOI:10.1007/s40687-018-0177-6. arXiv: 1808.05240.
- D. Yu, L. Deng: Automatic speech recognition: a deep learning approach. Signals and Communication Technology. Springer, New York (2015)