Analysis of a Laser Grating

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1 Introduction

We study the Grating which is commonly supplied with Green Laser Pointers, popularly known also as "Disco Laser". The grating can be rotated to change the pattern formed. First we look at what the patterns formed actually look like and then try to deduce the exact structure of the grating. Figure 1 and 2 show the grating.

Figure 1: Input side

Figure 2: Output side

2 Experimental Setup

The setup required is quite simple. We mount the laser on a steady surface and attach the grating. A small pointer is also attached to the grating to measure the angle of rotation. The wall acts as a screen and the pattern formed is recorded. Figure 3 shows the setup.

3 Analysis

Figure 4 shows the pattern formed at a certain angle which we shall define to be the starting point or 0 the reason for which will be clear from the analysis.
The pattern formed has roughly equispaced, well-resolved maximas in the plane of the screen. Now, we know that a 1-d Diffraction Grating produces a similar pattern in one dimension. This suggests that a "2-d grating" might be involved. The exact expression of intensity can be found in the references[1].
Further, rotating the grating appears to have the effect of producing more maximas around the ones present at $\theta = 0$. Two 2-d Diffraction Gratings one after the other, with the relative angle between them then

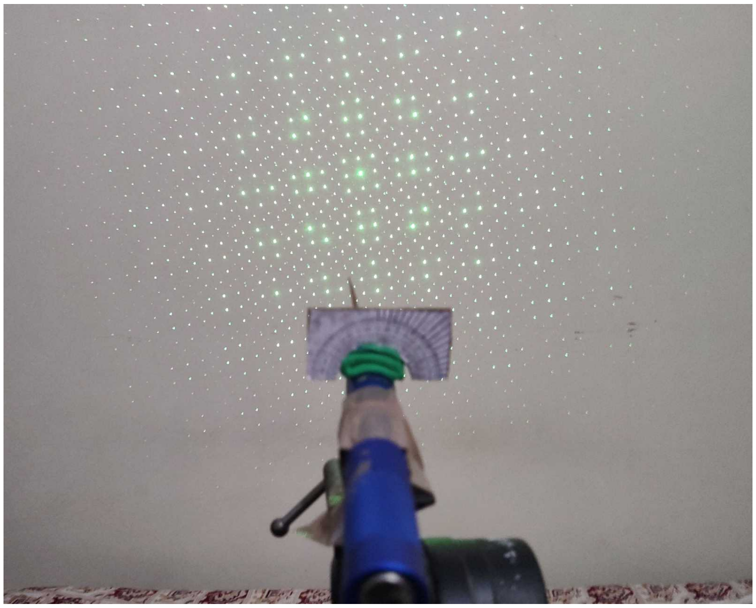

Figure 3: The setup

adjustable seems to be a reasonable structure. We shall assume this to be our working hypothesis and try to reproduce experimental observation computationally and analytically. Figure 4 shows the pattern at zero angle.

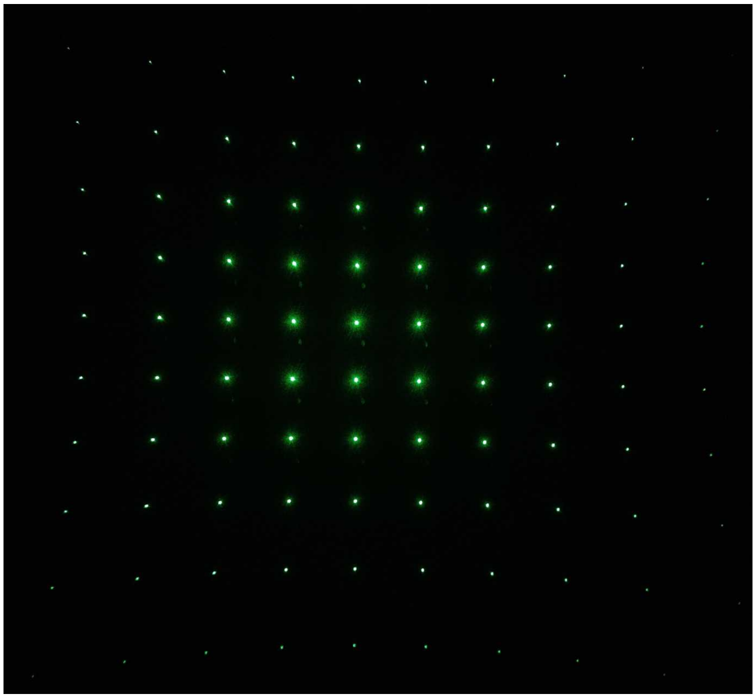

Figure 4: $\theta = 0°$

3.1 A crude attempt

A very simple but crude way to predict the position maximas can be as follows: the first 2-d grating splits the beam into equispaced maximas. Then, the second beam further splits each maxima further. Assuming the second grating to be at an angle of θ, we can give the position of maximas as(suitably scaling the axes):

$$x_c = i + k\cos(\theta) - l\sin(\theta)$$
$$y_c = j + k\sin(\theta) + l\cos(\theta)$$

where x_c and y_c are coordinates of the maximas and i, j, k, l are integers which can be varied independently. We can easily plot these points and check the patterns. Even this simple model matches quite well with the observed ones:

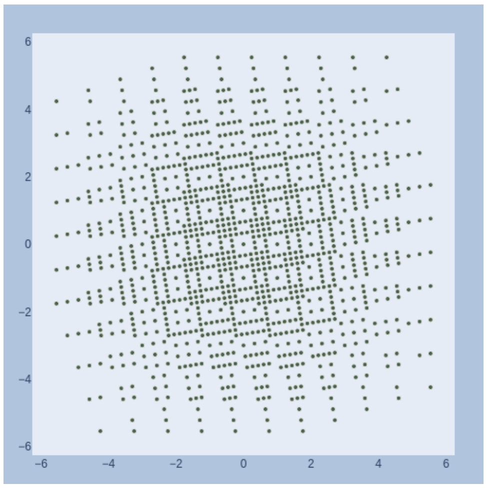

Figure 5: Plot, $\theta = 19°$

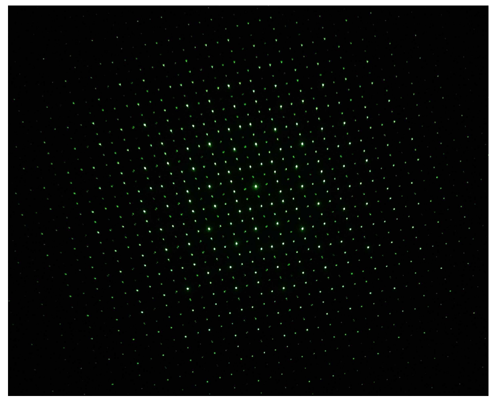

Figure 6: Actual, $\theta = 20°$

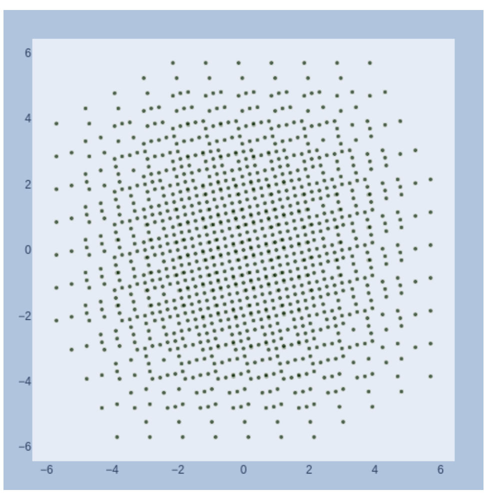

Figure 7: Plot, $\theta = 27.5°$

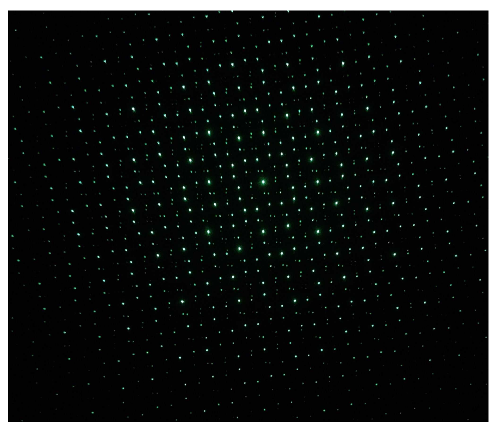

Figure 8: Actual, $\theta = 28°$

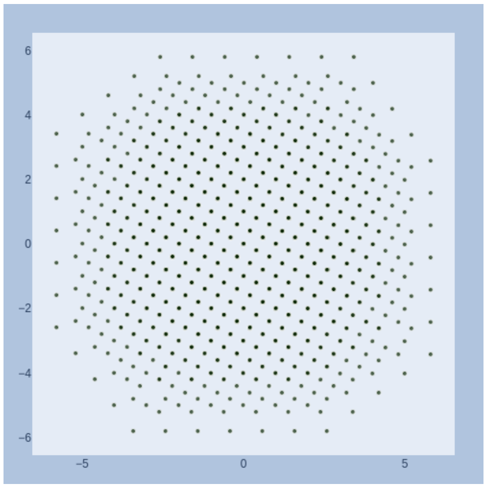

Figure 9: Plot, $\theta = 36.5°$

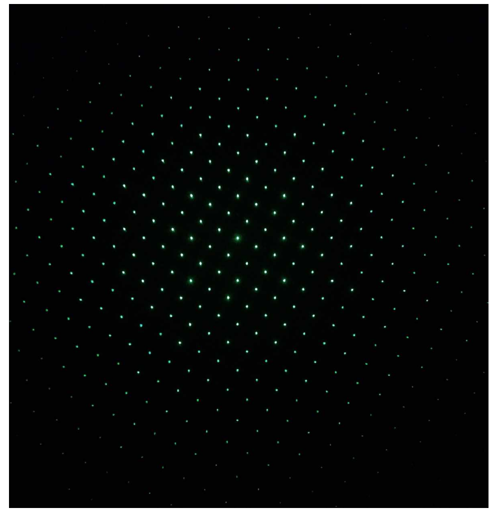

Figure 10: Actual, $\theta = 38°$

3.2 A more refined approach

We know that the beam incident on the first grating is gaussian. Now we want to find out the electric field profile at the second grating. Since the two are quite close by, Fraunhofer approximation is not applicable, but it is reasonable to assume that the Fresnel approximation holds. Once we have the electric field profile at the second grating, we can simply use Fraunhofer integral to find out the intensity profile at the screen since it is kept sufficiently far away. The Fresnel Integral is given by:

$$U_2(x', y') = c \int \int U_1(x'', y'') e^{\frac{k(x'-x'')^2}{2d}} e^{\frac{k(y'-y'')^2}{2d}} dx'' dy''$$

where c is a constant, $''$ and $'$ denote the coordinates at first and second grating respectively, U_1 corresponds to the first grating and d is the distance between the gratings.
Note that this is the 2-d convolution of two functions. We can take a 3d Fourier transform on both the sides which transforms the convolution to a simple multiplication of Fourier transforms. Proof can be found at [??]. The required function can then be obtained by taking an inverse Fourier Transform:

$$\mathcal{F}\left[U_2(x', y')\right] = c\mathcal{F}\left[U_1(x'', y'')\right] \cdot \mathcal{F}\left[e^{\frac{k(x'-x'')^2}{2d}} e^{\frac{k(y'-y'')^2}{2d}}\right]$$

$$\mathcal{F}\left[U_2(x', y')\right] = c'\mathcal{F}\left[U_1(x'', y'')\right] \cdot e^{\frac{df_x^2}{2k}} e^{\frac{df_y^2}{2k}}$$

$$\therefore U_2(x', y') = c'\mathcal{F}^{-1}\left[\mathcal{F}\left[U_1(x'', y'')\right] \cdot e^{\frac{df_x^2}{2k}} e^{\frac{df_y^2}{2k}}\right]$$

Now it is straightforward to obtain the profile at the screen:

$$U(x, y) = c'' \int U_2'(x', y') e^{\frac{-ikxx'}{2z}} e^{\frac{-ikyy'}{2z}} dx' dy'$$

where z is distance from the screen and x, y are screen coordinates. Note that the above expression involves U_2' instead of U_2. U_2' simply takes into account the second aperture/grating.
Now, the above calculation can be cumbersome to perform analytically, but should be straightforward to perform numerically. Also, this would then give us the complete intensity profile and not just the position of maximas as in the first method. The results from this numerical simulation are shown below. The source code can be found in References[2]:

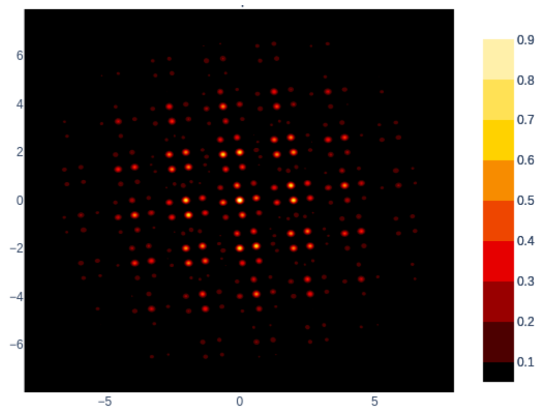

Figure 11: Plot, $\theta = 18°$

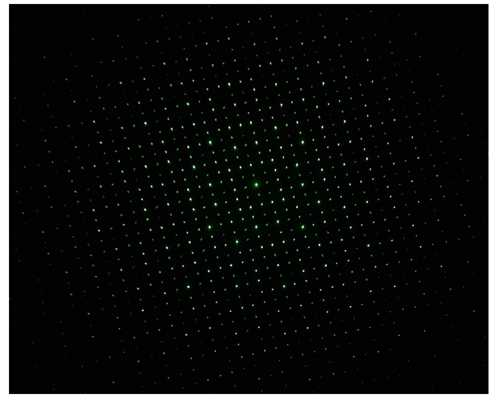

Figure 12: Actual, $\theta = 20°$

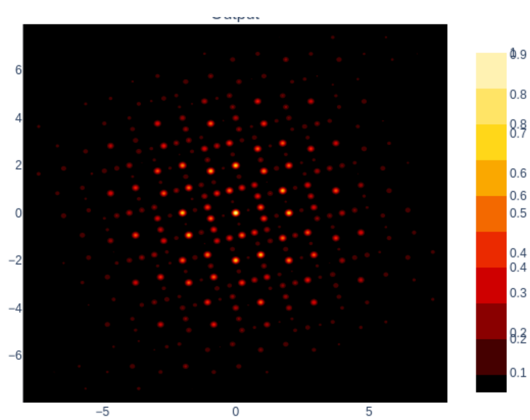

Figure 13: Plot, $\theta = 28°$

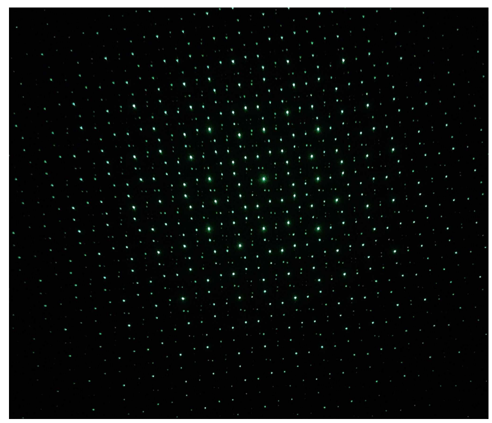

Figure 14: Actual, $\theta = 28°$

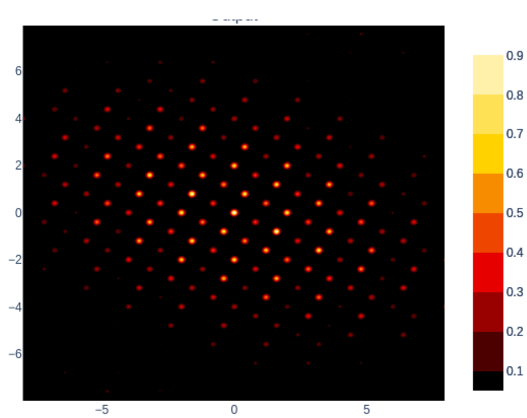

Figure 15: Plot, $\theta = 36.9°$

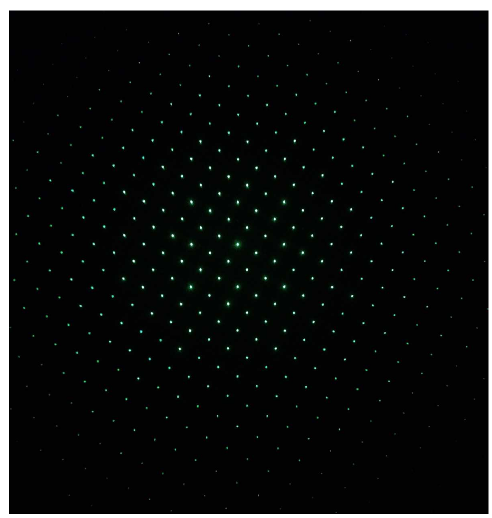

Figure 16: Actual, $\theta = 38°$

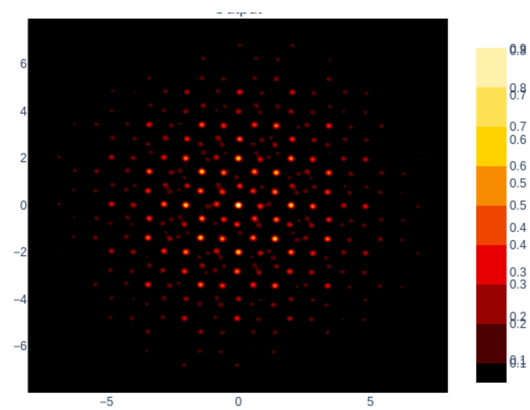

Figure 17: Plot, $\theta = 44°$

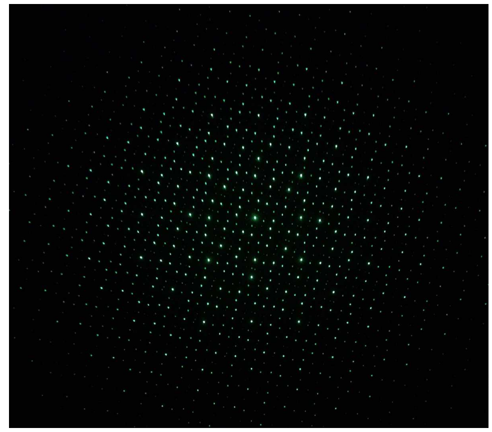

Figure 18: Actual, $\theta = 45°$

4 Conclusion and Comments

The above results show that our hypothesis matches very well with experimental observation and the device most likely has two 2-d diffraction gratings with relative angle between them being adjustable.
Note : As of now, a simplifying approximation has been assumed for numerical calculation, assuming the two gratings to be very close together, we write the field at the second grating to be simply a combination of the two apertures, and then apply the Fraunhofer integral. This shouldn't appreciably change the results but will be fixed as soon as possible.

References

1. Optics by Eugene Hecht, Addison-Wesley Publication (Chapter 10)

2. https://github.com/harishss3/Diffraction
   A web app has also been hosted at https://speckrel.pythonanywhere.com which can simulate Fraunhofer diffraction for a few difrent aperatures.