

University of Massachusetts Lowell — Comp 3010: Organization of Programming Languages  
Assignment 2

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NAME: **[[ HARISHWAR REDDY ERRI ]]**  
UML ID: **[[ 02148304 ]]**  
Collaborators: **[[Put your collaborators here, if any.]]**

*Make sure that the remaining pages of this assignment do not contain any identifying information.*

## 1 Small-Step Semantics

(25 points)

Let us define the calculator language below that performs arithmetic and boolean operations. It has three different syntactic classes: (1) arithmetic expressions  $a$  (2) boolean expressions  $b$ , and (3) final values  $v$ . The following questions require you define small-step semantics. That is, first you need to define the configuration. Then, you need to write inference rules that show one configuration small-steps to another configuration.

$$\begin{aligned} n &\in \mathbb{Z} \\ a &::= n \mid a_1 + a_2 \mid a_1 \times a_2 \\ b &::= \mathbf{true} \mid \mathbf{false} \mid a = a \mid a \neq a \\ &\quad \mid a \leq a \mid a > a \mid \neg b \mid b \& \& b \\ v &::= n \mid \mathbf{true} \mid \mathbf{false} \end{aligned}$$

(a) Write small-step semantics for the syntactic class of arithmetic expressions generated by  $a$ .

II

*(a) Small-step semantics for arithmetic expressions generated by the rules for arithmetic expressions:*

$$a ::= n \mid a_1 + a_2 \mid a_1 \times a_2$$

*We define small-step reduction rules for arithmetic expressions:*

*1. Addition: If the left-hand side of the addition is not a value, reduce it:*

$$\frac{a_1 \rightarrow a'_1}{a_1 + a_2 \rightarrow a'_1 + a_2}$$

*If the right-hand side is not a value:*

$$\frac{a_2 \rightarrow a'_2}{n + a_2 \rightarrow n + a'_2}$$

*When both sides are values:*

$$n_1 + n_2 \rightarrow n_1 + n_2$$

*2. Multiplication: If the left-hand side of the multiplication is not a value, reduce it:*

$$\frac{a_1 \rightarrow a'_1}{a_1 \times a_2 \rightarrow a'_1 \times a_2}$$

*If the right-hand side is not a value:*

$$\frac{a_2 \rightarrow a'_2}{n \times a_2 \rightarrow n \times a'_2}$$

*When both sides are values:*

$$n_1 \times n_2 \rightarrow n_1 \times n_2$$

*3. For all integer literals:*

$$n \rightarrow n (\text{where } x \in \mathbb{Z})$$

II

(b) Write small-step semantics for the syntactic class of boolean expressions generated by  $b$ .

II

*Boolean expressions are generated by the following grammar:*

$$b ::= \text{true} \mid \text{false} \mid a = a \mid a \neq a \mid a \leq a \mid a \rangle a \mid \neg b \mid b \wedge b$$

*The small-step semantics for boolean expressions are defined as:*

*1. Equality check:*

$$\frac{a_1 \rightarrow a'_1}{a_1 = a_2 \rightarrow a'_1 = a_2}$$

$$\frac{a_2 \rightarrow a'_2}{n = a_2 \rightarrow n = a'_2}$$

$$n_1 = n_2 \rightarrow \text{true} \quad \text{if } n_1 = n_2$$

$$n_1 = n_2 \rightarrow \text{false} \quad \text{if } n_1 \neq n_2$$

*2. Inequality check:*

$$\frac{a_1 \rightarrow a'_1}{a_1 \neq a_2 \rightarrow a'_1 \neq a_2}$$

$$n_1 \neq n_2 \rightarrow \text{true} \quad \text{if } n_1 \neq n_2$$

$$n_1 \neq n_2 \rightarrow \text{false} \quad \text{if } n_1 = n_2$$

*3. Relational operators ( $\langle, \rangle$ ):*

$$\frac{a_1 \rightarrow a'_1}{a_1 \leq a_2 \rightarrow a'_1 \leq a_2}$$

$$n_1 \leq n_2 \rightarrow \text{true} \quad \text{if } n_1 \leq n_2$$

$$n_1 \leq n_2 \rightarrow \text{false} \quad \text{if } n_1 \rangle n_2$$

*4. Negation:*

$$\frac{b \rightarrow b'}{\neg b \rightarrow \neg b'}$$

$$\neg \text{true} \rightarrow \text{false}$$

$$\neg \text{false} \rightarrow \text{true}$$

*5. Boolean conjunction (AND):*

$$\frac{b_1 \rightarrow b'_1}{b_1 \wedge b_2 \rightarrow b'_1 \wedge b_2}$$

$$\text{true} \wedge b_2 \rightarrow b_2$$

$$\text{false} \wedge b_2 \rightarrow \text{false}$$

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