University of Massachusetts Lowell — Comp 3010: Organization of Programming Languages Assignment 7

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Make sure that the remaining pages of this assignment do not contain any identifying information.

1 References (20 points)

$$\begin{array}{l} \text{let } y = \text{ref } \lambda x.\,x \text{ in} \\ \text{let } z = (y := \lambda x.\,!y\ 4) \text{ in} \\ !y\ 2 \end{array}$$

We will evaluate this program using small-step semantics under call-by-value (CBV) semantics.

1. Initial Expression:

$$\langle \text{let } y = \text{ref } \lambda x. x \text{ in let } z = (y := \lambda x. ! y 4) \text{ in } ! y 2, \rangle$$

2. Evaluate let $y = \text{ref}\lambda x.x$: - ref $\lambda x.x$ creates a reference containing the function $\lambda x.x$. - Resulting expression:

$$\langle \text{let } z = (y := \lambda x. ! y 4) \text{ in } ! y 2, \{ l_1 \mapsto \lambda x. x \} \rangle$$

3. Evaluate let $z = (y := \lambda x.!y 4)$: - Assigns $y := \lambda x.!y 4$, which updates the reference to a new function. - Resulting expression:

$$\langle !y \ 2, \{l_1 \mapsto \lambda x. x\} \rangle$$

4. Evaluate y: - Dereferences y, which now contains λx . y 4. - Resulting expression:

$$\langle \lambda x. ! y 42, \{l_1 \mapsto \lambda x. x\} \rangle$$

5. Apply the function: - Substitute x=2: - Resulting expression:

$$\langle !y \, 4, \, \{l_1 \mapsto \lambda x. \, x\} \rangle$$

6. Evaluate !y again: - Dereference y, which still points to λx . !y 4. - Resulting expression:

$$\langle \lambda x. ! y 44, \{l_1 \mapsto \lambda x. x\} \rangle$$

7. Apply the function again: - Substitute x=4: - Resulting expression:

$$\langle !y \, 4, \, \{l_1 \mapsto \lambda x. \, x\} \rangle$$

8. Conclusion: The evaluation will continue indefinitely as each application of y returns back to the same function y 4 without reaching a value.

2 Typing Derivation

(30 points)

Show the type-checking for the following terms using derivation trees to get credit.

(i)
$$y : \mathbf{int} \vdash (\lambda x : \mathbf{int}. \ y + 40) \ y : \mathbf{int}$$

$$\begin{array}{c} \text{T-Var} \frac{\text{T-Var} \frac{}{y:int \vdash y:int} \quad \text{T-Int} \frac{}{y:int \vdash 40:int}}{y:int \vdash y+40:int} \\ \text{T-Abs} \frac{\text{T-Abs} \frac{}{y:int \vdash \lambda x:int. (y+40):int}}{y:int \vdash (\lambda x:int. y+40)y:int} \\ \text{T-Var} \frac{}{y:int \vdash y:int} \\ \end{array}$$

The typing derivation for 2(a) is valid. This results in the entire expression being well-typed.

(ii)
$$\vdash (\lambda x\!:\!\mathsf{int}.\,x+40)\;(1+2)\!:\!\mathsf{int}$$

$$\begin{array}{c} \text{T-Add} & \begin{array}{c} \text{T-Var} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline + x : int & \overline{\text{T-Add}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline + x + 40 : int & \overline{\text{T-Add}} & \overline{\text{T-Add}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline \\ \text{T-APP} & \overline{\text{T-Add}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline & \overline{\text{T-Add}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} & \overline{\text{T-Int}} \\ \hline \end{array}$$

The typing derivation for 2(b) is not valid because y is not in the context, making it impossible to type-check y + 40. This results in the entire expression being ill-typed.