A report on

Black Scholes option pricing model

By

Harishankar Muppirala	2017A4PS0468G
Piyush Agarwal	2017B4A80812G
Dhruv Garg	2017B4A30409G
Shivam Sharma	2017B4A40707G
Shreyansh Gautam	2017B4A40807G

In partial fulfillment of

MATH F424 Stochastic Processes

November 2020

Acknowledgement

We would like to express our gratitude towards Prof Mayank Goel who gave us an opportunity to do this project and provide his valuable guidance and encouragement during the course of the project.

Introduction

Through this project, our objective is to explore the use of the Black-Scholes model to understand the strategies involved in trading stocks and options. Using the Black-Scholes Model we aim to get a theoretical value of the asset's option price and then compare it to the actual market price, and hence give valuable inferences.

Black Scholes Model

The Black Scholes model is a mathematical model for pricing an options contract. The model estimates the variation over time of financial instruments. It assumes these instruments (such as stocks or futures) will have a lognormal distribution of prices. Using this assumption and factoring in other important variables, the equation derives the price of a call option.

The model assumes that the price of heavily traded assets follows a geometric Brownian motion with constant drift and volatility. When applied to a stock option, the model incorporates the constant price variation of the stock, the time value of money, the option's strike price, and the time to the option's expiry. It's used to calculate the theoretical value of options using current stock prices, expected dividends, the option's strike price, expected interest rates, time to expiration and expected volatility.

The Black Scholes call option formula is calculated by multiplying the stock price by the cumulative standard normal probability distribution function. Thereafter, the net present value (NPV) of the strike price multiplied by the cumulative standard normal distribution is subtracted from the resulting value of the previous calculation.

Black Scholes Formula

$$C = S_t N(d_1) - K e^{-rt} N(d_2)$$

where:

$$d_1 = \frac{ln\frac{S_t}{K} + (r + \frac{\sigma_v^2}{2})\, t}{\sigma_s \sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s \sqrt{t}$$

where:

C = Call option price

S = Current stock (or other underlying) price

K = Strike price

r = Risk-free interest rate

t = Time to maturity

N = A normal distribution

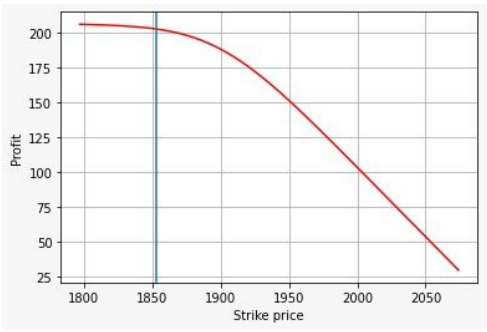
Strategy

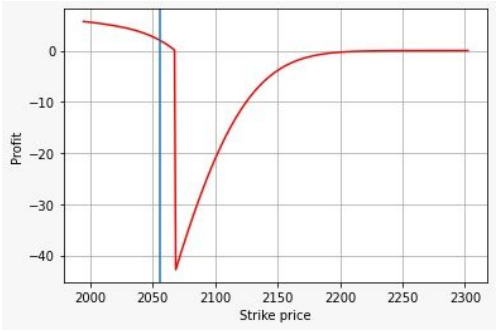
Our aim is to determine the profitability of buying an option of a certain company is profitable or not. Black Scholes model essentially determines the call option price/premium for a fixed value of strike price. We analyzed the variation of the **profit of the shares of HDFC bank over a period of 6 months**. We varied the strike price from **-3% to +12% of the present stock price**. The profits of the variations were calculated and plotted against the strike price.
After plotting the graphs of the profit over a period of time, the average value of profit was also plotted against time. The nature of this plot over the sum of several stochastic random processes would be approximately similar to a Normal distribution. Thus, using the properties of Normal distribution we determined the risk of not making profit if such an option is purchased.
All these calculations were done under the assumptions that:

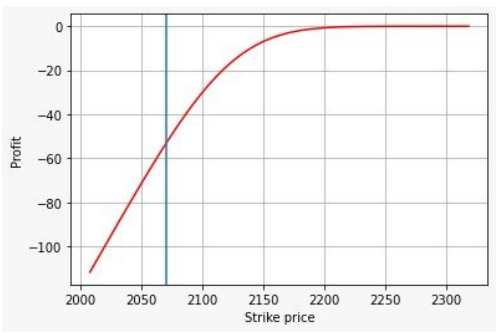
1. The option is exercised at expiration, which we have taken to be 6 months.

2. No dividends are paid during the option's life.
3. Market movements are only predicted using the previous data.
4. Risk free rate and volatility are constants.

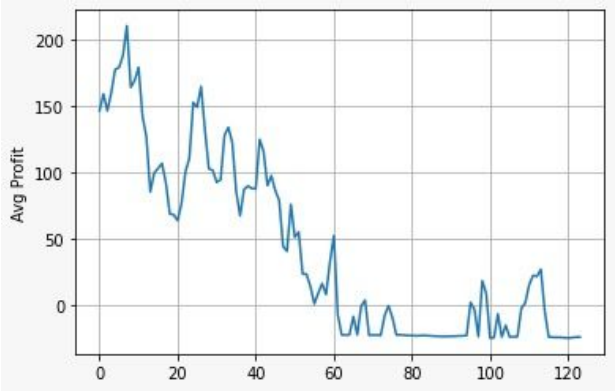
Result







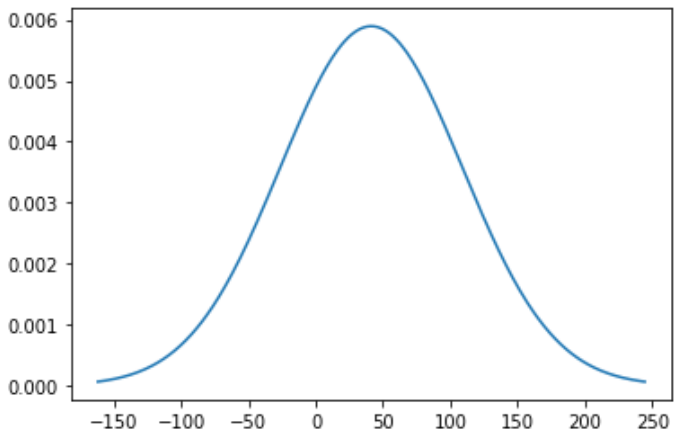
The above graphs represent the variation of profit while the strike price is varied from -3% to +12%. Every graph corresponds to the data on a particular day, and the blue line represents the stock price on the date of the purchase of the option. A graph is plotted for every day over the data of 6 months of **HDFC Bank** stock from **1st Jan 2019 to 1st July 2019**. The randomness in the shape and nature of the graph can be explained by the stochastic random variation of the stock.



The statistical measurements of the average profit were found to be:
Mean(μ) = 41.32
Standard deviation(σ) = 67.68

To estimate the overall profitability of the option when the strike price is in the range of -3% and 12%, the average value of every graph over the 6 months time period is plotted against time as in the above graph.
We claim that since the data is random, it follows that over the sum of several stochastic variables over time, the average profit would eventually follow Normal distribution. Thus determining the risk percentage of the profit being less than Rs

0 using the standard deviation and mean of the data previously obtained in the graph of average profit vs time.



Calculated risk = 27% (From the above graph)
I.e the percentage of the normal distribution that lies behind a profit of Rs 0.

Conclusion

Our method gives a way of predicting risks involved and the associated profits with stock options of companies based on a certain amount of recorded data. The risk associated is of significant importance to the investors, which is why our focus was to give a measure for that, and not only the profits. This clearly is impactful for our society, especially with the ever increasing investments in the stock market.

References

1) https://www.investopedia.com/terms/b/blackscholes.asp
2) http://janroman.dhis.org/stud/I2014/BS2/BS_Daniel.pdf
3) https://towardsdatascience.com/systematically-pricing-financial-options-with-black-scholes-4998c591ccbd?gi=6cdd5581ce1e
4) https://www.kaggle.com/rohanrao/nifty50-stock-market-data