Electromagnetic Characterisation of a Short-Stroke Ferromagnetic Actuator: Part B

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Abstract

This laboratory exercise is concerned with the electromechanical characterisation of a short-stroke ferromagnetic actuator. Students will work in pairs to study prescribed aspects of the actuator behaviour using analytical and numerical finite element analysis methods. Findings will be reported by means of an individual technical note conforming to an IEEE academic paper template. Students are expected to complete the tasks and address the questions raised in each section of this document in their technical note submission. Part A of the laboratory script is associated with the first three-hour laboratory session and focuses on familiarising students with the steps necessary to model the ferromagnetic actuator using Finite Element Method Magnetics (FEMM) and Matlab software. Part B of the laboratory script is associated with the second laboratory session and introduces the concepts necessary to analyse the behaviour of the actuator. Analytical modelling and analysis of results will be supported by lecture material as the unit progresses.

I. Introduction

In the previous laboratory session, [1], Matlab and FEMM software, [2], were used to construct a Finite Element (FE) model of a simple ferromagnetic linear actuator composed of a stator core, an armature (mover or plunger) and two independently connected electrical windings of N turns. An example of the pre- and post-processor of the FE model is given in Figs. 1 and 2. **Note: If the flux density plot from your model does not closely resemble Fig. 2 then check the definition of your windings including the current (turns) direction.** In this laboratory session, the FE model will be used alongside analytical methods taught in the lecture series, [3], to predict the behaviour of the actuator in the electrical, magnetic and mechanical domains. Findings will be reported by means of an individual technical note conforming to an IEEE academic paper template. Students are expected to complete the tasks and address the questions raised in each section of this document in their technical note submission. Details of the coursework requirements, submission format, means and deadline are given in Section VIII.

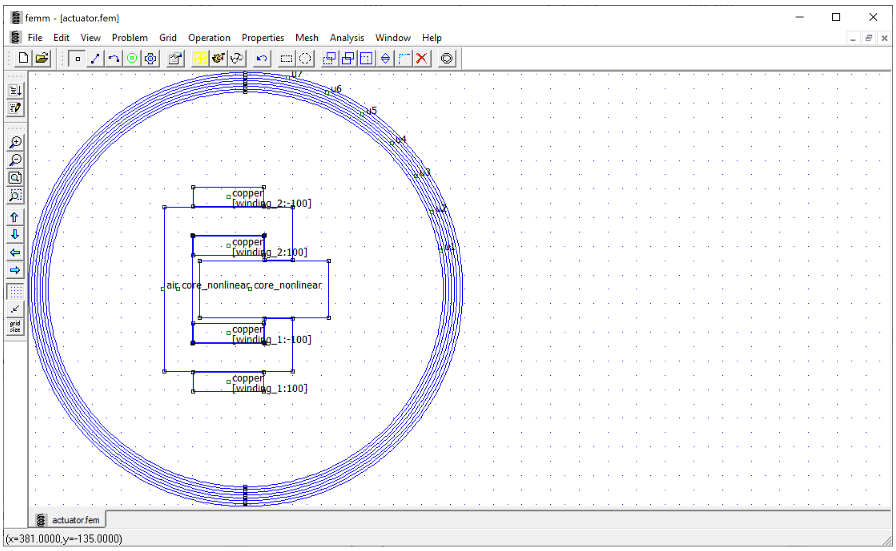

Fig. 1: 2-d actuator geometry implemented in FEMM with an Improvised Asymptotic Boundary Condition.

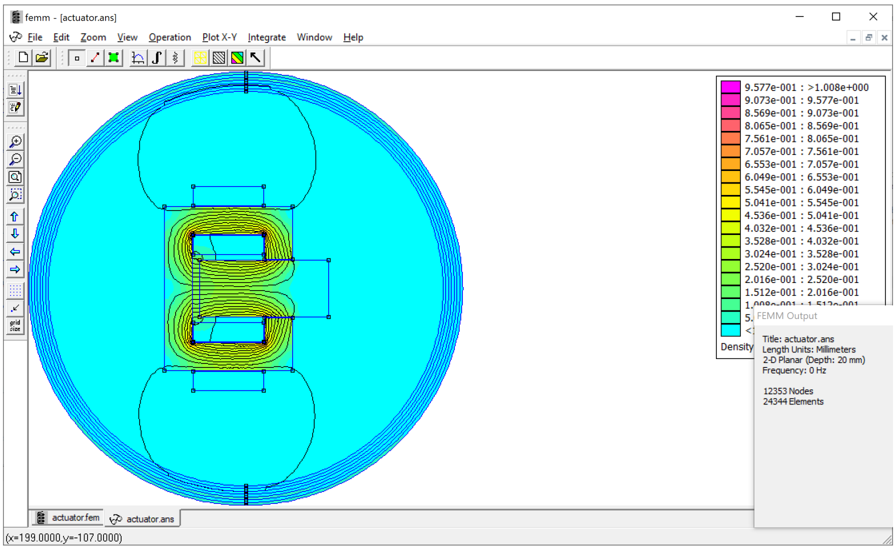

Fig. 2: Illustration of the FEM solution of the actuator.

II. Meshing the Model

The actuator geometry and surrounding region must be discretised using an appropriate method, in this case, Delaunay triangulation. FEMM uses Triangle, [4], to automatically generate a suitable mesh. To begin with, turn *Smartmesh* off using `smartmesh(0)` or through the `Problem` menu, then generate the mesh using the `mi_createmesh()` command or by clicking `Run mesh generator` from the FEMM toolbar, Fig. 3. Mesh visibility can be toggled using `mi_showmesh()`. Notice the irregular triangle areas which are generated, the quality i.e. the regularity of triangle shapes and mesh density i.e. the number of triangles in a given region influence the accuracy of the numerical solution to the magnetic vector potential. The Smartmesh feature uses meshing heuristics to automatically improve the quality of the mesh within the problem geometry. Enable Smartmesh, `smartmesh(1)`, and re-mesh the problem. Notice the change in geometric and area regularity and mesh density in the regions. However, the Smartmesh does not have an overarching understanding of the problem to be solved, hence, it is necessary to make manual adjustments to the mesh density. It is possible to assign a specific mesh spacing to line segments and arcs which influence local mesh density or to block labels which influences the mesh density over the whole region. It is important in this model to locally refine the mesh density in the variable air-gap. It is recommended that this be achieved programmatically by selecting the appropriate line segment, of which there are two, Fig. 4, `mi_selectsegment(x,y)`, applying a 0.5 mm element size, `mi_setsegmentprop('<None>', 0.5, 0, 0, group)`, and finally deselecting the line segment, `mi_clearselected()`.

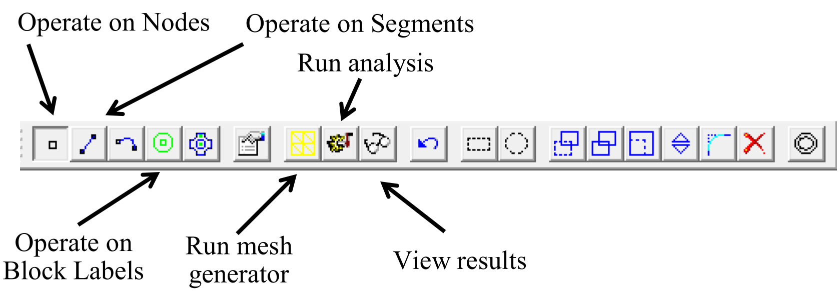

Fig. 3: Illustration of the main FEMM toolbar.

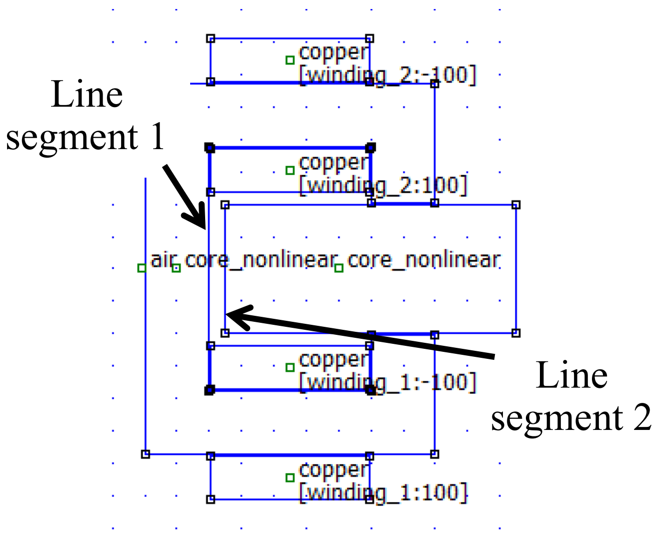

Fig. 4: Line segment selection for mesh density refinement.

Coursework Task 2: In your technical note, include a section on model meshing which documents the model mesh before and after the use of Smartmeshing and refinement along the designated line segments. Record the number of mesh elements in each mesh and qualitatively comment on the mesh quality in terms of edge length and area regularity along with mesh density in each region.

III. Armature Motion

In order to characterise the behaviour of the actuator, a magnetostatic analysis must be conducted at discrete armature positions at various applied winding current levels. The nodes and line segments of the model are assigned appropriate group numbers to facilitate the selection and geometric translation of the armature programmatically. To move the armature, use `mi_selectgroup(n)` to select the necessary group and `mi_movetranslate(dx,dy)` to translate the armature in the x-direction by `dx` millimetres, deselect the group using `mi_clearselected()`. Note that the armature position is initially at 5 mm which is the maximum travel, hence, the armature may move by up to -5 mm. However, the geometry of the armature and the stator core cannot interfere or overlap, as such, the maximum translation should be limited to 4.9 mm to maintain a minimum of 0.1 mm between the armature and stator core. **Note: The script must maintain a record of the armature position to enable movement back and forth without exceeding the travel end-stops of $l_{ag} = 0.1$ mm (limited by the FE model) and $l_{ag} = 5$ mm (limited physically).**

IV. Winding Resistance

Coursework Task 3: In your technical note, include a section on the winding resistance which reports the findings of the analyses performed in this part of the laboratory, including appropriate numerical data, figures and interpretation.

Ensure that 10 A is applied to each of the windings, `winding_1` and `winding_2` using the `mi_setcurrent('CircName',i)` command, see `FEMM_programming_manual.pdf`. Solve the model, `mi_analyse()` and load the solution in to the post-processor, `mi_loadsolution()`. Extract the circuit properties, `CP`, of each coil using `CP = mo_getcircuitproperties('circuit')`. Where `CP(1)`, `CP(2)` and `CP(3)` are the current, I, applied voltage, V, and flux linkage , ψ, respectively. Measure the cross-section, A_w, of a winding region, `mo_groupselectblock(n)`, `A = mo_blockintegral(5)`, `mo_clearblock()`. Measure the active length, l_w, (depth in to the page) via the volume, V_w, `mo_groupselectblock(n)`, `V = mo_blockintegral(10)`, `mo_clearblock()`, and (1). **Note: The reported dimensions are in metres.**

$$l_w = \frac{V_w}{A_w} \tag{1}$$

$$R_w = \frac{N l_w}{\sigma\left(\left(k_{PF} A_w\right)/N\right)} \tag{2}$$

Calculate the resistance of each winding using (2), taking account of the number of turns, N, the electrical conductivity, $\sigma = 58$ MS/m, and the packing factor, $k_{PF} = 0.6$, which relates to the available conductor area when insulation and geometric packing of the conductors is considered. Compare and comment on the analytical result and that reported from the FEM model using the measured voltage and current stored in `CP`. As the model is 2-d, the end-windings (those which run in the same plane as the page connecting the upper and lower winding sections) are not included in the prediction. Use (2) and the 3-d CAD model or otherwise to find the DC resistance of each winding. Plot the total winding power loss as a function of applied current from 0 to 10 A.

V. Winding Inductance

Coursework Task 4: In your technical note, include a section on the winding inductance which reports the findings of the analyses performed in this part of the laboratory, including appropriate numerical data, figures and interpretation.

In the steady state, (magnetostatic case), the winding resistance is independent of armature position. However, the inductance, L, is highly dependent upon the reluctance, $\mathcal{R}$, of the magnetic circuit and as such on the armature position. Derive a magnetic equivalent circuit for the actuator under consideration and find an analytical expression for the inductance as a function of armature position, (3), assume that the air-gap fringing flux is negligible and that the core is linear, $\mu_r = 1000$, as in `core_linear`. Find a second analytical expression for the inductance as a function of armature position, assuming that the core is linear, accounting for air-gap fringing flux using the effective air-gap area model, Eq B-5, given in [3].

$$L = \frac{N^2}{\mathcal{R}} \tag{3}$$

Use the FEM model to predict the inductance of the actuator as a function of armature position for both the `core_linear` and `core_nonlinear` material cases. Plot the four inductance trends (analytical and numerical), compare and contrast their form discussing the reasons for the differences. **Note: The inductance can be calculated as $L = \psi/I$.**

Astute students may realise that the non-linearity of the core material could be incorporated in to a magnetic equivalent circuit using current controlled resistances to model the effective reluctance increase as the material begins to saturate i.e. the electrical resistance increases as a function of current flow in accordance with the BH curve. This could be modelled using Simulink or SPICE and is left as an extended exercise for those interested.

VI. Force on the Armature

The force imparted on the armature as a result of applied winding current can be determined using an energy balance, (4), where, for an isolated system energy must be conserved. Hence, the electrical energy, sourced or sunk must be equal to the sum of the mechanical energy, work done and stored, the energy stored in the magnetic field and the system losses which in this case are composed of the Joule winding losses alone, [5], [6]. Winding AC loss, and core loss i.e. eddy current and hysteresis losses are neglected in this quasi-static analysis.

$$E_{elec} = E_{mech} + E_{mag} + E_{loss} \rightarrow \Delta E_{elec} = \Delta E_{mech} + \Delta E_{mag} + \Delta E_{loss} \tag{4}$$

First consider the case where the armature is fixed in the open position, i.e. $l_{ag} = 5$ mm such that the actuator behaves like an inductor. A winding current, I, is applied at time, $t = 0$, and ramped from 0 A to 10 A. The instantaneous flux linkage resulting from the applied current can be determined from (5) to form a $\Psi - I$ diagram, using analytical or numerical methods, as illustrated in Fig. 5. If there is no mechanical motion, $E_{mech} = 0$, and the winding loss is assumed negligible, $E_{loss} = 0$, then the energy supplied from the electrical supply, E_{elec} is stored in the magnetic field, E_{mag}, hence $E_{elec} = E_{mag}$, (4).

$$\Psi = LI \tag{5}$$

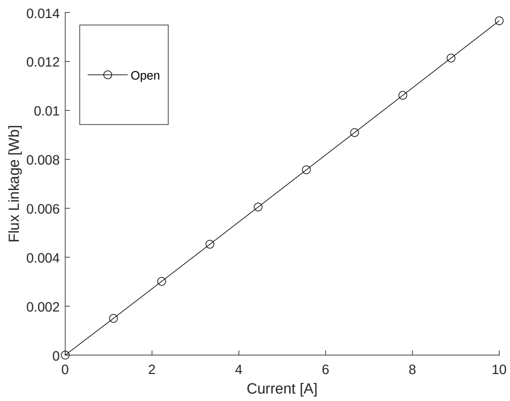

Fig. 5: $\Psi - I$ diagram corresponding to the open armature position.

The electrical energy supplied is determined from the time integral of the power, (6). In this case the applied voltage, V, is unknown, however, through Faraday's law the back EMF and hence, V, resulting from the time changing current is equal to the time derivative of flux linkage, $d\Psi/dt$, making the electrical energy the integral of the current with respect to the flux linkage, i.e. the area of the $Psi - I$ diagram between the y-axis and the $\Psi - I$ curve. The area of the $\Psi - I$ diagram between the x-axis and the $\Psi - I$ curve is known as the co-energy, E'_{elec}, which for linear $\Psi - I$ curves is numerically equal to the energy, $E_{elec} \equiv E'_{elec}$. Substituting the definition of flux linkage $\Psi = N\Phi$ and of $MMF = NI$, it is shown that the magnetic energy is given by the integral of the analogous electrical quantities, magnetic voltage, MMF, and magnetic current, Φ.

$$E_{elec} = \int IV dt = \int I \frac{d\Psi}{dt} dt = \int I d\Psi = \int NI d\Phi = \int MMF d\Phi \tag{6}$$

In a linear magnetic system, in which no saturation occurs, the stored magnetic energy can be determined from Fig. 5 and (6) by substituting (5) to yield (7) which is a familiar result.

$$E_{elec} = E_{mag} = \int I d\Psi = \int \frac{\Psi}{L} d\Psi = \frac{1}{L}\frac{\Psi^2}{2} = \frac{1}{2}\frac{(LI)^2}{L} = \frac{1}{2}LI^2 \tag{7}$$

Now consider the case where the armature is fixed in the closed position, i.e. $l_{ag} = 0.1$ mm such that the actuator behaves like an inductor. A winding current, I, is applied at time, $t = 0$, and ramped from 0 A to 10 A. In this case the variable air-gap is much smaller, hence, the induced flux becomes high enough to begin to saturate the core materials and the linear analytical expressions of inductance found in Section V no longer hold true. Instead, the instantaneous flux linkage, Ψ, is extracted from the circuit properties of `winding_1` and `winding_2` solved at appropriate winding current levels to form the $\Psi - I$ curve illustrated in Fig. 6.

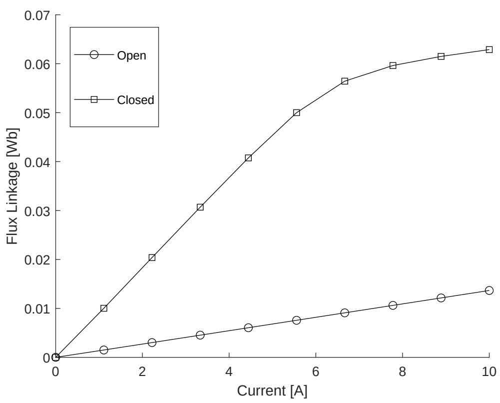

Fig. 6: $\Psi - I$ diagram corresponding to the open armature position.

If `Ivec` and `PSIvec` are Matlab current and flux linkage vectors respectively then the energy, $E_{elec} = E_{mag}$, can be determined using numerical trapezoidal integration as `Eelec = trapz(PSIvec, Ivec)`. Hence, in the armature open position 0.137 J are stored in the magnetic field, while in the armature closed position 0.456 J are stored, a difference of $\Delta E = 0.319$ J.

The energy exchange between the electrical supply and magnetic field has been studied for two cases, armature open and armature closed. It is clear that the armature closed position exhibits a smaller magnetic reluctance which results in a higher inductance, (3), and greater energy storage in the magnetic field than the armature open case, for equal applied winding current. It has been assumed that the armature moves instantaneously between the open and closed positions and no mechanical work is done, i.e. a quasi-static analysis. Now, the interaction of the electrical, magnetic and mechanical energy is studied to determine the static mechanical force acting upon the armature at a given applied winding current and armature position. The armature is assumed to move sufficiently slowly to allow highly dynamic effects to be neglected, thereby, simplifying the $\Psi - I$ diagrams and analysis.

Consider the case where the armature is in the fully open position, $l_{ag} = 5$ mm, with an applied winding current of, $I = 10$ A. From experience, an attractive magnetic force will act to draw the armature closer to the core. The armature is released and allowed to close under no external mechanical load and comes to rest in the closed position where $l_{ag} = 0.1 \approx 0$ mm. The electrical energy supplied to the system is the time integral of voltage and current, however, the voltage is unknown and so Faraday's law is used to give the voltage in terms of rate of change of flux linkage and change the subject of integration, (8). Now, the electrical energy supplied to the system, ΔE_{elec}, is given by the integral of applied current, $I = 10$ A, and flux linkage Ψ between limits of Ψ_o and Ψ_c representing the flux linkage in the open and closed positions respectively. From inspection of Fig. 7, the E_{elec} integral is equal to the area ***(A+B)***.

$$\Delta E_{elec} = \int_{t_1}^{t_2} IV dt = \int_{t_1}^{t_2} I\frac{d\Psi}{dt} dt = \int_{\Psi_o}^{\Psi_c} I d\Psi \tag{8}$$

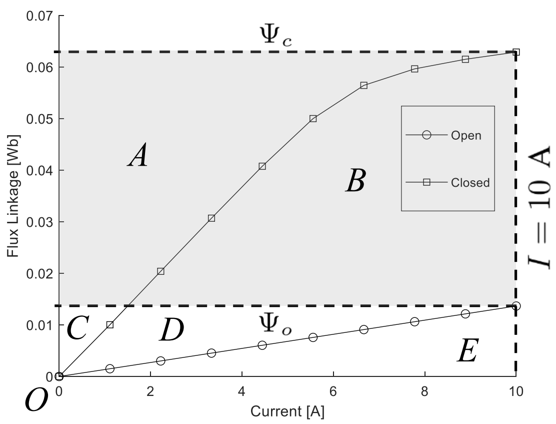

Fig. 7: $\Psi - I$ diagram corresponding to the open and closed armature positions.

As described by (6) and (7), the change in stored magnetic field energy, ΔE_{mag}, between the open and closed states is given by (9) which is equal to the area ***(A+C)*** with the area ***(C+D)*** subtracted to give ***(A+C)-(C+D)=(A-D)***.

$$\Delta E_{mag} = \int_0^{\Psi_c} I d\Psi - \int_0^{\Psi_o} I d\Psi \tag{9}$$

Neglecting the system losses, $\Delta E_{loss} = 0$, the energy balance given in (4) becomes (10), hence, the change in mechanical energy, ΔE_{mech}, when moving from the armature open to the armature closed position is given by the area ***(A+B)-(A-D)=(B+D)***, which is equal to the change in co-energy between the two states.

$$\Delta E_{elec} = \Delta E_{mag} + \Delta E_{mech} \rightarrow \Delta E_{mech} = \Delta E_{elec} - \Delta E_{mag} \tag{10}$$

The mechanical energy is defined as the integral of force, F, and displacement, x, (11), if the respective vectors are parallel and of the same sense. Hence, the numerical change in mechanical energy, derived from the $\Psi - I$ diagram resulting from a change in armature position, Δx, at a fixed winding current, I, can be used to determine a force-displacement curve of the actuator.

$$\Delta E_{mech} = \int_{dx_o}^{dx_c} F dx = Fx \rightarrow F = \frac{\Delta E_{mech}}{\Delta x} \tag{11}$$

Coursework Task 5: In your technical note, include a section on the force on the armature which reports the findings of the analyses performed in this part of the laboratory, including appropriate numerical data, figures and interpretation. Use the analytical expressions for inductance as a function of armature position derived in Section V to generate $\Psi - I$ diagrams for the open armature, closed armature and at least four intermediate positions over a winding current range of $I = 0$ A to $I = 10$ A, Fig. 7. Determine the change in co-energy between armature position steps at $I = 10$ A from the resulting $\Psi - I$ diagrams and find the force-displacement characteristic of the actuator, (11). Repeat this process using the FE numerical model for both `core_linear` and `core_nonlinear` materials. Compare and contrast the four $\Psi - I$ diagrams and force-displacement curves. Tabulate and plot the change in co-energy between armature positions of the `core_nonlinear` model derived from the $\Psi - I$ diagram, `trapz()`, and that measured from the FE model, `mo_groupselectblock()`, selects the entire computational domain, `Ec = mo_blockintegral(17)`, calculates the co-energy, `mo_clearblock()`, clears the selection.

VII. Conclusion

Coursework Task 6: Consider the four modelling approaches used, i.e. analytical equivalent circuit without air-gap fringing, analytical equivalent circuit with air-gap fringing, FE with linear materials and FE with non-linear materials, discuss the limitations of each and explain which method produces the most accurate results. Describe the limitations of this method, considering the 2-d nature of the model. Discuss one or two real world examples where such an actuator may be used. Include other relevant conclusions drawn from the analyses conducted.

VIII. Technical Note Submission

Students are expected to submit a two-column individual technical note of **no more than 8 pages excluding references** conforming to an IEEE academic template which can be found on Blackboard under `Electromagnetics Lab → Coursework Template` in either *Word* or LaTeX format along with instructions. Coursework tasks 1-6 should be addressed in the technical note. Examples of style, conventions, diagrams and figures can be found in published papers on IEEEXplore. Diagrams of the actuator may be derived from the CAD model. A high-level mark scheme is given in Fig. 8. The following sections are suggested as a guide.

1) Abstract
2) Introduction
3) Winding Resistance
4) Winding Inductance
5) Force on the Armature
6) Conclusion

The submission deadline is Friday 27th of March 23:59 via Blackboard where you must submit a PDF of your technical note along with a copy of your Matlab script, more details to follow.

References

[1] N. Simpson, "Electromagnetic characterisation of a short-stroke ferromagnetic actuator: Part a," 2019.
[2] D. Meeker. (2019) Finite element method magnetics (femm). [Online]. Available: www.femm.info
[3] N. Simpson and D. Drury, "Electro-mechanical energy conversion lecture notes," Lecture Notes, 2019.
[4] J. Shewchuck. A two-dimensional quality mesh generator and delaunay triangulator.triangle,. [Online]. Available: http://www.cs.cmu.edu/~quake/triangle.html
[5] P. Hammond, *Applied electromagnetism*. Elsevier, 2013.
[6] J. Pyrhonen, T. Jokinen, and V. Hrabovcova, *Design of rotating electrical machines*. John Wiley & Sons, 2013.

	Exceptional (20)	Above Average (16)	Good (12)	Bare Pass (8)	Unacceptable (4)
Presentation (15%)	• Professionally laid out title pages • Concise introduction • Clear and well though out layout • Professional use of formatting, titles, figures and table numbers • Equations professionally presented • Tables and graphs used where appropriate	As for average but with demonstration that specific thought has been put into how to convey the concepts resented to the reader in a way that eases reading and assimilation. Evidence of organization of thought and data.	Document is structured in an appropriate manner using a mixture of text, figures, tables, etc. and referring to these correctly in the text.	Presentation is mostly fine with use of titles, figure numbers, etc. but with no real thought into how the reader could best assimilate and understand the information presented. This might include : • lengthy text discussions where figures would be appropriate. • Multiple tables of data where a single table would suffice. • Little structure	Any or all of the following demonstrating a lack of understanding of the need for neat presentation as a tool for good communication. • Large amount of hand sketched figures • Equations copied and pasted from other sources. • No titles/figure numbers, etc. • Bad or inconsistent formatting • Flawed use of English
Achievement (25%)	All aspects of the lab have been fully explored and completed and the student has investigated aspects not specifically requested.	All aspects of the lab have been covered as requested. No additional work undertaken.	Most aspects of the work requested in the lab covered. Aspects missing such as • Ambient temperature measurements • Thermal images	Each part of the lab attempted with significant aspects of any of them missing.	Obvious observations reported only with significant aspects of the analysis either missing or incorrect.
Analysis/Discussion (40%)	As good but confident use of measured data to back up theories and the limitations of these. Observation of component behaviour and attempts to explain these behaviours with the data or supplementary investigations.	All required aspects of the laboratory discussed correctly and with reference to measurements, data, theory and laboratory experience. Little or no evidence of looking for anything other than those concepts requested. No curiosity demonstrated.	All aspects of the lab are discussed. Derivation of required expressions but no real use of these to understand electromechanical behaviour. No use of measurements to understand limitations of derived expressions.	Some discussion of the concepts presented that might demonstrate a limited understanding or lack of engagement with the material. Understanding of core concepts without correct application to the specific lab exercise. No analysis or comparison of the measurements taken with the theory developed.	Obvious observations reported only with significant aspects of the analysis either missing or incorrect.
Background (20%)	Each section of report makes reference to either a particular application or practical use or limitation of the lab exercise being discussed. The student has clearly understood the uses of the components outside of the laboratory. Bibliography of sources.	Applications and limitations or practicalities of use suggested for most of the lab exercises investigated whether requested or not. Attempts are made to understand why the applications or limitations are relevant/present.	Some reference to the applications or limitations is made when specifically asked for. No real insight as to why applications are relevant.	Applications requested for some of the lab experiments have been suggested with no real insight or expansion.	No reference to applications at all.

Fig. 8: High level mark scheme for the technical note - for guidance only.