Homework 7

2020-10-07

Question 10.1

Using the same crime data set uscrime.txt as in Questions 8.2 and 9.1, find the best model you can using

(a) a regression tree model, and

(b) a random forest model.

In R, you can use the tree package or the rpart package, and the randomForest package. For each model, describe one or two qualitative takeaways you get from analyzing the results (i.e., don't just stop when you have a good model, but interpret it too).

Answer:

a) Using regression tree model:

```
rm(list = ls())
uscrime <- read.delim("uscrime.txt")
```

Read the data.

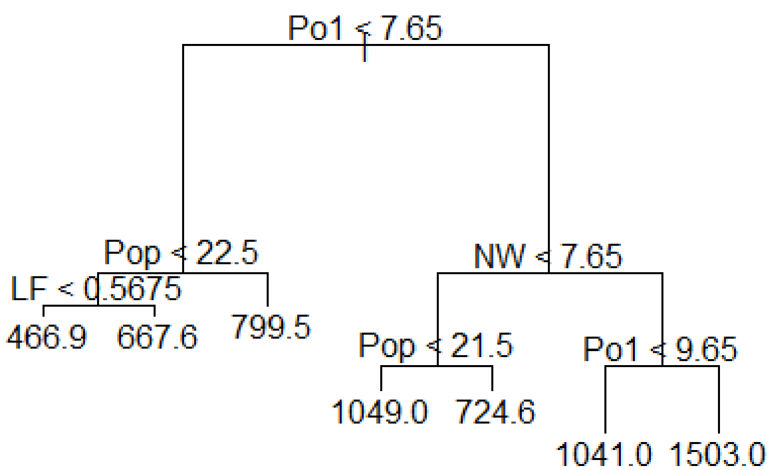
Fitting the regression tree function to the crime data.

```
 Regression Tree Model
library(tree)
uscrime_tree <- tree(Crime~., data = uscrime)
summary(uscrime_tree)

##
## Regression tree:
## tree(formula = Crime ~ ., data = uscrime)
## Variables actually used in tree construction:
## [1] "Po1" "Pop" "LF"  "NW"
## Number of terminal nodes:  7
## Residual mean deviance:  47390 = 1896000 / 40
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -573.900  -98.300   -1.545    0.000  110.600  490.100
```

Deviance: This is a quality of fit statistic that is a generalization of sum of squared residuals

Tree Visualization:

```r
plot(uscrime_tree)
text(uscrime_tree)
```

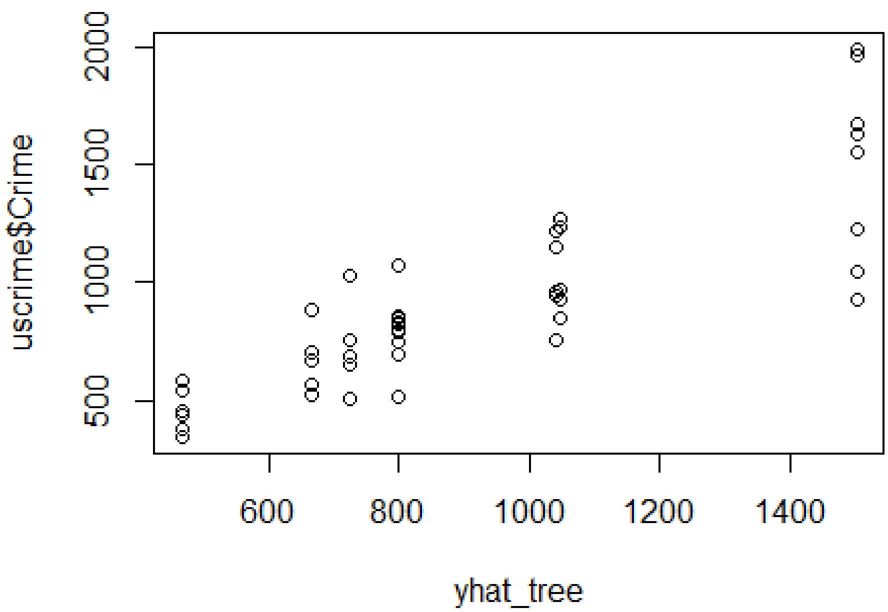



```r
uscrime_tree$frame
```

```
##        var  n         dev      yval splits.cutleft splits.cutright
## 1      Po1 47 6880927.66  905.0851          <7.65           >7.65
## 2      Pop 23  779243.48  669.6087          <22.5           >22.5
## 4       LF 12  243811.00  550.5000        <0.5675         >0.5675
## 8   <leaf>  7   48518.86  466.8571
## 9   <leaf>  5   77757.20  667.6000
## 5   <leaf> 11  179470.73  799.5455
## 3       NW 24 3604162.50 1130.7500          <7.65           >7.65
## 6      Pop 10  557574.90  886.9000          <21.5           >21.5
## 12  <leaf>  5  146390.80 1049.2000
## 13  <leaf>  5  147771.20  724.6000
## 7      Po1 14 2027224.93 1304.9286          <9.65           >9.65
## 14  <leaf>  6  170828.00 1041.0000
## 15  <leaf>  8 1124984.88 1502.8750
```

```r
uscrime_tree$where
```

```
##  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 
26 
##  6 13  4 13  9 10 12 13  6  5 13  6  6  5  6 12  4 13 10 13  6  4 12  9  5 
13 
## 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 
##  4 12 13  6  4 12  6  9 10  9  6  5  6 12  5  4  6 10  4 10  9

#Manually compute R2. Is this a good measure of the quality of fit?
#Notice that we can only use averages of each leaf to make
#predictions.

yhat_tree <- predict(uscrime_tree)
plot(yhat_tree,uscrime$Crime)
```




```
Let us prune the tree leaves and find the quality of fit of our model.

prune.tree(uscrime_tree)$size

## [1] 7 6 5 4 3 2 1

prune.tree(uscrime_tree)$dev

## [1] 1895722 2013257 2276670 2632631 3364043 4383406 6880928

set.seed(42)
```

The tree has 7 leaaves and observe that the deviance is highest for the leaf 1 while lowest deviance is for leaf7.

Run the k fold cross validation to find the deviance or number of misclassification as a function of cost complexity parameter k= 10, FUN is the function to do pruning.

```
cv.tree(object = uscrime_tree, FUN = prune.tree, k = 10)
```

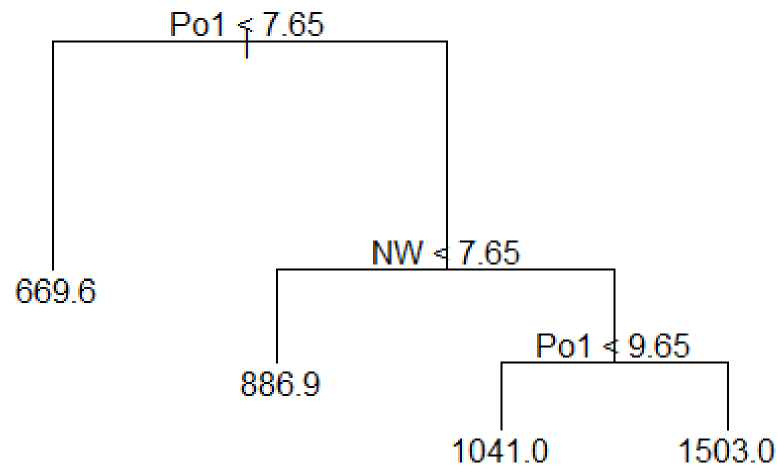
```
## $size
## [1] 7 6 5 4 3 2 1
##
## $dev
## [1] 7986688 7986688 7885099 7920849 7707536 7035494 8536032
##
## $k
## [1]      -Inf  117534.9  263412.9  355961.8  731412.1 1019362.7 2497521.7
##
## $method
## [1] "deviance"
##
## attr(,"class")
## [1] "prune"         "tree.sequence"
```

we can do cross validation.

Example of manaully pruning a tree in which we choose to only have 4 leaves

Tree with only 4 leafs :

```
uscrime_tree_prune4 <- prune.tree(uscrime_tree,best = 4)
plot(uscrime_tree_prune4)
text(uscrime_tree_prune4)
```

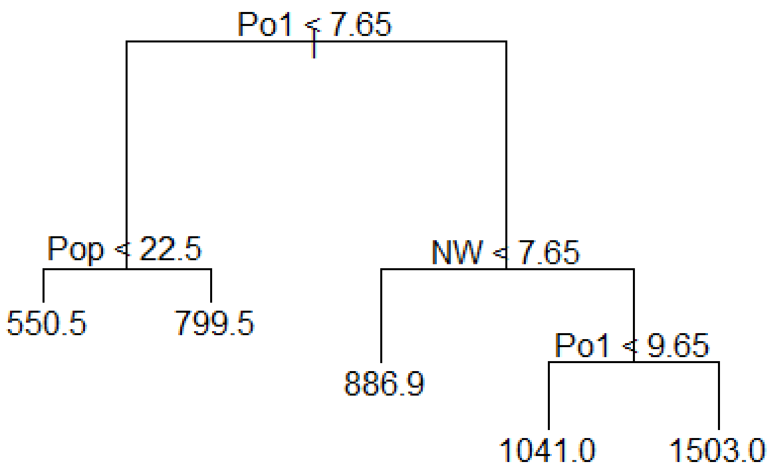



```r
summary(uscrime_tree_prune4)

##
## Regression tree:
## snip.tree(tree = uscrime_tree, nodes = c(6L, 2L))
## Variables actually used in tree construction:
## [1] "Po1" "NW"
## Number of terminal nodes:  4
## Residual mean deviance:  61220 = 2633000 / 43
## Distribution of residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -573.90 -152.60   35.39    0.00  158.90  490.10

crimeTree4Predict <- predict(uscrime_tree_prune4, data = uscrime[,1:15])
RSSofTree4 <- sum((crimeTree4Predict - uscrime[,16])^2)
TSS <- sum((uscrime[,16] - mean(uscrime[,16]))^2)
R2ofTree4 <- 1 - RSSofTree4/TSS
R2ofTree4

## [1] 0.6174017
```

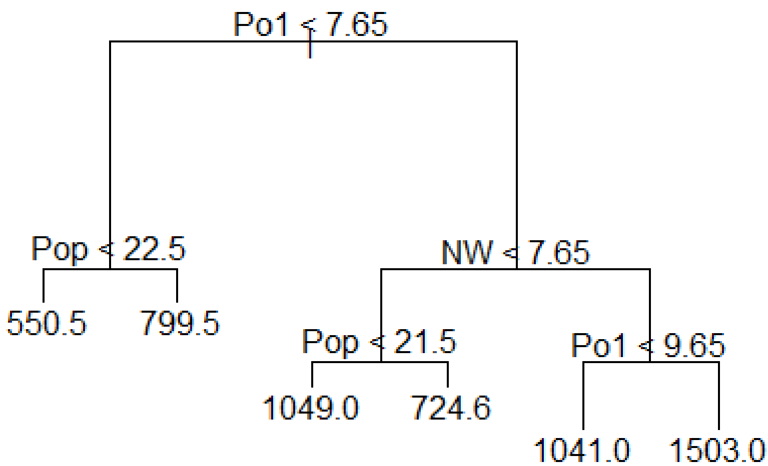
The R Squared error for Tree with 4 leaves is 0.6174017

Prune the tree to have 5 leaves

```r
uscrime_tree_prune5 <- prune.tree(uscrime_tree,best = 5)
plot(uscrime_tree_prune5)
text(uscrime_tree_prune5)
```




```
summary(uscrime_tree_prune5)

##
## Regression tree:
## snip.tree(tree = uscrime_tree, nodes = c(4L, 6L))
## Variables actually used in tree construction:
## [1] "Po1" "Pop" "NW"
## Number of terminal nodes:  5
## Residual mean deviance:  54210 = 2277000 / 42
## Distribution of residuals:
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -573.9  -107.5    15.5     0.0   122.8   490.1

crimeTree5Predict <- predict(uscrime_tree_prune5, data = uscrime[,1:15])
RSSofTree5 <- sum((crimeTree5Predict - uscrime[,16])^2)
R2ofTree5 <- 1 - RSSofTree5 /TSS
R2ofTree5

## [1] 0.6691333

uscrime_tree_prune6 <- prune.tree(uscrime_tree,best = 6)
plot(uscrime_tree_prune6)
text(uscrime_tree_prune6)
```




```
summary(uscrime_tree_prune6)

##
## Regression tree:
## snip.tree(tree = uscrime_tree, nodes = 4L)
## Variables actually used in tree construction:
## [1] "Po1" "Pop" "NW"
## Number of terminal nodes:  6
## Residual mean deviance:  49100 = 2013000 / 41
## Distribution of residuals:
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -573.900  -99.520   -1.545    0.000  122.800  490.100

crimeTree6Predict <- predict(uscrime_tree_prune6, data = uscrime[,1:15])
RSSofTree6 <- sum((crimeTree6Predict - uscrime[,16])^2)
R2ofTree6 <- 1 - RSSofTree6 /TSS
R2ofTree6

## [1] 0.7074149



Analysis :
I built 4 models with different number of leaves. Below are the results.
The RSquared error for tree with 7 leaves  = 0.47390
The RSquared error for tree with 6 leaves = 0.7074149
The Rsquared error for tree with 5 leaves = 0.6691333
```

```
The Rsquared error for tree with 4 leaves = 0.6174017

The model of the tree with 4 leaves has a lower R Squared error and so looks
like the best among the built models.

 Q10.1b : Using Random Forests
```

Grow Random trees and set the number of predictors that you want to consider at each split. Using number of predictors / 3 is a good number to choose. Since we have 15 apredictors, 15/3 ~ 4 would be a good value for the number of predictors at each split.

```
library(randomForest)

## randomForest 4.6-14

## Type rfNews() to see new features/changes/bug fixes.

set.seed(42)
num_pred <- 4
uscrime_rf <- randomForest(Crime~.,data = uscrime,mtry = num_pred,importance
= TRUE , ntree = 500)
uscrime_rf

##
## Call:
##  randomForest(formula = Crime ~ ., data = uscrime, mtry = num_pred,
importance = TRUE, ntree = 500)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 4
##
##           Mean of squared residuals: 83636.77
##                     % Var explained: 42.87

crime_rf_predict <- predict(uscrime_rf, data=uscrime[,-16])
RSS <- sum((crime_rf_predict - uscrime[,16])^2)
R2 <- 1 - RSS/TSS
R2

## [1] 0.4287212

The RSqaured error for mtry = 4 is 0.4287212

num_pred5 <- 5
uscrime_rf5 <- randomForest(Crime~.,data = uscrime,mtry =
num_pred5,importance = TRUE , ntree = 500)
uscrime_rf5

##
## Call:
```

```
##  randomForest(formula = Crime ~ ., data = uscrime, mtry = num_pred5,
importance = TRUE, ntree = 500)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 5
##
##           Mean of squared residuals: 88034.11
##                     % Var explained: 39.87
```

```
crime_rf_predict5 <- predict(uscrime_rf5, data=uscrime[,-16])
RSS5 <- sum((crime_rf_predict5 - uscrime[,16])^2)
R2_pre5 <- 1 - RSS5/TSS
R2_pre5
```

```
## [1] 0.3986853
```

```
The RSqaured error for mtry = 5 is 0.3986953
```

For mtry = 5 has the lowest RSqaured than the model with mtry = 4.

```
importance(uscrime_rf)
```

```
##           %IncMSE IncNodePurity
## M       2.4984854     200566.40
## So      1.3802135      33881.59
## Ed      4.8378328     198601.72
## Po1     9.7354718    1076933.25
## Po2    10.6715396    1268930.03
## LF      0.6449124     311872.13
## M.F     1.1555044     239897.22
## Pop     2.1893155     379760.15
## NW      8.7310286     542658.76
## U1      2.6422460     145760.60
## U2      1.6754487     190587.49
## Wealth  3.2683848     626353.30
## Ineq    2.1162044     238557.90
## Prob    8.6884908     812217.29
## Time    1.6622726     202467.06
```

Inference : Random Forest gives us the better model with the lowert RSquared error than the Random Tree. There is a problem of overfitting the data . Looks like Po2 , Po1, NW, are very important predictors which contribute to the Random Trees by the measure of %IncMSE . Increasing the number of variables at each split decreases the accuracy of the model.

Question 10.2

Describe a situation or problem from your job, everyday life, current events, etc., for which a logistic regression model would be appropriate. List some (up to 5) predictors that you might use.

Answer:

Logistic Regression method can be used to predict the probability of success(pass/fail) in the exam based on the number of hours of study, class attendance of the student, homework submission, homework grade and student's health.

```
##########Q10.3#########
```

Question 10.3

1. **Using the GermanCredit data set `germancredit.txt` from http://archive.ics.uci.edu/ml/machine-learning-databases/statlog/german / (description at http://archive.ics.uci.edu/ml/datasets/Statlog+%28German+Credit+Data%29), use logistic regression to find a good predictive model for whether credit applicants are good credit risks or not. Show your model (factors used and their coefficients), the software output, and the quality of fit. You can use the `glm` function in R. To get a logistic regression (logit) model on data where the response is either zero or one, use `family=binomial(link="logit")` in your `glm` function call.**

2. **Because the model gives a result between 0 and 1, it requires setting a threshold probability to separate between "good" and "bad" answers. In this data set, they estimate that incorrectly identifying a bad customer as good, is 5 times worse than incorrectly classifying a good customer as bad. Determine a good threshold probability based on your model.**

```
set.seed(10)

rm(list = ls())
germancredit <- read.table("germancredit.txt",header = FALSE)
str(germancredit)

## 'data.frame':    1000 obs. of  21 variables:
##  $ V1 : chr  "A11" "A12" "A14" "A11" ...
```

```
##  $ V2 : int  6 48 12 42 24 36 24 36 12 30 ...
##  $ V3 : chr  "A34" "A32" "A34" "A32" ...
##  $ V4 : chr  "A43" "A43" "A46" "A42" ...
##  $ V5 : int  1169 5951 2096 7882 4870 9055 2835 6948 3059 5234 ...
##  $ V6 : chr  "A65" "A61" "A61" "A61" ...
##  $ V7 : chr  "A75" "A73" "A74" "A74" ...
##  $ V8 : int  4 2 2 2 3 2 3 2 2 4 ...
##  $ V9 : chr  "A93" "A92" "A93" "A93" ...
##  $ V10: chr  "A101" "A101" "A101" "A103" ...
##  $ V11: int  4 2 3 4 4 4 4 2 4 2 ...
##  $ V12: chr  "A121" "A121" "A121" "A122" ...
##  $ V13: int  67 22 49 45 53 35 53 35 61 28 ...
##  $ V14: chr  "A143" "A143" "A143" "A143" ...
##  $ V15: chr  "A152" "A152" "A152" "A153" ...
##  $ V16: int  2 1 1 1 2 1 1 1 1 2 ...
##  $ V17: chr  "A173" "A173" "A172" "A173" ...
##  $ V18: int  1 1 2 2 2 2 1 1 1 1 ...
##  $ V19: chr  "A192" "A191" "A191" "A191" ...
##  $ V20: chr  "A201" "A201" "A201" "A201" ...
##  $ V21: int  1 2 1 1 2 1 1 1 1 2 ...

# Notice that we have many categorical predictors.

#Make the response variable binary in terms of 0 and 1.
germancredit$V21[germancredit$V21==1] <- 0
germancredit$V21[germancredit$V21==2] <- 1

head(germancredit)

##    V1 V2  V3  V4   V5  V6  V7 V8  V9  V10 V11  V12 V13  V14  V15 V16  V17
V18
## 1 A11  6 A34 A43 1169 A65 A75  4 A93 A101   4 A121  67 A143 A152   2 A173
1
## 2 A12 48 A32 A43 5951 A61 A73  2 A92 A101   2 A121  22 A143 A152   1 A173
1
## 3 A14 12 A34 A46 2096 A61 A74  2 A93 A101   3 A121  49 A143 A152   1 A172
2
## 4 A11 42 A32 A42 7882 A61 A74  2 A93 A103   4 A122  45 A143 A153   1 A173
2
## 5 A11 24 A33 A40 4870 A61 A73  3 A93 A101   4 A124  53 A143 A153   2 A173
2
## 6 A14 36 A32 A46 9055 A65 A73  2 A93 A101   4 A124  35 A143 A153   1 A172
2
##    V19  V20 V21
## 1 A192 A201   0
## 2 A191 A201   1
## 3 A191 A201   0
## 4 A191 A201   0
## 5 A191 A201   1
## 6 A192 A201   0
```

```r
#split the data into training and testing sets.
germancredit_train <- germancredit[1:800,]
germancredit_test <- germancredit[801:1000,]

#create a logistic regression model
germancredit_model = glm(V21~., family=binomial(link = "logit"),
                          data=germancredit_train)

summary(germancredit_model)

##
## Call:
## glm(formula = V21 ~ ., family = binomial(link = "logit"), data =
germancredit_train)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.7373  -0.6979  -0.3604   0.6663   2.5591
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)  3.978e-01  1.249e+00   0.318 0.750139
## V1A12       -2.701e-01  2.437e-01  -1.108 0.267784
## V1A13       -9.306e-01  4.018e-01  -2.316 0.020567 *
## V1A14       -1.737e+00  2.686e-01  -6.465 1.02e-10 ***
## V2           2.893e-02  1.042e-02   2.777 0.005479 **
## V3A31        2.255e-01  6.289e-01   0.359 0.719936
## V3A32       -7.639e-01  4.753e-01  -1.607 0.108022
## V3A33       -9.172e-01  5.233e-01  -1.753 0.079627 .
## V3A34       -1.487e+00  4.907e-01  -3.031 0.002440 **
## V4A41       -1.832e+00  4.425e-01  -4.141 3.46e-05 ***
## V4A410      -1.413e+00  8.263e-01  -1.710 0.087326 .
## V4A42       -9.368e-01  2.990e-01  -3.134 0.001727 **
## V4A43       -9.044e-01  2.799e-01  -3.230 0.001236 **
## V4A44       -8.312e-01  8.946e-01  -0.929 0.352807
## V4A45       -3.222e-01  6.092e-01  -0.529 0.596843
## V4A46        1.688e-02  4.255e-01   0.040 0.968354
## V4A48       -2.213e+00  1.219e+00  -1.816 0.069365 .
## V4A49       -8.368e-01  3.850e-01  -2.173 0.029760 *
## V5           1.138e-04  5.166e-05   2.202 0.027682 *
## V6A62       -3.991e-01  3.182e-01  -1.254 0.209771
## V6A63       -4.615e-01  4.762e-01  -0.969 0.332404
## V6A64       -1.222e+00  5.473e-01  -2.232 0.025592 *
## V6A65       -7.093e-01  2.929e-01  -2.421 0.015462 *
## V7A72       -2.017e-01  4.948e-01  -0.408 0.683485
## V7A73       -3.028e-01  4.706e-01  -0.643 0.519975
## V7A74       -1.105e+00  5.113e-01  -2.162 0.030623 *
## V7A75       -4.092e-01  4.712e-01  -0.869 0.385102
## V8           3.602e-01  9.933e-02   3.626 0.000287 ***
## V9A92       -4.434e-01  4.300e-01  -1.031 0.302374
```

```
## V9A93       -1.230e+00  4.245e-01  -2.897 0.003769 **
## V9A94       -4.630e-01  5.119e-01  -0.905 0.365705
## V10A102      7.521e-01  4.771e-01   1.576 0.114917
## V10A103     -9.329e-01  4.830e-01  -1.931 0.053423 .
## V11          3.282e-03  9.850e-02   0.033 0.973420
## V12A122      4.101e-01  2.897e-01   1.415 0.156969
## V12A123      1.536e-01  2.649e-01   0.580 0.562115
## V12A124      7.122e-01  4.714e-01   1.511 0.130827
## V13         -1.868e-02  1.055e-02  -1.770 0.076682 .
## V14A142     -1.442e-02  4.733e-01  -0.030 0.975695
## V14A143     -4.354e-01  2.724e-01  -1.599 0.109919
## V15A152     -3.967e-01  2.739e-01  -1.448 0.147576
## V15A153     -5.576e-01  5.303e-01  -1.051 0.293071
## V16          3.297e-01  2.124e-01   1.552 0.120602
## V17A172      5.151e-01  7.807e-01   0.660 0.509351
## V17A173      5.655e-01  7.507e-01   0.753 0.451267
## V17A174      8.202e-01  7.597e-01   1.080 0.280307
## V18          5.065e-01  2.854e-01   1.775 0.075972 .
## V19A192     -3.739e-01  2.323e-01  -1.610 0.107489
## V20A202     -1.498e+00  8.079e-01  -1.854 0.063779 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 975.68  on 799  degrees of freedom
## Residual deviance: 705.07  on 751  degrees of freedom
## AIC: 803.07
##
## Number of Fisher Scoring iterations: 5
```

```r
#consider doing some type of variable selection even though this has not
#been covered in the lectures yet. Also, notice how glm() implicitly
#creates dummy binary variables for each of the categorical variables.
#This is the correct way to do regression with categorical variables.
#however, if you want to do variable selection with these many dummy
#variables, you must re-define your categorical variables either manually
# or with an R function.

yhat<-predict(germancredit_model,germancredit_test[,-21],type= "response")
table(germancredit_test$V21, round(yhat))
```

```
##
##       0   1
##   0 115  24
##   1  29  32
```

```r
#Important to use type = "response" here because without this
# we are given predictions of log-odds in the default case.
```

```
#"round" your the yhat to get binary predictions from which
#you can compute an accuracy (classification rate). You may  want
#to try out differnt thresholds for rounding. You can also use AUC to
#estimate the quality of fit.

library(pROC)

## Type 'citation("pROC")' for a citation.

##
## Attaching package: 'pROC'

## The following objects are masked from 'package:stats':
##
##     cov, smooth, var

roc(germancredit_test$V21,round(yhat))

## Setting levels: control = 0, case = 1

## Setting direction: controls < cases

##
## Call:
## roc.default(response = germancredit_test$V21, predictor = round(yhat))
##
## Data: round(yhat) in 139 controls (germancredit_test$V21 0) < 61 cases
(germancredit_test$V21 1).
## Area under the curve: 0.676

#Look at different threshold probability values and then compute the
#cost that corresponds to each threshold.

thresh <- 0.8
yhat_thresh <- as.integer(yhat > thresh)
conf_matrix <- as.matrix(table(yhat_thresh,germancredit_test$V21))
conf_matrix

##
## yhat_thresh   0   1
##           0 134  53
##           1   5   8

accuracy <-
(conf_matrix[1,1]+conf_matrix[2,2])/(conf_matrix[1,1]+conf_matrix[1,2]+conf_m
atrix[2,1]+conf_matrix[2,2])
accuracy

## [1] 0.71

specificity <- (conf_matrix[1,1])/(conf_matrix[1,1]+conf_matrix[2,1])
specificity
```

```
## [1] 0.9640288
```

```r
thresh <- 0.7
yhat_thresh <- as.integer(yhat > thresh)
conf_matrix <- as.matrix(table(yhat_thresh,germancredit_test$V21))
conf_matrix
```

```
##
## yhat_thresh   0   1
##           0 132  43
##           1   7  18
```

```r
accuracy <-
(conf_matrix[1,1]+conf_matrix[2,2])/(conf_matrix[1,1]+conf_matrix[1,2]+conf_m
atrix[2,1]+conf_matrix[2,2])
accuracy
```

```
## [1] 0.75
```

```r
specificity <- (conf_matrix[1,1])/(conf_matrix[1,1]+conf_matrix[2,1])
specificity
```

```
## [1] 0.9496403
```

Conclusion:

Based on the confusion Matrix for threshold 0.8 has accuracy 0.71 and specificity = 0.9640288 and for threshold 0.7 accuracy is 0.75 and specificity = 0.9496403. With threshold 0.7 we are getting better accuracy and specificity.