Homework 8 by Haritha Pulletikurti

Question 11.1

Using the crime data set uscrime.txt from Questions 8.2, 9.1, and 10.1, build a regression model using:

1. Stepwise regression
2. Lasso
3. Elastic net

For Parts 2 and 3, remember to scale the data first – otherwise, the regression coefficients will be on different scales and the constraint won't have the desired effect.
For Parts 2 and 3, use the glmnet function in R.
Notes on R:

- For the elastic net model, what we called λ in the videos, glmnet calls "alpha"; you can get a range of results by varying alpha from 1 (lasso) to 0 (ridge regression) [and, of course, other values of alpha in between].
- In a function call like glmnet(x,y,family="mgaussian",alpha=1) the predictors x need to be in R's matrix format, rather than data frame format. You can convert a data frame to a matrix using as.matrix – for example, x <- as.matrix(data[,1:n-1])
- Rather than specifying a value of T, glmnet returns models for a variety of values of T.

Answer:

Stepwise Regression Techniques: As we all know,

Bias: is the difference between average prediction of our model and the correct value we are trying to predict.
Variance: is the variability of the model prediction based on different data sets i.e. Training / validation/Test data sets.
Best Model: We all know the best model is considered to have low bias and low variance.

If n = Number of data points and p = number of predictors

Case 1: if n is very large than p i.e. n >> p, then the least squared regression model tends to have low bias and low variance.

Case 2: if n is not much larger than p i.e. n >p, then there can be lot of variability in least squares fit resulting in overfitting.
Case 3: If n is smaller than p, i.e. n < p, then the variance is infinite as there is no longer a unique least squared estimate.

There are many approaches for variable selection i.e. excluding the irrelevant variables from a multiple regression model like the ones mentioned below

Forward Step wise Regression:

Step 1: Let Mo denote the null model which contains no predictors.
Step 2: For k = 0, …, p-1

a) Consider all p-k models that augment the predictors in the set Mk with one additional predictor.

b) Choose best among the p-k models, call it Mk+1. Here best is defined as having the smallest RSS or highest R^2.

Step3: Select the single best model from among the Mo, …, Mp using cross validated predictor error AIC, BIC or adjusted R^2.

Backward Stepwise Elimination:

Step 1: Let Mo denote the full model which contains all the predictors.

Step 2: For k = p,p-1, …,1

a) Consider all k models that contain all but one of the predictors in the set Mk, for a total of k-1 predictors.
b) Choose best among these k models, call it Mk-1. Here best is defined as having the smallest RSS or highest R^2.

Step3: Select the single best model from among the Mo, …, Mp using cross validated predictor error AIC, BIC or adjusted R^2.

Backward selection requires that the number of samples n is larger than number of variables in p so that full model can be fit. In contrast Forward selection model can be used even when n < p and so is the only viable subset method when p is large.

Stepwise Regression: is a combination of both forward selection and backward elimination.

1.Start with no predictors.

2.1 Find the best new predictor if it is good enough

a) add that factor, fit model with current set of factors.

b) Remove factors with high p value.

c) Check if we have enough factors. If not repeat from steps 2.1.

2.2 If the chosen new predictor is not good enough

a) remove factors with high p-value

b) fit the model with final set of factors.

Optimal Model: The model that contains all the predictors will always have the smallest RSS and the largest R^2, since these quantities are related to training error. We should choose the model that has the low-test error. Since these measurements are not suitable to choose between models with different number of predictors, the best way to choose the model is by checking AIC, BIC and Adjusted R^2.

[References: An Introduction to Statistical Learning Text book].

Forward Regression

```
rm(list = ls())
set.seed(82)
uscrime<- read.table("uscrime.txt", stringsAsFactors = FALSE, header = TRUE)

uscrime[1:3,]

##      M So   Ed  Po1 Po2    LF   M.F Pop   NW    U1  U2 Wealth Ineq     Prob
## 1 15.1  1  9.1  5.8 5.6 0.510  95.0  33 30.1 0.108 4.1   3940 26.1 0.084602
## 2 14.3  0 11.3 10.3 9.5 0.583 101.2  13 10.2 0.096 3.6   5570 19.4 0.029599
```

```
## 3 14.2  1  8.9  4.5 4.4 0.533  96.9  18 21.9 0.094 3.3   3180 25.0 0.083401
##      Time Crime
## 1 26.2011   791
## 2 25.2999  1635
## 3 24.3006   578

# Scale the data
Scaleduscrime <- as.data.frame(scale(uscrime[,c(1,3:15)]))
Scaleduscrime <- cbind(uscrime[,2],Scaleduscrime,uscrime[,16])
colnames(Scaleduscrime)[1] <- "So"
colnames(Scaleduscrime)[16] <- "Crime"

Scaleduscrime[1:3,]

##   So         M         Ed        Po1        Po2         LF        M.F
## 1  1 0.9886930 -1.3085099 -0.9085105 -0.8666988 -1.2667456 -1.1206050
## 2  0 0.3521372  0.6580587  0.6056737  0.5280852  0.5396568  0.9834175
## 3  1 0.2725678 -1.4872888 -1.3459415 -1.2958632 -0.6976051 -0.4758239
##          Pop          NW          U1         U2     Wealth     Ineq       Prob
## 1 -0.09500679 1.943738564  0.69510600  0.8313680 -1.3616094 1.679364  1.6497631
## 2 -0.62033844 0.008483424  0.02950365  0.2393332  0.3276683 0.000000 -0.7693365
## 3 -0.48900552 1.146296747 -0.08143007 -0.1158877 -2.1492481 1.403647  1.5969416
##          Time Crime
## 1 -0.05599367   791
## 2 -0.18315796  1635
## 3 -0.32416470   578

# Split the data into Training and Test Datasets.
library(caret)

## Loading required package: lattice

## Loading required package: ggplot2

randomrows <-  createDataPartition(y=1:nrow(Scaleduscrime),p=0.7, list = FALSE)
TrainingData = Scaleduscrime[randomrows,]
TestData = Scaleduscrime[-randomrows,]
dim(TrainingData)

## [1] 35 16

dim(TestData)

## [1] 12 16

library(olsrr)

##
## Attaching package: 'olsrr'

## The following object is masked from 'package:datasets':
##
##     rivers

## ------------------------------------------------------------------------------

# Perform Forward Regression using aic
model<-lm(Crime~.,data = TrainingData)
Forwardfit.aic <-ols_step_forward_aic(model, details = TRUE)

## Forward Selection Method
## ------------------------
```

```
##
## Candidate Terms:
##
## 1 . So
## 2 . M
## 3 . Ed
## 4 . Po1
## 5 . Po2
## 6 . LF
## 7 . M.F
## 8 . Pop
## 9 . NW
## 10 . U1
## 11 . U2
## 12 . Wealth
## 13 . Ineq
## 14 . Prob
## 15 . Time
##
##  Step 0: AIC = 507.0876
##  Crime ~ 1
##
## ------------------------------------------------------------------------------
## Variable     DF      AIC         Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Po1           1    495.841    1128626.869     2453269.016     0.315         0.294
## Po2           1    496.326    1094409.821     2487486.064     0.306         0.284
## Prob          1    497.633     999751.467     2582144.419     0.279         0.257
## Pop           1    499.351     869842.365     2712053.520     0.243         0.220
## Time          1    502.489     615451.183     2966444.703     0.172         0.147
## Wealth        1    502.537     611427.125     2970468.761     0.171         0.146
## U2            1    507.541     154871.969     3427023.917     0.043         0.014
## Ed            1    507.567     152323.822     3429572.063     0.043         0.014
## Ineq          1    508.547      54884.610     3527011.276     0.015        -0.015
## M.F           1    508.844      24841.604     3557054.282     0.007        -0.023
## So            1    508.940      15112.715     3566783.170     0.004        -0.026
## NW            1    508.988      10216.386     3571679.500     0.003        -0.027
## M             1    509.016       7344.408     3574551.478     0.002        -0.028
## U1            1    509.038       5105.396     3576790.489     0.001        -0.029
## LF            1    509.085        314.577     3581581.309     0.000        -0.030
## ------------------------------------------------------------------------------
##
##
## - Po1
##
##
##  Step 1 : AIC = 495.8411
##  Crime ~ Po1
##
## -----------------------------------------------------------------------------
## Variable     DF      AIC         Sum Sq          RSS         R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## Ineq          1    491.397    412570.973     2040698.043     0.430         0.395
## M             1    491.519    405405.508     2047863.508     0.428         0.393
## Time          1    494.656    213421.255     2239847.762     0.375         0.336
## So            1    494.849    201012.706     2252256.311     0.371         0.332
## NW            1    495.057    187592.705     2265676.311     0.367         0.328
## Prob          1    496.070    121066.144     2332202.872     0.349         0.308
## Pop           1    496.667     80947.348     2372321.668     0.338         0.296
## Wealth        1    497.570     18934.023     2434334.994     0.320         0.278
## M.F           1    497.651     13278.426     2439990.591     0.319         0.276
```

```
## U2             1    497.681     11227.973    2442041.043    0.318         0.276
## U1             1    497.733      7586.205    2445682.812    0.317         0.275
## Po2            1    497.746      6676.168    2446592.848    0.317         0.274
## Ed             1    497.764      5367.968    2447901.049    0.317         0.274
## LF             1    497.780      4263.764    2449005.252    0.316         0.274
## ------------------------------------------------------------------------------
##
## - Ineq
##
##
##  Step 2 : AIC = 491.3966
##  Crime ~ Po1 + Ineq
##
## ------------------------------------------------------------------------------
## Variable      DF      AIC        Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Wealth         1    484.774    445589.256    1595108.787    0.555         0.512
## Prob           1    486.606    359892.126    1680805.917    0.531         0.485
## Ed             1    487.472    317758.831    1722939.212    0.519         0.472
## M.F            1    490.587    157410.744    1883287.299    0.474         0.423
## M              1    490.768    147630.923    1893067.120    0.471         0.420
## Time           1    491.324    117321.510    1923376.533    0.463         0.411
## LF             1    491.955     82341.568    1958356.475    0.453         0.400
## U1             1    493.363      1980.571    2038717.472    0.431         0.376
## U2             1    493.379      1023.689    2039674.354    0.431         0.375
## Pop            1    493.380       951.595    2039746.449    0.431         0.375
## NW             1    493.390       357.619    2040340.424    0.430         0.375
## So             1    493.396        41.327    2040656.716    0.430         0.375
## Po2            1    493.397         2.564    2040695.479    0.430         0.375
## ------------------------------------------------------------------------------
##
## - Wealth
##
##
##  Step 3 : AIC = 484.7744
##  Crime ~ Po1 + Ineq + Wealth
##
## ------------------------------------------------------------------------------
## Variable      DF      AIC        Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## M              1    478.186    347075.378    1248033.409    0.652         0.605
## Prob           1    482.112    198933.020    1396175.767    0.610         0.558
## Time           1    483.484    143109.094    1451999.694    0.595         0.541
## Ed             1    484.913     82605.957    1512502.830    0.578         0.521
## M.F            1    485.267     67258.096    1527850.691    0.573         0.517
## U1             1    486.536     10834.128    1584274.660    0.558         0.499
## NW             1    486.625      6785.588    1588323.200    0.557         0.497
## So             1    486.685      4046.061    1591062.727    0.556         0.497
## LF             1    486.694      3659.550    1591449.238    0.556         0.496
## U2             1    486.707      3086.595    1592022.192    0.556         0.496
## Pop            1    486.719      2526.835    1592581.952    0.555         0.496
## Po2            1    486.745      1337.351    1593771.436    0.555         0.496
## ------------------------------------------------------------------------------
##
## - M
##
##
##  Step 4 : AIC = 478.1863
##  Crime ~ Po1 + Ineq + Wealth + M
##
## ------------------------------------------------------------------------------
```

```
## Variable      DF      AIC        Sum Sq           RSS         R-Sq      Adj. R-Sq
## ---------------------------------------------------------------------------------
## Prob           1     477.124    104553.541     1143479.868     0.681         0.626
## U1             1     477.869     79970.953     1168062.456     0.674         0.618
## Ed             1     478.051     73880.461     1174152.948     0.672         0.616
## U2             1     478.557     56776.806     1191256.603     0.667         0.610
## M.F            1     478.723     51097.530     1196935.879     0.666         0.608
## Time           1     478.757     49954.024     1198079.385     0.666         0.608
## NW             1     479.845     12104.006     1235929.403     0.655         0.595
## So             1     479.968      7776.684     1240256.725     0.654         0.594
## LF             1     480.134      1847.025     1246186.383     0.652         0.592
## Po2            1     480.142      1595.606     1246437.803     0.652         0.592
## Pop            1     480.144      1494.654     1246538.755     0.652         0.592
## ---------------------------------------------------------------------------------
##
## - Prob
##
##
##  Step 5 : AIC = 477.124
##  Crime ~ Po1 + Ineq + Wealth + M + Prob
##
## -------------------------------------------------------------------------------
## Variable      DF      AIC       Sum Sq           RSS         R-Sq      Adj. R-Sq
## -------------------------------------------------------------------------------
## Ed             1     476.040    96447.490     1047032.378     0.708         0.645
## U1             1     476.409    85335.284     1058144.584     0.705         0.641
## U2             1     477.242    59865.135     1083614.733     0.697         0.633
## M.F            1     477.574    49526.924     1093952.944     0.695         0.629
## LF             1     478.890     7619.524     1135860.344     0.683         0.615
## Time           1     478.950     5669.305     1137810.563     0.682         0.614
## Po2            1     479.035     2913.147     1140566.721     0.682         0.613
## Pop            1     479.098      841.502     1142638.366     0.681         0.613
## NW             1     479.105      626.205     1142853.662     0.681         0.613
## So             1     479.124        7.427     1143472.441     0.681         0.612
## -------------------------------------------------------------------------------
##
## - Ed
##
##
##  Step 6 : AIC = 476.04
##  Crime ~ Po1 + Ineq + Wealth + M + Prob + Ed
##
## ---------------------------------------------------------------------------------
## Variable      DF      AIC        Sum Sq           RSS         R-Sq      Adj. R-Sq
## ---------------------------------------------------------------------------------
## U2             1     473.509    127134.753      919897.625     0.743         0.677
## U1             1     475.456     74522.160      972510.218     0.728         0.658
## LF             1     476.002     59212.052      987820.326     0.724         0.653
## Time           1     477.007     30442.920     1016589.459     0.716         0.643
## NW             1     477.574     13847.376     1033185.003     0.712         0.637
## So             1     477.702     10061.297     1036971.081     0.710         0.635
## M.F            1     477.730      9223.807     1037808.571     0.710         0.635
## Pop            1     477.942      2924.112     1044108.266     0.709         0.633
## Po2            1     478.020       596.911     1046435.467     0.708         0.632
## ---------------------------------------------------------------------------------
##
## - U2
##
##
##  Step 7 : AIC = 473.5091
##  Crime ~ Po1 + Ineq + Wealth + M + Prob + Ed + U2
```

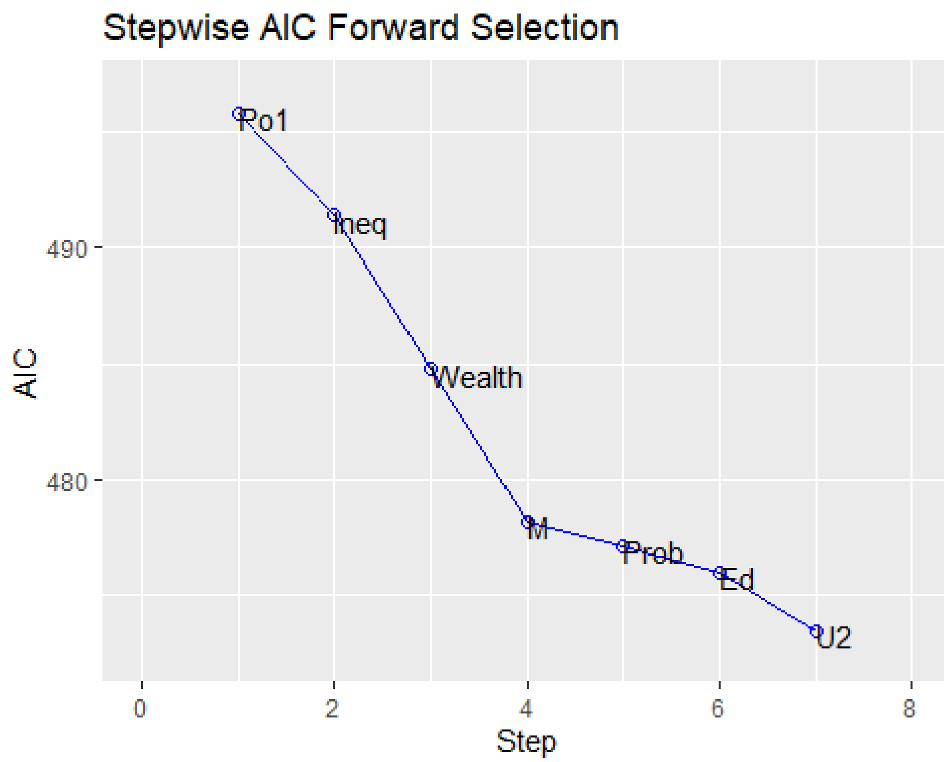
```
##
## ---------------------------------------------------------------------------
## Variable     DF      AIC       Sum Sq         RSS        R-Sq     Adj. R-Sq
## ---------------------------------------------------------------------------
## Time          1    474.294    31378.195    888519.430    0.752        0.676
## LF            1    474.787    18781.605    901116.020    0.748        0.671
## So            1    475.244     6935.270    912962.355    0.745        0.667
## NW            1    475.325     4822.279    915075.346    0.745        0.666
## U1            1    475.449     1579.265    918318.360    0.744        0.665
## Pop           1    475.492      462.964    919434.661    0.743        0.664
## M.F           1    475.505       99.901    919797.724    0.743        0.664
## Po2           1    475.507       50.328    919847.297    0.743        0.664
## ---------------------------------------------------------------------------
##
##
## No more variables to be added.
##
## Variables Entered:
##
## - Po1
## - Ineq
## - Wealth
## - M
## - Prob
## - Ed
## - U2
##
##
## Final Model Output
## ------------------
##
##                           Model Summary
## -----------------------------------------------------------------
## R                       0.862       RMSE                 184.581
## R-Squared               0.743       Coef. Var             20.622
## Adj. R-Squared          0.677       MSE                34070.282
## Pred R-Squared          0.504       MAE                  127.287
## -----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                  ANOVA
## -----------------------------------------------------------------------
##                   Sum of
##                  Squares        DF    Mean Square      F         Sig.
## -----------------------------------------------------------------------
## Regression    2661998.261         7     380285.466    11.162    0.0000
## Residual       919897.625        27      34070.282
## Total         3581895.886        34
## -----------------------------------------------------------------------
##
##                                      Parameter Estimates
## ---------------------------------------------------------------------------------------
--
##       model        Beta    Std. Error    Std. Beta      t        Sig        lower
upper
## ---------------------------------------------------------------------------------------
--
## (Intercept)     882.916        32.809                 26.911    0.000     815.598
950.234
```

```
##          Po1      164.745         67.323         0.481      2.447     0.021       26.610
302.880
##         Ineq      378.075         75.204         1.183      5.027     0.000      223.769
532.381
##       Wealth      266.440        114.685         0.827      2.323     0.028       31.125
501.755
##            M      140.702         45.849         0.457      3.069     0.005       46.628
234.776
##         Prob     -122.299         60.001        -0.321     -2.038     0.051     -245.410
0.813
##           Ed      129.626         59.133         0.408      2.192     0.037        8.294
250.958
##           U2       72.373         37.465         0.217      1.932     0.064       -4.500
149.245
## ---------------------------------------------------------------------------------------------
--
```

```
Forwardfit.aic
```

```
##
##                               Selection Summary
## -----------------------------------------------------------------------------
## Variable       AIC        Sum Sq           RSS          R-Sq      Adj. R-Sq
## -----------------------------------------------------------------------------
## Po1          495.841    1128626.869    2453269.016    0.31509      0.29434
## Ineq         491.397    1541197.843    2040698.043    0.43027      0.39467
## Wealth       484.774    1986787.099    1595108.787    0.55467      0.51158
## M            478.186    2333862.477    1248033.409    0.65157      0.60511
## Prob         477.124    2438416.018    1143479.868    0.68076      0.62572
## Ed           476.040    2534863.508    1047032.378    0.70769      0.64505
## U2           473.509    2661998.261     919897.625    0.74318      0.67660
## -----------------------------------------------------------------------------
```

```
plot(Forwardfit.aic)
```



Analysis : The Forward Selection Model started with no predictors and at each step added one predictor (selection was based on the least AIC) until the model can no longer be improved.

Here the best Model that the Forward Selection Model gave us is Crime ~ M + Ed + Po1 + U2 + Wealth + Ineq + Prob with AIC = 473.509.

Backward Elimination model using aic

```
BackwardFit.aic <- ols_step_backward_aic(model, details = TRUE)

## Backward Elimination Method
## ---------------------------
##
## Candidate Terms:
##
## 1 . So
## 2 . M
## 3 . Ed
## 4 . Po1
## 5 . Po2
## 6 . LF
## 7 . M.F
## 8 . Pop
## 9 . NW
## 10 . U1
## 11 . U2
## 12 . Wealth
## 13 . Ineq
## 14 . Prob
## 15 . Time
##
```

```
##  Step 0: AIC = 484.7026
##  Crime ~ So + M + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 + U2 + Wealth + Ineq + Prob + 
Time
## 
## -----------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq          RSS         R-Sq     Adj. R-Sq 
## -----------------------------------------------------------------------------
## Pop           1     482.714       250.498     802109.470    0.776        0.619 
## Po2           1     482.759      1293.475     803152.447    0.776        0.619 
## So            1     482.772      1583.973     803442.946    0.776        0.619 
## Po1           1     483.175     10902.572     812761.545    0.773        0.614 
## NW            1     483.487     18171.034     820030.007    0.771        0.611 
## U1            1     483.552     19705.007     821563.980    0.771        0.610 
## Time          1     484.499     42220.203     844079.176    0.764        0.599 
## U2            1     484.625     45268.764     847127.737    0.763        0.598 
## LF            1     484.721     47608.402     849467.375    0.763        0.597 
## Prob          1     484.765     48676.878     850535.851    0.763        0.596 
## M.F           1     485.236     60195.359     862054.332    0.759        0.591 
## M             1     486.290     86547.099     888406.072    0.752        0.578 
## Wealth        1     486.643     95547.301     897406.274    0.749        0.574 
## Ed            1     489.051    159477.034     961336.007    0.732        0.544 
## Ineq          1     498.583    460403.892    1262262.865    0.648        0.401 
## -----------------------------------------------------------------------------
## 
## 
## Variables Removed: 
## 
## - Pop 
## 
## 
##   Step 1 : AIC = 482.7135 
##  Crime ~ So + M + Ed + Po1 + Po2 + LF + M.F + NW + U1 + U2 + Wealth + Ineq + Prob + Time 
## 
## -----------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq          RSS         R-Sq     Adj. R-Sq 
## -----------------------------------------------------------------------------
## Po2           1     480.761      1092.172     803201.643    0.776        0.637 
## So            1     480.795      1857.929     803967.400    0.776        0.637 
## Po1           1     481.208     11404.397     813513.867    0.773        0.632 
## NW            1     481.517     18630.550     820740.020    0.771        0.629 
## U1            1     481.632     21336.603     823446.074    0.770        0.628 
## U2            1     482.671     46130.102     848239.573    0.763        0.617 
## Time          1     482.682     46415.770     848525.240    0.763        0.616 
## LF            1     482.918     52137.655     854247.125    0.762        0.614 
## Prob          1     483.000     54156.417     856265.887    0.761        0.613 
## M.F           1     483.382     63543.681     865653.151    0.758        0.609 
## M             1     484.493     91466.915     893576.385    0.751        0.596 
## Wealth        1     484.643     95304.099     897413.570    0.749        0.594 
## Ed            1     487.051    159226.701     961336.172    0.732        0.565 
## Ineq          1     497.712    501540.763    1303650.234    0.636        0.411 
## -----------------------------------------------------------------------------
## 
## - Po2 
## 
## 
##   Step 2 : AIC = 480.7612 
##  Crime ~ So + M + Ed + Po1 + LF + M.F + NW + U1 + U2 + Wealth + Ineq + Prob + Time 
## 
## -----------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq          RSS         R-Sq     Adj. R-Sq 
## -----------------------------------------------------------------------------
```

```
## So             1      478.826      1489.580      804691.223    0.775           0.653
## NW             1      479.526     17745.888      820947.531    0.771           0.646
## U1             1      479.641     20448.211      823649.854    0.770           0.645
## U2             1      480.675     45147.670      848349.313    0.763           0.634
## LF             1      480.962     52120.699      855322.342    0.761           0.631
## Prob           1      481.015     53419.316      856620.959    0.761           0.630
## Time           1      481.166     57138.700      860340.343    0.760           0.629
## M.F            1      481.409     63130.298      866331.940    0.758           0.626
## M              1      482.616     93524.644      896726.287    0.750           0.613
## Wealth         1      482.900    100836.714      904038.357    0.748           0.610
## Po1            1      484.060    131278.377      934480.019    0.739           0.597
## Ed             1      485.551    171951.057      975152.700    0.728           0.579
## Ineq           1      496.058    513397.425     1316599.068    0.632           0.432
## ------------------------------------------------------------------------------
##
## - So
##
##
##   Step 3 : AIC = 478.826
##  Crime ~ M + Ed + Po1 + LF + M.F + NW + U1 + U2 + Wealth + Ineq + Prob + Time
##
## ------------------------------------------------------------------------------
## Variable      DF       AIC          Sum Sq           RSS          R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## NW             1      477.619     18436.438      823127.661    0.770           0.660
## U1             1      477.683     19940.065      824631.288    0.770           0.660
## U2             1      478.721     44775.544      849466.766    0.763           0.649
## Prob           1      479.053     52859.310      857550.533    0.761           0.646
## LF             1      479.394     61251.778      865943.000    0.758           0.643
## Time           1      479.412     61702.685      866393.907    0.758           0.642
## M.F            1      479.425     62022.316      866713.538    0.758           0.642
## M              1      480.623     92197.866      896889.088    0.750           0.630
## Wealth         1      480.985    101529.518      906220.741    0.747           0.626
## Po1            1      482.791    149525.604      954216.827    0.734           0.606
## Ed             1      483.714    175023.558      979714.780    0.726           0.596
## Ineq           1      494.433    526093.088     1330784.311    0.628           0.451
## ------------------------------------------------------------------------------
##
## - NW
##
##
##   Step 4 : AIC = 477.6188
##  Crime ~ M + Ed + Po1 + LF + M.F + U1 + U2 + Wealth + Ineq + Prob + Time
##
## ------------------------------------------------------------------------------
## Variable      DF       AIC          Sum Sq           RSS          R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## U1             1      476.087     11090.766      834218.426    0.767           0.670
## Prob           1      477.222     38573.581      861701.242    0.759           0.659
## U2             1      477.261     39549.547      862677.208    0.759           0.659
## M.F            1      477.473     44785.628      867913.289    0.758           0.657
## LF             1      477.653     49263.265      872390.925    0.756           0.655
## Time           1      478.203     63076.242      886203.902    0.753           0.649
## Wealth         1      481.272    144278.457      967406.117    0.730           0.617
## Ed             1      481.717    156678.104      979805.765    0.726           0.612
## Po1            1      482.173    169529.771      992657.432    0.723           0.607
## M              1      482.893    190144.934     1013272.594    0.717           0.599
## Ineq           1      498.234    747540.956     1570668.617    0.561           0.379
## ------------------------------------------------------------------------------
##
## - U1
```

```
##
##
##   Step 5 : AIC = 476.0873
##  Crime ~ M + Ed + Po1 + LF + M.F + U2 + Wealth + Ineq + Prob + Time
##
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS          R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## M.F            1      475.513      34675.622      868894.049     0.757         0.670
## Prob           1      475.603      36928.143      871146.569     0.757         0.669
## U2             1      475.700      39342.180      873560.606     0.756         0.668
## LF             1      475.733      40167.458      874385.885     0.756         0.668
## Time           1      476.664      63728.189      897946.616     0.749         0.659
## Ed             1      479.718     145606.358      979824.784     0.726         0.628
## Po1            1      480.497     167650.813     1001869.239     0.720         0.620
## Wealth         1      481.909     208893.018     1043111.445     0.709         0.604
## M              1      482.058     213358.113     1047576.540     0.708         0.602
## Ineq           1      499.484     889266.996     1723485.423     0.519         0.346
## ------------------------------------------------------------------------------
##
## - M.F
##
##
##   Step 6 : AIC = 475.5127
##  Crime ~ M + Ed + Po1 + LF + U2 + Wealth + Ineq + Prob + Time
##
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS          R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## LF             1      474.294      19625.381      888519.430     0.752         0.676
## Time           1      474.787      32221.971      901116.020     0.748         0.671
## Prob           1      476.232      70188.887      939082.935     0.738         0.657
## U2             1      476.848      86876.467      955770.516     0.733         0.651
## Po1            1      478.776     141000.520     1009894.568     0.718         0.631
## Wealth         1      480.625     195800.679     1064694.728     0.703         0.611
## Ed             1      481.134     211377.795     1080271.844     0.698         0.606
## M              1      483.066     272702.069     1141596.117     0.681         0.583
## Ineq           1      497.983     879375.429     1748269.477     0.512         0.362
## ------------------------------------------------------------------------------
##
## - LF
##
##
##   Step 7 : AIC = 474.2944
##  Crime ~ M + Ed + Po1 + U2 + Wealth + Ineq + Prob + Time
##
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS          R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Time           1      473.509      31378.195      919897.625     0.743         0.677
## Prob           1      474.594      60326.174      948845.604     0.735         0.666
## U2             1      477.007     128070.028     1016589.459     0.716         0.643
## Wealth         1      478.818     182046.544     1070565.974     0.701         0.624
## Po1            1      478.957     186295.061     1074814.491     0.700         0.622
## Ed             1      479.144     192054.947     1080574.377     0.698         0.620
## M              1      481.507     267525.139     1156044.570     0.677         0.594
## Ineq           1      496.044     862794.994     1751314.425     0.511         0.384
## ------------------------------------------------------------------------------
##
## - Time
##
```

```
##
##   Step 8 : AIC = 473.5091
##  Crime ~ M + Ed + Po1 + U2 + Wealth + Ineq + Prob
##
## -----------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq          RSS        R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## U2           1     476.040    127134.753    1047032.378    0.708         0.645
## Prob         1     476.519    141548.376    1061446.001    0.704         0.640
## Ed           1     477.242    163717.108    1083614.733    0.697         0.633
## Wealth       1     477.888    183890.591    1103788.216    0.692         0.626
## Po1          1     478.520    204022.573    1123920.198    0.686         0.619
## M            1     481.982    320862.878    1240760.503    0.654         0.579
## Ineq         1     494.632    861088.259    1780985.884    0.503         0.396
## -----------------------------------------------------------------------------
##
##
## No more variables to be removed.
##
## Variables Removed:
##
## - Pop
## - Po2
## - So
## - NW
## - U1
## - M.F
## - LF
## - Time
##
##
## Final Model Output
## ------------------
##
##                          Model Summary
## -----------------------------------------------------------------
## R                        0.862       RMSE                184.581
## R-Squared                0.743       Coef. Var            20.622
## Adj. R-Squared           0.677       MSE               34070.282
## Pred R-Squared           0.504       MAE                 127.287
## -----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
##
##                                ANOVA
## -----------------------------------------------------------------------
##                   Sum of
##                  Squares        DF    Mean Square      F         Sig.
## -----------------------------------------------------------------------
## Regression    2661998.261        7     380285.466    11.162    0.0000
## Residual       919897.625       27      34070.282
## Total         3581895.886       34
## -----------------------------------------------------------------------
##
##                                    Parameter Estimates
## ------------------------------------------------------------------------------------------
--
##       model        Beta    Std. Error    Std. Beta      t        Sig        lower
upper
## ------------------------------------------------------------------------------------------
```

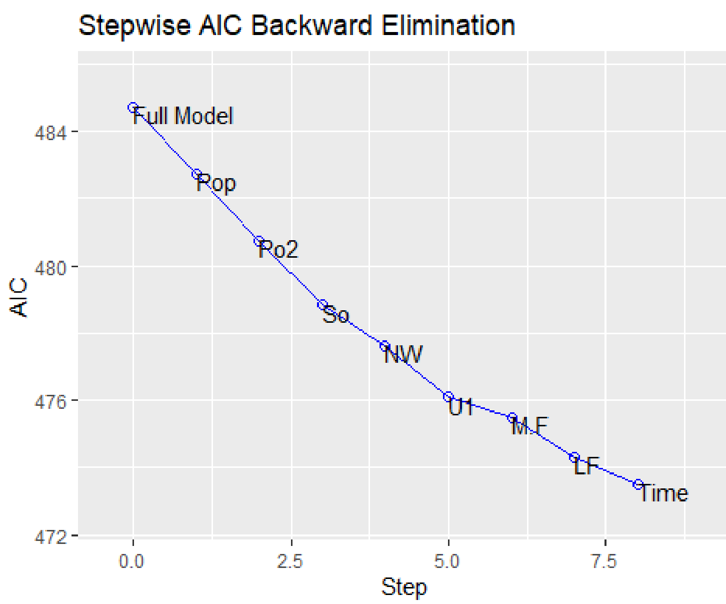
```
--
## (Intercept)     882.916        32.809                 26.911     0.000     815.598
950.234
##           M     140.702        45.849        0.457     3.069     0.005      46.628
234.776
##          Ed     129.626        59.133        0.408     2.192     0.037       8.294
250.958
##         Po1     164.745        67.323        0.481     2.447     0.021      26.610
302.880
##          U2      72.373        37.465        0.217     1.932     0.064      -4.500
149.245
##      Wealth     266.440       114.685        0.827     2.323     0.028      31.125
501.755
##        Ineq     378.075        75.204        1.183     5.027     0.000     223.769
532.381
##        Prob    -122.299        60.001       -0.321    -2.038     0.051    -245.410
0.813
## ---------------------------------------------------------------------------------------
--
```

```
BackwardFit.aic
```

```
##
##
##                        Backward Elimination Summary
## -------------------------------------------------------------------------
## Variable        AIC          RSS          Sum Sq        R-Sq      Adj. R-Sq
## -------------------------------------------------------------------------
## Full Model    484.703     801858.973     2780036.913     0.77614       0.59940
## Pop           482.714     802109.470     2779786.415     0.77607       0.61931
## Po2           480.761     803201.643     2778694.243     0.77576       0.63695
## So            478.826     804691.223     2777204.663     0.77534       0.65281
## NW            477.619     823127.661     2758768.225     0.77020       0.66029
## U1            476.087     834218.426     2747677.459     0.76710       0.67006
## M.F           475.513     868894.049     2713001.837     0.75742       0.67009
## LF            474.294     888519.430     2693376.455     0.75194       0.67562
## Time          473.509     919897.625     2661998.261     0.74318       0.67660
## -------------------------------------------------------------------------
```

Analysis: The Backward Elimination Summary method removed 8 predictors which are listed above. It started from model with full predictors, at each step it removed the predictor which resulted the highest AIC. It continued until it removed Pop, Po2, So, NW, U1, MF, LF, Time. This is shown below in the Elimination Plot.

```
plot(BackwardFit.aic)
```



Analysis: ***The Backward Elimination Model, suggests that the Best Model is*** **Crime ~ M + Ed + Po1 + U2 + Wealth + Ineq + Prob with AIC = 473.5091.**

Stepwise Regression using both directions and aic

```
model = model<-lm(Crime~.,data = TrainingData)

StepwiseBothFit.aic<- ols_step_both_aic(model, details = TRUE)

## Stepwise Selection Method
## -------------------------
##
## Candidate Terms:
##
## 1 . So
## 2 . M
## 3 . Ed
## 4 . Po1
## 5 . Po2
## 6 . LF
## 7 . M.F
## 8 . Pop
## 9 . NW
## 10 . U1
## 11 . U2
## 12 . Wealth
## 13 . Ineq
## 14 . Prob
## 15 . Time
##
##  Step 0: AIC = 507.0876
##  Crime ~ 1
##
```

```
##
## Variables Entered/Removed:
##
##                                Enter New Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Po1            1     495.841    1128626.869     2453269.016     0.315         0.294
## Po2            1     496.326    1094409.821     2487486.064     0.306         0.284
## Prob           1     497.633     999751.467     2582144.419     0.279         0.257
## Pop            1     499.351     869842.365     2712053.520     0.243         0.220
## Time           1     502.489     615451.183     2966444.703     0.172         0.147
## Wealth         1     502.537     611427.125     2970468.761     0.171         0.146
## U2             1     507.541     154871.969     3427023.917     0.043         0.014
## Ed             1     507.567     152323.822     3429572.063     0.043         0.014
## Ineq           1     508.547      54884.610     3527011.276     0.015        -0.015
## M.F            1     508.844      24841.604     3557054.282     0.007        -0.023
## So             1     508.940      15112.715     3566783.170     0.004        -0.026
## NW             1     508.988      10216.386     3571679.500     0.003        -0.027
## M              1     509.016       7344.408     3574551.478     0.002        -0.028
## U1             1     509.038       5105.396     3576790.489     0.001        -0.029
## LF             1     509.085        314.577     3581581.309     0.000        -0.030
## ------------------------------------------------------------------------------
##
## - Po1 added
##
##
##  Step 1 : AIC = 495.8411
##  Crime ~ Po1
##
##                                Enter New Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Ineq           1     491.397    1541197.843     2040698.043     0.430         0.395
## M              1     491.519    1534032.377     2047863.508     0.428         0.393
## Time           1     494.656    1342048.124     2239847.762     0.375         0.336
## So             1     494.849    1329639.575     2252256.311     0.371         0.332
## NW             1     495.057    1316219.575     2265676.311     0.367         0.328
## Prob           1     496.070    1249693.014     2332202.872     0.349         0.308
## Pop            1     496.667    1209574.218     2372321.668     0.338         0.296
## Wealth         1     497.570    1147560.892     2434334.994     0.320         0.278
## M.F            1     497.651    1141905.295     2439990.591     0.319         0.276
## U2             1     497.681    1139854.842     2442041.043     0.318         0.276
## U1             1     497.733    1136213.074     2445682.812     0.317         0.275
## Po2            1     497.746    1135303.037     2446592.848     0.317         0.274
## Ed             1     497.764    1133994.837     2447901.049     0.317         0.274
## LF             1     497.780    1132890.633     2449005.252     0.316         0.274
## ------------------------------------------------------------------------------
##
## - Ineq added
##
##
##  Step 2 : AIC = 491.3966
##  Crime ~ Po1 + Ineq
##
##                            Remove Existing Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Ineq           1     495.841    1128626.869     2453269.016     0.315         0.294
```

```
## Po1           1    508.547      54884.610    3527011.276    0.015       -0.015
## -----------------------------------------------------------------------------
##
##                               Enter New Variables
## -----------------------------------------------------------------------------
## Variable     DF      AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## Wealth        1    484.774    1986787.099    1595108.787    0.555        0.512
## Prob          1    486.606    1901089.969    1680805.917    0.531        0.485
## Ed            1    487.472    1858956.673    1722939.212    0.519        0.472
## M.F           1    490.587    1698608.587    1883287.299    0.474        0.423
## M             1    490.768    1688828.766    1893067.120    0.471        0.420
## Time          1    491.324    1658519.352    1923376.533    0.463        0.411
## LF            1    491.955    1623539.410    1958356.475    0.453        0.400
## U1            1    493.363    1543178.414    2038717.472    0.431        0.376
## U2            1    493.379    1542221.531    2039674.354    0.431        0.375
## Pop           1    493.380    1542149.437    2039746.449    0.431        0.375
## NW            1    493.390    1541555.461    2040340.424    0.430        0.375
## So            1    493.396    1541239.170    2040656.716    0.430        0.375
## Po2           1    493.397    1541200.407    2040695.479    0.430        0.375
## -----------------------------------------------------------------------------
##
## - Wealth added
##
##
##  Step 3 : AIC = 484.7744
##  Crime ~ Po1 + Ineq + Wealth
##
##                           Remove Existing Variables
## -----------------------------------------------------------------------------
## Variable     DF      AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## Po1           1    487.950    1732586.652    1849309.234    0.484        0.451
## Wealth        1    491.397    1541197.843    2040698.043    0.430        0.395
## Ineq          1    497.570    1147560.892    2434334.994    0.320        0.278
## -----------------------------------------------------------------------------
##
##                               Enter New Variables
## -----------------------------------------------------------------------------
## Variable     DF      AIC         Sum Sq            RSS        R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## M             1    478.186    2333862.477    1248033.409    0.652        0.605
## Prob          1    482.112    2185720.119    1396175.767    0.610        0.558
## Time          1    483.484    2129896.192    1451999.694    0.595        0.541
## Ed            1    484.913    2069393.056    1512502.830    0.578        0.521
## M.F           1    485.267    2054045.195    1527850.691    0.573        0.517
## U1            1    486.536    1997621.226    1584274.660    0.558        0.499
## NW            1    486.625    1993572.686    1588323.200    0.557        0.497
## So            1    486.685    1990833.159    1591062.727    0.556        0.497
## LF            1    486.694    1990446.648    1591449.238    0.556        0.496
## U2            1    486.707    1989873.693    1592022.192    0.556        0.496
## Pop           1    486.719    1989313.933    1592581.952    0.555        0.496
## Po2           1    486.745    1988124.450    1593771.436    0.555        0.496
## -----------------------------------------------------------------------------
##
## - M added
##
##
##  Step 4 : AIC = 478.1863
##  Crime ~ Po1 + Ineq + Wealth + M
##
```

```
##                              Remove Existing Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Po1            1     482.752    2076346.586    1505549.300    0.580         0.539
## M              1     484.774    1986787.099    1595108.787    0.555         0.512
## Wealth         1     490.768    1688828.766    1893067.120    0.471         0.420
## Ineq           1     492.661    1583622.645    1998273.240    0.442         0.388
## ------------------------------------------------------------------------------
##
##                                  Enter New Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Prob           1     477.124    2438416.018    1143479.868    0.681         0.626
## U1             1     477.869    2413833.430    1168062.456    0.674         0.618
## Ed             1     478.051    2407742.938    1174152.948    0.672         0.616
## U2             1     478.557    2390639.283    1191256.603    0.667         0.610
## M.F            1     478.723    2384960.007    1196935.879    0.666         0.608
## Time           1     478.757    2383816.501    1198079.385    0.666         0.608
## NW             1     479.845    2345966.483    1235929.403    0.655         0.595
## So             1     479.968    2341639.161    1240256.725    0.654         0.594
## LF             1     480.134    2335709.502    1246186.383    0.652         0.592
## Po2            1     480.142    2335458.083    1246437.803    0.652         0.592
## Pop            1     480.144    2335357.131    1246538.755    0.652         0.592
## ------------------------------------------------------------------------------
##
## - Prob added
##
##
##  Step 5 : AIC = 477.124
##  Crime ~ Po1 + Ineq + Wealth + M + Prob
##
##                              Remove Existing Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Prob           1     478.186    2333862.477    1248033.409    0.652         0.605
## Po1            1     479.725    2277777.427    1304118.458    0.636         0.587
## M              1     482.112    2185720.119    1396175.767    0.610         0.558
## Wealth         1     486.619    1993842.141    1588053.745    0.557         0.498
## Ineq           1     492.983    1677190.103    1904705.783    0.468         0.397
## ------------------------------------------------------------------------------
##
##                                  Enter New Variables
## ------------------------------------------------------------------------------
## Variable      DF       AIC         Sum Sq           RSS         R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Ed             1     476.040    2534863.508    1047032.378    0.708         0.645
## U1             1     476.409    2523751.302    1058144.584    0.705         0.641
## U2             1     477.242    2498281.153    1083614.733    0.697         0.633
## M.F            1     477.574    2487942.942    1093952.944    0.695         0.629
## LF             1     478.890    2446035.542    1135860.344    0.683         0.615
## Time           1     478.950    2444085.323    1137810.563    0.682         0.614
## Po2            1     479.035    2441329.165    1140566.721    0.682         0.613
## Pop            1     479.098    2439257.520    1142638.366    0.681         0.613
## NW             1     479.105    2439042.223    1142853.662    0.681         0.613
## So             1     479.124    2438423.445    1143472.441    0.681         0.612
## ------------------------------------------------------------------------------
##
## - Ed added
```

```
##
##
##  Step 6 : AIC = 476.04
##  Crime ~ Po1 + Ineq + Wealth + M + Prob + Ed
##
##                             Remove Existing Variables
## ------------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq           RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## Ed            1    477.124    2438416.018    1143479.868    0.681         0.626
## Prob          1    478.051    2407742.938    1174152.948    0.672         0.616
## Wealth        1    480.146    2335313.915    1246581.971    0.652         0.592
## M             1    481.161    2298631.492    1283264.394    0.642         0.580
## Po1           1    481.348    2291757.391    1290138.495    0.640         0.578
## Ineq          1    494.862    1683760.669    1898135.217    0.470         0.379
## ------------------------------------------------------------------------------
##
##                                 Enter New Variables
## ------------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq           RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## U2            1    473.509    2661998.261     919897.625    0.743         0.677
## U1            1    475.456    2609385.668     972510.218    0.728         0.658
## LF            1    476.002    2594075.559     987820.326    0.724         0.653
## Time          1    477.007    2565306.427    1016589.459    0.716         0.643
## NW            1    477.574    2548710.883    1033185.003    0.712         0.637
## So            1    477.702    2544924.805    1036971.081    0.710         0.635
## M.F           1    477.730    2544087.315    1037808.571    0.710         0.635
## Pop           1    477.942    2537787.620    1044108.266    0.709         0.633
## Po2           1    478.020    2535460.419    1046435.467    0.708         0.632
## ------------------------------------------------------------------------------
##
## - U2 added
##
##
##  Step 7 : AIC = 473.5091
##  Crime ~ Po1 + Ineq + Wealth + M + Prob + Ed + U2
##
##                             Remove Existing Variables
## ------------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq           RSS        R-Sq     Adj. R-Sq
## ------------------------------------------------------------------------------
## U2            1    476.040    2534863.508    1047032.378    0.708         0.645
## Prob          1    476.519    2520449.884    1061446.001    0.704         0.640
## Ed            1    477.242    2498281.153    1083614.733    0.697         0.633
## Wealth        1    477.888    2478107.670    1103788.216    0.692         0.626
## Po1           1    478.520    2457975.688    1123920.198    0.686         0.619
## M             1    481.982    2341135.382    1240760.503    0.654         0.579
## Ineq          1    494.632    1800910.001    1780985.884    0.503         0.396
## ------------------------------------------------------------------------------
##
##                                 Enter New Variables
## -----------------------------------------------------------------------------
## Variable     DF      AIC        Sum Sq          RSS        R-Sq     Adj. R-Sq
## -----------------------------------------------------------------------------
## Time          1    474.294    2693376.455    888519.430    0.752         0.676
## LF            1    474.787    2680779.866    901116.020    0.748         0.671
## So            1    475.244    2668933.530    912962.355    0.745         0.667
## NW            1    475.325    2666820.540    915075.346    0.745         0.666
## U1            1    475.449    2663577.526    918318.360    0.744         0.665
## Pop           1    475.492    2662461.225    919434.661    0.743         0.664
```

```
## M.F           1    475.505    2662098.161    919797.724    0.743         0.664
## Po2           1    475.507    2662048.589    919847.297    0.743         0.664
## ----------------------------------------------------------------------------
## 
## 
## No more variables to be added or removed.
## 
## Final Model Output
## ------------------
## 
##                           Model Summary
## ----------------------------------------------------------------
## R                          0.862       RMSE                184.581
## R-Squared                  0.743       Coef. Var            20.622
## Adj. R-Squared             0.677       MSE               34070.282
## Pred R-Squared             0.504       MAE                 127.287
## ----------------------------------------------------------------
##  RMSE: Root Mean Square Error
##  MSE: Mean Square Error
##  MAE: Mean Absolute Error
## 
##                                  ANOVA
## ----------------------------------------------------------------------
##                    Sum of
##                   Squares        DF    Mean Square      F         Sig.
## ----------------------------------------------------------------------
## Regression    2661998.261         7     380285.466    11.162    0.0000
## Residual       919897.625        27      34070.282
## Total         3581895.886        34
## ----------------------------------------------------------------------
## 
##                                       Parameter Estimates
## ------------------------------------------------------------------------------------------
--
##       model        Beta    Std. Error    Std. Beta      t        Sig        lower
upper
## ------------------------------------------------------------------------------------------
--
## (Intercept)     882.916        32.809                 26.911    0.000     815.598
950.234
##         Po1     164.745        67.323        0.481     2.447    0.021      26.610
302.880
##        Ineq     378.075        75.204        1.183     5.027    0.000     223.769
532.381
##      Wealth     266.440       114.685        0.827     2.323    0.028      31.125
501.755
##           M     140.702        45.849        0.457     3.069    0.005      46.628
234.776
##        Prob    -122.299        60.001       -0.321    -2.038    0.051    -245.410
0.813
##          Ed     129.626        59.133        0.408     2.192    0.037       8.294
250.958
##          U2      72.373        37.465        0.217     1.932    0.064      -4.500
149.245
## ------------------------------------------------------------------------------------------
--
```

```
StepwiseBothFit.aic
```

```
## 
## 
```

```
##                                   Stepwise Summary
## -------------------------------------------------------------------------------------
## Variable      Method        AIC          RSS           Sum Sq        R-Sq      Adj. R-Sq
## -------------------------------------------------------------------------------------
## Po1          addition     495.841    2453269.016    1128626.869    0.31509      0.29434
## Ineq         addition     491.397    2040698.043    1541197.843    0.43027      0.39467
## Wealth       addition     484.774    1595108.787    1986787.099    0.55467      0.51158
## M            addition     478.186    1248033.409    2333862.477    0.65157      0.60511
## Prob         addition     477.124    1143479.868    2438416.018    0.68076      0.62572
## Ed           addition     476.040    1047032.378    2534863.508    0.70769      0.64505
## U2           addition     473.509     919897.625    2661998.261    0.74318      0.67660
## -------------------------------------------------------------------------------------

plot(StepwiseBothFit.aic)
```

#Analysis:

Stepwise Regression is a combination of both Forward and Backward Regression.

All the three methods, Forward Regression, Backward Elimination and Stepwise Regression in both Directions returned the model with lm(Crime~Po1 + Ineq + Wealth + Prob + M + Ed + U2) using the Scaled Training Data.

```
BestModelWithTrainingData<- lm(Crime~Po1 + Ineq  + Wealth  + M + Ed + U2 +Prob, data =
Training Data)
summary(BestModelWithTrainingData)

##
## Call:
## lm(formula = Crime ~ Po1 + Ineq + Wealth + M + Ed + U2 + Prob,
##     data = Training Data)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -267.58 -125.25   -6.28   96.12  451.18
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   882.92      32.81  26.911  < 2e-16 ***
## Po1           164.74      67.32   2.447  0.02119 *
## Ineq          378.08      75.20   5.027 2.83e-05 ***
## Wealth        266.44     114.69   2.323  0.02794 *
## M             140.70      45.85   3.069  0.00485 **
## Ed            129.63      59.13   2.192  0.03717 *
## U2             72.37      37.47   1.932  0.06395 .
## Prob         -122.30      60.00  -2.038  0.05143 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 184.6 on 27 degrees of freedom
## Multiple R-squared:  0.7432, Adjusted R-squared:  0.6766
## F-statistic: 11.16 on 7 and 27 DF,  p-value: 1.476e-06
```

```
BestModelwithTestData<- lm(Crime~Po1 + Ineq  + Wealth  + M + Ed + U2 + Prob, data = TestData)
summary(BestModelwithTestData)
```

```
##
## Call:
## lm(formula = Crime ~ Po1 + Ineq + Wealth + M + Ed + U2 + Prob,
##     data = TestData)
##
## Residuals:
##          2          5          8         18         21         22         25
##  163.97555  -73.52705   81.19548   57.51035    0.72107  -64.95555   -0.07717
##         26         27         42         43         45
## -128.98342  -28.64515   20.95836   15.66399  -43.83645
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   919.86      43.57  21.111 2.98e-05 ***
## Po1           399.55      60.51   6.604  0.00273 **
## Ineq          108.45     102.27   1.060  0.34872
## Wealth       -113.01     100.27  -1.127  0.32277
## M             131.94      62.07   2.126  0.10070
## Ed            294.87      94.78   3.111  0.03583 *
## U2            163.74      50.80   3.223  0.03218 *
## Prob          -86.44      35.89  -2.409  0.07366 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 128.9 on 4 degrees of freedom
## Multiple R-squared:  0.9798, Adjusted R-squared:  0.9444
## F-statistic: 27.67 on 7 and 4 DF,  p-value: 0.003117
```

Analysis:

The Initial Model had 15 predictors. The Stepwise Selection model, forward regression and backward elimination models , all these three methods removed the same predictors and suggested best model as Crime~Po1 + Ineq + Wealth + Prob + M + Ed + U2 using the Scaled Training Data.

The Adjusted R^2 for Training Data Set is 0.6766 While for the test Data Set it is *0.9444. This shows that the Predictor elimination using stepwise Regression has lowered the Variance.*

```
b. Lasso Regression
c. Elastic.Net Regression

library(glmnet)

## Loading required package: Matrix

## Loaded glmnet 4.0-2

set.seed(82)

# The cv part means we want to use Cross validation to
#obtain the optimalvalues for Lambda.
model_lasso <- cv.glmnet(x=as.matrix(TrainingData[,-16]),
                         y = as.matrix(TrainingData[,16]),
                         alpha = 1 ,
                         nfolds = 8,
                         nlambda = 20,
                         type.measure = "mse",
                         family ="gaussian",
                         standardize = TRUE)
model_lasso.predicted<-predict(model_lasso,s=model_lasso$lambda.1se,newx=as.matrix(TestData[,-
16]))
#Lambda.1se is the value of lambda,that resulted in the simplest model(model with few non zero
parameters)
#and was within 1 standard error of the lambda that had the smallest sum.
model_lasso.predicted

##           1
## 2  895.0571
## 5  895.0571
## 8  895.0571
## 18 895.0571
## 21 895.0571
## 22 895.0571
## 25 895.0571
## 26 895.0571
## 27 895.0571
## 42 895.0571
## 43 895.0571
## 45 895.0571

# Find the accuracy
sse = sum((model_lasso.predicted - TestData[,16])^2)
totalSumofSquares = sum((TestData[,16]-mean(TestData[,16]))^2)
RSquared = 1- (sse/totalSumofSquares)
RSquared

## [1] -0.005634715

#Elastic.Net Regression

# The cv part means we want to use Cross validation to
#obtain the optimalvalues for Lambda.
model_elasticnet_alpha0.5 <- cv.glmnet(x=as.matrix(TrainingData[,-16]),
                                       y = as.matrix(TrainingData[,16]),
```

```
                                        alpha = 0.5 ,
                                        nfolds = 8,
                                        nlambda = 20,
                                        type.measure = "mse",
                                        family ="gaussian",
                                        standardize = TRUE)
model_elasticnet_alpha0.5.predicted<-
predict(model_elasticnet_alpha0.5,s=model_elasticnet_alpha0.5$lambda.1se,newx=as.matrix(TestDa
ta[,-16]))
#Lambda.1se is the value of lambda,that resulted in the simplest model(model with few non zero
parameters)
#and was within 1 standard error of the lambda that had the smallest sum.
model_elasticnet_alpha0.5.predicted

##            1
## 2  1111.1050
## 5   971.0996
## 8  1087.6278
## 18  691.4622
## 21  928.0997
## 22  743.8468
## 25  568.1519
## 26 1292.4630
## 27  631.7364
## 42  357.4658
## 43 1015.9012
## 45  772.1347

# Find the accuracy
sse = sum((model_elasticnet_alpha0.5.predicted - TestData[,16])^2)
totalSumofSquares = sum((TestData[,16]-mean(TestData[,16]))^2)
RSquared = 1- (sse/totalSumofSquares)
AdjustedRSqaured = RSquared - (1-RSquared)*15/(nrow(TestData)-15-1)
AdjustedRSqaured

## [1] 2.25108

RSquared

## [1] 0.5450617

#Lets try more values of alpha
# We create the Elastic.NET fit using the cv.glmnet() function,
#which takes alpha values from 0.0,0.1,..1.0.
list_of_fits <- list()
for(i in 0:10)
{
  fit.name <- paste0("alpha",i/10)
  list_of_fits[[fit.name]] <- cv.glmnet(x=as.matrix(TrainingData[,-16]),
                                        y = as.matrix(TrainingData[,16]),
                                        alpha = i/10 ,
                                        nfolds = 8,
                                        nlambda = 20,
                                        type.measure = "mse",
                                        family ="gaussian",
                                        standardize = FALSE)
}

results <- data.frame()
# This for loop will give us the error values for each model from above.
for(i in 0:10)
```

```
{
  fit.name <- paste0("alpha",i/10)
  predicted <- predict(list_of_fits[[fit.name]],
                       s=list_of_fits[[fit.name]]$lambda.1se,newx=as.matrix(TestData[,-16]))

  # Find the accuracy
  sse = sum((predicted - TestData[,16])^2)
  totalSumofSquares = sum((TestData[,16]-mean(TestData[,16]))^2)
  RSquared = 1- (sse/totalSumofSquares)
  temp <- data.frame(alpha=i/10, Rsqaured=RSquared, fit.name)
  results <- rbind(results, temp)
}

results

##    alpha     Rsqaured fit.name
## 1    0.0  0.192802377   alpha0
## 2    0.1  0.151978925 alpha0.1
## 3    0.2 -0.005634715 alpha0.2
## 4    0.3 -0.005634715 alpha0.3
## 5    0.4 -0.005634715 alpha0.4
## 6    0.5  0.281999903 alpha0.5
## 7    0.6  0.151110822 alpha0.6
## 8    0.7 -0.005634715 alpha0.7
## 9    0.8 -0.005634715 alpha0.8
## 10   0.9  0.218213133 alpha0.9
## 11   1.0 -0.005634715   alpha1

model_elasticnet_alpha0.5$glmnet.fit

##
## Call:  glmnet(x = as.matrix(TrainingData[, -16]), y = as.matrix(TrainingData[,      16]),
alpha = 0.5, nlambda = 20, family = "gaussian", standardize = TRUE)
##
##    Df  %Dev Lambda
## 1   0  0.00 359.10
## 2   4 19.67 221.20
## 3   5 31.22 136.20
## 4   7 38.91  83.89
## 5  13 51.28  51.66
## 6  13 63.84  31.82
## 7  13 70.37  19.59
## 8  14 73.82  12.07
## 9  15 75.76   7.43
## 10 15 76.79   4.58
## 11 14 77.25   2.82
## 12 13 77.42   1.74
## 13 13 77.51   1.07
## 14 14 77.55   0.66
## 15 14 77.57   0.41
## 16 15 77.58   0.25
## 17 15 77.60   0.15
## 18 15 77.61   0.09
## 19 15 77.61   0.06
## 20 15 77.61   0.04
```

Analysis:

For Using Lasso and Elastic.Net Regression in R, we used glmnet library.

Lasso Regression Penalty = Sum of Squared Residuals + Lambda1(|var1| + … + |varx|) + Lambda2(var1^2+…+varx^2).

Glmnet interprets these Lambda's differently.
Glmnet has a single Lambda as shown:

Regression Penalty =Sum of Squared Residuals +
Lambda*[alpha*(|var1| +…+ |varx|) + (1-alpha) (var1^2+..varx^2)].

When alpha = 0, Lasso penalty goes to zero and the model reduces to ridge regression.
When alpha = 1, Rigde regression penalty goes to zero and the model reduces to Lasso regression.
When 0< alpha < 1 , then the model reduces to Elastic.Net.

Lambda controls how much penalty to apply to the regression.
When Lambda = 0, the model reduces to Linear Regression as penalty = 0.
When Lambda > 0, then Elastis.Net penalty kicks in.
Run glmnet for finding Lasso and ElasticNet Best Models and these results are obtained

```
##    alpha     Rsqaured fit.name
## 1    0.0  0.192802377   alpha0   -> This fit is Ridge Regression
## 2    0.1  0.151978925 alpha0.1   -> This fit is Elastic.Net Regression
## 3    0.2 -0.005634715 alpha0.2   -> This fit is Elastic.Net Regression
## 4    0.3 -0.005634715 alpha0.3   -> This fit is Elastic.Net Regression
## 5    0.4 -0.005634715 alpha0.4   -> This fit is Elastic.Net Regression
## 6    0.5  0.281999903 alpha0.5   -> This fit is Elastic.Net Regression
## 7    0.6  0.151110822 alpha0.6   -> This fit is Elastic.Net Regression
## 8    0.7 -0.005634715 alpha0.7   -> This fit is Elastic.Net Regression
## 9    0.8 -0.005634715 alpha0.8   -> This fit is Elastic.Net Regression
## 10   0.9  0.218213133 alpha0.9   -> This fit is Elastic.Net Regression
## 11   1.0 -0.005634715   alpha1   -> This fit is Lasso Regression
```

The Highest RSquared error is for model fit with alpha = 0.5 which is an Elastic.Net Regression. This model seems to be better than the rest.

Next best model is model fit with alpha = 0.9 which is also from Elastic.net Regression.

The Alpha = 0 is the Ridge Regression Model and Alpha = 1 is the Lasso Model.

Based on the Rsquared, Elastic.Net wins, then Ridge Regression is the next best and the next will be Lasso.

For the Best Elastic.Net Model, the lambda values ranged between 0.04 to 359.10 in the cv model.