Homework6.R

2020-09-30

Question 9.1

Using the same crime data set `uscrime.txt` as in Question 8.2, apply Principal Component Analysis and then create a regression model using the first few principal components. Specify your new model in terms of the original variables (not the principal components), and compare its quality to that of your solution to Question 8.2. You can use the R function `prcomp` for PCA. (**Note** that to first scale the data, you can include `scale. = TRUE` to scale as part of the PCA function. Don't forget that, to make a prediction for the new city, you'll need to unscale the coefficients (i.e., do the scaling calculation in reverse)!)

Answer:

Principle component Analysis is used to reduce the number of factors to be used in the final model. It also removes the factor correlation by rotating the data into a new dimension and the factors in the new dimension with highest variance will be more important for the model predictions.

Step1: Read the crime data.

```
rm(list = ls())
set.seed(1)

#Read the crime data
crimedata <- read.table("uscrime.txt",stringsAsFactors = FALSE, header = TRUE)
```

Step 2: Perform "Prinicipal component analysis" on the crime data using PRCOMP(). The prcomp function takes all predictor columns except for the response column (Crime) on scaled data.

```
principalcomponents <- prcomp(crimedata[,-16],center=TRUE,scale=TRUE)
summary(principalcomponents)

## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6     PC7
## Standard deviation     2.4534 1.6739 1.4160 1.07806 0.97893 0.74377 0.56729
## Proportion of Variance 0.4013 0.1868 0.1337 0.07748 0.06389 0.03688 0.02145
## Cumulative Proportion  0.4013 0.5880 0.7217 0.79920 0.86308 0.89996 0.92142
##                            PC8     PC9    PC10    PC11    PC12    PC13   PC14
## Standard deviation     0.55444 0.48493 0.44708 0.41915 0.35804 0.26333 0.2418
## Proportion of Variance 0.02049 0.01568 0.01333 0.01171 0.00855 0.00462 0.0039
## Cumulative Proportion  0.94191 0.95759 0.97091 0.98263 0.99117 0.99579 0.9997
##                           PC15
```

```
## Standard deviation     0.06793
## Proportion of Variance 0.00031
## Cumulative Proportion  1.00000
```

Analysis: Observe the descending order of the variances of Principle Components. The PCS are shown in the order of importance.

In the above listed output, Standard Deviation is the "square roots of the eigenvalues of the covariance/correlation matrix"

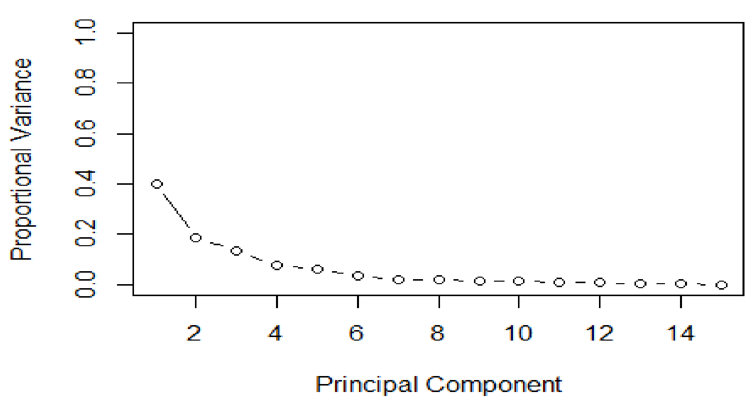
Step 3: Choosing the number of the principle components to use in performing multiple linear regression: If we plot the fraction of total variance retained VS. Number of Eigen Values, The point where the plot shows a steady downward curve is where we will know from that point the eigen values do not contribute much to the model.

```
library(DAAG)
varianceofeachEigenColumn<-principalcomponents$sdev^2

proportionalvariance <- varianceofeachEigenColumn/sum(varianceofeachEigenColumn)
proportionalvariance

##  [1] 0.401263510 0.186789802 0.133662956 0.077480520 0.063886598 0.036879593
##  [7] 0.021454579 0.020493418 0.015677019 0.013325395 0.011712360 0.008546007
## [13] 0.004622779 0.003897851 0.000307611

plot(proportionalvariance,xlab="Principal Component", ylab="Proportional Variance",
     ylim=c(0,1),type = "b")
```



Analysis: The Above plot shows that from PC6 the proportional variance is almost close to zero. So the first 5 Principle components will be chosen to perform the linear regression.

```
# Hence K = 5
k =5
```

Step 4: Bind the response column to the first 5 principle components into a data frame.

```
# Create a data frame with first 5 principle components and the response column
PCData= cbind(principalcomponents$x[,1:k],crimedata[,16])
PCData

##              PC1         PC2         PC3         PC4          PC5     
##  [1,] -4.1992835 -1.09383120 -1.11907395  0.67178115  0.055283376  791
##  [2,]  1.1726630  0.67701360 -0.05244634 -0.08350709 -1.173199821 1635
##  [3,] -4.1737248  0.27677501 -0.37107658  0.37793995  0.541345246  578
##  [4,]  3.8349617 -2.57690596  0.22793998  0.38262331 -1.644746496 1969
##  [5,]  1.8392999  1.33098564  1.27882805  0.71814305  0.041590320 1234
##  [6,]  2.9072336 -0.33054213  0.53288181  1.22140635  1.374360960  682
##  [7,]  0.2457752 -0.07362562 -0.90742064  1.13685873  0.718644387  963
##  [8,] -0.1301330 -1.35985577  0.59753132  1.44045387 -0.222781388 1555
##  [9,] -3.6103169 -0.68621008  1.28372246  0.55171150 -0.324292990  856
## [10,]  1.1672376  3.03207033  0.37984502 -0.28887026 -0.646056610  705
## [11,]  2.5384879 -2.66771358  1.54424656 -0.87671210 -0.324083561 1674
## [12,]  1.0065920 -0.06044849  1.18861346 -1.31261964  0.358087724  849
## [13,]  0.5161143  0.97485189  1.83351610 -1.59117618  0.599881946  511
## [14,]  0.4265556  1.85044812  1.02893477 -0.07789173  0.741887592  664
## [15,] -3.3435299  0.05182823 -1.01358113  0.08840211  0.002969448  798
## [16,] -3.0310689 -2.10295524 -1.82993161  0.52347187 -0.387454246  946
## [17,] -0.2262961  1.44939774 -1.37565975  0.28960865  1.337784608  539
## [18,] -0.1127499 -0.39407030 -0.38836278  3.97985093  0.410914404  929
## [19,]  2.9195668 -1.58646124  0.97612613  0.78629766  1.356288600  750
## [20,]  2.2998485 -1.73396487 -2.82423222 -0.23281758 -0.653038858 1225
## [21,]  1.1501667  0.13531015  0.28506743 -2.19770548  0.084621572  742
## [22,] -5.6594827 -1.09730404  0.10043541 -0.05245484 -0.689327990  439
## [23,] -0.1011749 -0.57911362  0.71128354 -0.44394773  0.689939865 1216
## [24,]  1.3836281  1.95052341 -2.98485490 -0.35942784 -0.744371276  968
## [25,]  0.2727756  2.63013778  1.83189535  0.05207518  0.803692524  523
## [26,]  4.0565577  1.17534729 -0.81690756  1.66990720 -2.895110075 1993
## [27,]  0.8929694  0.79236692  1.26822542 -0.57575615  1.830793964  342
## [28,]  0.1514495  1.44873320  0.10857670 -0.51040146 -1.023229895 1216
## [29,]  3.5592481 -4.76202163  0.75080576  0.64692974  0.309946510 1043
## [30,] -4.1184576 -0.38073981  1.43463965  0.63330834 -0.254715638  696
## [31,] -0.6811731  1.66926027 -2.88645794 -1.30977099 -0.470913997  373
## [32,]  1.7157269 -1.30836339 -0.55971313 -0.70557980  0.331277622  754
## [33,] -1.8860627  0.59058174  1.43570145  0.18239089  0.291863659 1072
## [34,]  1.9526349  0.52395429 -0.75642216  0.44289927  0.723474420  923
## [35,]  1.5888864 -3.12998571 -1.73107199 -1.68604766  0.665406182  653
## [36,]  1.0709414 -1.65628271  0.79436888 -1.85172698  0.020031154 1272
## [37,] -4.1101715  0.15766712  2.36296974 -0.56868399 -2.469679496  831
## [38,] -0.7254706  2.89263339 -0.36348376 -0.50612576  0.028157162  566
## [39,] -3.3451254 -0.95045293  0.19551398 -0.27716645  0.487259213  826
## [40,] -1.0644466 -1.05265304  0.82886286 -0.12042931 -0.645884788 1151
## [41,]  1.4933989  1.86712106  1.81853582 -1.06112429  0.009855774  880
## [42,] -0.6789284  1.83156328 -1.65435992  0.95121379  2.115630145  542
## [43,] -2.4164258 -0.46701087  1.42808323  0.41149015 -0.867397522  823
## [44,]  2.2978729  0.41865689 -0.64422929 -0.63462770 -0.703116983 1030
## [45,] -2.9245282 -1.19488555 -3.35139309 -1.48966984  0.806659622  455
## [46,]  1.7654525  0.95655926  0.98576138  1.05683769  0.542466034  508
## [47,]  2.3125056  2.56161119 -1.58223354  0.59863946 -1.140712406  849
```

Step 5: Perform Linear Regression on the first 5 PC data

```
LinearRegressionPCmodel <- lm(V6~.,data = as.data.frame(PCData))
summary(LinearRegressionPCmodel)

##
## Call:
## lm(formula = V6 ~ ., data = as.data.frame(PCData))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -420.79 -185.01   12.21  146.24  447.86
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   905.09      35.59  25.428  < 2e-16 ***
## PC1            65.22      14.67   4.447 6.51e-05 ***
## PC2           -70.08      21.49  -3.261  0.00224 **
## PC3            25.19      25.41   0.992  0.32725
## PC4            69.45      33.37   2.081  0.04374 *
## PC5          -229.04      36.75  -6.232 2.02e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 244 on 41 degrees of freedom
## Multiple R-squared:  0.6452, Adjusted R-squared:  0.6019
## F-statistic: 14.91 on 5 and 41 DF,  p-value: 2.446e-08
```

Analysis: I am choosing the threshold of p-value as “0.1”. The above all Principle Components contribute to the linear regression model.

Step 6: Now we need to go back to the Original Coordinate System.

1. We need to scale back the data. From the Office Hours, we know that
   xscaled = x-mean(x)/sigma(x) => x = xscaled*sigma(x) + mean(x)
   yscaled = y-mean(y)/sigma(y) => y = yscaled*sigma(y) + mean(y)

Intercept= Intercept(scaled) - ScaledCoefficients (Mean(x)/sigma(y) - ScaledCoefficients(Mean(y)/Sigma(y))

```
#Intercept
Interceptscaled <- LinearRegressionPCmodel$coefficients[1]
Interceptscaled

## (Intercept)
##    905.0851

#Scaled Coefficents
scaledcoefs <-LinearRegressionPCmodel$coefficients[2:(k+1)]

# Rotate the scaled data back.
#OriginalRotatationScaledData = RotationMatrix * Scaleddata
OriginalRotatationScaledData <- principalcomponents$rotation[,1:k]%*%scaledcoefs

summary(OriginalRotatationScaledData)
##        V1
##  Min.   :-34.64
##  1st Qu.: 24.65
##  Median : 37.85
##  Mean   : 51.64
```

```
##  3rd Qu.: 87.16
##  Max.   :117.34
```

mu <- sapply(crimedata[,1:15],mean)
sigma <-sapply(crimedata[,1:15],sd)

originalCoeff = OriginalRotatationScaledData/sigma
OriginalIntercept = Interceptscaled - sum(OriginalRotatationScaledData*mu/sigma)

```
original = as.matrix(crimedata[,1:15]) %*% originalCoeff+OriginalIntercept

# Find the accuracy
sse = sum((original - crimedata[,16])^2)
totalSumofSquares = sum((crimedata[,16]-mean(crimedata[,16]))^2)
RSquared = 1- (sse/totalSumofSquares)
AdjustedRSqaured = RSquared - (1-RSquared)*k/(nrow(crimedata)-k-1)


AdjustedRSqaured
## [1] 0.601925
RSquared
## [1] 0.6451941
```

Analysis: Observe that Adjusted R Squared, RSquared values from Principle Components Linear regression model from Step 5 and the Original Coordinate Model from Step-6 are same. This Confirms that the method used to get back the original data and dimension is correct.

Step 7: Let us use the test data from the Homework 5 to analyse the prediction results. Last week homework, the prediction was 1304.

```
testpt <- data.frame(M = 14.0,So = 0,Ed = 10.0,Po1 = 12.0,
                     Po2 = 15.5,LF = 0.640, M.F = 94.0, Pop = 150,
                     NW = 1.1,U1 = 0.120,U2 = 3.6,Wealth = 3200,
                     Ineq = 20.1,Prob = 0.04,Time = 39.0)
#Predict the crime rate for the data point
# Replace PCA data into the test point
PCATestPoint <- data.frame(predict(principalcomponents,testpt))


predict_model <-predict(LinearRegressionPCmodel,PCATestPoint)
predict_model

##        1
## 1388.926
```

Conclusion:

For homework 5 - 8.2 , I used the threshold of p-value = 0.1 and R-Squared values from the linear regression result to pick the best 5 predictors and got the test data prediction to be 1304. I used an iterative method of removing few factors each time and determined the following result.

Model	lm() Removed Factors	P-Value	RSE	F	Multiple R-Squared	Y-Hat
0	None – All 15 are present	3.539e-07	209.1	8.829(15,31)DF	0.8031	155.4349
1	Crime~.-Po2	1.67e-07	208.6313	9.01(14,32)DF	0.797576	724.8202
2	Crime~.-Po2-Wealth-U1-So-NW	6.08e-09	207	12.4(10,36)DF	0.775	1254
3	Crime~.-Po2-Wealth-U1-So-NW LF-M.F-Pop-Time	3.42e-11	201	21.8(6,40)DF	0.766	1304
4	Crime~.-Po2-Wealth-U1-So-NW LF-M.F-Pop-Time -Ineq	6.24e-08	250	13.8(5,41)DF	0.628	1250

The best model is model-3 with predicted value of 1304.

Using the Principle Component Analysis Method is a better and easy way than the above method, to reduce the number of factors and remove correlation among the data.

In this Assignment, the prediction is now 1399.926 which is close to the earlier predicted model. This shows that using PCA is a better option to reduce the number of factors to be used in the final model.