GATE ASSIGNMENT 1

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Download all python codes from

https://github.com/harithar1234/EE3900−Haritha/blob/main/gateassignment1/gateassignment1.py

QUESTION

EC-2019 Q.29

It is desired to find a three-tap causal filter which gives zero signal as an output to an input of the form

$$x[n] = c_1 \exp\left(\frac{-j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right)$$

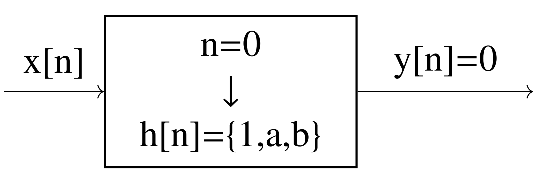
where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by

$$h[0] = 1, h[1] = a, h[2] = b$$

and

$$h[n] = 0 \text{ for } n < 0 \text{ or } n > 2.$$

What are the values of the filter taps a and b if the output is y[n] = 0 for all n, when x[n] is as given above?



(A)a=1,b=1
(B)a=0,b=-1
(C)a=-1,b=1
(D)a=0,b=1

SOLUTION

given: y[n] = 0 for all n.

$$x[n] = c_1 e^{\frac{-j\pi n}{2}} + c_2 e^{\frac{j\pi n}{2}} \tag{0.0.1}$$

h[0] = 1, h[1] = a, h[2] = b and
h[n] = 0 for n < 0 or n > 2.
answer: A discrete-time LTI system with impulse response h[n] has a transfer function $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$
An N-tap FIR filter with coefficients h[k] in general has Z-transform given by $\sum_{n=0}^{N-1} h(n) z^{-n}$

for the 3-tap FIR filter, the Z-transform is:

$$H(z) = \sum_{n=0}^{2} h(n) z^{-n} \tag{0.0.2}$$

The 3-tap filter's frequency response $H(e^{jw})$ obtained by substituting $z = e^{jw}$ is:

$$H(e^{jw}) = \sum_{n=0}^{2} h(n) e^{-jnw} \tag{0.0.3}$$

$$H(e^{jw}) = h(0)e^{-0jw} + h(1)e^{-1jw} + h(2)e^{-2jw} \tag{0.0.4}$$

$$H(e^{jw}) = 1 + ae^{-jw} + be^{-2jw} \tag{0.0.5}$$

output of discrete-time LTI system y[n] for a given input sequence x[n] is given by the convolution sum : $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

output of discrete-time LTI system with exponential input is $y[n] =$

$\left|H\left(e^{j\omega}\right)\right| x(n)\, e^{(j\omega+\Phi)}$

at $w = \frac{-\pi}{2}$

$$H\left(e^{-j\frac{\pi}{2}}\right) = 1 + ae^{-j\left(\frac{-\pi}{2}\right)} + be^{-j2(-\pi/2)} \tag{0.0.6}$$

$$H\left(e^{j\frac{\pi}{2}}\right) = (1-b) + ja \tag{0.0.7}$$

at $w = \frac{\pi}{2}$

$$H\left(e^{j\frac{\pi}{2}}\right) = 1 + ae^{-j\left(\frac{\pi}{2}\right)} + be^{-j2(\pi/2)} \tag{0.0.8}$$

$$H\left(e^{j\frac{\pi}{2}}\right) = (1-b) - ja \tag{0.0.9}$$

$$\left|H\left(e^{j\frac{\pi}{2}}\right)\right| = \left|H\left(e^{-j\frac{\pi}{2}}\right)\right| = \sqrt{(1-b)^2 + a^2} \tag{0.0.10}$$

Expression of output y[n]:

$$y(n) = \left[(1-b)^2 + a^2\right]^{1/2}\left[c_1 e^{-j\left(\frac{n\pi}{2}+\Phi_1\right)} + c_2 e^{j\left(\frac{\pi}{2}n+\Phi_2\right)}\right] \tag{0.0.11}$$

so y[n]=0 always, if $\left|H\left(e^{j\omega}\right)\right| = 0$

$$\sqrt{(1-b)^2 + a^2} = 0 \tag{0.0.12}$$

from the options
b = 1
a = 0
option D