ASSIGNMENT 3

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Download all python codes from

https://github.com/harithar1234/EE3900−Haritha/blob/main/assignment3/assignment3.py

QUESTION

Construction/Q 2.18

Draw a circle with centre B and radius 6. If C be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.

SOLUTION

Lemma 0.1. *For Circle with radius r with centre B and two tangents(AC,DC) drawn from point C which is d units away from centre ,the co-ordinates are represented by :*

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.1}$$

$$\mathbf{C} = \begin{pmatrix} d \\ 0 \end{pmatrix} \tag{0.0.2}$$

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \tag{0.0.4}$$

$$\text{where } a_1 = \frac{r^2}{d} \tag{0.0.5}$$

$$a_2 = r\sqrt{(1 - \frac{r^2}{d^2})} \tag{0.0.6}$$

$$d_1 = \frac{r^2}{d} \tag{0.0.7}$$

$$d_2 = -r\sqrt{(1 - \frac{r^2}{d^2})} \tag{0.0.8}$$

$$\tag{0.0.9}$$

Theorem 0.1. *The points of intersection of the line*

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{0.0.10}$$

with a general conic are given by

$$\mathbf{x}_i = \mathbf{q} + \mu_i\mathbf{m} \tag{0.0.11}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\Big(-\mathbf{m}^T\left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{V}\mathbf{q} + \mathbf{u}\right)\right]^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right)}\Big) \tag{0.0.12}$$

The general form of a conic is given by

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.0.13}$$

for a circle, $\mathbf{V} = \mathbf{I}$ and $\mathbf{u} = -\mathbf{B}$ where $\mathbf{B}$ is the center of the circle.
Let $\mathbf{q}$ be the locus of the point of tangency from point $\mathbf{C}$, the distance of $\mathbf{C}$ from $\mathbf{B}$ is d

$$(\mathbf{q} - \mathbf{B})^T(\mathbf{q} - \mathbf{C}) = 0 \tag{0.0.14}$$

$$(\mathbf{q} + \mathbf{u})^T(\mathbf{q} - \mathbf{C}) = 0 \tag{0.0.15}$$

$$\mathbf{q}^T\mathbf{q} + \mathbf{u}^T\mathbf{q} - \mathbf{u}^T\mathbf{C} - \mathbf{q}^T\mathbf{C} = 0 \tag{0.0.16}$$

Using (0.0.13)

$$(\mathbf{u} + \mathbf{C})^T\mathbf{q} = -f - \mathbf{u}^T\mathbf{C} \tag{0.0.17}$$

Let $\mathbf{n} = \mathbf{u} + \mathbf{C}$ and $c = -f - \mathbf{u}^T\mathbf{C}$ This is equation of a line, let $\mathbf{q} = \mathbf{a}$ be a point that lies on this line

$$\therefore \mathbf{q} = \mathbf{a} + \lambda\begin{pmatrix} -\mathbf{e_1}^T\mathbf{n} \\ \mathbf{e_2}^T\mathbf{n} \end{pmatrix} \tag{0.0.18}$$

We need to find the intersection point of this with the given circle.
Using (0.0.12)

$$\mathbf{q} = \mathbf{a} + \mu_i\mathbf{m} \tag{0.0.19}$$

$$\mu_i = \frac{1}{d^2}\Big(-\mathbf{m}^T\left(\mathbf{a} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{a} + \mathbf{u}\right)\right]^2 - \left(\mathbf{a}^T\mathbf{a} + 2\mathbf{u}^T\mathbf{a} + f\right)d^2}\Big) \tag{0.0.20}$$

Now, we can confirm the solution by checking for

$\mathbf{u} = \mathbf{0}$, $\mathbf{C} = d\mathbf{e}_1$, $f = -r^2$

$$\implies \mathbf{n} = d\mathbf{e}_1, \ \mathbf{m} = d\mathbf{e}_2 \tag{0.0.21}$$

An arbitrary choice of $\mathbf{a}$ could be $\frac{r^2}{d}\mathbf{e}_1$,

$$\mu_i = \pm\frac{1}{d^2}\sqrt{(r^2d^2 - r^4)} \tag{0.0.22}$$

$$\mathbf{q} = \frac{r^2}{d}\mathbf{e}_1 \pm r\sqrt{1 - \frac{r^2}{d^2}}\mathbf{e}_2 \tag{0.0.23}$$

$\mathbf{A}, \mathbf{D}$ are the corresponding points of tangency from $\mathbf{C} = 10\mathbf{e_1}$

Using (0.0.23), we obtain all the points of tangency.
d=10 and r=6.
Therefore, $\mathbf{A} = \begin{pmatrix} 3.6 \\ 4.8 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3.6 \\ -4.8 \end{pmatrix}$

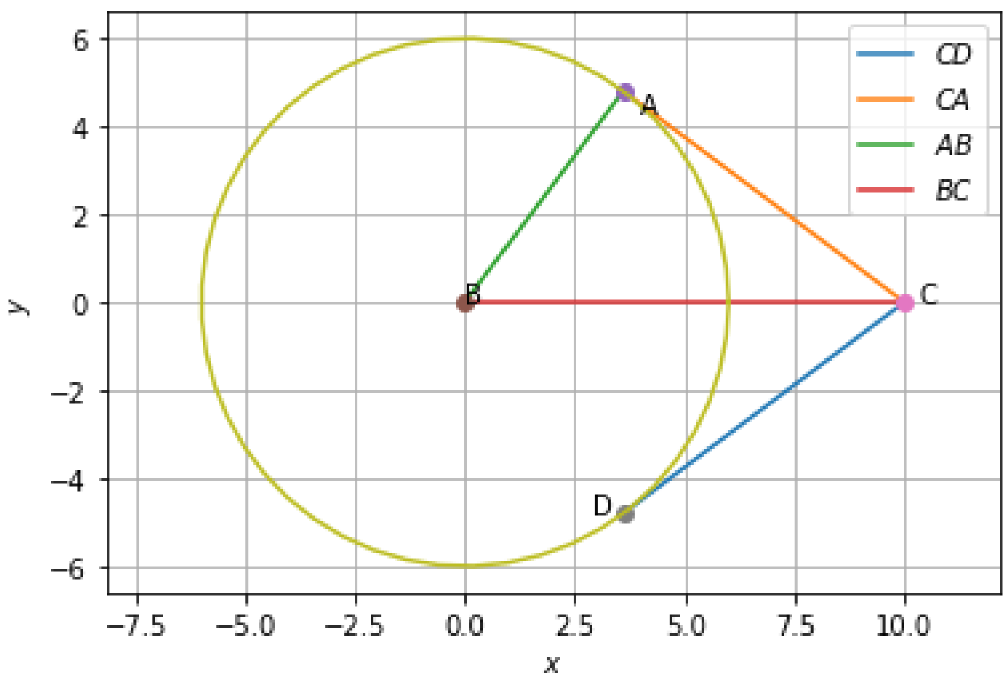

Fig. 1: Plot of the tangents