Quiz 2

Haritha R
AI20BTECH11010

Download python codes from

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https://github.com/harithar1234/EE3900−Haritha/
    tree/main/quiz2/code
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QUESTION

The z-transform/3.3(b)

Determine the z-transform of each of the following sequences. Include with your answer the region of convergence in the z-plane and a sketch of the pole-zero plot. Express all sums in closed form.

$$(b)x_b[n] = \begin{cases} 1, & 0 \le \text{n} \le \text{N-1} \\ 0, & \text{otherwise} \end{cases}$$

SOLUTION

The z-transform of a sequence x[n] is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{0.0.1}$$

The z-transform of the sequence $x_b[n]$ is:

$$X_b(z) = \sum_{n=-\infty}^{\infty} x_b[n]z^{-n} \tag{0.0.2}$$

since the sequence $x_b[n]$ is non-zero only for $0 \le$ n $\le$ N-1, The z-transform of the sequence $x_b[n]$ is:

$$X_b(z) = \sum_{n=0}^{N-1} x_b[n]z^{-n} \tag{0.0.3}$$

$$X_b(z) = \sum_{n=0}^{N-1} z^{-n} \tag{0.0.4}$$

$$X_b(z) = \frac{1-(z^{-N})}{1-(z^{-1})} = \frac{1}{z^{N-1}}\frac{z^N-1}{z-1} \tag{0.0.5}$$

The region of convergence, known as the ROC for a given $x_b[n]$, is defined as the range of z for which the z-transform converges.The condition to be satisfied for convergence is:

$$\sum_{n=-\infty}^{\infty} |x_b[n]z^{-n}| < \infty \tag{0.0.6}$$

$$\sum_{n=0}^{N-1} |x_b[n]z^{-n}| < \infty \tag{0.0.7}$$

$$\sum_{n=0}^{N-1} |z^{-n}| < \infty \tag{0.0.8}$$

the sum $\sum_{n=0}^{N-1} |z^{-n}|$ will finite as z^{-1} is finite, which requires $z \neq 0$. The ROC of $X_b(z)$ includes the entire z-plane, with the exception of origin.

$$\text{ROC of } X_b(z) : |z| > 0$$

$$X_b(z) = \frac{1-(z^{-N})}{1-(z^{-1})} \tag{0.0.9}$$

The zeros of $X_b(z)$ will be the solutions of the equation $z^N = 1$.

The zeros of $X_b(z)$ will vary depending on N. ($N \ge 2$).however z=1 will always be zero irrespective of N. The pole of $X_b(z)$ is z=1 always.

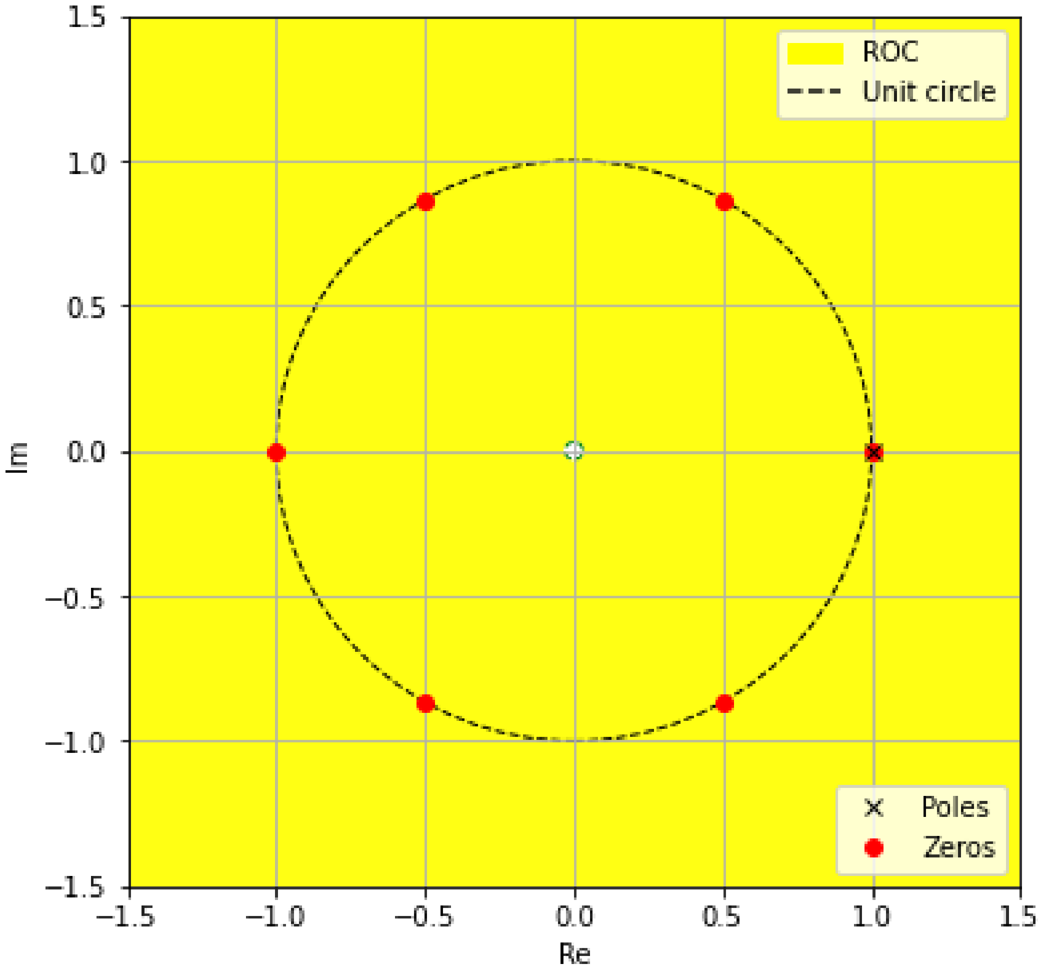

Fig. 0: Plot of zeros for N=6,plot of poles and region of convergence