ASSIGNMENT 3

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Download all python codes from

https://github.com/harithar1234/EE3900−Haritha/blob/main/assignment3/assignment3.py

QUESTION

Construction/Q 2.18

Draw a circle with centre B and radius 6. If C be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.

SOLUTION

Lemma 0.1. *For Circle with radius r with centre B and two tangents(AC,DC) drawn from point C which is d units away from centre ,the co-ordinates are represented by :*

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.1}$$

$$\mathbf{C} = \begin{pmatrix} d \\ 0 \end{pmatrix} \tag{0.0.2}$$

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{D} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \tag{0.0.4}$$

$$\text{where } a_1 = \frac{r^2}{d} \tag{0.0.5}$$

$$a_2 = r\sqrt{(1 - \frac{r^2}{d^2})} \tag{0.0.6}$$

$$d_1 = \frac{r^2}{d} \tag{0.0.7}$$

$$d_2 = -r\sqrt{(1 - \frac{r^2}{d^2})} \tag{0.0.8}$$

$$\tag{0.0.9}$$

Proof. Let the center of the circle be

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.10}$$

and radius = 6 units.
Therefore the equation of the circle is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 6\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{0.0.11}$$

The tangent are drawn from a point $\mathbf{C}\begin{pmatrix} 10 \\ 0 \end{pmatrix}$ on the x-axis which intersect the circle at point $\mathbf{A}$ and $\mathbf{D}$.
Since, point $\mathbf{A}$ lie on the circle ,

$$\|\mathbf{AB}\| = 6 \tag{0.0.12}$$

$$\implies \|\mathbf{A} - \mathbf{B}\| = 6 \tag{0.0.13}$$

$$\implies \|\mathbf{A}\| = 6 \tag{0.0.14}$$

In any circle tangent is perpendicular to the radius. So in $\triangle ABC$, $\mathbf{BA}$ and $\mathbf{CA}$ are perpendicular

$$(\mathbf{BA})^\top(\mathbf{AC}) = 0 \tag{0.0.15}$$

$$\implies (\mathbf{B} - \mathbf{A})^\top(\mathbf{A} - \mathbf{C}) = 0 \tag{0.0.16}$$

$$\implies \mathbf{B}^\top\mathbf{A} - \mathbf{B}^\top\mathbf{C} - \|\mathbf{A}\|^2 + \mathbf{A}^\top\mathbf{C} = 0 \tag{0.0.17}$$

since $\mathbf{B}^\top\mathbf{A} = 0$ and $\mathbf{B}^\top\mathbf{C} = 0$

$$-\|\mathbf{A}\|^2 + \mathbf{C}^\top\mathbf{A} = 0 \tag{0.0.18}$$

$$\implies \mathbf{C}^\top\mathbf{A} = \|\mathbf{A}\|^2 \tag{0.0.19}$$

$$\implies \mathbf{C}^\top\mathbf{A} = 36 \tag{0.0.20}$$

Now, (0.0.20) can be rewritten as

$$\begin{pmatrix} 10 & 0 \end{pmatrix}\mathbf{A} = 36 \tag{0.0.21}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{A} = 3.6 \tag{0.0.22}$$

$$\implies \mathbf{A} = \begin{pmatrix} 3.6 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.23}$$

$$\implies \mathbf{A} = \mathbf{q} + \lambda\mathbf{m} \tag{0.0.24}$$

where, $\mathbf{q} = \begin{pmatrix} 3.6 \\ 0 \end{pmatrix}$ and $\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Now, we know that

$$\|\mathbf{A}\|^2 = 36 \tag{0.0.25}$$

$$\implies \|\mathbf{q} + \lambda\mathbf{m}\|^2 = 36 \tag{0.0.26}$$

$$\implies (\mathbf{q} + \lambda\mathbf{m})^\top(\mathbf{q} + \lambda\mathbf{m}) = 36 \tag{0.0.27}$$

$$\implies \|\mathbf{q}\|^2 + 2\mathbf{q}^\top\lambda\mathbf{m} + \lambda^2\|\mathbf{m}\|^2 = 36 \tag{0.0.28}$$

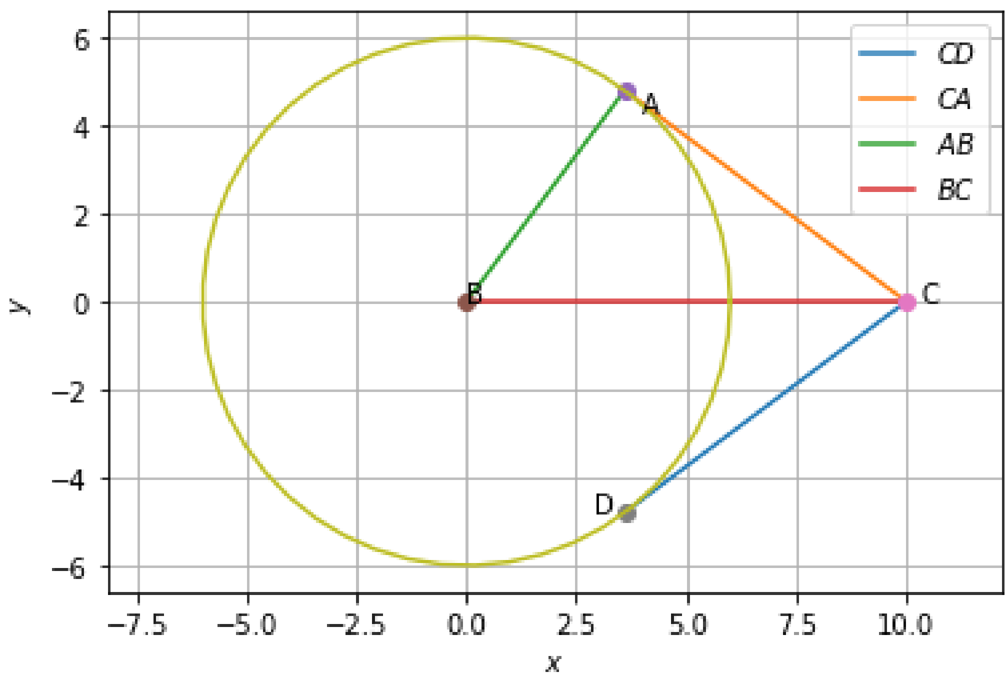
Since, $\mathbf{q}^\top\mathbf{m} = 0 \implies 2\mathbf{q}^\top\lambda\mathbf{m} = 0$

$$\implies \lambda^2 = \frac{36 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{0.0.29}$$

$$\implies \lambda^2 = \frac{36 - (3.6)^2}{1} \tag{0.0.30}$$

$$\implies \lambda = \pm 4.8 \tag{0.0.31}$$

Therefore, $\mathbf{A} = \begin{pmatrix} 3.6 \\ 4.8 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3.6 \\ -4.8 \end{pmatrix}$



$\square$