Question 1

(a) $B \cup C = \{m, n, p, q, s, t\}$

$A \cup (B \cup C) = \{p, q, s, t, m, n\}$

$(A \cup B) = \{m, n, p, q, s, t\}$

$(A \cup B) \cup C = \{m, n, p, q, s, t\} = A \cup (B \cup C)$ (shown)

(b) ~~A~~ Question not complete.

(c) $2^2 = 4$, $4 < 16$, $4 < 23$

$3^2 = 9$, $9 < 16$, $9 < 23$

$4^2 = 16$, $16 < 23$

$5^2 = 25$

$R = \{(2,16), (2,23), (3,16), (3,23), (4,23)\}$

(d) Let A = number of student in art class = 35

B = number of student in science class = 57

Find $A \cup B$

(i) $A \cap B = 12$, $A \cup B = 35 + 57 - 12 = 80$ students

(ii) $A \cup B = A + B$

$= 35 + 57 = 92$ students

Question 2

(a)(i) p: You try hard

q: You have a talent

r: You get rich

$(p \vee q) \rightarrow r$

(ii)

(iii)

(b)

p	q	r	$r \rightarrow p$	$q \wedge (r \rightarrow p)$	$p \vee (q \wedge (r \rightarrow p))$	$\neg(p \vee (q \wedge (r \rightarrow p)))$
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	F	F	F	T
F	F	F	T	F	T	F

p	q	r	$q \to r$	$\neg p$	$\neg p \wedge (q \to r)$
T	T	T	T	F	F
T	T	F	F	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	T	T

A & B not equivalent.

(i)

(c) $x = 1 - 2y$

not all x is true when $P(x)$.

$$y = \frac{1 - x}{2}$$

When x is odd, y is integer, and when x is even, y will become fraction.

Thus, x must be odd. When x is even, $P(x,y)$ will become False

Thus, $\neg \forall x P(x)$ is true since not all x will lead to true in $P(x,y)$.

(c) (iii) $\exists x P(x)$

Existent of x that $P(x,y)$ is true.

Let $x=1$, $1+2y=1$

$y=0$ (integer)

Since when $x=1, y=0$, $P(x,y)$ is true, thus $\exists x P(x)$ is true since there is at least one true in $P(x,y)$.

(d) Question missing symbol.

Question 3

(a) (i) $R = \{(s,s), (s,t), (t,t), (t,u), (t,v), (u,u), (u,s), (v,v)\}$

(ii)

$$M_R = \begin{array}{c} \\ s \\ t \\ u \\ v \end{array} \begin{array}{c} \begin{array}{cccc} s & t & u & v \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(iii) Not antisymmetric since consist (t,v) and (t,u). (More than one arrow from node t)

Not asymmetric since exist of (s,s), (u,u), (t,t) and (v,v). (reflexive)

Not partial order since no antisymmetric

↳ no transitive also since exist (u,s), (s,t) but there is no (u,t)

↳ but reflexive. since exise (s,s), (u,u), (t,t) and (v,u).

(b)(i) True. This is because all a map to b.

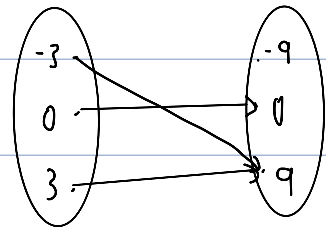
(ii) False. This is because a can only have one b, but it exist (1, 4) and (1, 8).

(iii) False. This is because no b for a = 2.

(c) For $v(x)$, $-3 = 9$

$0 = 0$

$3 = 9$

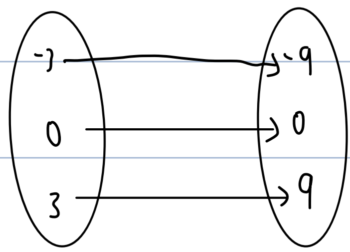


Not one-to-one, not onto Y, no bijectio

For $w(x)$, $-3 = -9$

$0 = 0$

$3 = 9$



$w(x)$ is one-to-one, onto Y and bijection.

(d) (i) Let $y = 2x+3$

$x = \frac{y-3}{2}$

$f^{-1}(x) = \frac{x-3}{2}$

Let $y = x+5$

$x = y-5$

$g^{-1}(x) = x-5$

(ii) $f \circ g \circ f = f(2x+3+5)$

$= f(2x+8)$

$= 2(2x+8)+3$

$= 4x+16+3$

$= 4x+19$

Question 4

(a) $F_0 = 3.0$, $F_1 - 2.5$

$$F_t = F_{t-1} + \frac{1}{4} F_{t-2} \quad , \text{for } t \geq 2.$$

(b) $F_0 = 3.0$, $F_1 = 2.5$

$$F_2 = F_1 + \frac{1}{4} F_0 \qquad F_3 = F_2 + \frac{1}{4} F_1$$

$$= 2.5 + \frac{3}{4} \qquad = 3.25 + \frac{1}{4}(2.5)$$

$$= 3.25 \qquad = 3.875$$

(c)
```
warm( (t){
    if t=0
        return 3.0
  else if t=1
        return 2.5
  else   return  warm( (t-1)+0.25 * warm( (t-2);
}
```

Question 5

(a)(i) $(5-1)! = 4!$

$= 24$ ways

(ii) Treat Captain, two vice-captain as one block, thus remain 3 block

Arrangement for 3 block $= (3-1)!$

$= 2$

Arrangement for 3 person within single block $= 3!$

$= 6$

Total arrangement $= 6 \times 2 = 12$ ways

(b) Total arrangement $= 5!$

$= 120$ ways

Head and asistant as one block $= 2! = 2$ (next to each other)

Four block exist $= 4! = 24$

Total number of arrangement for head & asistant that next to each other $= 2 \times 24$

$= 48$

Valid arrangement $= 120 - 48$ (48 is invalid arrangement)

$= 72$ ways.

Question 5

(c) Buy 6 out of 10

(i) No restriction

$C(n+r-1, r) = C(10+6-1, 6)$

$= C(15, 6)$

$= \frac{15!}{6!\,9!}$

$= 5005$ ways

(ii) At least 4 hazelnut flavoured

$6-4 = 2$ chocolate type from 9 type

$C(9+2-1, 2) = C(10, 2)$

$= \frac{10!}{8!\,2!}$

$= 45$ ways.

(iii) $C(10, 6) = \frac{10!}{6!\,4!}$

$= 210$ ways.

Question 5

(d) 13 player

(i) ${}^{13}C_{11} = 78$ ways

(ii) ${}^{13}P_{11} = 3\,113\,510\,400$ ways

(iii) ${}^{3}C_{1} \times {}^{10}C_{10} = 3$

${}^{3}C_{2} \times {}^{10}C_{9} = 3 \times 10 = 30$

${}^{3}C_{3} \times {}^{10}C_{8} = 45$

Total ways = 78 ways.

Question 6

(a) Since there is three colour ball, at least need to take 4 balls.

(b) $10 \times 8 = 80$ pieces cheesecakes.

number of people $= 30 + 2 = 32$ people

$32 \times 3 = 96$ pieces

Since the number of pieces of cheesecake is small than number of cheesecake required by people for at least 3 pieces each person, not all participants can get three pieces.

(c) Combination sum of 10 = $\{(2,8), (3,7), (4,6)\}$

To have at least one pair sum of 10, it is numbers 6, which is the 4th integer in the set.