QUESTION 1 **[15 Marks]**

You are conducting a study on the choice of university among Sijil Pelajaran Malaysia (SPM) leavers in Malaysia. Your data includes a sample of 500 students who have recently enrolled in universities across the country. Specifically, you are interested in whether these students chose to study in private universities or local public universities. The data you collected is as follows:

- 175 students chose private universities.
- 325 students chose local public universities

a) Calculate the probability that a randomly selected student from your sample chose to study in a private university. (1 Mark)

(a) Let $P(A)$ = probability students choose private uni.
$P(B)$ = " local public uni.

$$P(A) = \frac{175}{500}$$
$$= 0.35$$

b) Determine the probability that a randomly selected student from your sample chose to study in a local public university. (1 Mark)

c) Are these two events mutually exclusive? Explain your reasoning. (3 Marks)

(b) $P(B) = \frac{325}{500}$
$= 0.65$

(c) Yes. Student can only choose one uni to study, they won't study in both uni. $P(A) + P(B) = 1$ also prove that there is $P(A \cap B) = 0$, that indicate mutually exclusive.

Now, let's consider additional factors that may influence the choice of university. You have gathered the following information:

- Among the students who chose private universities, 60% majored in business- related fields, while the rest majored in other disciplines.
- Among the students who chose local public universities, 40% majored in business-related fields, while the rest majored in other disciplines.
- 70% of the students who majored in business-related fields chose private universities.

d) Calculate the probability that a student chosen at random for the following :

i. Majored in a business-related field and chose a private university (3 Marks)

ii. Majored in a business-related field (3 Marks)

iii. Majored in a business-related field but chosen a local public university (4 Marks)

(d) (i) Let $P(C)$: probability student major in business.

$P(D)$: probability student major in other disciplines.

$$P(A \cap C) = \frac{\text{no. of student major in business \& choose private uni}}{\text{total student}}$$

$$= \frac{0.6 \times 175}{500}$$

$$= 0.21$$

(ii) $P(C) = P(A \cap C) + P(B \cap C)$

$$= 0.21 + \frac{0.4 \times 325}{500}$$

$$= 0.47$$

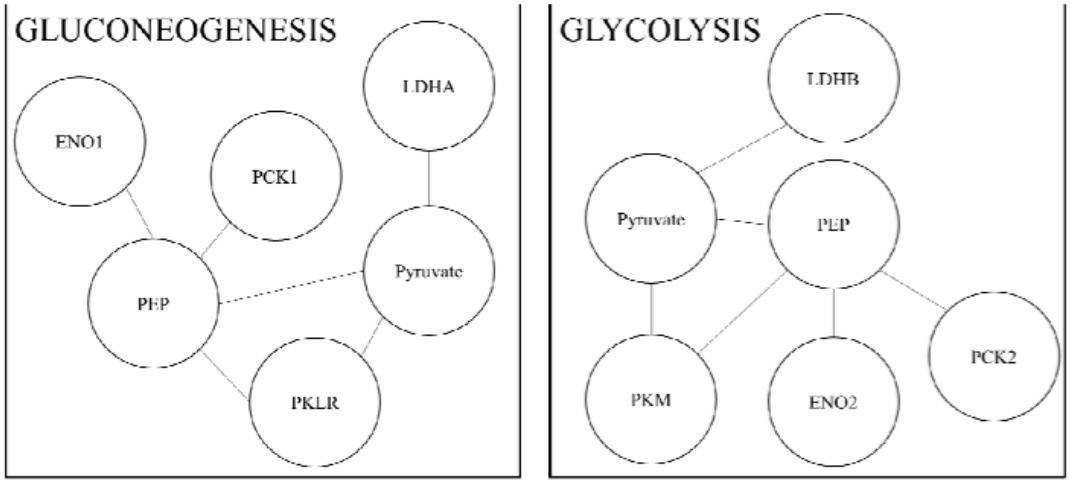
(iii) $P(B \cap C) = \frac{0.4 \times 325}{500}$

$$= 0.26$$

QUESTION 2 **10 MARKS**

For the following pair of graphs GLUCONEOGENESIS and GLYCOLYSIS, show that both graphs are isomorphic by

- Define the one-to-one and onto functions from the vertices of graph GLUCONEOGENESIS and the vertices of graph GLYCOLYSIS.
- Construct the adjacency matrix.



→ Both have 6 vertices & 6 edges

→ Both are simple & connected graph

→ Both have 1 vertices with 4 degrees, 1 vertices with 3 degrees, 1 vertices with 2 degrees and 3 vertices with 1 degrees.

→ $f(PEP) = f(PEP)$ $f(PKLR) = f(PKM)$

$f(ENO1) = f(ENO2)$ $f(Pyruvate) = f(Pyruvate)$

$f(PCK1) = f(PCK2)$ $f(LDHA) = f(LDHB)$

Since all vertices in GLUCONEOGENESIS match with GLYCOLYSIS, they are one-to-one & onto.

→ Let GLUCONEOGENESIS = GN, GLYCOLYSIS = G

A_{GN} =

	PEP	ENO1	PK1	PKLR	Pyruvate	LDHA
PEP	0	1	1	1	1	0
ENO1	1	0	0	0	0	0
PK1	1	0	0	0	0	0
PKLR	1	0	0	0	1	0
Pyruvate	1	0	0	1	0	1
LDHA	0	0	0	0	1	0

A_G =

	PEP	ENO2	Pk2	Pkm	Pyruvate	LDHB
PEP	0	1	1	1	1	0
ENO2	1	0	0	0	0	0
PK2	1	0	0	0	0	0
PKM	1	0	0	0	1	0
Pyruvate	1	0	0	1	0	1
LDHB	0	0	0	0	1	0

∴ Both graph are isomorphic.

QUESTION 3 **[31 Marks]**

Suppose Amina needs to spread a message among friends. Because of a restriction in the flow of communications, her 6 friends can only talk to specified friend as in Table 1:

Table 1

Friends	Can Talk to
Heidi	Nadia
Nadia	Heidi, Adam
Ali	Faiz, Bara,Adam
Faiz	Bara, Ali
Amina	Heidi, Faiz
Bara	Faiz, Ali
Adam	Heidi, Nadia,Ali

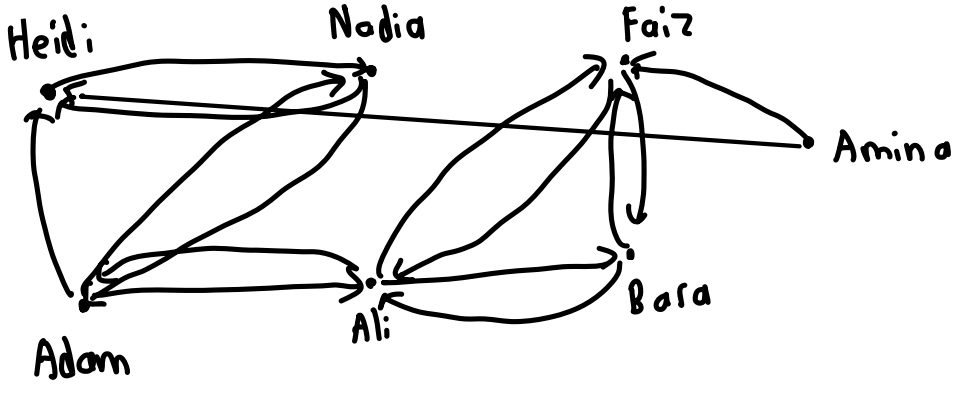
a) Draw a directed graph to represent the flow of communications for Amina and her friends. (4 Marks)

b) If Amina initiates the message and a continuous path is created, it there a path that will possible the message delivered to everybody exactly once? If so, execute the path flow of the communication. (4 Marks)

c) What theorem in graph theory implemented to solved problem in (b) (2 Marks)

d) Towns on a mountainous island are linked by railway lines as shown Figure 1. The distances (in kilometres) between stations are shown on the map

(a)



(b) Yes.
Amina → Heidi → Nadia → Adam → Ali → Faiz → Bara.

(c) Hamiltonian Path. Each vertex will included exactly once.

d) Towns on a mountainous island are linked by railway lines as shown Figure 1. The distances (in kilometres) between stations are shown on the map

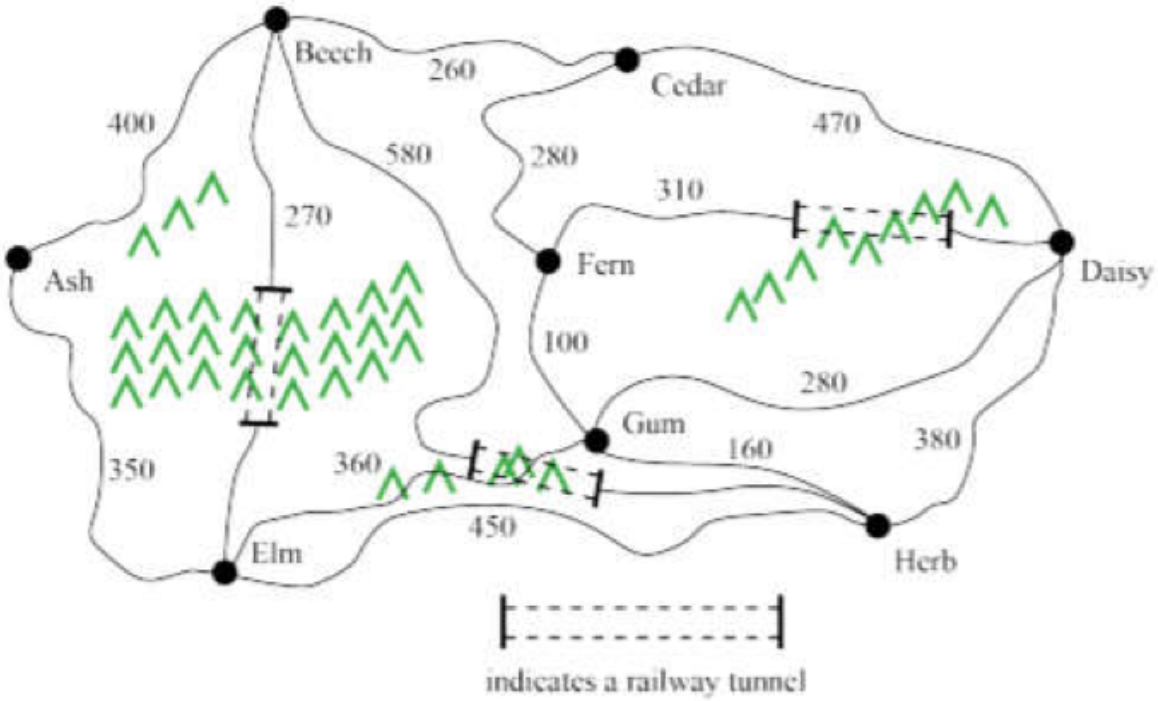


Figure 1

i) Construct a graph for the network of the railway system. Label the stations with the first letter and show the distances on the lines. (3 Marks)

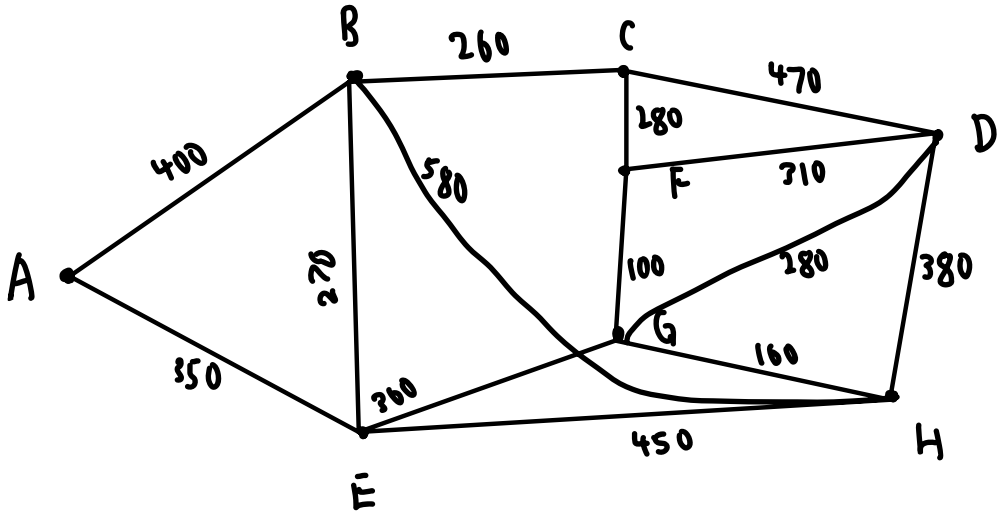
ii) Determine the degrees of the vertices in the graph. (3 Marks)

iii) It is possible to plan a trip that travels all sections of the railway line without travelling on any section of the line more than once. (You may have to visit towns more than once.). If so, determine the specific station to start and end the trip? Explain. (3 Marks)

iv) The railway company decides to close one line to reduce the maintenance cost but the tourist still possible to travel all sections of the railway line once from any station and return back to the same station. Identify which line that can be closed? Explain

(2 Marks)

(d)(i)



(ii)

Vertex	A	B	C	D	E	F	G	H
Degree	2	4	3	4	4	3	4	4

(iii) To meet the requirement, Euler trail theorem used, where pass through every vertex (town) at least once, every edge (line) exactly one.

Since there are only 2 vertex consist odd degree, Euler trail exist, it is possible.

C → B → A → E → B → H → E → G → H → D → G → F → C → D → F

(iv) The line from town Cedar to Fern. This is because after close the line, all degree of vertex will become even, and makes it as Euler circuit.

In Euler circuit, tourist will return to same section after travel all section by only travel railway line once.

v) Determine the shortest route and the minimum total length of track to travel from Ash to Daisy using Dijkstra algorithm by completing Table 2. (10 Marks)

S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
{}	{A,B,C,D,E,F,G,H}	0	∞	∞	∞	∞	∞	∞	∞
{A}	{B,C,D,E,F,G,H}		400	∞	∞	350	∞	∞	∞
{A,E}	{B,C,D,F,G,H}		400	∞	∞		∞	710	800
{A,E,B}	{C,D,F,G,H}			660	∞		∞	710	800
{A,E,B,C}	{D,F,G,H}				1130		940	710	800
{A,E,B,C,G}	{D,F,H}				990		810		800
{A,E,B,C,G,H}	{D,F}				990		810		
{A,E,B,C,G,H,F}	{D}				990				
{A,E,B,C,G,H,F,D}									

A → E → G → D