3.8 PROPERTIES OF CONTEXT FREE LANGUAGES (NOV/DEC'10'11)

The context free languages are closed under some operation which means after performing the particular operation on those CFLs the resultant language is context free language and the properties are given below

1. The context free languages are closed under union.
2. The context free languages are closed under concatenction.
3. The context free languages are closed under kleen closure.
4. The context free languages are not closed under intersection.
5. The context free languages are not closed under complement.

Theorem 1

If L_1 and L_2 are context free languages then $L = L_1 \cup L_2$ is also context free. That is, the CFLs are closed under union.

Proof:

Consider the two languages L_1 and L_2 which are context free languages. We can give these languages using context free grammars G_1 and G_2 such that $G_1 \in L_1$ and $G_2 \in L_2$. Then the grammar G_1 can be given as $G_1 = \{V_1, \Sigma, P_1, S_1\}$ where P_1 can be given as

$P_1 = \{$

$$S_1 \rightarrow A_1S_1A_1 \mid B_1S_1B_1 \mid \varepsilon$$
$$A_1 \rightarrow a$$
$$B_1 \rightarrow b$$

$\}$

Here $V_1 = \{S_1, A_1, B_1\}$ and S_1 is a start symbol.

Similarly, we can write for $G_2 = \{V_2, \Sigma, P_2, S_2\}$

$P_2 = \{$

$$S_2 \rightarrow a\,A_2A_2 \mid b\,B_2B_2$$
$$A_2 \rightarrow b$$
$$B_2 \rightarrow a$$

$\}$

Where $V_2 = \{S_2, A_2, B_2\}$ and S_2 is a start symbol.

Now $L = L_1 \cup L_2$ gives $G \in L$.

Then G is given as,

$$G = \{V, \Sigma, P, S\}$$
$$V = \{S_1, A_1, B_1, S_2, A_2, B_2\}$$
$$P = \{P_1 \cup P_2\}$$

S is a start symbol

$P = \{$